Local Thermalization of $\mathrm{SU}(2)$ Lattice Gauge Fields on Quantum Computers

Ghanashyam Meher

National Taiwan University

Based on arXiv:2603.23948v4

Presented at: Taiwan-Korea Joint Workshop on Nuclear Physics 2026

Organized by IoP, Academia Sinica

September 17, 2026

Collaborators

Prof. Jiunn-Wei Chen

National Taiwan University

Yu-Ting Chen

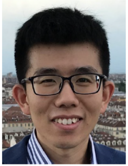
National Taiwan University

Prof. Berndt Müller

Duke University

Prof. Andreas Schäfer

Universität Regensburg



Dr. Xiaojun Yao

University of Washington

Contents

1. Thermalization and ETH
2. 2+1D square plaquette chain in SU(2) LGT
3. Hamiltonian of 1D spin chain
4. Quantum Circuit for Time Evolution
5. Reduced Density Matrix and Rényi-2 Entropy of a Subsystem
6. Results
7. Summary and Next Goal

Thermalization: The Puzzle

- **Thermalization:** Local observables approach equilibrium values (thermal value) with small fluctuations at late times.
- **Classical systems:** Thermalization arises from **collisions and randomness**.
- **Quantum systems:** Time evolution is **unitary** and **reversible** $\Rightarrow$ information is preserved (entropy is zero)

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle,$$

- **Key question:** How can an **isolated quantum system** appear thermal?
- **Example (Heavy-ion collisions):** Single-hadron energy spectrum look **thermal** at very early times

Eigenstate Thermalization Hypothesis (ETH)

☞ **Answer:** Thermal behavior emerges from **entanglement growth**.

☞ **Eigenstate Thermalization Hypothesis (ETH):** Local observables in energy eigenstates match thermal values.

$$\langle E_n|\mathcal{O}_{\rm local}|E_n\rangle \simeq \langle \mathcal{O}_{\rm local}\rangle_{\rm micro}(E).$$

☞ **Physical picture:**

- Total system: **pure quantum state**
- Subsystem: **Mixed** and can appear **thermal** due to entanglement
- Local observables $\rightarrow$ **thermal values** or **micro-canonical value**

☞ **Motivation:** Real-time dynamics of non-Abelian gauge theories are **hard for classical computation**.

☞ **This work:** Growth of Rényi-2 entropy $\Rightarrow$ probe of local thermalization

2+1D square plaquette chain in SU(2) LGT

- Study Rényi-2 entropy of 2+1D chain of square plaquettes in SU(2) lattice gauge theory

- Each plaquette forms a **closed loop of gauge links**

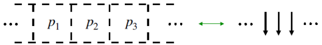
- Each link characterized by SU(2) representation **(truncated Hilbert Space):**

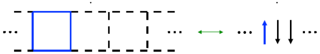
$$j = 0, \tfrac{1}{2} \quad (j_{\max} = \tfrac{1}{2})$$

- **Spin mapping:**

$$j = 0 \;\rightarrow\; |\downarrow\rangle, \quad j = \tfrac{1}{2} \;\rightarrow\; |\uparrow\rangle$$

- **Plaquettes chain $\Rightarrow$ Effective 1D spin chain**





Hamiltonian of 1D spin chain

- Hamiltonian:

$$\begin{aligned}
\mathcal{H}_{\text{sc}} &= J\sum_{i=0}^{N-2} Z_i Z_{i+1} + h_z \sum_{i=0}^{N-1} Z_i + \frac{h_x}{16}\sum_{i=0}^{N-1} X_i \\
&\quad -\frac{3h_x}{16}\sum_{i=0}^{N-2}[Z_i X_{i+1} + X_i Z_{i+1}] + \frac{9h_x}{16}\sum_{i=0}^{N-3} Z_i X_{i+1} Z_{i+2}\,, \qquad (1)\\
\mathcal{H}_{\text{sc}} &\equiv H_{ZZ} + H_Z + H_X + H_{ZX} + H_{XZ} + H_{ZXZ}\,. \qquad (2)
\end{aligned}$$

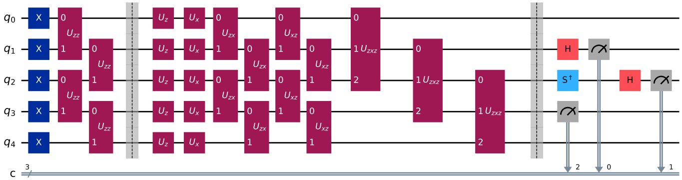
- We begin with strong-coupling vacuum state

$$|\psi(t=0)\rangle = |\downarrow\rangle^{\otimes N} \equiv |1\rangle^{\otimes N}. \qquad (3)$$

- Pure state: No entanglement at beginning.

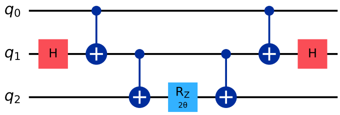
$$|\psi(t)\rangle = \left(\prod_{n=1}^{t/\delta t} e^{-i\,\mathcal{H}_{\text{sc}}\,\delta t}\right)|\psi(0)\rangle \qquad (4)$$

Quantum Circuit for Time Evolution



☞ $U_z = R_Z(\theta, i)$, and
$U_x = R_X(\theta, i)$,

☞ $U_{zz} = R_{ZZ}(\theta, i, i+1)$,
$U_{zx} = R_{ZX}(\theta, i, i+1)$, and
$U_{xz} = R_{ZX}(\theta, i+1, i)$



☞ $U_{zxz} = e^{-i\theta Z_i X_{i+1} Z_{i+2}}$

Reduced Density Matrix and Rényi-2 Entropy of a Subsystem

☞ Reduced density matrix of a subsystem A of size k:

$$\rho_A \quad = \quad \frac{1}{2^k} \sum_{P_A \in \{I,X,Y,Z\}^k} \langle \psi(t) | P_A \otimes I_{\bar{A}} | \psi(t) \rangle P_A \,, \tag{5}$$

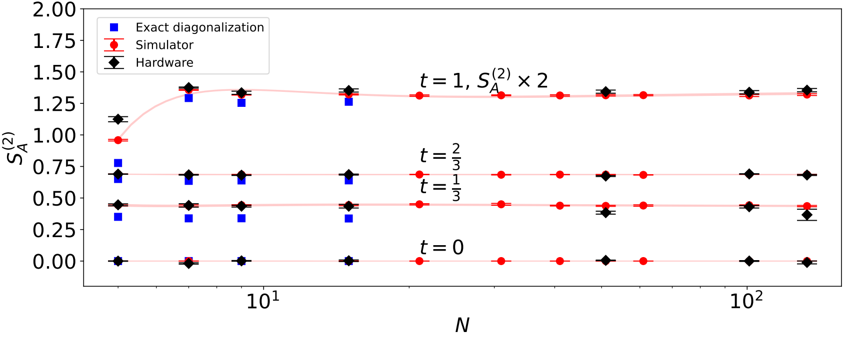
☞ The Rényi-2 entropy:

$$S_A^2 = -\log(\mathrm{Tr}[\rho_A^2]) = -\log\left(\frac{1}{2^k} \sum_{P_A \in \{I,X,Y,Z\}^k} \langle \psi(t) | P_A | \psi(t) \rangle^2 \right), \tag{6}$$

☞ $\langle P_A \rangle$ by changing measurement axis as $X \to Z = HXH$ and $Y \to Z = HS^\dagger YSH$,

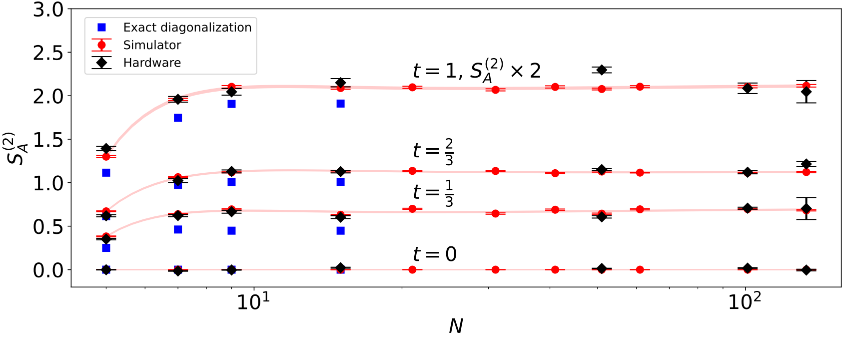
☞ The Rényi-2 entropy is obtained for subsystem size $N_A = 1, 2$, and 3 for various system size N

Results For $N_A = 1$



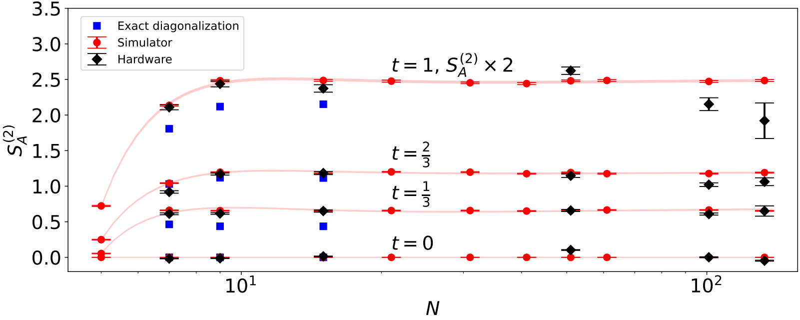
- Classical simulation (20K shots) and quantum hardware (4K shots)
- Classical data fitted with $S_A^{(2)} = a + \frac{b}{N} + \frac{c}{N^2} + \frac{d}{N^3}$
- Good agreement between quantum hardware and classical results

Result For $N_A = 2$



☞ Similar behavior observed as in $N_A = 1$.

Result For $N_A = 3$



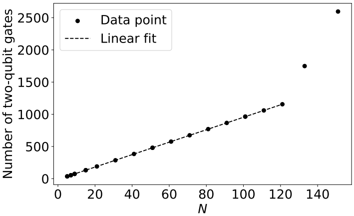
- ☞ $N \geq 101$: deviate from classical simulation.
- ☞ $N = 133$: **Larger error bars**, reason is **break down of linear connectivity** between nearest neighbors → larger 2-qubits gate depth

Error mitigation techniques used:

- **1.** Dynamical Decoupling **(DD)** **2.** Pauli Twirling **(PT)** **3.** Operator Decoherence Renormalization **(ODR)**

Circuit Depth and Two-Qubit Gate Counts

System size	Total depth	Two-qubit depth	Two-qubit gate count
$N = 5$	94	24	36
$N = 7$	92	24	54
$N = 9$	95	24	74
$N = 15$	95	24	132
$N = 51$	96	24	480
$N = 101$	96	24	964
$N = 133$	394	149	1749
$N = 151$	523	198	2596



2-qubit gate counts vs **N**

Key observation:

The circuit depth remains nearly constant up to $N = 101$. For $N \geq 133$, **break down of linear connectivity** introduces additional routing, substantially increasing both the depth and gate count.

Qubits Layout of ibm_aachen

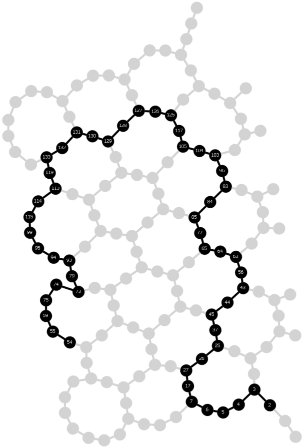


Figure: N=51

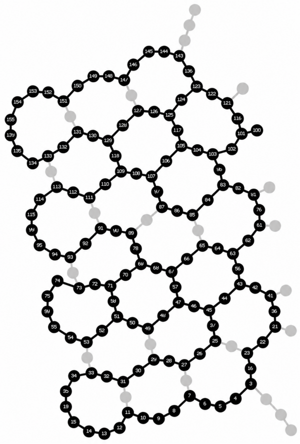
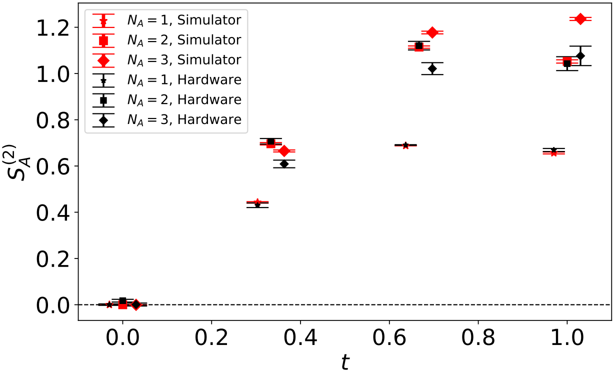


Figure: N=133

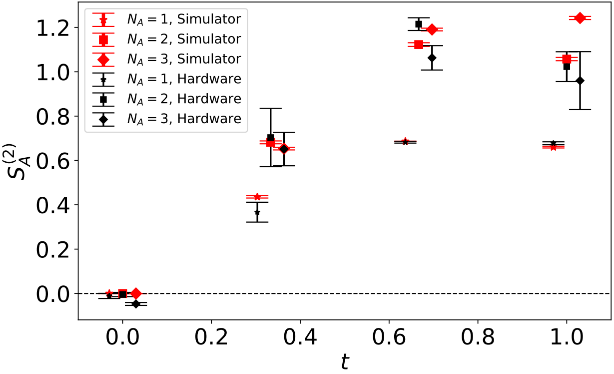
☞ Breakdown of linear connectivity between the nearest qubits for $N = 133$

Results for N=101



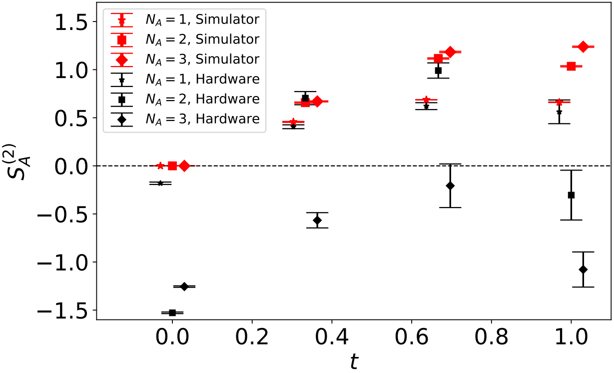
- $S_A^{(2)}$ measured at $t = 0$, $1/3$, $2/3$, and 1
- Initial product state: $S_A^{(2)}(t = 0) = 0$
- $S_A^{(2)}$ increases and approaches a thermal value after a steady growth ($t \gtrsim 2/3$)
- Statistical uncertainty $\lesssim 2.3\%$, 3%, 4% for $N_A = 1, 2, 3$

Results for N=133



- Similar behavior observed as for $N = 101$
- Good agreement with classical simulation within error bars
- Increased hardware connectivity overhead $\rightarrow$ larger two-qubit gate depth $\rightarrow$ higher errors
- Statistical uncertainty $\lesssim$ 12.3%, 18.5%, 13.5% for $N_A = 1, 2, 3$

Results for N=151



- Large deviations observed for $N_A > 1$ (only $N_A = 1$ remains reliable)
- Scaling to larger systems ($\sim$ 156 qubits) requires more advanced error mitigation

Summary and Next Goal

- Studied **local thermalization** via time evolution of **Rényi-2 entropy**.
- The local thermalization begins to emerge for $t \geq 2/3$.
- Good agreement between **classical simulation** and **quantum hardware**.
- Statistical noise increases for larger subsystems ($N_A \geq 3$) $\Rightarrow$ requires more measurement shots.
- For $N \geq 130$ break down of liner connectivity $\rightarrow$ Increase in gate depth and counts $\rightarrow$ Increase error bar.

☞ **Next goal:** Compute $S_A^{(2)}$ for larger subsystems ($N_A = 8$) using **classical shadow techniques**

Thank you for your attention . . .

Thank You...