Is right-handed current contribution to $\bar{B} \to X_u l\nu$ decays corrected by non-trivial topology in QCD vacuum?

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arXiv:2208.11896 [hep-ph]

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Introduction

Semi-leptonic $\bar{B}$ decays ➡ Determinations of $|V_{cb}|, |V_{ub}|$

$$-\mathcal{L} = \frac{g}{\sqrt{2}}(V_{cb}\bar{c}_L\gamma^\mu b_L + V_{ub}\bar{u}_L\gamma^\mu b_L)W_\mu + \text{H.c.}$$

(1) Exclusive processes — Theory: difficult

$\bar{B} \to D^{(*)}l\nu, \bar{B} \to \pi l\nu, \bar{B} \to \rho l\mu$, etc.

(2) Inclusive processes $\overset{?}{=}$ sum of exclusive decays — Theory: easy?

(quark-hadron duality)

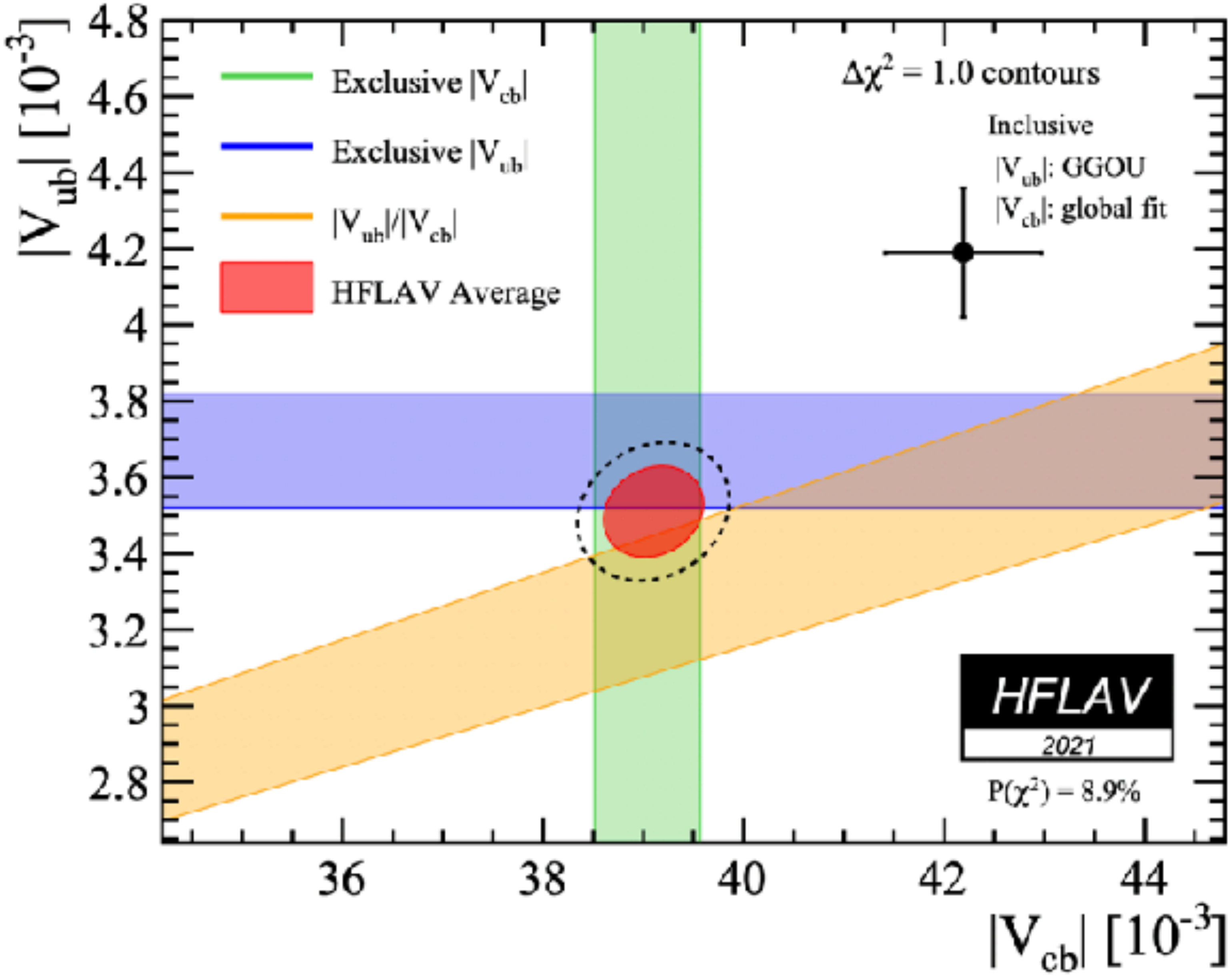
Hadronic final states are not specified.

Operator product expansion (OPE): $\Gamma \propto m_b^5\left(1 + \frac{c_2}{m_b^2} + \frac{c_3}{m_b^3} + \cdots\right)$

$1/m_b$: expansion parameter

Duality violation ⬅ A source of uncertainty in (2)

Determinations of $|V_{cb}|$ and $|V_{ub}|$



Inclusive vs Exclusive

$$\begin{cases} |V_{cb}| : 3.3\sigma \\ |V_{ub}| : 3.3\sigma \end{cases}$$

Implication of new physics?

Crivellin, Pokorski [1407.1320]

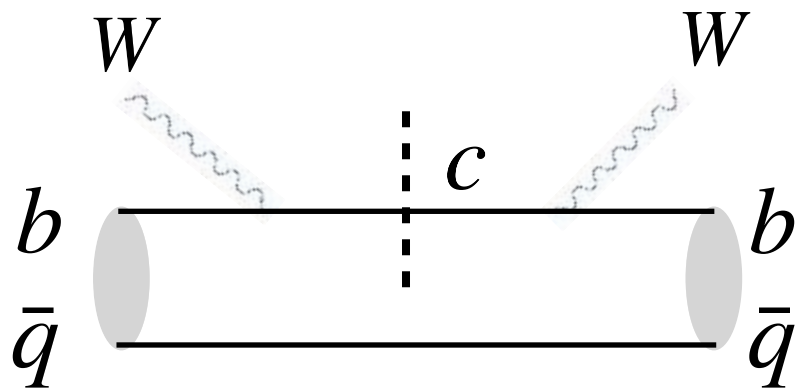
- This interpretation is disfavored, shown in the absence of the light right-handed neutrino.

✔Theoretical uncertainty is underestimated?

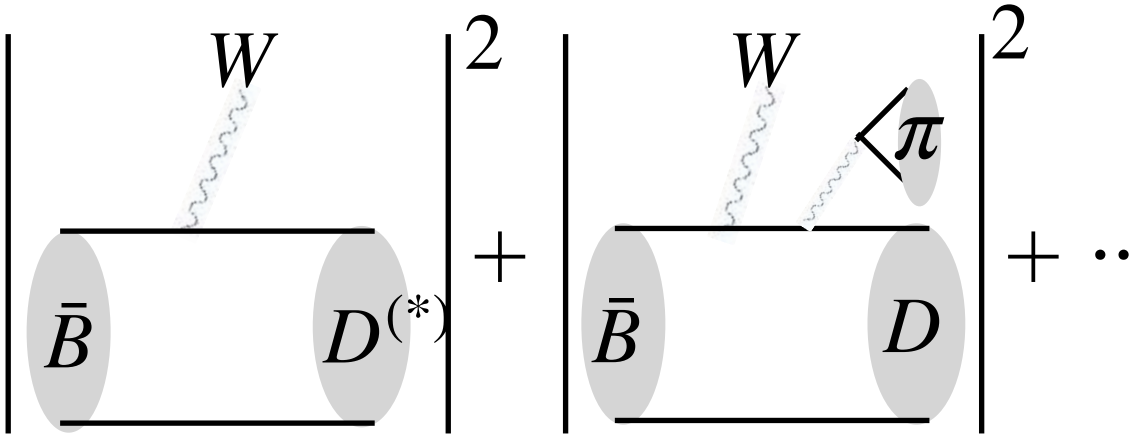
- Duality violation is hard to quantify.

Inclusive processes

Theory (OPE) $\qquad \bar{B} \to X_c l \bar{\nu}$ $\qquad$ **Experiment**



duality



+ QCD corrections, power corrections

Possible sources of errors

◉ Ultimate accuracy of the OPE is limited due to divergences in perturbative series.

(1) Proliferation of Feynman diagrams (2) Renormalons ✔(3) power series

Approaches to duality violation

(1) Resonance-based model

[9510366, 9705390, 9708396, 9805404, 9805241, 9902315, 9903258, 0006346, 0106205, 0112323, 0605248, 2106.06215, 2111.01401]

- The large-N_c limit and linear Regge tragectory are considered as a starting point of discussion.

Pros: Theoretical predictions are rigorous.

Cons: This is mostly a toy model of QCD.

✔(2) Instanton-based model [9605465, 2208.11896]

- This method takes account of a (fixed-sized) background instanton, more or less as an orientational direction to capture contributions related to the factorial divergence in power corrections.

Pros: Some realistic contributions could be examined (not a toy model).

Cons: Detail of the QCD vacuum is involved in a non-trivial way.

(3) Lattice QCD [2005.13730, 2203.11762]

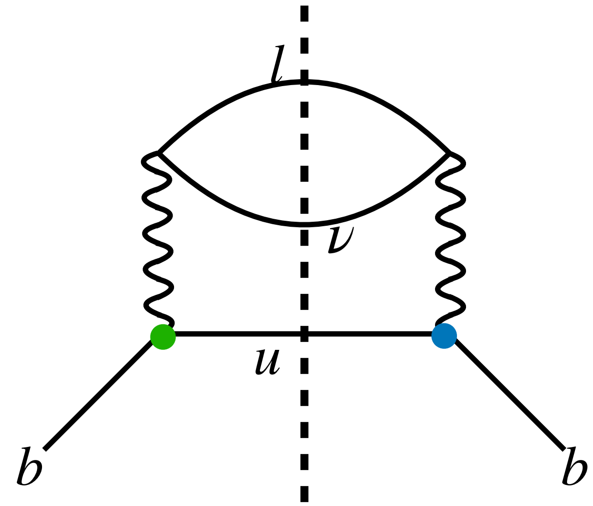
- Reconstruction of the spectral function is considered for inclusive $\bar{B}$ decays.

Pros: It is based on *ab initio* evaluation of the hadronic correlator.

Cons: Specific duality-violating components are suppressed in the Euclid space.

Fourmlas for inclusive width

Partonic decay:
$b \to u l \nu$



$q^\mu = p_l^\mu + p_{\bar{\nu}}^\mu$

$c_\mathrm{L} = 1,\ c_\mathrm{R} = 0$ in the SM

$$\mathscr{H}_\mathrm{eff} = \frac{4G_F}{\sqrt{2}} V_{ub}[c_\mathrm{L}(\bar{u}\gamma^\mu P_\mathrm{L} b) + c_\mathrm{R}\underbrace{(\bar{u}\gamma^\mu P_\mathrm{R} b)}_{\text{Right-handed current (RHC)}}]\bar{l}\gamma_\mu P_\mathrm{L}\nu$$

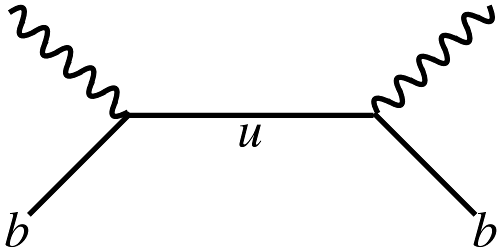
Four contributions: (X, Y) = (L, L), (L, R), (R, L), (R, R)

$$\begin{cases} \Gamma_\mathrm{XY} = -16G_F^2|V_{ub}|^2 \mathrm{Im}(T_\mathrm{XY}^{\mu\nu}L_{\mu\nu}) \\ T_\mathrm{XY}^{\mu\nu} = -i\int \mathrm{d}^4x e^{-iq\cdot x} < \bar{B}_v | J_\mathrm{X}^{\dagger\mu}(x) J_\mathrm{Y}^\nu(0) | \bar{B}_v > \quad \text{Hadronic tensor} \\ L_{\mu\nu} = -\frac{1}{2\pi^4 x^8}(2x_\mu x_\nu - x^2 g_{\mu\nu}) \quad \text{Leptonic tensor} \end{cases}$$

Width: $\Gamma = \underbrace{c_\mathrm{L}^2\Gamma_\mathrm{LL} + c_\mathrm{L}c_\mathrm{R}\Gamma_\mathrm{LR}}_{\text{Discussed in the remaining part of this talk}} + c_\mathrm{R}c_\mathrm{L}\Gamma_\mathrm{RL} + c_\mathrm{R}^2\Gamma_\mathrm{RR}$

For perturbative contributions, LL is chirally-favored while LR is chirally-suppressed by m_u/m_b.

Duality violation in hadronic tensor



Hadronic tensor: $T_{\mu\nu} = \int \mathrm{d}^4 x e^{iQx} S(x) \times (\cdots)$ $\quad S(x) = -\dfrac{\hat{x}}{2\pi^2 x^4}$

$\rho, Q > 0$

$$f(Q) = \int_{-\infty}^{+\infty} \mathrm{d}x \frac{e^{iQx}}{x^2 + \rho^2}$$ Difference: pole at $x^2 = -\rho^2$

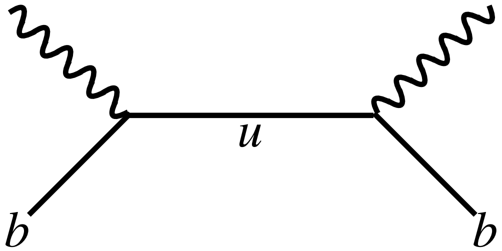
(1) Exact evalution: $f_1(Q) = \dfrac{\pi}{\rho} e^{-Q\rho}$

(2) Short-distance expansion at $x^2 = 0$ first (OPE): $f_2(Q) = 0$

$$\frac{1}{x^2 + \rho^2} = \frac{1}{\rho^2}\left[1 - \frac{x^2}{\rho^2} + \left(\frac{x^2}{\rho^2}\right)^2 - \cdots\right]$$

➡ An error is induced in (2), if there are poles at $x^2 = -\rho^2$.

Duality violation in hadronic tensor



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An example of the pole: instanton $A^a_\mu = \bar{\eta}_{a\mu\nu} \dfrac{(x-z)_\nu \rho^2}{(x-z)^2[(x-z)^2+\rho^2]}$ $\quad z_\mu$: instanton center, ρ : instanton size

propagator: $S(x,y,z) = \displaystyle\sum_{n=0}^{\infty} \frac{\psi_0(x)\psi_0^\dagger(y)}{\lambda_n - m_u} = S_{\mathrm{zm}} + S_{\mathrm{nzm}} + \mathcal{O}(m_u)$

✔zero mode: $S_{\mathrm{zm}} = -\dfrac{\rho^2}{8\pi^2 m_u} \dfrac{\tilde{\not{x}}\gamma_\mu\gamma_\nu\tilde{\not{y}}}{\sqrt{\tilde{x}^2\tilde{y}^2}(\tilde{x}^2+\rho^2)^{3/2}(\tilde{y}^2+\rho^2)^{3/2}} \tau_\mu^- \tau_\nu^+ P_{\mathrm{L}}$ Diakonov, Petrov, 1985

non-zero mode:
$$S_{\mathrm{nzm}} = -\frac{\not{\Delta}}{2\pi^2\Delta^4} \frac{\sqrt{\tilde{x}^2\tilde{y}^2}}{\sqrt{\tilde{x}^2+\rho^2}\sqrt{\tilde{y}^2+\rho^2}} \left[1 + \frac{\rho^2}{\tilde{x}^2\tilde{y}^2}(\tau^-\cdot\tilde{x})(\tau^+\cdot\tilde{y})\right] - \frac{i}{4\pi^2\tilde{x}^2\tilde{y}^2\Delta^2} \frac{\sqrt{\tilde{x}^2\tilde{y}^2}}{\sqrt{\tilde{x}^2+\rho^2}\sqrt{\tilde{y}^2+\rho^2}}$$
$$\times(\tau^-\cdot\tilde{x})\left[\frac{\rho^2}{\tilde{x}^2+\rho^2}\not{\tau}^+(\tau^-\cdot\Delta)P_{\mathrm{R}} + \frac{\rho^2}{\tilde{y}^2+\rho^2}(\tau^+\cdot\Delta)\not{\tau}^- P_{\mathrm{L}}\right](\tau^+\cdot\tilde{y}),$$

$\Delta = x - y$, $\tilde{x} = x - z$, $\tilde{y} = y - z$.

Brown, Carlitz, Creamer, Lee 1978

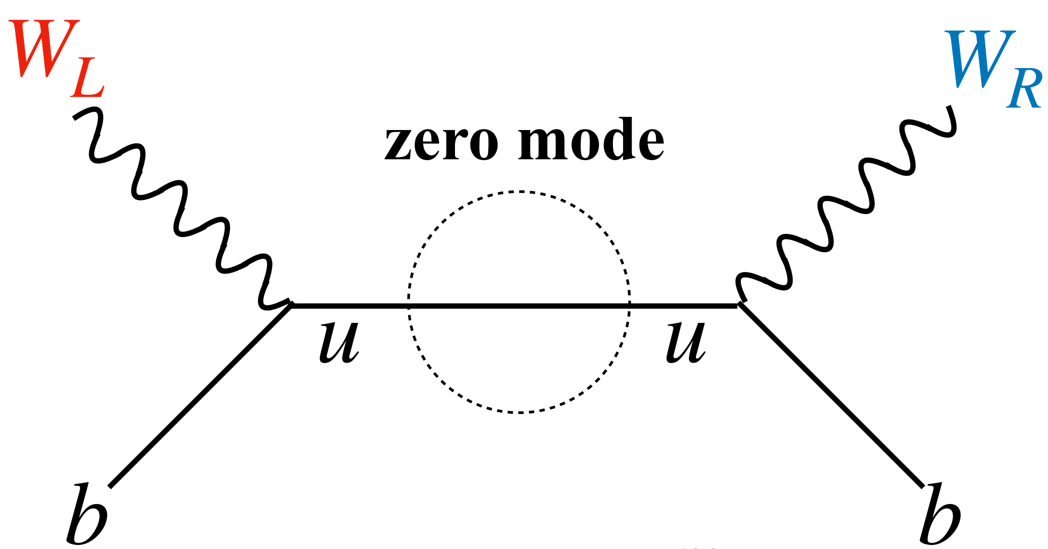
Single instanton approximation (SIA)

- Due to the diluteness of the instanton vacuum, the leading contribution arises from the nearest instanton at the short-distance region.
- Multiple instantons effects are taken into account by,

$$S_{\rm zm} = -\frac{1}{m_u}\psi_0\psi_0^\dagger \to -\frac{1}{m^*}\psi_0\psi_0^\dagger \qquad m^*:\ \text{effective mass}$$

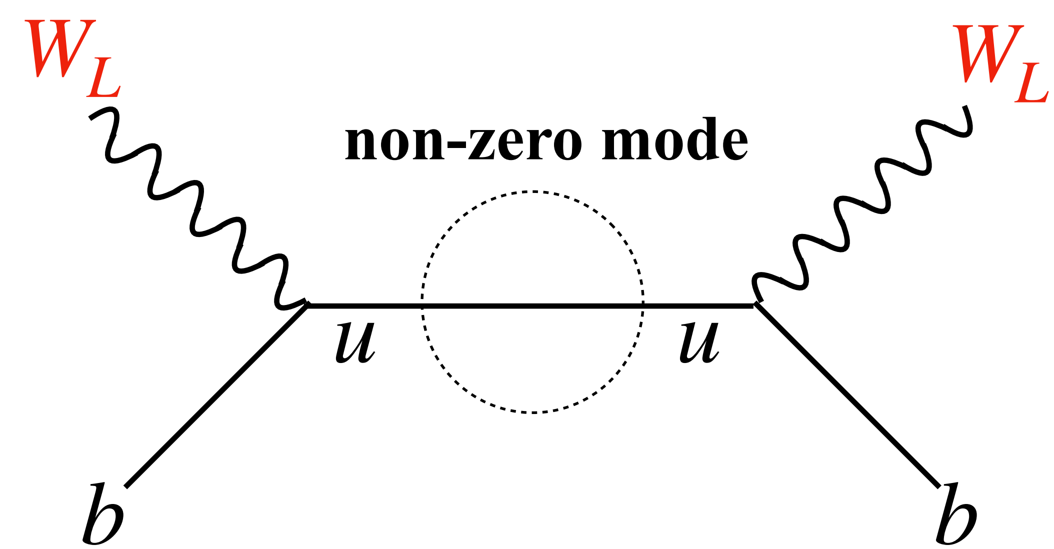
- The pole at $x^2 \simeq -4\rho^2$ is analytically captured.

This work



enhanced by $\dfrac{m_b}{m^*}$

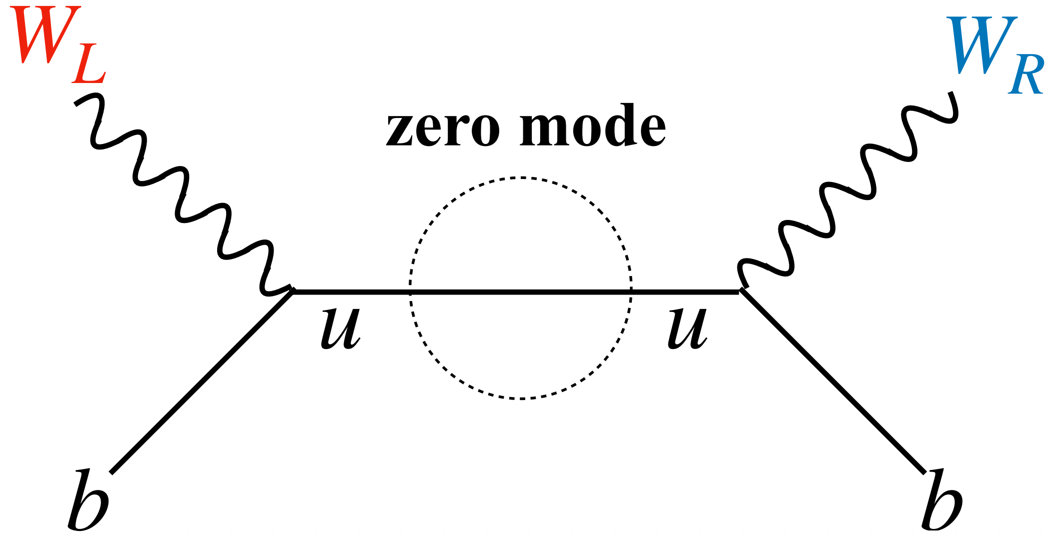
Previous work

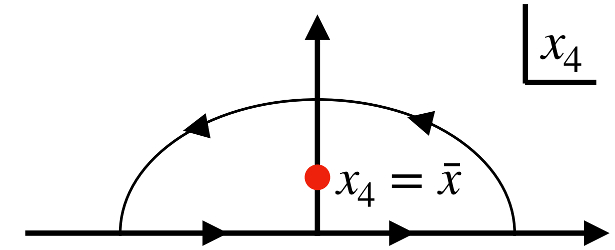


Chibisov, Dikeman, Shifman and Uraltsev [9605465]

Duality violation is tested for $\begin{cases} \text{(1) Total width for semileptonic decay} \\ \text{(2) Lepton energy distribution} \end{cases}$

Width for RHC





$$\bar{x} = \sqrt{\frac{(z-\alpha x)^2+\rho_0^2}{\alpha(1-\alpha)} + \vec{x}^2}$$

$$T_{\mu\nu} = i\int \frac{\mathrm{d}\rho}{\rho^5} d(\rho)\mathrm{d}U\mathrm{d}^4z\mathrm{d}^4x e^{iQ\cdot x}\langle \bar{B}_v|\,\bar{b}_v(x)\gamma_\mu P_\mathrm{L} S_\mathrm{zm}(x,0,z)\gamma_\nu P_\mathrm{R} b_v(0)\,|\bar{B}_v\rangle\,.$$

$$d(\rho) = \frac{n_I\rho^4}{2}\rho\delta(\rho-\rho_0)$$

instanton liquid model
Shuryak (1982)

$$\begin{cases} \rho_0 = \dfrac{1}{3}\mathrm{fm} \\ n_I = 1\ \mathrm{fm}^{-4} \end{cases}$$

$$\int_{-\infty}^{+\infty} dx_4(\cdots)e^{iQ\cdot x}S_\mathrm{zm} \propto \underline{e^{-Q_4\bar{x}}}f(Q_4) \longrightarrow e^{iQ_0\bar{x}}f(-iQ_0)$$

Analytic continuation
$Q_4 = -iQ_0$

The saddle point approximation can be applied to evaluate the other integrals such as the instanton center.

$$\begin{cases} \text{(1) For total semileptonic width:} & Q_\mu = m_b v_\mu \\ \text{(2) For lepton energy distribution:} & Q_\mu = m_b v_\mu - p_l \end{cases}$$

Numerical resutls for width

Ratio: $R^{\rm tot}_{\rm XY}[\,B \to X_u l\nu\,] = \dfrac{\tilde{\Gamma}^{\rm I}_{\rm XY}}{\Gamma^{\rm pert}_{\rm XY}}$ (X, Y) = (L, L), (L, R)

$$\begin{cases} \text{RHC} \quad R^{\rm tot}_{\rm LR} \simeq +\dfrac{2}{3} d_0 \left(\dfrac{m_b^2}{m^* m_u}\right) \dfrac{96\pi}{(m_b\rho_0)^8} \cancel{\cos(2m_b\rho_0)} \\ \text{SM} \quad R^{\rm tot}_{\rm LL} \simeq -\dfrac{2}{3} d_0 \dfrac{96\pi}{(m_b\rho_0)^8} \cancel{\sin(2m_b\rho_0)} \end{cases}$$

$m_b \to m_c, \quad m_u \to m_d$

for $D \to X_d l\nu$

	Mass inputs	(m_b, m^*) [GeV]	$(4.18, 0.120)$	$(4.78, 0.120)$	$(4.18, 0.177)$	$(4.78, 0.177)$
$\bar{B}$ decays	RHC	$R^{\rm tot}_{\rm LR}[\bar{B} \to X_u l\nu]$	2×10^{-2}	7×10^{-3}	1×10^{-2}	5×10^{-3}
	SM	$R^{\rm tot}_{\rm LL}[\bar{B} \to X_u l\nu]$	2×10^{-7}	7×10^{-8}	2×10^{-7}	7×10^{-8}

	Mass inputs	(m_c, m^*) [GeV]	$(1.27, 0.120)$	$(1.67, 0.120)$	$(1.27, 0.177)$	$(1.67, 0.177)$
D decays	RHC	$R^{\rm tot}_{\rm LR}[D \to X_d l\nu]$	7	1	5	1
	SM	$R^{\rm tot}_{\rm LL}[D \to X_d l\nu]$	3×10^{-3}	3×10^{-4}	3×10^{-3}	3×10^{-4}

The importance of duality violation for the RHC is higher than that of the SM.

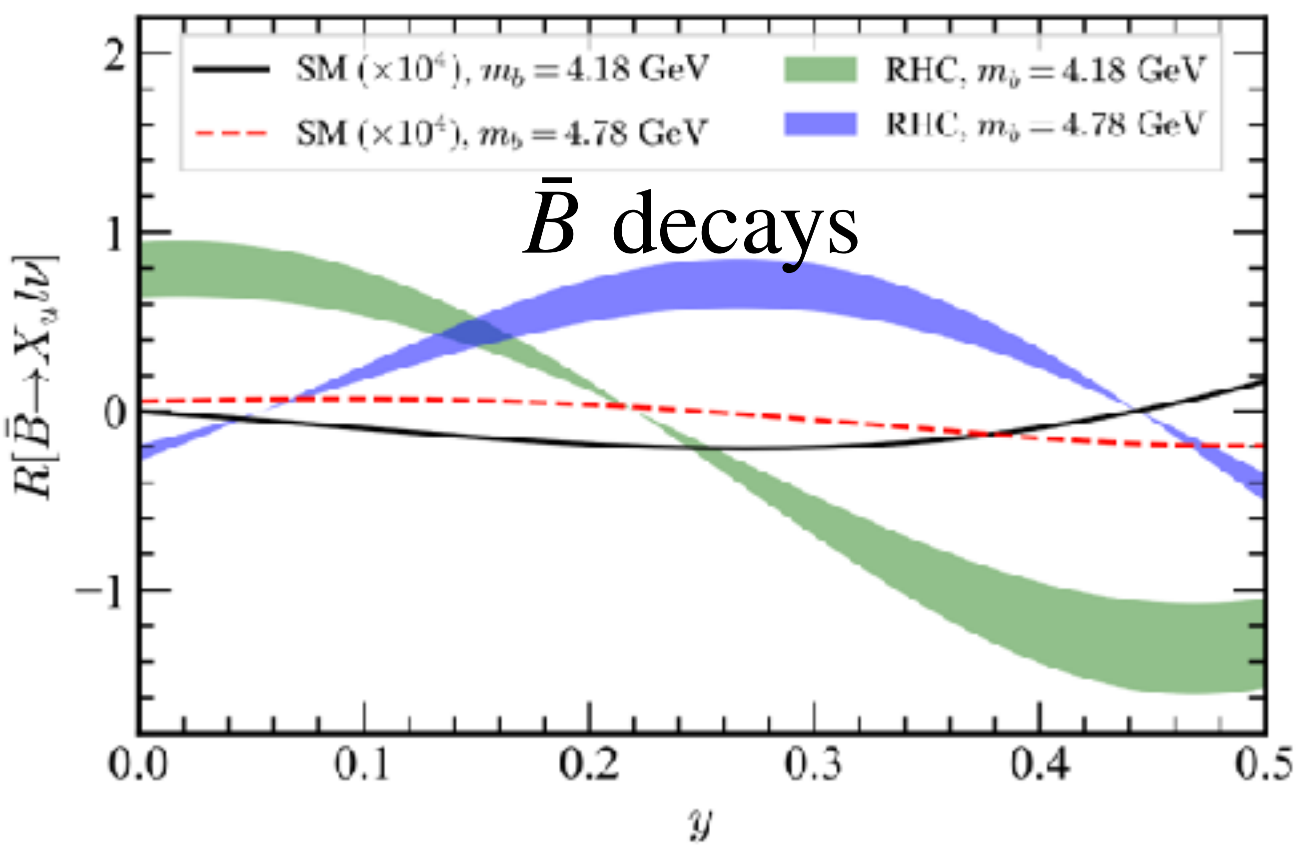
Numerical results for lepton energy spectra

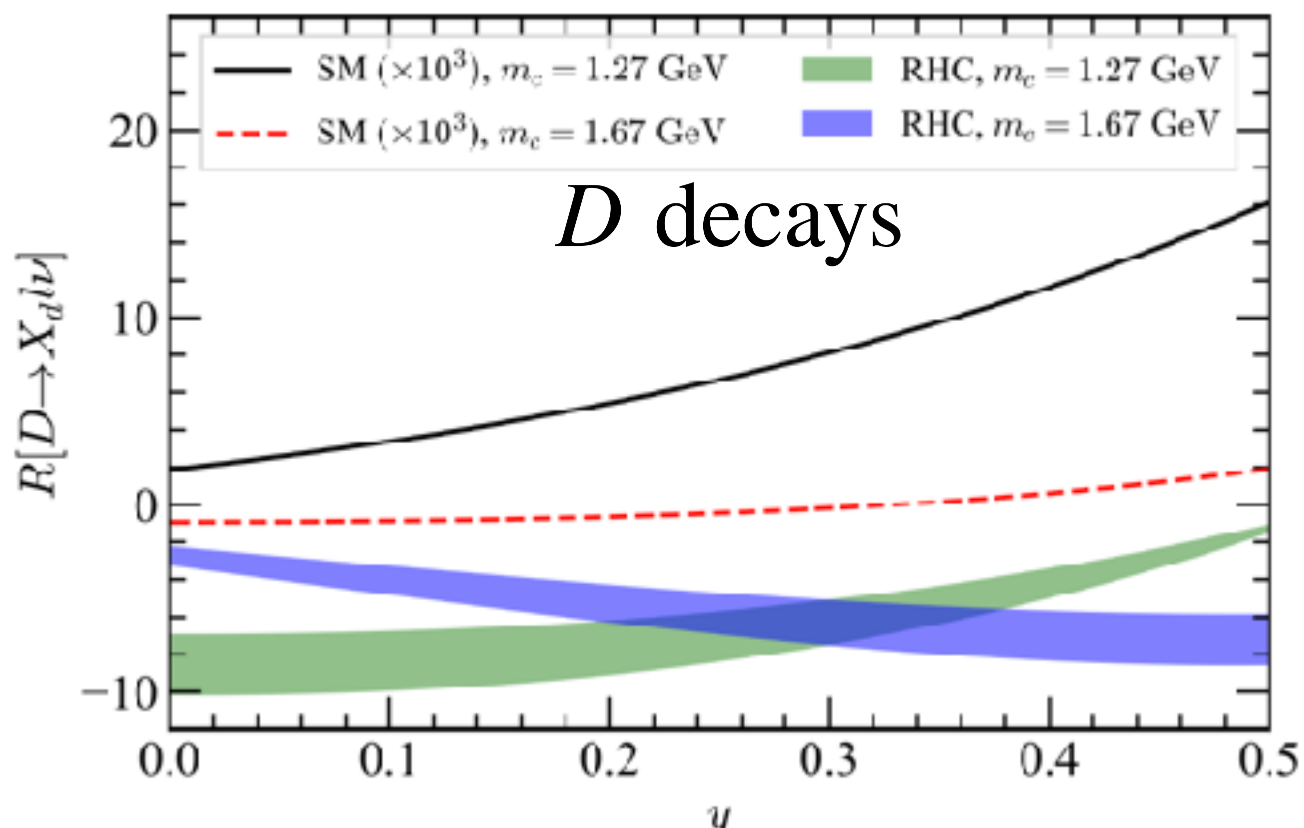
Ratio: $R_{\rm XY}[B \to X_u l\nu\ ] = \dfrac{(\mathrm{d}\Gamma/\mathrm{d}y)|^{\rm I}_{\rm XY}}{(\mathrm{d}\Gamma/\mathrm{d}y)|^{\rm pert}_{\rm XY}}$ $(\mathrm{X},\mathrm{Y}) = (\mathrm{L},\mathrm{L}), (\mathrm{L},\mathrm{R})$

$y = 2E_l/m_b$: dimensionless lepton energy

RHC $R_{\rm LR} \simeq +\dfrac{2}{3} d_0 \dfrac{m_b}{m^*} \dfrac{16\pi}{(m_b\rho_0)^5} \dfrac{1}{\left(1-\frac{y}{2}\right)^2} \sin\left(2m_b\rho\left(1-\dfrac{y}{2}\right)\right)$, $120\ \mathrm{MeV} \le m^* \le 177\ \mathrm{MeV}$

SM $R_{\rm LL} \simeq -\dfrac{2}{3} d_0 \dfrac{48\pi}{(m_b\rho_0)^5} \dfrac{1}{\left(1-\frac{y}{2}\right)^3 (3-2y)} \cos\left(2m_b\rho\left(1-\dfrac{y}{2}\right)\right)$





Conclusion

- We have studied an instanton-induced correction to $\bar{B} \to X_u l\nu$ decays, finding that the patterns of duality violation depend on the effective operators.

- The zero-mode induced duality violation gives rise to the chirally-favored contribution in the presence of the RHC.

- Within the SIA for the RHC, the duality-violating component of the width is at most $\mathcal{O}(10^{-2})$ and $\mathcal{O}(1)$ relative to the chirally-suppressed perturbative contributions in $\bar{B}$ and D decays, respectively.

- As for the lepton energy distributions, the duality violation is maximally $\mathcal{O}(1)$ and $\mathcal{O}(10)$ for $0 \leq y \leq 0.5$ in $\bar{B}$ and D decays, respectively. $y = \dfrac{2E_l}{m_b}$

Backup

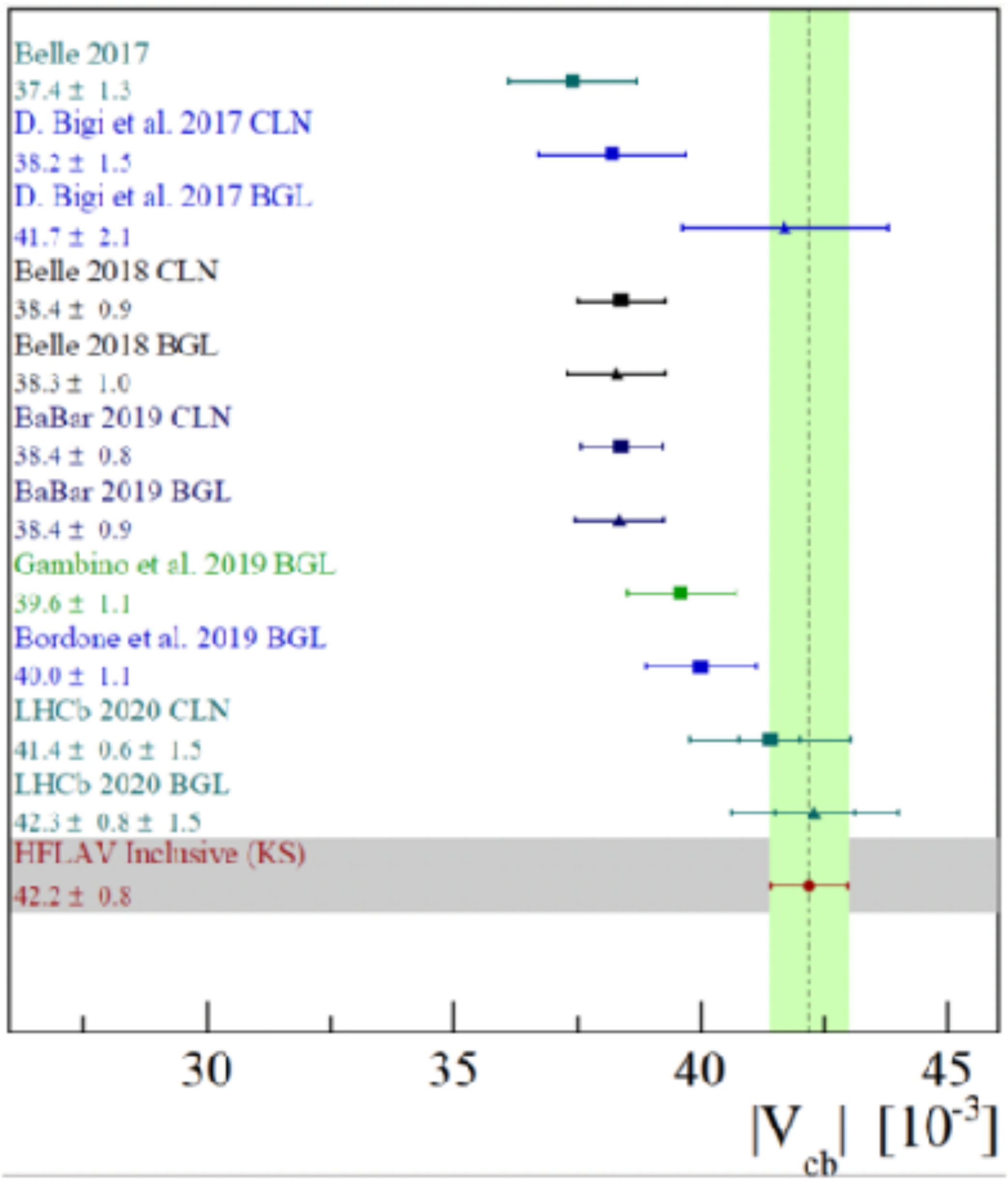
Inclusive determinations of $|V_{ub}|$

HFLAV2022 Experimental and theoretical errors for $|V_{ub}|$

	BLNP	DGE	GGOU	ADFR	BLL
Input parameters					
scheme	SF	$\overline{MS}$	kinetic	$\overline{MS}$	$1S$
Ref.	[598, 599]	Ref. [600]	see Sec. 7.2.2	Ref. [601]	Ref. [581]
m_b (GeV)	4.582 ± 0.026	4.188 ± 0.043	4.554 ± 0.018	4.188 ± 0.043	4.704 ± 0.029
μ_π^2 (GeV^2)	$0.202\,^{+0.091}_{-0.098}$	-	0.464 ± 0.076	-	-
Ref.			$\lvert V_{ub}\rvert$ values $[10^{-3}]$		
CLEO E_e [589]	$4.22 \pm 0.49^{+0.29}_{-0.34}$	$3.86 \pm 0.45^{+0.25}_{-0.27}$	$4.23 \pm 0.49^{+0.22}_{-0.31}$	$3.42 \pm 0.40^{+0.17}_{-0.17}$	-
Belle M_X, q^2 [591]	$4.51 \pm 0.47^{+0.27}_{-0.29}$	$4.43 \pm 0.47^{+0.19}_{-0.21}$	$4.52 \pm 0.48^{+0.25}_{-0.28}$	$3.93 \pm 0.41^{+0.18}_{-0.17}$	$4.68 \pm 0.49^{+0.30}_{-0.30}$
Belle E_e [590]	$4.93 \pm 0.46^{+0.26}_{-0.29}$	$4.82 \pm 0.45^{+0.23}_{-0.23}$	$4.95 \pm 0.46^{+0.16}_{-0.21}$	$4.48 \pm 0.42^{+0.20}_{-0.20}$	-
BABAR E_e [585]	$4.41 \pm 0.12^{+0.27}_{-0.27}$	$3.85 \pm 0.11^{+0.08}_{-0.07}$	$3.96 \pm 0.10^{+0.17}_{-0.17}$	-	-
BABAR E_e, s_h^{max} [588]	$4.71 \pm 0.32^{+0.33}_{-0.38}$	$4.35 \pm 0.29^{+0.28}_{-0.30}$	-	$3.81 \pm 0.19^{+0.19}_{-0.18}$	
Belle E_ℓ^B, (M_X, q^2) fit [593]	$4.05 \pm 0.23^{+0.18}_{-0.20}$	$4.16 \pm 0.24^{+0.12}_{-0.11}$	$4.15 \pm 0.24^{+0.08}_{-0.09}$	$4.05 \pm 0.23^{+0.18}_{-0.18}$	-
BABAR M_X [580]	$4.24 \pm 0.19^{+0.25}_{-0.25}$	$4.47 \pm 0.20^{+0.19}_{-0.24}$	$4.30 \pm 0.20^{+0.20}_{-0.21}$	$3.83 \pm 0.18^{+0.20}_{-0.19}$	-
BABAR M_X [580]	$4.03 \pm 0.22^{+0.22}_{-0.22}$	$4.22 \pm 0.23^{+0.21}_{-0.27}$	$4.10 \pm 0.23^{+0.16}_{-0.17}$	$3.75 \pm 0.21^{+0.18}_{-0.18}$	-
BABAR M_X, q^2 [580]	$4.32 \pm 0.23^{+0.26}_{-0.28}$	$4.24 \pm 0.22^{+0.18}_{-0.21}$	$4.33 \pm 0.23^{+0.24}_{-0.27}$	$3.75 \pm 0.20^{+0.17}_{-0.17}$	$4.50 \pm 0.24^{+0.29}_{-0.29}$
BABAR P_+ [580]	$4.09 \pm 0.25^{+0.25}_{-0.25}$	$4.17 \pm 0.25^{+0.28}_{-0.37}$	$4.25 \pm 0.26^{+0.26}_{-0.27}$	$3.57 \pm 0.22^{+0.19}_{-0.18}$	-
BABAR p_ℓ^*, (M_X, q^2) fit [580]	$4.33 \pm 0.24^{+0.19}_{-0.21}$	$4.45 \pm 0.24^{+0.12}_{-0.13}$	$4.44 \pm 0.24^{+0.09}_{-0.10}$	$4.33 \pm 0.24^{+0.19}_{-0.19}$	-
BABAR p_ℓ^* [580]	$4.34 \pm 0.27^{+0.20}_{-0.21}$	$4.43 \pm 0.27^{+0.13}_{-0.13}$	$4.43 \pm 0.27^{+0.09}_{-0.11}$	$4.28 \pm 0.27^{+0.19}_{-0.19}$	-
Belle M_X, q^2 [592]	-	-	-	-	$5.01 \pm 0.39^{+0.32}_{-0.32}$
Average	$4.28 \pm 0.13^{+0.20}_{-0.21}$	$3.93 \pm 0.10^{+0.09}_{-0.10}$	$4.19 \pm 0.12^{+0.11}_{-0.12}$	$3.92 \pm 0.12^{+0.18}_{-0.12}$	$4.62 \pm 0.20^{+0.29}_{-0.29}$

Exclusive average: $|V_{ub}| = (3.51 \pm 0.12) \times 10^{-3}$

Recent exclusive determinations of $|V_{cb}|$



Ricciardi [2103.06099]

2017: Belle published unfolded $B \to D^* l\nu$ data [1702.01521]

This enables theorists to perform their own fit.

$$\begin{cases} |V_{cb}|_{\mathbf{CLN}} = (\mathbf{38.2 \pm 1.5}) \times \mathbf{10^{-3}} \\ |V_{cb}|_{\mathbf{BGL}} = (\mathbf{41.7^{+2.0}_{-2.1}}) \times \mathbf{10^{-3}} \end{cases}$$

Bigi, Gambino and Schacht [1703.06124]

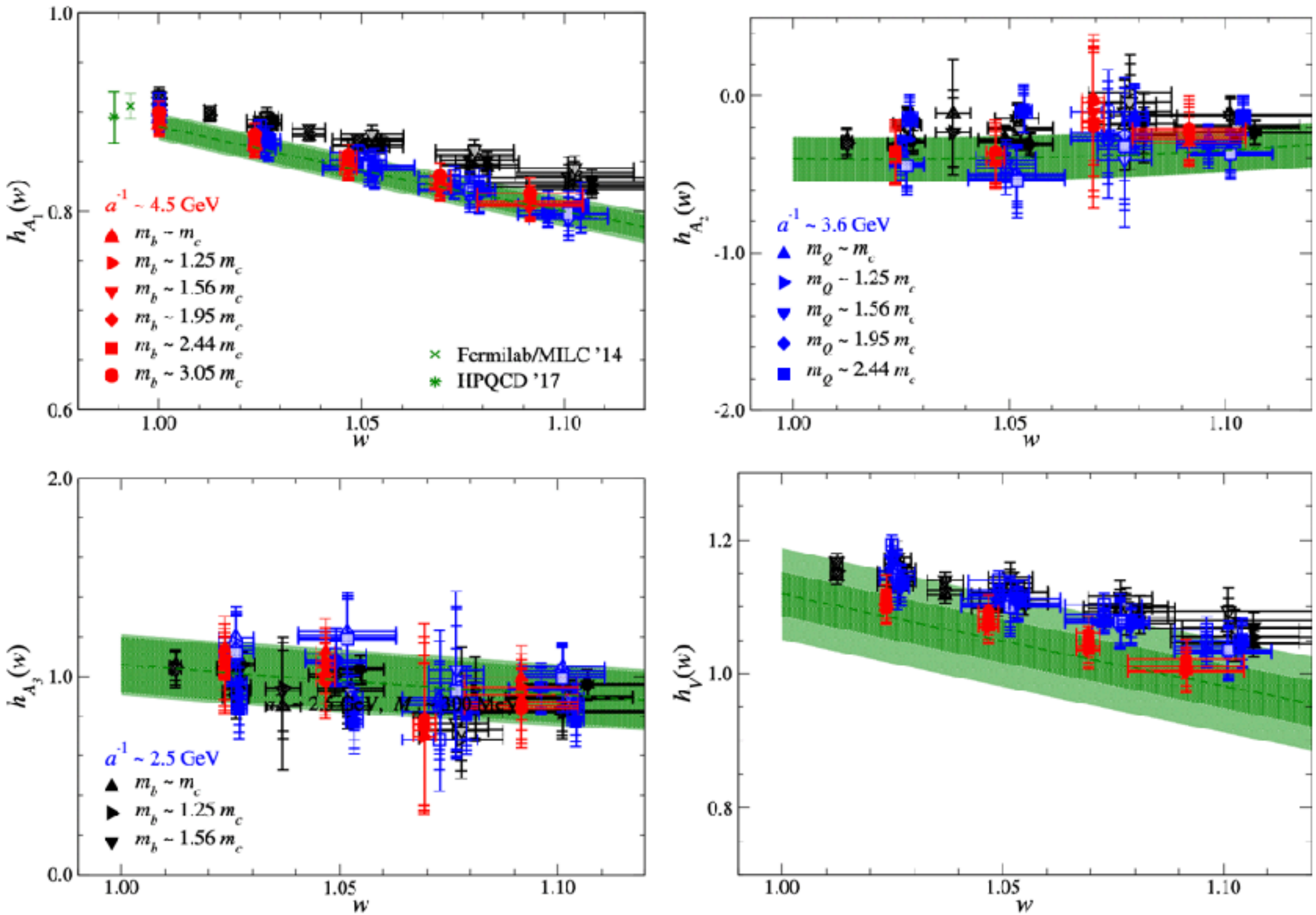
BGL is consistent with both CLN and inclusive.

2018: New Belle data published [1809.03290]

2019: New BaBar data published [1903.10002]

Both CLN and BGL are deviated from the inclusive value.

Form factor in exclusive $B \to D^*$ decays [2112.13775]



On universality of effective mass in SIA [0106019]

$$S(x,y) = \frac{\psi_0(x)\psi_0^\dagger(y)}{im^*}$$

m^* should be universal for two-quark condensate, four-quark condensates, meson correlators, etc.

TABLE I: Quark condensates evaluated in the full instanton ensemble and from the leading-instanton, only.

condensate	complete calculation	LI
χ_{uu}	$(-232 \pm 5MeV)^3$	$(-198 \pm 1MeV)^3$
χ_{uudd}	$(310 \pm 7MeV)^6$	$(309 \pm 3MeV)^6$

An uncertainty is involved for the process where only one zero-mode contributes.

Two-quark condensate: $\chi_{uu} = \left\langle Tr\left[\sum_{I,J} \psi_{0\,I}(x) \left(\frac{1}{T}\right)_{I\,J} \psi_{0\,J}^\dagger(x)\right]\right\rangle$

Four-quark condensate: $\chi_{uudd} := < 0|Tr\,[\bar{u}(x)u(x)] \cdot Tr\,[\bar{d}(x)d(x)]|0 > = < [Tr\,S(x,x)]^2 >$

Overlap matrix element: $T_{IJ} = \int d^4z \psi^\dagger(z)_I (i\not{\partial})\psi(z)_J$

Part larger than whole paradox [9605465]

Total width: $$\left.\frac{\Gamma}{\Gamma_0}\right|_{\mathrm{LR}}^{\mathrm{zm}} = -\frac{2}{3}d_0\left(\frac{m_b}{m^*}\right)\frac{192\pi}{(m_b\rho_0)^8}\cos(2m_b\rho_0)$$

Differential width: $$\left.\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}y}\right|_{\mathrm{LR}}^{\mathrm{zm}} = -\frac{2}{3}d_0\left(\frac{m_b}{m^*}\right)\frac{96\pi}{(m_b\rho_0)^5}\frac{y^2}{\left(1-\frac{y}{2}\right)^2}\sin\left(2m_b\rho_0\left(1-\frac{y}{2}\right)\right)$$

- Absolute size of the differential width is parametrically larger than the total width by $\sim (m_b\rho)^3$.
- The similar behavior has been already found in [9605465] for non-zero mode contribution in the SM.
- In the endpoint region, the semi-classical approximation is not valid *per se*.

Treatment to obtain the lepton energy distribution

In [9605465]: the dispersion relation is imposed via the OPE-like counterterms.

In this work: the momentum (position) space propagator is used for charged lepton (neutrino) to avoid the endpoint integral.

Detail of finite distance singularity

$$f_1(Q) = \int_0^\infty \frac{e^{iQx}}{x^2+\rho^2}\mathrm{d}x \qquad f_1(Q) + f_1(-Q) = \int_{-\infty}^{+\infty} \frac{e^{iQx}}{x^2+\rho^2}\mathrm{d}x$$

Below : $\rho, Q > 0$

(1) Exact evaluation

Duality violation

$$\begin{cases} f_1(+Q) = +\frac{i}{\rho}\sum_{k=0}^{\infty}\frac{(2k)!}{(Q\rho)^{2k+1}} + \frac{\pi}{2\rho}e^{-Q\rho}, \\ f_1(-Q) = -\frac{i}{\rho}\sum_{k=0}^{\infty}\frac{(2k)!}{(Q\rho)^{2k+1}} + \frac{\pi}{2\rho}e^{-Q\rho}. \end{cases}$$

$$f_1(Q) + f_1(-Q) = \frac{\pi}{\rho}e^{-|Q|\rho}$$

(2) Short-distance expansion first (OPE)

$$\begin{cases} f_2(+Q) = +\frac{i}{\rho}\sum_{k=0}^{\infty}\frac{(2k)!}{(Q\rho)^{2k+1}} - \frac{i}{\rho}\sum_{k=0}^{\infty}\frac{\lim_{b\to\infty}\Gamma(2k+1,-ib)}{(Q\rho)^{2k+1}}, \\ f_2(-Q) = -\frac{i}{\rho}\sum_{k=0}^{\infty}\frac{(2k)!}{(Q\rho)^{2k+1}} + \frac{i}{\rho}\sum_{k=0}^{\infty}\frac{\lim_{b\to\infty}\Gamma(2k+1,+ib)}{(Q\rho)^{2k+1}}. \end{cases}$$

$$f_2(Q) + f_2(-Q) = 0$$

$\Gamma(k, z)$: the upper imcomplete gamma function