Application of the background field approach to matching relations for gluon pseudo-distributions

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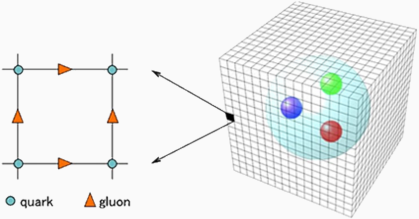
Motivation: Lattice QCD



Source: http://lpc-clermont.in2p3.fr/IMG/theorie/LQCD2.jpg

- PDFs from lattice a subject of considerable interest
- Nonperturbative regularization
- Lattice field theory uses a discretized version of Euclidean spacetime, $t \to it$ [Wilson, 1974]
- $z^2 = 0$ not accessible in Euclidean spacetime
- Historically, lattice calculations involved extracting moments of PDFs

Quasidistributions

- X. Ji [2013] proposed to calculate equal time correlation functions at purely spacelike separations

- qPDFs are defined through matrix elements of bilocal operators with purely spacelike ($z = (0, 0, 0, z_3)$) separation

$$q\left(x, \mu^2, p_z\right) = \int \frac{\mathrm{d}z}{4\pi} e^{ixzp_z} \langle p| \bar{\psi}(z)\gamma^z \exp\left(-ig\int_0^z \mathrm{d}z' A^z(z')\right) \psi(0) |p\rangle + \mathcal{O}\left(\Lambda^2_{\mathsf{QCD}}/x^2p_z^2, m^2/x^2p_z^2\right)$$

- Approach PDF in large p^z limit

- Large momentum factorization, LaMET:

$$q\left(x, \mu^2, p^z\right) = \int_x^1 \frac{\mathrm{d}y}{y} Z\left(\frac{x}{y}, \frac{\mu}{p^z}\right) q(y, \mu^2) + \mathcal{O}\left(\Lambda^2_{\mathsf{QCD}}/x^2p_z^2, m^2/x^2p_z^2\right)$$

[X. Ji, 2013]

Pseudodistributions

- Coordinate-space oriented approach [Radyushkin, 2017]

- For singlet quark PDF:

$$M^\mu(z,p) = \sum_f \langle p| \bar\psi_f(z)\gamma^\mu[z,0]\psi_f(0)|p\rangle = 2p^\mu \underbrace{\mathcal{M}_p\left(\nu,-z^2\right)}_{\substack{\text{Ioffe-time}\\ \text{pseudodistribution}\\ \text{(pseudo-ITD)}}} + z^\mu \underbrace{\mathcal{M}_z\left(\nu,-z^2\right)}_{\substack{\text{Higher ``twist''}\\ \text{contamination}}}$$

$$\nu = -(pz) \quad \text{[Ioffe, 1969]}$$

- On the lightcone, $M^+(z,p)$, $z^+=0$, $\nu = p^+ z^-$:

$$\mathcal{M}_p(\nu,0) = \underbrace{\mathcal{I}_p(\nu)}_{\text{ITD}} = \int_{-1}^{1} \mathrm{d}x e^{-ix\nu} f_S(x) = \frac{1}{2}\frac{1}{p^+}\sum_f \langle p|\bar\psi_f(z^-)\gamma^+\psi_f(0)|p\rangle$$

[Braun, *et al.*, 1995]

- Spacelike $z=(0,0,0,z_3)$, $\nu = p_3 z_3$:

$$M^0(z,p) = 2p^0\mathcal{M}_p\left(\nu, z_3^2\right)$$

Pseudodistributions

- Linear and logarithmic UV divergences from gauge link
- Ratio method: UV divergences cancel, multiplicative renormalizability:

$$\underbrace{\mathfrak{M}\left(\nu, z_3^2\right)}_{\text{reduced ITD (rITD)}} = \frac{\mathcal{M}\left(\nu, z_3^2\right)}{\mathcal{M}\left(0, z_3^2\right)}$$

[Orginos, *et al.*, 2017], [Radyushkin, 2017]

- z_3^2 dependence in evolution logs → short distance evolution:

$$\frac{\mathrm{d}}{\mathrm{d}\ln z_3^2}\mathfrak{M}\left(\nu, z_3^2\right) = -\frac{\alpha_s}{2\pi}C_F \int_0^1 \mathrm{d}u\, \underbrace{P_{qq}^{(0)}(u)}_{\substack{\text{DGLAP}\\ \text{kernel}}} \mathfrak{M}\left(u\nu, z_3^2\right) + \mathcal{O}\left(z_3^2 m^2, z_3^2 \Lambda_{\mathrm{QCD}}^2\right)$$

- Short distance factorization:

$$\underbrace{\mathfrak{M}\left(\nu, z_3^2\right)}_{\text{Lattice QCD}} = \int_{-1}^1 \mathrm{d}u\, \underbrace{\mathcal{C}\left(u, z_3^2\mu^2, \alpha_s\right)}_{\substack{\text{Perturbation}\\ \text{theory}}} \mathcal{I}\left(u\nu, \mu^2\right) + \mathcal{O}\left(z_3^2 m^2, z_3^2 \Lambda_{\mathrm{QCD}}^2\right)$$

Method of calculation for gluon

- Coordinate space calculation using the background field method along with the Schwinger parametrization of the propagator via the QCD heat kernel [Balitsky, Braun, 1989]
- Allows for the calculation of corrections to gluon operators in an explicitly gauge invariant form
- Employs the $\overline{\mathrm{MS}}$ renormalization scheme $\rightarrow$ calculations were performed in d spacetime dimensions

Background field method

Background fields: $(\bar{\psi}, \psi, A)$, virtualities below μ_1^2

Quantum fields: $(\bar{\phi}, \phi, \mathcal{A})$, virtualities between μ_1^2 and μ_2^2

$$(gA, g\mathcal{A}) \to (A, \mathcal{A})$$

Integrating over the quantum fields produces a result in terms of the external (or background) fields at the lower renormalization point μ_1^2.

$$\mathcal{L} = -\frac{1}{4g^2}\left(G^a_{\mu\nu} + D_\mu \mathcal{A}^a_\nu - D_\nu \mathcal{A}^a_\mu + f^{abc}\mathcal{A}^b_\mu \mathcal{A}^c_\nu\right)^2 + \mathcal{L}_{\mathrm{GF}}$$
$$+ \left(\bar{\phi} + \bar{\psi}\right)\left(i\not{D} + \mathcal{A}^a_\mu \gamma^\mu t^a\right)(\phi + \psi) + \mathcal{L}_{gh}$$

$$D_\mu = \partial_\mu - iA_\mu$$
$$G^a_{\mu\nu} = D_\mu A^a_\nu - D_\nu A^a_\mu$$

Background field (BF) gauge applies to quantum fields:

$$\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2g^2}\left(D^\mu \mathcal{A}^a_\mu\right)^2 \quad \to \quad \mathcal{A}^a_\mu \to \mathcal{A}^a_\mu + f^{abc}\mathcal{A}^b_\mu \alpha^c$$

Background field maintains local gauge invariance $\quad \to \quad A^a_\mu \to A^a_\mu + D_\mu \alpha^a$

Background field method

Background field gluon propagator:

$$\overbrace{\mathcal{A}_\mu^a(z)\mathcal{A}}_\nu^b(0) = g^2 \langle z| \left(\frac{-i}{P^2 g_{\mu\nu} + 2iG_{\mu\nu} + i\epsilon} \right)^{ab} |0\rangle$$

using the notation:

$$\langle x| B |y\rangle \equiv \int \frac{\mathrm{d}^d p}{(2\pi)^4} B(p) e^{-ip(x-y)} \ .$$

Straight line gauge link:

$$\begin{aligned}
[x,y] = [x,y]_c &+ i \int_0^1 \mathrm{d}u (x-y)^\rho [x, ux+\bar{u}y]_c \mathcal{A}_\rho(ux+\bar{u}y)[ux+\bar{u}y, y]_c \\
&+ i^2 \int_0^1 \mathrm{d}u \int_0^u \mathrm{d}v (x-y)^\rho (x-y)^\sigma [x, ux+\bar{u}y]_c \mathcal{A}_\rho(ux+\bar{u}y) \\
&\times [ux+\bar{u}y, vx+\bar{v}y]_c \mathcal{A}_\sigma(vx+\bar{v}y)[vx+\bar{v}y, y]_c + \mathcal{O}\left(g^3\right)
\end{aligned}$$

$$\bar{u} = 1-u \qquad \mathcal{A}_\rho = T^a \mathcal{A}_\rho^a$$

Fock-Schwinger gauge and Schwinger parametrization

$$\langle z|\left(\frac{-i}{P^2 g_{\mu\nu}+2iG_{\mu\nu}+i\epsilon}\right)^{ab}|0\rangle = -\int_0^\infty \mathrm{d}s\,\langle z|\,e^{is\left(P^2 g_{\mu\nu}+2iG_{\mu\nu}+i\epsilon\right)}\,|0\rangle^{ab}\ ,$$

Expand in 's' and integrate → lightcone expansion in external fields.

$$\int_0^\infty \mathrm{d}s s^n\,\langle z|\,e^{isp^2}\,|0\rangle = \frac{(-i)^n\Gamma\left(d/2-n-1\right)}{4^{n+1}\pi^2\left(-z^2\right)^{d/2-n-1}}\ .$$

Fock-Schwinger (FS) gauge:

$$(z-z_0)^\mu A_\mu(z)=0 \quad\rightarrow\quad A_\nu(z)=\int_0^1 \mathrm{d}w\,wz^\mu G_{\mu\nu}(wz)$$

Gluon propagator in external fields:

$$\begin{aligned}
&\langle z|\,\frac{1}{P^2 g_{\alpha\beta}+2iG_{\alpha\beta}}\,|0\rangle \\
&= -ig_{\alpha\beta}\frac{\Gamma(d/2-1)}{4\pi^2\left(-z^2\right)^{d/2-1}}+\frac{\Gamma(d/2-2)}{16\pi^2\left(-z^2\right)^{d/2-2}}\int_0^1 \mathrm{d}u\Big\{2G_{\alpha\beta}(uz)-\bar{u}uz^\nu D^\mu G_{\mu\nu}(uz)g_{\alpha\beta} \\
&\quad -2ig_{\alpha\beta}\int_0^u \mathrm{d}v\bar{u}vz^\lambda G_{\lambda\xi}(uz)z^\rho G_\rho^{\ \xi}(vz)\Big\} \\
&\quad -\frac{i\Gamma(d/2-3)}{16\pi^2\left(-z^2\right)^{d/2-3}}\int_0^1 \mathrm{d}u\int_0^u \mathrm{d}v\,\Big[G_{\alpha\xi}(uz)G_\beta^{\ \xi}(vz)-\tfrac{1}{2}i\bar{u}D^2 G_{\alpha\beta}(uz)\Big]+\mathcal{O}\left(\text{"twist 3"}\right)
\end{aligned}$$

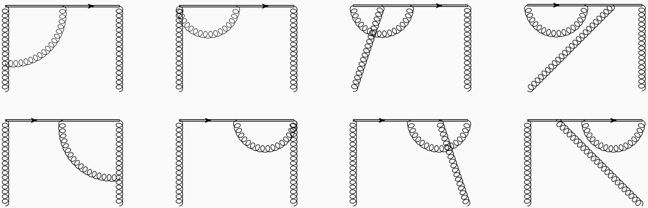
Gluon bilocal operator

Uncontracted gluon bilocal operator at one loop in terms of quantum gluon fields:

$$
\begin{aligned}
& G^a_{\mu\alpha}(z)[z,0]G^a_{\nu\beta}(0)\Big|^{\mu_2^2} \\
& \to \underbrace{G^a_{\mu\alpha}(z)[z,0]G^a_{\nu\beta}(0)}_{\text{LO, gauge link}} + \underbrace{\left(D_\mu\mathcal{A}^a_\alpha - D_\alpha\mathcal{A}^a_\mu\right)(z)[z,0]\left(D_\nu\mathcal{A}^a_\beta - D_\beta\mathcal{A}^a_\nu\right)(0)}_{\text{Handbag diagrams}} \\
& \quad + \underbrace{G^a_{\mu\alpha}(z)[z,0]\left(D_\nu\mathcal{A}^a_\beta - D_\beta\mathcal{A}^a_\nu\right)(0) + \left(D_\mu\mathcal{A}^a_\alpha - D_\alpha\mathcal{A}^a_\mu\right)(z)[z,0]G^a_{\nu\beta}(0)}_{\text{Vertex diagrams}} \\
& \quad + \underbrace{G^a_{\mu\alpha}(z)[z,0]f^{abc}\mathcal{A}^b_\nu(0)\mathcal{A}^c_\beta(0) + f^{abc}\mathcal{A}^b_\mu(z)\mathcal{A}^c_\alpha(z)[z,0]G^a_{\nu\beta}(0)}_{\text{Self energy type diagrams}}\Big|^{\mu_1^2}
\end{aligned}
$$

- Gluon self energy requires the insertion of a next order vertex, and consideration of contributions from ghost fields
- Result for dual field easily obtained by: $\widetilde{G}^{\rho\sigma} = \frac{1}{2}\epsilon^{\rho\sigma\nu\beta}G_{\nu\beta}$
- Operator level calculation with general Lorentz indices $\to$ applicability to nonforward matrix elements: GPDs, DAs, etc.

Gluon bilocal operator, vertex



$$O^{\mathrm{V}}_{\mu\alpha;\nu\beta}(z)$$

$$\to \frac{g^2 C_A \Gamma(d/2-1)}{4\pi^2(-z^2)^{d/2-1}} \int_0^1 \mathrm{d}u \int_0^u \mathrm{d}v$$

$$\times \left\{ \delta(\bar{u}) \left(\frac{v^{3-d}-v}{d-2} \right) G^a_{\mu\alpha}(uz) \left(z_\beta G^a_{z\nu}(vz) - z_\nu G^a_{z\beta}(vz) \right) \right.$$

$$\left. + \delta(v) \left(\frac{\bar{u}^{3-d}-\bar{u}}{d-2} \right) \left(z_\alpha G^a_{z\mu}(uz) - z_\mu G^a_{z\alpha}(uz) \right) G^a_{\nu\beta}(vz) \right\}$$

$$+ \frac{g^2 C_A \Gamma(d/2-2)}{8\pi^2(-z^2)^{d/2-2}} \int_0^1 \mathrm{d}u \int_0^u \mathrm{d}v \left\{ \delta(\bar{u}) \left[\frac{v^{3-d}-1}{d-3} \right]_{+(0)} + \delta(v) \left[\frac{\bar{u}^{3-d}-1}{d-3} \right]_{+(1)} \right\}$$

$$\times G^a_{\mu\alpha}(uz) G^a_{\nu\beta}(vz)$$

Gluon bilocal operator, vertex

UV divergence in $\Gamma(d/2-1)$:

$$\int_0^1 \mathrm{d}v\, \frac{v^{3-d}-v}{d-2} = \frac{1}{2(4-d)} = \frac{1}{4\epsilon_{UV}}\ ,$$

UV singular part:

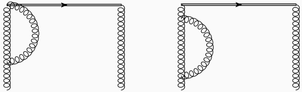
$$\frac{g^2 C_A}{16\pi^2(-z^2)}\left(\frac{1}{\epsilon_{\mathrm{UV}}} + \ln\left(-z^2\mu_{\mathrm{UV}}^2 e^{2\gamma_{\mathrm{E}}}/4\right)\right)\Big\{G^a_{\mu\alpha}(z)\left(z_\beta G^a_{z\nu}(0) - z_\nu G^a_{z\beta}(0)\right) + \left(z_\alpha G^a_{z\mu}(z) - z_\mu G^a_{z\alpha}(z)\right)G^a_{\nu\beta}(0)\Big\}$$

UV finite part:

$$\frac{g^2 C_A}{8\pi^2(-z^2)}\int_0^1 \mathrm{d}u \int_0^u \mathrm{d}v \Big\{\delta(\bar{u})\left(\frac{1}{v} - v\right)_{+(0)} G^a_{\mu\alpha}(uz)\left(z_\beta G^a_{z\nu}(vz) - z_\nu G^a_{z\beta}(vz)\right) + \delta(v)\left(\frac{1}{\bar{u}} - \bar{u}\right)_{+(1)}\left(z_\alpha G^a_{z\mu}(uz) - z_\mu G^a_{z\alpha}(uz)\right)G^a_{\nu\beta}(vz)\Big\}$$

Linear UV divergence cancels between diagrams

Gluon bilocal operator, self energy



- Self energy calculation involves:

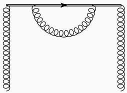
$$\langle z| \frac{-i}{P^2 g_{\mu\nu} + 2iG_{\mu\nu}} |z\rangle \to -2\int \frac{\mathrm{d}^d p}{(2\pi)^4} \frac{1}{p^4} G_{\mu\nu}(z)$$
$$= -\frac{i}{8\pi^2} \frac{\pi^{-\epsilon}}{\Gamma(1-\epsilon)} G_{\mu\nu}(z) \int \mathrm{d}p_\perp^2 \frac{p_\perp^{-2\epsilon}}{p_\perp^2}$$

- Formally zero in dimensional regularization.

- Introduce scale dependent logarithm: $\ln\left(\mu_{\mathrm{UV}}^2/\mu_{\mathrm{IR}}^2\right)$
$= \ln\left(-z^2\mu_{\mathrm{UV}}^2 e^{2\gamma_E}/4\right) - \ln\left(-z^2\mu_{\mathrm{IR}}^2 e^{2\gamma_E}/4\right)$

$$O^S_{\mu\alpha;\nu\beta}(z) \to -\frac{g^2 C_A}{8\pi^2}\left(\frac{1}{\epsilon_{\mathrm{IR}}} - \ln\left(-z^2\mu_{\mathrm{IR}}^2 e^{2\gamma_E}/4\right) + \frac{1}{\epsilon_{\mathrm{UV}}} + \ln\left(-z^2\mu_{\mathrm{UV}}^2 e^{2\gamma_E}/4\right)\right)$$
$$\times \left[2 - \frac{\beta_0}{2C_A}\right] G_{\mu\alpha}(z) G_{\nu\beta}(0)$$

Gluon bilocal operator, link self energy



$$O^{\mathrm{L}}_{\mu\alpha;\nu\beta}(z) = \frac{g^2 N_c \Gamma(d/2-1)}{4\pi^2(-z^2)^{d/2-2}} \frac{-1}{\underbrace{(d-3)}_{\text{UV linear}}\underbrace{(d-4)}_{\text{UV log}}} G_{\mu\alpha}(z) G_{\nu\beta}(0)$$

- Dimensional regularization insensitive to linear divergence:

$$\frac{g^2 C_A}{8\pi^2}\left(\frac{1}{\epsilon_{\mathrm{UV}}} + \ln\left(-z^2\mu_{\mathrm{UV}}^2 e^{2\gamma_{\mathrm{E}}}/4\right) + 2\right) G_{\mu\alpha}(z) G_{\nu\beta}(0)$$

- Linear divergence is nontrivial in lattice calculation

Matrix element, unpolarized

Nucleon spin-averaged matrix elements with non-contracted indices:

$$M_{\mu\alpha;\nu\beta}(z,p) \equiv \langle p|\, G_{\mu\alpha}(z)\,[z,0]\,G_{\nu\beta}(0)\,|p\rangle$$

Lorentz decomposition:

$$\begin{aligned}
&M_{\mu\alpha;\nu\beta}(z,p)\\
&= (g_{\mu\nu}p_\alpha p_\beta - g_{\mu\beta}p_\alpha p_\nu - g_{\alpha\nu}p_\mu p_\beta + g_{\alpha\beta}p_\mu p_\nu)\,\mathcal{M}_{pp}(\nu,z^2)\\
&+ (g_{\mu\nu}z_\alpha z_\beta - g_{\mu\beta}z_\alpha z_\nu - g_{\alpha\nu}z_\mu z_\beta + g_{\alpha\beta}z_\mu z_\nu)\,\mathcal{M}_{zz}(\nu,z^2)\\
&+ (g_{\mu\nu}z_\alpha p_\beta - g_{\mu\beta}z_\alpha p_\nu - g_{\alpha\nu}z_\mu p_\beta + g_{\alpha\beta}z_\mu p_\nu)\,\mathcal{M}_{zp}(\nu,z^2)\\
&+ (g_{\mu\nu}p_\alpha z_\beta - g_{\mu\beta}p_\alpha z_\nu - g_{\alpha\nu}p_\mu z_\beta + g_{\alpha\beta}p_\mu z_\nu)\,\mathcal{M}_{pz}(\nu,z^2)\\
&+ (p_\mu z_\alpha p_\nu z_\beta - p_\alpha z_\mu p_\nu z_\beta - p_\mu z_\alpha p_\beta z_\nu + p_\alpha z_\mu p_\beta z_\nu)\,\mathcal{M}_{ppzz}(\nu,z^2)\\
&+ (g_{\mu\nu}g_{\alpha\beta} - g_{\mu\beta}g_{\alpha\nu})\,\mathcal{M}_g(\nu,z^2)
\end{aligned}$$

Matrix element, unpolarized

On the lightcone, $z = (0, z^-, 0, 0)$:

$$-g^{\alpha\beta} M_{+\alpha,+\beta}(z_-, p) = -2p_+^2 \mathcal{M}_{pp}(\nu, 0) = 2p_+^2 \frac{1}{2} \int_{-1}^{1} \mathrm{d}x e^{-ix\nu} x f_g(x)$$

Multiplicatively renormalizable projections for spacelike separations, $z = (0, 0, 0, z_3)$:

$$\langle p| G_{3i}(z) G_{3i}(0) |p\rangle = 2\mathcal{M}_g - 2p_3^2 \mathcal{M}_{pp} - 2z_3^2 \mathcal{M}_{zz} - 2z_3 p_3 (\mathcal{M}_{zp} + \mathcal{M}_{pz})$$

$$\langle p| G_{0i}(z) G_{0i}(0) |p\rangle = -2\mathcal{M}_g - 2p_0^2 \mathcal{M}_{pp}$$

$$\langle p| G_{0i}(z) G_{3i}(0) + G_{3i}(z) G_{0i}(0) |p\rangle = -4p_0 p_3 \mathcal{M}_{pp} - 2p_0 z_3 (\mathcal{M}_{pz} + \mathcal{M}_{zp})$$

$$\langle p| G_{ji}(z) G_{ji}(0) |p\rangle = 2\mathcal{M}_g$$

[Zhang et. al, 2019]

Isolate leading "twist" through:

$$M_{0i;0i} + M_{ji;ji} = -2p_0^2 \mathcal{M}_{pp}$$

$M_{0i;0i}$ and $M_{ji;ji}$ have same anomalous dimension.

One-loop result, unpolarized

$$
\begin{aligned}
&\frac{M_{0i;0i}(z,p)+M_{ji;ji}(z,p)}{2p_0^2} = -\mathcal{M}_{pp}\left(\nu, z_3^2\right) \\
&\to \frac{g^2 C_A}{8\pi^2}\int_0^1 \mathrm{d}u \Bigg\{ \left(\frac{5}{6}\ln\left(z_3^2\mu_{\mathrm{UV}}^2 e^{2\gamma}/4\right)+2\right)\delta\left(\bar{u}\right) \\
&\qquad\qquad - \left(\frac{1}{2}\delta\left(\bar{u}\right) + \left[\frac{2}{3}\left(1-u^3\right)+\frac{4u+4\ln(\bar{u})}{\bar{u}}\right]_{+(1)}\right) \\
&\qquad\qquad - \ln\left(z_3^2\mu_{\mathrm{IR}}^2 e^{2\gamma}/4\right)\underbrace{\left[2\bar{u}(1+u^2)+2\left[\frac{u^2}{\bar{u}}\right]_{+(1)}+\frac{1}{2}\left(\frac{\beta_0}{C_A}-6\right)\delta(\bar{u})\right]}_{\text{Gluon DGLAP kernel}} \Bigg\} \\
&\qquad\qquad \times\left(-\mathcal{M}_{pp}\left(u\nu, z_3^2\right)\right) \\
&\quad + \frac{g^2 C_A}{8\pi^2}\int_0^1 \mathrm{d}u \left\{-\frac{2}{3}\left(1-u^3\right)-\ln\left(z_3^2\mu_{\mathrm{IR}}^2 e^{2\gamma}/4\right)\bar{u}\left(u^2+1\right)\right\} u^2 z_3^2 \mathcal{M}_{ppzz}\left(u\nu, z_3^2\right) \\
&\quad + \frac{g^2 C_F}{8\pi^2}\int_0^1 \mathrm{d}u \Bigg\{ -\ln\Big(z_3^2\mu_{\mathrm{IR}}^2 e^{2\gamma_E}/4\Big)\underbrace{\left(2\bar{u}+\delta(\bar{u})\right)}_{\text{GQ DGLAP kernel}}\Bigg\}\mathcal{M}_p(u\nu, z_3^2)/\nu \\
&\quad + \frac{g^2 C_F}{8\pi^2}\int_0^1 \mathrm{d}u \Bigg\{ \ln\Big(z_3^2\mu_{\mathrm{IR}}^2 e^{2\gamma_E}/4\Big)\, 6\Big[u^2-\bar{u}u\Big]_{+(1)}\Bigg\}\frac{z_3^2}{\nu^2}\mathcal{M}_z(u\nu, z_3^2)
\end{aligned}
$$

Evolution and matching, unpolarized

- Pseudo-rITD simply defined as:

$$\mathfrak{M}(\nu, z_3^2) = \frac{M_{0i;0i}(z,p) + M_{ji;ji}(z,p)}{M_{0i;0i}(z,p_3=0) + M_{ji;ji}(z,p_3=0)} = \frac{\mathcal{M}_{pp}\left(\nu, z_3^2\right)}{\mathcal{M}_{pp}\left(0, z_3^2\right)}$$

- Short distance evolution equation:

$$\begin{aligned}\frac{\mathrm{d}\mathfrak{M}(\nu, z_3^2)}{\mathrm{d}\ln z_3^2} &= -\frac{\alpha_s C_A}{2\pi}\int_0^1 \mathrm{d}u B_{gg}(u)\mathfrak{M}\left(u\nu, z_3^2\right) \\ &\quad - \frac{\alpha_s C_F}{2\pi}\int_0^1 \mathrm{d}u \mathcal{B}_{gq}(u)\left(\mathfrak{M}_S(u\nu, z_3^2) - \mathfrak{M}\left(\nu, z_3^2\right)\mathfrak{M}_S(0, z_3^2)\right) \\ &\quad + \mathcal{O}\left(z_3^2 m^2, z_3^2\Lambda_{\mathrm{QCD}}\right)\end{aligned}$$

$$B_{gg}(u) = \left[\frac{2(1-u\bar{u})^2}{\bar{u}}\right]_{+(1)}, \qquad \mathcal{B}_{gq}(u) = 1 + \bar{u}^2$$

Evolution and matching, unpolarized

Relating reduced Ioffe-time pseudo-distribution to light-cone Ioffe time distribution

$$\mathfrak{M}(\nu, z_3^2) = \frac{\mathcal{I}_g(\nu,\mu^2)}{\mathcal{I}_g(0,\mu^2)} - \frac{\alpha_s C_A}{2\pi} \int_0^1 \mathrm{d}u \Bigg\{ \left[\frac{2}{3}\left(1-u^3\right) + \frac{4u + 4\ln(\bar{u})}{\bar{u}}\right]_{+(1)} + \ln\left(z_3^2\mu^2 e^{2\gamma}/4\right) B_{gg}(u) \Bigg\} \frac{\mathcal{I}_g(u\nu,\mu^2)}{\mathcal{I}_g(0,\mu^2)}$$

$$- \frac{\alpha_s C_F}{2\pi} \ln\Big(z_3^2\mu^2 e^{2\gamma_E}/4\Big) \int_0^1 \mathrm{d}u \mathcal{B}_{gq}(u) \left(\frac{\mathcal{I}_S(u\nu,\mu^2)}{\mathcal{I}_g(0,\mu^2)} - \frac{\mathcal{I}_g(\nu,\mu^2)}{\mathcal{I}_g(0,\mu^2)} \frac{\mathcal{I}_S(0,\mu^2)}{\mathcal{I}_g(0,\mu^2)} \right) + \mathcal{O}\left(z_3^2 m^2, z_3^2 \Lambda_{\mathrm{QCD}}\right)$$

Gluon momentum fraction and singlet quark momentum fraction:

$$\mathcal{I}_g\left(0,\mu^2\right) = \langle x \rangle_{\mu^2}\,, \quad \mathcal{I}_S(0,\mu^2) = \langle x_S \rangle_{\mu^2}$$

Can be directly related to light-cone PDF using:

$$\mathcal{I}_g\left(\nu,\mu^2\right) = \frac{1}{2}\int_{-1}^{1} \mathrm{d}x e^{ix\nu} x f_g\left(x,\mu^2\right) = \int_0^1 \mathrm{d}x \cos(x\nu) x f_g\left(x,\mu^2\right)$$

Evolution and matching, unpolarized

New kernels found by cosine transformation only, because sine part integrates to zero $\big(xf_g(x,\mu^2)$ is even$\big)$:

$$R_{gg}(x\nu, z_3^2\mu^2) = \int_0^1 \mathrm{d}u \cos(ux\nu) \left(\delta(\bar{u}) - \frac{\alpha_s C_A}{2\pi}\left\{ \left(\left[\frac{2}{3}\left(1-u^3\right) + \frac{4u + 4\ln(\bar{u})}{\bar{u}}\right]_{+(1)}\right) + \ln\left(z_3^2\mu^2 e^{2\gamma}/4\right) B_{gg}(u)\right\}\right)$$

$$R_{gq}(x\nu, z_3^2\mu^2) = \int_0^1 \mathrm{d}u \cos(ux\nu) \left(-\frac{\alpha_s C_F}{2\pi} \ln\Big(z_3^2\mu^2 e^{2\gamma_E}/4\Big)\mathcal{B}_{gq}(u)\right)$$

$$R_r(x\nu, z_3^2\mu^2) = \frac{4}{3}\cos(x\nu)\frac{\alpha_s C_F}{2\pi}\ln\Big(z_3^2\mu^2 e^{2\gamma_E}/4\Big)$$

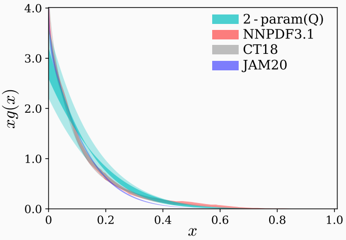
R kernels are given by explicit perturbatively calculable expressions.

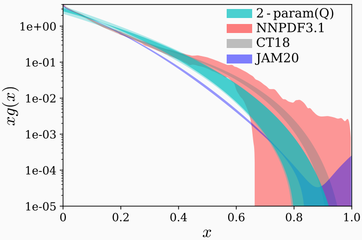
New kernel form:

$$\mathfrak{M}(\nu, z_3^2) = \int_0^1 \mathrm{d}x \frac{xf_g(x,\mu^2)}{\langle x\rangle_{\mu^2}}\left(R_{gg}(x\nu, z_3^2\mu^2) + R_r(x\nu, z_3^2\mu^2)\frac{\langle x_S\rangle_{\mu^2}}{\langle x\rangle_{\mu^2}}\right) + \int_0^1 \mathrm{d}x \frac{xf_S(x,\mu^2)}{\langle x\rangle_{\mu^2}} R_{gq}(x\nu, z_3^2\mu^2) + \mathcal{O}\left(z_3^2 m^2, z_3^2\Lambda_{\rm QCD}\right)$$

Lattice implementation

Lattice extraction of the gluon PDF by the HadStruc collaboration at Jefferson Lab [Khan, *et al.*, 2021], and the lattice group at Michigan State University [Fan, Lin, 2021], [Fan, Zhang, Lin, 2021].





Unpolarized gluon PDF (cyan band) extracted from HadStruc lattice data using the 2-param (Q) model. Results are compared to gluon PDFs extracted from global fits to experimental data, CT18 [Hou, *et al.*, 2021], NNPDF3.1 [Ball, *et al.*, 2017], and JAM20 [Moffat, *et al.*, 2021]. The gluon momentum fraction used in the calculation was $\langle x \rangle_{\mu^2=4\,\mathrm{GeV}^2} = 0.427(92)$ from [Alexandrou, *et al.*, 2020]. The left figure uses a linear scale, while the right figure uses a lograthmic scale in order to enhance the view of the large-x region. [Khan, *et al.*, 2021]

Conclusions

- Methods and results of computation of one-loop corrections to the gluon bilocal operator were outlined
- Matching relation between gluon pseudo-ITDs and lightcone PDFs was given for unpolarized case
- Similar procedure for polarized case
- Matching relations have already been used in the lattice extraction of light cone PDFs by the HadStruc collaboration at Jefferson Lab, and the lattice group at Michigan State University
- A key feature of the general result for the gluon bilocal operator is its process independence, and therefore its applicability to nonforward matrix elements
- Result can be used in the future calculation of matching conditions for the extraction of gluon GPDs and DAs from lattice calculations, in addition to other distributions that may not be experimentally accessible

Thank you!