Collective Neutrino Oscillations

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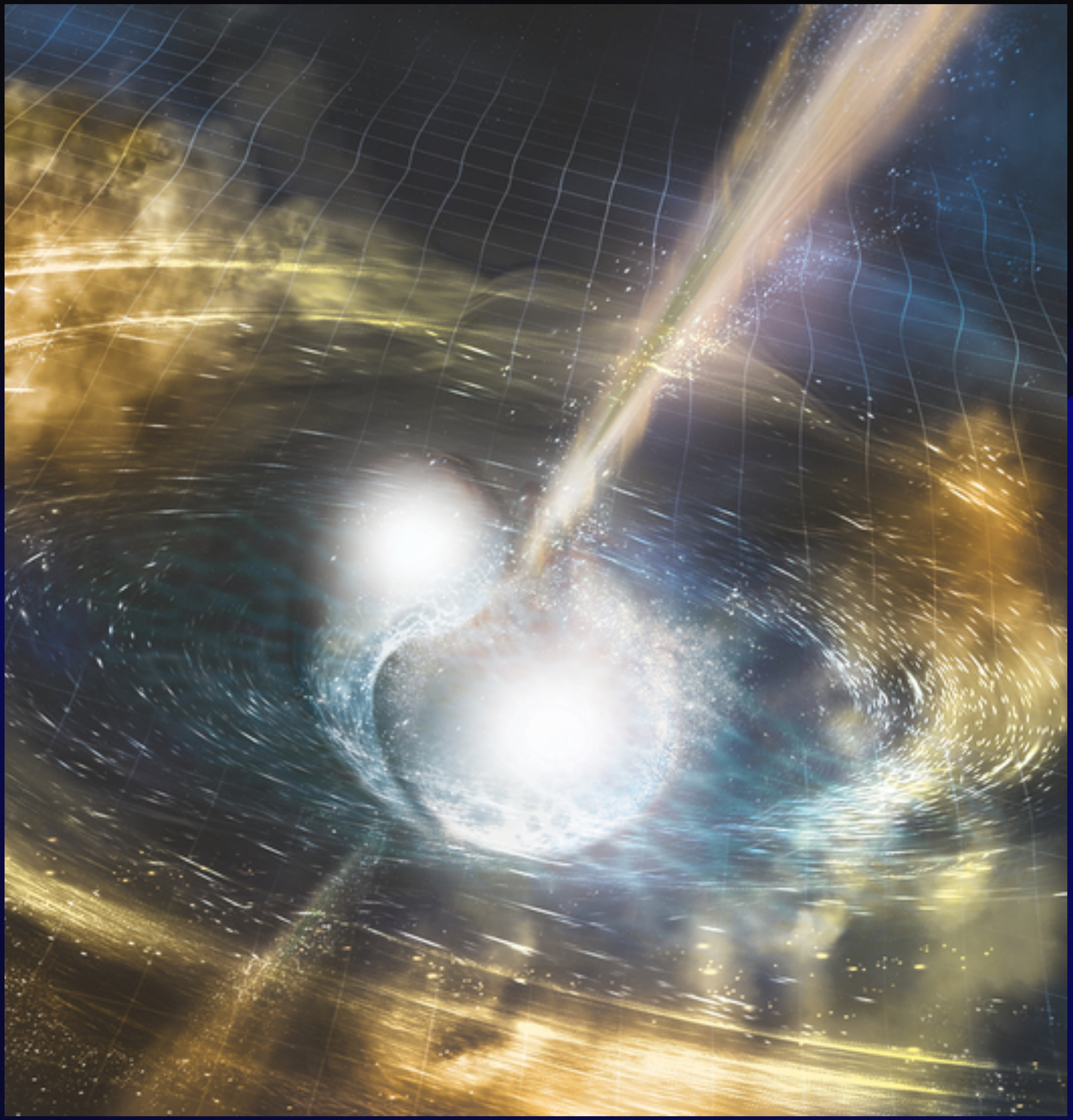
U.S. DEPARTMENT OF ENERGY | Office of Science

Supernova

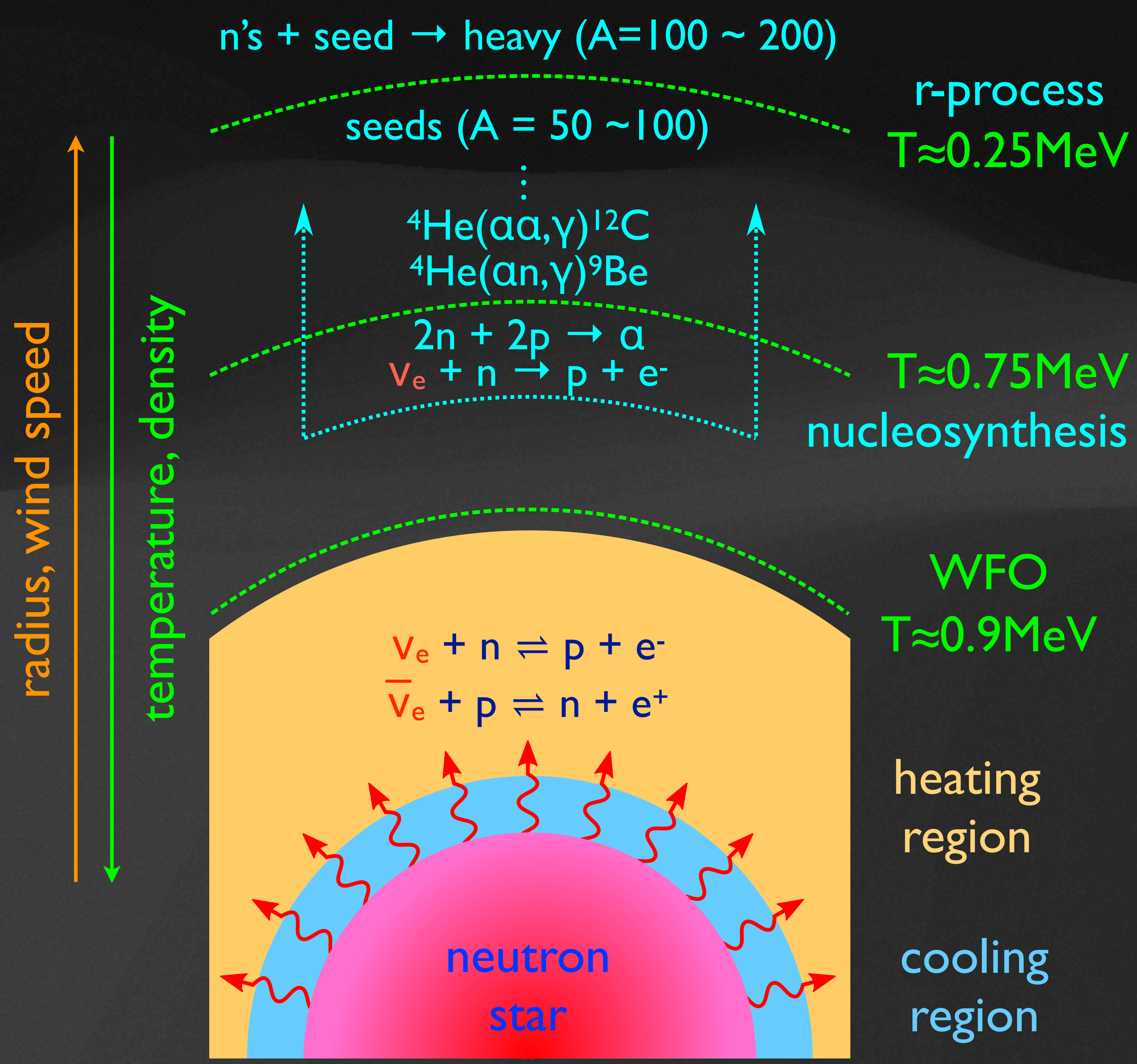
David Malin / AAO

Neutron Star Merger



NSF/LIGO/Sonoma State University/A. Simonnet

Neutrinos in Supernovae



- ~10^{53} ergs, 10^{58} neutrinos in ~10 seconds
- All neutrino species, 10~30 MeV
- Dominate energetics
- Influence nucleosynthesis

Outline

- Introduction
  - Physics and numerical modeling
- Some Theories
  - Flavor instabilities and flavor crossing
  - Collective oscillation waves
- Effects of Collisions
  - Neutral- and charged-current scattering
- Summary

Incomplete references!!

Introduction

Neutrino Mixing

WEAK FLAVOR STATES

VACUUM MASS EIGENSTATES

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12}e^{\mathrm{i}\phi} - c_{12}s_{13}s_{23} & c_{12}c_{23}e^{\mathrm{i}\phi} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{23}s_{12}e^{\mathrm{i}\phi} - c_{12}c_{23}s_{13} & -c_{12}s_{23}e^{\mathrm{i}\phi} - c_{23}s_{12}s_{13} & c_{13}c_{23} \end{pmatrix}^* \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

$$\delta m_{12}^2 \simeq \delta m_\odot^2 \simeq 7\text{–}8 \times 10^{-5}\mathrm{eV}^2, \quad \theta_{12} \simeq \theta_\odot \simeq 0.6$$

$$|\delta m_{23}^2| \simeq \delta m_{\mathrm{atm}}^2 \simeq 2\text{–}3 \times 10^{-3}\mathrm{eV}^2, \quad \theta_{23} \simeq \theta_{\mathrm{atm}} \simeq \frac{\pi}{4}$$

$$|\delta m_{13}^2| \simeq |\delta m_{23}^2| \simeq 2\text{–}3 \times 10^{-3}\mathrm{eV}^2, \quad \theta_{13} \simeq 0.15$$

ϕ is unknown ← CP VIOLATION PHASE

Vacuum Oscillations

neutrino mass eigenstates ≠ weak interaction states

$$|\nu_1\rangle = \cos\theta_{\rm v}|\nu_e\rangle + \sin\theta_{\rm v}|\nu_\mu\rangle \quad \text{with mass } m_1$$

$$|\nu_2\rangle = -\sin\theta_{\rm v}|\nu_e\rangle + \cos\theta_{\rm v}|\nu_\mu\rangle \quad \text{with mass } m_2$$

vacuum mixing angle

initially $|\psi(x=0)\rangle = |\nu_e\rangle$

$$P_{\nu_e\nu_e}(x) \equiv |\langle\nu_e|\psi(x)\rangle|^2 = 1 - \sin^2 2\theta_{\rm v} \sin^2\left(\frac{\delta m^2 x}{4E_\nu}\right)$$

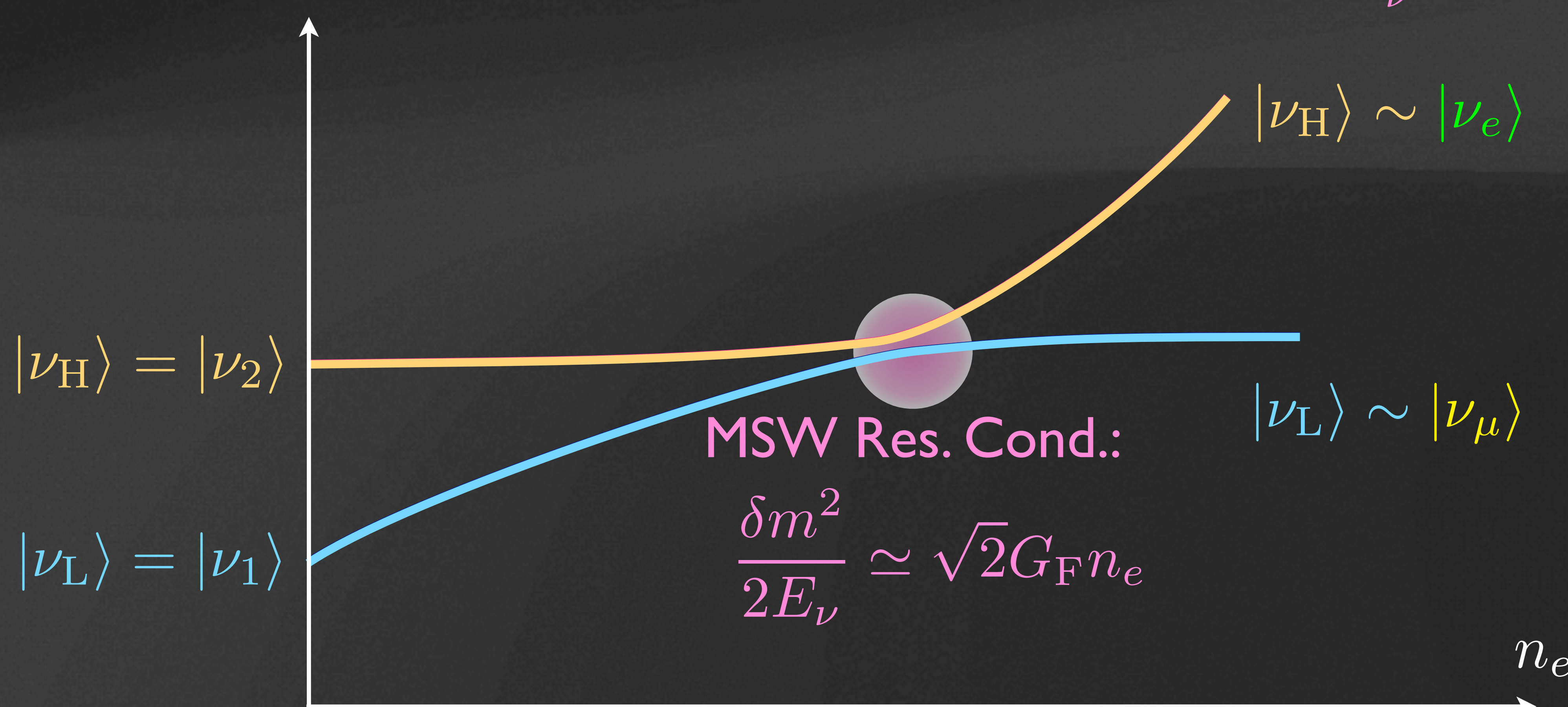
neutrino survival probability

Matter Effect

electron number density

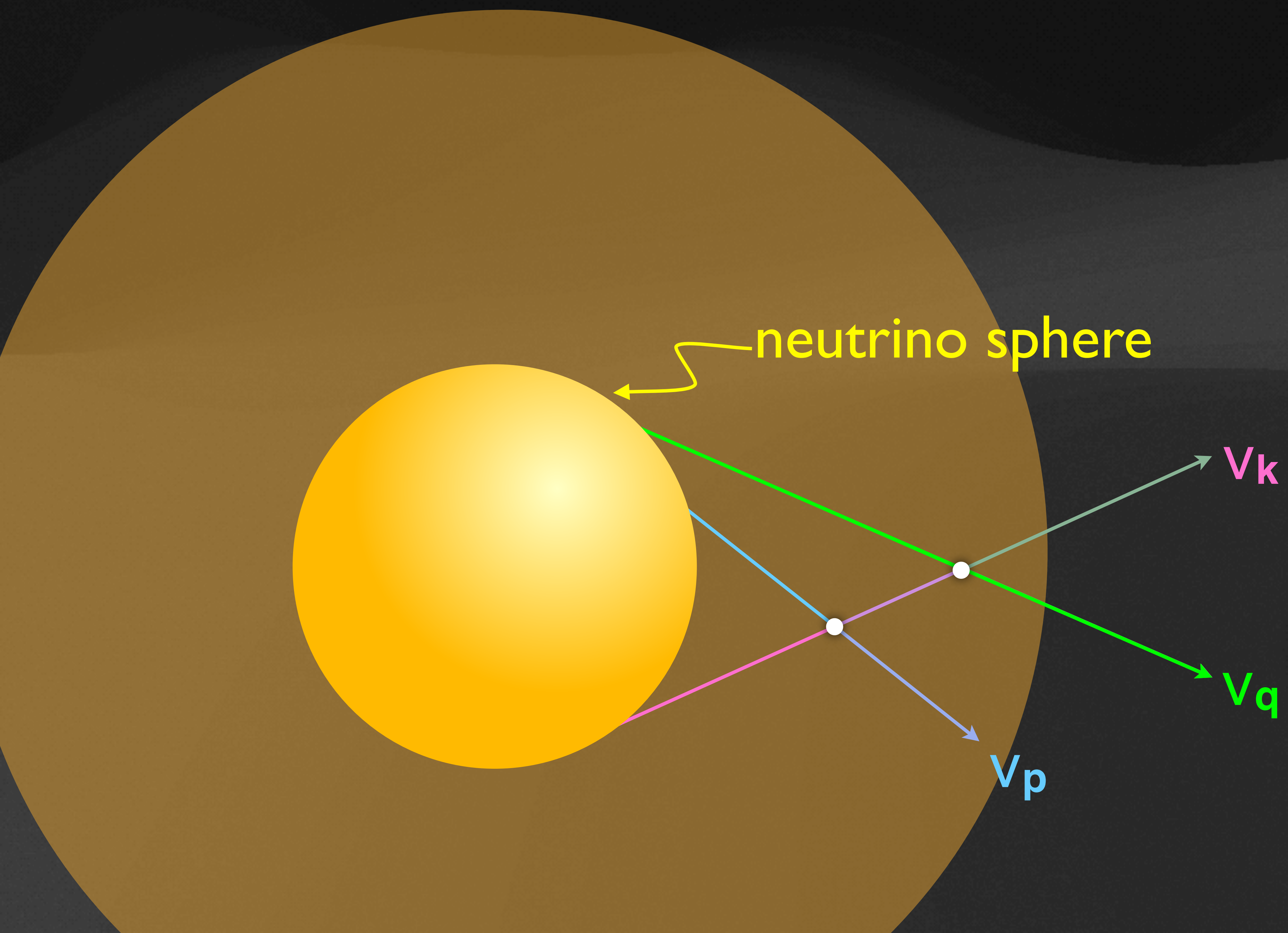
$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x}\begin{bmatrix}\langle\nu_e|\psi_\nu\rangle\\ \langle\nu_\mu|\psi_\nu\rangle\end{bmatrix} = \frac{1}{2}\begin{bmatrix}2\sqrt{2}G_\mathrm{F}n_e - \omega\cos 2\theta_\mathrm{v} & \omega\sin 2\theta_\mathrm{v}\\ \omega\sin 2\theta_\mathrm{v} & \omega\cos 2\theta_\mathrm{v}\end{bmatrix}\begin{bmatrix}\langle\nu_e|\psi_\nu\rangle\\ \langle\nu_\mu|\psi_\nu\rangle\end{bmatrix}$$

vac. osc. freq. $\omega = \frac{\delta m^2}{2E_\nu}$



Mikheyev & Smirnov (1985)

Dense Neutrino Gas



Mean-Field Approximation

Flavor density matrix

$$[\rho_{\mathbf{p}}(t,\mathbf{r})]_{\alpha\beta} = \int e^{i\mathbf{q}\cdot\mathbf{r}} \langle \Psi(t) \,|\, \hat{a}_\alpha^\dagger(\mathbf{p}-\mathbf{q}/2)\, \hat{a}_\beta(\mathbf{p}+\mathbf{q}/2) \,|\, \Psi(t) \rangle \frac{d^3 q}{(2\pi)^3}$$

$$\rho = \begin{bmatrix} f_{\nu_e} & S \\ S^* & f_{\nu_x} \end{bmatrix}$$

f_{ν_α} : occupation

S : coherence

Beyond MF: See Roggero (Thursday) & Martin (Friday)

Sigl & Raffelt (1993)

Flavor Transport

Equation of motion

$$(\partial_t + \hat{\mathbf{v}} \cdot \nabla)\rho = -\,\mathrm{i}[\mathrm{H}, \rho] + \mathscr{C}$$

← Collision

mass matrix → M^2

electron density → n_e

$$\mathrm{H} = \frac{\mathrm{M}^2}{2E} + \sqrt{2}G_\mathrm{F}\,\mathrm{diag}[n_e, 0, 0] + \mathrm{H}_{\nu\nu}$$

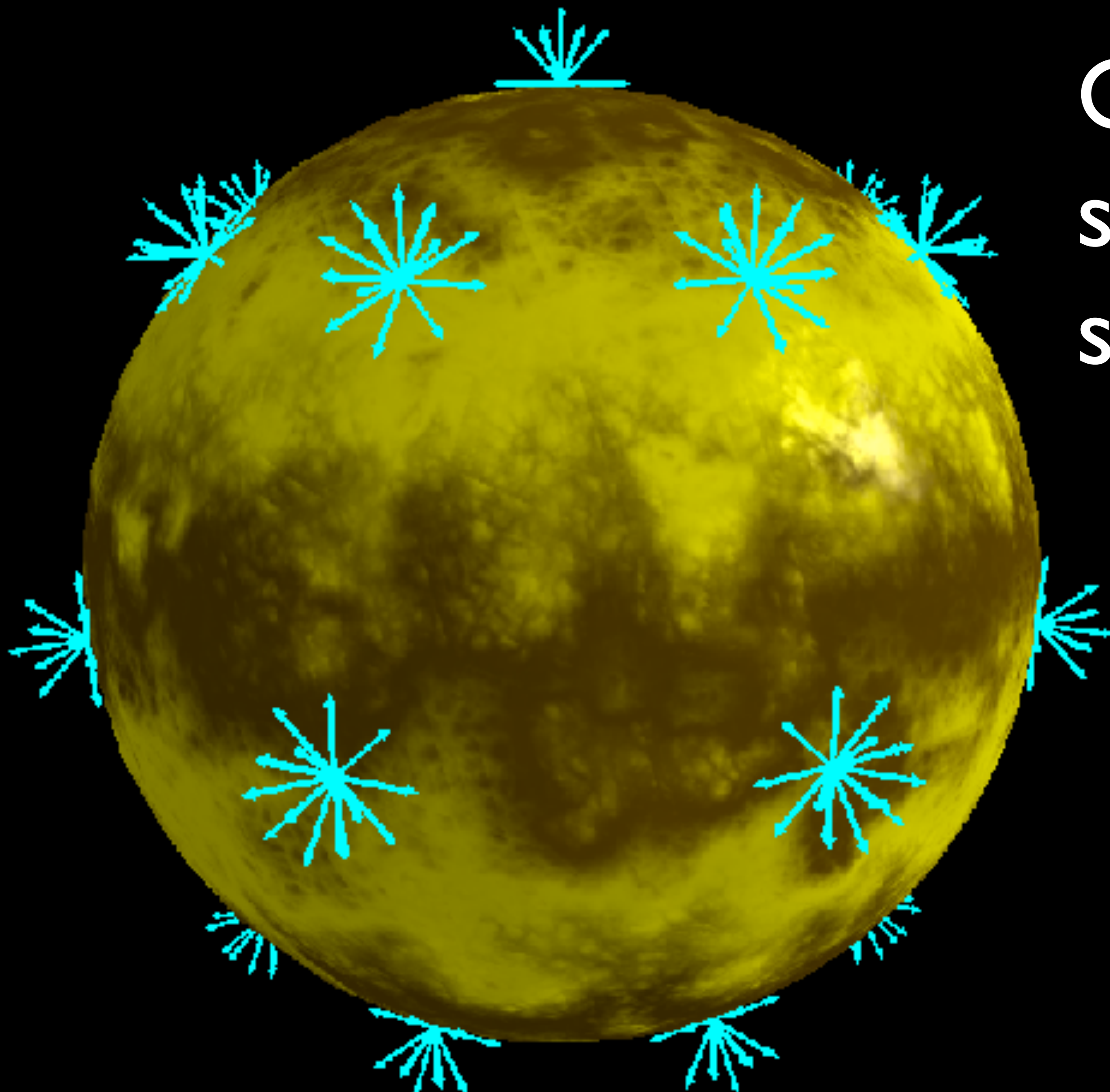
neutrino energy → E

v-v forward scattering (self-coupling) → $\mathrm{H}_{\nu\nu}$

$$\mathrm{H}_{\nu\nu} = \sqrt{2}G_\mathrm{F} \int \mathrm{d}^3\mathbf{p}'(1 - \hat{\mathbf{v}} \cdot \hat{\mathbf{v}}')(\rho_{\mathbf{p}'} - \bar{\rho}_{\mathbf{p}'})$$

Sigl & Raffelt (1993)

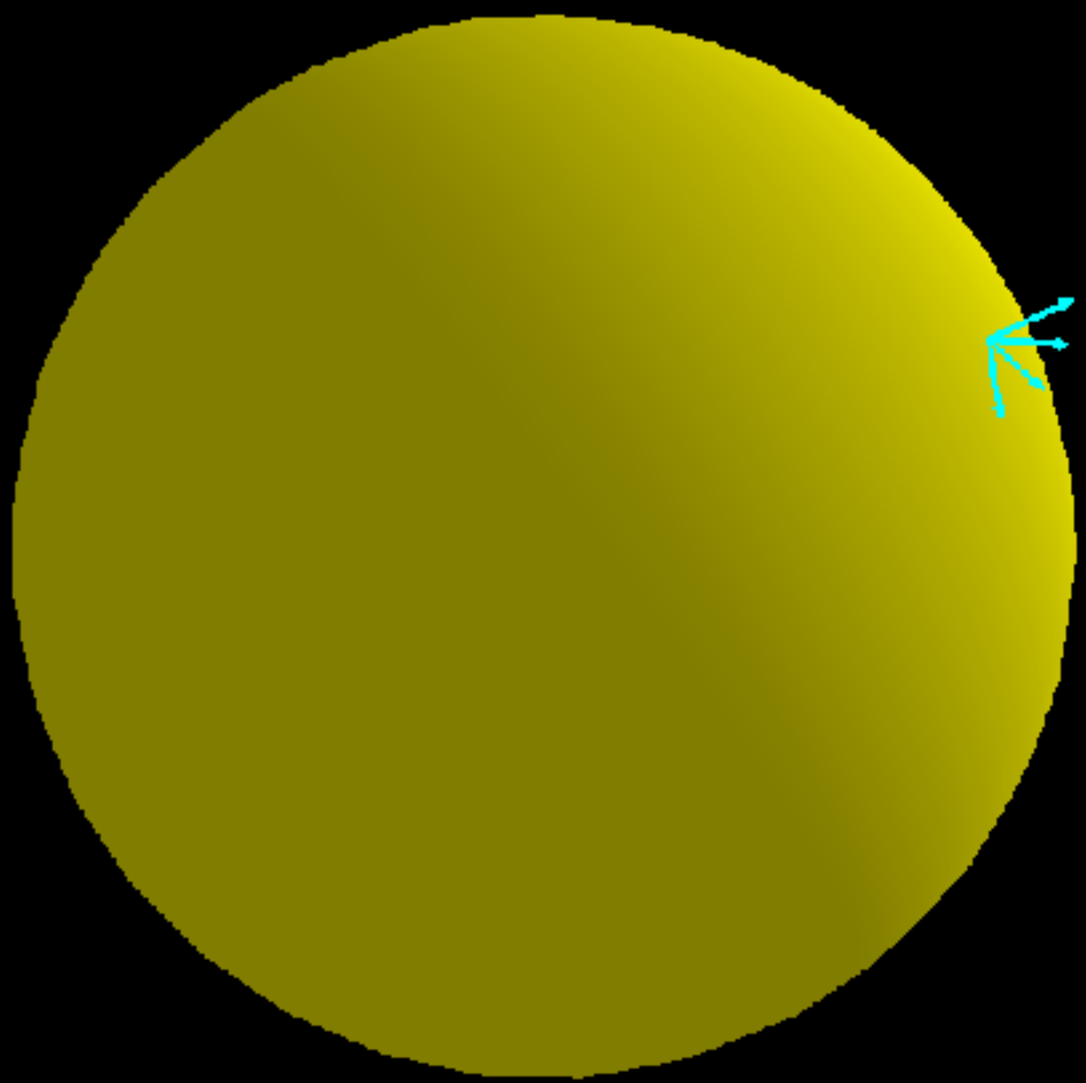
7D Problem



Coherent forward scattering outside neutrino sphere

$$\rho_{\mathbf{p}}(t, \mathbf{r})$$

Bulb Model

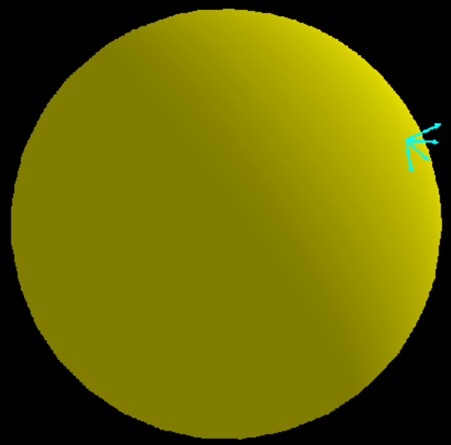


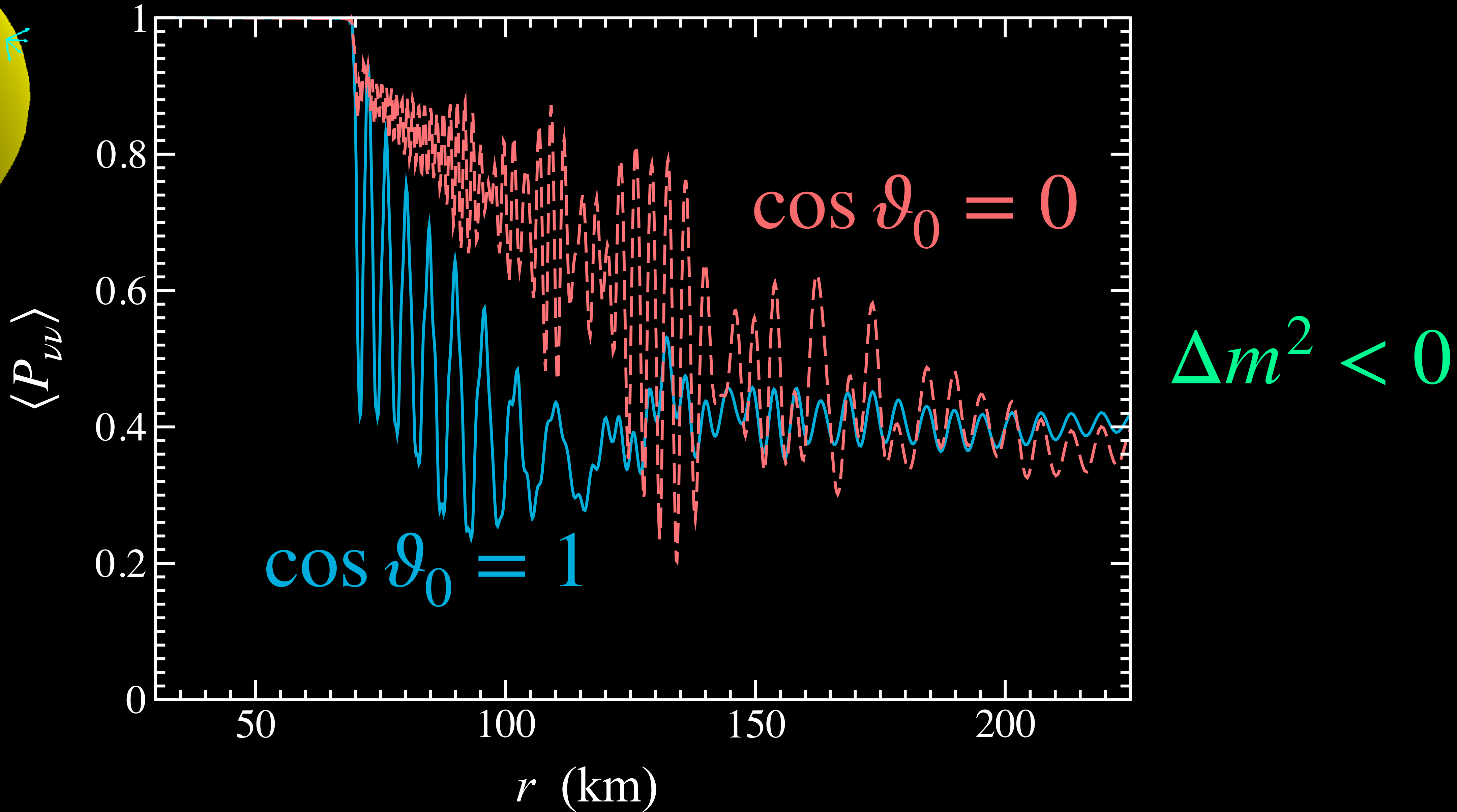
Azimuthal symmetry around any radial direction

$$\rho_{E,\vartheta}(r)$$

Numerical Results

Bulb model (late time)

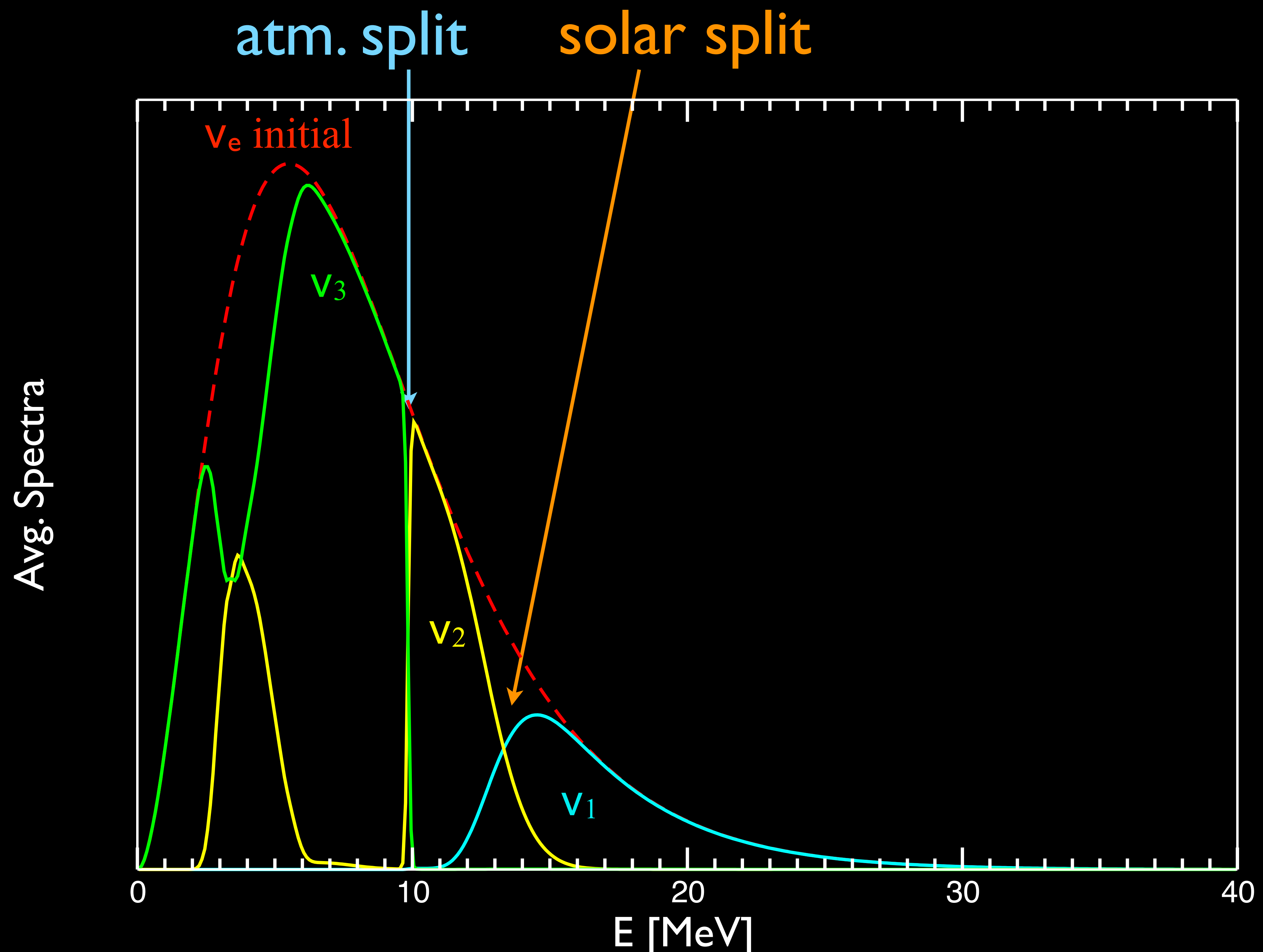




HD+ (2006)

Numerical Results

Neutronization burst



Duan+ (2007), Cherry+ (2010)

Some Theories

Linear Stability Analysis

Homogeneous and isotropic gas

Electron flavor neutrinos and antineutrinos initially

$$\rho \propto \begin{bmatrix} 1 & S \\ S^* & 0 \end{bmatrix} \qquad \bar{\rho} \propto \begin{bmatrix} 1 & \bar{S} \\ \bar{S}^* & 0 \end{bmatrix}$$

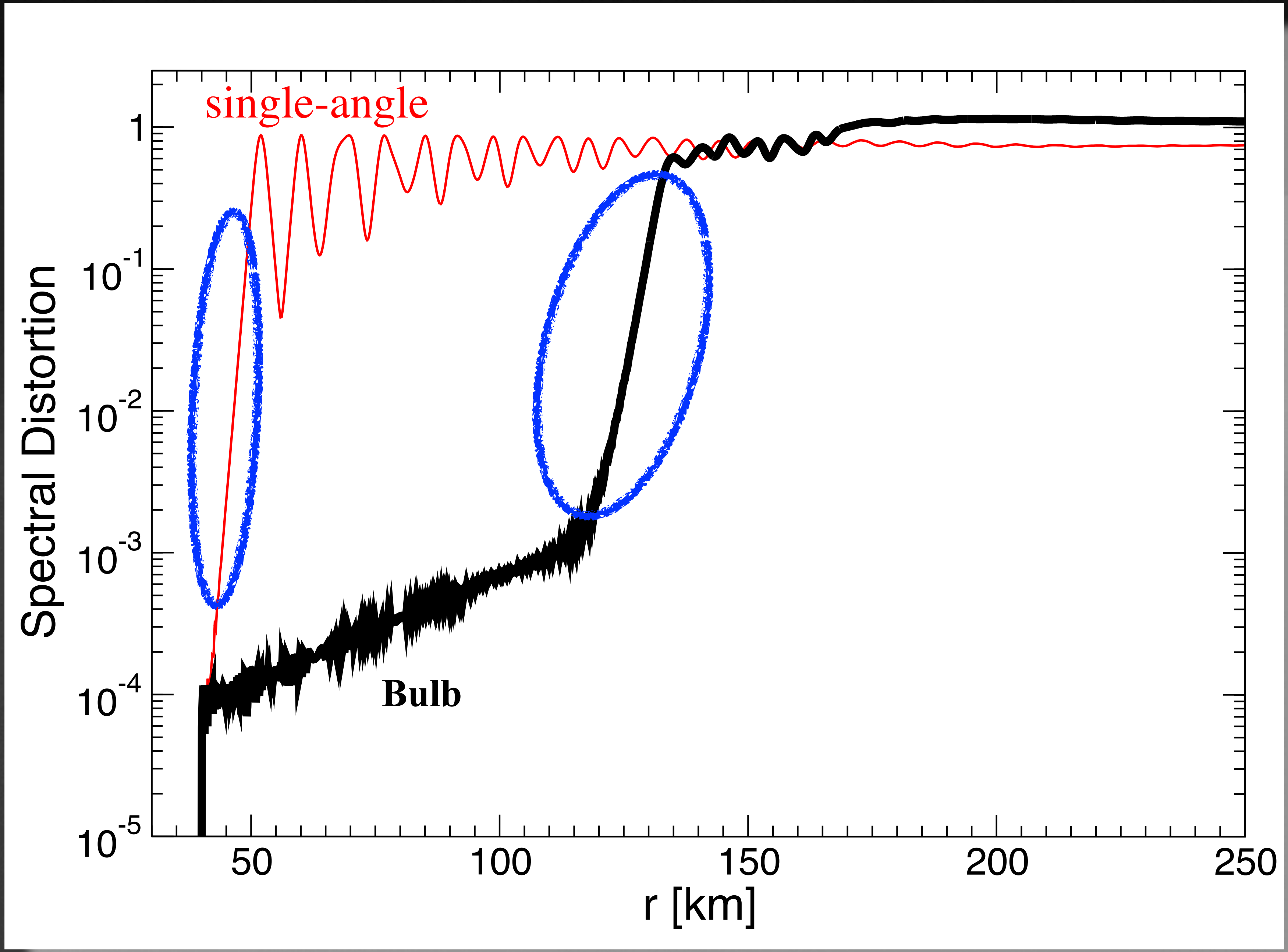
$$\mathrm{i} \begin{bmatrix} \dot{S} \\ \dot{\bar{S}} \end{bmatrix} \approx \begin{bmatrix} -\omega - \alpha\mu & \alpha\mu \\ -\mu & \omega + \mu \end{bmatrix} \begin{bmatrix} S \\ \bar{S} \end{bmatrix}$$

$\omega = \Delta m^2 / 2E$
$\alpha = n_{\bar{\nu}} / n_\nu$
$\mu \propto n_\nu$

- Normal modes —> Collective oscillations ($S, \bar{S} \sim e^{-\mathrm{i}\Omega t}$)
- $\mathrm{Im}(\Omega) > 0$ —> Flavor instabilities

Banerjee+ (2011)

Flavor Instabilities



HD & Friedland (2010)

Collective Oscillation Wave

$$S_{\mathbf{p}}(t, \mathbf{r}) \propto e^{-\mathrm{i}(\Omega t - \mathbf{K}\cdot\mathbf{r})}$$

- Collective flavor oscillations are the collective wave modes in the neutrino gas with the dispersion relation $\Omega(\mathbf{K})$.
- $\mathrm{Im}(\Omega) > 0$ —> Flavor instabilities.
- **Slow** oscillations occur on the distance scale of **1 km** ($\sim 10\,\mathrm{MeV}/\Delta m^2_{\mathrm{atm}}$).
- **Fast** oscillations can occur on the distance scale of **1 cm** ($\sim 1/G_{\mathrm{F}} n_\nu$), independent of the neutrino energies (Sawyer, 2016).
- Collective oscillations spontaneously break the spatial symmetries in the nonlinear regime.

HD+ (2008)
Izaguirre+ (2017)

Neutrino Flavor-Spin and Distribution

Aka flavor isospin and ELN distribution

$$G(E, \hat{\mathbf{v}}) = \begin{cases} f_{\nu_e} - f_{\nu_x} & \text{if } E > 0 \\ f_{\bar{\nu}_x} - f_{\bar{\nu}_e} & \text{if } E < 0 \end{cases}$$

$E < 0$ $\longleftarrow\longrightarrow$ $E > 0$

$$\begin{bmatrix} \bar{\nu}_x \\ -\bar{\nu}_e \end{bmatrix} \qquad \begin{bmatrix} \nu_e \\ \nu_x \end{bmatrix}$$

HD+ (2005)
Dasgupta+ (2009)

Flavor Instability and Crossing

$$G(E, \hat{\mathbf{v}}) = \begin{cases} f_{\nu_e} - f_{\nu_x} & \text{if } E > 0 \\ f_{\bar{\nu}_x} - f_{\bar{\nu}_e} & \text{if } E < 0 \end{cases}$$

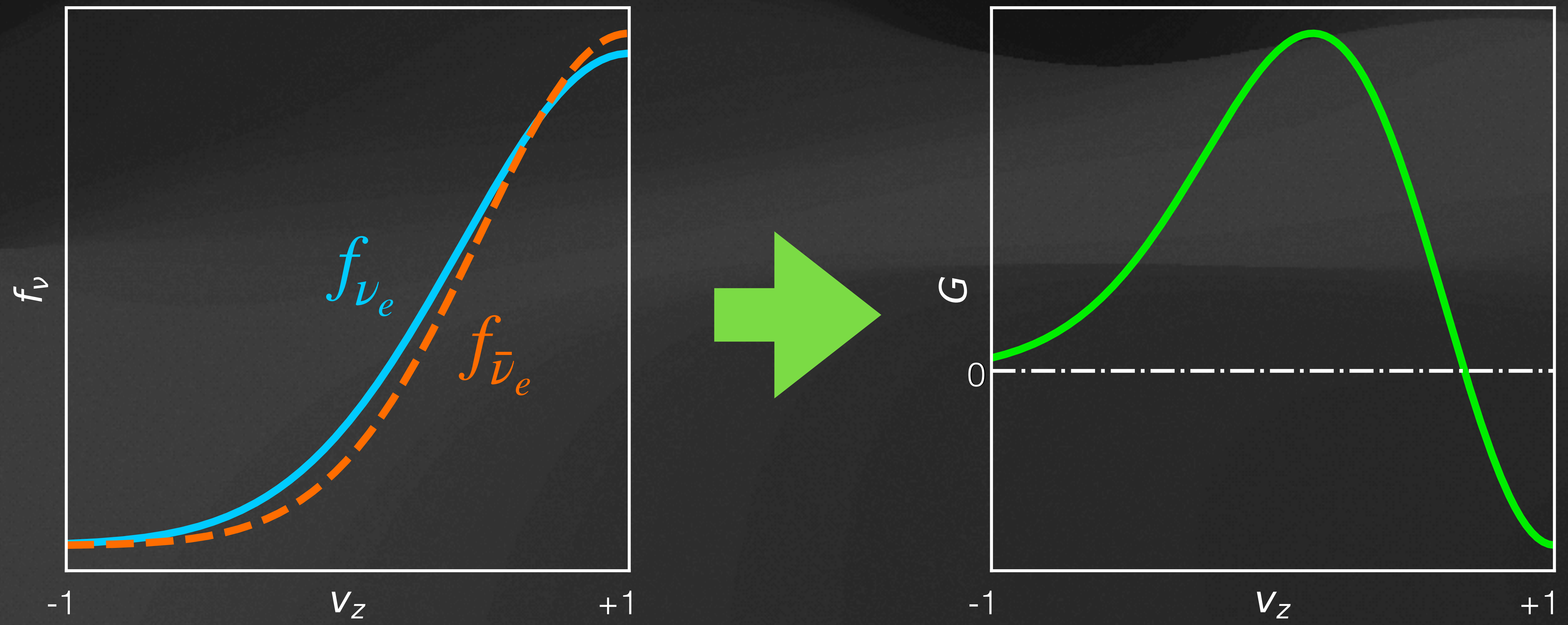
- Identical neutrino angular distribution: **Slow** flavor instability requires crossing in $G(E)$.
- **Fast** flavor instability requires crossing in $G(\hat{\mathbf{v}})$.
- Mixing of the fast and slow instabilities?

Dasgupta+ (2009)
Izaguirre+ (2017)
Airen+ (2018)

Flavor Crossing

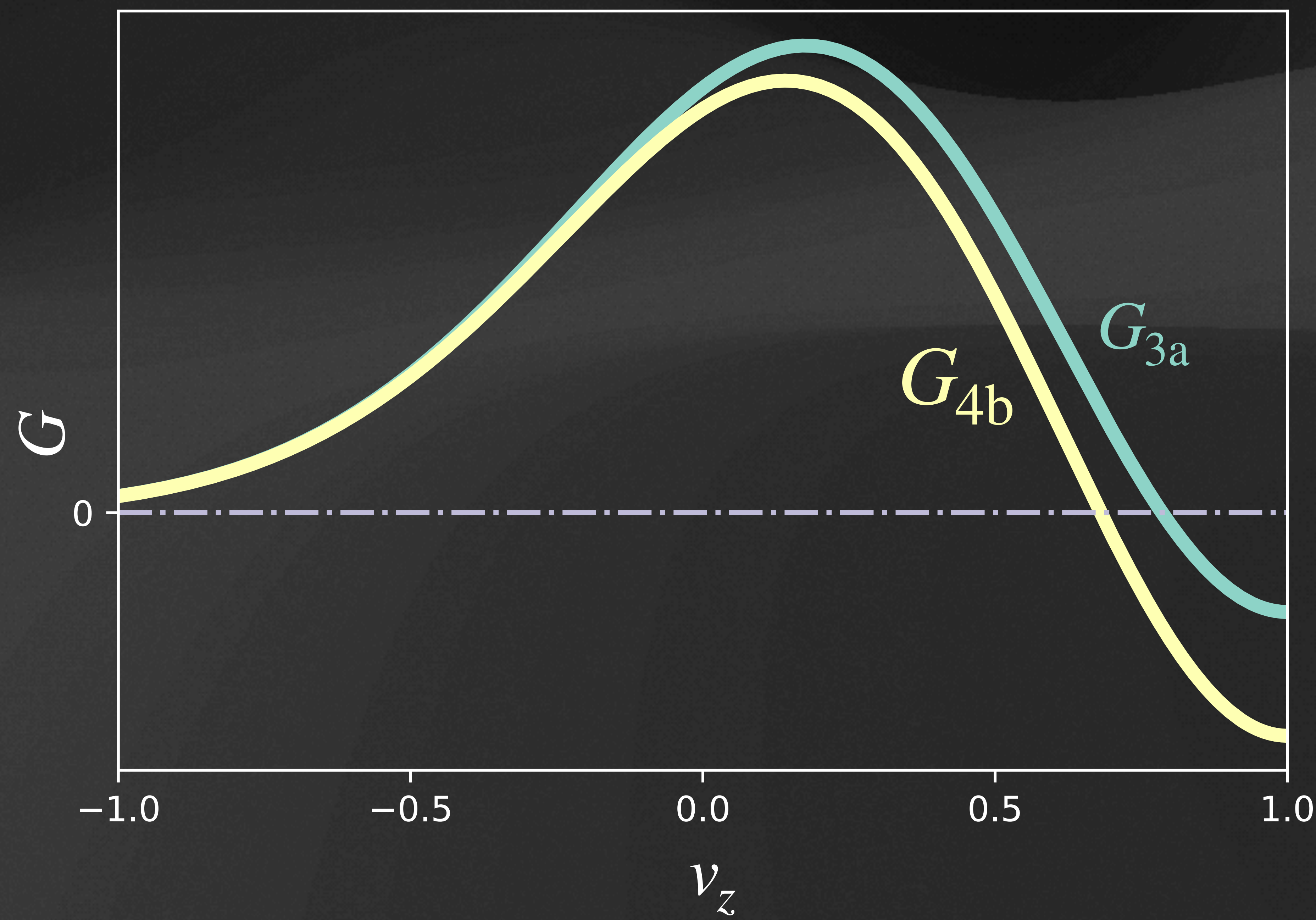
1D axisymmetric neutrino gas

$$G \sim (f_{\nu_e} - f_{\bar{\nu}_e}) - (f_{\nu_x} - f_{\bar{\nu}_x})$$



Flavor Distribution Crossing

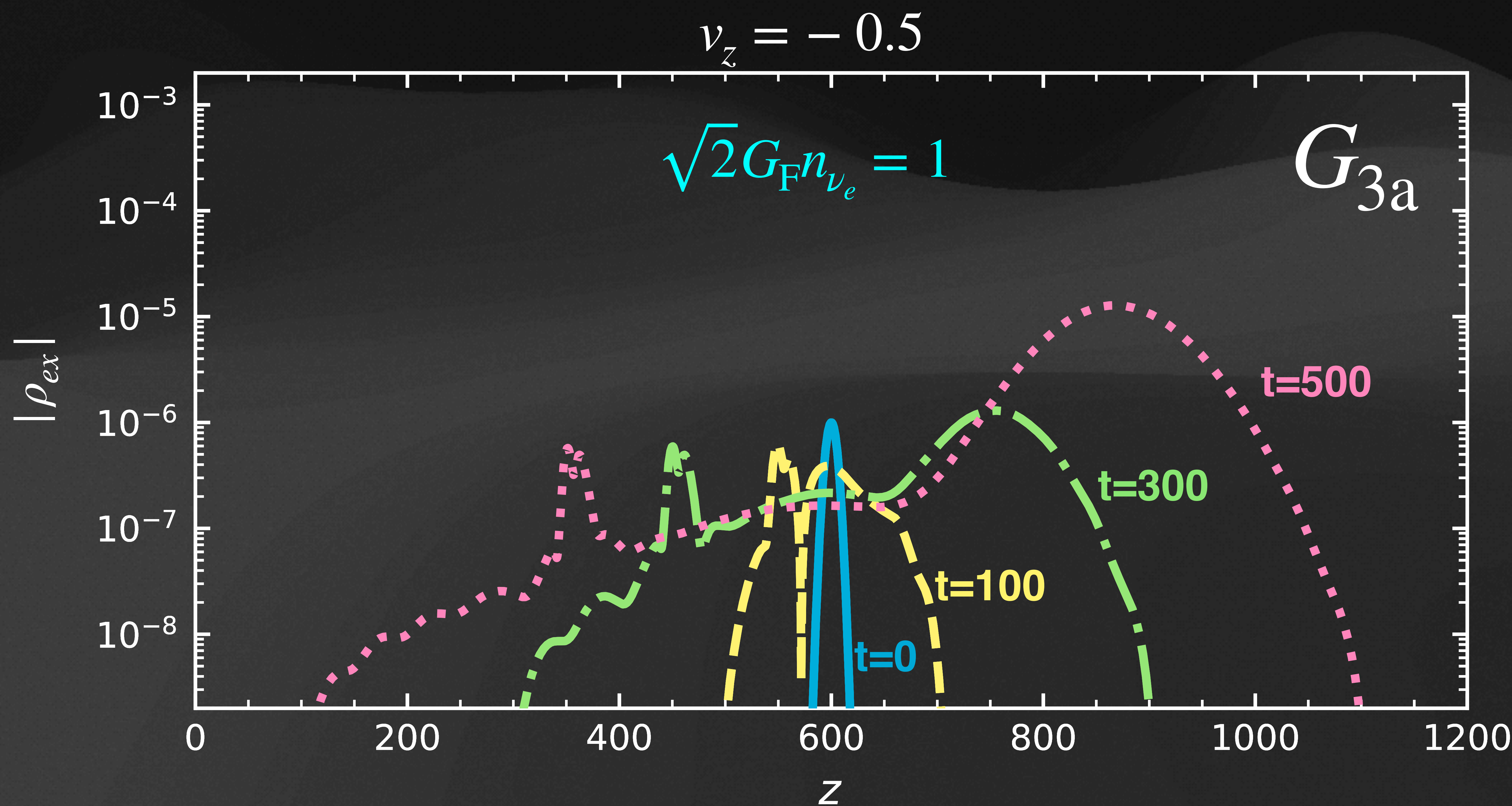
1D axisymmetric neutrino gas



Martin+ (2020)

Convective Instability

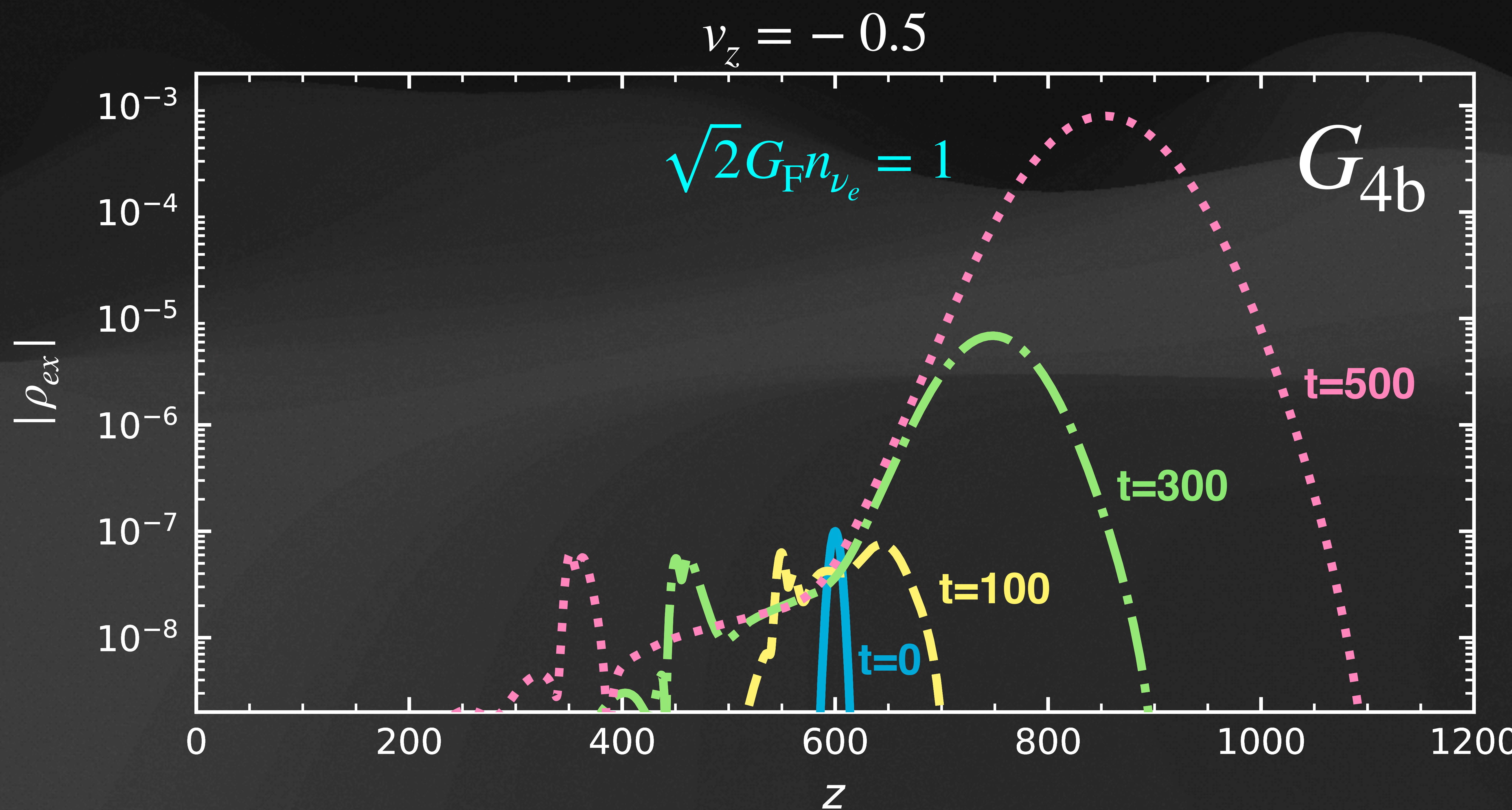
1D axisymmetric neutrino gas



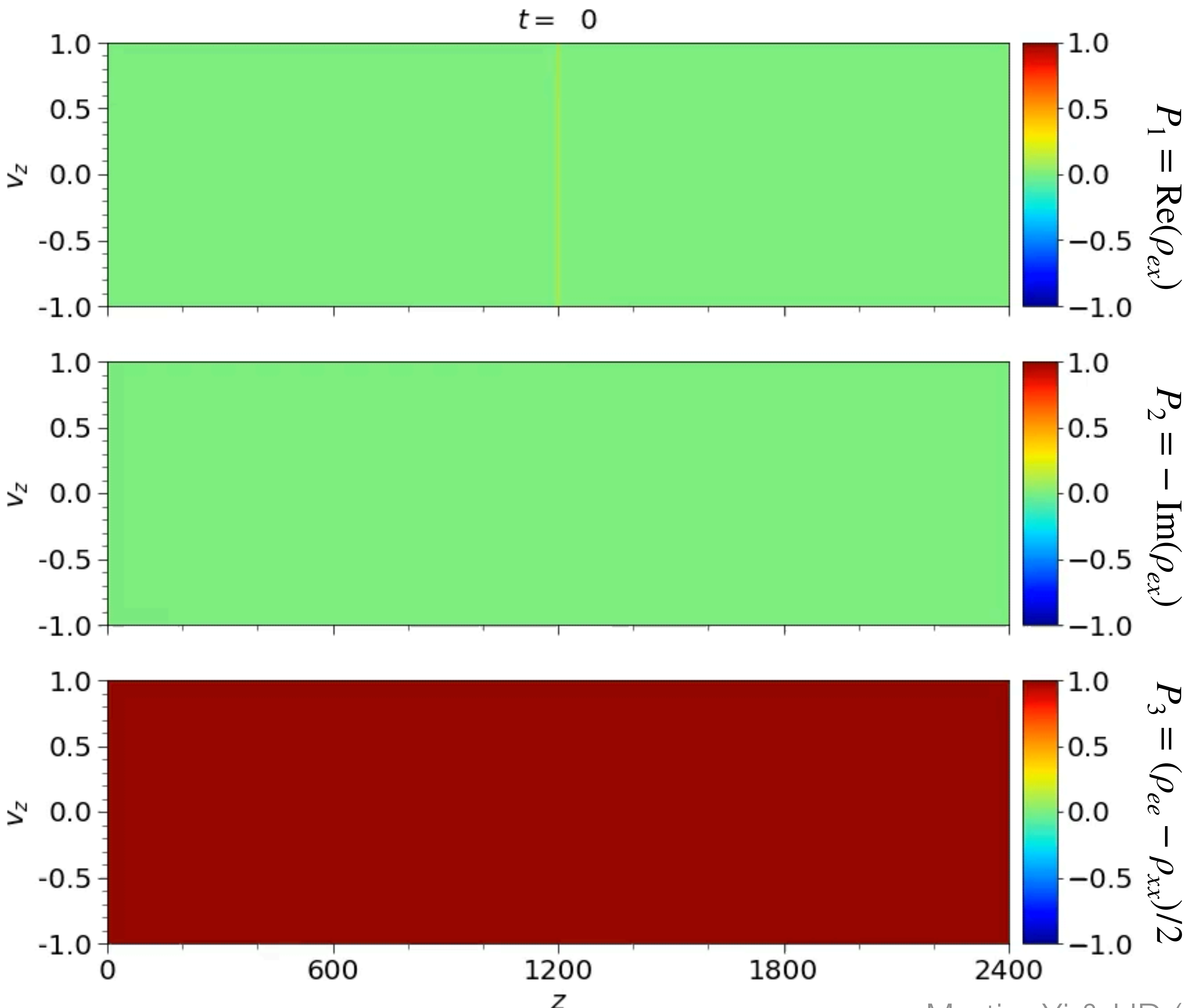
Martin+ (2020)

Absolute Instability

1D axisymmetric neutrino gas

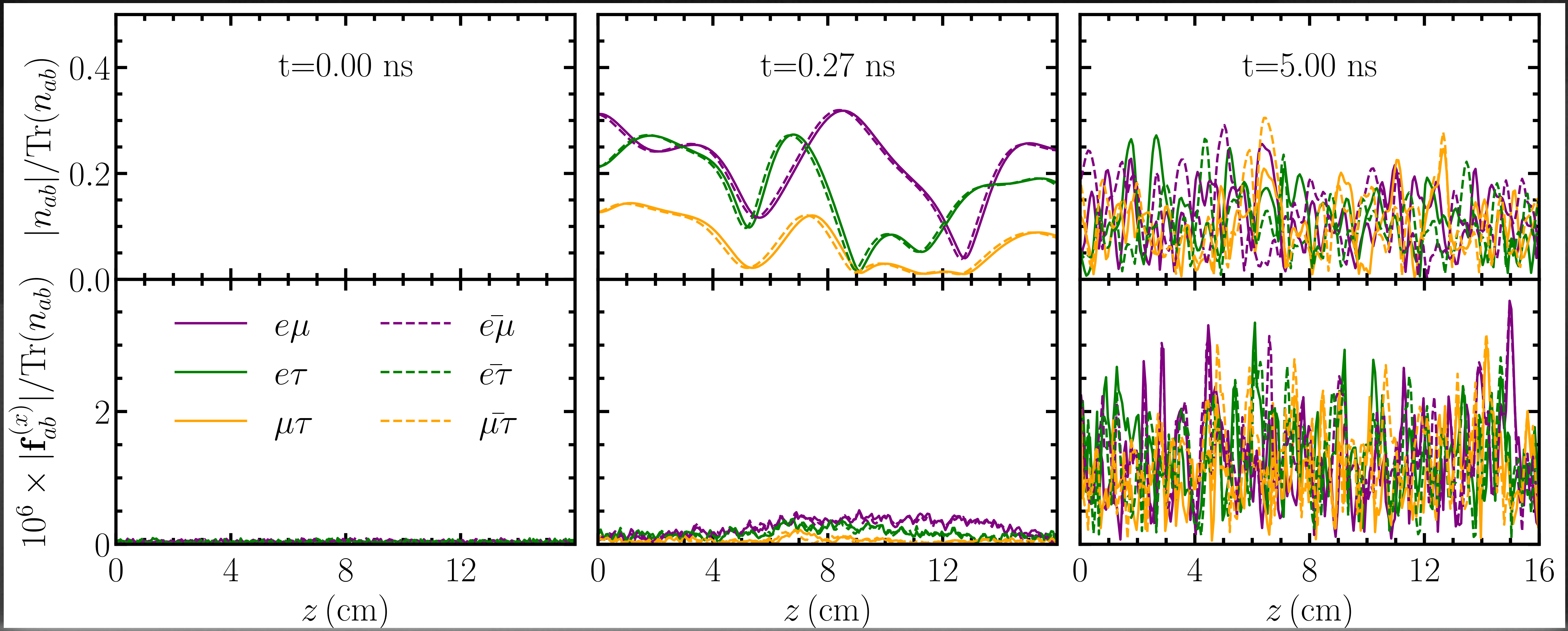


Martin+ (2020)



Martin, Yi & HD (2020)

Kinematic Decoherence



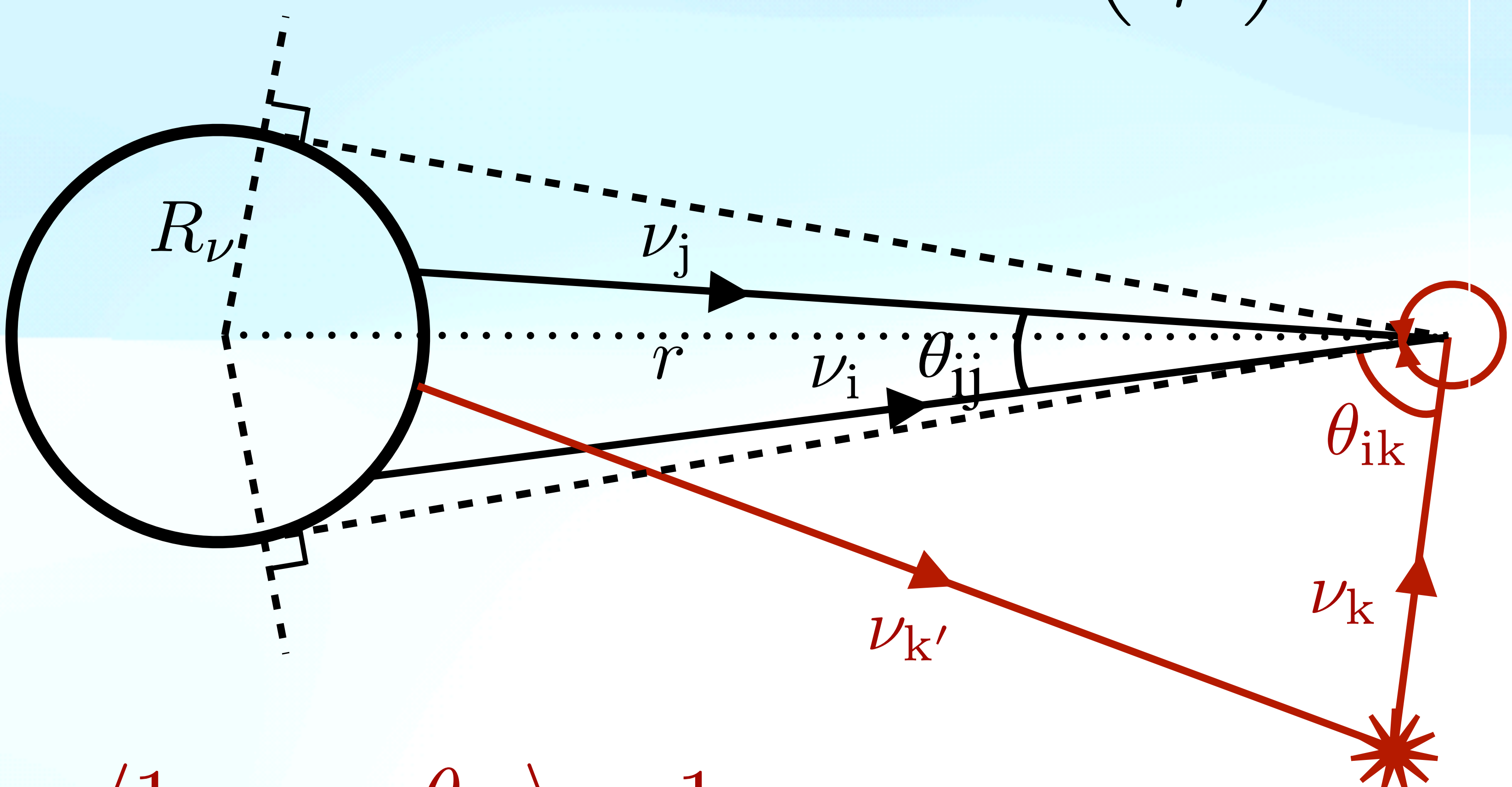
Richers+ (2021)
Wu+ (2021)

Effects of Collisions

Neutrino Halo

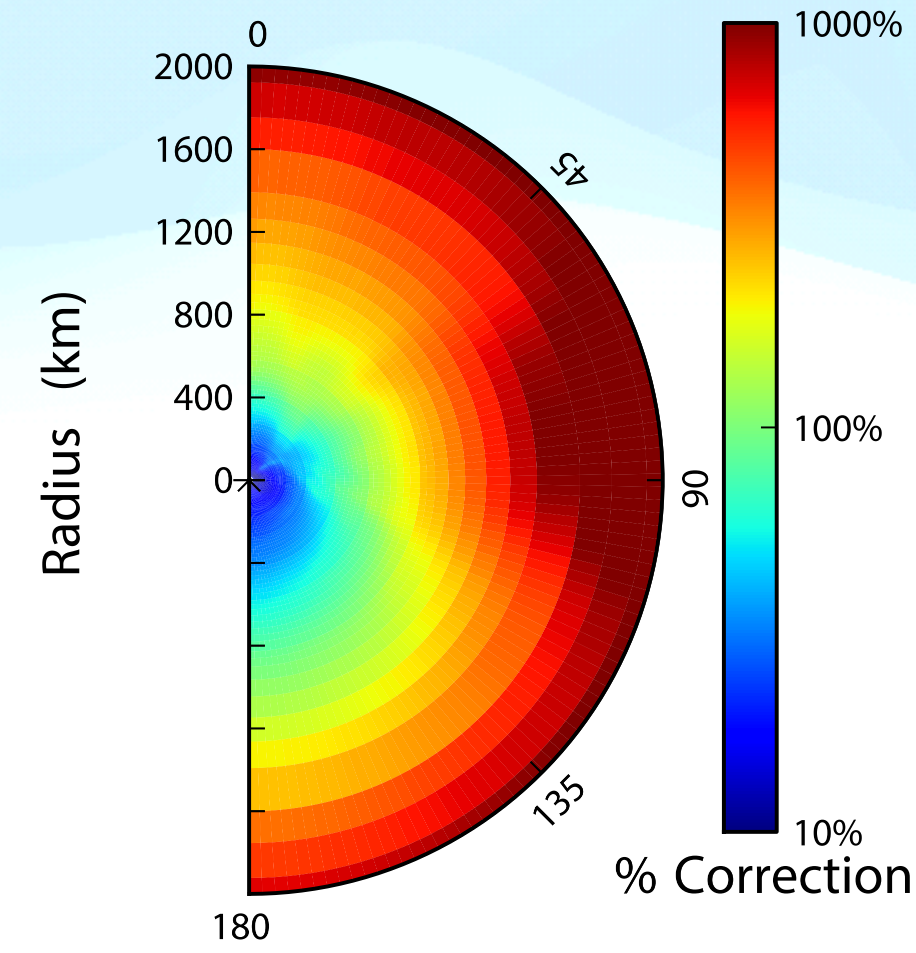
νN neutral-current scattering

$$r \gg R_\nu \Rightarrow \langle 1 - \cos\theta_{\rm ij} \rangle \propto \left(\frac{R_\nu}{r}\right)^2$$



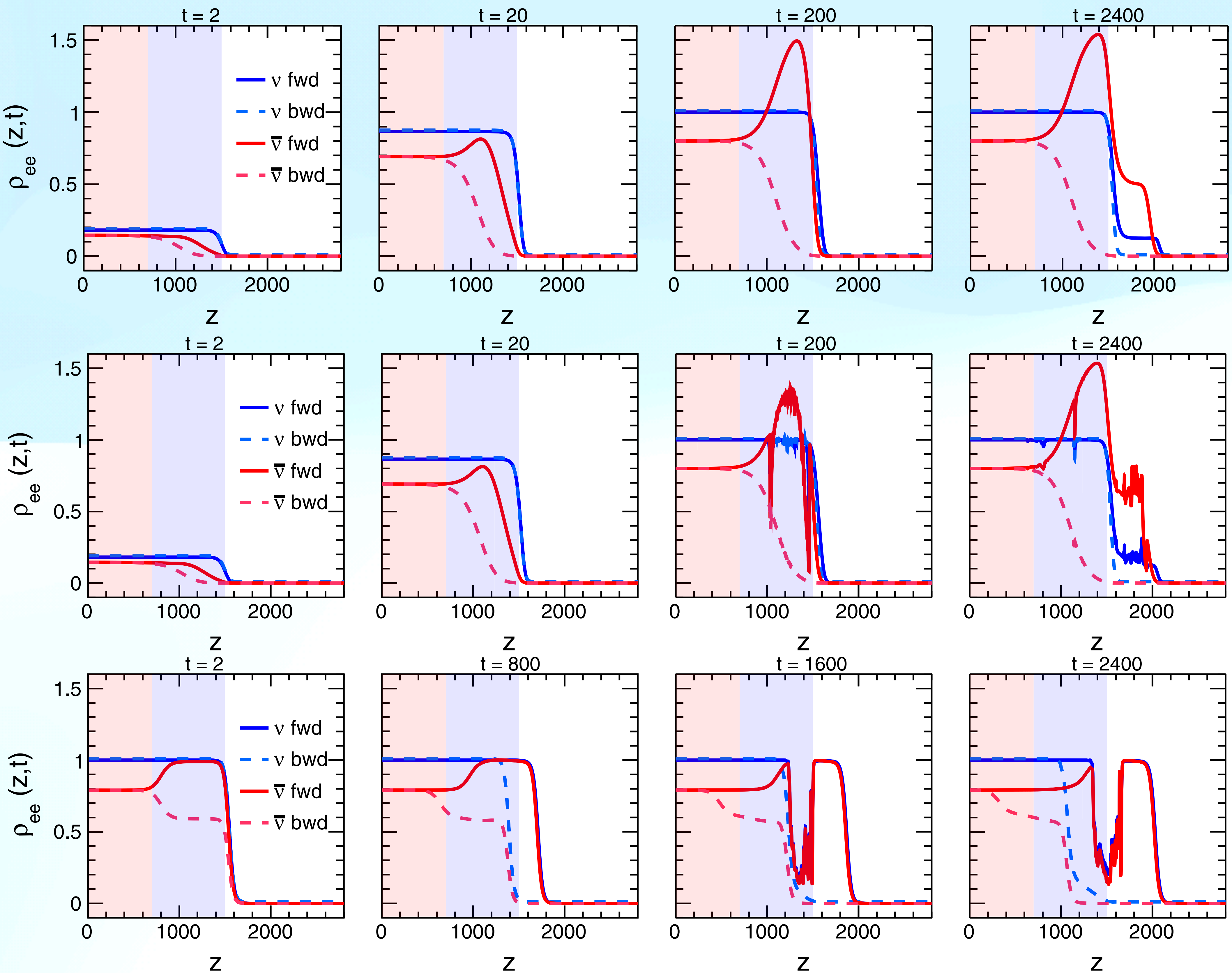
$$\langle 1 - \cos\theta_{\rm ik} \rangle \approx 1$$

$\sim 10^{-3}$ of all ν's



Cherry+ (2013)

Collision and Fast Oscillations



w/o $\nu\nu$ coupling

strong collision

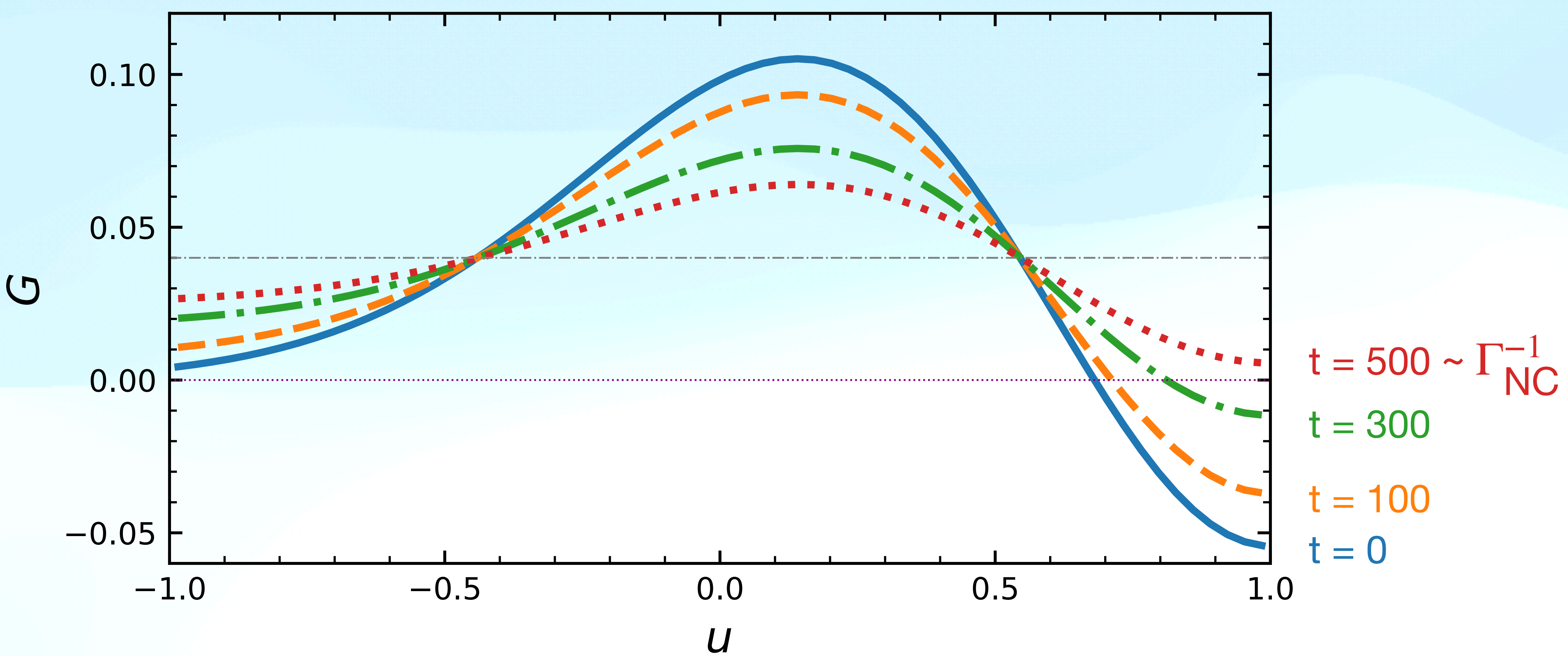
w/ $\nu\nu$ coupling

weak collision

Capozzi+ (2019)

Collision and Fast Oscillations

νN neutral-current collisions

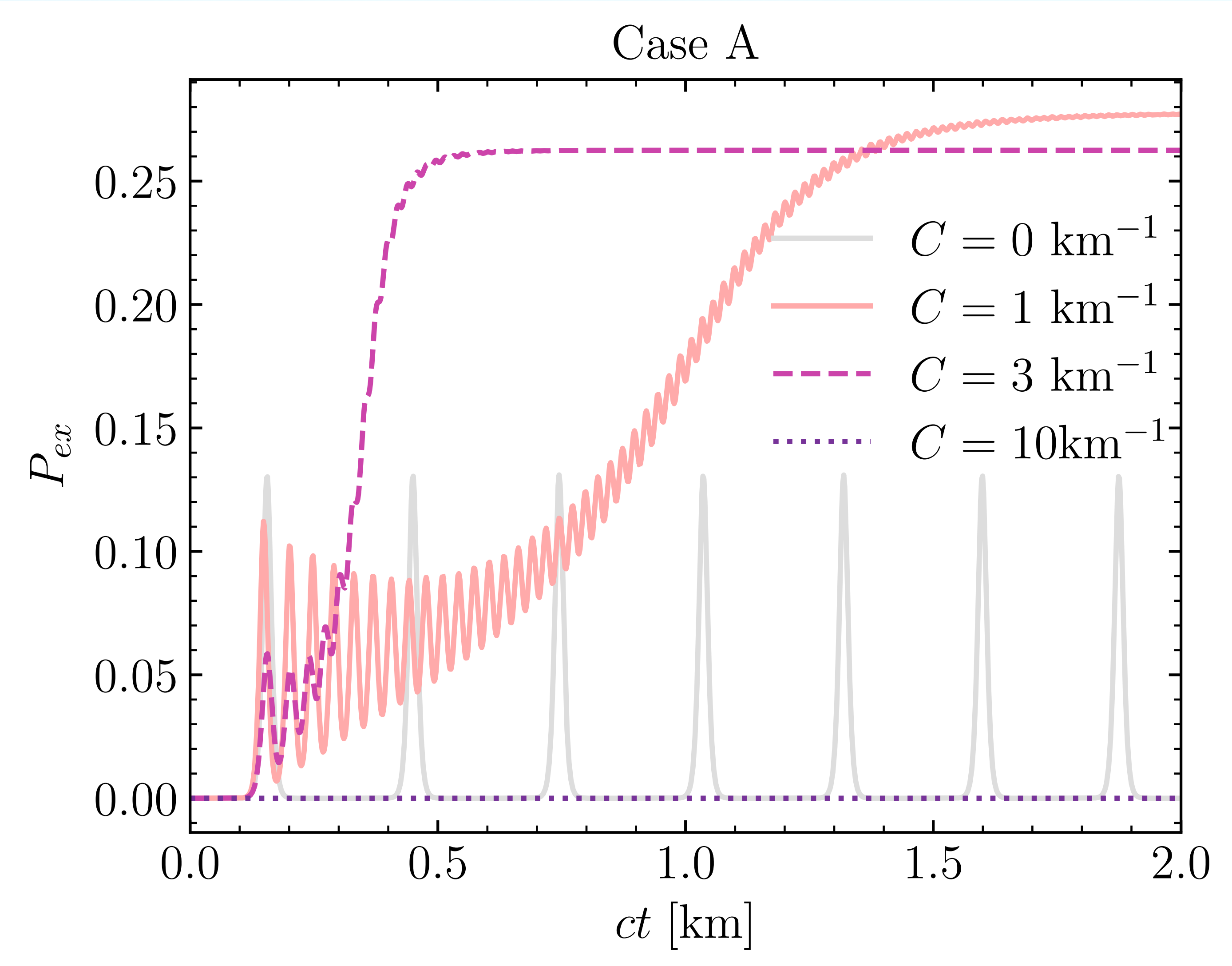


Neutral-current νN collisions tend to damp/destroy fast flavor instabilities

Martin+ (2021)

Collision and Fast Oscillations

νN neutral-current collisions

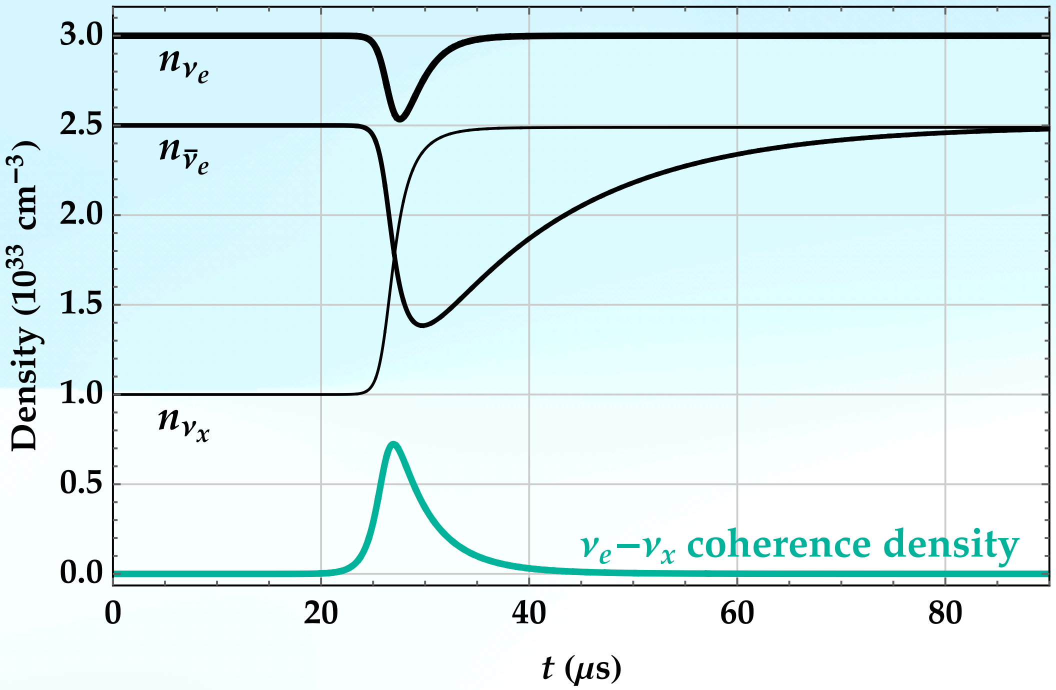


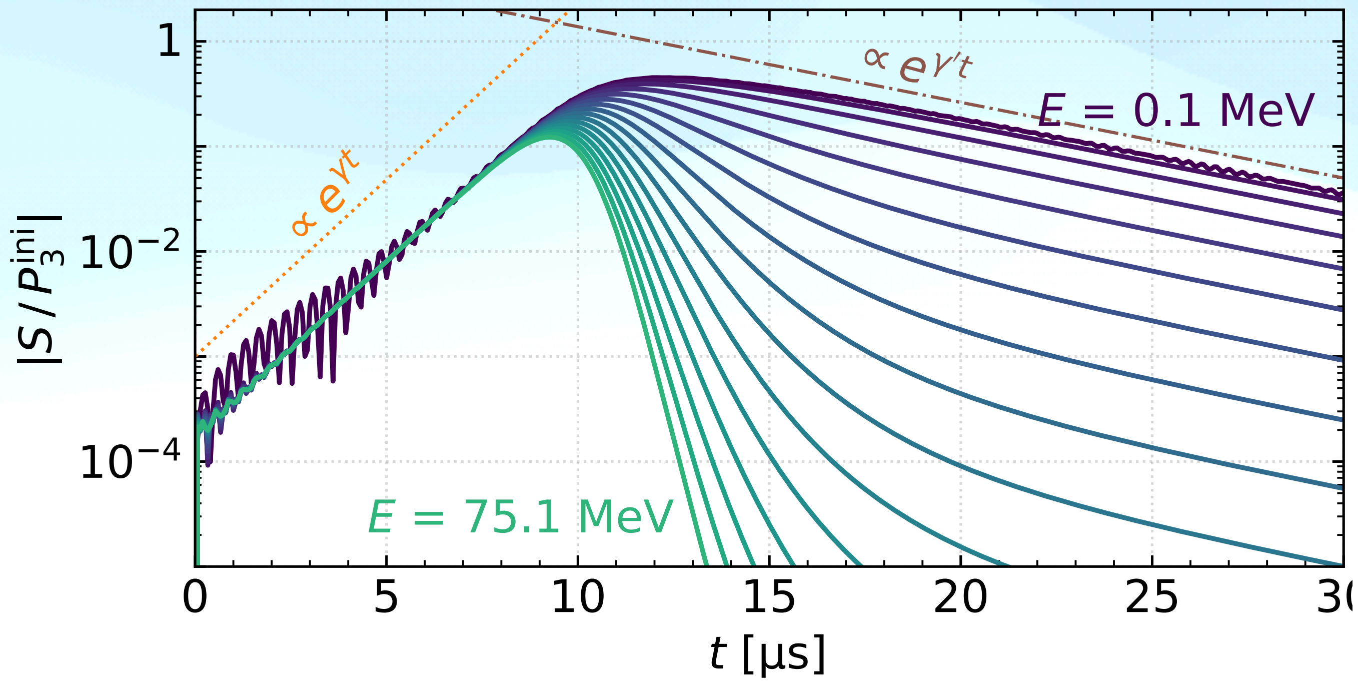
Neutral-current νN collisions can also enhance fast flavor conversion?

Shalgar & Tamborra (2021)
Sasaki &Takiwaki (2021)
Hansen+ (2022)

Collision and Fast Oscillations

Charged-current collisions





Charged-current collisions can induce collisional flavor instability

Johns (2021)
Lin & HD (2022)
Xiong+ (2022)

Summary

- A dense neutrino gas can experience collective flavor oscillations because of the $\nu\nu$ coupling.
- A crossing in the neutrino flavor distribution can produce flavor instabilities.
- Neutrino collisions can also induce/enhance flavor instabilities/conversions.
- Flavor oscillation waves are produced as the flavor instabilities grow out of the linear regime.
- Kinematic decoherence of the oscillation wave can lead to “flavor equilibrium”.