Collective neutrino oscillations: from symmetry to simulation

Zewei Xiong
GSI Helmholtzzentrum für Schwerionenforschung GmbH

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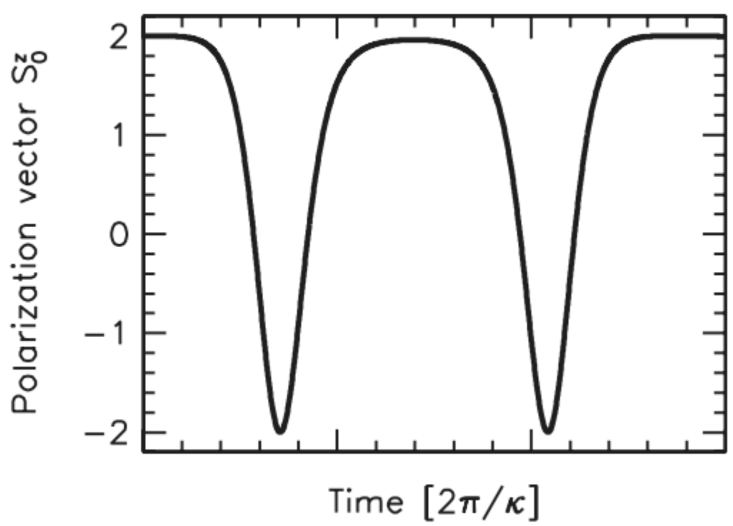
Collective neutrino oscillations: a challenging problem

- Many-body entanglements and correlations
- Three-flavor nature of oscillations
- Advection in an inhomogeneous environment
- Non-forward scatterings (collisions)

Various behaviors in homogeneous model

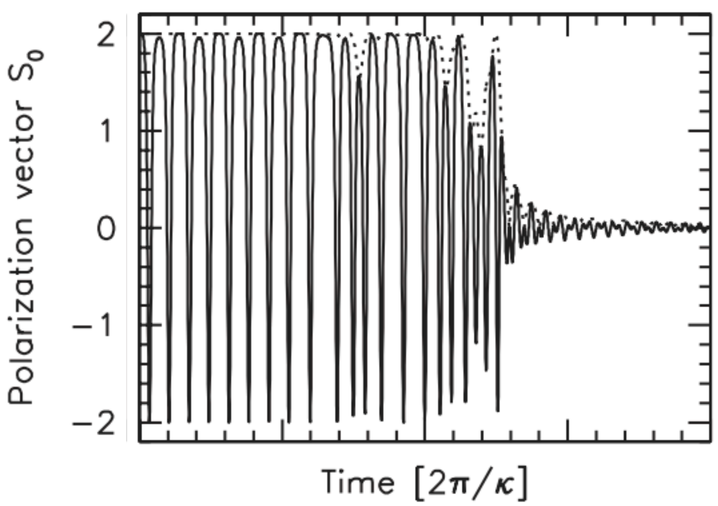
Similarity in terms of gyroscopic pendulum or Gaudin invariants
[I. Padilla-Gay, I. Tamborra, G. G. Raffelt, 2022], [D. F. G. Fiorillo, G. G. Raffelt, 2023]

Single-angle slow mode



periodic bipolar motion

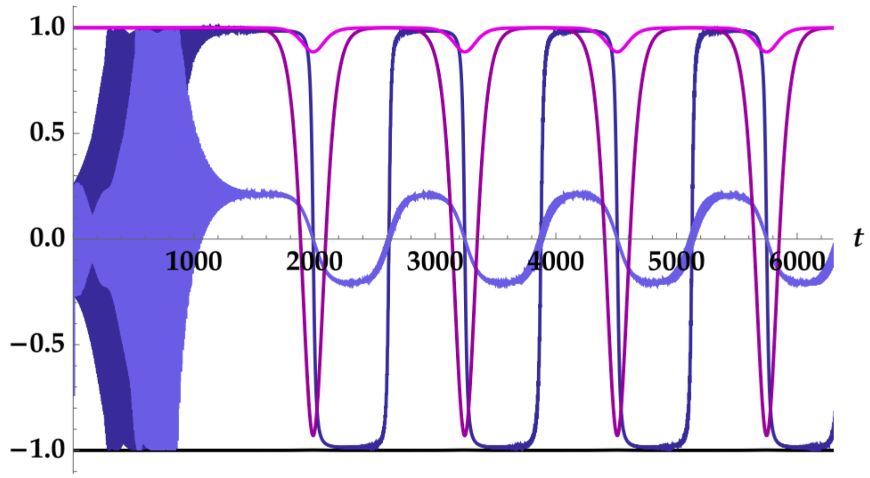
Multi-zenith-angle (MZA) slow mode



[G. G. Raffelt, G. Sigl, 2007]

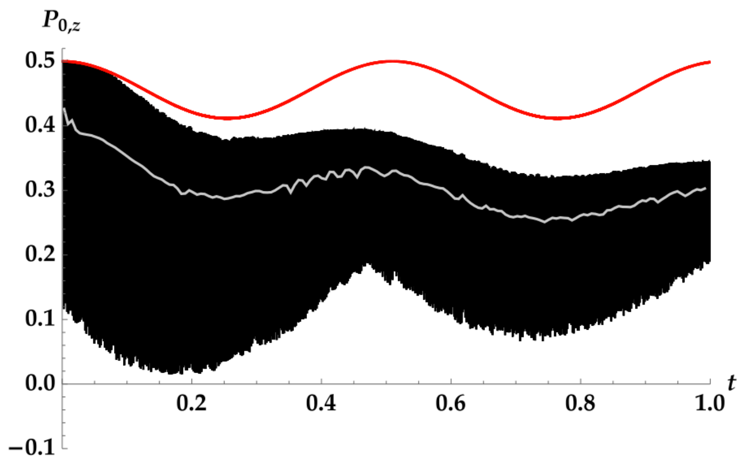
kinematic decoherence

Single-energy axisymmetric (AS) fast mode



periodic bipolar motion

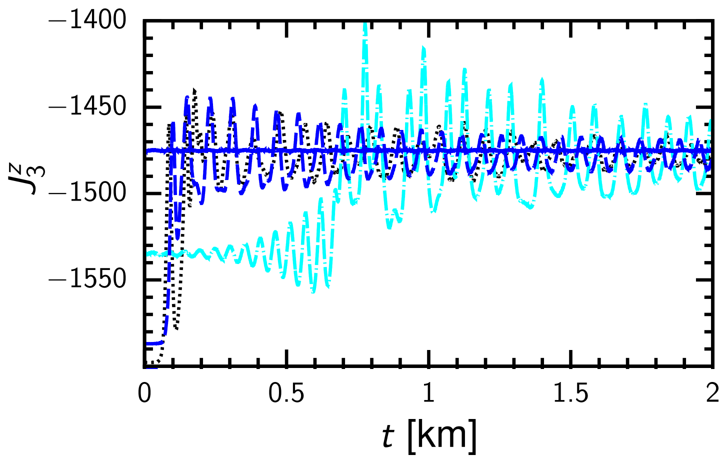
Multi-energy AS fast mode



[L. Johns, H. Nagakura, G. M. Fuller, A. Burrows, 2020]

relaxation and cascade

Axial-symmetry-breaking (SB) fast mode



[ZX, Y.-Z. Qian, 2021]

oscillations around stationary state

Geometric symmetry in bipolar flavor evolution

[ZX, M.-R. Wu, Y.-Z. Qian, arXiv: 2303.05906]

Collective neutrino oscillations

- Flavor evolution equation:

$$\partial_t \mathbf{P}(\omega, \vec{v}) = \mathbf{H}(\omega, \vec{v}) \times \mathbf{P}(\omega, \vec{v})$$

neutrinos / antineutrinos $\pm \frac{\delta m^2}{2E}$ $\quad \frac{\vec{q}}{|\vec{q}|}$ Hamiltonian vector polarization vector

$$\mathbf{H}(\omega, \vec{v}) = \omega \mathbf{B} + v_\rho(\vec{v})(\lambda^\rho \hat{\mathbf{e}}_3 + \mu \mathbf{J}^\rho)$$

vacuum term (negligible mixing angle) matter term self-induced term

$$\mathbf{J}^\rho = \int v^\rho(\vec{v}')F(\omega', \vec{v}')\mathbf{P}(\omega', \vec{v}')\, d\omega'\, d\vec{v}'$$

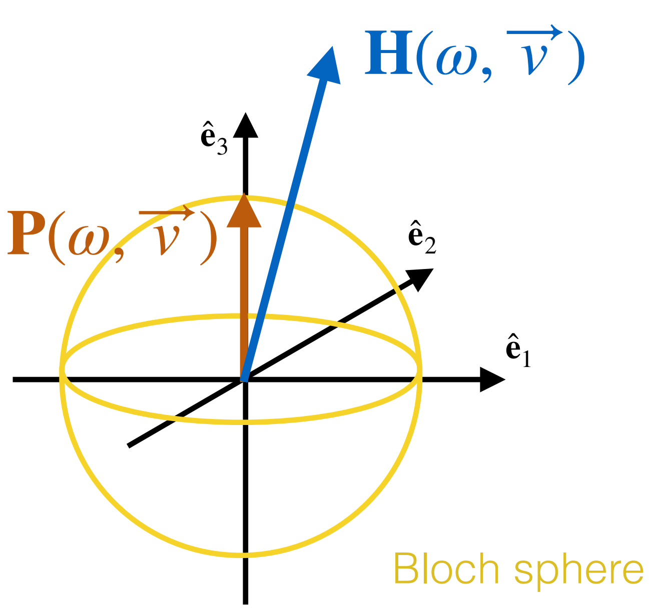
polarization current

- Decomposed for vertical component, P_3, and horizontal component expressed as a complex function, $\mathsf{P}_\perp \equiv P_1 - iP_2$, with $\mathsf{H}_\perp \equiv H_1 - iH_2$

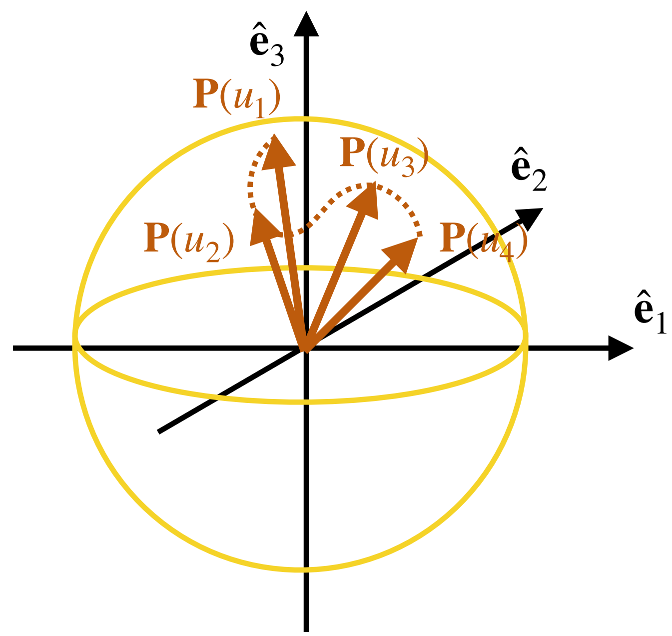
$$i\partial_t \mathsf{P}_\perp = H_3 \mathsf{P}_\perp - P_3 \mathsf{H}_\perp, \quad \partial_t P_3 = \mathrm{Im}(\mathsf{H}_\perp \mathsf{P}_\perp^*)$$

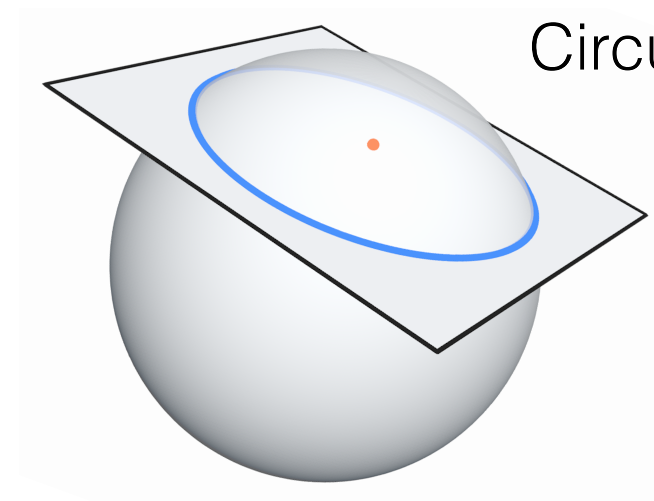
- Linear stability analysis ($|\mathsf{P}_\perp| \ll P_3 \approx 1$):
  Assuming $\mathsf{P}_\perp(t) = \mathsf{Q}e^{-i\Omega t}$, we have

$$\mathsf{P}_\perp = \frac{\mathsf{H}_\perp}{H_3 - \Omega}$$

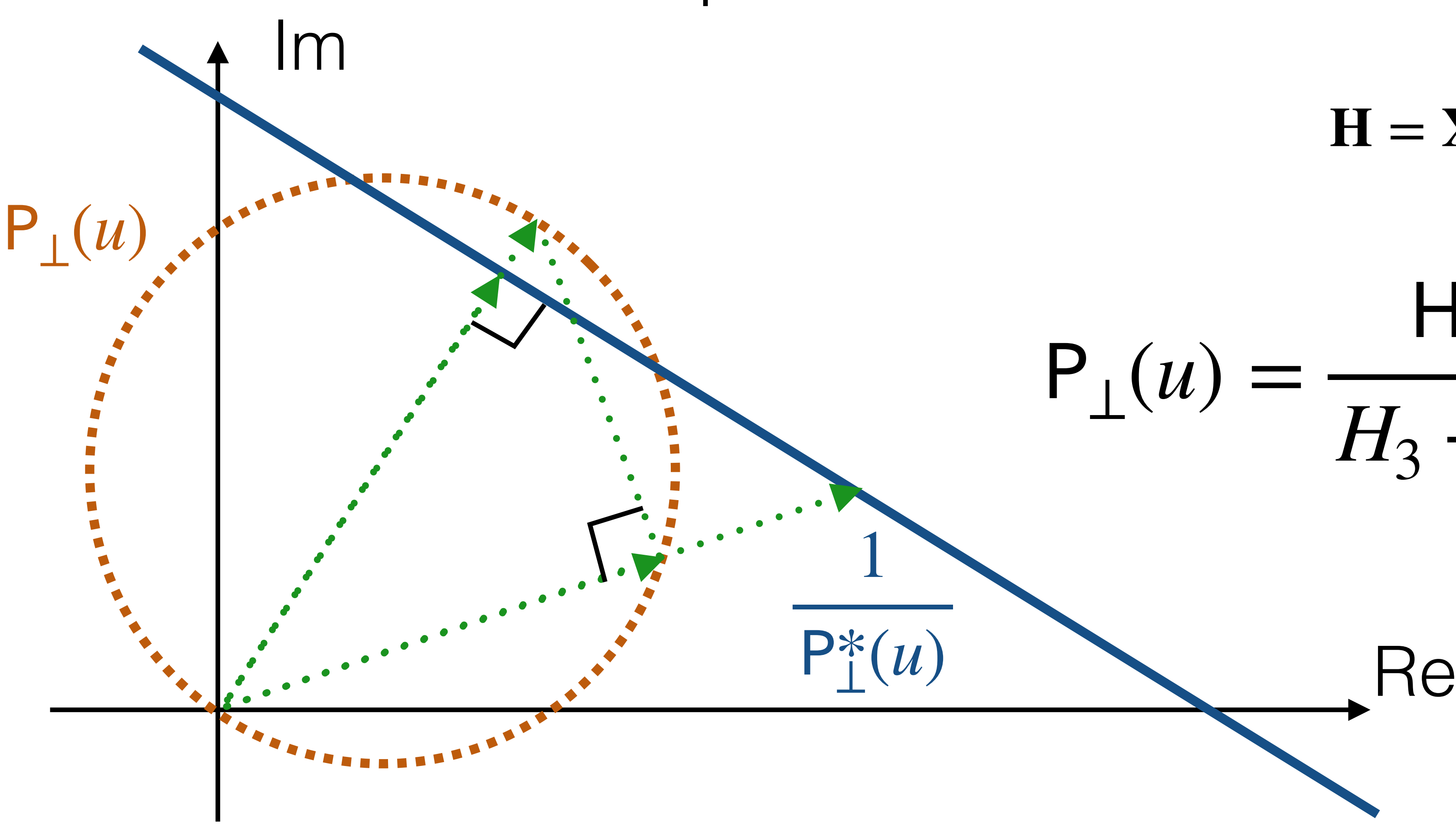


Circular configuration in linear regime





Circular!



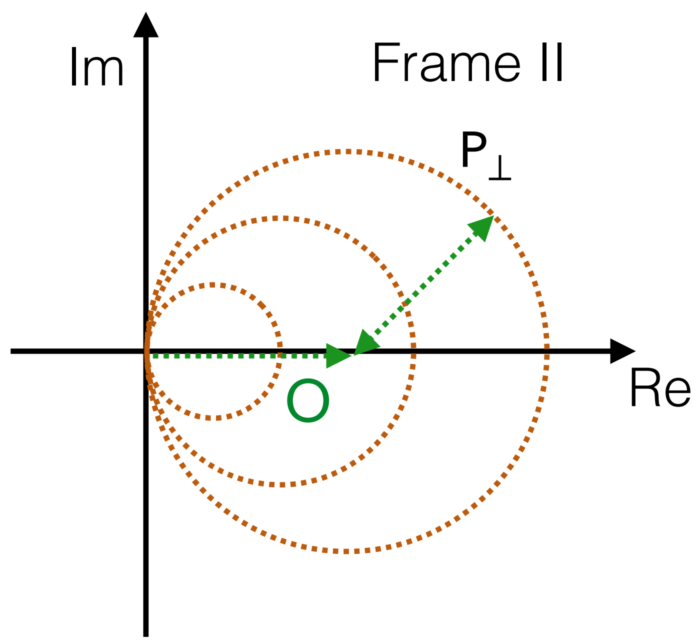
$$\mathbf{H} = \mathbf{X} + u\mathbf{Y} \qquad \begin{cases} \mathbf{H}_{\mathrm{slow}} = \mu \mathbf{J}^t + \omega \mathbf{B} \\ \mathbf{H}_{\mathrm{AS\ fast}} = \mu \mathbf{J}^t - v^z \mu \mathbf{J}^z \end{cases}$$

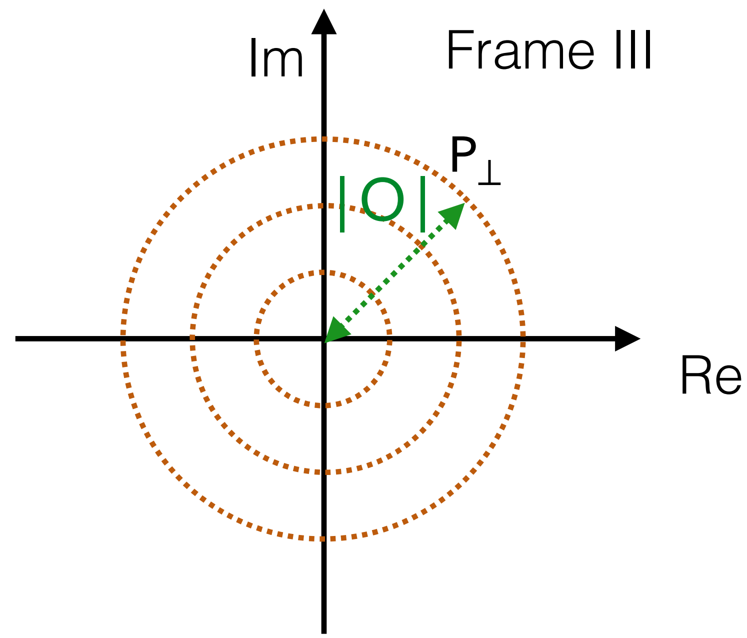
$$\mathrm{P}_\perp(u) = \frac{\mathrm{H}_\perp}{H_3 - \Omega} = \frac{\mathrm{X}_\perp + \mathrm{Y}_\perp u}{X_3 - \Omega + Y_3 u}$$

The circular configuration passes through the origin if $\mathrm{Im}(\mathrm{X}_\perp \mathrm{Y}^*_\perp) = 0$.

Geometric symmetry

$e^{i\Omega t}$ $\quad$ $\Omega = \Omega_r + i\Omega_i$ $\quad$ $i\partial_t \mathrm{P}_\perp = H_3 \mathrm{P}_\perp - P_3 \mathrm{H}_\perp$





$\sim \Theta$

$$i\partial_t \mathrm{P}_\perp^{\mathrm{III}} = (\tilde{X}_3 + u\tilde{Y}_3 - \Omega_r)\mathrm{P}_\perp^{\mathrm{III}}$$
$$-(\mathrm{X}_\perp + u\mathrm{Y}_\perp + \Omega \mathrm{O})$$

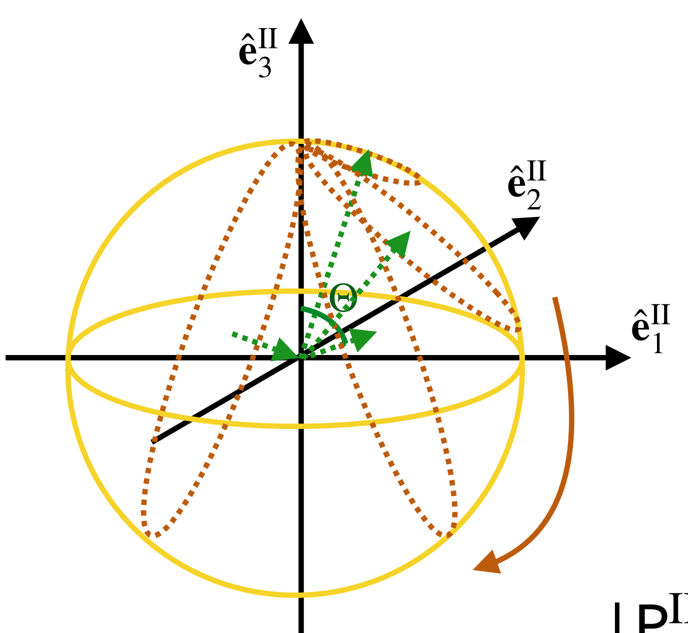
$\sim \Theta \quad \sim \Theta \quad \sim \Theta$

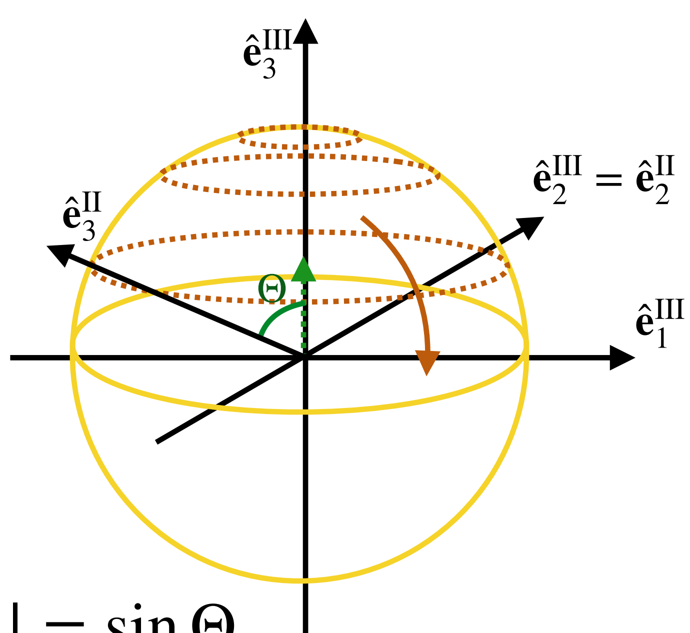
$$= i\Omega_i \mathrm{P}_\perp^{\mathrm{III}}$$

$$\mathbf{H}^{\mathrm{II}} = \mathbf{X}^{\mathrm{II}} + u\mathbf{Y}^{\mathrm{II}} - \Omega_r \hat{\mathbf{e}}_3^{\mathrm{II}}$$

$$\mathbf{H}^{\mathrm{III}} = \mathbf{X}^{\mathrm{III}} + u\mathbf{Y}^{\mathrm{III}} - \Omega_r \cos\Theta\, \hat{\mathbf{e}}_3^{\mathrm{III}}$$
$$+\Omega_r \sin\Theta\, \hat{\mathbf{e}}_1^{\mathrm{III}} - (\partial_t \Theta)\, \hat{\mathbf{e}}_2^{\mathrm{III}}$$

$$\partial_t \Theta = \Omega_i \sin\Theta$$





$$|\mathrm{P}_\perp^{\mathrm{III}}| = \sin\Theta$$

$\sim \cos\Theta \quad \sim \cos\Theta \quad \sim \sin\Theta$

$$i\partial_t \mathrm{P}_\perp^{\mathrm{III}} = (X_3^{\mathrm{III}} + uY_3^{\mathrm{III}} - \Omega_r \cos\Theta)\mathrm{P}_\perp^{\mathrm{III}}$$
$$-P_3^{\mathrm{III}}(\mathrm{X}_\perp + u\mathrm{Y}_\perp + \Omega \sin\Theta)$$

$\sim \cos\Theta \sim \sin\Theta \sim \sin\Theta$

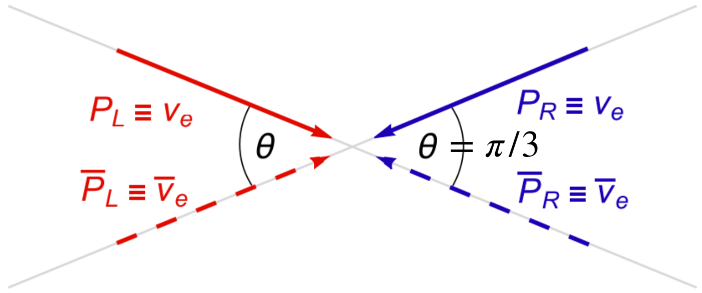
$$= i\Omega_i \mathrm{P}_\perp^{\mathrm{III}} \cos\Theta$$

Eight representative models

If $H_{\rm mat}$ is included we take $\lambda^\rho = (\mu, 0, 0, 0.5\mu)$. For cases where $H_{\rm vac}$ is included, the neutrino mass ordering is taken to be inverted ordering. ($\mathbf{B} \approx \hat{\mathbf{e}}_3$)

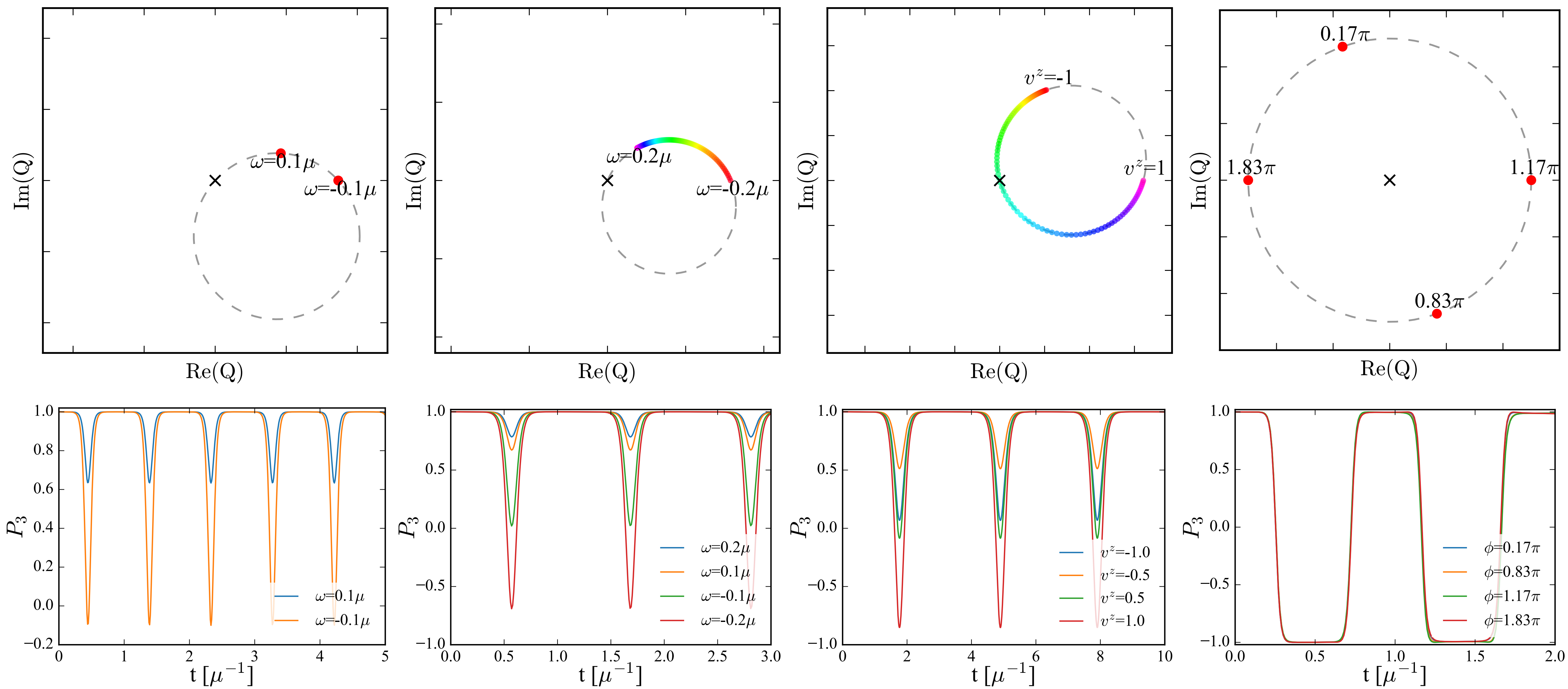
model	$N_{\rm beam}$	$H_{\rm vac}$	$H_{\rm mat}$	discretization schemes	$F_{\omega,\vec{v}}$
Two-beam slow	2	✓	–	$v^x = v^y = v^z = 0$ effectively; ω is either 0.1μ or -0.1μ	$F_\omega = \mathrm{sgn}(\omega) + 0.5$
Single-angle slow	10000	✓	–	$v^x = v^y = v^z = 0$ effectively; 10000 bins for $-0.2\mu < \omega < 0.2\mu$	$F_\omega = \mathrm{sgn}(\omega) + 0.5$
Single-energy AS fast	10000	–	–	$v^x = v^y = 0$ effectively; 10000 bins for $-1 < v^z < 1$	$F_{v^z} = g(v^z, 0.9)$
Four-beam coplanar fast [B. Dasgupta+, 2018]	4	–	–	$v^z = 0$; ϕ takes $\pi/6$, $5\pi/6$, $7\pi/6$, and $11\pi/6$ respectively	$F_{v^x,v^y} = \mathrm{sgn}(v^y)$
Eight-beam coplanar fast	8	–	–	$v^z = 0$; 8 bins for $0 < \phi < 2\pi$	$F_{v^x,v^y} = \mathrm{sgn}(v^y)$
AS fast with non-zero matter bulk velocity [I. Padilla-Gay+, 2021]	10000	–	✓	$v^x = v^y = 0$ effectively; 10000 bins for $-1 < v^z < 1$	$F_{v^z} = g\,(v^z, 0.9)$
MZA slow	40000	✓	–	$v^x = v^y = 0$ effectively; 200 bins for $-0.2\mu < \omega < 0.2\mu$; 200 bins for $-1 < v^z < 1$	$F_{\omega,v^z} = [\mathrm{sgn}(\omega) + 0.5] \times (1 + 0.5v^z)$
SB fast	38400	–	–	300 bins for $-1 < v^z < 1$; 128 bins for $0 < \phi < 2\pi$	$F_{v^x,v^y,v^z} = g(v^z, 1.1)$

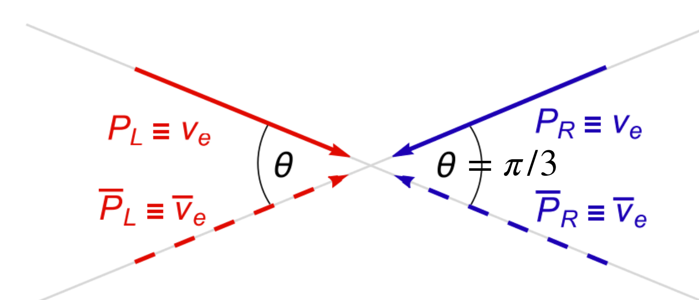
$$g(v^z, \alpha) = \frac{20}{\sqrt{\pi}}\left[\sigma_\nu^{-1} e^{-\left(\frac{1-v^z}{\sigma_\nu}\right)^2} - \alpha\sigma_{\bar{\nu}}^{-1} e^{-\left(\frac{1-v^z}{\sigma_{\bar{\nu}}}\right)^2}\right]$$ with $\sigma_\nu = 0.6\sqrt{2}$ and $\sigma_{\bar{\nu}} = 0.5\sqrt{2}$.



Periodic bipolar flavor evolution

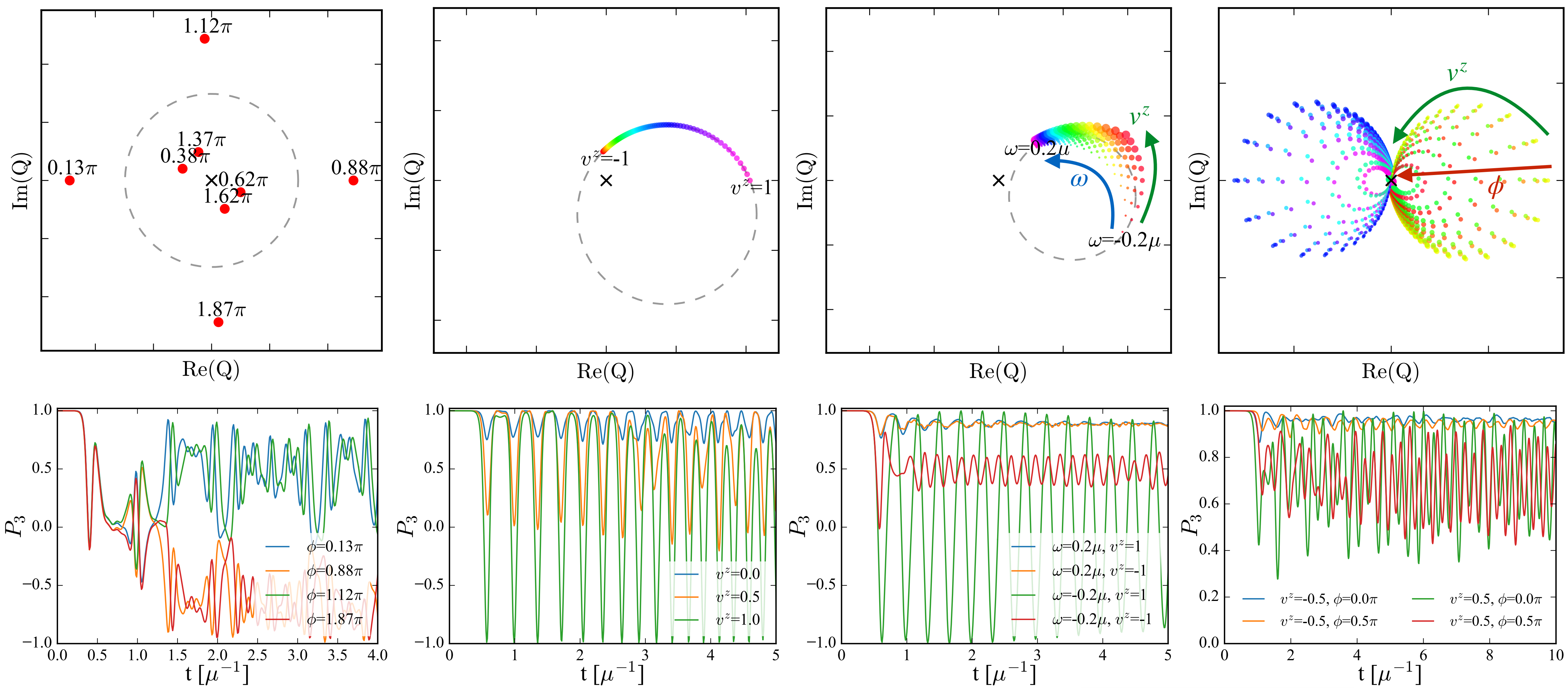
Two-beam slow | Single-angle slow | Single-energy AS fast | Four-beam coplanar fast





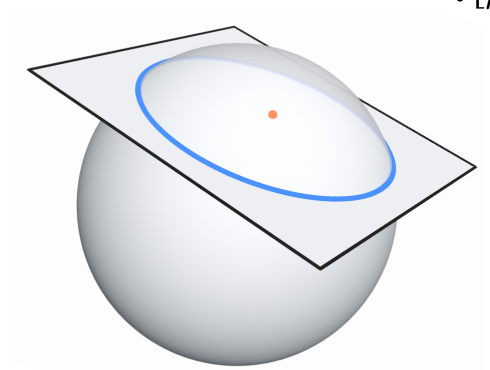
Breaking-down of bipolar evolution

Eight-beam coplanar fast | AS fast with bulk velocity | MZA slow | SB fast



$$\partial_t \mathbf{J}^t = \lambda^t \hat{\mathbf{e}}_3 \times \mathbf{J}^t - \lambda^z \hat{\mathbf{e}}_3 \times \mathbf{J}^z$$

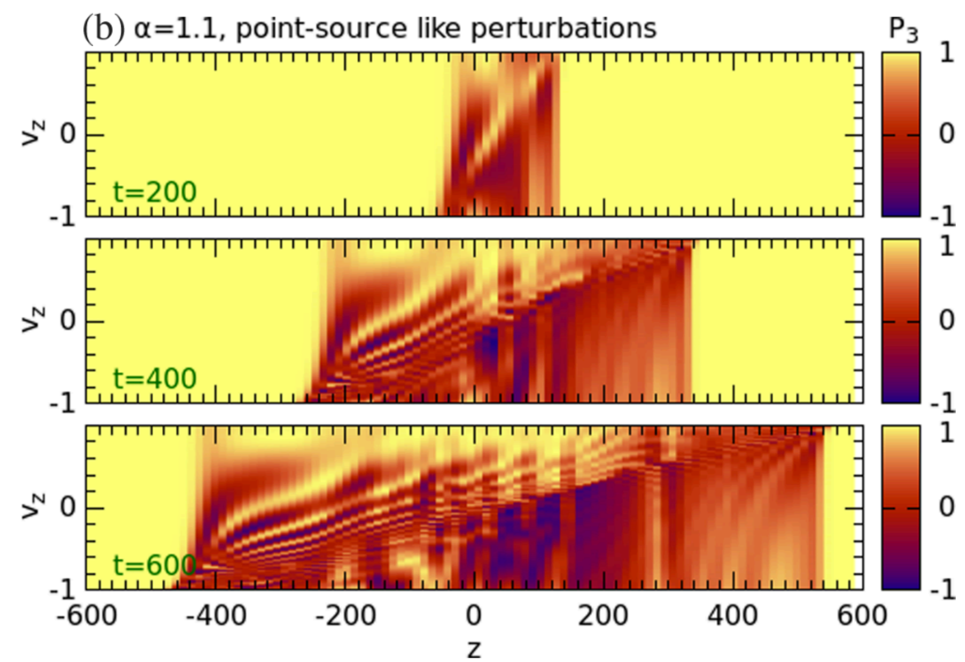
$$\mathrm{Im}(\mathrm{J}_\perp^t \mathrm{J}_\perp^{z*}) \not\approx 0$$



Inhomogeneous neutrino gas

- Equation of motion:

$$v^\rho \partial_\rho \mathbf{P} = \quad \partial_t \mathbf{P} + v^z \partial_z \mathbf{P} = \mathbf{H} \times \mathbf{P}$$



[M.-R. Wu, M. George, C.-Y. Lin, ZX, 2021]

- Even if only allow one K mode to develop from linear regime to non-linear regime [$\mathsf{P}_\perp(z) \propto e^{iK^z z}$]

$$\partial_t \mathbf{P} - v^z K^z \hat{\mathbf{e}}_3 \times \mathbf{P} = \mathbf{H} \times \mathbf{P} \quad = \lambda^t \hat{\mathbf{e}}_3 \times \mathbf{P}$$

$$\partial_t \mathbf{J}^t = \lambda^t \hat{\mathbf{e}}_3 \times \mathbf{J}^t + K^z \hat{\mathbf{e}}_3 \times \mathbf{J}^z$$

$$\mathrm{Im}(\mathsf{J}^t_\perp \mathsf{J}^{z*}_\perp) \not\approx 0$$

- This symmetry relies on the geometry of two-flavor Bloch sphere. Three flavors? Incoherent collisions?

Collisional flavor instability simulated in spherically symmetric supernova model

[ZX, M.-R. Wu, G. Martínez-Pinedo, T. Fischer, M. George, C.-Y. Lin, L. Johns, arXiv: 2210.08254]

Collisional flavor instability in quantum kinetic equation

- Flavor evolution equation with advection in spherically symmetric supernova model and incoherent collisions:

$$(\partial_t + v_r \partial_r + \frac{1-v_r^2}{r}\partial_{v_r})\varrho(E, v_r) = -i[\mathscr{H}(E, v_r), \varrho(E, v_r)] + \mathscr{C}\{\varrho(E, v_r)\}$$

EA	Emission and absorption	$\nu_e + n \leftrightarrows p + e^-$
		$\bar{\nu}_e + p \leftrightarrows n + e^+$
		$\bar{\nu}_e + p + e^- \leftrightarrows n$
NNS	Neutrino-nucleon scattering	$\nu + N \leftrightarrows \nu + N$
		$\bar{\nu} + N \leftrightarrows \bar{\nu} + N$
NES	Neutrino-electron scattering	$\nu + e^\pm \leftrightarrows \nu + e^\pm$
NPR	Neutrino pair reactions	$\nu + \bar{\nu} \leftrightarrows e^- + e^+$
		$\nu + \bar{\nu} + N + N \leftrightarrows N + N$

Classical neutrino transport:

$$\mathrm{d}f_{\nu_e}/\mathrm{d}t = j_e(1 - f_{\nu_e}) - \chi_e f_{\nu_e}$$

emissivity opacity

Quantum kinetic equation:

[A. Vlasenko, G. Fuller, V. Cirigliano, 2014;
D.N. Blaschke, V. Cirigliano, 2016; ....]

$$\mathscr{C}_{\mathrm{EA}} \sim \begin{pmatrix} j_e(1-\varrho_{ee}) - \chi_e \varrho_{ee} & -(j_e + \chi_e)\varrho_{e\mu}/2 \\ -(j_e + \chi_e)\varrho_{e\mu}^*/2 & 0 \end{pmatrix}$$

Collisional flavor instability

[Lucas Johns, 2021]

Models with advection

$$(\partial_t + v_r \partial_r + \frac{1 - v_r^2}{r} \partial_{v_r}) \varrho = -i[a_{\nu\nu} \mathscr{H}_{\nu\nu}, \varrho] + \mathscr{C}(\varrho)$$

- Iso-energetic NNS:

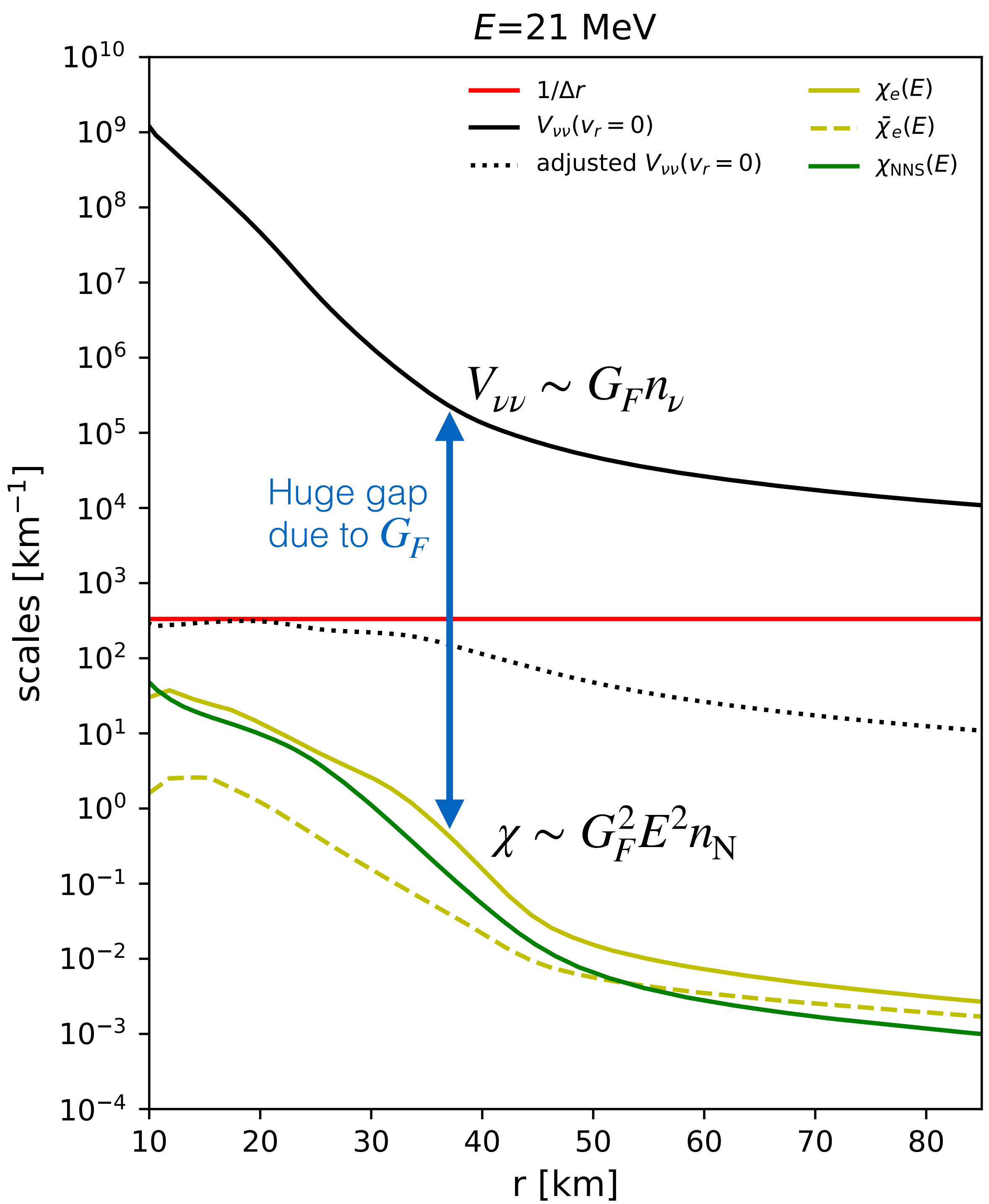
$$\mathscr{C}_{\mathrm{NNS}} = \int dv_r' \, R_{\mathrm{NNS}}(E, v_r, v_r')[\varrho(E, v_r') - \varrho(E, v_r)]$$

with the opacity

$$\chi_{\mathrm{NNS}}(E) = \int dv_r' R_{\mathrm{NNS}}(E, v_r, v_r').$$

- Attenuation factor

$$a_{\nu\nu}(r) = \frac{a_1}{1 + e^{(a_2 - r)/a_3}}$$



Models with advection

- Use background profiles from 1-D CCSN simulations ($\texttt{AGILE} - \texttt{BOLTZTRAN}$) with an $18M_\odot$ progenitor at the post-bouncing time $t_{\rm pb} \approx$ 250ms

- Solve the neutrino flavor evolution equation

$$(\partial_t + v_r\partial_r + \frac{1-v_r^2}{r}\partial_{v_r})\varrho(E, v_r) = -i[a_{\nu\nu}\mathscr{H}_{\nu\nu}, \varrho(E, v_r)] + \mathscr{C}_{\rm EA} + \mathscr{C}_{\rm NNS}$$

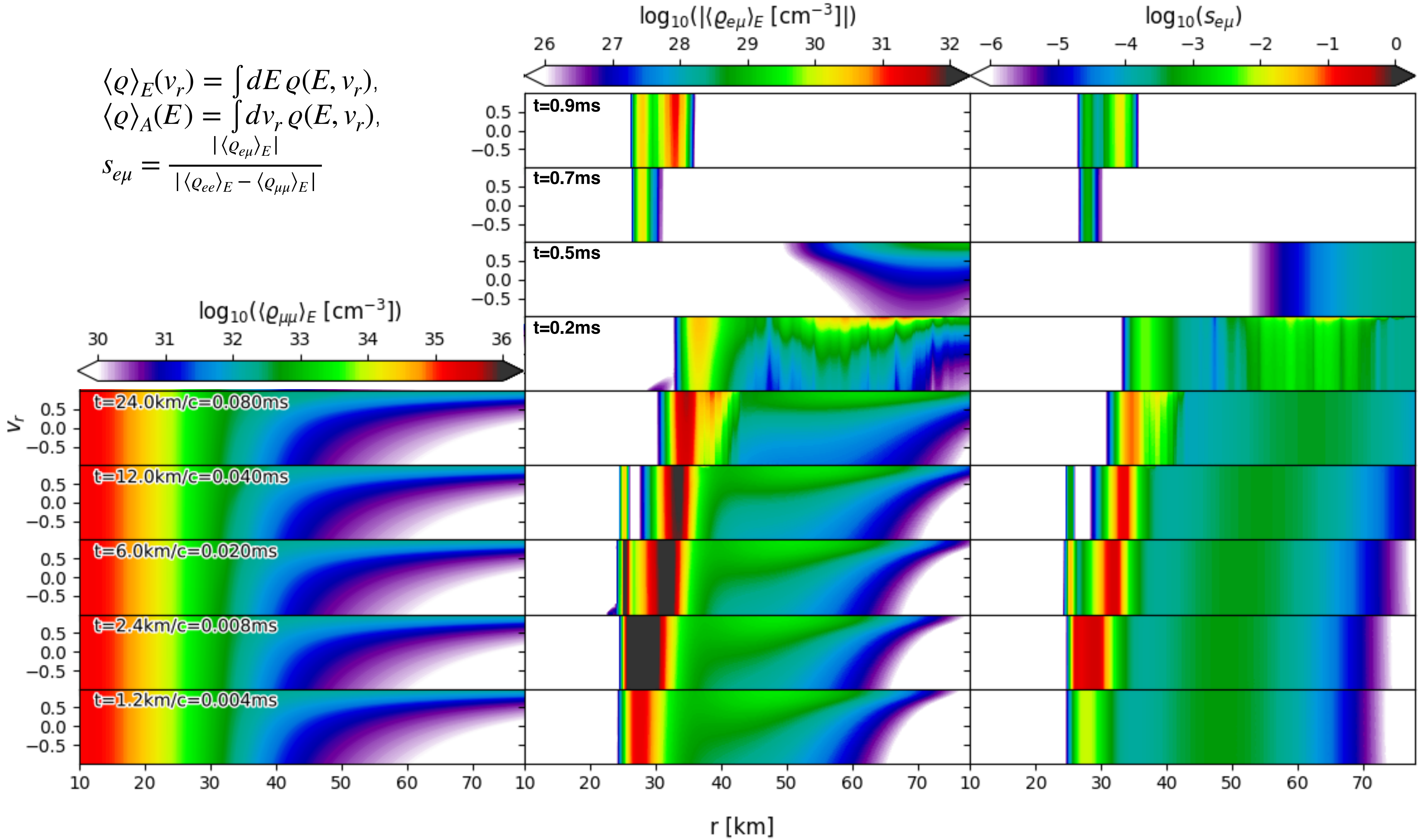
  - in $\texttt{COSE}\nu$ for two flavors ν_e and ν_μ up to ~1 ms
  - study the collisional flavor instability in the absence of fast flavor instability
  - NES and NPR are more computationally expensive because of $R_{\rm NES/NPR}(E, E', v_r, v_r')$

- Boundary conditions:
  - Inner boundary: neutrinos in thermal equilibrium with matter between 10 and 16 km to mimic NPR
  - Outer boundary: freely stream out at 85 km

- Initial perturbation (flavor mixing seed): radial-dependent Gaussian function

Evolution of collisional flavor instability

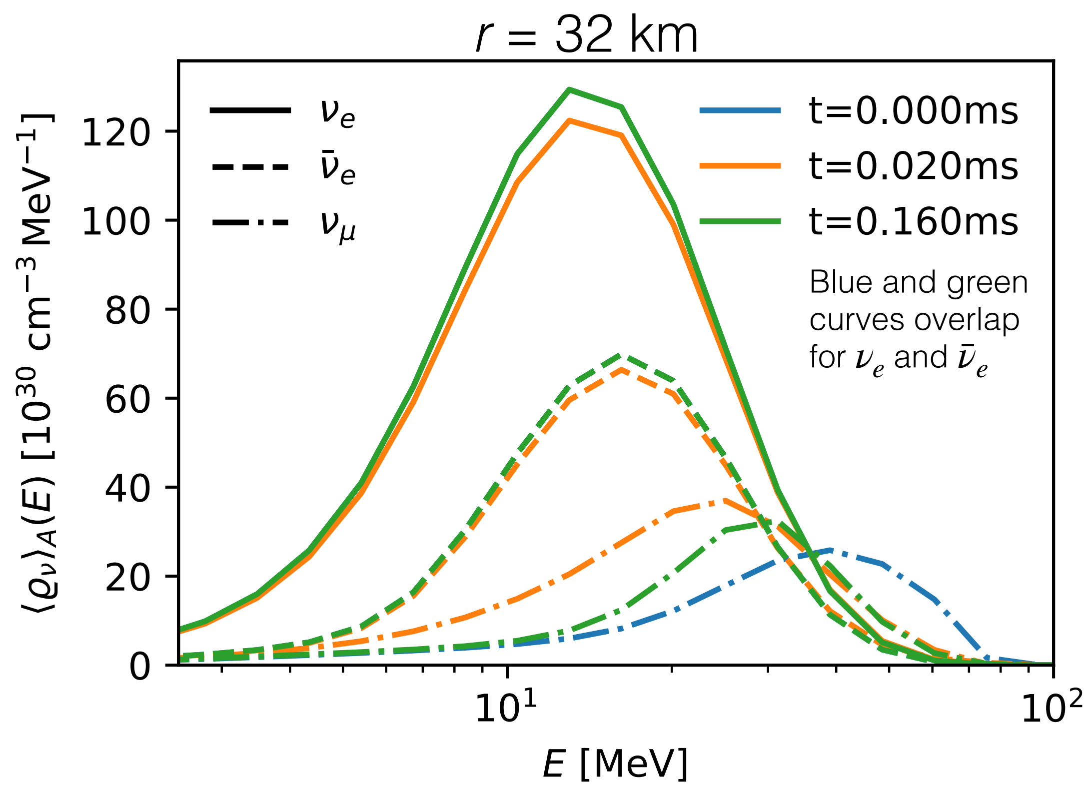
$$\langle \varrho \rangle_E(v_r) = \int dE\, \varrho(E, v_r),$$
$$\langle \varrho \rangle_A(E) = \int dv_r\, \varrho(E, v_r),$$
$$s_{e\mu} = \frac{|\langle \varrho_{e\mu} \rangle_E|}{|\langle \varrho_{ee} \rangle_E - \langle \varrho_{\mu\mu} \rangle_E|}$$

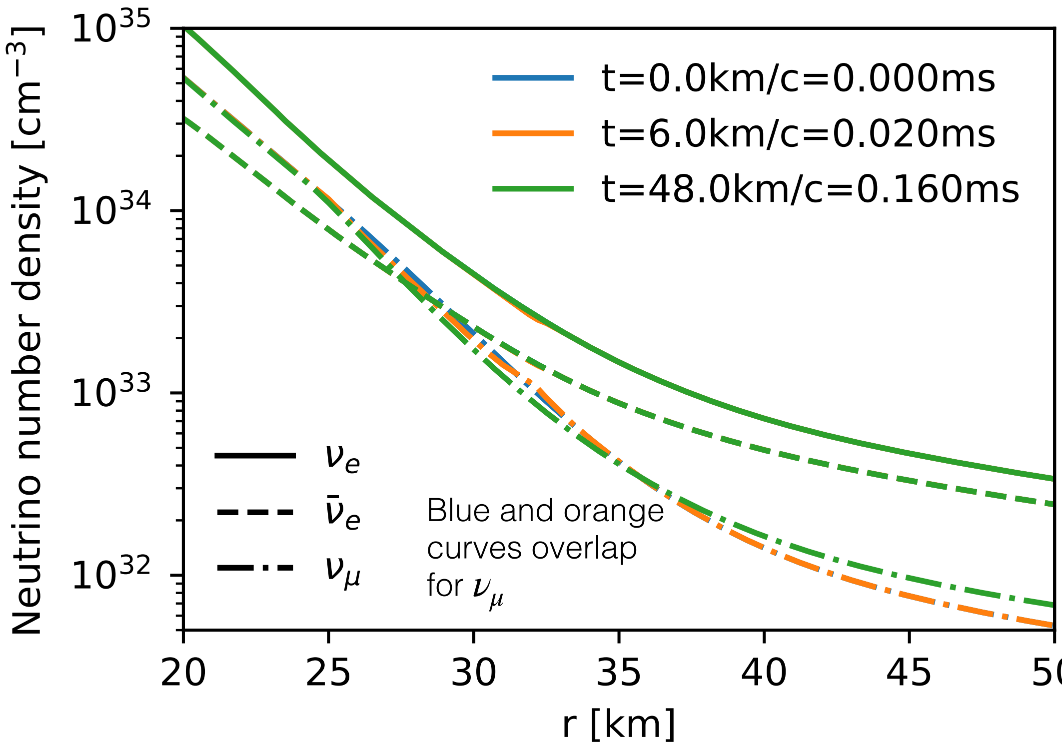


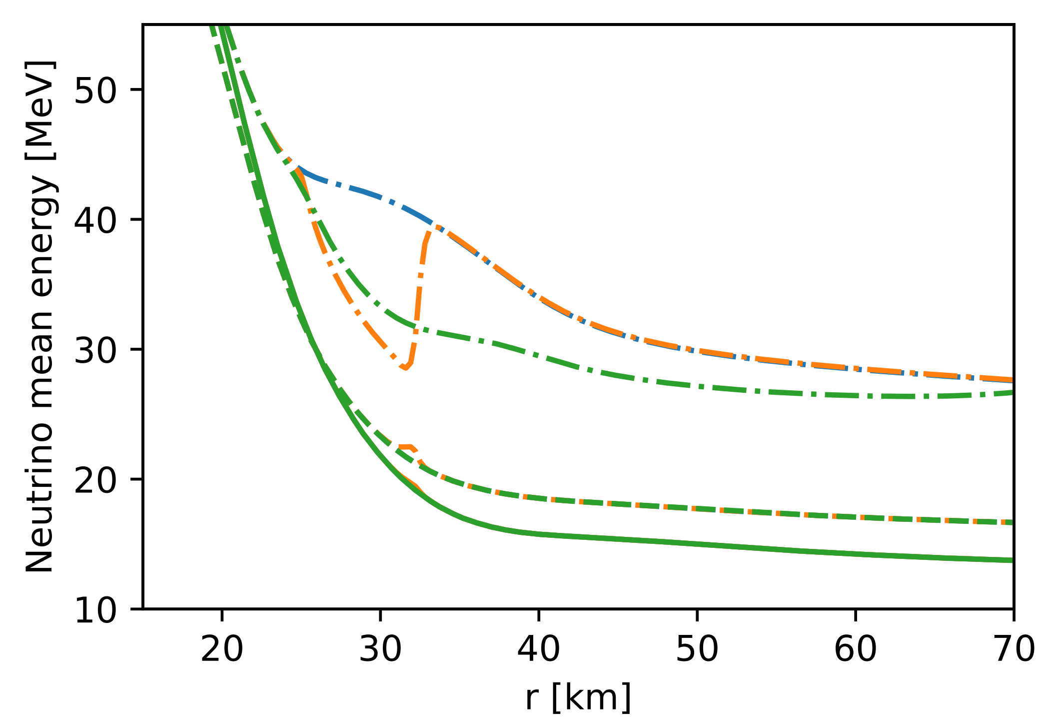
Evolution of collisional flavor instability

- distributions of ν_e and $\bar{\nu}_e$ are affected at the onset of the flavor conversion, but quickly restored by large EA rates

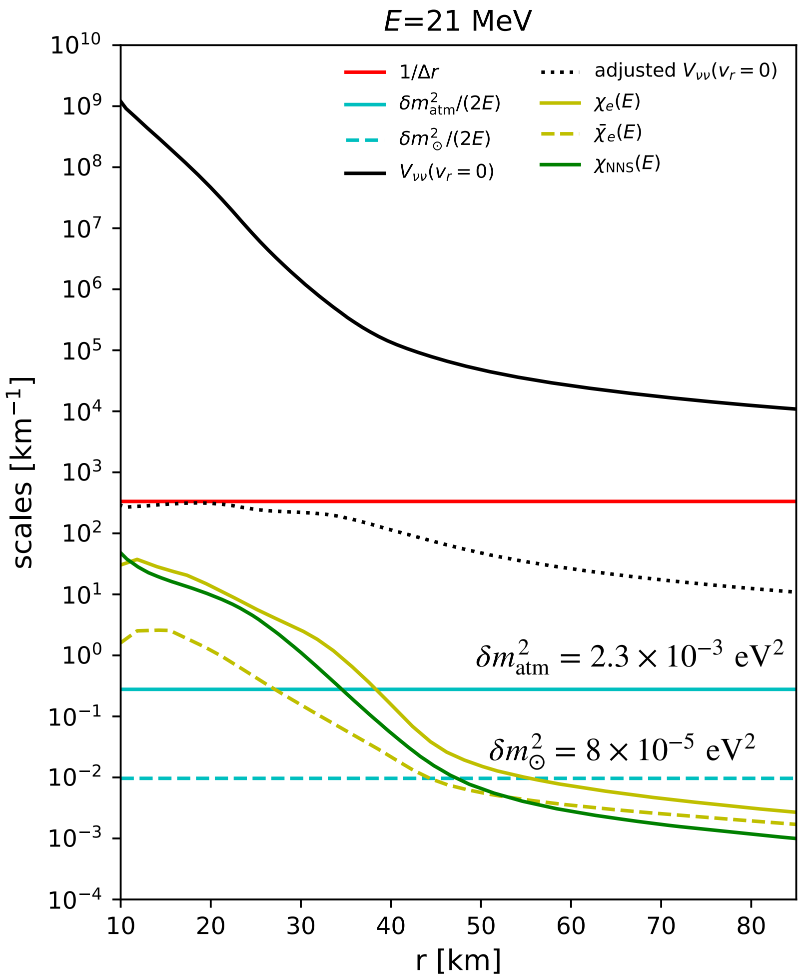
- leave imprints in the spectra of heavy-lepton (anti)neutrinos at the free-streaming regime



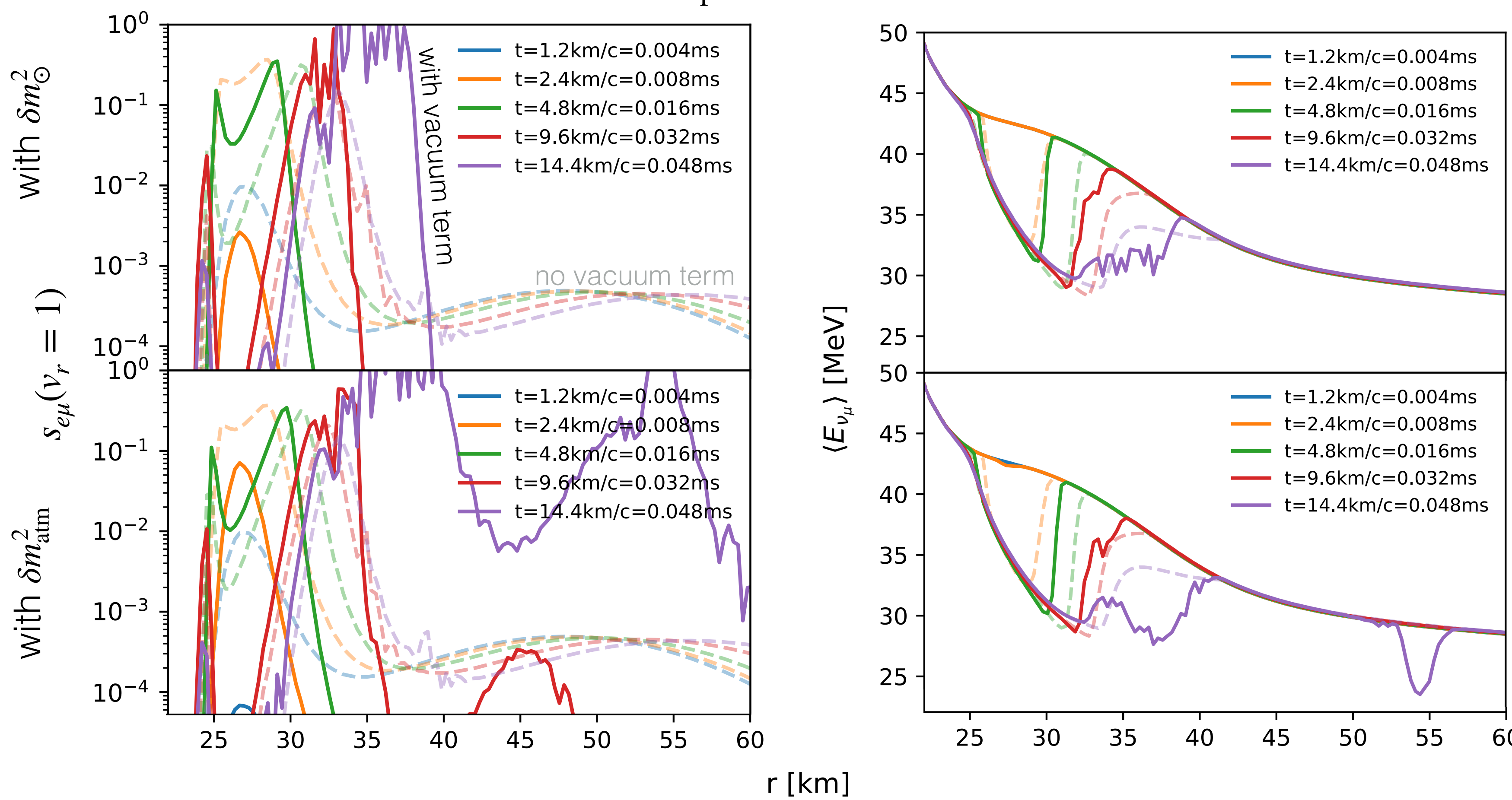




Effects of vacuum term



$t_{\rm pb} \approx 250$ ms

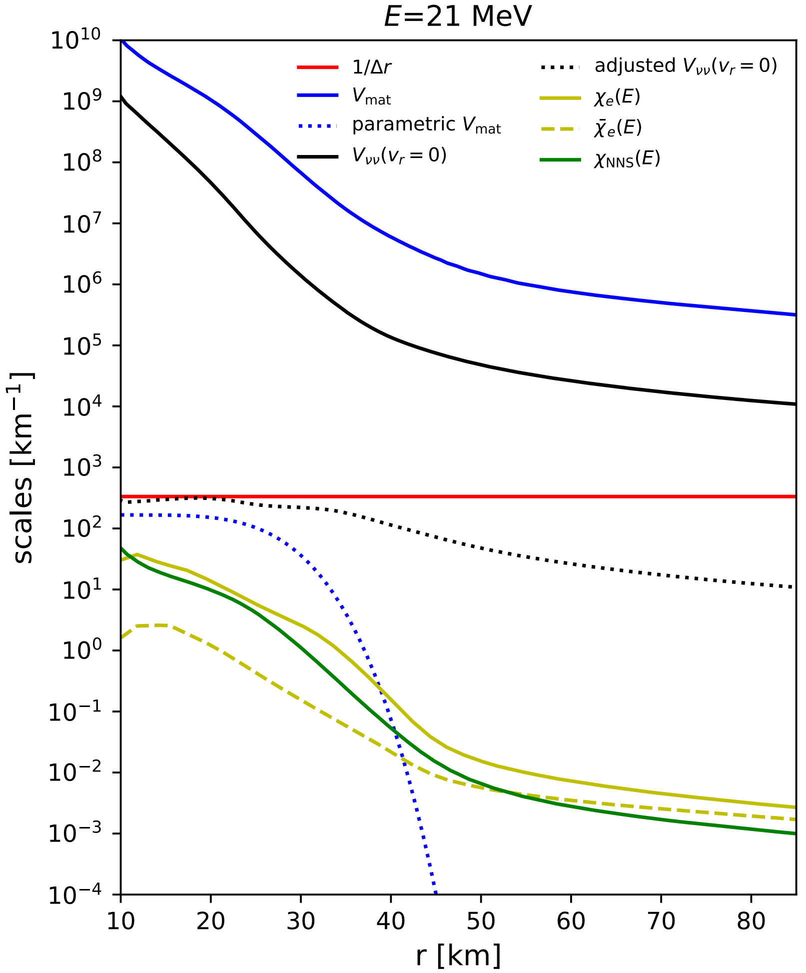


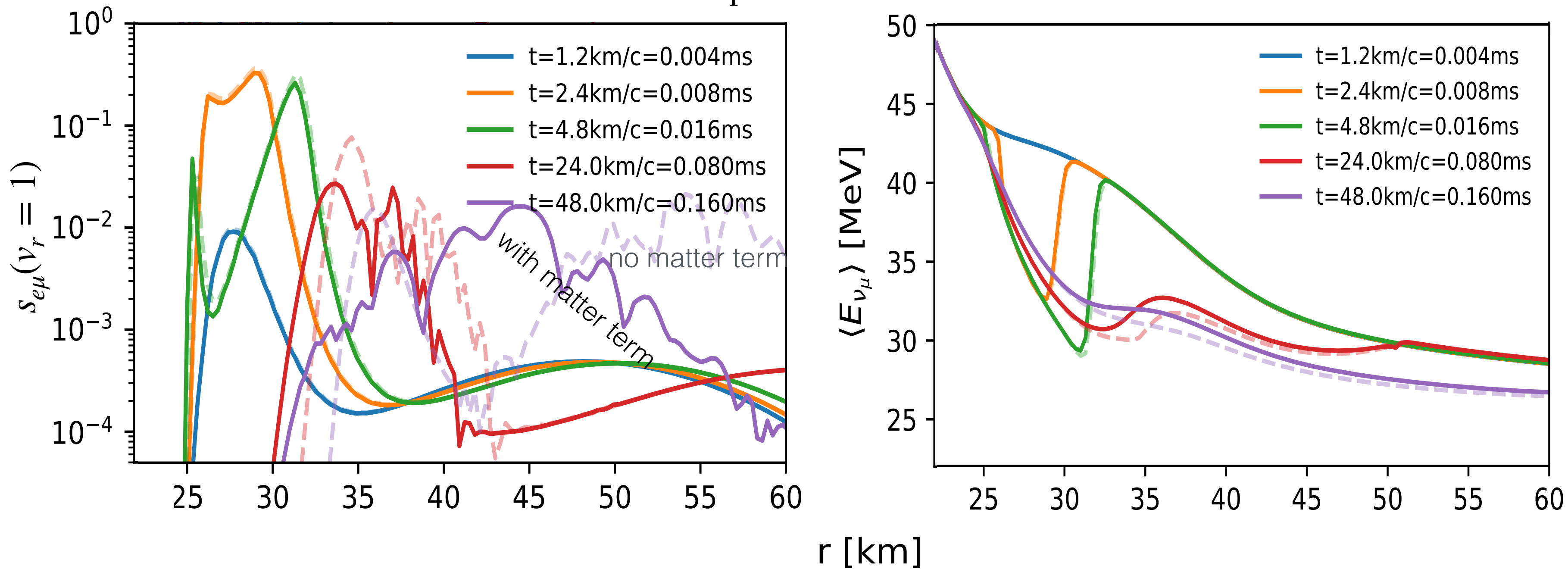
$$\mathbf{H}_{\rm vac} = \frac{\delta m^2}{4E}\begin{pmatrix} -\cos 2\theta_V & \sin 2\theta_V \\ \sin 2\theta_V & \cos 2\theta_V \end{pmatrix}$$

with $\theta_V = 10^{-6}$ and assuming normal mass hierarchy

- amplify flavor mixing at ~35 km
- more reduction of neutrino mean energy
- artificial slow flavor instability appears at ~45 km for the case with $\delta m^2_{\rm atm}$

Effects of matter term





$$\mathbf{H}_{\rm mat} = \begin{pmatrix} V_{\rm mat} & 0 \\ 0 & 0 \end{pmatrix} = \sqrt{2}G_F \begin{pmatrix} n_e & 0 \\ 0 & 0 \end{pmatrix}$$

Use parametric function instead:

$$V_{\rm mat} \to \frac{1}{2\Delta r} \exp\left[-\left(\frac{r\ [\rm km] - 10}{18}\right)^4\right]$$

- This parametric function mainly varies between 20 km and 50 km where the collisional instability occurs.
- does not affect the initial evolution of flavor instability for $t \lesssim 0.016$ ms
- affect the later transport of flavor mixing by mainly reducing the group velocity
- affect the reduced amount of neutrino mean energy

Summary and outlook

- We identify a geometric symmetry
  - to understand the periodic bipolar flavor evolution.
  - We show in numerical examples absence of this symmetry leads to kinematic decoherence.
  - Periodic bipolar evolution is special.
- We implement a multi-energy and multi-angle simulator
  - including advection on a global scale in a spherically symmetric model
  - with realistic EA & NNS collisional rates
  - to study collisional flavor instability.
  - Collisional instability leads to flavor conversions of heavy lepton neutrinos near decoupling region.
  - Artificial attenuation? include NES & NPR? matter feedback? GR effect?