Angular Momentum Inheritance from the Schwinger Effect in (Chromo)electromagnetic Fields

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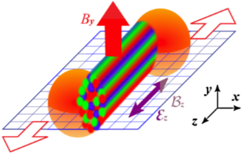
Based on: PC and Yoshimasa Hidaka, [2203.10917 (hep-ph)]

- In off-central heavy ion collisions a large magnetic field and *angular momentum* are created.
- → Quark gluon plasma most vortical fluid.
- **Global Λ hyperon polarization** is seen - STAR [Nature 548 (2017) 62; PRC 98 (2018) 014910.] Also a local polarization.

Schwinger produced pairs inherent properties of the background fields, e.g.,

1. Energy - Momentum
2. *CP* violation



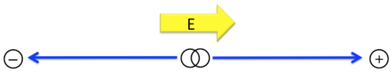
Pair production in color flux tube
[K. Fukushima, D.E. Kharzeev, and H.J. Warringa, *PRL* 104, 212001 (2010).]

What about Angular Momentum?

Schwinger Effect
Background

Under a background electric field the QFT vacuum is unstable against the production of particle anti-particle pairs:

Schwinger effect [J. Schwinger, *Phys. Rev.* 82, 664 (1951).]



Study the effect using, ↑, heuristic **Splitting Condensate Model**, and the **In-In Formalism** to one-loop.

Splitting Condensate Model

- *Serves as a simple and intuitive check of pair production!*

1. Consider a virtual particle anti-particle pair just produced from the vacuum.
2. Probability of occurance
   $\mathcal{W} = 1 - |\int \mathcal{D}\bar{\psi}\mathcal{D}\psi \, e^{i\int d^4x\bar{\psi}(i\not{D}-m)\psi}|^2 \sim \exp\left(-\frac{\pi m^2}{eE}\right)$
3. Pair evolves **classically** for long times, $\mathcal{T}$, under fields.

Background Fields (Homogeneous) with Lorentz Invariants

Look at simplest background with angular momentum density:

Homogeneous and (Abelian-like) Fields

Generalize setup to $SU(2)\times U(1)$ → simple to get to U(1).

- Background fields Abelian and SU(2) non-Abelian:

$$\mathbf{E} = F^{i0}\,, \quad \mathbf{B} = \widetilde{F}^{i0}\,, \quad \boldsymbol{\mathcal{E}} = \mathrm{tr}[IG^{i0}]\,, \quad \boldsymbol{\mathcal{B}} = \mathrm{tr}[I\widetilde{G}^{i0}]\,.$$

- Combined field strength: Isospin, Usual SU(2) field strength

$$\mathcal{F}_{\mu\nu} := eF_{\mu\nu} + g\,\mathrm{tr}[I\,G_{\mu\nu}]\,.$$

- Some Lorentz invariants: $I_{\tilde{F}F} = -\frac{1}{4}\widetilde{\mathcal{F}}_{\mu\nu}\mathcal{F}^{\mu\nu}\,,\ \ I_{FF} = \frac{1}{2}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}$

$$\lambda_E = \frac{1}{\sqrt{2}}\sqrt{\sqrt{I_{FF}^2 + 4I_{\tilde{F}F}^2} - I_{FF}}\,, \quad \lambda_B = \frac{1}{\sqrt{2}}\sqrt{\sqrt{I_{FF}^2 + 4I_{\tilde{F}F}^2} + I_{FF}}$$

Splitting Condensate Model

In SU(2)×U(1) two types of pairs possible:

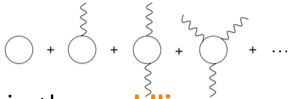
$$(\pm e, \pm g) \text{ as well as } (\pm e, \mp g)$$

In U(1) just one: $\pm e$.

Find the classical e.o.m.→ **Wong's Equations (Lorentz Force)**

- Let's begin with the partition function in SU(2)×U(1) for massive fermions [C. Schubert, *Phys. Reports* 355 (2001) 73.] for $\mathcal{D}_\mu = \partial_\mu + ieA_\mu(x) + ig\mathcal{A}_\mu(x)$:

$$e^{i\Gamma[A,\mathcal{A}]} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\Big[i\int d^4x\bar{\psi}(i\not{\mathcal{D}} - m)\psi\Big]$$

$\Gamma[A] =$ 

- The effective action in the **worldline representation** is [M. G. Schmidt and C. Schubert, *Phys. Lett. B* 318 (1993) 438; *Phys. Lett. B* 331 (1994) 69.]

$$\Gamma[A,\mathcal{A}] = \frac{i}{2}\int_0^\infty \frac{dT}{T}\oint \mathcal{D}x\mathcal{D}h\Phi[h]$$
$$\times\,\mathrm{tr}\mathcal{P}e^{-i\int_0^T d\tau\left[m^2h+\frac{1}{4h}\dot{x}^2+eA^\mu\dot{x}_\mu+g\mathcal{A}^\mu\dot{x}_\mu+\frac{h}{2}(eF_{\mu\nu}+gG_{\mu\nu})\sigma^{\mu\nu}\right]}$$

- The worldline "action" has matrix weight...
- *Coherent state method* converts Wilson loop to path integral.

1. Fermion d.o.f. with grassman variables: θ_μ [H. Kleinert, (2009).]
2. Color d.o.f. with coherent states: $u \in \mathrm{SU}(2)$
   Non-Abelian Stokes Theorem [K. Kondo, *PRD* 58, 105016 (1998).]

$$\Gamma[A,\mathcal{A}] = \frac{i}{2}\int_0^\infty \frac{dT}{T}\oint \mathcal{D}x\mathcal{D}u\mathcal{D}\theta\, e^{iS}$$

$$S = -\int_0^T d\tau\Big[\frac{m}{2}+\frac{m}{2}\dot{x}^2+eA_\mu\dot{x}^\mu+\frac{g}{2}tr(\sigma_3 u\mathcal{A}_\mu\dot{x}^\mu u^\dagger)-\frac{i}{2}tr(\sigma_3 u\dot{u}^\dagger)-\frac{i}{4}\theta_\mu\dot{\theta}^\mu+\frac{i}{4m}\mathcal{F}_{\mu\nu}\theta^\mu\theta^\nu\Big]$$

Equations of motion (**Wong's Equations** [S. Wong, (1970) 689.]):

$$\ddot{x}_\mu = \frac{1}{m}\mathcal{F}_{\mu\nu}\dot{x}^\nu\,, \quad \dot{\theta}_\mu = \frac{1}{m}\mathcal{F}_{\mu\nu}\theta^\nu\,, \quad \dot{I} = -[ig\mathcal{A}_\mu\dot{x}^\mu, I]\,, \quad I := \frac{1}{2}u^\dagger\sigma_3 u$$

→ Lorentz force solution! [D.M. Fradkin, *J. Phys. A* 11 (1978) 1069.]

$$\dot{x}(\tau) = \Big\{\big[\cosh\big(m^{-1}\lambda_E\tau\big) + \lambda_E^{-1}\mathcal{F}\sinh\big(m^{-1}\lambda_E\tau\big)\big]P_E + \big[\cos\big(m^{-1}\lambda_B\tau\big) + \lambda_B^{-1}\mathcal{F}\sin\big(m^{-1}\lambda_B\tau\big)\big]P_B\Big\}\dot{x}(0)$$

(matrix notation: $\mathcal{F} = \mathcal{F}^\mu_\nu$ and $\dot{x} = \dot{x}^\mu$)

- Projection operators: $P_E = \frac{\lambda_B^2+\mathcal{F}^2}{\lambda_E^2+\lambda_B^2}\,, \quad P_B = \frac{\lambda_E^2-\mathcal{F}^2}{\lambda_E^2+\lambda_B^2}\,.$
- From the grassman variable write a spin tensor

$$S_{\mu\nu} := -\frac{i}{2}\theta_\mu\theta_\nu\,, \quad \dot{S}_{\mu\nu} = \frac{1}{m}\mathcal{F}_{\mu\sigma}S^{\sigma}_{\ \nu} - \frac{1}{m}S_\mu^{\ \sigma}\mathcal{F}_{\sigma\nu}$$

Then the **Pauli-Lubanski vector** is

$$W_\mu := \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}S^{\nu\alpha}\dot{x}^\beta$$

- **BMT equation** [V. Bargmann, L. Michel, and V. L. Telegdi, *PRL* 2, 435 (1959).] can be found as (for $s = 2$)

$$\dot{W}_\mu = \frac{1}{m}\mathcal{F}_{\mu\nu}W^\nu$$

- Solution is the same as for the Lorentz force!

$$W(\tau) = \Big\{\Big[\cosh\big(m^{-1}\lambda_E\tau\big) + \lambda_E^{-1}\mathcal{F}\sinh\big(m^{-1}\lambda_E\tau\big)\Big]P_E + \Big[\cos\big(m^{-1}\lambda_B\tau\big) + \lambda_B^{-1}\mathcal{F}\sin\big(m^{-1}\lambda_B\tau\big)\Big]P_B\Big\}W(0)$$

- Vector current reads for $\mathcal{W} \propto \exp\big(-\frac{\pi m^2}{\lambda_E}\big)$ for $(\pm e, \pm g)$ pair,

$$\mathbf{J}_{(\pm e,\pm g)} = \frac{e}{2}\mathcal{W}\int d^3y d\tau \Big\{\Big[\delta^4(y-x(\tau))\dot{\mathbf{x}}(\tau)\Big|_{e,g\to e,g} - \Big["\Big|_{e,g\to -e,-g}\Big\}\,.$$

- Only electric fields in the direction $\hat{\mathbf{x}}_E$: $\lambda_E = |g\mathcal{E}|$, and $\lambda_B = 0$:

$$\mathbf{J}_{(\pm e,\pm g)} = \mathrm{sgn}(g)\frac{e}{2}\frac{\lambda_E^2\mathcal{V}\mathcal{T}}{\pi^3}\exp\Big(-\frac{\pi m^2}{\lambda_E}\Big)\hat{\mathbf{x}}_E\,.$$

However, $\mathbf{J}_{(\pm e,\mp g)} = -\mathbf{J}_{(\pm e,\pm g)}$: **sum of the two currents vanishes**! [N. Tanji, *PRD* 92 (2015) 125012.]

- Likewise we can determine the polarization as

$$P^\mu_{(\pm e,\pm g)} = \frac{\mathcal{W}}{2}\Big\{\Big[\frac{W^\mu(y_0)}{|\dot{x}^0(y_0)|}\Big|_{e,g\to e,g} + \Big[\frac{W^\mu(y_0)}{|\dot{x}^0(y_0)|}\Big|_{e,g\to -e,-g}\Big\}\,.$$

- Polarization is in parallel fields:

$$P^0 = P^0_{(\pm e,\pm g)} + P^0_{(\pm e,\mp g)} = I_{\tilde{F}F}\frac{\mathcal{T}\mathcal{V}}{\pi^2}e^{-\frac{\pi m^2}{\lambda_E}}$$

In agreement with the **Chiral Anomaly**!

- Take for the energy-momentum tensor of the pair:

$$\mathcal{T}^{\mu\nu}_{(\pm e,\pm g)} = \frac{m\mathcal{W}}{2}\left\{\left[\frac{\dot{x}^{\mu}(y_0)\dot{x}^{\nu}(y_0)}{|\dot{x}^{0}(y_0)|}\right|_{e,g\to e,g} + \left[\frac{\dot{x}^{\mu}(y_0)\dot{x}^{\nu}(y_0)}{|\dot{x}^{0}(y_0)|}\right|_{e,g\to -e,-g}\right\}.$$

- With **orbital angular momentum** given by

$$L^{\mu\nu\sigma} = x^{\nu}\mathcal{T}^{\mu\sigma} - x^{\sigma}\mathcal{T}^{\mu\nu}.$$

- We find for the total spatial angular momentum

$$\mathbf{L} = \frac{\mathcal{T}^2\mathcal{V}}{2\pi^2}\frac{\mathcal{E}_{\parallel}^2|g\mathcal{B}_{\parallel}|}{\mathcal{B}_{\parallel}^2+\mathcal{E}_{\parallel}^2}\coth\left(\frac{|\mathcal{B}_{\parallel}|\pi}{|\mathcal{E}_{\parallel}|}\right)e^{-\frac{m^2\pi}{|g\mathcal{E}_{\parallel}|}}g^2\mathbf{z}\times\left[\boldsymbol{\mathcal{E}}(z)\times\boldsymbol{\mathcal{B}}(z)+\boldsymbol{\mathcal{E}}(z)\times\boldsymbol{\mathcal{B}}_{\parallel}+\boldsymbol{\mathcal{E}}_{\parallel}\times\boldsymbol{\mathcal{B}}(z)\right].$$

$\propto$ **Net Angular Momentum of Fields**

- Let's confirm similar behavior at the full quantum level!
- For the non-equilibrium Schwinger effect use **in-in or Schwinger Keldysh** formalism [J. Schwinger, *J. Math. Phys.* (1961).]

In-in propagator is an augmentation of the proper time integral [E. Fradkin, G. Gitman, and S. Shvartsman, (1991).]:

$$S^c_{\rm in}(x,y) = i\langle {\rm in}|\mathcal{T}\psi(x)\bar{\psi}(y)|{\rm in}\rangle = (i\not{D} + m)\int_{\rm in} dT\,\mathcal{K}(x,y,T)\,,$$

$$\int_{\rm in} dT := \int_0^\infty dT - \int_{0-i\frac{\pi}{\lambda_E}}^{\infty-i\frac{\pi}{\lambda_E}} dT - \Theta(z)\oint_{-i\frac{\pi}{\lambda_E}} dT$$

$$\Theta(z) := \theta(n^{-\,T}_{\lambda_E} z)\theta((n^{-\,T}_{\lambda_E} z)^2 - (n^{+\,T}_{\lambda_E} z)^2)$$

Electromagnetic tensor electric eigenvectors:

$$\mathcal{F} n^\pm_E = \pm\lambda_E n^\pm_E\,, \quad n^+_{\lambda_E} = \frac{1}{2}(n^-_E + n^+_E)\,, \quad n^-_{\lambda_E} = \frac{1}{2}(n^-_E - n^+_E)$$

$\mathcal{K}(x, y, T) = i\langle \mathrm{x}| e^{-i(\hat{\not{\mathcal{P}}}^2 + m^2)T} |\mathrm{y}\rangle$ can be solved exactly [J. Schwinger, *Phys. Rev.* 82, 664 (1951).] for arbitrary homogeneous fields, even SU(2)×U(1)! (Here $^+$ refers to the $g \to +g$ part of the color):

$$\mathcal{K}^+(x, y, T) = \frac{\lambda_E \lambda_B \exp\left[-im^2 T + i\varphi(x, y, T)\right]}{(4\pi)^2 \sinh(\lambda_E T) \sin(\lambda_B T)} \Phi(T),$$

$$\varphi(x, y, T) = \frac{1}{2} x^T \mathcal{F} y - \frac{1}{4} z^T \coth(\mathcal{F}T) \mathcal{F} z,$$

$$\Phi(T) = \cos(\lambda_B T) \cosh(\lambda_E T) + i\gamma_5 \mathrm{sgn}(I_{\tilde{F}F}) \sin(\lambda_B T) \sinh(\lambda_E T)$$
$$- \Big\{ \Big[\frac{1}{2} (\lambda_B + i\gamma_5 \mathrm{sgn}(I_{\tilde{F}F}) \lambda_E) \frac{\mathcal{F}_{\mu\nu} \sigma^{\mu\nu}}{\lambda_B^2 + \lambda_E^2} \Big]$$
$$\times \left[i \sin(\lambda_B T) \cosh(\lambda_E T) + \gamma_5 \mathrm{sgn}(I_{\tilde{F}F}) \cos(\lambda_B T) \sinh(\lambda_E T) \right] \Big\}$$

1. Φ contains all the spin information of the system.
2. Poles of $\sinh(\lambda_E)$ contain Schwinger pair production info.

Vector current [E. Fradkin, G. Gitman, and S. Shvartsman, (1991).]

$$j^\mu = e\langle \mathrm{in}|\bar{\psi}\gamma^\mu\psi|\mathrm{in}\rangle = ie \lim_{x\to y} \mathrm{tr}[\gamma^\mu S^c_{\mathrm{in}}(x,y)]$$

$$= e\frac{\lambda_E \lambda_B T}{2\pi^2} e^{-\frac{m^2\pi}{\lambda_E}} \coth\left(\frac{\lambda_B \pi}{\lambda_E}\right) n^{-\mu}_{\lambda_E} + [g \to -g]\,.$$

- For simple parallel fields used before, however, $n^{-\mu}_{\lambda_E}$ is odd under g, therefore $j^\mu = 0$.

Spin Angular Momentum (Axial Vector Current) [PC, K. Fukushima and S. Pu, *PRL* 121 (2018) 261602.]

$$S^{\mu\nu\sigma} = \langle \mathrm{in}|\frac{1}{2}\bar{\psi}\gamma^\mu\sigma^{\nu\sigma}\psi|\mathrm{in}\rangle = \frac{i}{2}\lim_{x\to y} \mathrm{tr}[\gamma^\mu\sigma^{\nu\sigma} S^c_{\mathrm{in}}(x,y)]$$

$$S^{0ij} = -\varepsilon^{kij}\frac{I_{\tilde{F}F} T}{2\pi^2} e^{-\frac{m^2\pi}{\lambda_E}} n^{+k}_{\lambda_E} + [g \to -g]\,.$$

- *However*, $n^{+k}_{\lambda_E}$ is a time-like vector, and we can always find a frame in which **there is no spin angular momentum**.

- The orbital angular momentum is for fully symmetric stress energy tensor

$$l^{\mu\nu\sigma} = x^{\nu}\mathbb{T}_{\rm C}^{\mu\sigma} - x^{\sigma}\mathbb{T}_{\rm C}^{\mu\nu}\,, \quad \mathbb{T}^{\mu\nu} = (1/4)\big(T^{(\mu\nu)} + [T^{(\mu\nu)}]^*\big)$$

$$T^{\mu\nu} = \langle {\rm in}|\bar{\psi}\gamma^{\mu} i\mathcal{D}^{\nu}\psi|{\rm in}\rangle = T_{\rm C}^{\mu\nu} + T_{\Omega}^{\mu\nu}$$

- Expand around UV divergence and take *conduction current part*, $T_{\rm C}^{\mu\nu}$ (ignore polarization current part T_{Ω})

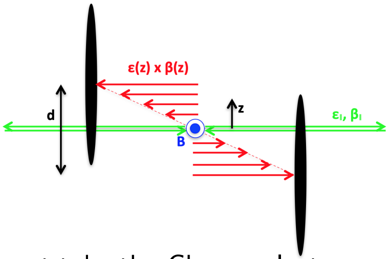
$$\mathbb{T}_{\rm C}^{\mu\nu+} = \coth\Big(\frac{\pi\lambda_B}{\lambda_E}\Big)\frac{\lambda_B}{4\pi^2}e^{-\frac{\pi m^2}{\lambda_E}}\Big\{(P_E^{\mu\nu} + 2n_{\lambda_E}^{-\mu}n_{\lambda_E}^{-\nu})\frac{\lambda_E^2\mathcal{T}^2}{4} + P_E^{\mu\nu}m^2\ln\Big(\frac{\lambda_E\mathcal{T}}{m}\Big)\Big\}$$

- Spatial angular momentum: $(\mathbf{l})^a = \frac{1}{2}\varepsilon^{aij}l^{0ij} \rightarrow$

$$\mathbf{l} \approx \frac{\lambda_B}{(4\pi)^2}e^{-\frac{\pi m^2}{\lambda_E}}\coth\Big(\frac{\pi\lambda_B}{\lambda_E}\Big)\frac{(\lambda_E\mathcal{T})^2}{\lambda_E^2+\lambda_B^2}\mathbf{x}\times(g\boldsymbol{\mathcal{E}}\times(e\mathbf{B}+g\boldsymbol{\mathcal{B}})) + [g\rightarrow -g]$$

<u>Angular momentum of produced pairs is acquired from fields.</u>

HIC Motivated Background Field Setup



- *For a given event* take the Chromoelectromagnetic fields to be decomposed with homogeneous contribution:

$$\boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{E}}(z) + \boldsymbol{\mathcal{E}}_{\parallel}\,, \quad \boldsymbol{\mathcal{B}} = \boldsymbol{\mathcal{B}}(z) + \boldsymbol{\mathcal{B}}_{\parallel}\,.$$

- Chromoelectromagnetic field w/ momentum $\boldsymbol{\mathcal{E}}(z), \boldsymbol{\mathcal{B}}(z)$

$$\boldsymbol{\mathcal{E}}(z) \times \boldsymbol{\mathcal{B}}(z) = \mathcal{E}_z \mathcal{B}_z \frac{2z}{d} \hat{\mathbf{x}}_{\parallel}\,, \quad |\boldsymbol{\mathcal{E}}(z)| \sim |\boldsymbol{\mathcal{B}}(z)|\,.$$

- CP violating chromoelectromagnetic fields $\mathcal{E}_{\parallel}, \mathcal{B}_{\parallel}$

$$g^2 \mathcal{E}_{\parallel} \mathcal{B}_{\parallel} \sim Q_s^2 \quad \boldsymbol{\mathcal{E}}_{\parallel} = \pm \mathcal{E}_{\parallel} \hat{\mathbf{x}}_{\parallel}\,, \quad \boldsymbol{\mathcal{B}}_{\parallel} = \pm \mathcal{B}_{\parallel} \hat{\mathbf{x}}_{\parallel}\,.$$

- Magnetic field, $\mathbf{B} = B\hat{\mathbf{x}}_{\perp}$.

HIC Motivated Background Field Setup

Average over all events:

$$\langle\!\langle o \rangle\!\rangle \coloneqq \frac{1}{4} \sum_{\pm\mathcal{E}_\parallel, \pm\mathcal{B}_\parallel} \int_{-d/2}^{d/2} dz\, o(z)\,.$$

$$\mathbf{l}_{\mathcal{F}} = \mathbf{z} \times (\boldsymbol{\mathcal{E}} \times \boldsymbol{\mathcal{B}})\,, \quad \langle\!\langle \mathbf{l}_{\mathcal{F}} \rangle\!\rangle = \mathcal{E}_z \mathcal{B}_z \frac{d^2}{6} \hat{\mathbf{x}}_\perp\,, \quad \langle\!\langle l_{\tilde{F}F} \rangle\!\rangle \approx g^2 \langle\!\langle \boldsymbol{\mathcal{E}}_\parallel \cdot \boldsymbol{\mathcal{B}}_\parallel \rangle\!\rangle = 0\,.$$

Splitting Condensate Model

$$\langle\!\langle J^\mu \rangle\!\rangle = 0\,, \quad \langle\!\langle P^0 \rangle\!\rangle = 0\,, \quad \langle\!\langle \mathbf{z} \times \mathbf{P} \rangle\!\rangle = 0\,,$$

$$\langle\!\langle \mathbf{L} \rangle\!\rangle = \frac{|g\mathcal{B}_\parallel|}{2\pi^2} \frac{\mathcal{E}_\parallel^2 \mathcal{T}^2 \mathcal{V}}{\mathcal{B}_\parallel^2 + \mathcal{E}_\parallel^2} \coth\!\left(\frac{|\mathcal{B}_\parallel|\pi}{|\mathcal{E}_\parallel|}\right) e^{-\frac{m^2\pi}{|g\mathcal{E}_\parallel|}} g^2 \langle\!\langle \mathbf{l}_{\mathcal{F}} \rangle\!\rangle$$

In-In Formalism

$$\langle\!\langle j^\mu \rangle\!\rangle = 0\,, \quad \langle\!\langle S^{0ij} \rangle\!\rangle = 0\,,$$

$$\langle\!\langle \mathbf{l} \rangle\!\rangle = \frac{|g\mathcal{B}_\parallel|}{8\pi^2} e^{-\frac{\pi m^2}{|g\mathcal{E}_\parallel|}} \coth\!\left(\frac{\pi|\mathcal{B}_\parallel|}{|\mathcal{E}_\parallel|}\right) \frac{\mathcal{E}_\parallel^2 \mathcal{T}^2}{\mathcal{E}_\parallel^2 + \mathcal{B}_\parallel^2} g^2 \langle\!\langle \mathbf{l}_{\mathcal{F}} \rangle\!\rangle$$

Conclusions

Angular Momentum from the Schwinger Mechanism

1. Use a heuristic **splitting condensate model** to easily and intuitively understand physics of Schwinger pair production.
2. Confirm out-of-equilibrium observables using **in-in formalism**.
3. Orbital angular momentum + CP violation background→ **orbital angular momentum + chiral density of pairs**.
4. In HIC motivated model just orbital angular momentum is transported.

Thank you for your time and attention!