Symmetries of Meson Correlators in High Temperature QCD with N_f=2+1+1 Physical Domain-Wall Quarks

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Outline

- Introduction
- Symmetries and Meson Correlators
- Gauge Ensembles
- The Temporal Correlators of $\bar{u}d$ Mesons
- The Spatial Correlators of $\bar{u}d$ Mesons
- Conclusions

Introduction

What is the nature of quark matter at high T QCD ?

It is relevant to the matter creation in the early universe, and the heavy-ion collision expt. at RHIC and LHC.

A first step is to find out the sym. of high T QCD, since the nature of matter can be unveiled from its sym.

At $T > T_c \approx 150$ MeV, the $SU(2)_L \times SU(2)_R$ chiral sym. of u and d quarks is effectively restored.

Is $U(1)_A$ symmetry is also restored at $T_1 \approx T_c$?

Introduction (cont)

At $T > T_1 \approx T_c$, the $U(1)_A \times SU(2)_L \times SU(2)_R$ chiral sym. of u and d quarks is effectively restored.

Are they the only symmetries of QCD from T_1 to $T \gg T_c$ such that $g^2_{\text{eff}}(T) \approx 0$ and the quarks and gluons behaving like a gas of free particles and forming the quark-gluon plasma ?

Are there any emerging sym. which are manifested in observables but not in the QCD lagrangian ?

Introduction (cont)

Recently a larger symmetry group $SU(2)_{CS}$ chiral-spin [Glozman, 1407.2798; Glozman & Pak, 1504.02323] is observed to be approximately manifested in the meson correlators, in $N_f = 2$ lattice QCD with DWF, for $T \approx 220 - 500$ MeV.

[C. Rohrhofer et al. 1902.0319; 1909.00927]

This suggests the possible existence of hadron-like objects which are predominantly bounded by the chromoelectric field into color singlets.

Introduction (cont)

What would be the scenarios of emergence of $SU(2)_{CS}$ color-spin symmetry in *QCD* with dynamical light and heavy quarks: (u, d, s); (u, d, s, c); (u, d, s, c, b) ?

I study the meson correlators in $N_f = 2+1+1$ lattice QCD with domain-wall quarks at the physical point, for $T \approx 190-770$ MeV. The meson correlators include a complete set of Dirac bilinears, and each for six combinations of quark flavors ($\bar{u}d$, $\bar{u}s$, $\bar{u}c$, $\bar{s}c$, $\bar{s}s$, $\bar{c}c$).

In this talk, I will focus on the $\bar{u}d$ meson correlators.

Symmetries and Meson Correlators

The correlation function of meson interpolator $\bar{q}_1 \Gamma q_2$

$$C_\Gamma(t,\vec{x}) = \left\langle (\bar{q}_1 \Gamma q_2)_x (\bar{q}_1 \Gamma q_2)_0^\dagger \right\rangle$$

$$= \left\langle \mathrm{tr}\left[\Gamma (D_c + m_1)^{-1}_{x,0} \Gamma (D_c + m_2)^{-1}_{0,x}\right] \right\rangle_{\mathrm{confs}}$$

$(D_c + m_q)^{-1}$ quark propagator

$$C_\Gamma(t,\vec{p}) = \int_0^\infty \frac{d\omega}{2\pi} \frac{\cosh\left[\omega\left(t - \frac{1}{2T}\right)\right]}{\sinh\left(\frac{\omega}{2T}\right)} \rho_\Gamma(\omega,\vec{p})$$

spectral function

$$C_\Gamma(t) = \sum_{x_1,x_2,x_3} C_\Gamma(t,\vec{x})$$

temporal *t*-correlator

$$\Rightarrow \rho(\omega, \vec{p} = 0)$$

$$C_\Gamma(z) = \sum_{x_1,x_2,x_4} C_\Gamma(t,\vec{x})$$

spatial *z*-correlator

$$\Rightarrow \rho(\omega = 0, p_1 = p_2 = 0)$$

Symmetries and Meson Correlators (cont)

The classification of meson interpolators $\bar{q}_1 \Gamma q_2$:

Name and notation	Γ (for t-correlators)	Γ (for z-correlators)
Scalar (S)	$\mathbb{1}$	$\mathbb{1}$
Pseudocalar (P)	γ_5	γ_5
Vector (V_k)	γ_k ($k = 1, 2, 3$)	γ_k ($k = 1, 2, 4$)
Axial vector (A_k)	$\gamma_5 \gamma_k$ ($k = 1, 2, 3$)	$\gamma_5 \gamma_k$ ($k = 1, 2, 4$)
Tensor vector (T_k)	$\gamma_4 \gamma_k$ ($k = 1, 2, 3$)	$\gamma_3 \gamma_k$ ($k = 1, 2, 4$)
Axial-tensor vector (X_k)	$\gamma_5 \gamma_4 \gamma_k$ ($k = 1, 2, 3$)	$\gamma_5 \gamma_3 \gamma_k$ ($k = 1, 2, 4$)

Symmetries and Meson Correlators (cont)

global $U(1)_A$ rotation

$$q(x) \to \exp(i\gamma_5\theta)q(x), \quad \bar{q}(x) \to \bar{q}(x)\gamma_4 \exp(-i\gamma_5\theta)\gamma_4$$

If $U(1)_A$ symmetry is effectively restored for $T \geq T_1 \approx T_c$

$C_S = C_P$ (correlators of scalar and pseudoscalar are degenerate)

$C_{T_k} = C_{X_k}$ (also those of tensor and axial-tensor vectors.)

Symmetries and Meson Correlators (cont)

flavor doublet $q = (q_1, q_2)^T$

vector bilinears (V_k)

$$\bar{q}(x)\gamma_k \frac{\tau_\pm}{2} q(x), \quad \tau_\pm = \tau_1 \pm i\tau_2 \quad \{\tau_1, \tau_2, \tau_3\} \text{ are Pauli matrices}$$

axial-vector bilinears (A_k) $\quad \bar{q}(x)\gamma_5\gamma_k \frac{\tau_\pm}{2} q(x)$

flavor non-singlet axial rotations

$$q(x) \to \exp\left(i\gamma_5 \frac{\vec{\tau}}{2} \cdot \vec{\theta}\right) q(x), \quad \bar{q}(x) \to \bar{q}(x)\gamma_4 \exp\left(-i\gamma_5 \frac{\vec{\tau}}{2} \cdot \vec{\theta}\right) \gamma_4$$

The effective restoration $SU(2)_L \times SU(2)_R$ chiral sym. for $T \ge T_c$ $\Leftrightarrow$

$C_{V_k} = C_{A_k}$ (correlators of vector and axial-vector are degenerate)

Symmetries and Meson Correlators (cont)

The $SU(2)_{CS}$ (chiral-spin) transformations

$$q(x) \to \exp\left(i\frac{\vec{\Sigma}_\mu}{2}\cdot\vec{\theta}\right) q(x), \quad \bar{q}(x) \to \bar{q}(x)\gamma_4 \exp\left(-i\frac{\vec{\Sigma}_\mu}{2}\cdot\vec{\theta}\right)\gamma_4, \quad \mu = 1,2,3,4$$

$\vec{\Sigma}_\mu = \{\gamma_5, \gamma_\mu, i\gamma_\mu\gamma_5\}$, and $\vec{\theta}$ can be global or local parameters

$SU(2)_{CS}$ contains $U(1)_A$ as a subgroup.

$SU(2)_{CS}$ is not a sym. of the QCD lagrangian, but the color charge $Q^a = \int d^3x\, \psi^\dagger T^a \psi$. In a given frame, the quark-gluon interaction can be composed into temporal and spatial parts: $\bar{\psi}(\gamma_4 D_4 + \gamma_i D_i)\psi$.

In the temporal part $\bar{\psi}\gamma_4 D_4 \psi = \psi^\dagger(\partial_4 + igT^a A_4^a)\psi$

the interaction term $ig\psi^\dagger T^a A_4^a \psi$ is invariant under $SU(2)_{CS}$.

Symmetries and Meson Correlators (cont)

This allows $SU(2)_{CS}$ chiral-spin symmetry to distinguish between chromoelectric and chromomagnetic interactions in a given frame.

The emergence of $SU(2)_{CS}$ suggests the possible existence of hadron-like objects which are predominantly bounded by the chromoelectric field into color singlets.

For the t-correlators, $\mu = 4$ (not mixing op. with different spins)

$SU(2)_{CS} \times S_3$ transformations generate one triplet and one nonet

$$(A_1, A_2, A_3);\ (V_1, V_2, V_3, T_1, T_2, T_3, X_1, X_2, X_3)$$

If the $SU(2)_{CS}$ is effectively restored for $T \geq T_{CS} > T_1$

$$C_{V_k} = C_{T_k} = C_{X_k}$$

Symmetries and Meson Correlators (cont)

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If the $SU(2)_{CS}$ is effectively restored for $T \geq T_{CS} > T_1$

$$C_{A_k} = C_{V_k} = C_{T_k} = C_{X_k} \qquad SU(2)_L \times SU(2)_R \times SU(2)_{CS}$$

Symmetries and Meson Correlators (cont)

For the z-correlators, the $SU(2)_{CS} \times S_2$ transformations generate the following multiplets:

$$(V_1, V_2);\ (A_1, A_2, T_4, X_4), \qquad \mu = 1$$

$$V_4;\ (A_4, T_1, T_2, X_1, X_2). \qquad \mu = 2$$

If the $SU(2)_{CS}$ is effectively restored for $T \geq T_{CS} > T_1$

$$C_{A_k} = C_{T_4} = C_{X_4}$$

$$C_{A_4} = C_{T_k} = C_{X_k}$$

Symmetries and Meson Correlators (cont)

For the z-correlators, the $SU(2)_{CS} \times S_2$ transformations generate the following multiplets:

$$(V_1, V_2);\ (A_1, A_2, T_4, X_4), \qquad \mu = 1$$

$$V_4;\ (A_4, T_1, T_2, X_1, X_2). \qquad \mu = 2$$

If the $SU(2)_{CS}$ is effectively restored for $T \geq T_{CS} > T_1$

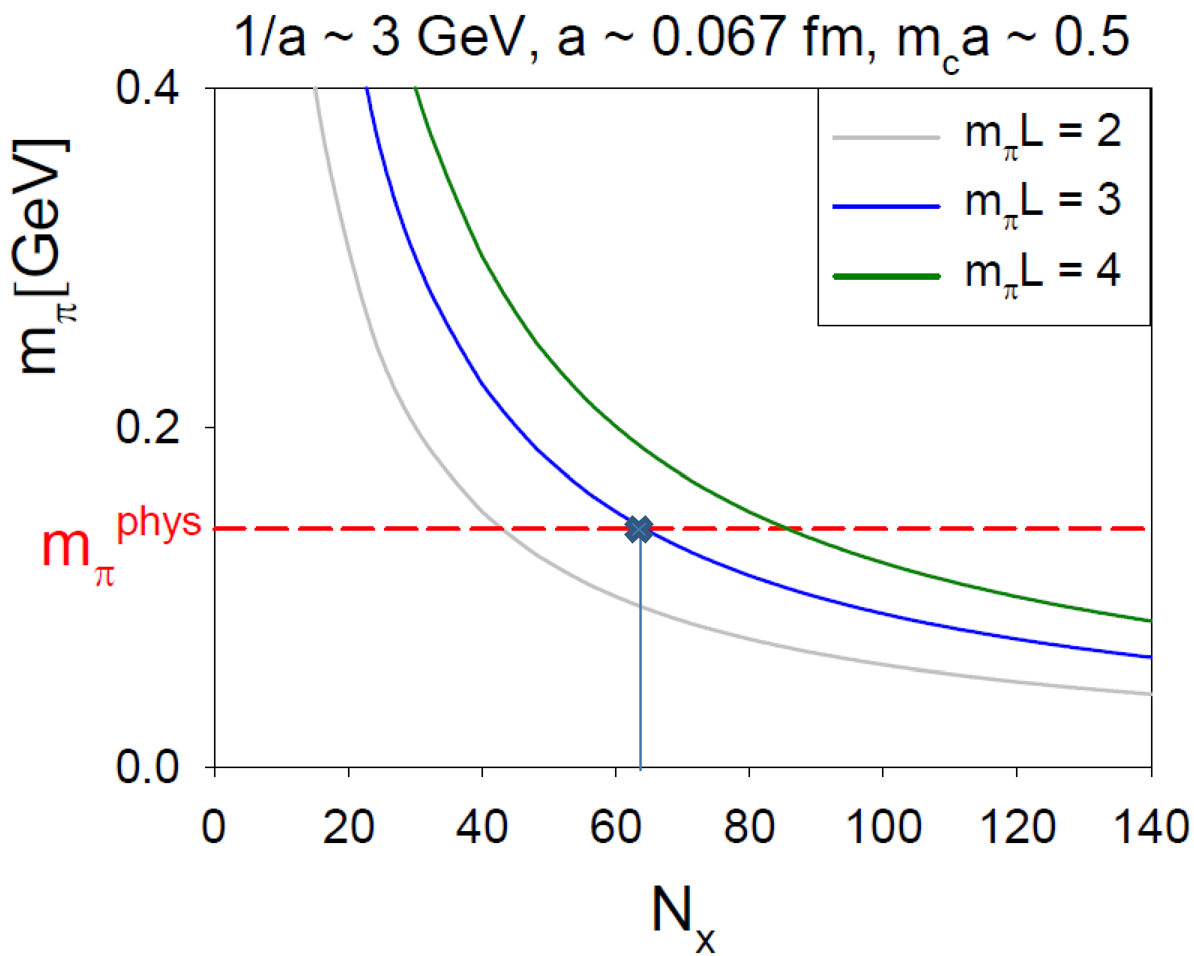
$$C_{V_k} = C_{A_k} = C_{T_4} = C_{X_4}$$

$$C_{V_4} = C_{A_4} = C_{T_k} = C_{X_k}$$

$$SU(2)_L \times SU(2)_R \times SU(2)_{CS}$$

Design lattice QCD with physical (u,d,s,c) quarks

TWC, arXiv:1811.08095



For the $64^3 \times 64$ lattice, $M_\pi L \approx 3$, $M_\pi \approx 140$ MeV, $L \approx 4.3$ fm

Actions and Algorithms

- Quarks: optimal DWF [TWC, PRL 2003] with $N_s = 16$, $\lambda_{\max}/\lambda_{\min} = 6.20 / 0.05$.

  Gluons: plaquette gauge action at $\beta = 6/g^2 = (6.20,\ 6.18,\ 6.15)$

- For the one-flavor, use the Exact One-Flavor pseudofermion Action (EOFA)

  [Y.C. Chen & TWC, Phys. Lett. B738 (2014) 55; TWC, Phys. Lett. B744 (2015) 95]

- For the 2-flavor, use the two-flavor algorithm for DWF.

  [TWC, T.H. Hsieh, Y.Y. Mao, Phys. Lett. B702 (2012) 131]

Gauge Ensembles of Nf=2+1+1 QCD

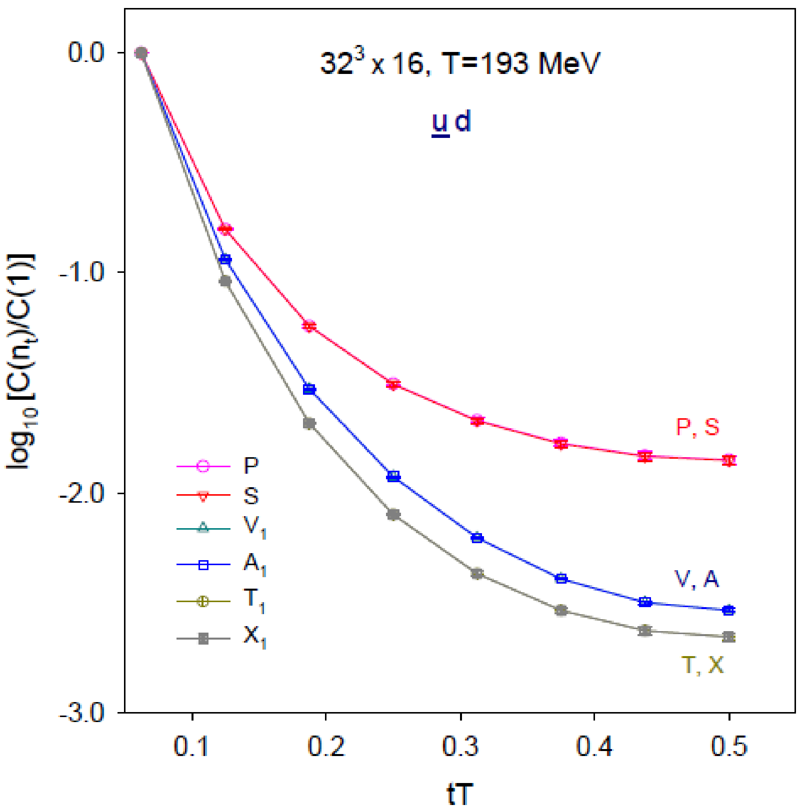
- Lattice sizes : $(64^3, 32^3) \times (64,\ 20,\ 16,\ 12,\ 10,\ 8,\ 6,\ 4)$
- Lattice spacings : (0.064, 0.068, 0.075) fm
- Spatial volume : $L^3 > (4\ \text{fm})^3,\ M_\pi L > 3;\ \ L^3 > (2\ \text{fm})^3, M_\pi L > 1.5$
- Number of gauge ensembles : $(1,1) \times 3 \times (8,7) = 45$
- Temperatures $T = (N_t a)^{-1}$: $\sim 0 - 770$ MeV
- Statistics : ~200 – 2000 configurations per ensemble
- The lattice spacings are determined by the Wilson flow, using $t^2 \langle E \rangle \big|_{t=t_0} = 0.3$ with $\sqrt{t_0} = 0.1416(8)$ fm.
- The physical $(u/d,\ s,\ c)$ masses are obtained by tuning their masses on the 64^4 lattices such that the masses of the lowest-lying states of the time-correlation function of $(\bar{u}\gamma_5 d,\ \bar{s}\gamma_i s,\ \bar{c}\gamma_i c)$ are in good agreement with $\pi(140)$, $\phi(1020)$ and $J/\psi(3097)$ respectively.

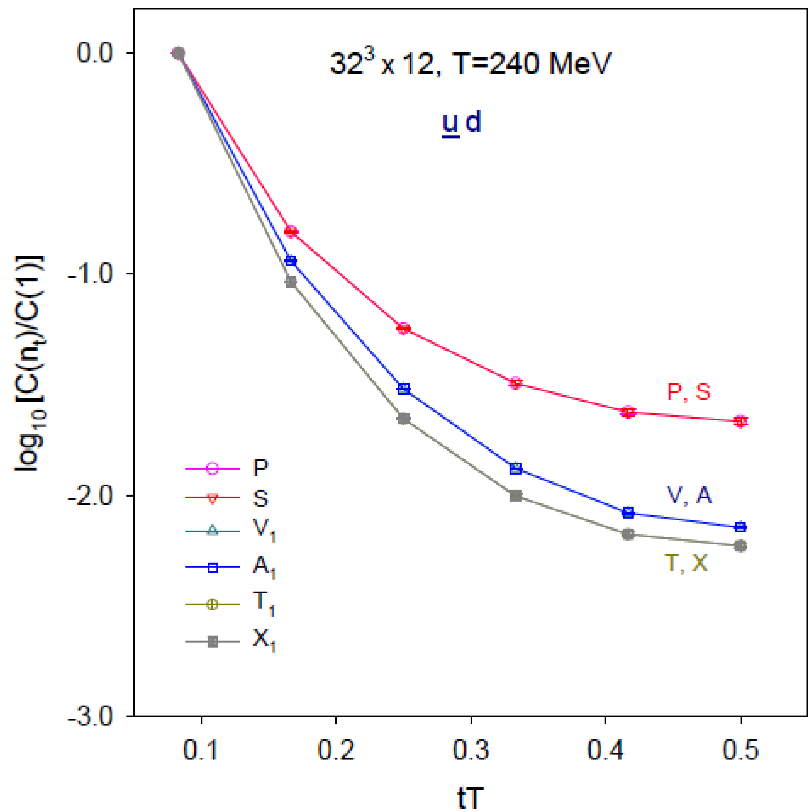
Gauge Ensembles of N_f=2+1+1 QCD (cont)

The 6 gauge ensembles for the meson correlators in this study.

$\beta = 6/g^2$	a[fm]	N_x	N_t	$m_{u/d}a$	$m_s a$	$m_c a$	T[MeV]	$N_{\rm confs}$
6.20	0.0641	32	16	0.00125	0.0400	0.55000	192	583
6.18	0.0685	32	12	0.00180	0.0580	0.62600	240	781
6.20	0.0641	32	10	0.00125	0.0400	0.55000	307	358
6.20	0.0641	32	8	0.00125	0.0400	0.55000	384	468
6.20	0.0641	32	6	0.00125	0.0400	0.55000	512	431
6.20	0.0641	32	4	0.00125	0.0400	0.55000	768	991

The Temporal t-Correlators of $\bar{u}d$ Mesons

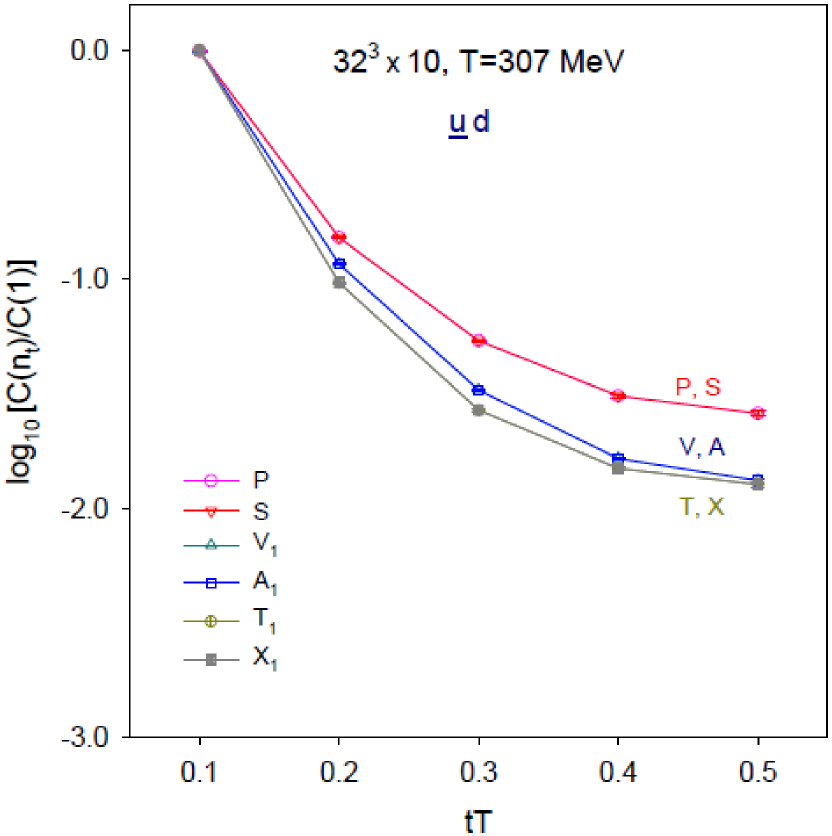




$C_{V_k} = C_{A_k} \Rightarrow SU(2)_L \times SU(2)_R$ is effectively restored.

$C_S = C_P$ and $C_{T_k} = C_{X_k} \Rightarrow U(1)_A$ is effectively restored.

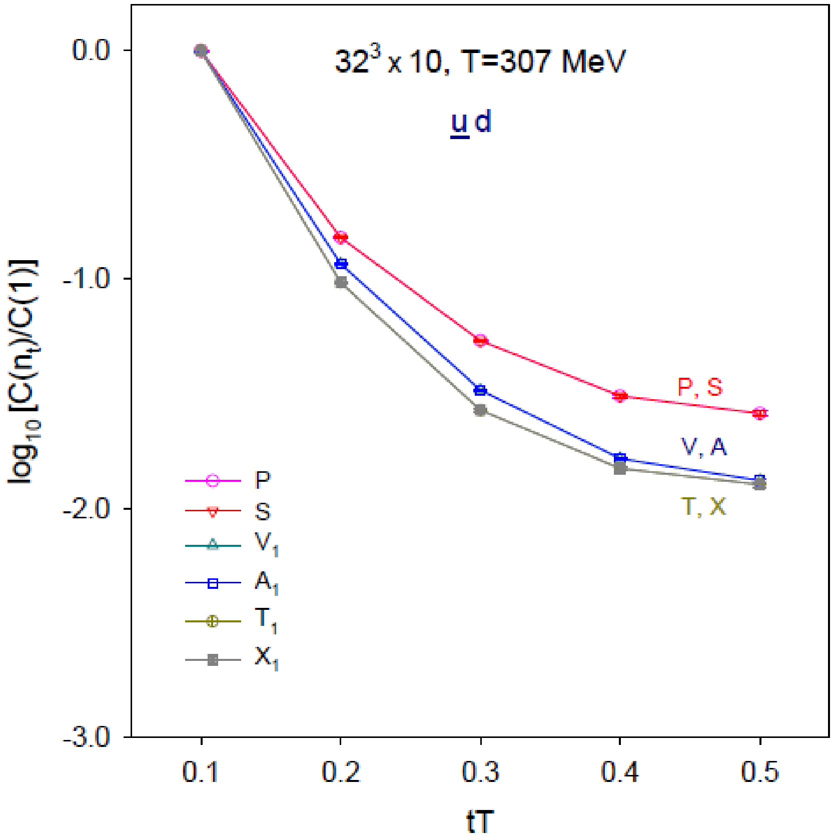
The Temporal t-Correlators of $\bar{u}d$ Mesons (cont)

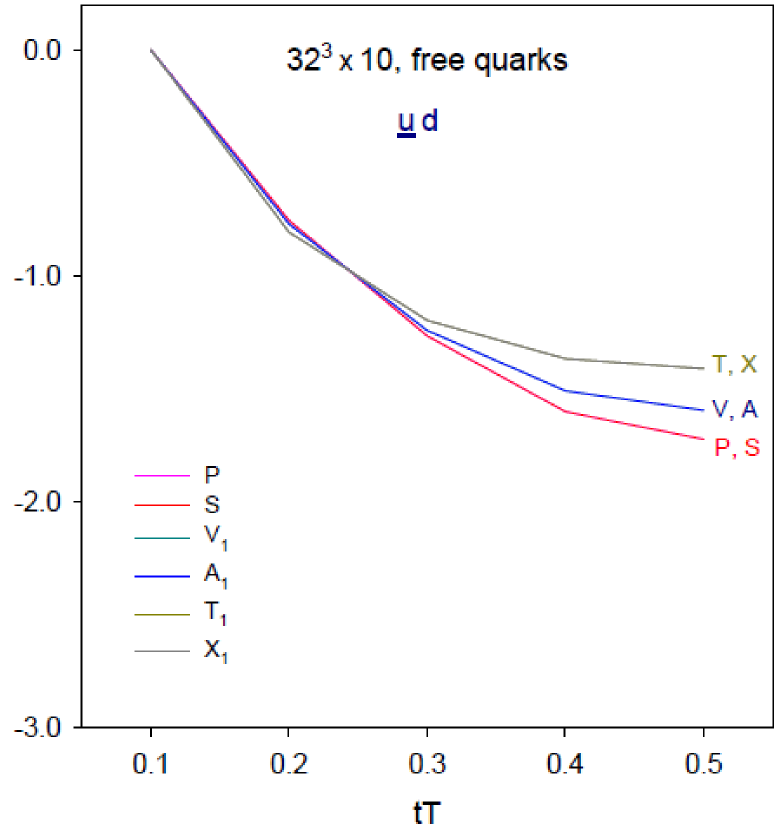


As T is increased from $190 \to 240 \to 310$ MeV, multiplets $\{V_k, A_k\}$ and $\{T_k, X_k\}$ are converging to form a single multiplet $\{V_k, A_k, T_k, X_k\}$

$\Rightarrow$ The emergence of $SU(2)_{CS}$ chiral-spin symmetry.

Comparison with the t-Correlators of Free Quarks





Comparing the meson *t*-correlators with those computed with free quark propagators on the same lattice and the same u/d mass, it shows that the u/d quarks in *QCD* have NOT deconfined at $T \approx 310$ MeV, and the existence of meson-like objects bounded by the chromoelectric fields.

The Symmetry Breaking Parameters for t-Correlators

To measure the breaking of $U(1)_A$ chiral sym :

$$\kappa_{PS}(t) = 1 - \frac{C_S(t)}{C_P(t)}, \quad n_t > 1$$

$$\kappa_{TX}(t) = 1 - \frac{C_{X_1}(t)}{C_{T_1}(t)}, \quad n_t > 1$$

To measure the breaking of $SU(2)_L \times SU(2)_R$ chiral sym :

$$\kappa_{VA}(t) = 1 - \frac{C_{A_1}(t)}{C_{V_1}(t)}, \quad n_t > 1$$

To measure the breaking of $SU(2)_{CS}$ chiral-spin sym :

$$\kappa_{AT}(t) = \frac{C_{A_1}(t)}{C_{T_1}(t)} - 1, \quad n_t > 1$$

The Symmetry Breaking Parameters for t-Correlators (cont)

To measure the splitting in the $SU(2)_{CS}$ multiplet $M_1 = \{A_k, V_k, T_k, X_k\}$ relative to the distance between M_1 and $M_0 = \{P, S\}$:

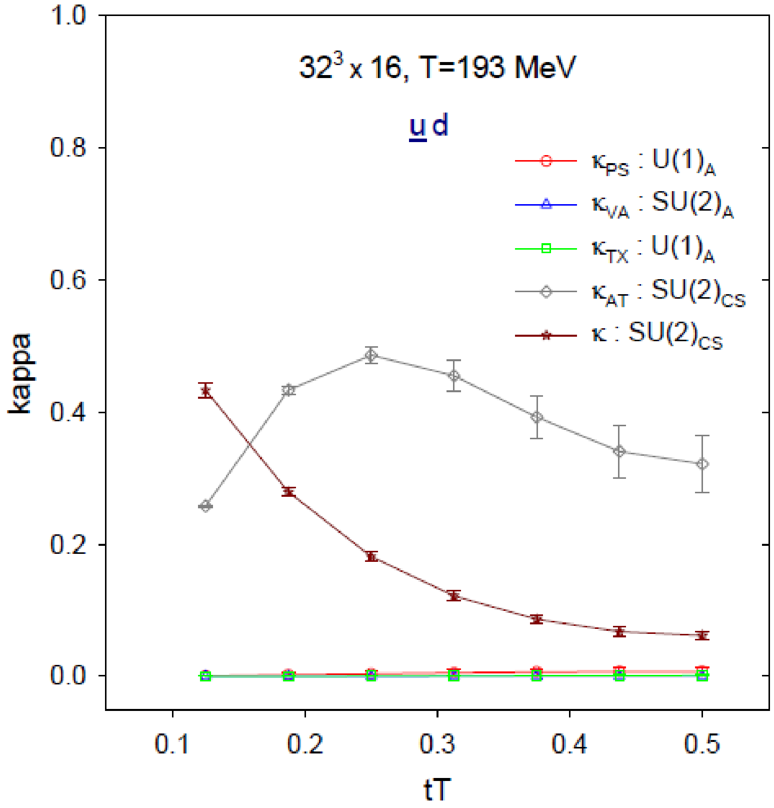
$$\kappa(t) = \frac{|C_{A_1}(t) - C_{T_1}(t)|}{|C_1(t) - C_0(t)|}$$

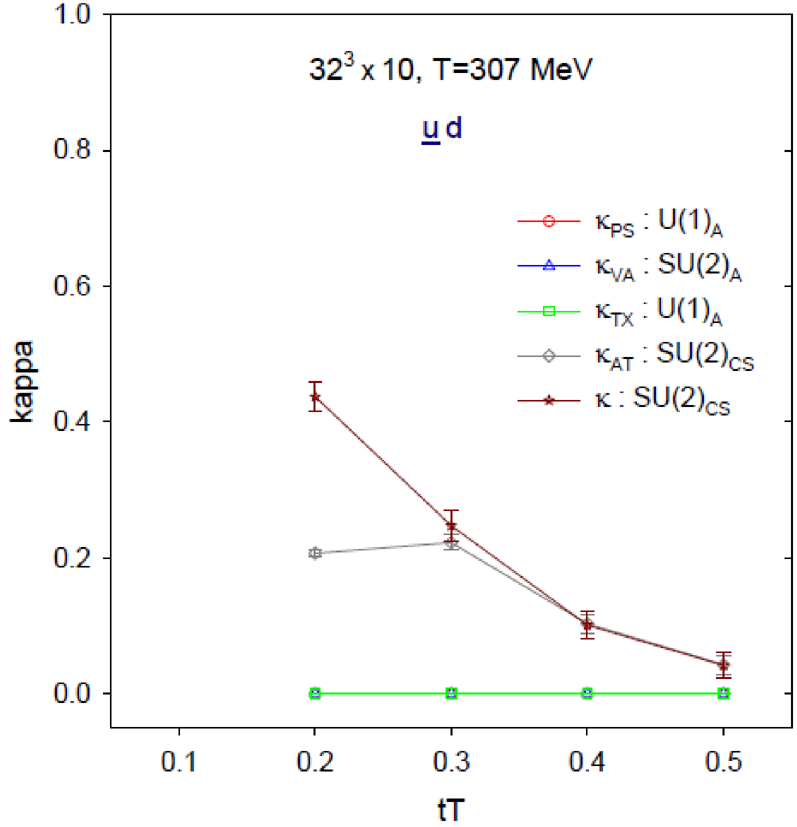
$$C_0(t) \equiv \frac{1}{2}\left[C_P(t) + C_S(t)\right]$$

$$C_1(t) \equiv \frac{1}{4}\left[C_{V_1}(t) + C_{A_1}(t) + C_{T_1}(t) + C_{X_1}(t)\right]$$

If $\kappa_{AT}(t) < 1$ and $\kappa(t) > 1$, the $SU(2)_{CS}$ multiplet M_1 converges with M_0 and they form a single multiplet, then the $SU(2)_{CS}$ chiral-spin sym. is washed away. This occurs for $T > T_s > 770$ MeV in $N_f = 2+1+1$ QCD.

The Symmetry Breaking Parameters for t-Correlators (cont)





$\kappa_{VA} < 2.6(8)\times 10^{-4}$,

$\kappa_{PS} < 7.6(6.5)\times 10^{-3}$,

$\kappa_{TX} < 1.4(1.3)\times 10^{-3}$,

$\kappa_{AT} < 0.5, \quad \kappa < 0.5$

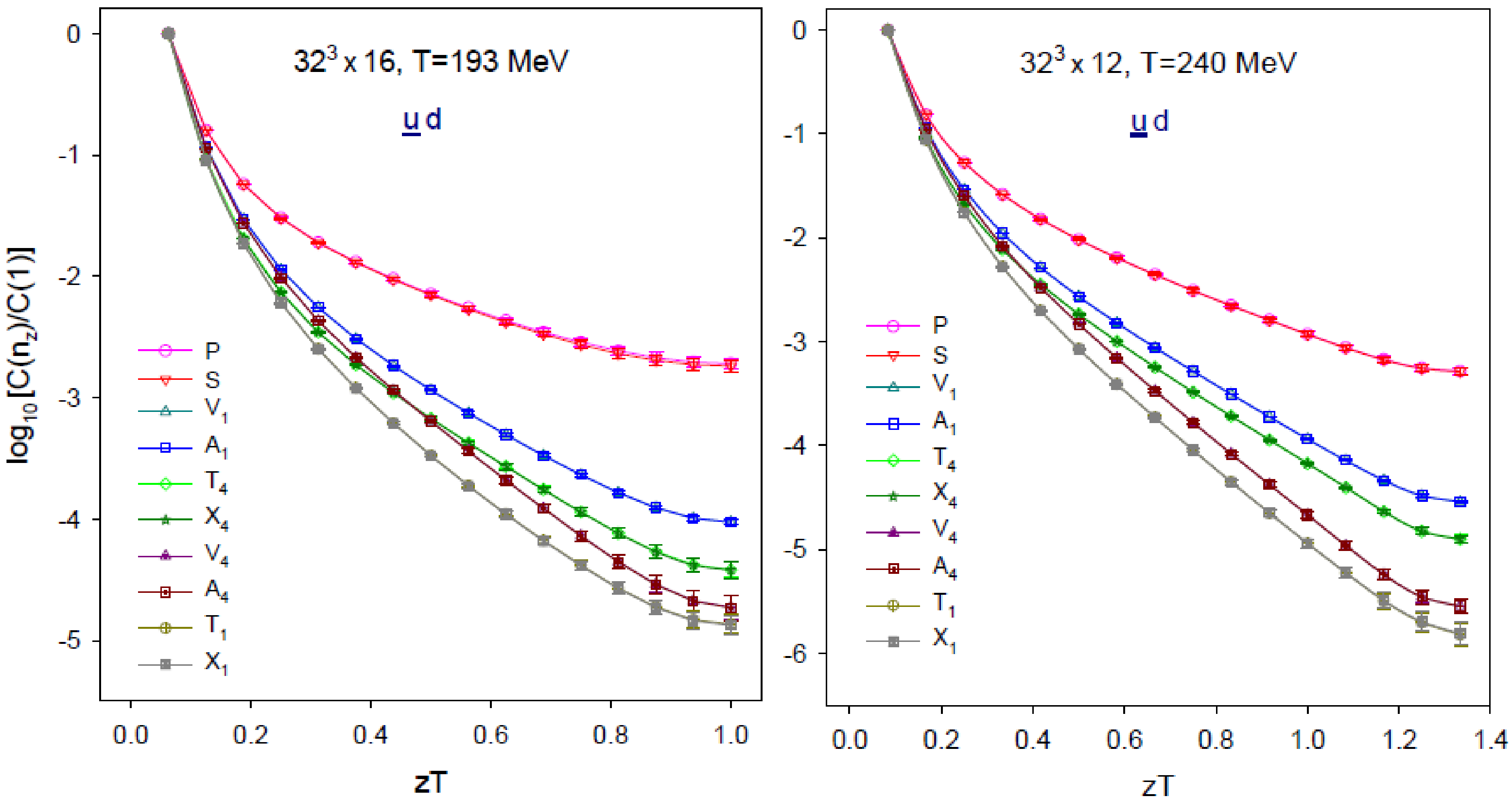
$\kappa_{VA} < 6.8(3)\times 10^{-6}$,

$\kappa_{PS} < 3.9(1.2)\times 10^{-5}$,

$\kappa_{TX} < 8.2(4)\times 10^{-6}$,

$\kappa_{AT} < 0.2, \quad \kappa < 0.5$

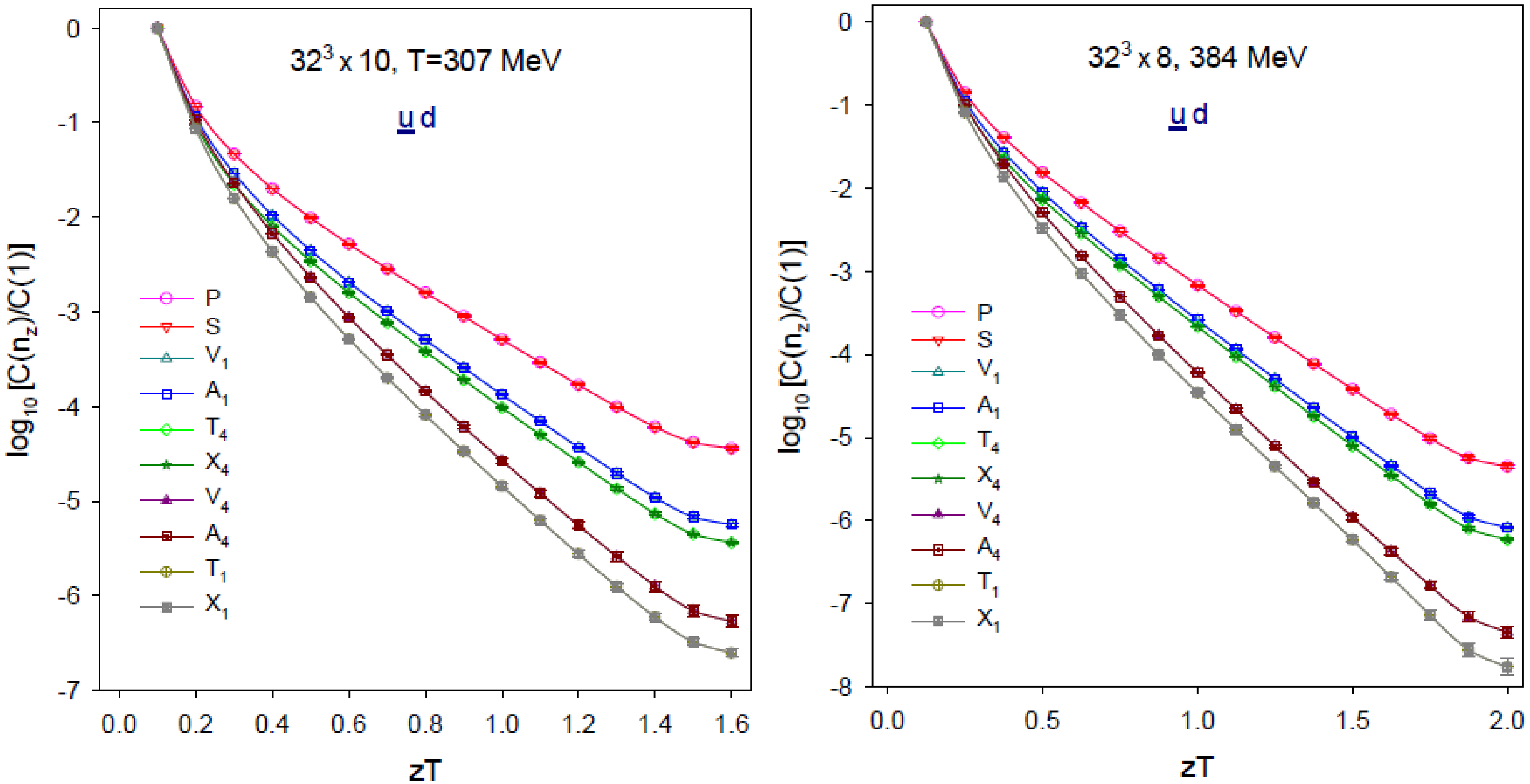
The Spatial z-Correlators of $\bar{u}d$ Mesons



$C_{V_k} = C_{A_k} \;\Rightarrow\; SU(2)_L \times SU(2)_R$ is effectively restored.

$C_S = C_P$ and $C_{T_k} = C_{X_k} \;\Rightarrow\; U(1)_A$ is effectively restored.

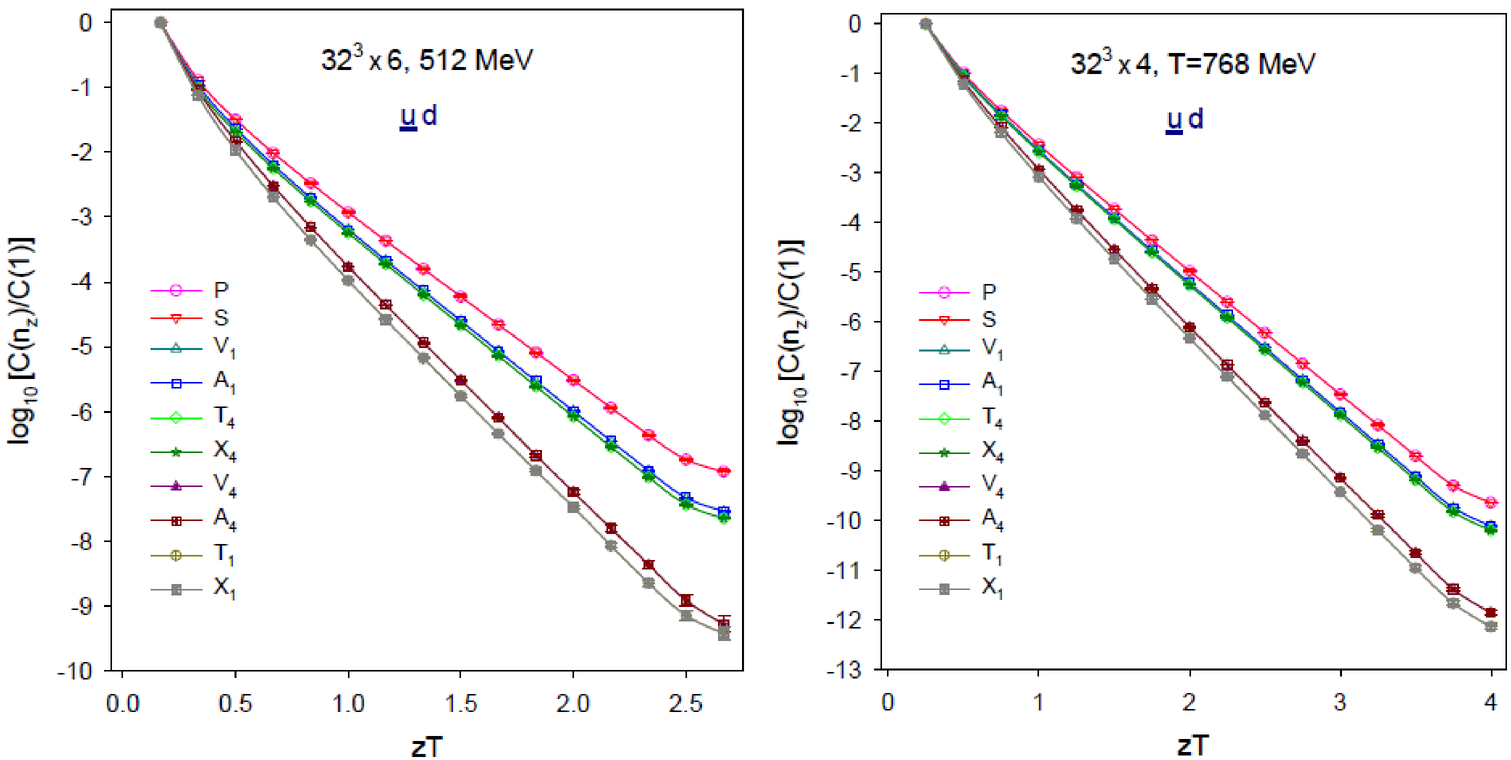
The Spatial z-Correlators of $\bar{u}d$ Mesons (cont)



As T is increased from $190 \to 240 \to 310 \to 385$ MeV, the emergence of $SU(2)_{CS}$ multiplets $\{V_1, V_2, A_1, A_2, T_4, X_4\}$ and $\{V_4, A_4, T_1, T_2, X_1, X_2\}$ are getting more and more pronounced.

$\Rightarrow$ The emergence of $SU(2)_{CS}$ chiral-spin symmetry.

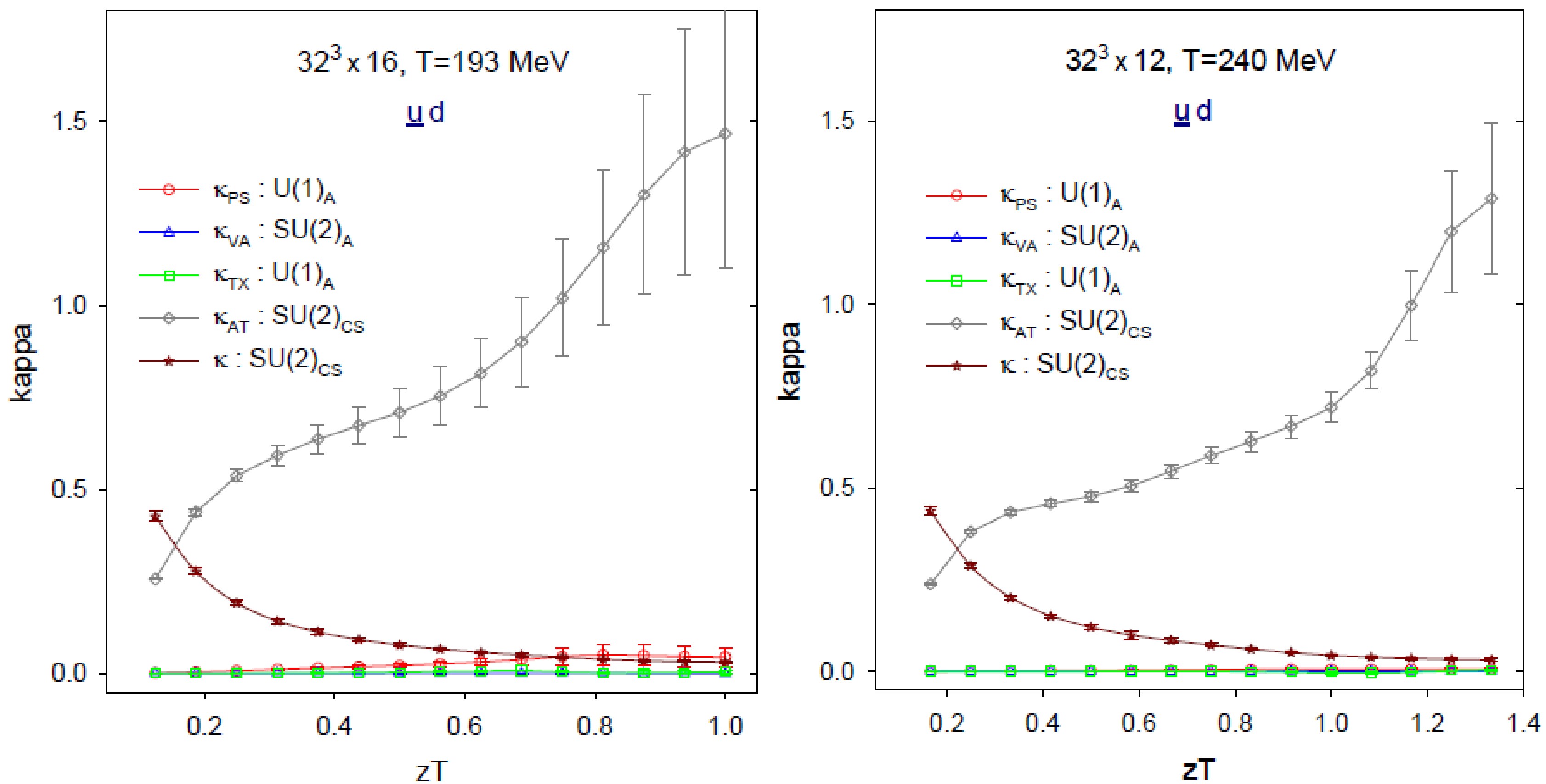
The Spatial z-Correlators of $\bar{u}d$ Mesons (cont)



As T is increased from $385 \to 510 \to 770$ MeV, the $SU(2)_{CS}$ multiplet $\{V_1, V_2, A_1, A_2, T_4, X_4\}$ are converging with $\{P, S\}$.

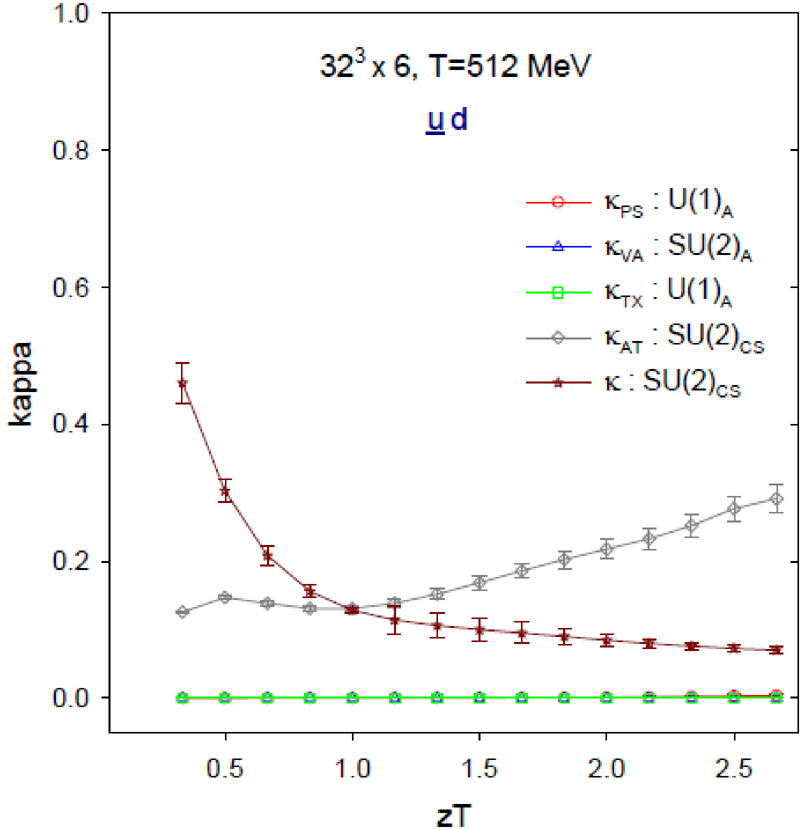
$\Rightarrow$ The $SU(2)_{CS}$ chiral-spin symmetry will be washed away for $T > T_s > 770$ MeV, and only the $\mathrm{U}(1)_A \times SU(2)_L \times SU(2)_R$ remains.

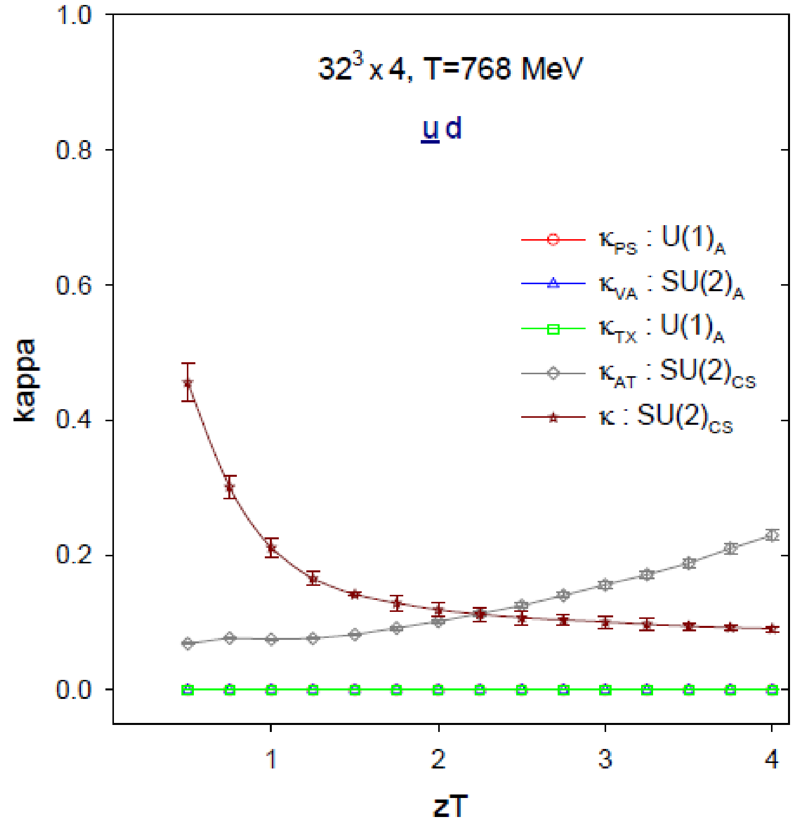
The Symmetry Breaking Parameters for z-Correlators



At $T = 193$ MeV, $\kappa_{PS} < 0.05(3)$ indicates that $U(1)_A$ is slightly broken in $\{P, S\}$ channel, while $\kappa_{TX} < 8.5(4.5) \times 10^{-3}$ seems to suggest that $U(1)_A$ is effectively restored in $\{T_1, X_1\}$ channel.

The Symmetry Breaking Parameters for z-Correlators (cont)





$\kappa_{VA} \simeq \kappa_{PS} \simeq \kappa_{TX} \approx 0,$

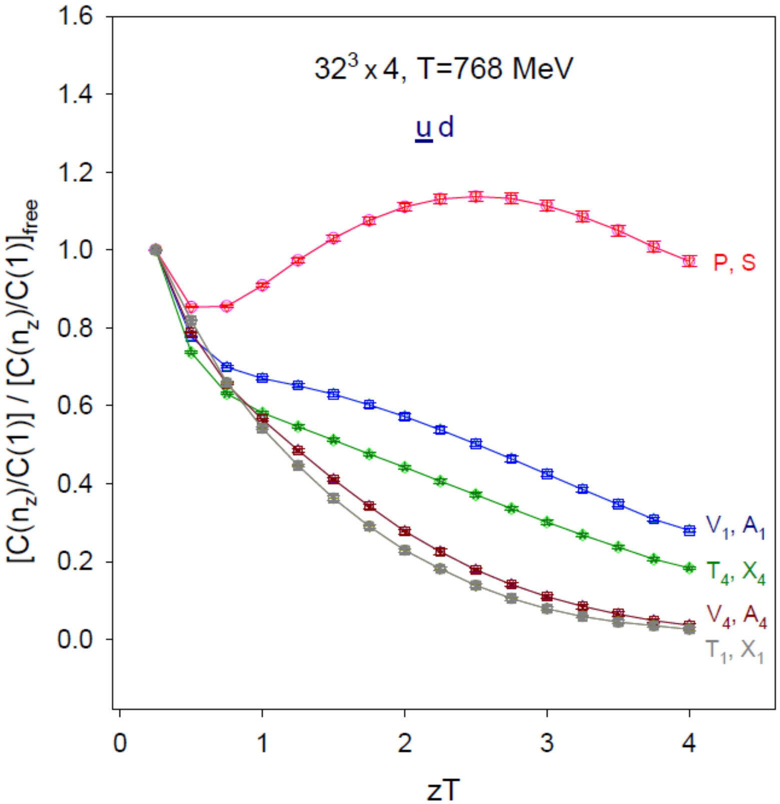
$\kappa_{AT} < 0.3, \quad \kappa < 0.5$

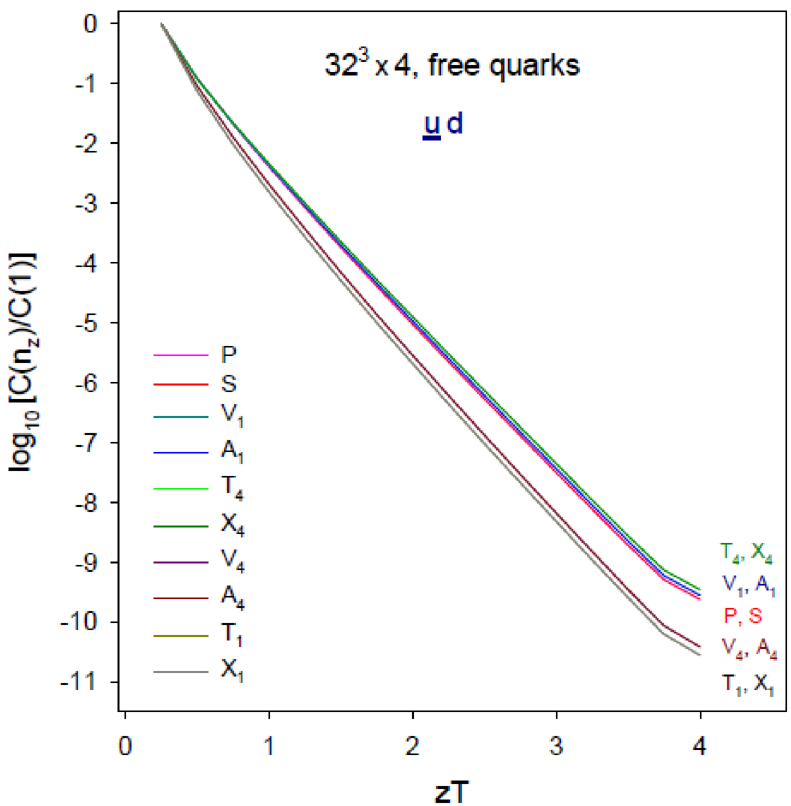
$\kappa_{VA} \simeq \kappa_{PS} \simeq \kappa_{TX} \approx 0,$

$\kappa_{AT} < 0.23, \quad \kappa < 0.5$

Even at $T \approx 770$ MeV, the $SU(2)_{CS}$ is still a rather approximate sym. comparing with the $U(1)_A \times SU(2)_L \times SU(2)_R$ chiral symmetry.

Comparison with the z-Correlators of Free Quarks





Even at $T \approx 770$ MeV, the meson z-correlators in $N_f = 2+1+1$ QCD are still quite different from those of the free quarks. This implies that quarks in QCD are not deconfined at such high T and the meson-like objects could be predominantly bounded by the chromoelectric fields.

Concluding Remarks

- From the symmetries of the $\bar{u}d$ meson correlators in $N_f = 2+1+1$ QCD at the physical point for $T \approx 190-770$ MeV, the $SU(2)_L \times SU(2)_R$ chiral symmetry is restored for $T \geq 190$ MeV, but $U(1)_A$ seems to be restored for $T > T_1 > 190$ MeV, in the (P, S) channel.
- Besides the $U(1)_A \times SU(2)_L \times SU(2)_R$ chiral symmetry, an approximate $SU(2)_{CS}$ chiral-spin sym. emerges for $T \approx 240-770$ MeV $\approx (1.6-5.1)T_c$. The emergence of $SU(2)_{CS}$ suggests the possible existence of hadron-like objects which are predominantly bounded by the chromoelectric field into color singlets.
- For $T \approx 190-770$ MeV, the $\bar{u}d$ meson correlators in $N_f = 2+1+1$ QCD are quite different from those of the free quarks. This implies that u/d quarks in QCD are not yet deconfined at $T \approx 770$ MeV, consistent with the emergence of $SU(2)_{CS}$ of chiral-spin symmetry.

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