Nucleon structure and multi-hadron dynamics from lattice QCD

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R. Edwards (JLab), A. Jackura (W&M),
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- We thank DOE for computer time allocations at NERSC and OLCF.
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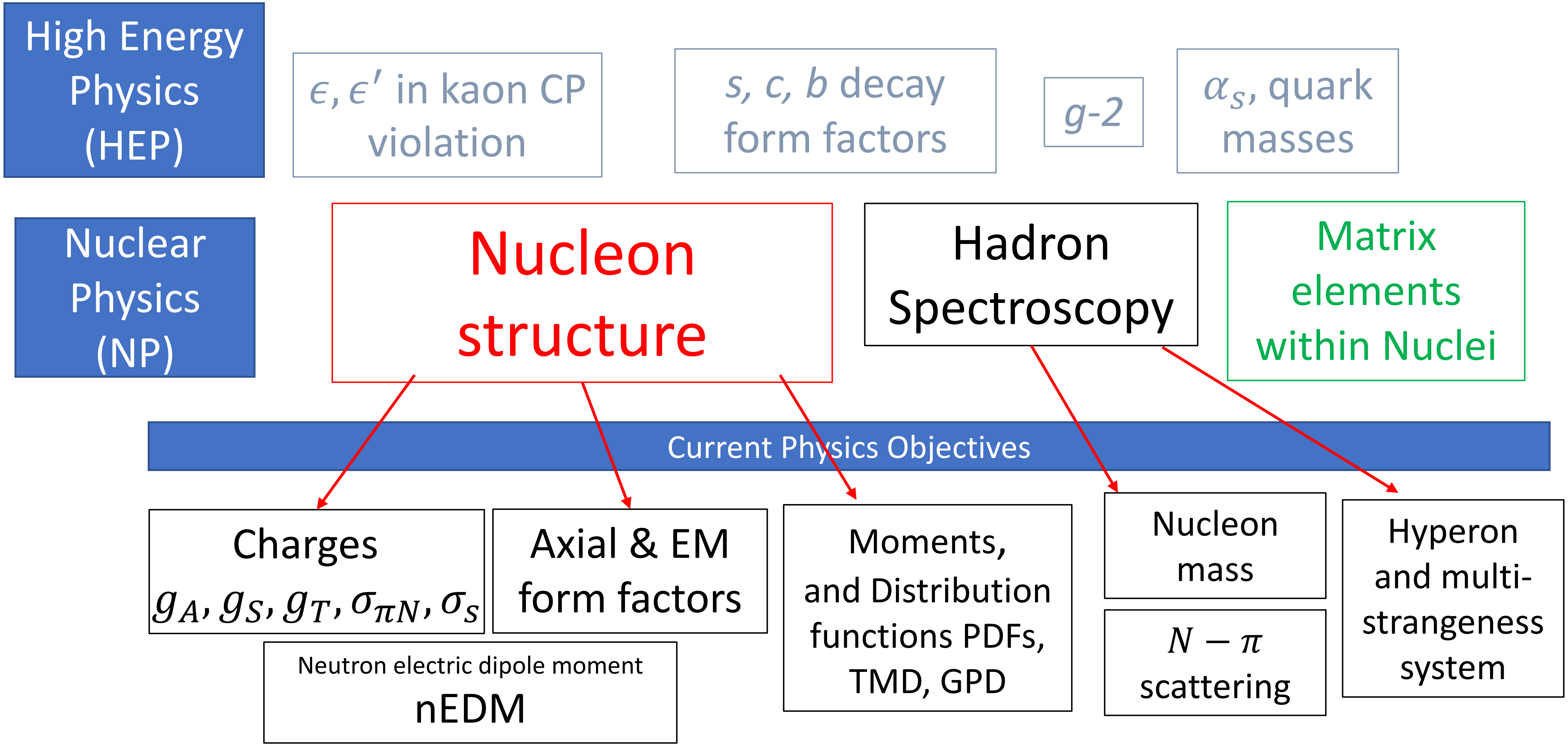
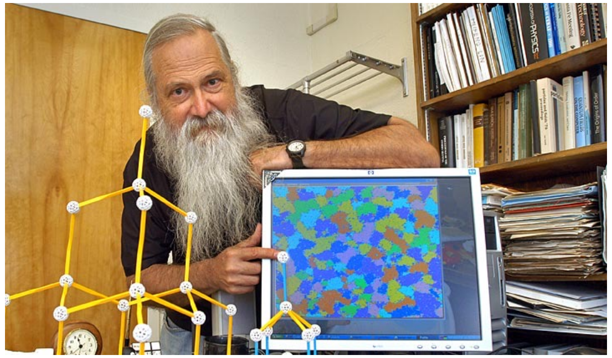
Rich Landscape of LQCD calculations



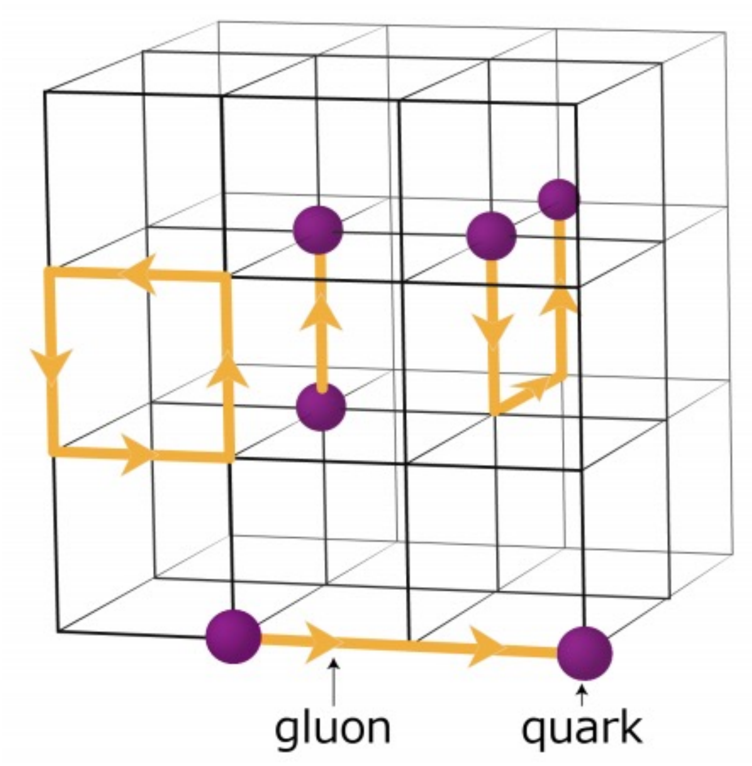
Flavor Lattice Average Group (FLAG) reviews 1902.08191, 2111.09849, 2411.04268

Lattice QCD

Formulated by K. Wilson (1974)



Numerical computation opened by M. Creutz (1979)



Lattice QCD is QCD defined on a 4-dimensional Euclidean space-time lattice

- Finite lattice spacing: (a)
- Quark fields $(q, \bar{q})$, Gauge fields (gluons): (U_μ)
- Perturbative & **Numerical (nonperturbative) calculations**

Major systematic errors coming from:

- Finite lattice spacing a (UV cut-off effect)
- Chiral fit to get value at physical pion mass
- Finite Volume
- Statistical errors
- Excited state contaminations
- Renormalization

Three 2+1+1-flavor physical pion mass HISQ Ensembles

Ensemble ID	a [fm]	M_π [MeV]	$M_\pi L$	$L^3\ T$	N_{conf}
a09m135	**0.0873**	**135**	**3.95**	$\mathbf{64^3 96}$	**1290 -> 6060 / 8000**
a06m135	**0.0564**	**135**	**3.95**	$\mathbf{96^3 192}$	**675 -> 8269 / 10000**
a04m135	0.0426	135	3.95	$144^3 288$	0 / 2000

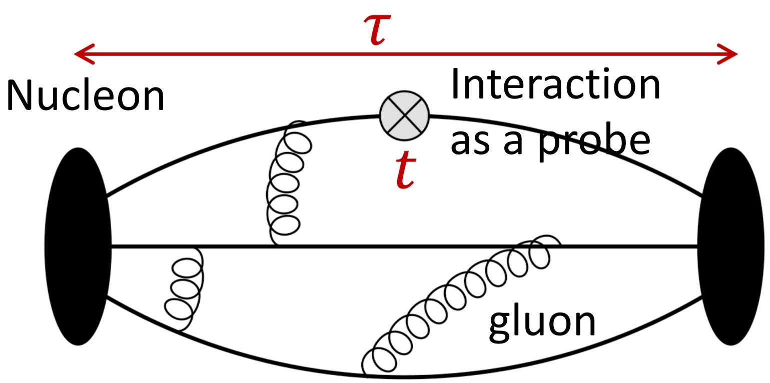
- Ensembles generated by MILC/CalLat/LANL Collaborations
- Improved source smearing
- Precise quark mass parameter tuning
- Larger source-sink time separations to remove excited states
- Doubled range of momentum transfer $\vec{q} = 2\pi\vec{n}/L$ Largest spacelike 4-momentum transfer $Q^2 = -q^2$) increased from $0.45\ \mathrm{GeV}^2$ to $0.82\ \mathrm{GeV}^2$

- **Will improve all nucleon results in the continuum limit very significantly (2-3X)**

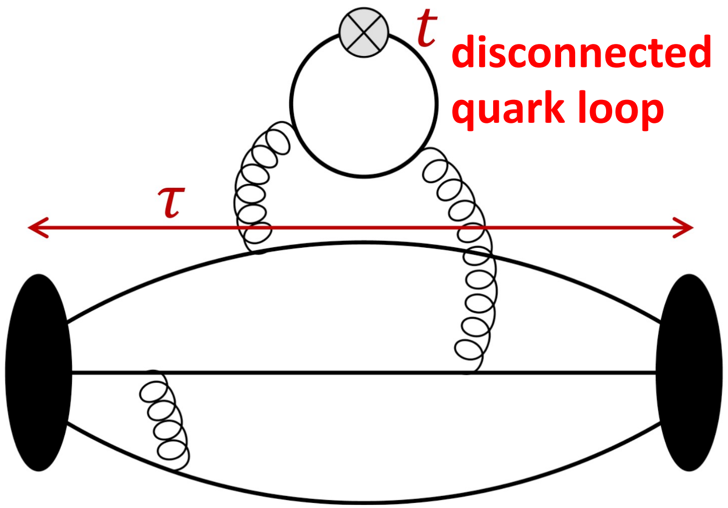
[PNDME Collab.]

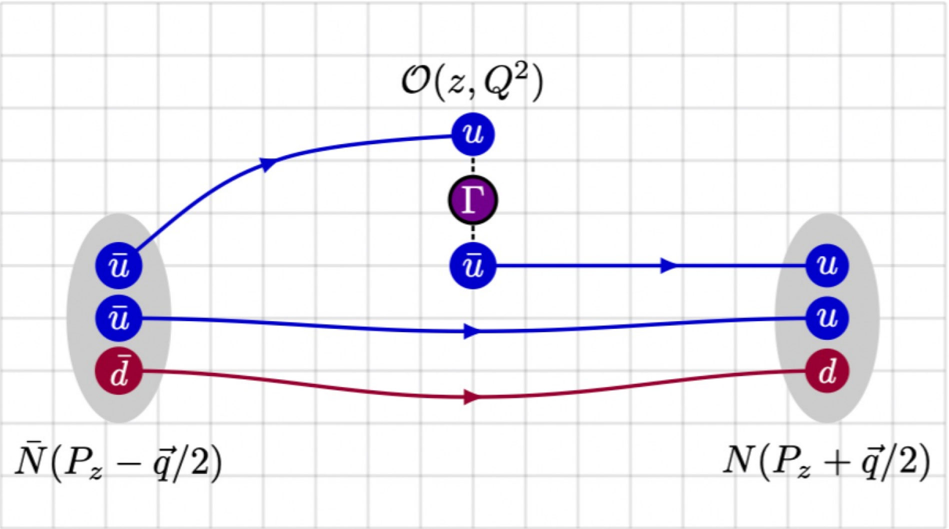
Nucleon structure from LQCD correlation functions

$$C_\Gamma^q(t,\tau) = C_\Gamma^{q,\mathrm{conn}}(t,\tau) + C_\Gamma^{q,\mathrm{disc}}(t,\tau)$$ → ***Charges, form factors, moments, PDFs and GPDs***



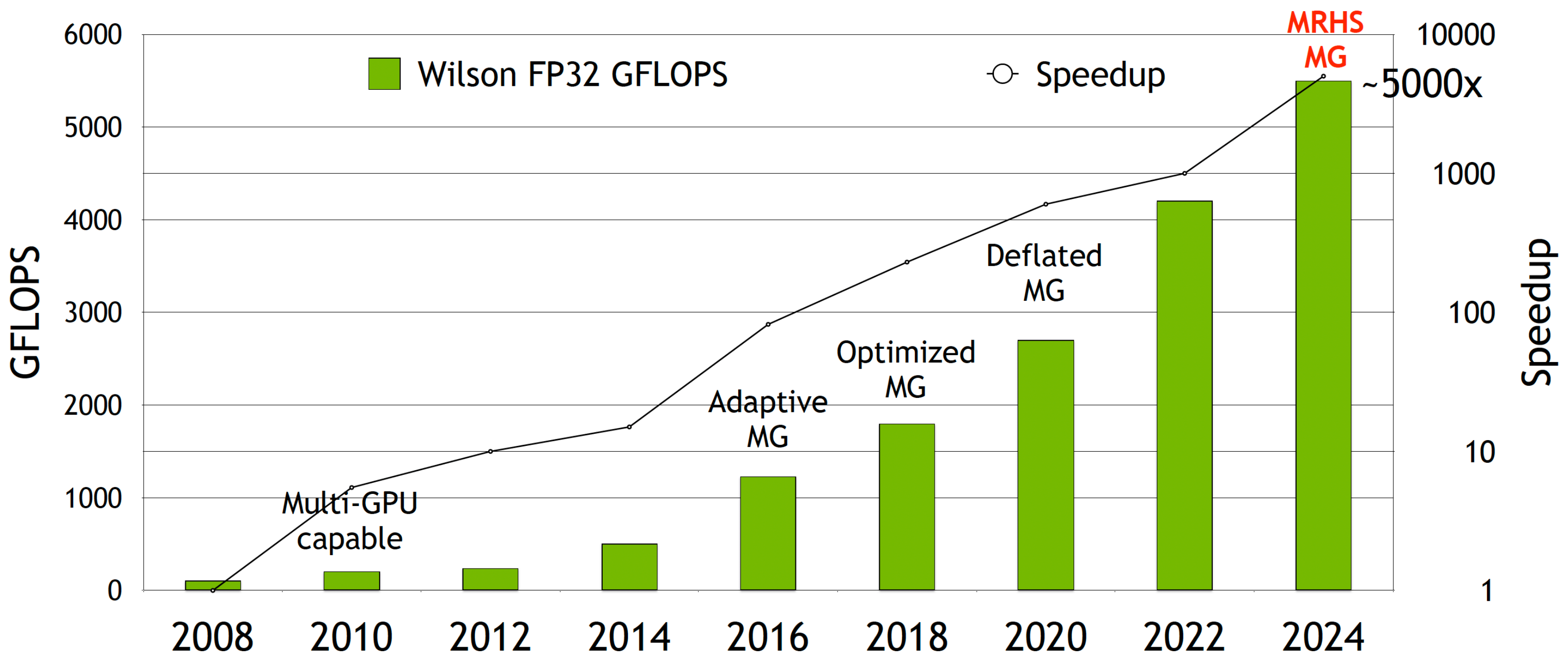
quark propagator
: inverse of large (O(10^9)) sparse Dirac matrix





Algorithm + GPU advances make these high-statistics calculations possible

- QUDA node performance over time



Speedup determined by measured time to solution for solving the Wilson operator against a random source on a $V=24^3 64$ lattice, $\beta=5.5$, $M_\pi=416$ MeV. One node is defined to be 3 GPUs.

Kate Clark, NVIDIA, Lattice 2024 Conference

Nucleon spin from quarks: Axial charge $g_A^q = \Delta q$

$g_A^q = \Delta q$: *Quark contributions to the nucleon spin*

$$\frac{1}{2} = \sum_{u,d,s,\cdots} \left(\frac{1}{2} \Delta q + L_q \right) + J_g$$

X. Ji, PRL 78 (1997) 610

L_q: orbital angular momentum of the quark

J_g: total angular momentum of the gluons

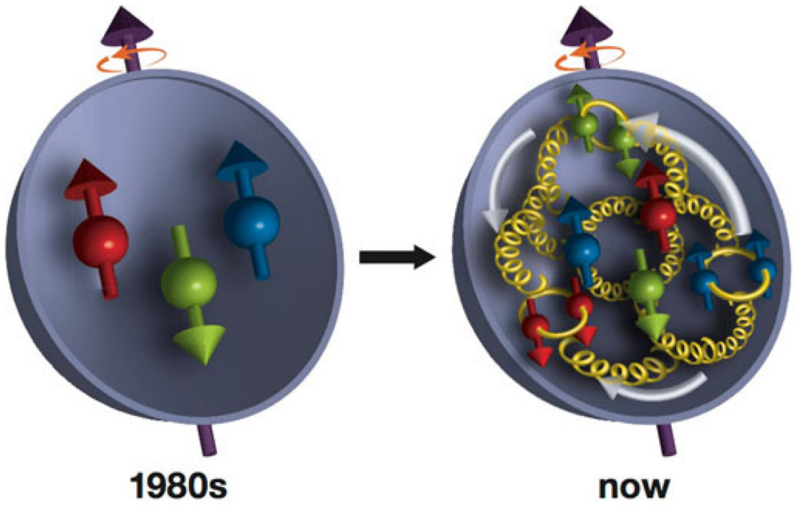
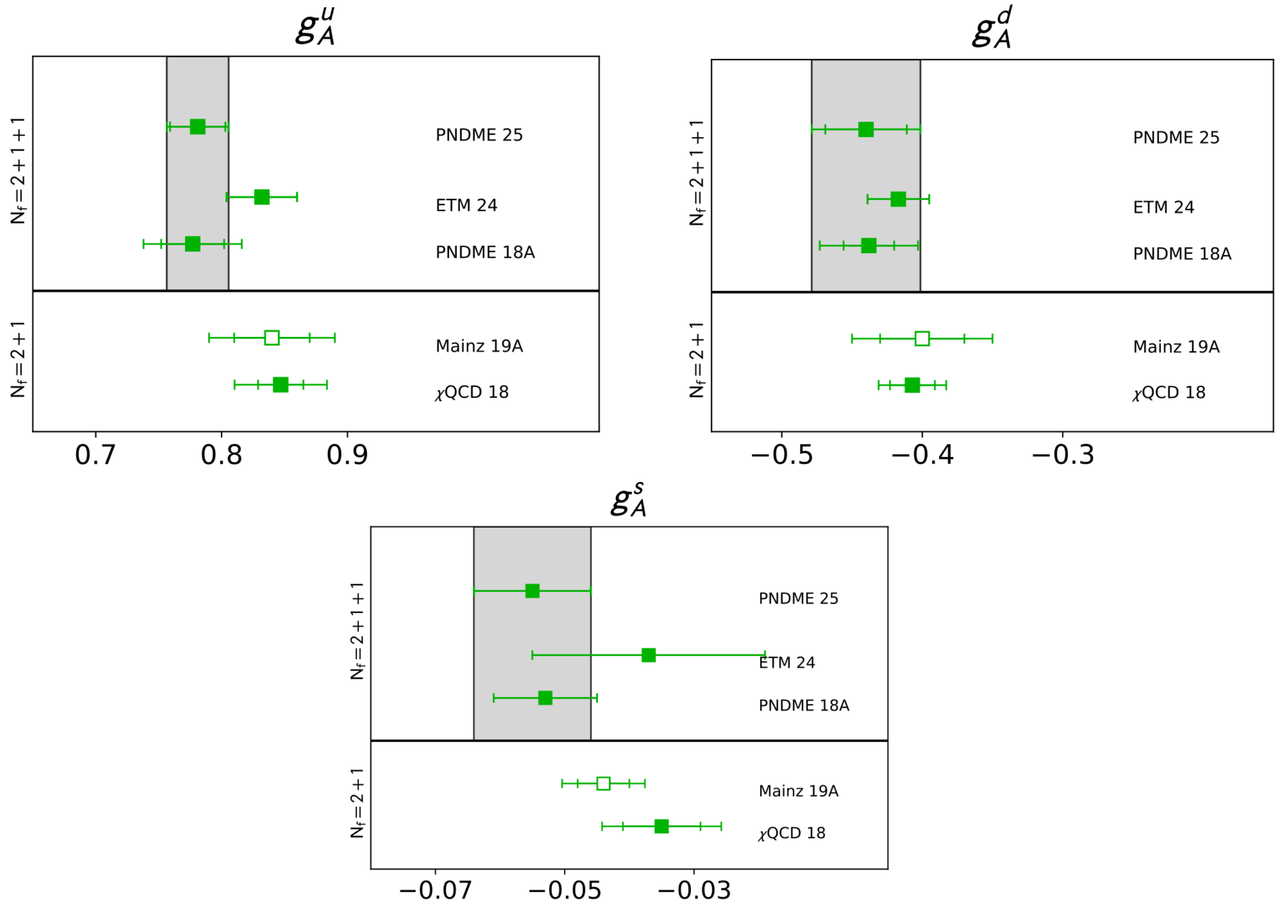


Photo courtesy of Brookhaven National Laboratory

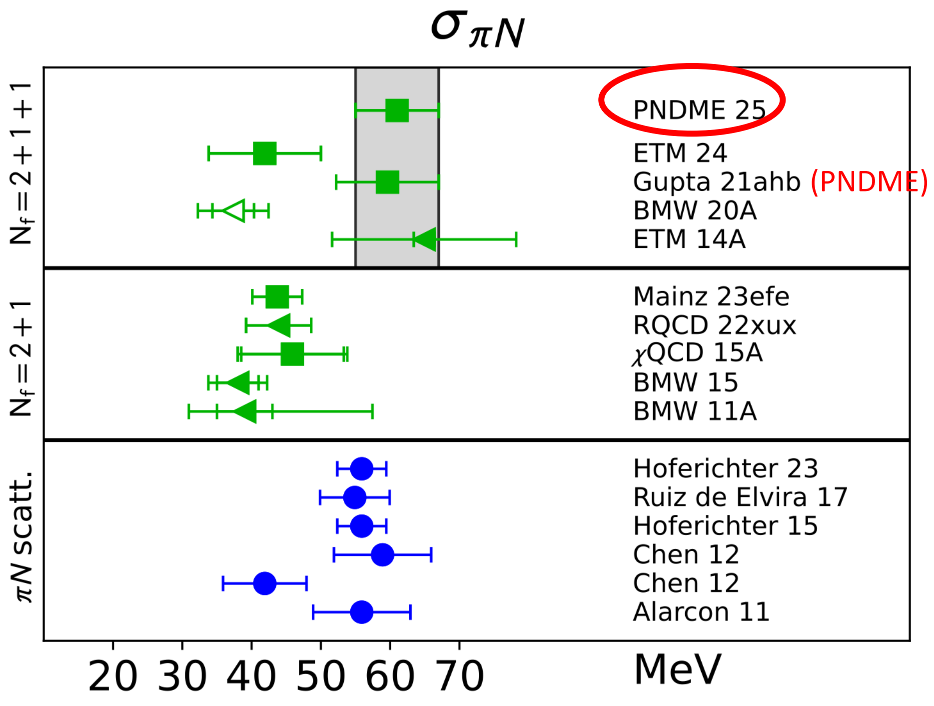


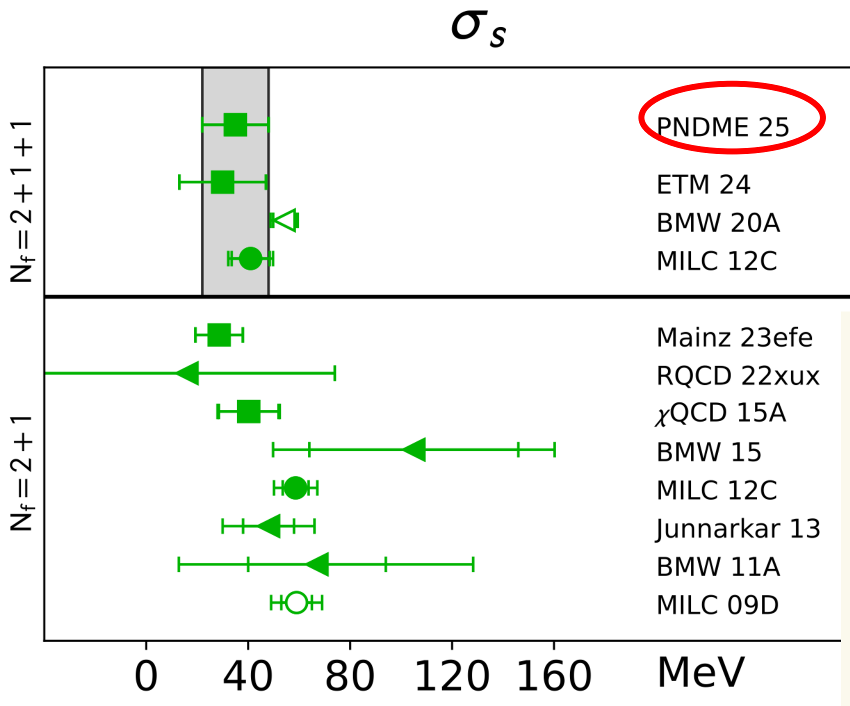
$0.13 \le \sum_q 0.5\Delta q \le 0.18$ COMPASS, Phys. Lett. B753, 18 (2016)

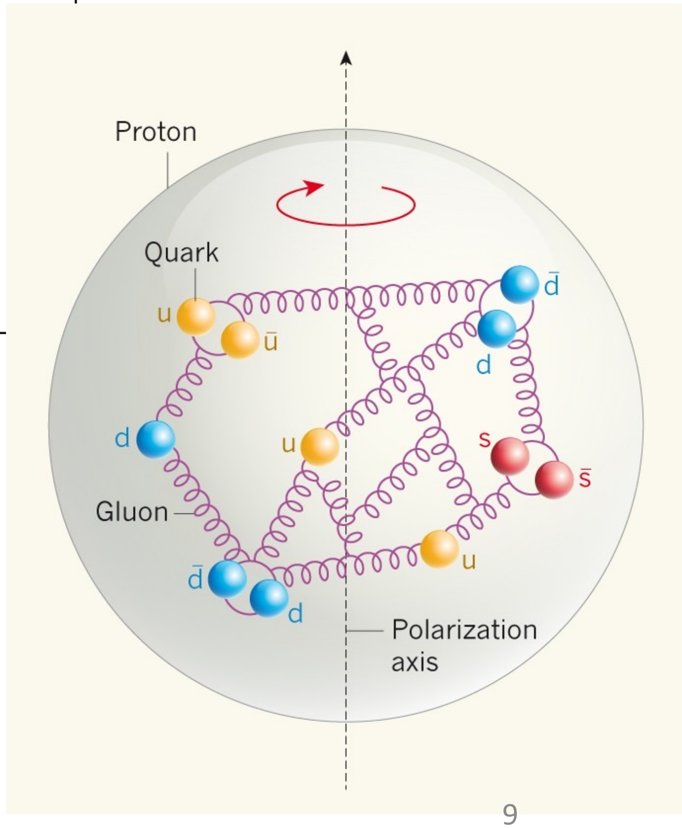
$\sum_q 0.5\Delta q = 0.143(24)$ Park et al. [PNDME] PRD112 054508 (2025)

Park et al. [PNDME] PRD112 054508 (2025)

Sigma terms: Quark contributions to the nucleon mass





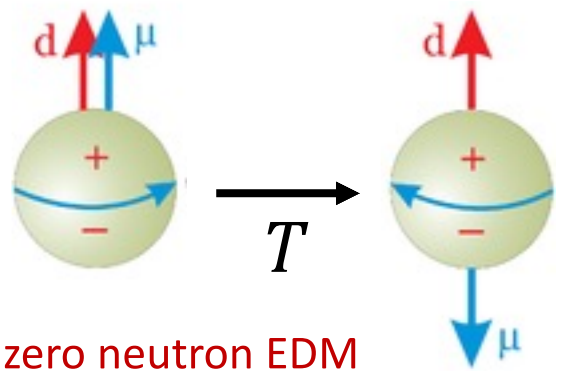


A value around 60 MeV corresponds to only about 6% of the nucleon mass; **most of the mass arises from nonperturbative QCD dynamics.**

Strange-quark contribution to the nucleon mass

Park et al. [PNDME] PRD112 054508 (2025)

Neutron electric dipole moment d_n: Tensor charge $\boldsymbol{g_T}$



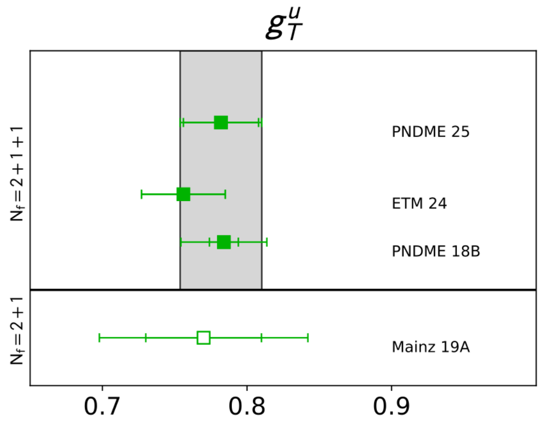
A nonzero neutron EDM
is a sensitive probe of new sources of CP violation

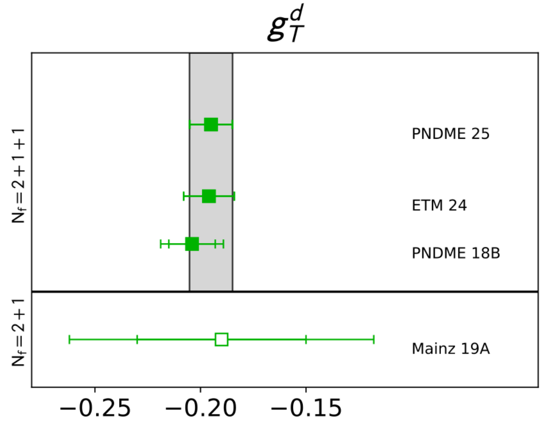
- Contribution of the Quark electric dipole moment (qEDM) operator

$$|d_n| = |d_u^\gamma \boldsymbol{g_T^u} + d_d^\gamma \boldsymbol{g_T^d} + d_s^\gamma \boldsymbol{g_T^s} + \cdots|$$

$\boldsymbol{g_T^q}$: **Tensor charges**

d_q^γ: **BSM couplings of CP violating qEDM operators**





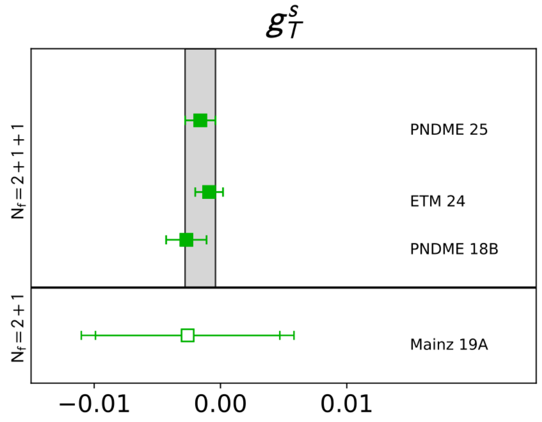
- standard model (CKM) contribution to **neutron EDM d_n** is tiny

$$|d_n| < 1.8\times 10^{-26}\ e\cdot\mathrm{cm}\ (90\%\ \mathrm{C.L.})$$

C. Abel et al., PRL 124 (2020) 081803

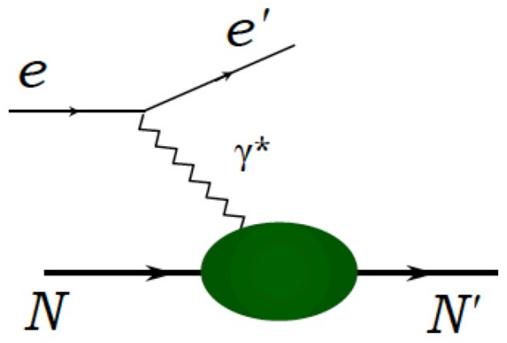
whereas $|d_n^{CKM}| < 6\times 10^{-32}\ e\cdot\mathrm{cm}$

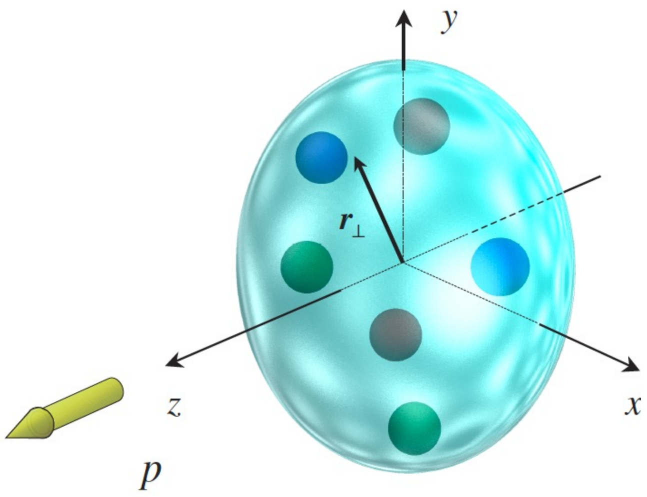
C.-Y. Seng, PRC 91 (2015) 025502

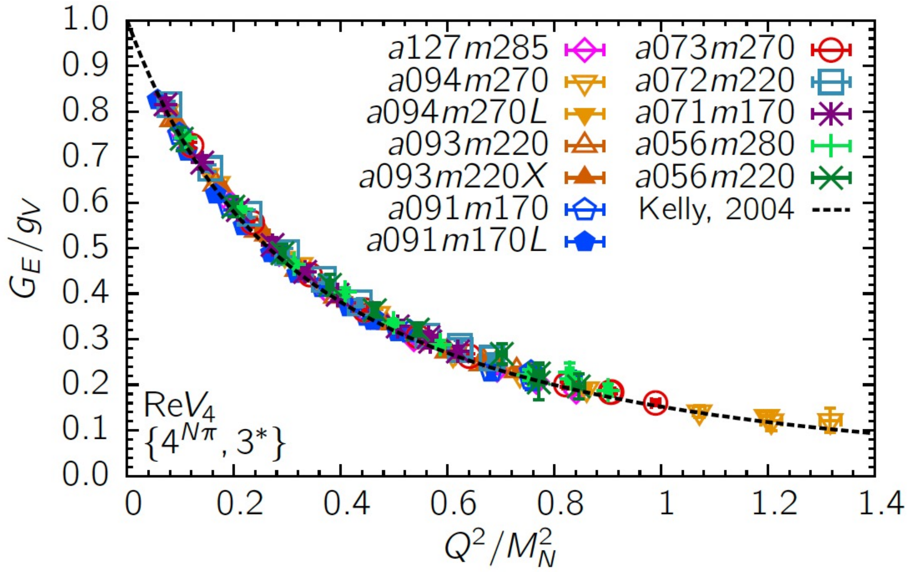


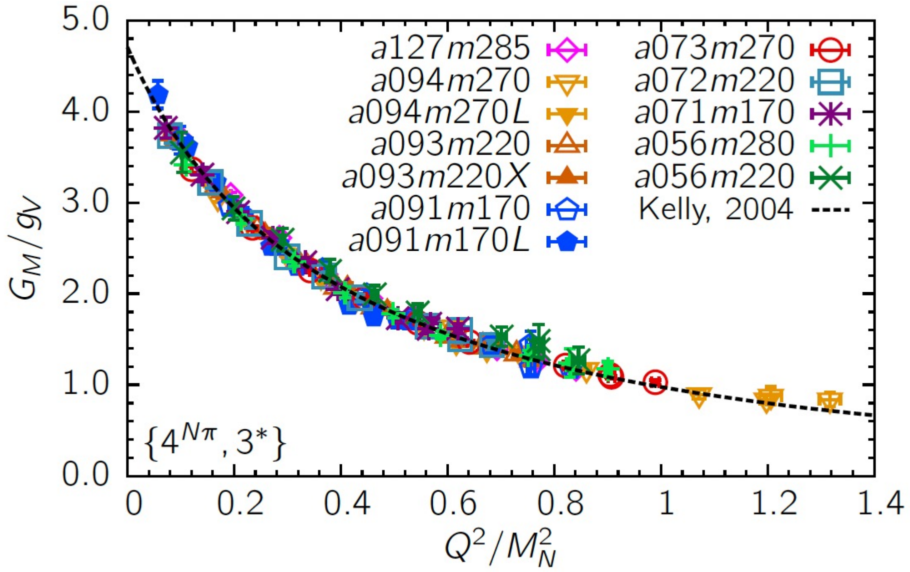
Park et al. [PNDME] PRD112 054508 (2025)

Nucleon electromagnetic form factor: charge distribution inside the nucleon

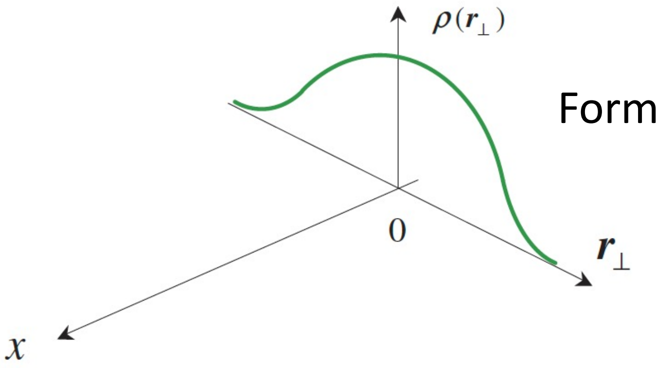








NME Collab. [S.Park, J.Yoo et al., in preparation]

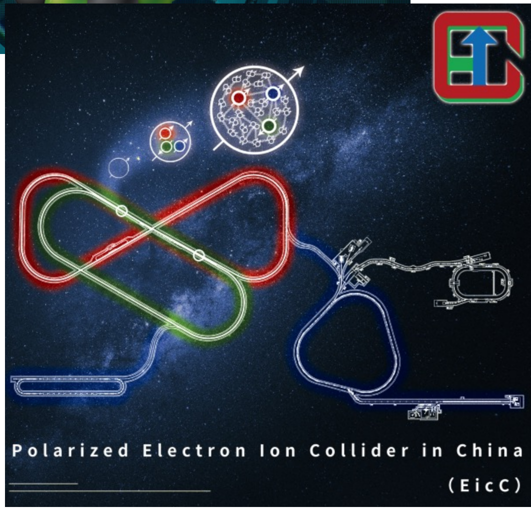


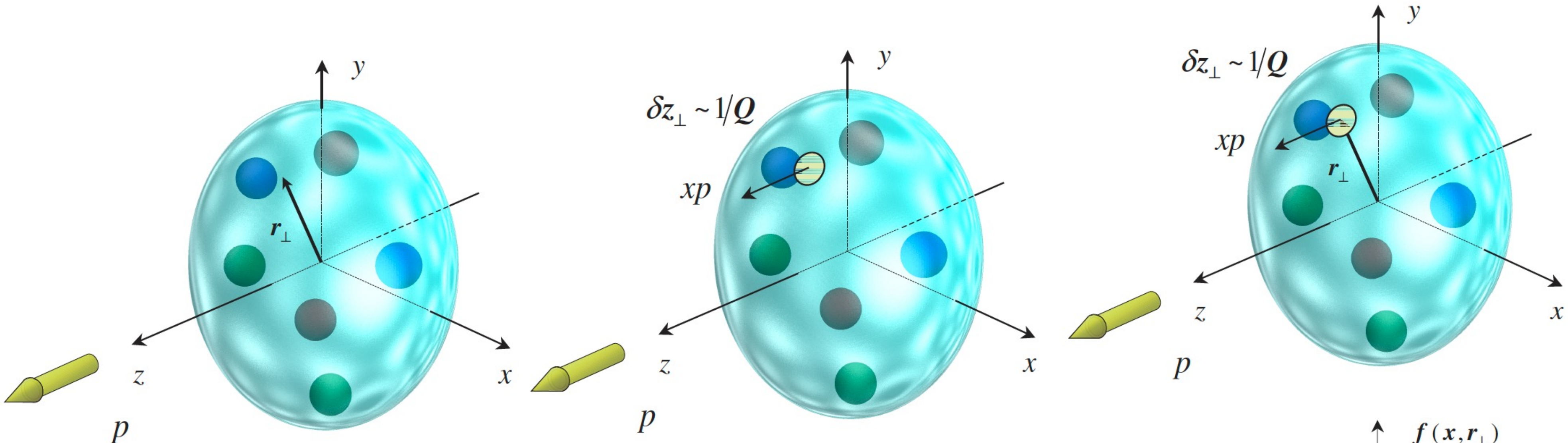
Form Factor

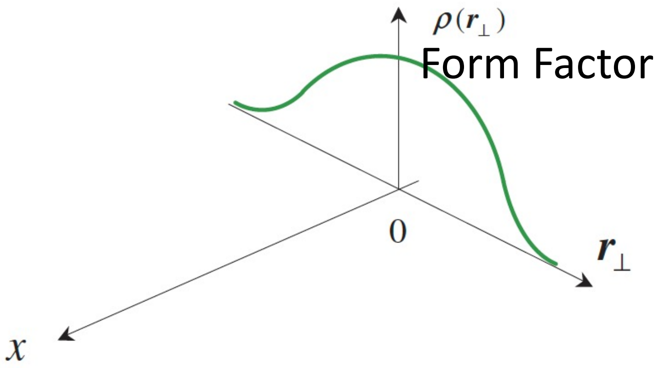
Preliminary results on the nucleon electric (G_E) and magnetic (G_M) form factors from 12 N_f=2+1 clover ensembles.

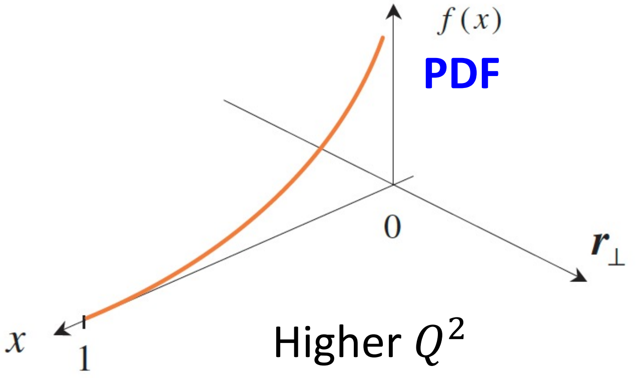
With highly improved statistics, these results show excellent agreement with experimental data (Kelly, 2004)

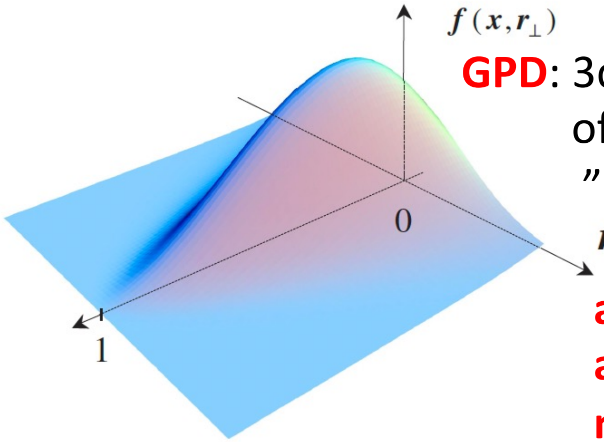
Beyond the nucleon form factor:
Parton Distribution Function (PDF)
Generalized Parton Distribution (GPD)











GPD: 3d tomographic image of the nucleon
"Main Goal of EIC / EicC"

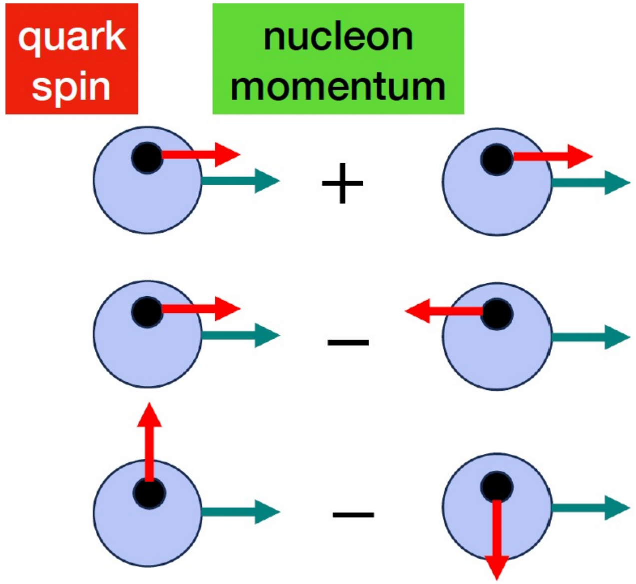
access to quark/gluon angular momentum and mechanical structure

Higher Q^2

Elastic scattering → Deep Inelastic Scattering (DIS) in EIC

Nucleon Parton distribution function (PDF)

how the nucleon's momentum and spin are divided among quarks and gluons

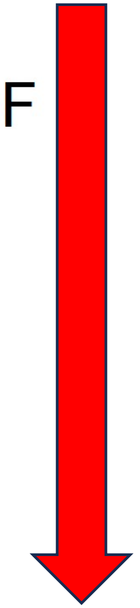


x: longitudinal momentum fraction

$q(x) = q_{\uparrow}(x) + q_{\downarrow}(x)$
: helicity averaged, spin independent, unpolarized PDF

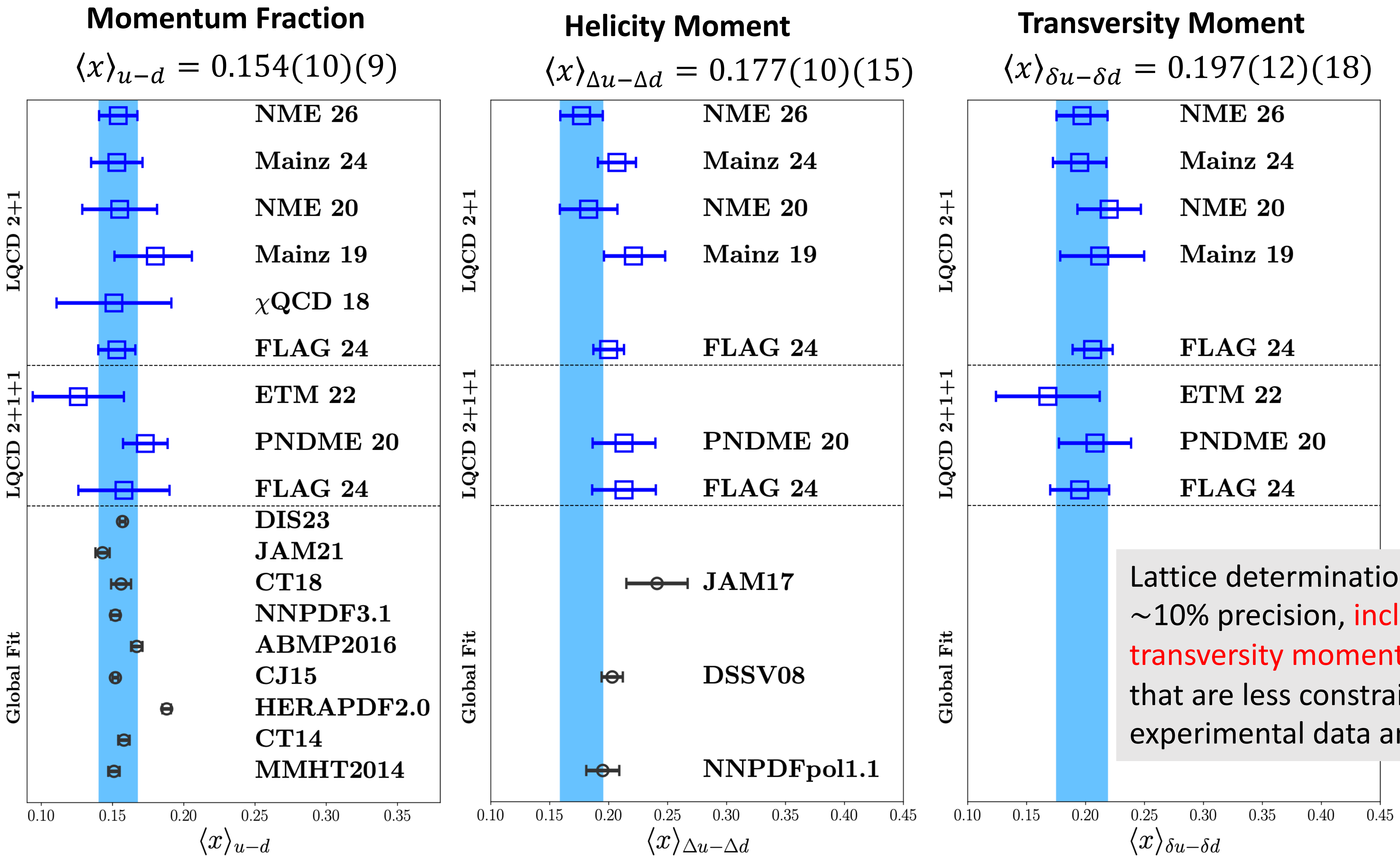
$\Delta q(x) = q_{\uparrow}(x) - q_{\downarrow}(x)$
: helicity-dependent, polarized PDF

$\delta q(x) = q_{\perp}(x) + q_{\top}(x)$
: spin transversely aligned PDF



Status of isovector moments

[NME 26] arXiv:2608.02836;
[NME 20] JHEP 21, 004; [PNDME 20] PRD 102, 054512



Lattice determinations are reaching ~10% precision, including helicity and transversity moments that are less constrained by experimental data and global analysis

Removing Excited State effects from Nucleon Matrix Element

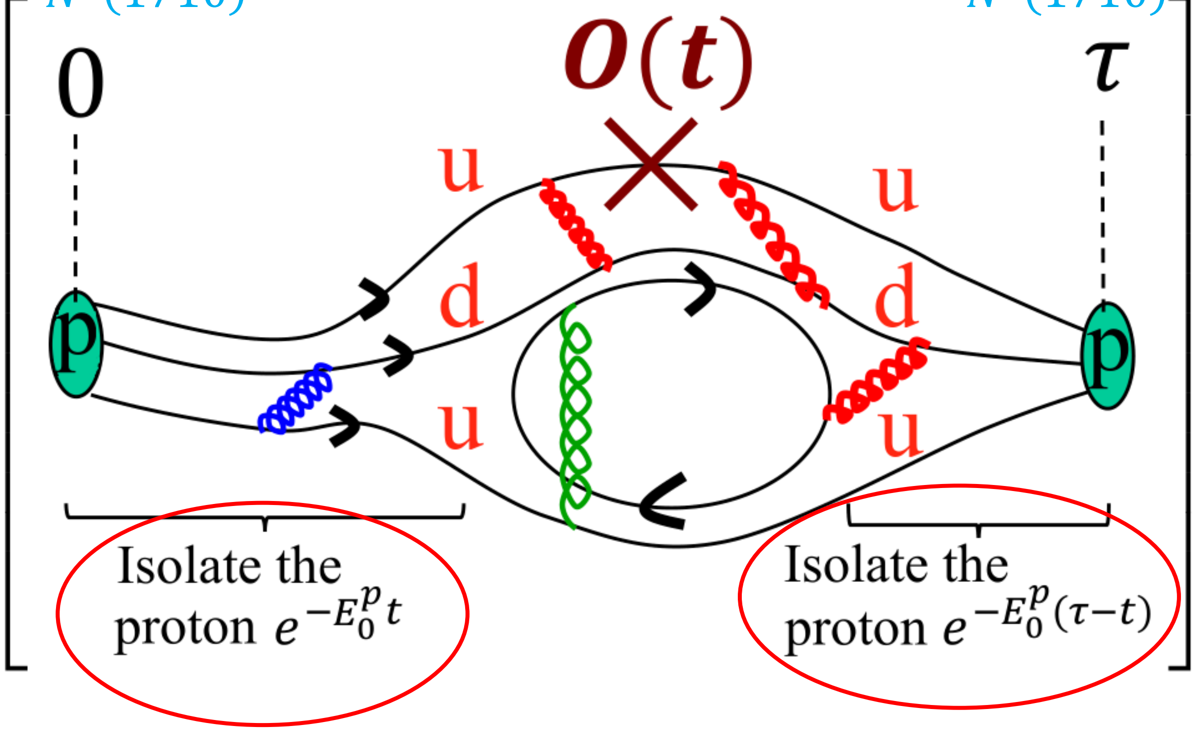
- Nucleon signal/noise decays $\propto e^{-(E-1.5M_\pi)\tau}$ with Euclidean time τ. [G.P.Lepage 1989]
  → Cannot go to large $\boldsymbol{\tau}$

Created Excited states must be removed!
But can we identify them?

$N\pi, N\pi\pi,$
$N\rho,$
$N^*(1440),$
$N^*(1710)$

$N\pi, N\pi\pi,$
$N\rho,$
$N^*(1440),$
$N^*(1710)$

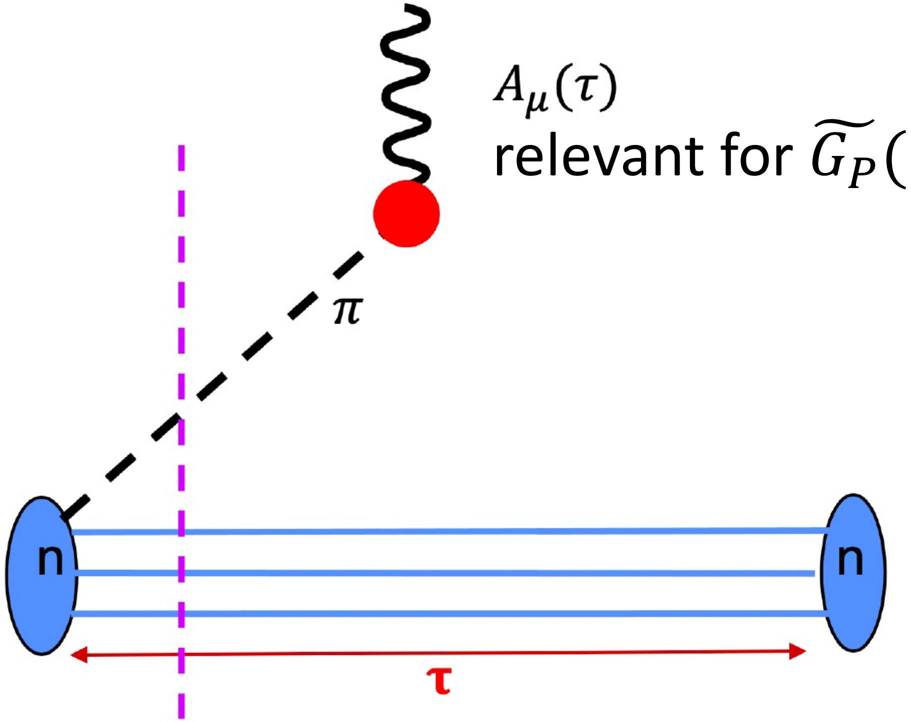
$$C(t,\tau) = \frac{1}{N_{\text{samp}}} \sum_{\{U\}}$$



Charges,
form factors,
PDFs,
GPDs

$N\pi$ Excited-state contribution can be enhanced

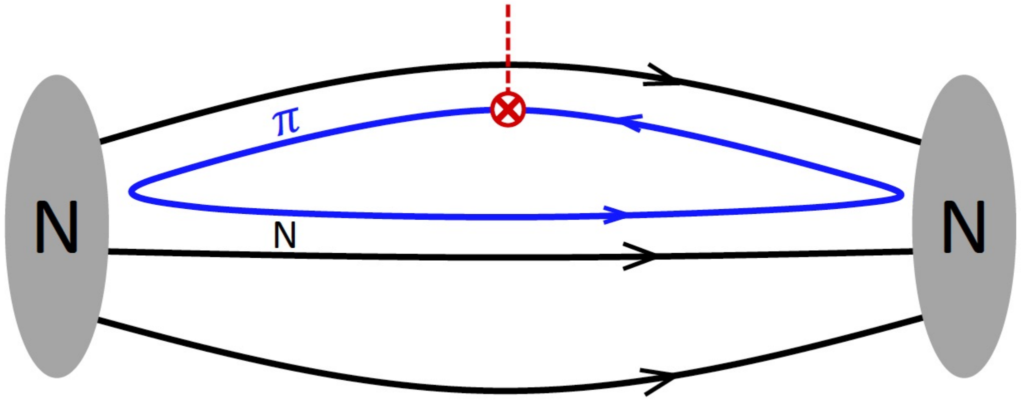
- To control ESC, we need to first determine the spectrum
- In nucleon 2pt functions, $1/L^3$-suppressed coupling to $N\pi$-state makes it challenging to extract its spectrum
- In certain 3pt functions (matrix elements) , ChPT suggests $N\pi$-state contributions are enhanced and compensate for the $1/L^3$-suppression



$A_\mu(\tau)$
relevant for $\widetilde{G}_P(Q^2)$

Jang et al., [PNDME 2020]
Park, Gupta et al., [NME 2022]

Scalar operator
relevant for $g_S^{u,d}$ and $\sigma_{\pi N} = m_{ud} g_S^{u+d}$



Gupta, Park et al., [PNDME 2021]

Nucleon scalar charge

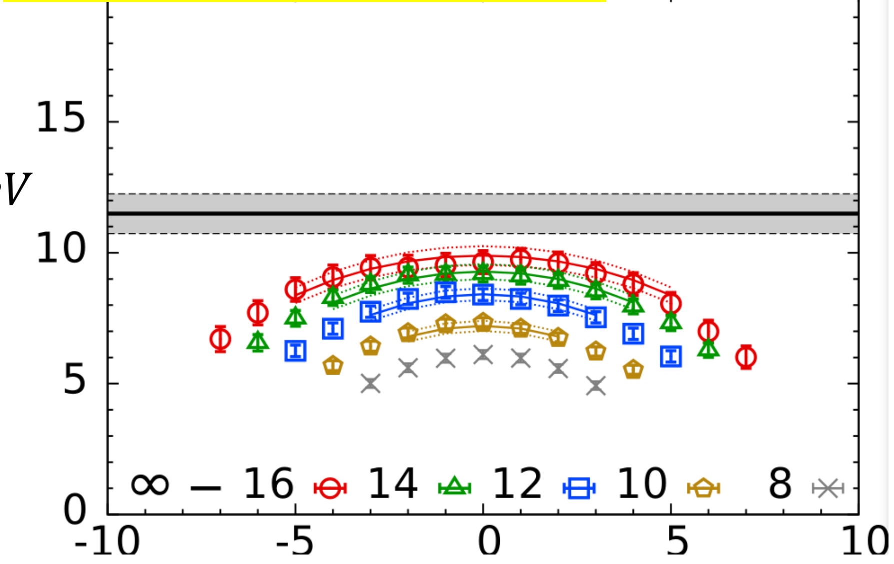
g_S^{u+d}: Excited state (ES) effect

$a \approx 0.09 fm$
$M_\pi \approx 135 MeV$

[PNDME] Gupta, Park et al., PRL 127 (2021) 242002

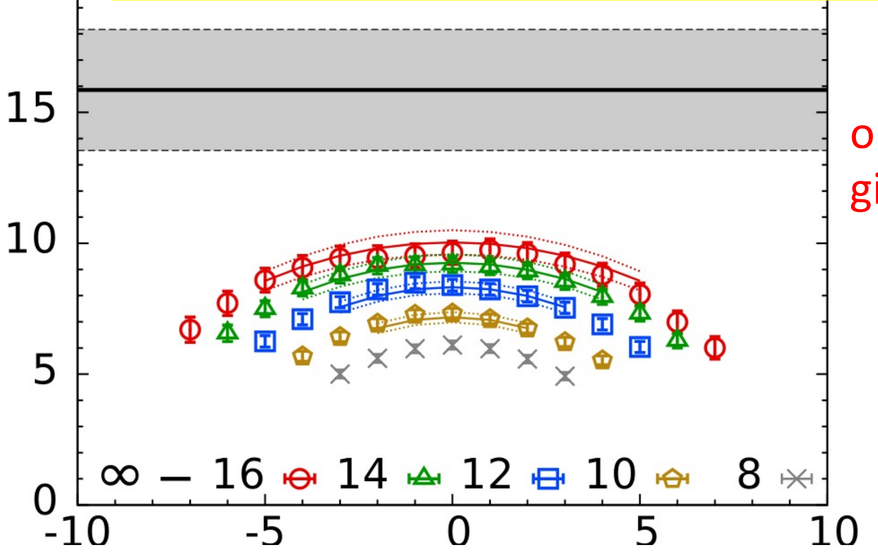
Standard analysis

$M_1 \approx 1.6\ GeV$



$\sigma_{\pi N} = m_l g_S^{u+d} \sim 40$ MeV, $\frac{\chi^2}{dof} = 1.1$

$N\pi$-motivated ES analysis



$N(1)\pi(-1)$
or, $N(0)\pi(0)\pi(0)$
gives $M_1 \approx 1.2\ GeV$

$\sigma_{\pi N} = m_l g_S^{u+d} \sim 60$ MeV, $\frac{\chi^2}{dof} = 1.2$

- Scalar is sensitive to $N\pi$ state
- Result is consistent with the phenomenological determination

[PNDME Collab.]

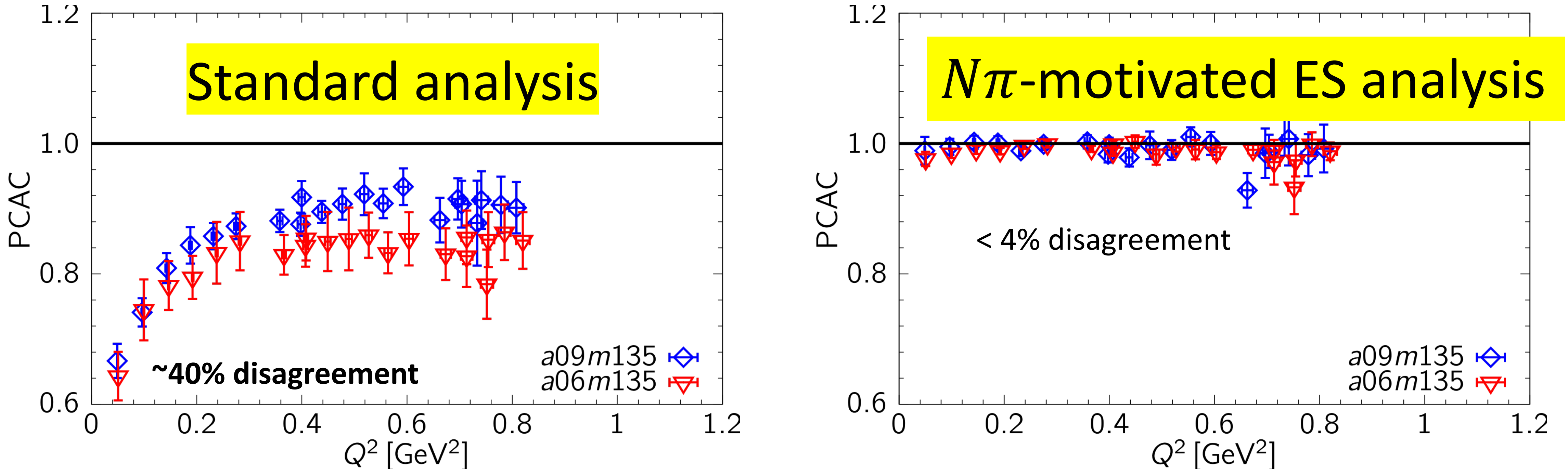
Axial Channel: large $N\pi$ excited-state (ES) effects

There are 3 axial form factors $G_A, \widetilde{G_P}, G_P$

Must satisfy the **PCAC relation:** $\partial_\mu A_\mu(x) = 2\hat{m}P(x)$ where $\hat{m} = Z_m m_{ud} Z_P Z_A^{-1}$

Applied to nucleon ground state, it relates the 3 nucleon form factors

$$\frac{\widetilde{G}_P(Q^2)}{G_A(Q^2)}\frac{Q^2}{4M^2} + \frac{G_P(Q^2)}{G_A(Q^2)}\frac{2\hat{m}}{2M} = 1$$



[PNDME Collab., Preliminary]

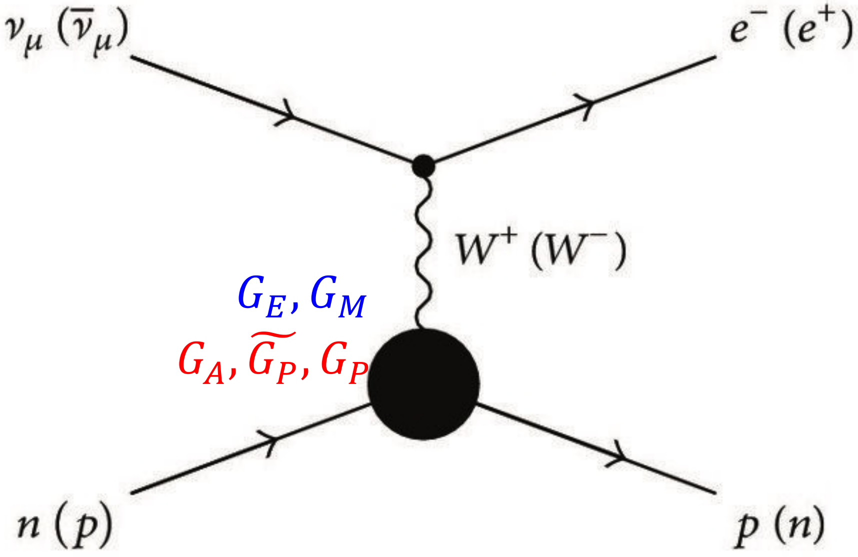
Neutrino Oscillation Experiment

DUNE
DEEP UNDERGROUND NEUTRINO EXPERIMENT

- Differential cross-section:

$$\frac{d\sigma}{dQ^2} \propto f(Q^2, [\text{VFF}], [\text{AFF}], \cdots)$$

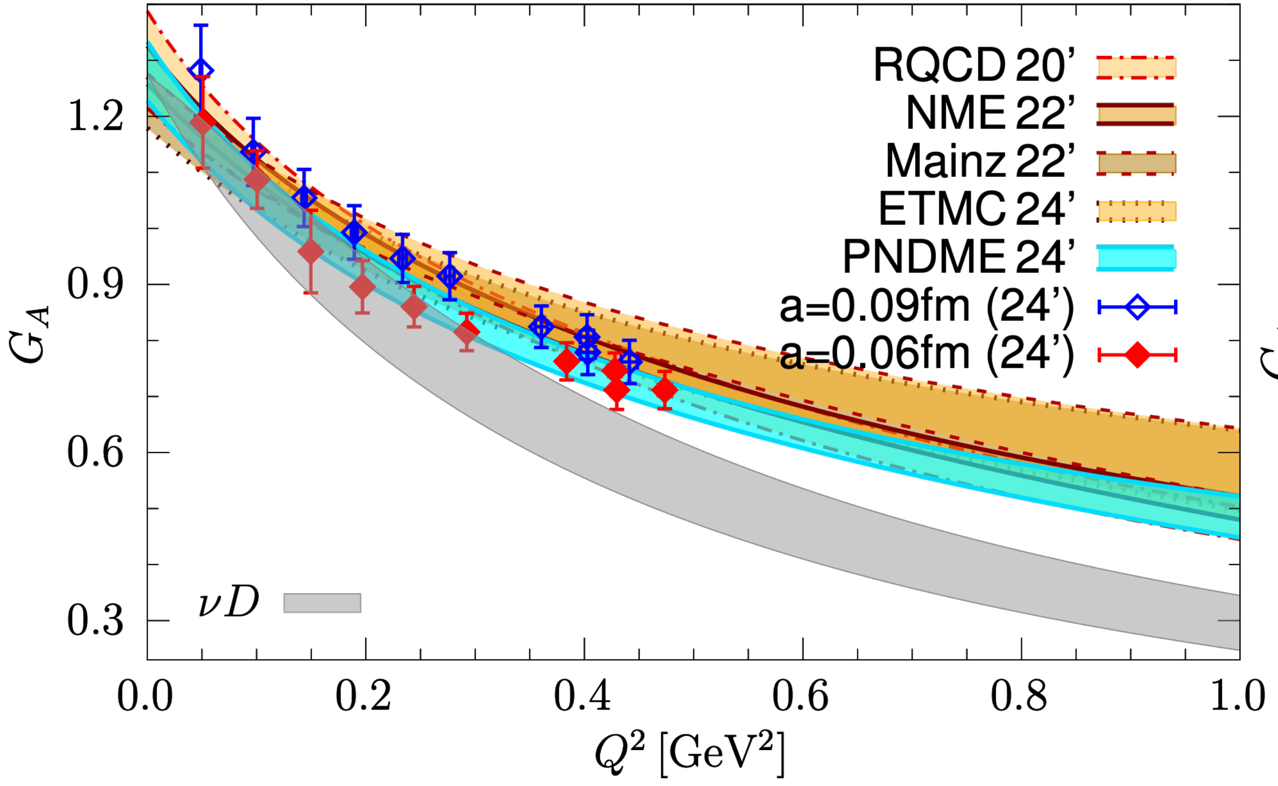
- Weak interaction (V-A): Low statistics
  - Vector form factor (VFF): G_E, G_M
    → high-statistics electron scattering experiments
  - Axial vector form factor (AFF): $G_A, \widetilde{G_P}, G_P$
    → **Lattice QCD** can provide direct first-principles access, can help constrain neutrino cross sections over a range of Q^2
    → require a complete parametrization of $G_A(Q^2)$ (e.g. z-expansion)



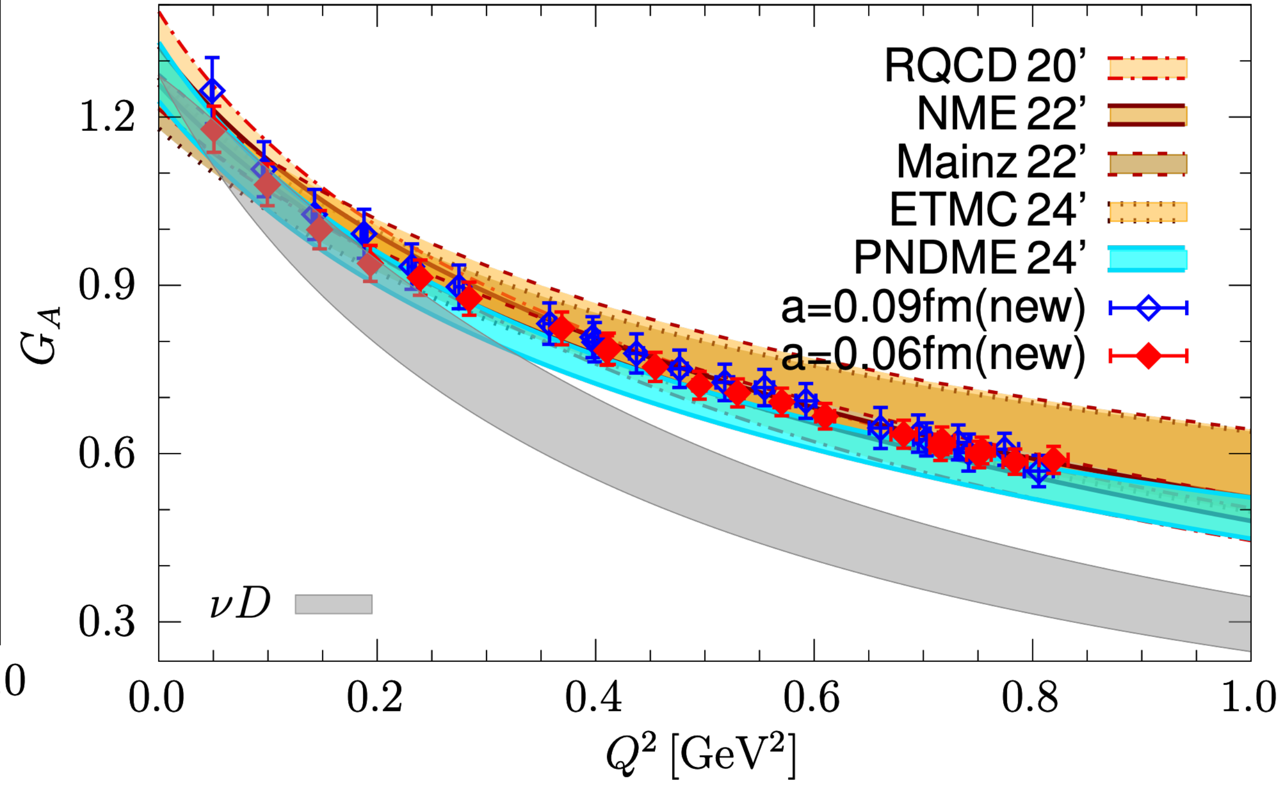
Quasi-Elastic (QE) scattering

Improved determination of G_A on the two physical pion mass ensembles

PNDME 24' PRD 109, 014503



Significantly Improved results
Park et al. [PNDME], in preparation

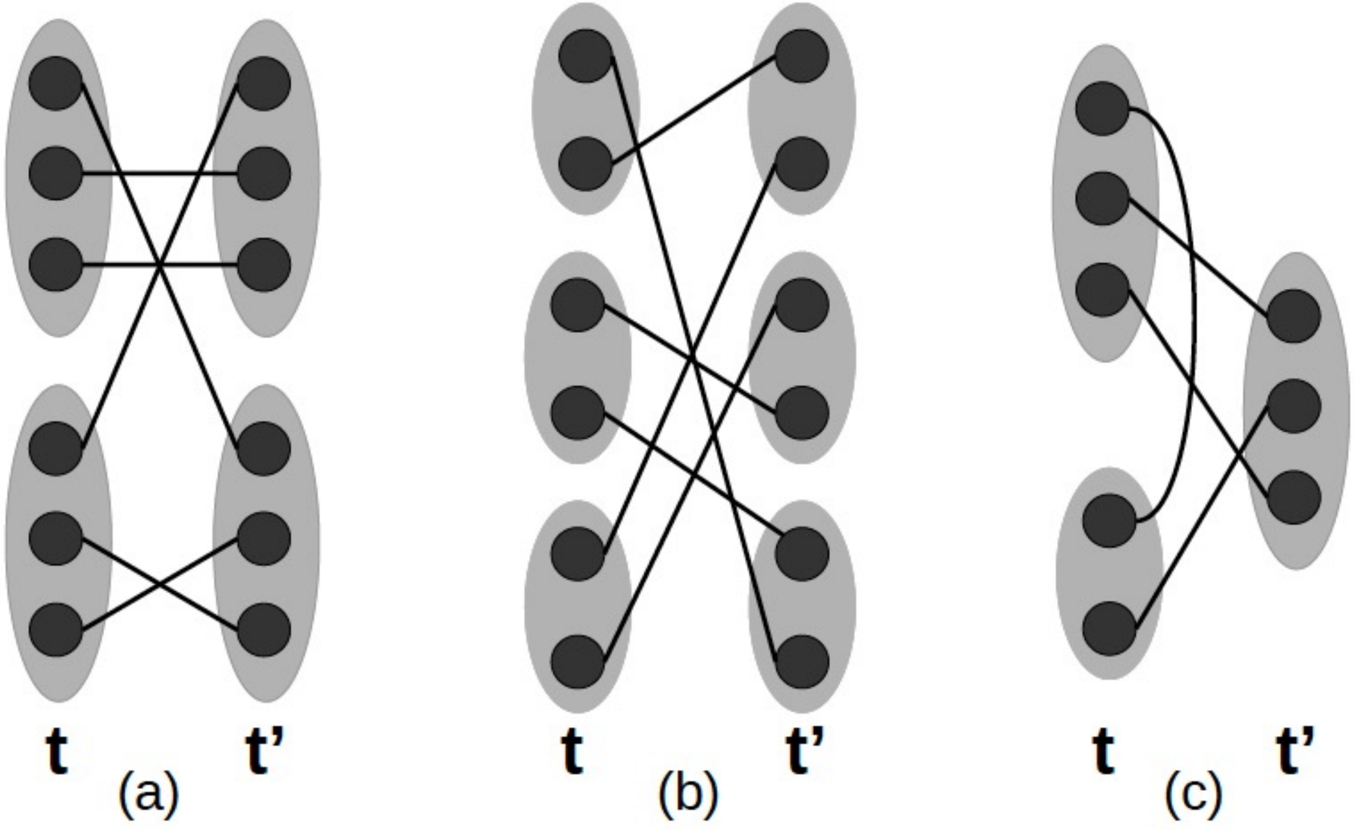


[PNDME Collab.]

$N\pi$: excited-state contamination in nucleon matrix elements → a signal in hadron spectroscopy

Multi-hadron spectrum in a finite box

Multi-hadron systems are computationally challenging

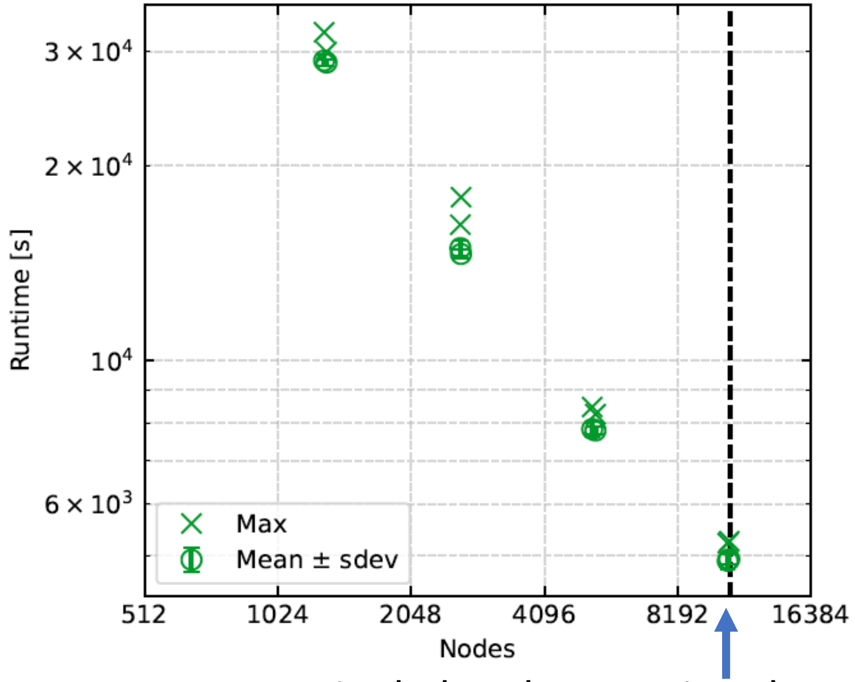


- Factorial growth of Wick-contraction permutations with the number of quarks
- "***Distillation***": spatial space → N_{eig} Laplacian eigenvectors → tensor multiplication with multiple distillation indices → cost grows as a high power of N_{eig}

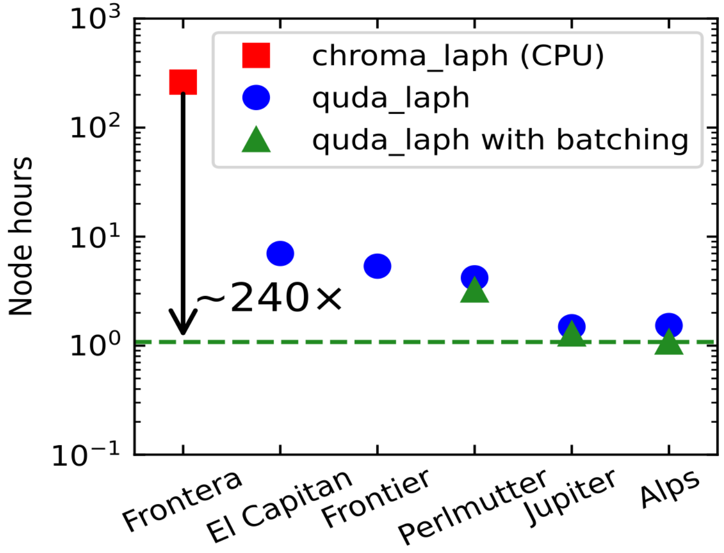
[J.Chen, R.Edwards, W. Mao, PASC '23: Proceedings of the Platform for Advanced Scientific Computing Conference]

Towards nucleon scattering at M_{π}^{Phys} using Exascale computing on El Capitan

- Our new GPU-based code, transforms our nucleon-nucleon studies from not being practical to tractable with a 240x improvement in required-node hours from latest CPU-GPU architectures. [Park et al., arXiv:2510.26313 Entry to the 2025 Gordon Bell Prize competition]



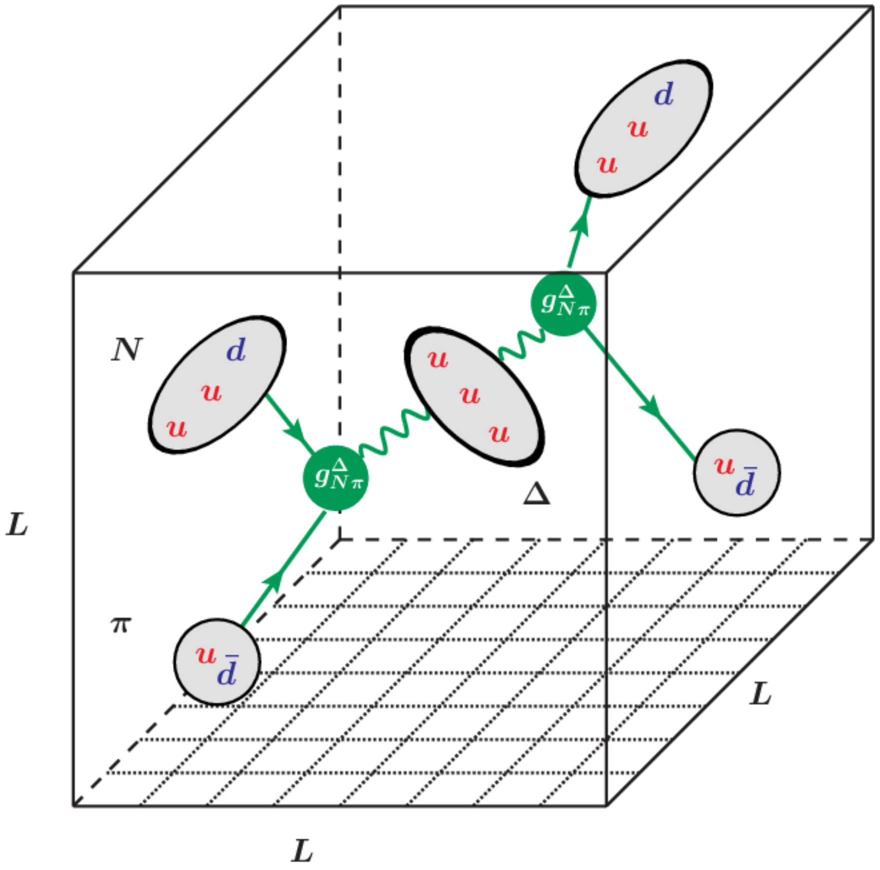
Single batch run using the entire El Capitan system (~11,000 nodes)!



Comparison of the previous state-of-the-art (red) with some of today's exascale-class supercomputers (blue/green).

[CosMoN Collab.]

$N - \pi$ scattering parameters from the Lattice

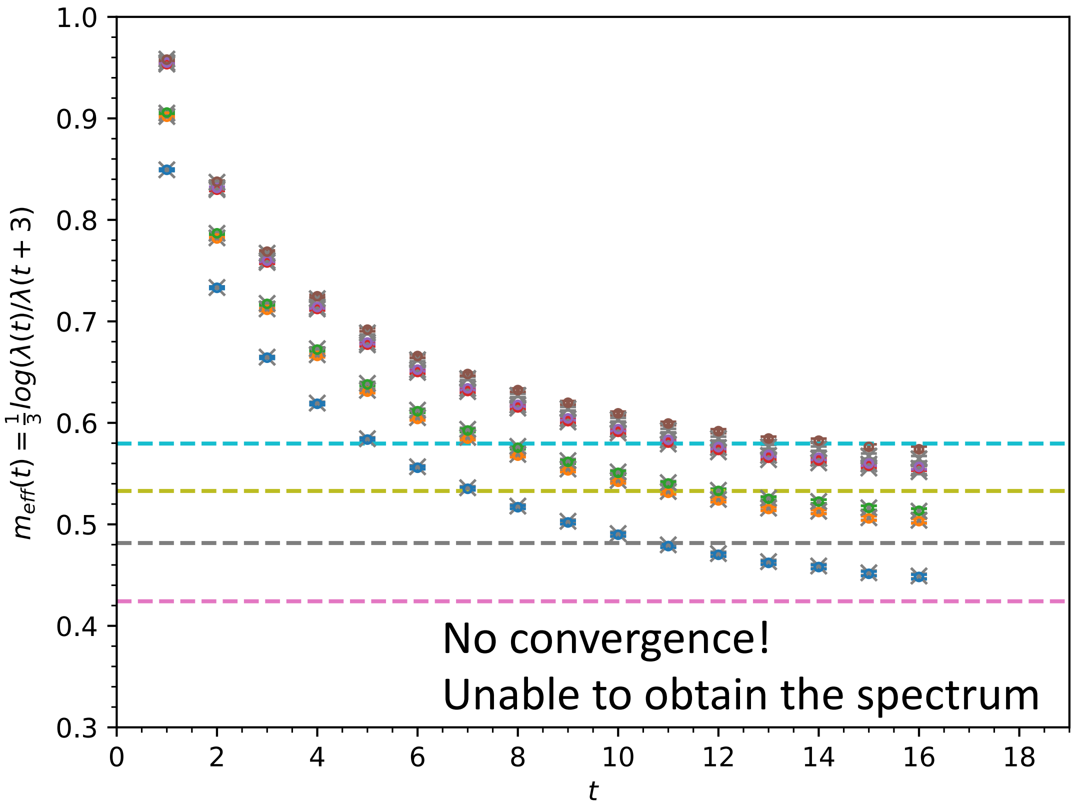


centre.eu/results/elementaryparticlephysics/the-d-resonance-in-elastic-nucleon-pion-scattering

- Why study **$N - \pi$ scattering**
  - The $N - \pi$ interaction is one of the most fundamental processes in nuclear physics
  - Phenomenologically related to the pion-nucleon sigma term $\sigma_{\pi N}$
  - The study of Δ resonances and $N \rightarrow \Delta$ transition form factors which can play crucial role in nucleon-neutrino scattering experiment
- Lüscher's method

Finite Volume Lattice Spectra $E_n(L, a)$ → Infinite Volume Scattering Phase Shift $\delta(E_{cm})$

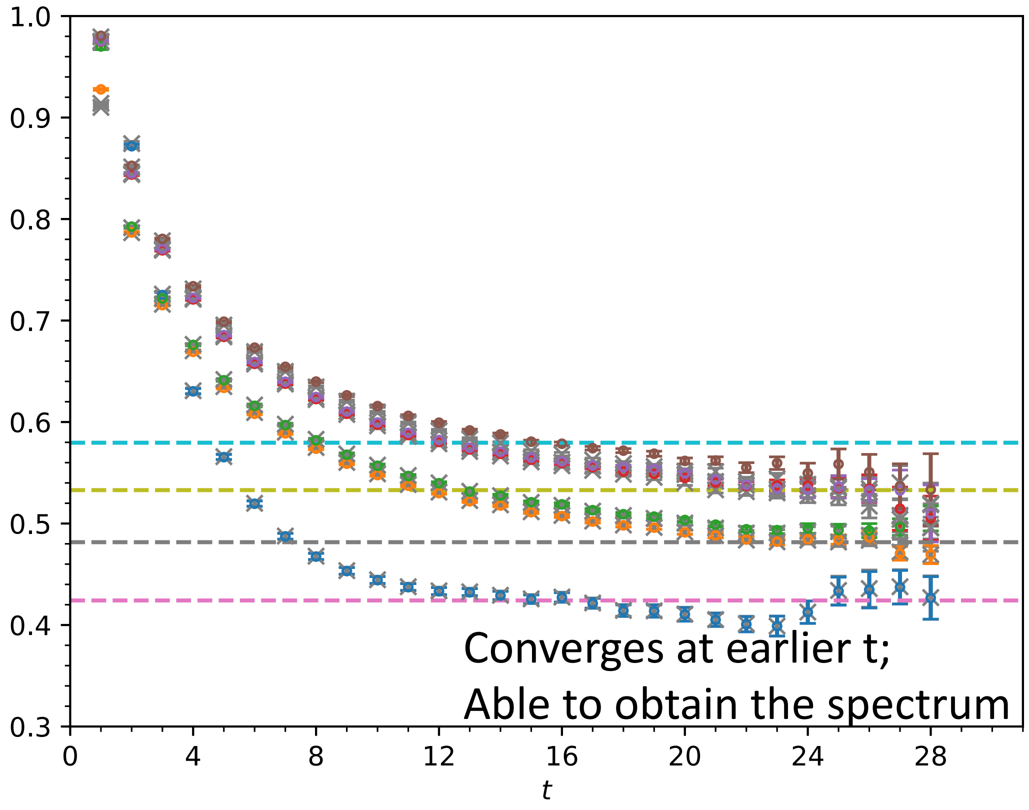
A large operator basis is essential for multi-hadron spectroscopy: $\boldsymbol{N} - \boldsymbol{N}$ Example for deuteron ($I = 0$) at rest



Unimproved nucleon operator

(Noninteracting Spectrum)

N(3)N(-3)
N(2)N(-2)
N(1)N(-1)
N(0)N(0)



Improved nucleon operator obtained by solving the GEVP (Generalized Eigenvalue Problem) over a large operator basis.

[HadSpec Collab., R.Edwards, A.Jackura, SP, C.Thomas]

Summary: take-home messages

- ***Precision nucleon structure***
  High-statistics, physical-pion-mass, fine-lattice calculations are enabling precision nucleon charges, moments, form factors and etc.
- ***Excited states matter***
  Low-lying $N\pi$ states can strongly affect nucleon observables:

  $\sigma_{\pi N}$ from ~40 → ~60 MeV, PCAC violation: ~40% → <4%
- ***Multi-hadron***
  Resolving low-lying $N\pi$ states with multi-hadron operators is an essential step toward quantitative studies of scattering and resonances.
- ***Outlook:***
  Extend this framework toward nucleon resonances and more complex/exotic hadronic systems.

(Since 2024)

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Korean lattice community has seen a significant expansion in recent years!