Chiral transport and chiral kinetic theory in supernovae

Naoki Yamamoto (Keio University)

Focus Workshop on Collective Oscillations and Chiral Transport of Neutrinos (March 15, 2023)

Neutrinos: Basics

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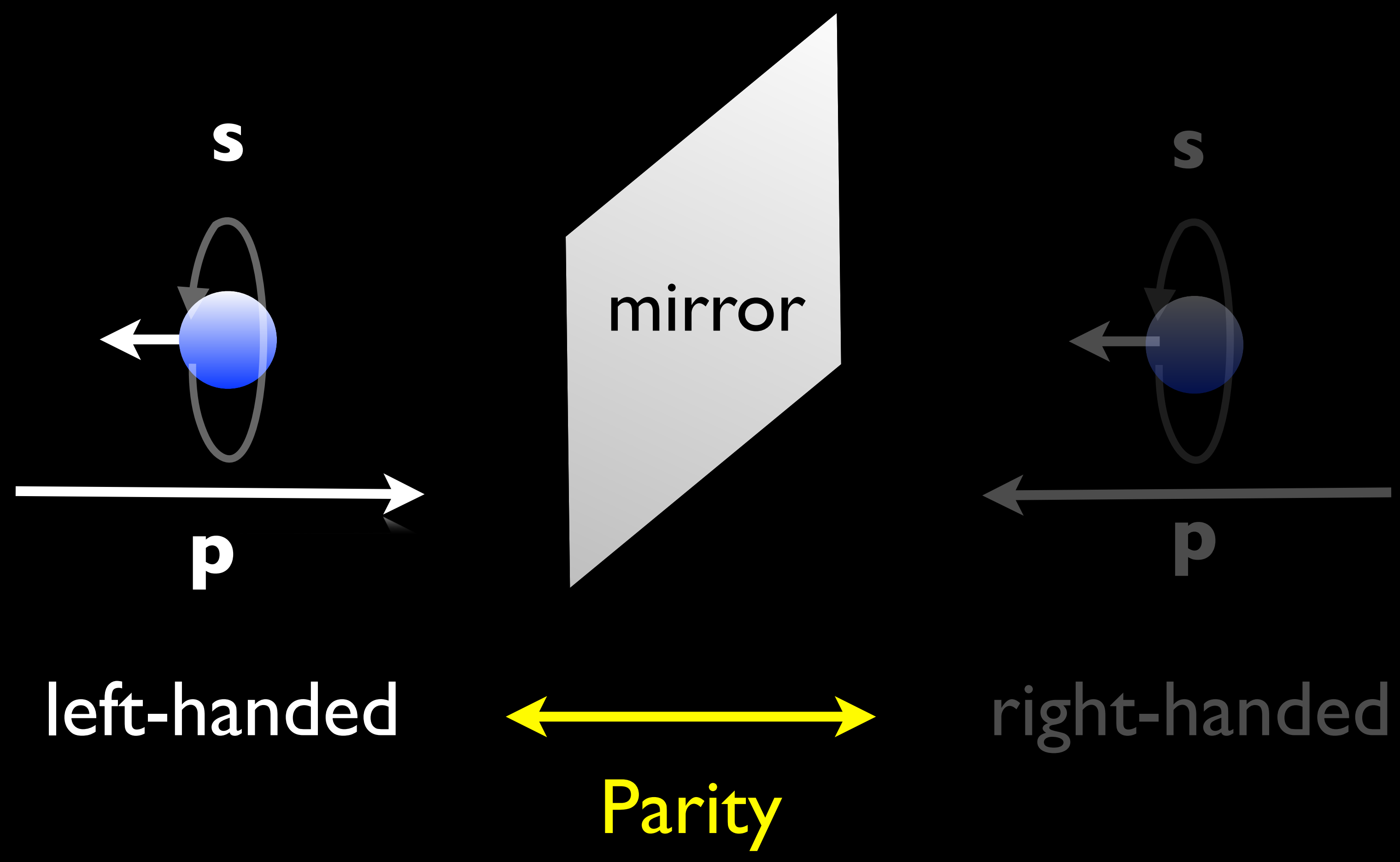
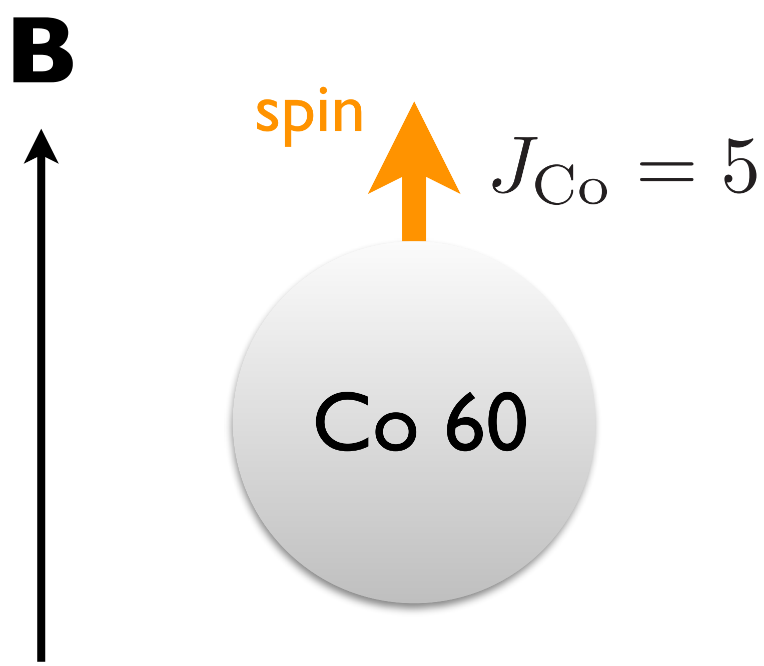
Neutrino

Composition	Elementary particle
Statistics	Fermionic
Family	Leptons, antileptons
Generation	First (ν_e), second (ν_μ), and third (ν_τ)
Interactions	Weak interaction and gravitation
Symbol	ν_e, ν_μ, ν_τ, $\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$
Particle	spin: $\pm\frac{1}{2}\hbar$, chirality: Left, weak isospin: $+\frac{1}{2}$, lepton nr.: +1, "flavour" in { e, μ, τ }

Chirality of SN neutrinos



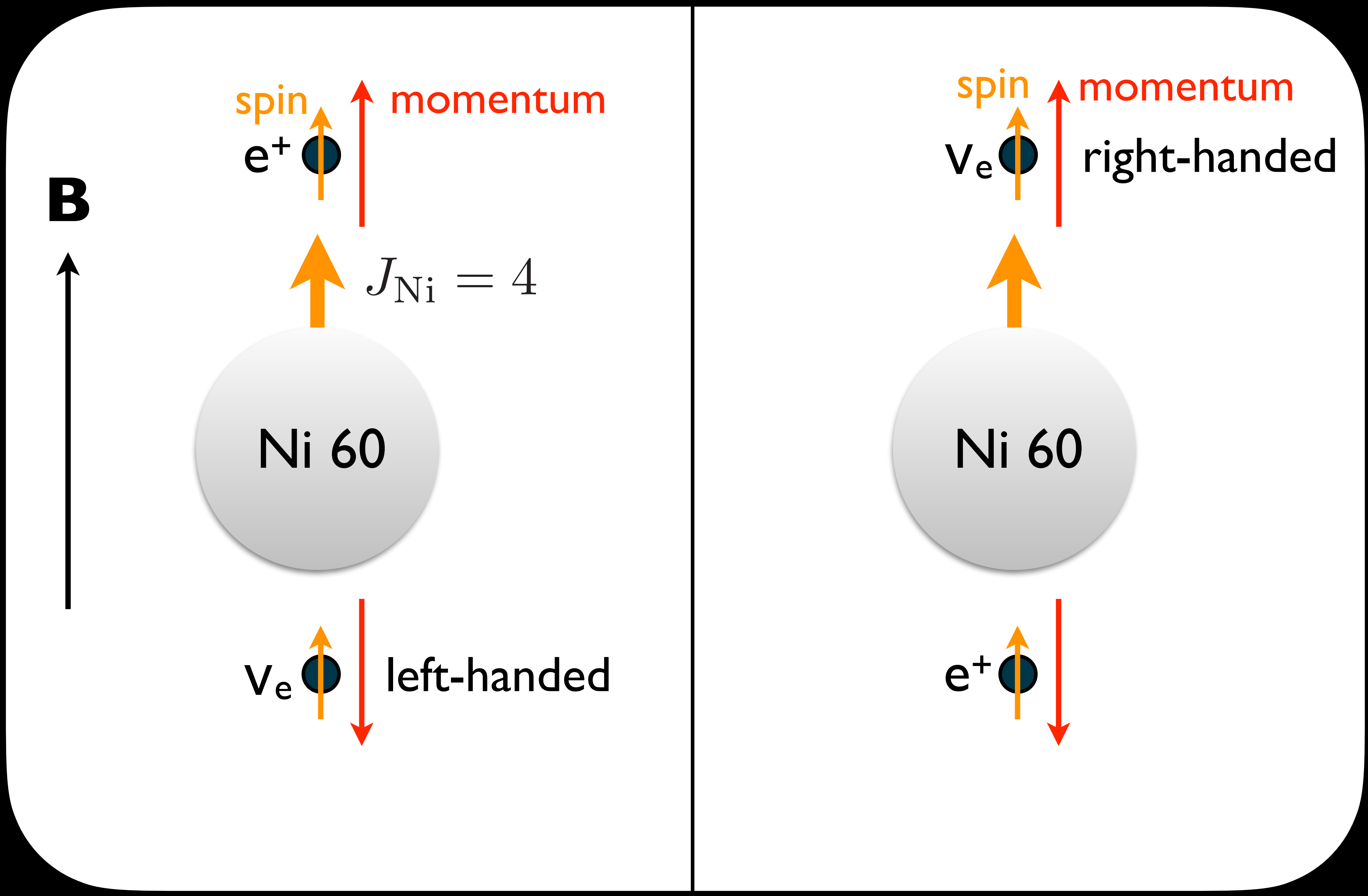
Wu experiment



Chien-Shiung Wu

Wu et al., (1957)

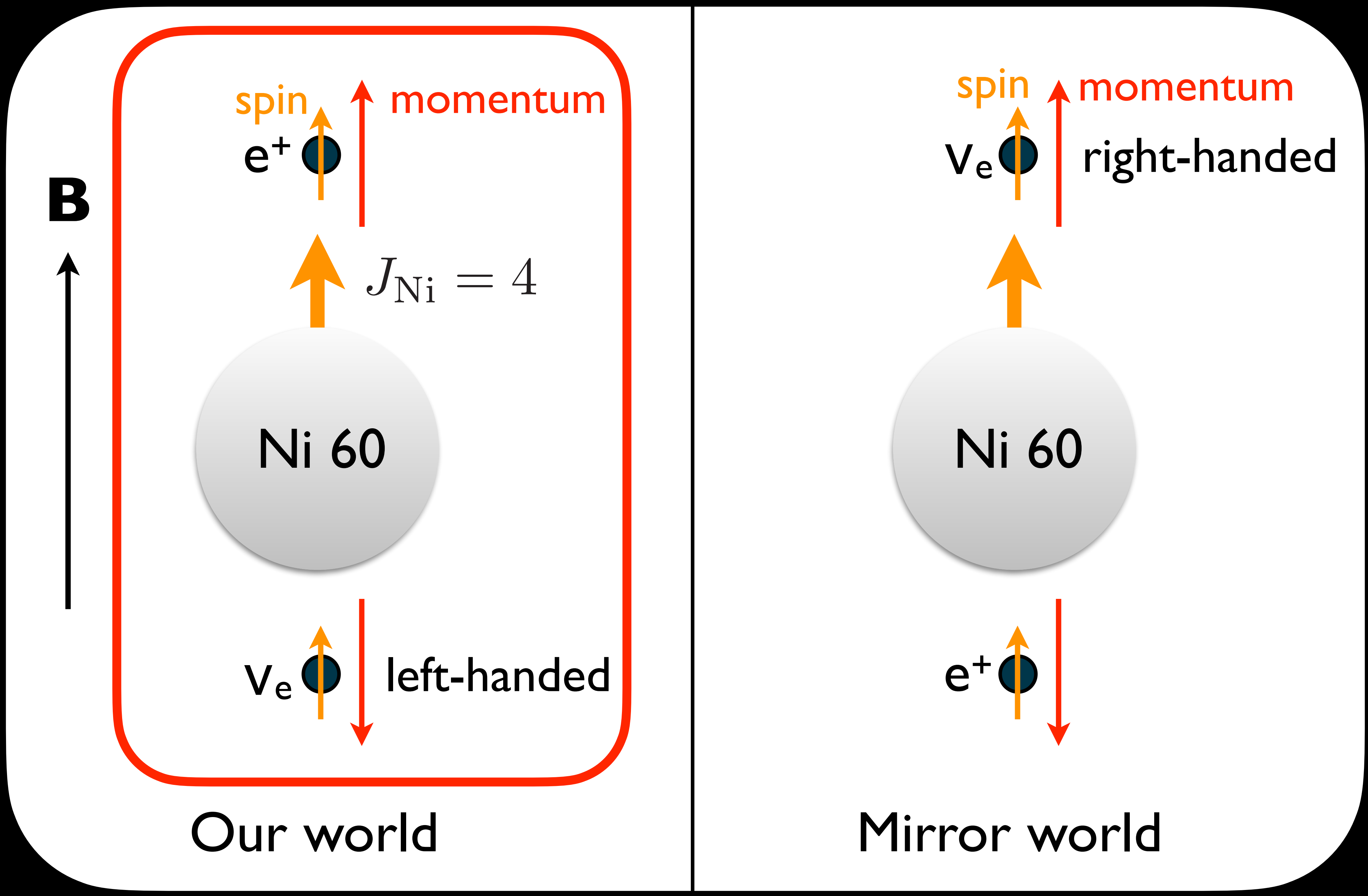
Wu experiment



$$5 = 4 + \frac{1}{2} + \frac{1}{2}$$

Wu et al., (1957)

Wu experiment



Wu et al., (1957)

Why is "God" left-handed?

- The laws of physics are (almost) left-right symmetric.
- Exception: weak interaction acts only on left-handed particles.

W. Pauli

"God is just a weak left-hander."

Contents

- Chiral transport phenomena
- Chiral kinetic theory
- Some applications in core-collapse supernovae:

  chiral plasma instability and inverse cascade

Units: $\hbar = c = k_{\mathrm{B}} = e = 1$

Kohei Kamada, Naoki Yamamoto, and Di-Lun Yang,
"*Chiral Effects in Astrophysics and Cosmology,*"
Prog. Part. Nucl. Phys. (2023) 2207.09184 [astro-ph.CO]

Chiral transport phenomena (general perspective)

Transport phenomena

- Classical and familiar examples:
  - Ohm's law: $\boldsymbol{j}_{\mathrm{e}} = \sigma \boldsymbol{E}$
  - Fourier's law: $\boldsymbol{j}_Q = \kappa(-\boldsymbol{\nabla} T)$

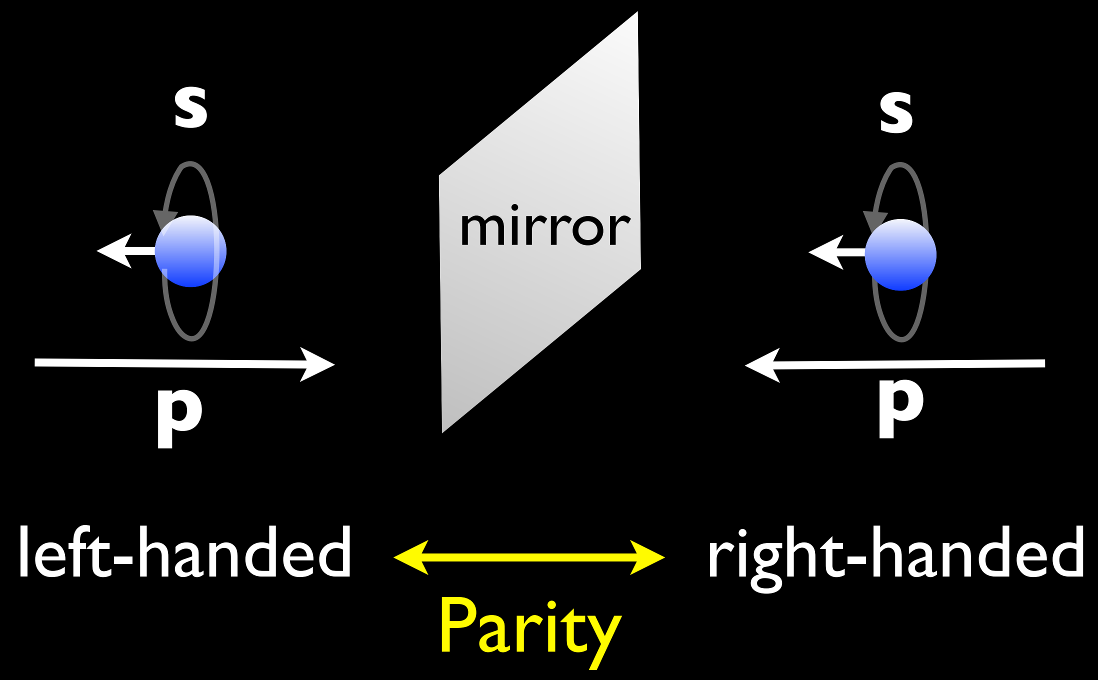
$$j_{\mathrm{e}} \sim \boldsymbol{B}?$$

Parity

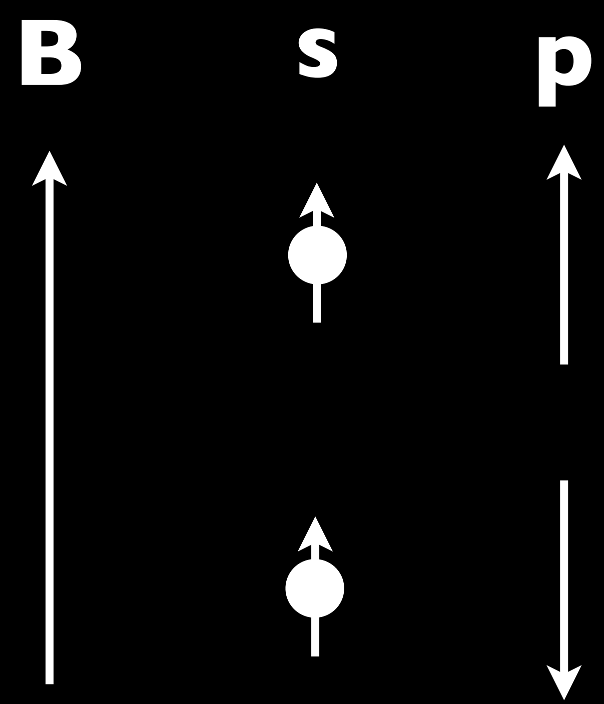
- Assume the relation: $\boldsymbol{j}_{\mathrm{e}} = \xi \boldsymbol{B}$
- Under the parity, $-\boldsymbol{j}_{\mathrm{e}} = \xi \boldsymbol{B}$ ($\because$ ***B*** is axial-vector)
- It is consistent with parity when $\xi = 0$.

Chirality

- Possible in chiral matter: $\boldsymbol{j}_{\mathrm{e}} \sim (\mu_{\mathrm{R}} - \mu_{\mathrm{L}})\boldsymbol{B}$
- This is the chiral magnetic effect (CME).



Chiral magnetic effect



right-handed $j_{\rm R} = \frac{\mu_{\rm R}}{4\pi^2}\boldsymbol{B}$

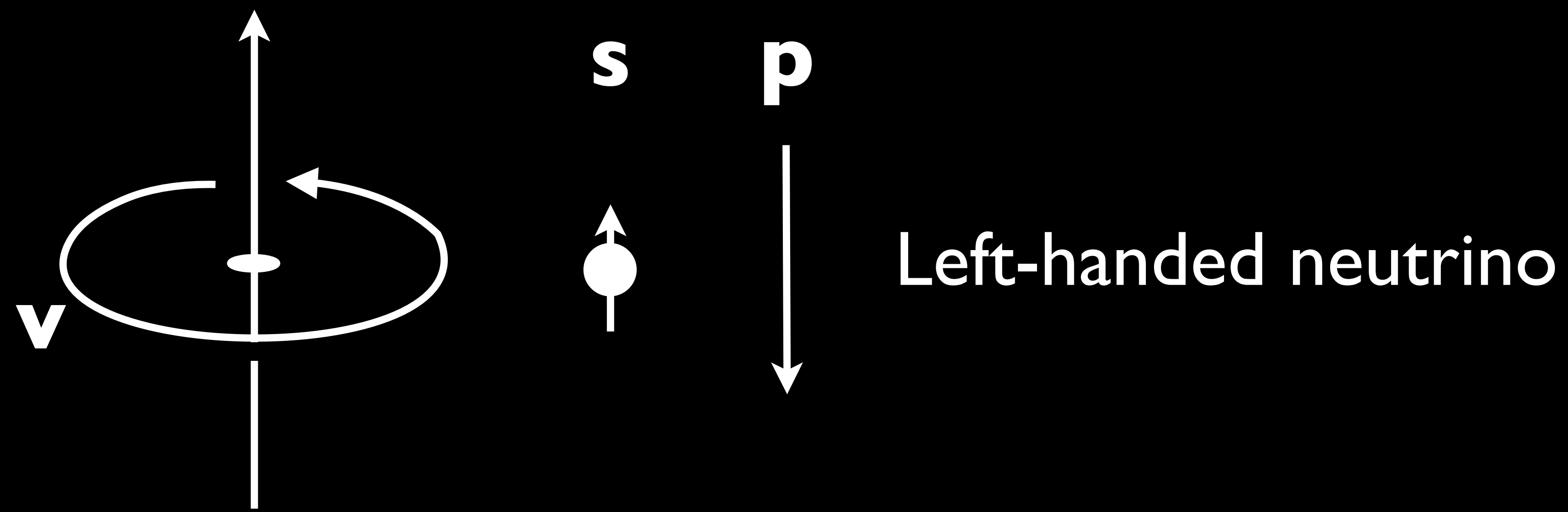
left-handed $j_{\rm L} = -\frac{\mu_{\rm L}}{4\pi^2}\boldsymbol{B}$

$$\boldsymbol{j} = \frac{\mu_{\rm R} - \mu_{\rm L}}{4\pi^2}\boldsymbol{B} \equiv \frac{\mu_5}{2\pi^2}\boldsymbol{B}$$

Vilenkin (1980); Nielsen, Ninomiya (1983); Fukushima, Kharzeev, Warringa (2008), …

Chiral vortical effect

$$\boldsymbol{\omega} \equiv \frac{1}{2}\nabla \times \boldsymbol{v}$$



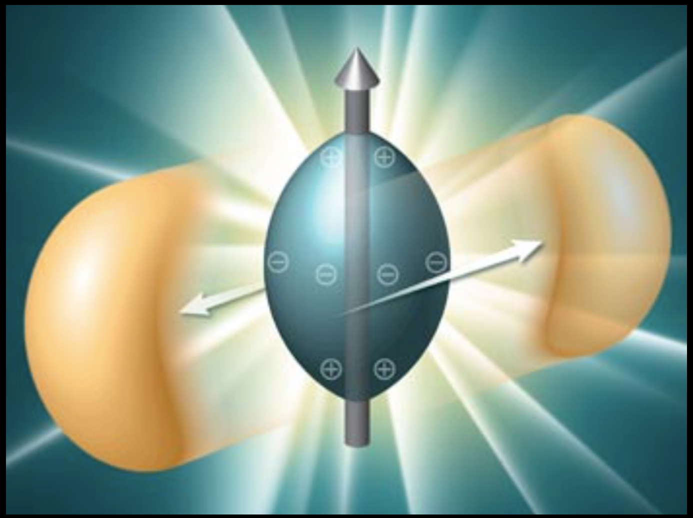
$$\boldsymbol{j} = -\left(\frac{\mu^2}{4\pi^2} + \frac{T^2}{12}\right)\boldsymbol{\omega}$$

Vilenkin (1979); Erdmenger et al. (2009); Banerjee et al. (2011); Son, Surowka (2009); Landsteiner et al. (2011)

Chiral matter

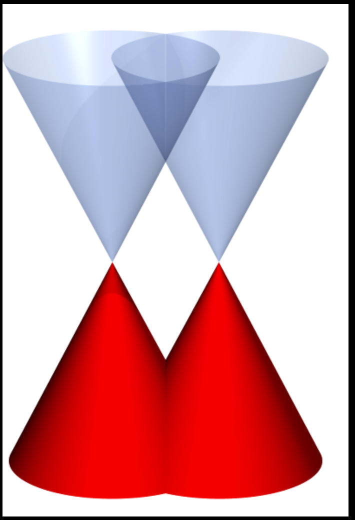
See talk by Jennifer Schober

- Electroweak plasma in early Universe — Joyce, Shaposhnikov (1997), …
- Quark-gluon plasma in heavy ion collision — Fukushima, Kharzeev, Warringa (2008), …
- Weyl semimetal (“3D graphene”) — Nielsen, Ninomiya (1983), …
- Neutrino matter in supernovae — Yamamoto (2016), …



Quark-Gluon Plasma

http://www0.bnl.gov/rhic/news2/



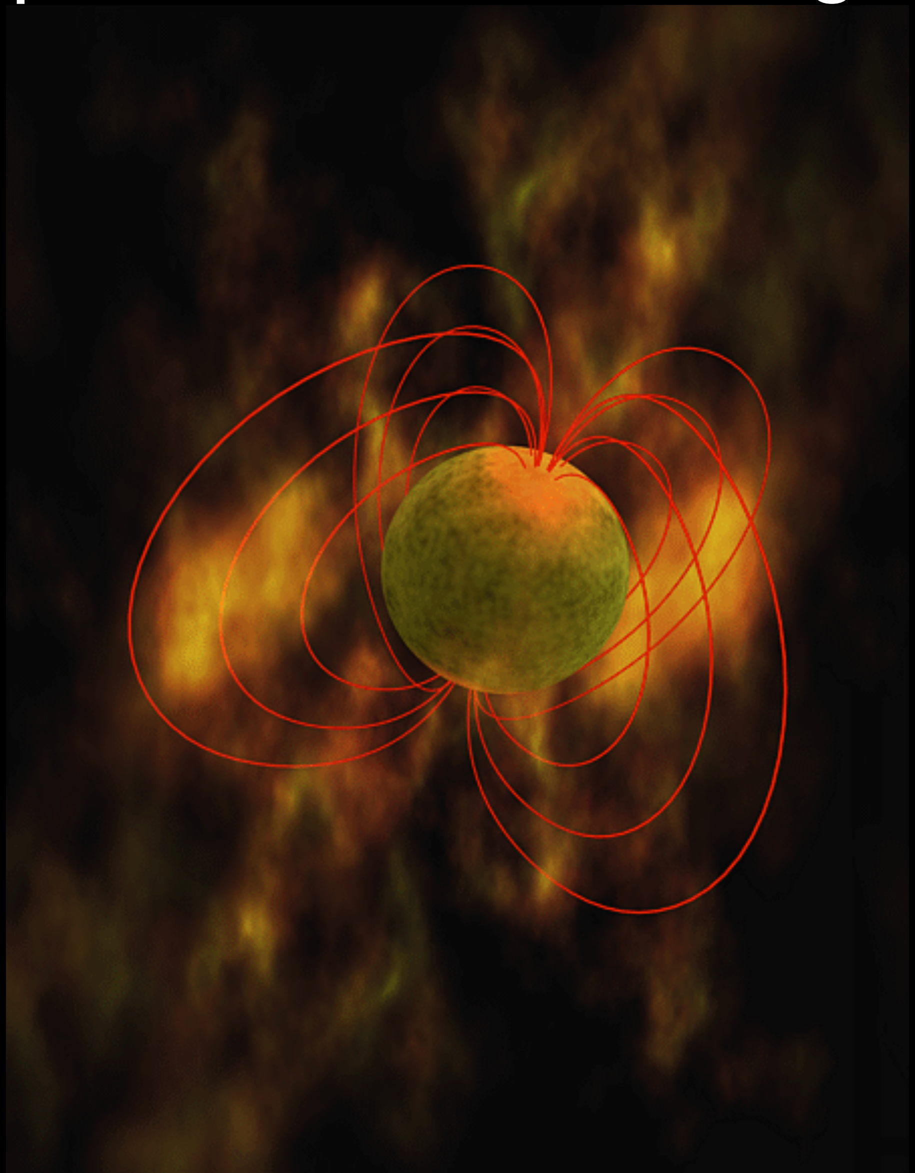
Weyl semimetal

Supernovae

Core-collapse supernova explosion

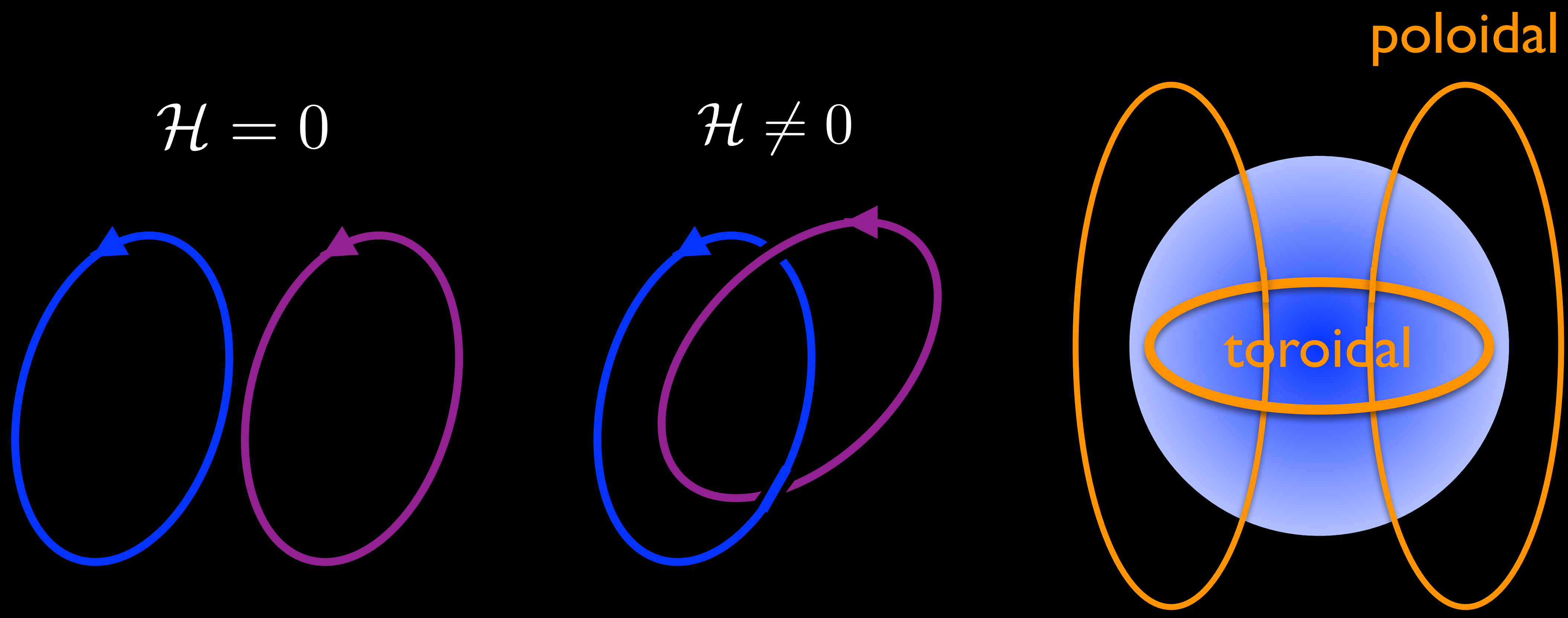
Puzzle of magnetars

- Surface magnetic field ~10^{15} G ("the strongest magnet")
- Origin of such a powerful and stable magnetic field?



Magnetic helicity

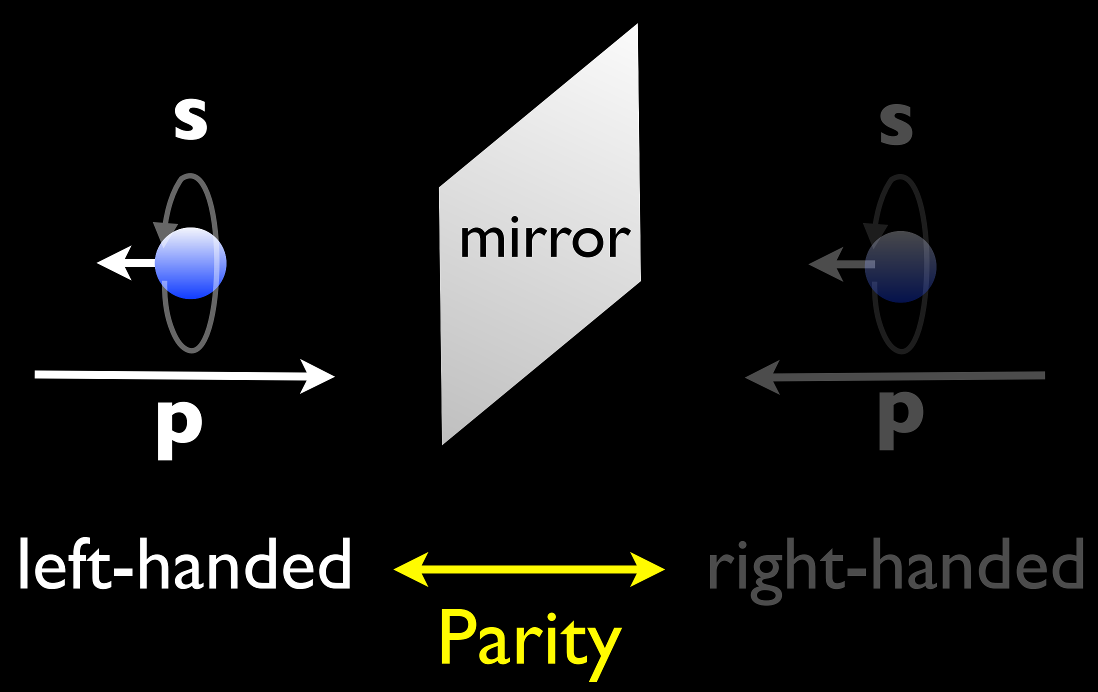
- $\mathcal{H} = \int \mathrm{d}^3 \boldsymbol{x} \boldsymbol{A} \cdot \boldsymbol{B}$: linking of magnetic fluxes (topological stability)
- Typically assumed as initial conditions, but its origin is unclear (global $\mathscr{H}$ cannot be generated by parity-even MHD).



Drawback of the theory

- Conventional transport theory ignores the left-handedness of ν.

→ *not* qualified as a correct low-energy effective theory

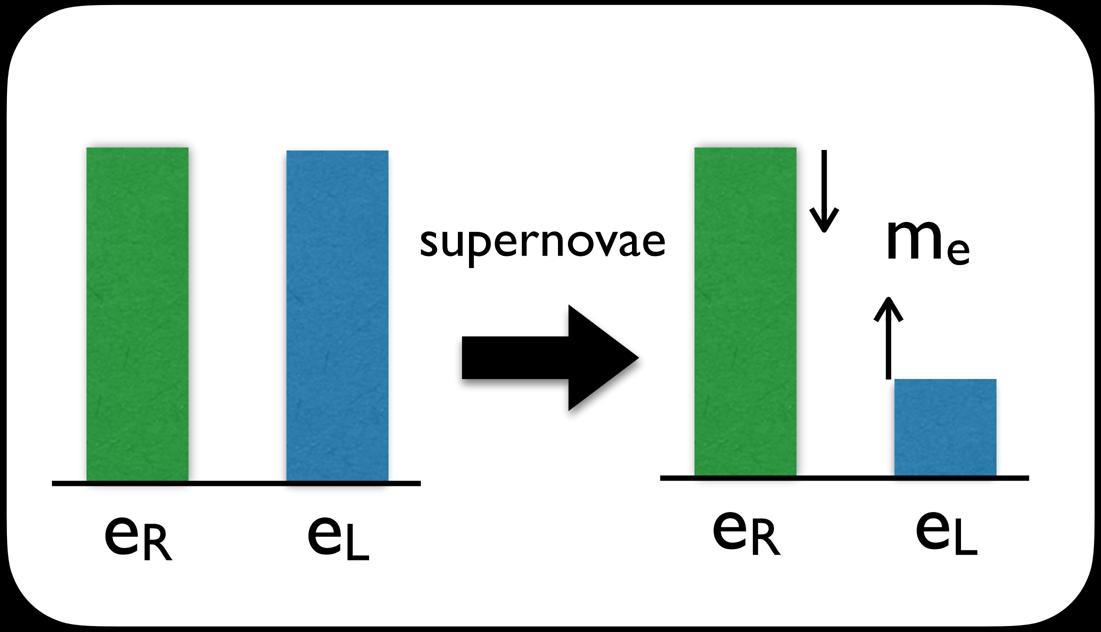


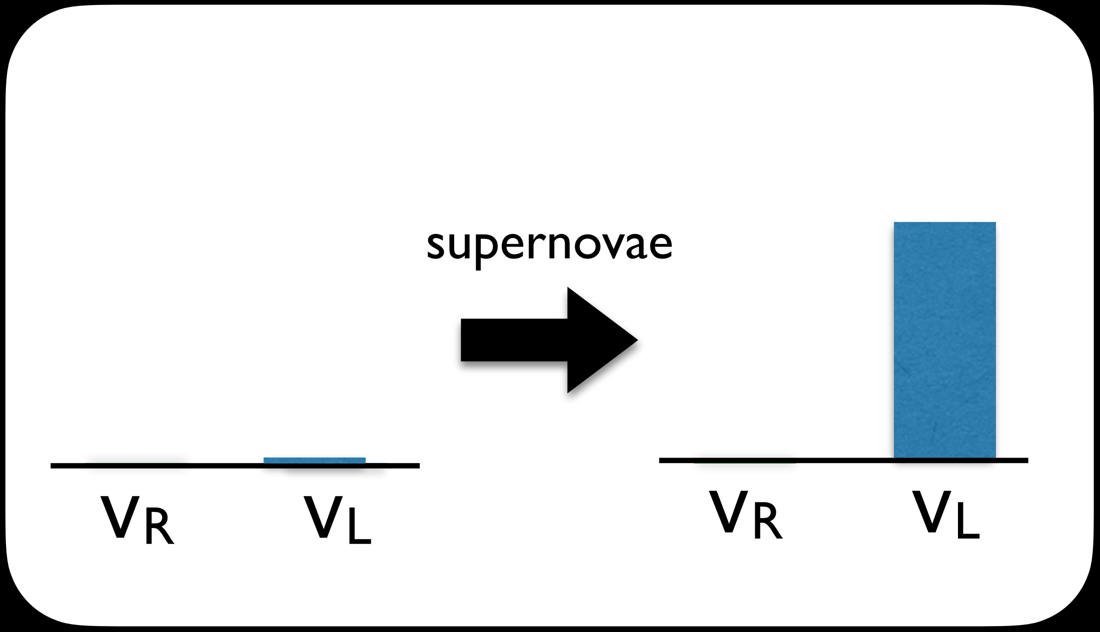
Why one should care?

- All the laws of physics are based on symmetry principle:
  - Standard Model of particle physics, hydrodynamics for (super)fluids, Ginzburg-Landau theory, …
- Examples of the importance of chirality:
  - $\pi^0 \rightarrow 2\gamma$ due to the chirality of quarks
  - Baryon number violation due to the chirality of quarks/leptons

Supernova = Giant Parity Breaker

$$p + e^{L} \rightarrow n + \nu_e^{L}$$





Ohnishi, Yamamoto (2014); Grabowska, Kaplan, Reddy (2015); Sigl, Leite (2016), …

From micro to macro

Micro — Standard Model of Particle Physics

↓

Nonequilibrium effective theory for ν

↓

Macro — Hydrodynamic evolution of core-collapse supernovae

From micro to macro

Micro Standard Model of Particle Physics

↓ ← Systematic derivation from SM?

Nonequilibrium effective theory for ν

↓

Macro Hydrodynamic evolution of core-collapse supernovae

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{x}} = C[f]$$

?

Conventional derivation

for neutral massless scalar field (spin 0)

- Green's function: $S^{<}(x,y) = \langle \phi^{\dagger}(y)\phi(x)\rangle, \quad S^{>}(x,y) = \langle \phi(x)\phi^{\dagger}(y)\rangle$
- Equation of motion: $\Box_x S^{<}(x,y) = 0$
- Wigner function: $S^{<}(q,X) = \int_s \mathrm{e}^{-\mathrm{i}q\cdot s} S^{<}\left(X+\frac{s}{2}, X-\frac{s}{2}\right) \sim f(q,X)$
- Derivative expansion: $\partial_X \ll q \longrightarrow q\cdot\partial_X f(q,X) = 0$

collisionless Boltzmann equation

Chiral radiation transport theory for neutrinos

From QFT to chiral kinetic theory

see, e.g., Hidaka, Pu, Yang, PRD (2017)

- Wigner function: $S^<(q,x) = \int_y \mathrm{e}^{-\mathrm{i}q\cdot y}\langle\psi^\dagger(x+y/2)\psi(x-y/2)\rangle \equiv \sigma^\mu \mathcal{L}^<_\mu$
- Equations of motion:

$$\mathcal{D}_\mu \mathcal{L}^{<\mu} = 0, \quad \cdots(1)$$

$$q_\mu \mathcal{L}^{<\mu} = 0, \quad \cdots(2)$$

$$\mathcal{D}_\mu \mathcal{L}^<_\nu - \mathcal{D}_\nu \mathcal{L}^<_\mu = -2\epsilon_{\mu\nu\rho\sigma}q^\rho \mathcal{L}^{<\sigma} \quad \cdots(3)$$

where $\mathcal{D}_\mu \mathcal{L}^<_\nu \equiv \partial_\mu \mathcal{L}^<_\nu - \Sigma^<_\mu \mathcal{L}^>_\nu + \Sigma^>_\mu \mathcal{L}^<_\nu$

- Solution of (2), (3): $\mathcal{L}^{<\mu} = 2\pi\delta(q^2)\big(q^\mu - S^{\mu\nu}\mathcal{D}_\nu\big)f^<$

frame vector

where $S^{\mu\nu} = \dfrac{\epsilon^{\mu\nu\alpha\beta}q_\alpha n_\beta}{2q\cdot n}$

- Inserting it into (1) → transport equation with collisions

$$J^\mu = 2\int_q \mathcal{L}^{<\mu}, \quad T^{\mu\nu} = \int_q (\mathcal{L}^{<\mu}q^\nu + \mathcal{L}^{<\nu}q^\mu)$$

Chiral radiation transport theory

Yamamoto, Yang, ApJ (2020)

General Relativity + Standard Model + Nonequilibrium Field Theory

emission absorption

$$\left[q^\mu D_\mu - (D_\mu S^{\mu\nu})\partial_\nu + S^{\mu\nu} q^\rho R^\lambda_{\rho\mu\nu} \partial_{q\lambda}\right] f = (1-f)\Gamma^{<} - f\Gamma^{>}$$

$$D_\mu = \nabla_\mu - \Gamma^\lambda_{\mu\nu} q^\nu \partial_{q\lambda}, \quad \Gamma^{\lessgtr} = (q^\nu - D_\mu S^{\mu\nu})\,\Sigma^{\lessgtr}_\nu, \quad S^{\mu\nu} = \frac{\epsilon^{\mu\nu\alpha\beta} q_\alpha n_\beta}{2q\cdot n}$$

- Systematic derivation from the underlying Standard Model
- New terms explicitly break spherical and axi-symmetries

Chiral radiation transport theory

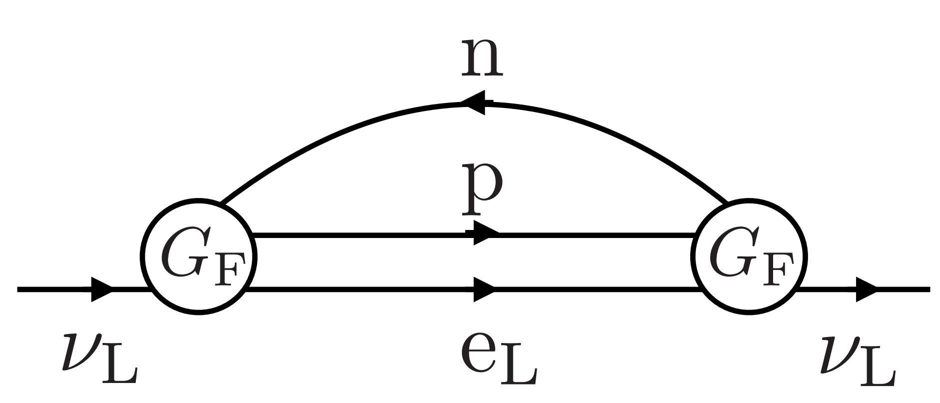
Yamamoto, Yang, ApJ (2020)

A practical version (with $n^\mu = (1,\mathbf{0})$ ignoring curvature)

$$q^\mu D_\mu f = \underbrace{(1-f)\Gamma^<}_{\text{emission}} - \underbrace{f\Gamma^>}_{\text{absorption}}$$

$$D_\mu = \nabla_\mu - \Gamma^\lambda_{\mu\nu} q^\nu \partial_{q\lambda}, \quad \Gamma^\lessgtr = (q^\nu - S^{\mu\nu} D_\mu)\,\Sigma^\lessgtr_\nu, \quad S^{\mu\nu} = \frac{\epsilon^{\mu\nu\alpha\beta} q_\alpha n_\beta}{2q\cdot n}$$

Neutrino self-energy



Chiral radiation transport theory

Yamamoto, Yang, ApJ (2020)

A practical version (with $n^\mu = (1,\mathbf{0})$ ignoring curvature)

emission absorption

$$q^\mu D_\mu f = (1-f)\Gamma^< - f\Gamma^>, \quad \Gamma^{\lessgtr} \approx \Gamma^{(0)\lessgtr} + \Gamma^{(\omega)\lessgtr}(q\cdot\omega) + \Gamma^{(B)\lessgtr}(q\cdot B)$$

$$\omega^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu \partial_\alpha u_\beta, \quad B^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta}$$

$$\Gamma^{(0)>} \approx \frac{G_{\rm F}^2}{\pi}(g_{\rm V}^2 + 3g_{\rm A}^2)E_\nu^3(1-f^{(\rm e)})\left(1-\frac{3E_\nu}{M_{\rm N}}\right)\frac{n_{\rm p}-n_{\rm n}}{1-{\rm e}^{\beta(\mu_{\rm n}-\mu_{\rm p})}}$$

$$\Gamma^{(B)>} \approx \frac{G_{\rm F}^2}{2\pi M_{\rm N}}(g_{\rm V}^2 + 3g_{\rm A}^2)E_\nu(1-f^{(\rm e)})\left(1-\frac{8E_\nu}{3M_{\rm N}}\right)\frac{n_{\rm p}-n_{\rm n}}{1-{\rm e}^{\beta(\mu_{\rm n}-\mu_{\rm p})}}$$

$$\Gamma^{(\omega)>} \approx \frac{G_{\rm F}^2}{2\pi}(g_{\rm V}^2 + 3g_{\rm A}^2)E_\nu(1-f^{(\rm e)})\left(2+\beta E_\nu f^{(\rm e)}\right)\frac{n_{\rm p}-n_{\rm n}}{1-{\rm e}^{\beta(\mu_{\rm n}-\mu_{\rm p})}}$$

$\Gamma^{(0)}$ was computed in Reddy, Prakash, Lattimer, PRD (1998)

Chiral radiation transport theory

Yamamoto, Yang, ApJ (2020)

A practical version (with $n^\mu = (1,\mathbf{0})$ ignoring curvature)

emission absorption

$$q^\mu D_\mu f = (1-f)\Gamma^< - f\Gamma^>, \quad \Gamma^{\lessgtr} \approx \Gamma^{(0)\lessgtr} + \Gamma^{(\omega)\lessgtr}(q\cdot\omega) + \Gamma^{(B)\lessgtr}(q\cdot B)$$

$$\omega^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu\partial_\alpha u_\beta, \quad B^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta}$$

Neutrino current and energy-momentum tensor (ignoring nonlinear in *f*)

$$J^\mu = \int_{\boldsymbol{q}} \frac{1}{|\boldsymbol{q}|}(q^\mu - S^{\mu\nu}D_\nu)f, \quad T^{\mu\nu} = \int_{\boldsymbol{q}} \frac{1}{|\boldsymbol{q}|}\left[q^\mu q^\nu - \frac{1}{2}\left(q^\mu S^{\nu\rho} + q^\nu S^{\mu\rho}\right)D_\rho\right] f$$

$$D_\mu = \nabla_\mu - \Gamma^\lambda_{\mu\nu}q^\nu\partial_{q\lambda}, \quad S^{\mu\nu} = \frac{\epsilon^{\mu\nu\alpha\beta}q_\alpha n_\beta}{2q\cdot n}$$

& corresponding chiral corrections in matter sector (back reaction)

Nonequilibrium chiral effects by ***B***

Yamamoto, Yang, PRD (2021), arXiv:2211.14465

- Neutrino current near equilibrium:

$$T_\nu^{i0} \approx \mu_\nu j_\nu^i \approx -\frac{1}{72\pi M_{\rm N} G_{\rm F}^2 (g_{\rm V}^2 + 3g_{\rm A}^2)} \frac{{\rm e}^{2\beta(\mu_{\rm n}-\mu_{\rm p})}}{n_{\rm n} - n_{\rm p}} (\boldsymbol{\nabla}\cdot\boldsymbol{v})\mu_\nu B^i$$

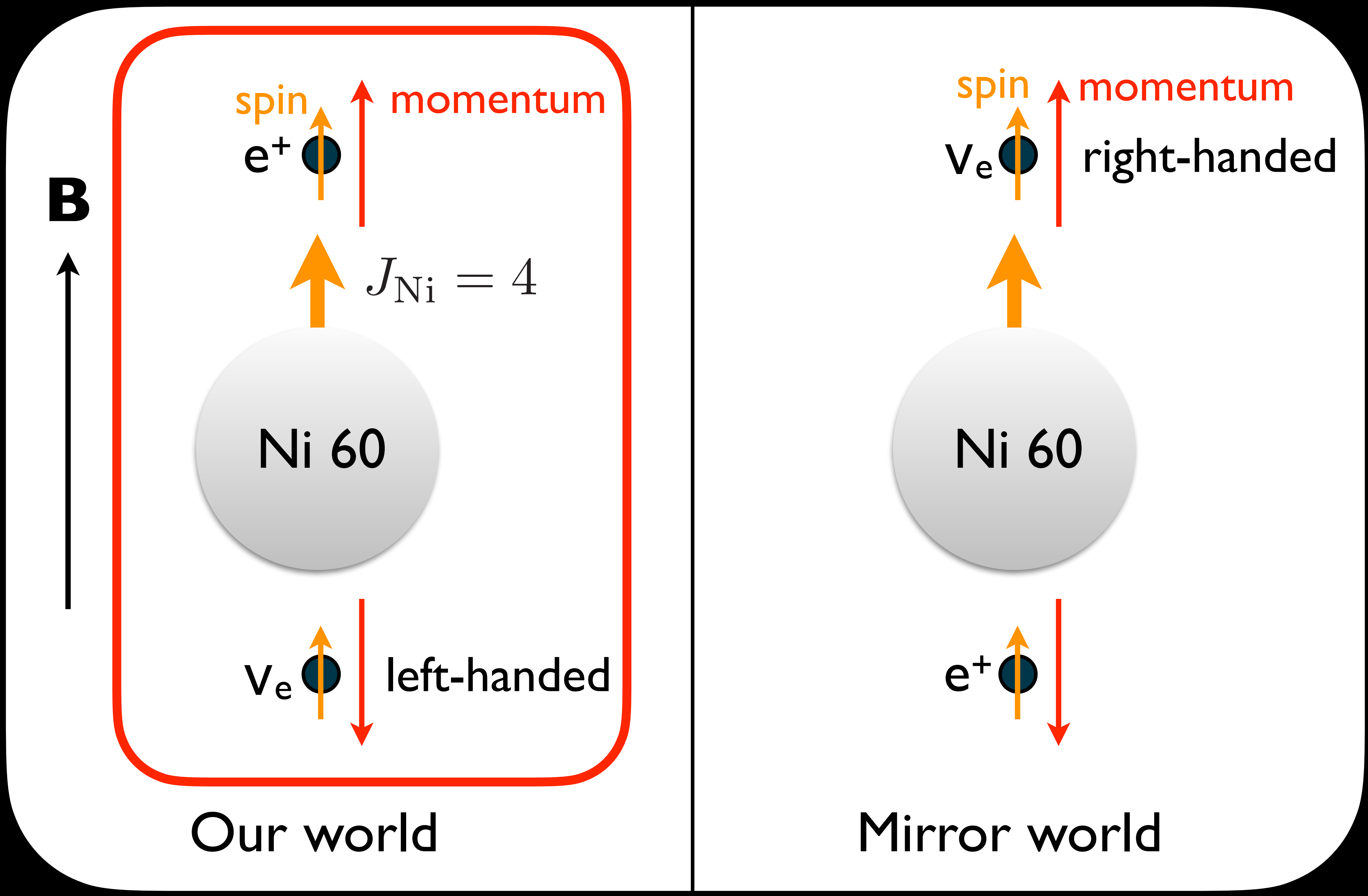
- Electric current induced by generic nonequilibrium neutrinos:

$$\boldsymbol{j}_{\rm e} = \xi_B \boldsymbol{B}$$

$$\dot{\xi}_B = \frac{1}{4\pi^3}(g_{\rm V}^2 + 3g_{\rm A}^2) G_{\rm F}^2 (n_{\rm p} - n_{\rm n}) \int_0^\infty p^2 {\rm d}p \left[\frac{\bar{f}_{\rm e}(1 - f_\nu)}{1 - {\rm e}^{\beta(\mu_{\rm n}-\mu_{\rm p})}} + \frac{(1 - \bar{f}_{\rm e}) f_\nu}{1 - {\rm e}^{\beta(\mu_{\rm p}-\mu_{\rm n})}} \right]$$

Effective chiral magnetic effect (without μ_5)

Wu experiment



$J_{e,\nu} \propto B$: nonequilibrium many-body manifestation of the chiral effect

Effective chiral magnetic effect

$$\boldsymbol{j}_{\rm e} = [\#\mu_5 + \#\boldsymbol{v}\cdot\boldsymbol{\omega} + \xi_B(f_\nu) + \cdots]\boldsymbol{B}$$

- μ_5 generated by the electron capture $\mathrm{p} + \mathrm{e}^{\mathrm{L}} \leftrightarrow \mathrm{n} + \nu_{\mathrm{e}}^{\mathrm{L}}$
  $\to$ may be erased by chirality flipping ($e_R \leftrightarrow e_L$) due to finite m_e
  Ohnishi, Yamamoto (2014); Grabowska, Kaplan, Reddy (2015); Sigl, Leite (2016), …

- Kinetic helicity generated by hydro evolution with CVE
  → *globally* present unlike turbulent generation (α effect)
  Yamamoto (2016) For the conventional α effect, see talk by Matsumoto

- $\xi_B(f_\nu)$ due to scattering with nonequilibrium neutrinos
  Yamamoto, Yang, arXiv:2211.14465

Effective chiral magnetic effect

$$\boldsymbol{j}_{\rm e} = [\#\mu_5 + \#\boldsymbol{v}\cdot\boldsymbol{\omega} + \xi_B(f_\nu) + \cdots]\boldsymbol{B}$$

- $\xi_B(f_\nu)$ due to scattering with nonequilibrium neutrinos

$$\xi_B = \frac{1}{4\pi^3}(g_{\rm V}^2 + 3g_{\rm A}^2)G_{\rm F}^2(n_{\rm p} - n_{\rm n})t\int_0^\infty p^2{\rm d}p\left[\frac{\bar{f}_{\rm e}(1-f_\nu)}{1-{\rm e}^{\beta(\mu_{\rm n}-\mu_{\rm p})}} + \frac{(1-\bar{f}_{\rm e})f_\nu}{1-{\rm e}^{\beta(\mu_{\rm p}-\mu_{\rm n})}}\right] + ({\rm antiparticle's})$$

$\sim 0.1\text{-}1\,{\rm MeV}$ in the gain region

$$Y_{\rm e} \simeq 0.4,\ \rho\sim 10^{10}\,{\rm g}\cdot{\rm cm}^{-3},\ T\sim 10^{11}\,{\rm K},\ \mu_{\rm n}-\mu_{\rm p}\simeq 3\,{\rm MeV},\ t\sim 0.1\,{\rm s}$$

Matter sector gains not only energy but also helicity

Local simulation for supernovae

Masada et al., arXiv:1805.10419; Matsumoto et al, arXiv:2202.09205

Chiral magnetohydrodynamic (MHD) equations

$$\partial_t \rho + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0$$

$$\partial_t (\rho \boldsymbol{v}) + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v} \boldsymbol{v}) = -\boldsymbol{\nabla} P + \boldsymbol{J} \times \boldsymbol{B} + \text{(dissipation)}$$

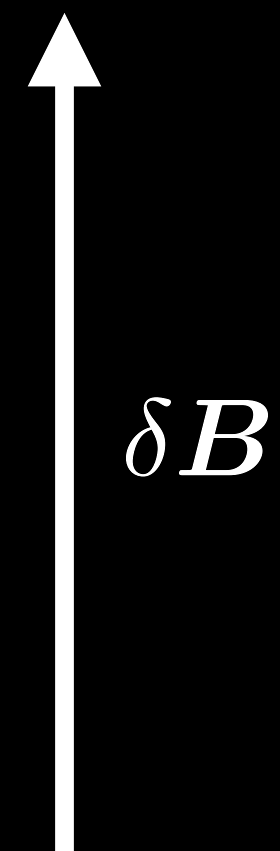
$$\partial_t \boldsymbol{B} = \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \boldsymbol{\nabla}^2 \boldsymbol{B} + \eta \boldsymbol{\nabla} \times \underline{(\xi_B \boldsymbol{B})}$$

$$\partial_t \mathcal{H}(\xi_B) = \frac{\eta}{2\pi^2} (\boldsymbol{\nabla} \times \boldsymbol{B} - \xi_B \boldsymbol{B}) \cdot \boldsymbol{B}$$

see also Rogachevskii et al. (2017), Brandenburg et al. (2017), Schober et al. (2018)

See talks by Schober and Matsumoto

Chiral plasma instability



Consider a perturbation of a seed magnetic field.

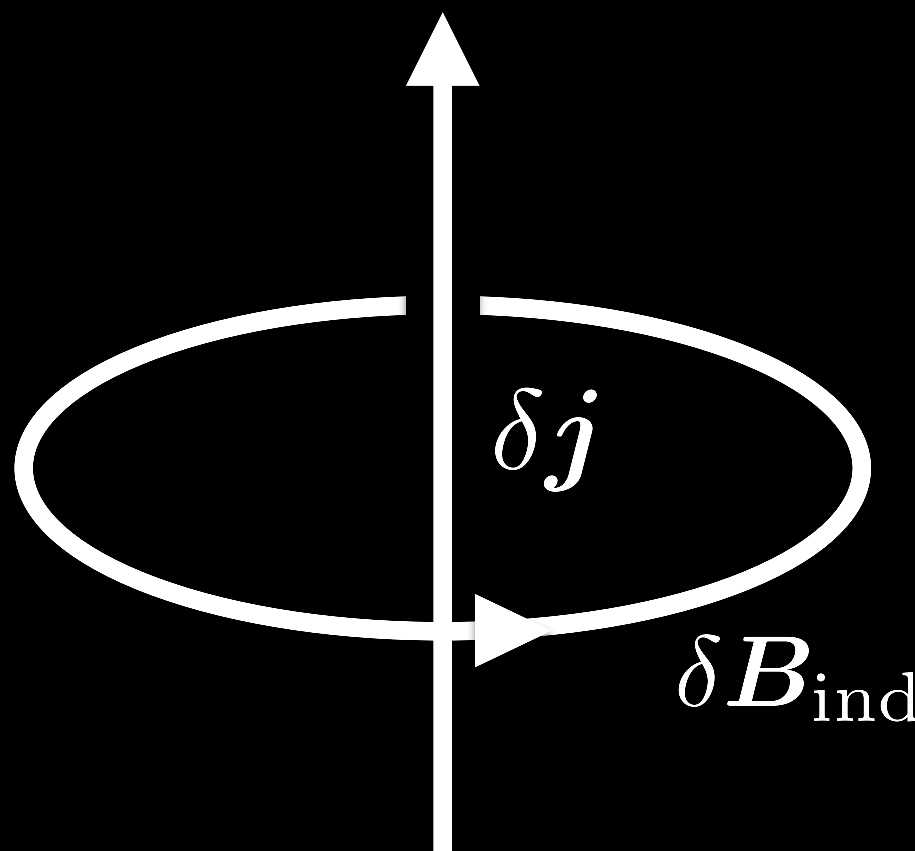
Joyce, Shaposhnikov (1997); Akamatsu, Yamamoto (2013)

Chiral plasma instability

$$\delta \boldsymbol{j} \propto \delta \boldsymbol{B}$$ effective CME

Joyce, Shaposhnikov (1997); Akamatsu, Yamamoto (2013)

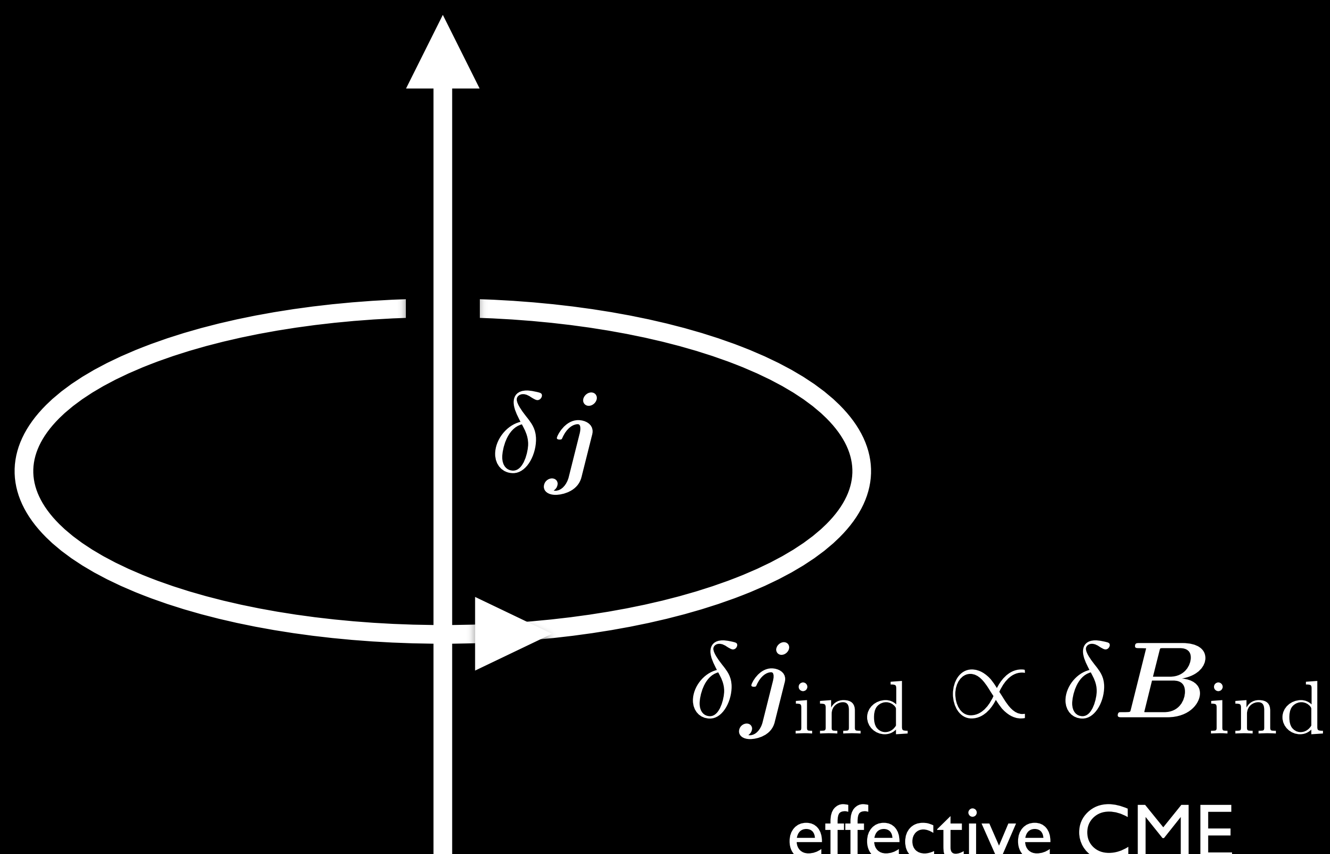
Chiral plasma instability



Ampère's law

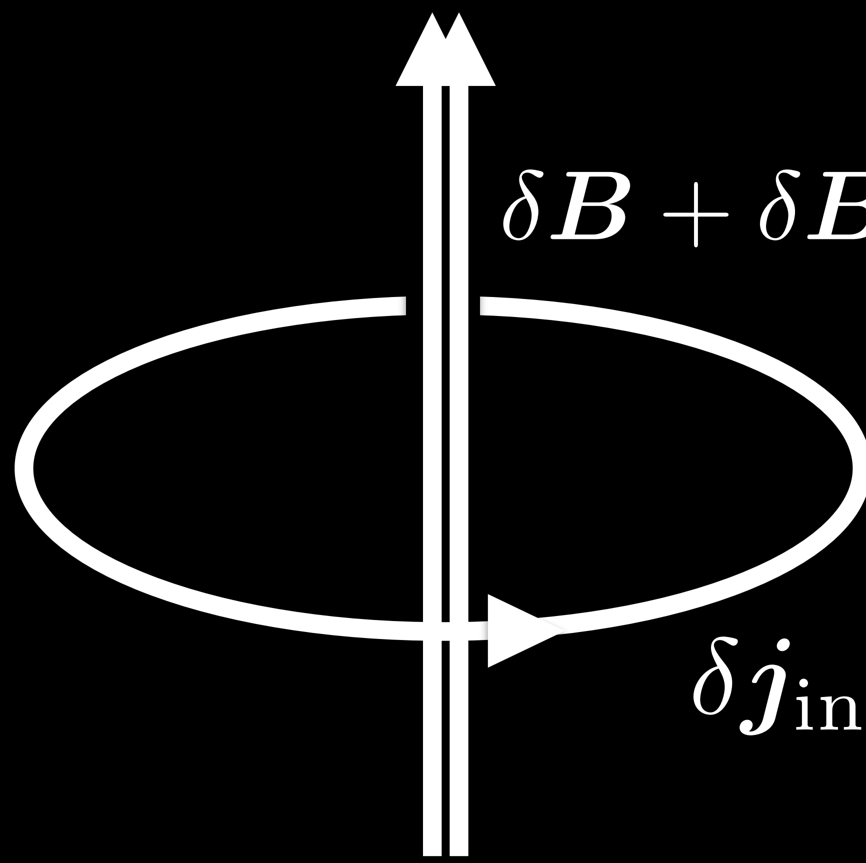
Joyce, Shaposhnikov (1997); Akamatsu, Yamamoto (2013)

Chiral plasma instability



Joyce, Shaposhnikov (1997); Akamatsu, Yamamoto (2013)

Chiral plasma instability

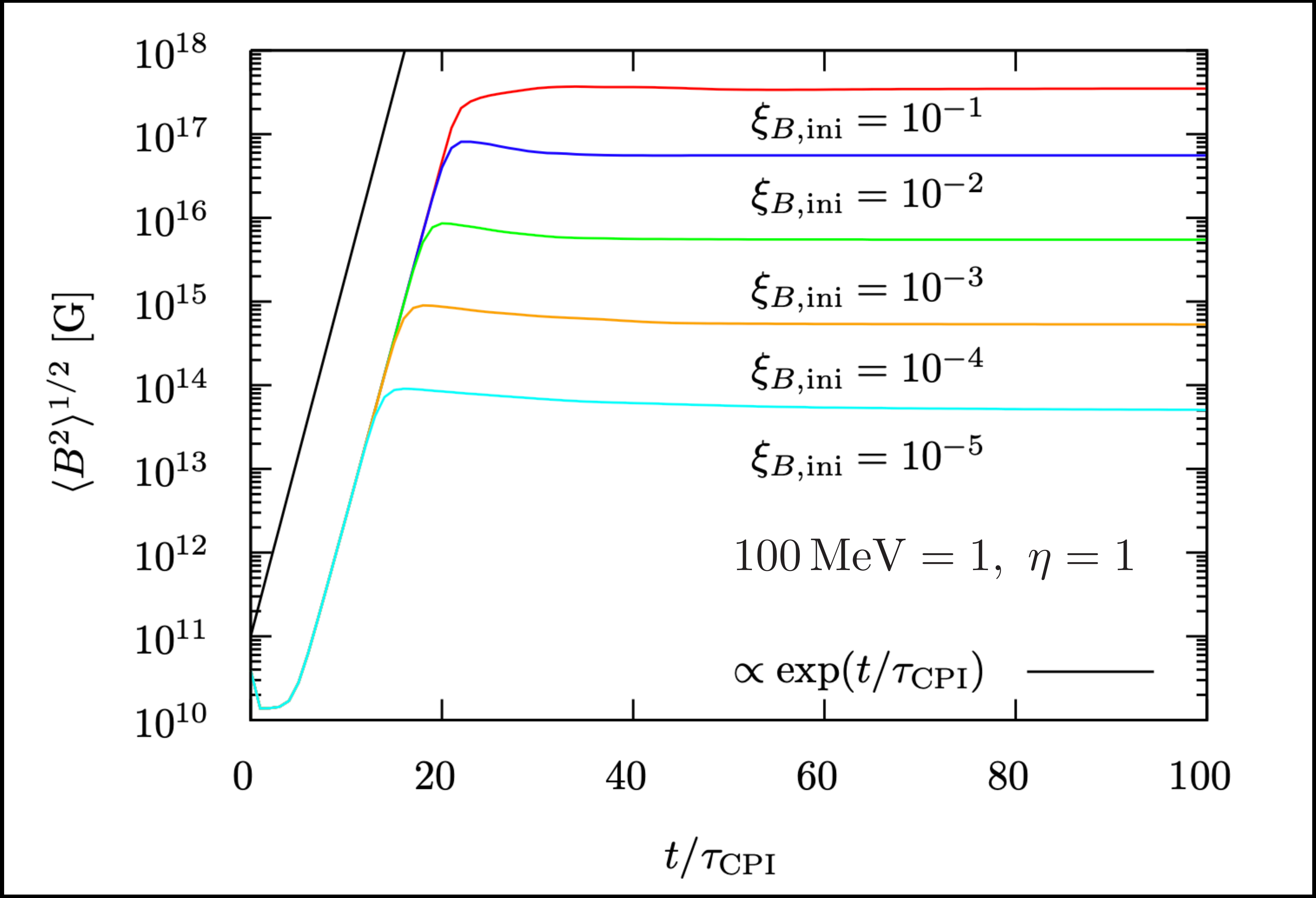


Positive feedback: instability → generation of magnetic field with $\mathscr{H} \neq 0$

Joyce, Shaposhnikov (1997); Akamatsu, Yamamoto (2013)

Time evolution of ***B***

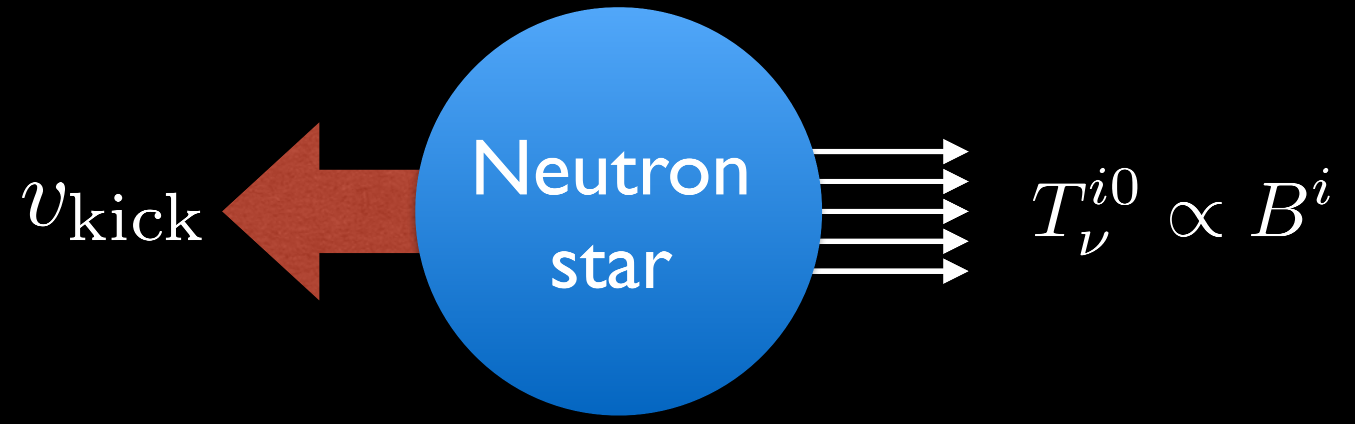
Matsumoto, Yamamoto, Yang, arXiv:2202.09205



Possible new mechanism for magnetars?

Contribution to pulsar kicks

Neutrino energy current provides a “kick” to neutron stars.



$$v_{\rm kick} \sim 100 \left(\frac{B}{10^{15}\ {\rm G}} \right)\ {\rm km/s}$$

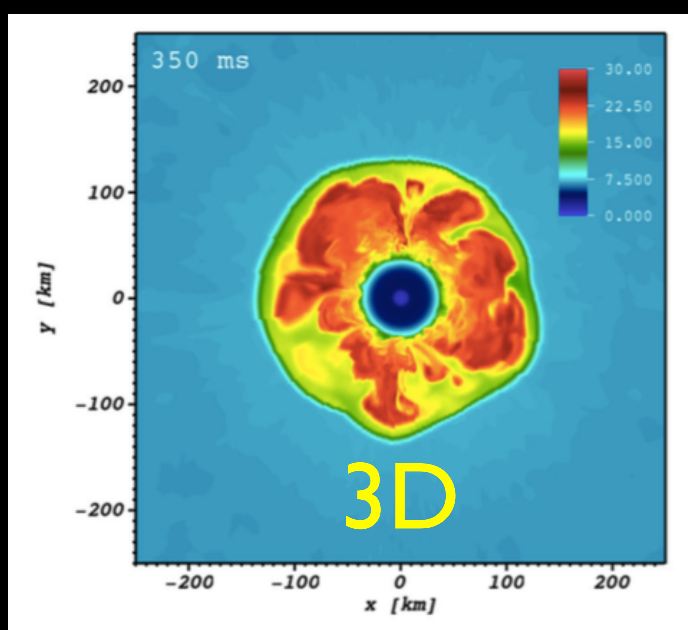
This can be comparable to the observed magnitude for B~10^{15} G.

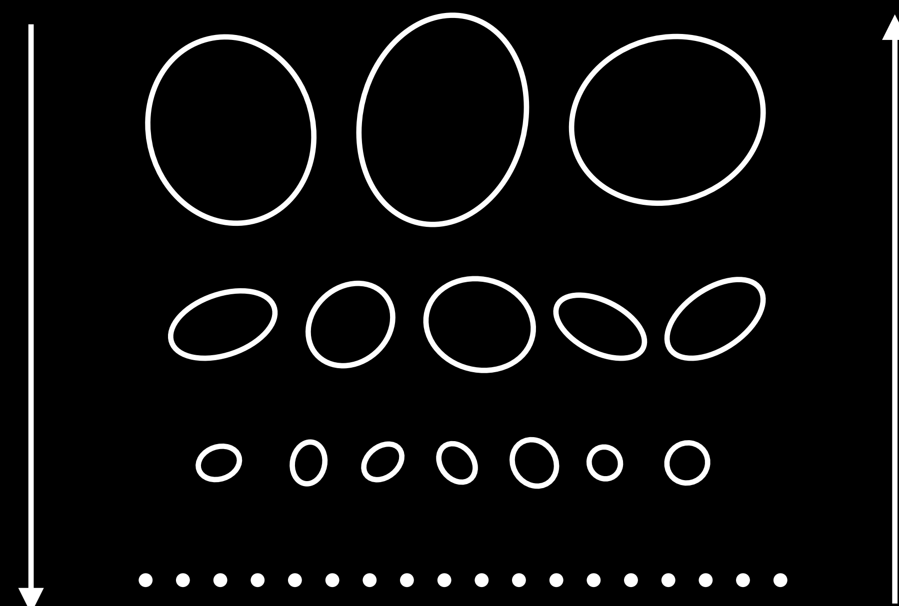
Yamamoto, Yang, PRD (2021); see also Vilenkin, ApJ (1995)

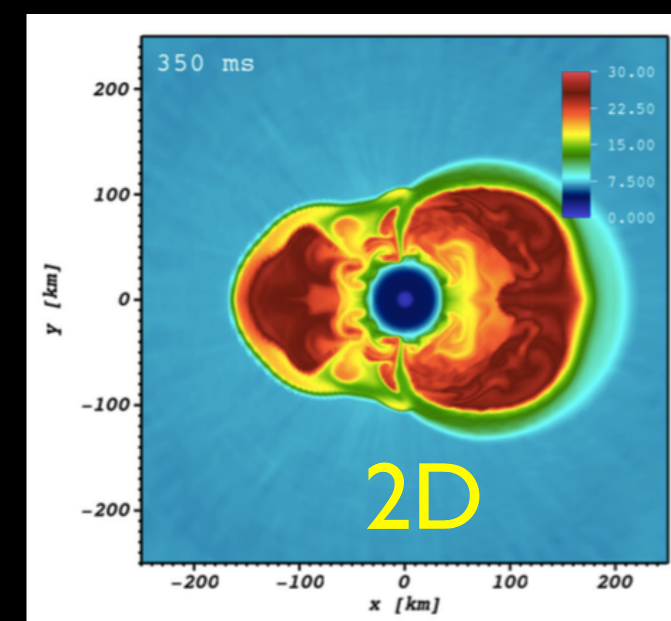
Turbulent cascade

Direct cascade (3D):
energy

Inverse cascade (2D):
energy & enstrophy







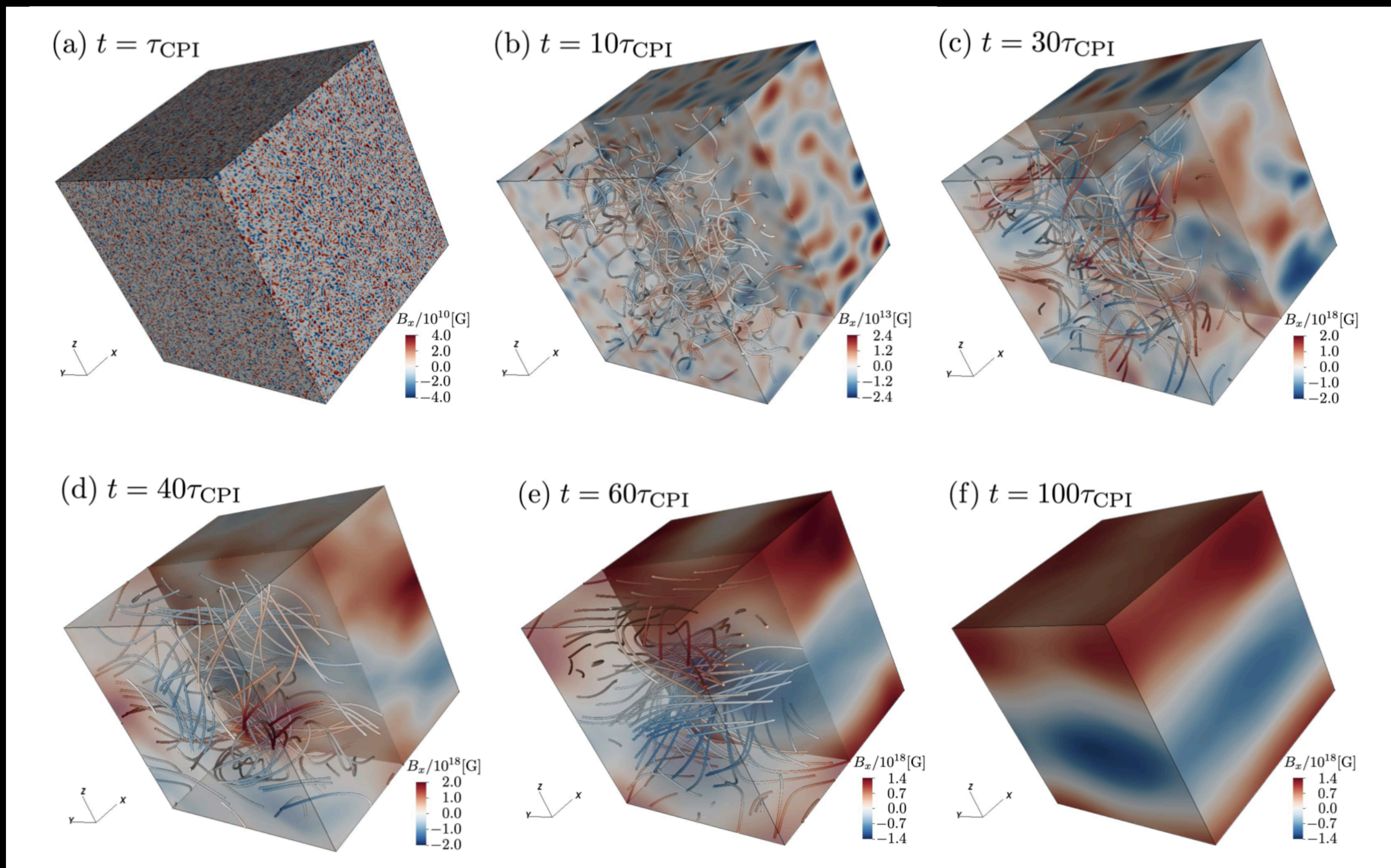
Hanke (2014)

What about 3D chiral matter?: energy & helicity

Time evolution of *B*

$\xi_{B,\mathrm{ini}} = 10^{-1}$

Matsumoto, Yamamoto, Yang, arXiv:2202.09205

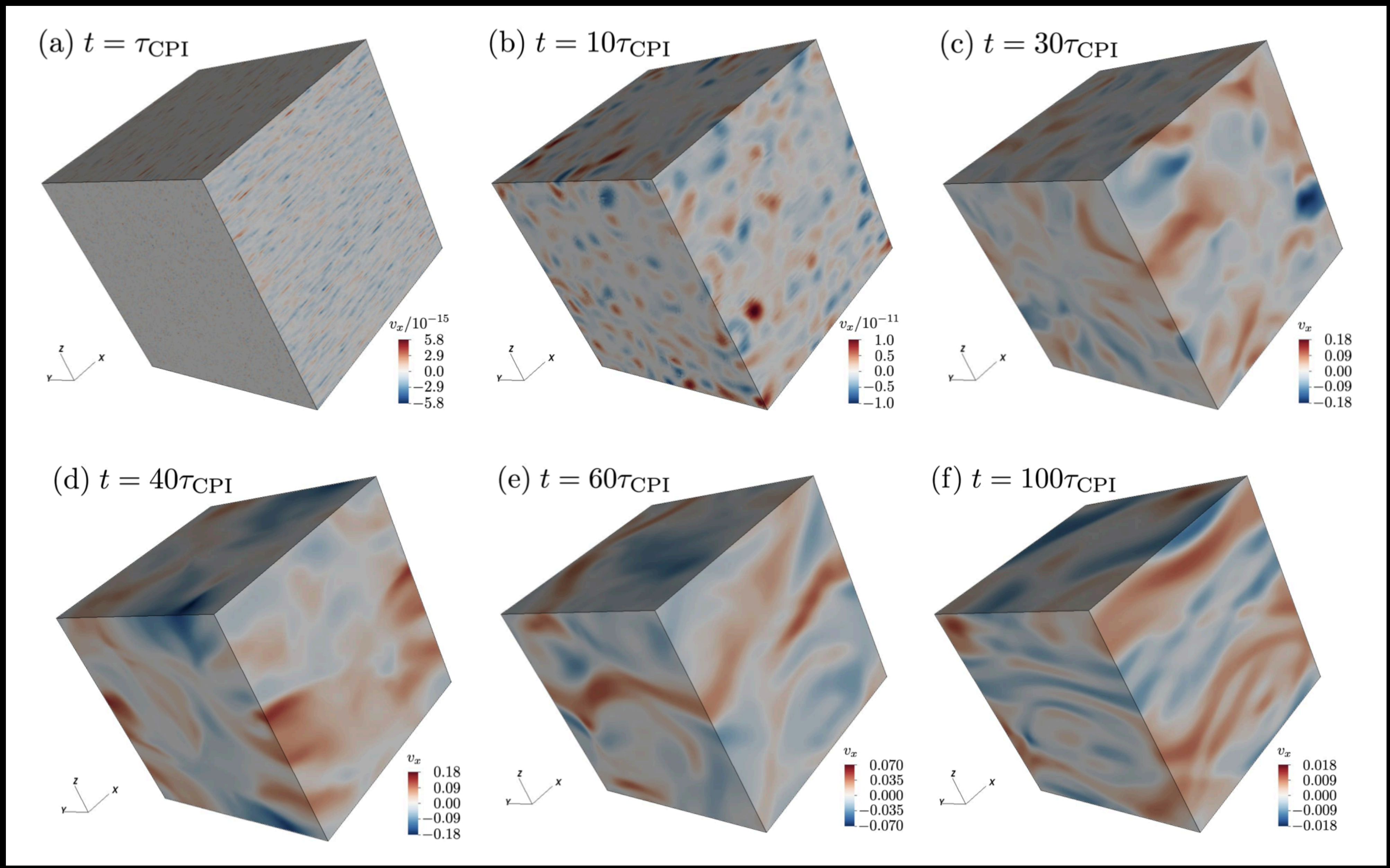


see also Brandenburg et al., arXiv:1707.03385; Masada et al., arXiv:1805.10419

Time evolution of **v**

$\xi_{B,\mathrm{ini}} = 10^{-1}$

Matsumoto, Yamamoto, Yang, arXiv:2202.09205



Chiral effects lead to inverse cascade, which may affect explosion dynamics

Summary & Outlook

- Parity violation in the weak theory is fundamental, yet ignored in the conventional supernova computations.
- Nonequilibrium chiral effects modify hydrodynamic behaviors: chiral plasma instability, inverse cascade, …
- Possible contributions to magnetars and pulsar kicks
- Relevance of other effects? (chiral vortical, spin Hall effects, …)
- Future global simulations would be important.