Effective QCD model with consistent quasi-gluon treatment 2606.09633 [hep-ph]

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Why effective model ?

- The theoretical description based on first principle Quantum chromodynamics (QCD):
  - Lattice QCD
  - Perturbative QCD

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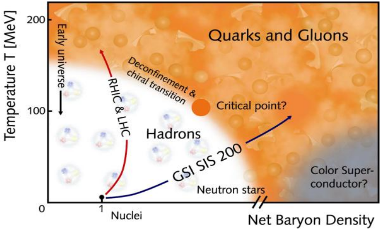

Figure: Source :arXiv:2606.09633 [hep-ph]

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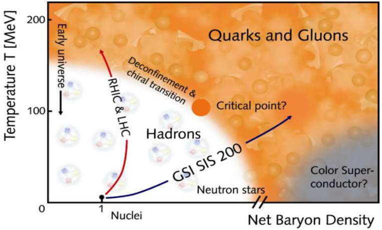
  - Perturbative QCD Phase transition properties are dominantly nonperturbative.

- A large region of QCD phase diagram can be explored in terms of effective models of QCD : Based on global symmetry properties of QCD, Intutive , Mathematically simpler to work with. The results are qualitative and parameter dependent.



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Polyakov Nambu—Jona-Lasinio model

- Model Lagrangian:

$$\mathcal{L}(\psi, \bar{\psi}, \Phi, \bar{\Phi}) = \bar{\psi}(i\not{D} - m_0)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2] - U(\Phi, \bar{\Phi}) \, .$$

- Quark contribution:

$$\mathcal{L}_q = \bar{\psi}(i\not{D} - m_0)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2]$$

  - $m_0 \equiv$ explicit chiral symmetry breaking $\rightarrow m_\pi \neq 0$.
  - Model parameters: m_0, G and 3 momentum cutoff Λ are fitted to reproduce vacuum physics.

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- Quarks are coupled to the temporal background gauge field:

$$D^\mu = \partial^\mu - iA^\mu \text{ and } \; A^\mu = \delta_{\mu 0}A^0 \,.$$

- Traced Polyakov loop acts as an order parameter for the confinement de-confinement phase transition:

$$\Phi = \frac{1}{3}\mathrm{Tr}_c\, L(x), \quad L(x) = \mathcal{P}\exp\left[-i\int_0^\beta d\tau A_4(\boldsymbol{x},\tau)\right]$$

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- Polyakov contribution :

$$U(\bar{\Phi},\Phi) = -\frac{\alpha_1(T)}{2}\Phi\bar{\Phi} - \frac{\alpha_2}{6}(\Phi^3+\bar{\Phi}^3) + \frac{\alpha_3}{4}(\Phi\bar{\Phi})^2$$

Thermodynamic properties

- Thermodynamic potential :

$$\Omega = -\frac{T}{V}\ln Z = \Omega_q + U(\bar{\Phi}, \Phi)$$

$$\Omega_q = -4N_cN_f T\int \frac{d^3p}{(2\pi)^3}\ln[1 + e^{-3\beta\varepsilon_q} + N_c(\Phi + \bar{\Phi}e^{-\beta\varepsilon_q})e^{-\beta\varepsilon_q}]$$

$$\frac{\sigma^2}{2G} - 6N_f\int_0^\Lambda \varepsilon_p$$

- $\varepsilon_q = \sqrt{p^2 + m^{*2}}$, where m^* is constituent quark mass

$$m^* = m_0 - G\langle\bar{\psi}\psi\rangle = m_0 + \langle\sigma\rangle$$

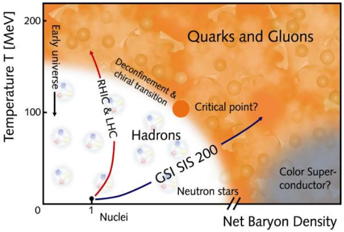
- Φ, $\bar{\Phi}$ provides statistical suppression to quark degrees of freedom as $\langle\Phi\rangle, \langle\bar{\Phi}\rangle \to 0$.

Saddle point approximation

- Chiral symmetry breaking $\implies$ dynamically generated quark mass $m^* \neq m_0 \implies \langle\sigma\rangle \neq 0.$
- Deconfinement $\implies \langle\Phi\rangle, \langle\bar{\Phi}\rangle \neq 0$
- Saddle point equations :

$$\left.\frac{\partial\Omega}{\partial\sigma}\right|_{\sigma=\sigma_{\rm mf}} = 0 \, ; \quad \left.\frac{\partial\Omega}{\partial\Phi}\right|_{\Phi=\Phi_{\rm mf}} = 0 \, ; \quad \left.\frac{\partial\Omega}{\partial\bar{\Phi}}\right|_{\bar{\Phi}=\bar{\Phi}_{\rm mf}} = 0 \, .$$

- The thermodynamic quantities , quark distribution functions f_q can be obtained from $\Omega(\sigma_{\rm mf}, \Phi_{\rm mf}, \bar{\Phi}_{\rm mf})$.
- No dynamical information of gluonic sector can be obtained from the model.



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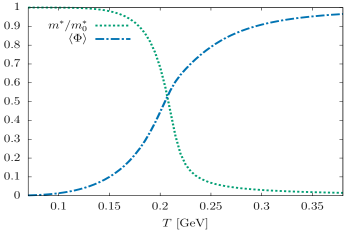

Transport Coefficients

- For a system of quasiparticles the transport coefficients of shear (η) and bulk (ζ) can be obtained from the relaxation time approximation (r = 1 for fermions and -1 for bosons) Kapusta et al. Phys. Rev. C 83, 014906 (2011)

$$\eta = \frac{2d\beta}{15}\int \frac{d^3p}{(2\pi)^3}\tau\left(\frac{|\vec{p}|^2}{\varepsilon}\right)^2 f(1-rf),$$

$$\zeta = 2d\beta\int \frac{d^3p\tau}{(2\pi)^3}\left(\frac{\left(\frac{1}{3}-v_s^2\right)|\vec{p}|^2 - v_s^2\frac{d}{d\beta^2}(\beta^2 m(T)^2)}{\varepsilon}\right)^2 f(1-rf),$$

- The quasiparticle distribution functions f_q and the constituent quark mass m^* the speed of sound v_s^2 can be obtained from the partition function using the saddle point approximation.
- The relaxation time can be obtained from the interaction information available from the Lagrangian.

Why gluon quasiparticle description is required?

- Transport coefficients estimated from chiral effective model description miss out on explicit contribution from the gluonic sector.

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- Transport coefficients estimated from chiral effective model description miss out on explicit contribution from the gluonic sector.

- Lattice calculation of QCD equation of states in presence of finite Ω is restricted small values due to sign problem.
- The difference between the Lattice results and effective model outcome for phase diagram at finite Ω can be attributed to the gluonic contribution.

Chernodub et al 10.1103/PhysRevD.110.014511

- Incorporating the rotational effects on the gluonic sector is beyond the scope of present Polyakov chiral effective model set up.

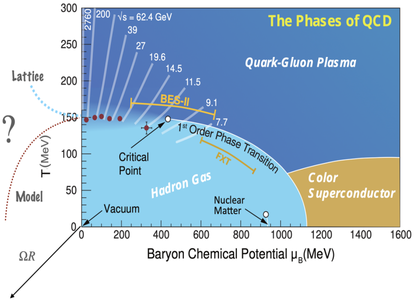


Figure: Sources : $T - \mu_B$ pd : arXiv:2606.09633 [hep-ph] Lattice result : Chernodub et al 10.1103/PhysRevD.110.014511 Model result : PS, V.E. Ambrus, M. N. Chernodub Phys.Rev.D 110 (2024) 9, 094053

Gluon quasiparticle model

- The thermodynamic potential : Sasaki et al. PhysRevD.86.014007 (2012)

$$\Omega_{\rm gqp} = 2T\int\frac{d^3p}{(2\pi)^3}\ln\det\left(1-\hat{L}_A e^{-\frac{|\vec{p}|}{T}}\right) = 2T\int\frac{d^3p}{(2\pi)^3}\ln\left(1+\sum_{n=1}^{8}a_n e^{-\frac{n|\vec{p}|}{T}}\right),$$

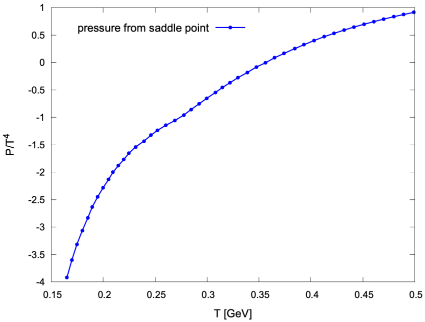
- The coefficients can be written in terms of the Polyakov loop as

$$\begin{aligned}
a_8 &= 1;\quad a_1 = a_7 = 1-9\bar{\Phi}\Phi;\\
a_2 &= a_6 = 1-27\bar{\Phi}\Phi+27(\bar{\Phi}^3+\Phi^3);\\
a_3 &= a_5 = -2+27\bar{\Phi}\Phi-81(\bar{\Phi}\Phi)^2;\\
a_4 &= 2[-1+9\bar{\Phi}\Phi-27(\bar{\Phi}^3+\Phi^3)+81(\bar{\Phi}\Phi)^2]\,.
\end{aligned}$$

- In Saddle point approximation pure gauge thermodyamics and gluon distribution function can be obtained from $\Omega_{\rm gqp}(\Phi_{\rm mf},\bar{\Phi}_{\rm mf})$.

Integral approach

- The saddle point approximation of the quasiparticle model leads to unphysical thermodynamics. Sasaki et al. PhysRevD.86.014007 (2012)



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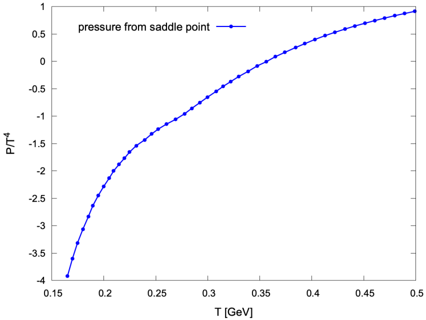
$$Z_g = \int \prod_{\mathbf{x}} \frac{1}{24\pi^2} d\theta_1(\mathbf{x}) d\theta_2(\mathbf{x}) \mathrm{Det}_{\mathrm{VDM}}$$

$$\exp\left[-\frac{1}{T}\int d^3x \Omega_{\mathrm{gqp}}[\theta_1(\mathbf{x}), \theta_2(\mathbf{x}),]\right].$$



- The Vandermonde determinant term :

$$\mathrm{Det}_{\mathrm{VDM}} = 27[1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2].$$

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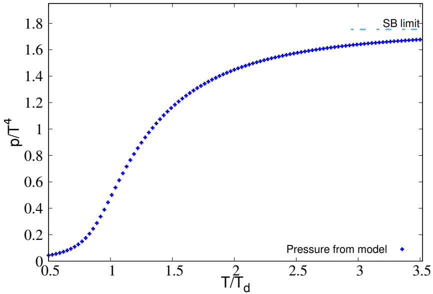

- Split the configuration space into $N \to \infty$ independent and equivalent points such that $Z_g = z_g^N$

$$z_g = \int \frac{1}{24\pi^2} d\theta_1 d\theta_2 \mathrm{Det}_{\mathrm{VDM}} \exp\left[-\frac{v}{T}\omega_{gqp}[\theta_1, \theta_2]\right]$$

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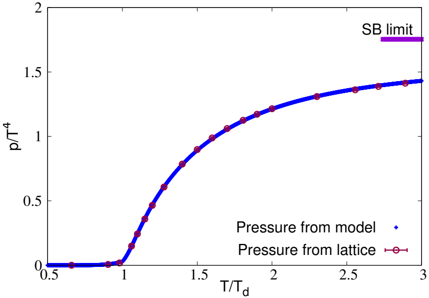


$$z_{\mathrm{g}} = \int \frac{1}{24\pi^2} d\theta_1 d\theta_2 \mathrm{Det}_{\mathrm{VDM}} \exp\left[-\frac{v}{T}\omega_{gqp}[\theta_1, \theta_2]\right]$$

Introducing gluon mass:

$$m_g/T = a + b/\log(cT/T_d) \quad \text{for } T > T_d;$$
$$= d(T_d/T)^2 \quad \text{for } T \le T_d;$$

Coupling to the quark sector

- We will consider the integral approach for the Polyakov sector while keeping the saddle point approximation for σ.
- Partition function can be written as $Z_{\text{PNJL}}[\sigma] = z_{\text{PNJL}}^N[\sigma]$,

$$z_{\text{PNJL}}[\sigma] = \int \frac{1}{24\pi^2} d\theta_1 d\theta_2 \text{Det}_{\text{VDM}} \exp\left[-\frac{v}{T}\omega_{\text{PNJL}}[\theta_1, \theta_2, \sigma]\right].$$

- The order parameters are obtained as,

$$\left.\frac{\partial \omega_{\text{PNJL}}}{\partial \sigma}\right|_{\sigma=\sigma_{\text{mf}}} = 0;$$

$$< O[\Phi, \bar{\Phi}] > = \frac{1}{z} \int \frac{1}{24\pi^2} d\theta_1 d\theta_2 \text{Det}_{\text{VDM}} O[\Phi, \bar{\Phi}] \exp\left[-\frac{v}{T}\omega[\theta_1, \theta_2, \sigma_{\text{mf}}]\right].$$

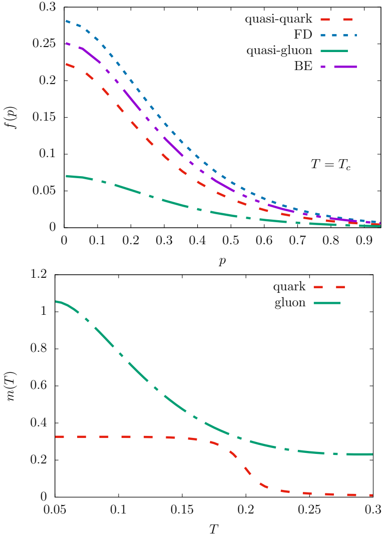
- All thermodynamic quantities can be obtained from $p = -\omega_{\text{PNJL}}[\sigma_{\text{mf}}] = -(T/v)\ln z_{\text{PNJL}}$.

Application

- Distribution functions:

$$f_g(p_1,T) = -\frac{2}{d_g 24\pi^2 z}\int d\theta_1 d\theta_2 \mathrm{Det}_{\mathrm{VDM}} \mathrm{e}^{-\frac{v}{T}\omega[\theta_1,\theta_2,\sigma_{\mathrm{mf}}]} \times \frac{\sum_{n=1}^{8} n a_n \exp\left[-\frac{n\varepsilon_g(p_1)}{T}\right]}{1+\sum_{n=1}^{8} a_n e^{-\frac{n\varepsilon_g(p_1)}{T}}},$$

$$f_q(p_1,T) = \frac{4N_f N_c}{d_q 24\pi^2 z}\int d\theta_1 d\theta_2 \mathrm{Det}_{\mathrm{VDM}} \mathrm{e}^{-\frac{v}{T}\omega[\theta_1,\theta_2,\sigma_{\mathrm{mf}}]} \times \frac{e^{-3\frac{\varepsilon_q(p_1)}{T}} + \left(\Phi + 2\bar{\Phi} e^{-\frac{\varepsilon_q(p_1)}{T}}\right) e^{-\frac{\varepsilon_q(p_1)}{T}}}{1+e^{-3\frac{\varepsilon_q(p_1)}{T}} + N_c\left(\Phi + \bar{\Phi} e^{-\frac{\varepsilon_q(p_1)}{T}}\right) e^{-\frac{\varepsilon_q(p_1)}{T}}}$$



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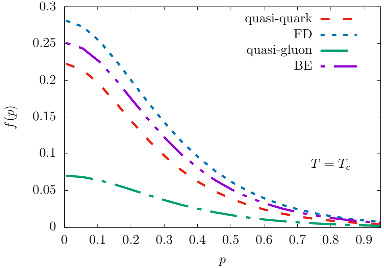
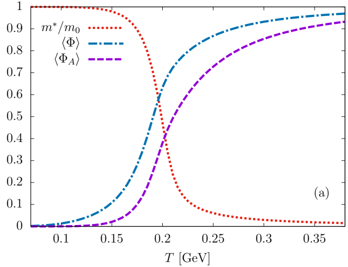
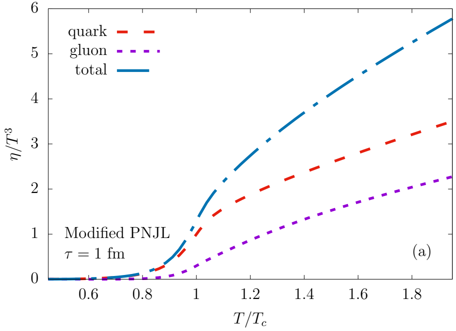
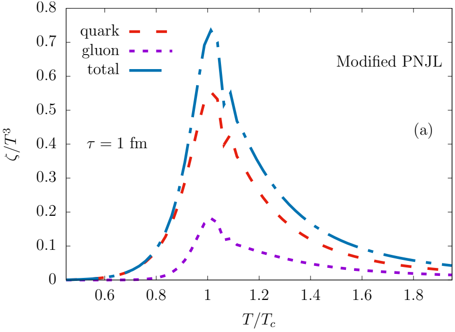




Transport coefficients



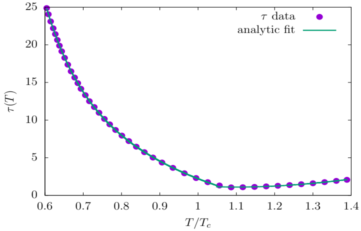


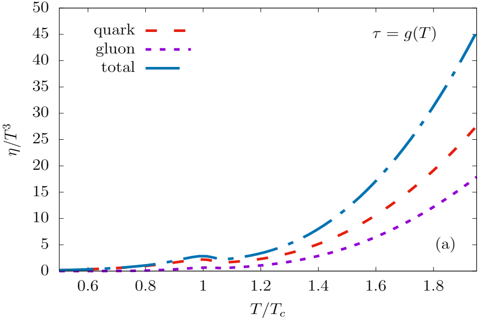
- Gluonic contributions are suppressed due to the effect of adjoint Polyakov loop and higher mass.
- For bulk viscosity the result depends on the prefactor,

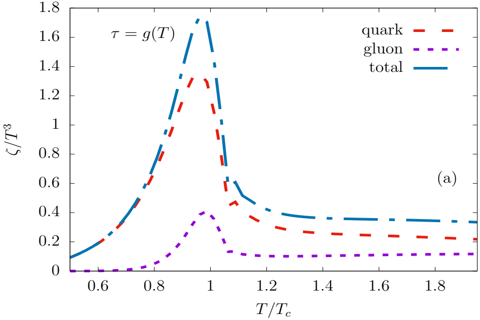
$$\zeta = 2d\beta \int \frac{d^3 p \tau}{(2\pi)^3} \left(\frac{\left(\frac{1}{3} - v_s^2\right) |\vec{p}|^2 - v_s^2 \frac{d}{d\beta^2} (\beta^2 m(T)^2)}{\varepsilon} \right)^2 f(1 - rf),$$

Transport Coefficients

- For simplicity we took a generic τ calculated withing the NJL model Deb et. al Phys. Rev. D 94, 094002 (2016).
- At low temperature the confinement effect provided by the Polyakov loop dominates.







Comparison with existing models

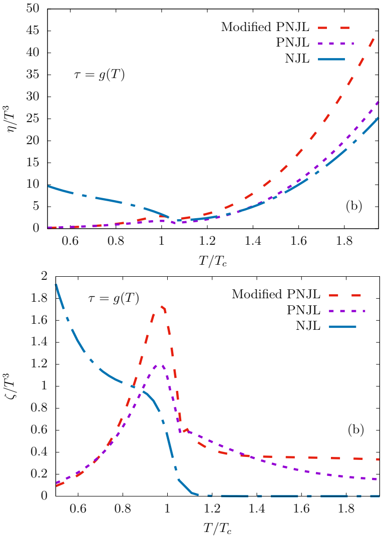
- For Usual PNJL model

$$f_q(p_1, T) = \frac{e^{-3\frac{\varepsilon q(p_1)}{T}} + (\Phi + 2\bar{\Phi}e^{-\frac{\varepsilon q(p_1)}{T}})e^{-\frac{\varepsilon q(p_1)}{T}}}{1 + e^{-3\frac{\varepsilon q(p_1)}{T}} + N_c(\Phi + \bar{\Phi}e^{-\frac{\varepsilon q(p_1)}{T}})e^{-\frac{\varepsilon q(p_1)}{T}}}$$

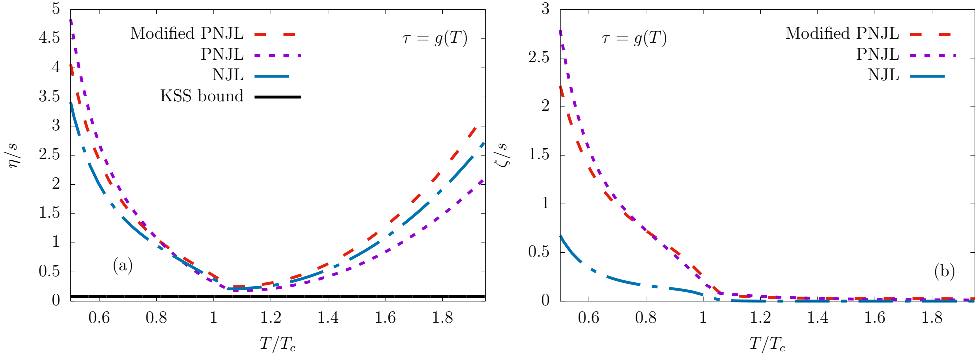
- For NJL Fermi-Dirac distribution with $m^* = m_0 + \sigma_{\rm mf}$ obtained from

$$\left.\frac{\partial \Omega_q^{\Phi,\bar{\Phi}\to 1}}{\partial \sigma}\right|_{\sigma=\sigma_{\rm mf}} = 0,$$

- As there is no Polyakov effect the temperature dependence of τ is reflected in the results of NJL model.



Transport coefficents

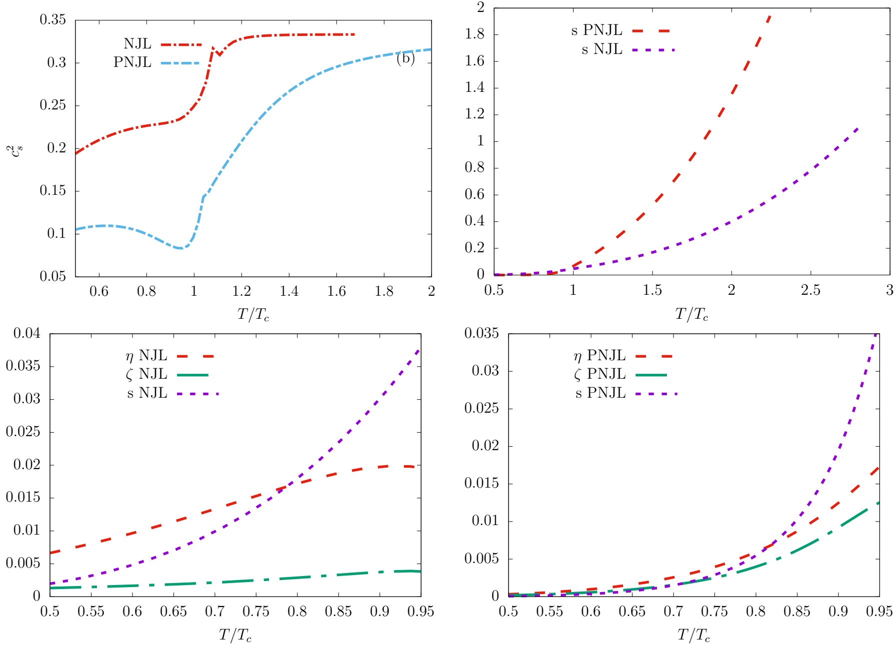


- The minima in η/s is at a temperature > T_c which is a direct consequence of the τ considered.
- The statistical suppression due to the presence of Polyakov loop is nullified by the entropy.
- The peak structure in the bulk viscosity completely vanishes.

Discussion

- We have obtained a physically consistent quasiparticle model for 2 flavor QCD where gluon and quarks are both coupled to background Polyakov field.
- Present work can be further modified by studying the 2+1 flavor QCD and and using the continuum limit lattice data to obtain temperature dependent gluon mass.
- This model enables us to calculate the gluonic contribution to the medium interaction. One can obtain the relaxation time τ incorporating those effects which will significantly modify the estimation of transport coefficients within this model framework.
- One can calculate the effect of the external parameters such as rotation on the gluonic sector of the medium.

Thank You For Your Attention!!


NJL
PNJL
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c_s^2
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