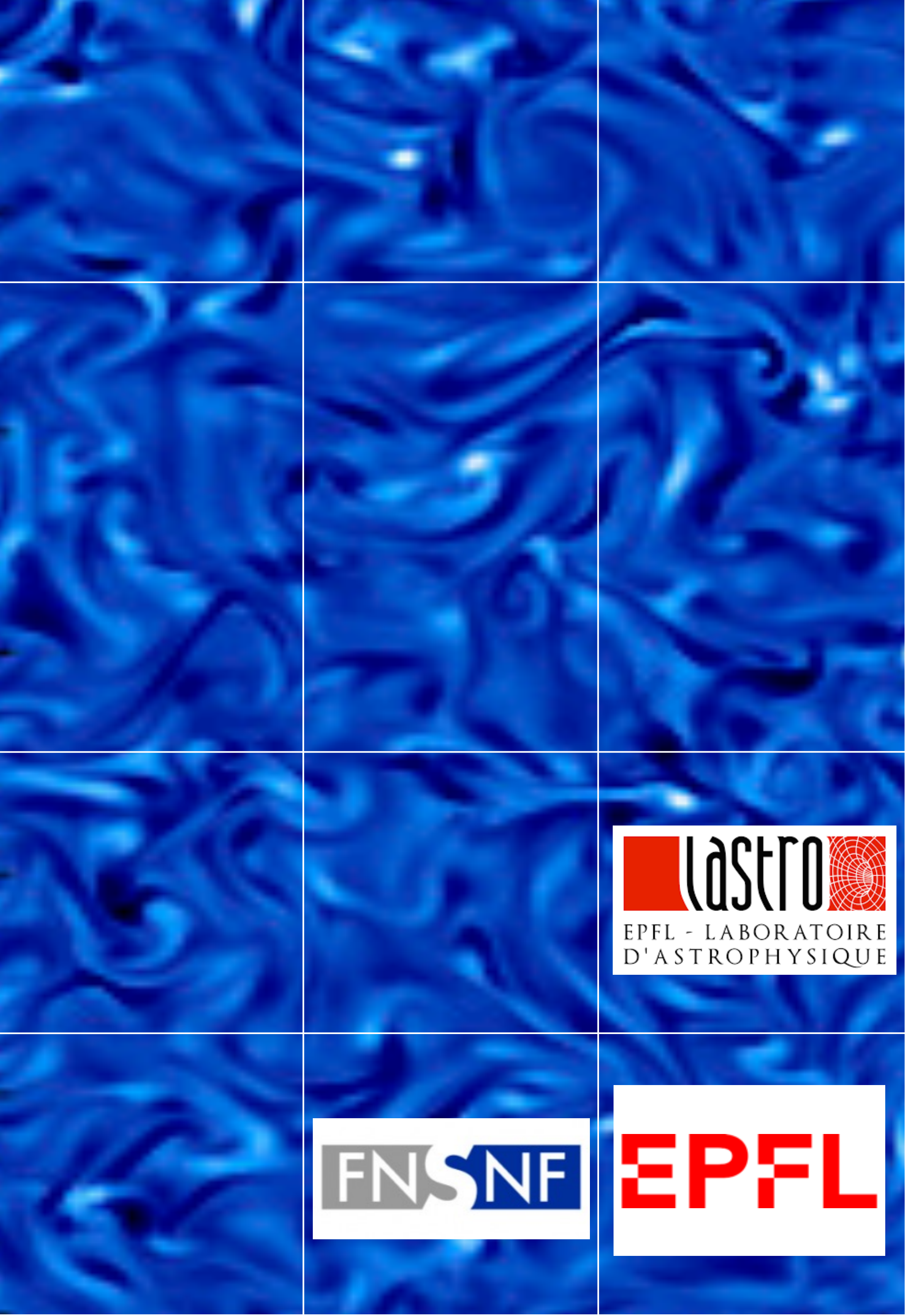

Focus workshop on collective oscillations and chiral transport of neutrinos

Taipei ❖ 15/03/2023

"Simulations of chiral magnetohydrodynamics"

Jennifer Schober

& collaborators:

Alexey Boyarsky, Axel Brandenburg,
Ruth Durrer, Tomohiro Fujita,
Jürg Fröhlich, Tina Kahniashvili,
Kohei Kamada, Kyohei Mukaida, Nathan Kleeorin,
Igor Rogachevskii, Oleg Ruchayskiy, Kai Schmitz

lastro
EPFL - LABORATOIRE D'ASTROPHYSIQUE

FNSNF

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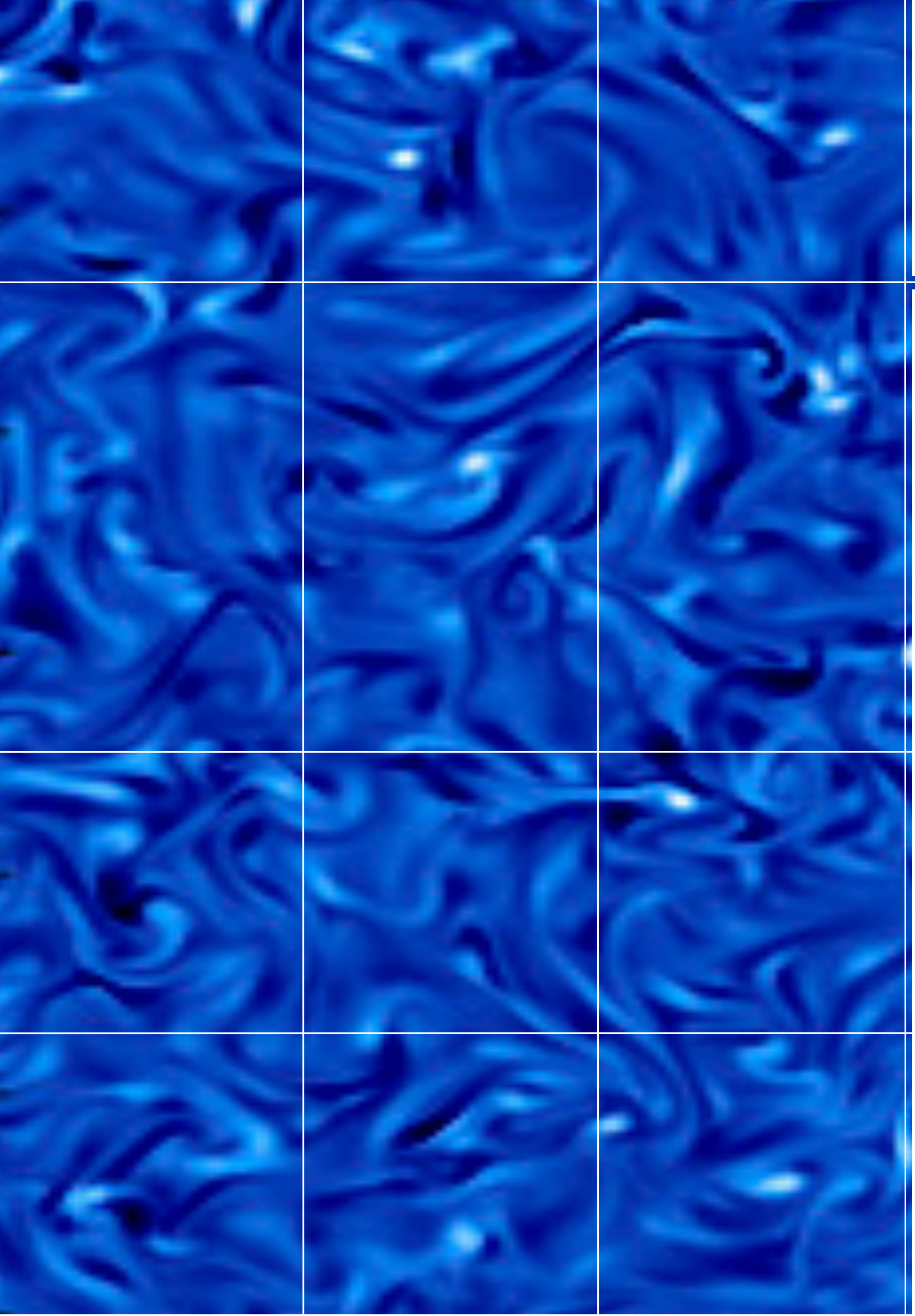
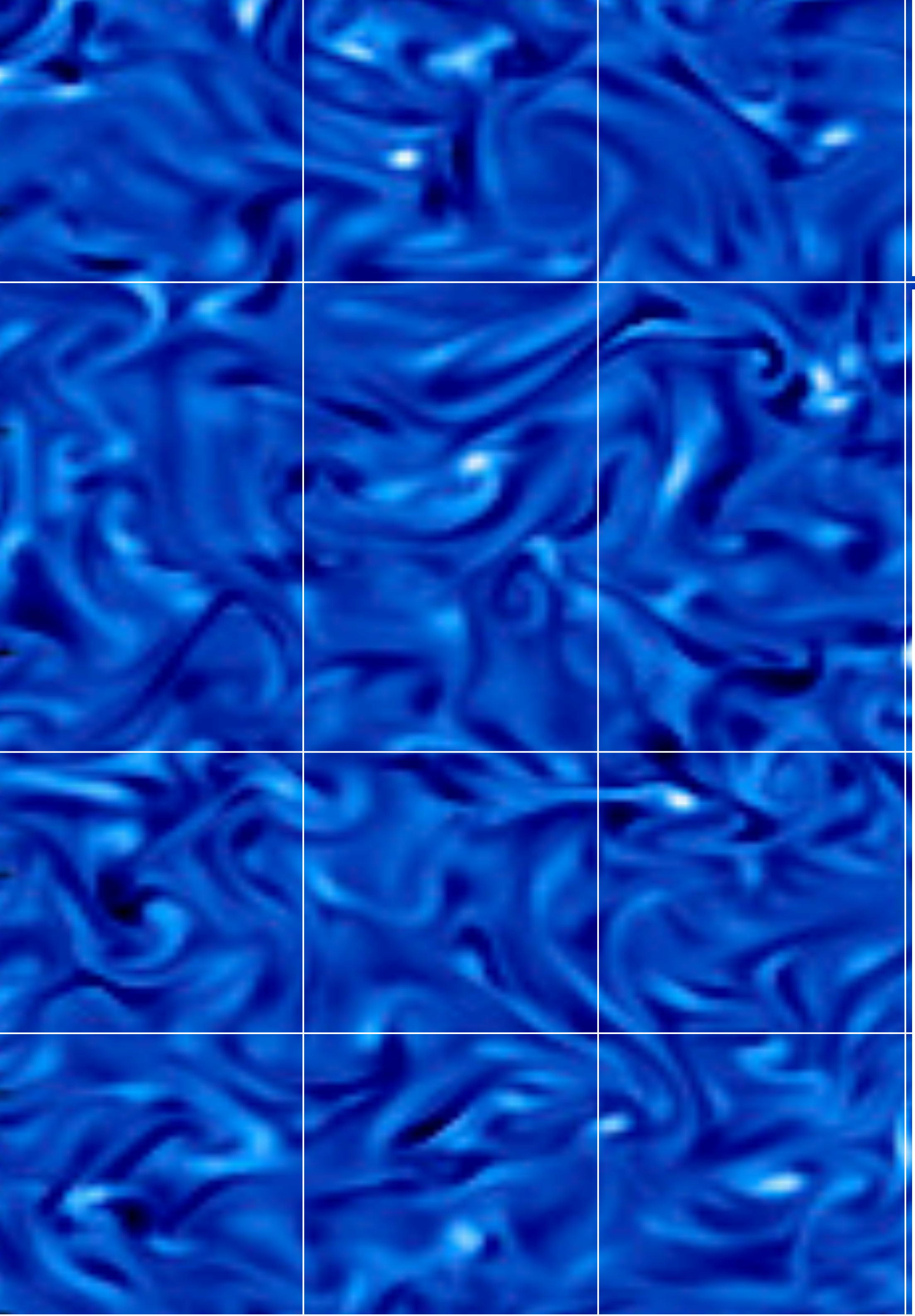


"Simulations of chiral magnetohydrodynamics"

-Outline-

1) Motivation: Observing the Big Bang through cosmic magnetic fields
2) Chiral magnetohydrodynamics and numerical simulations of its nonlinear dynamics
3) Insights from simulations
4) Conclusions



"Simulations of chiral magnetohydrodynamics"

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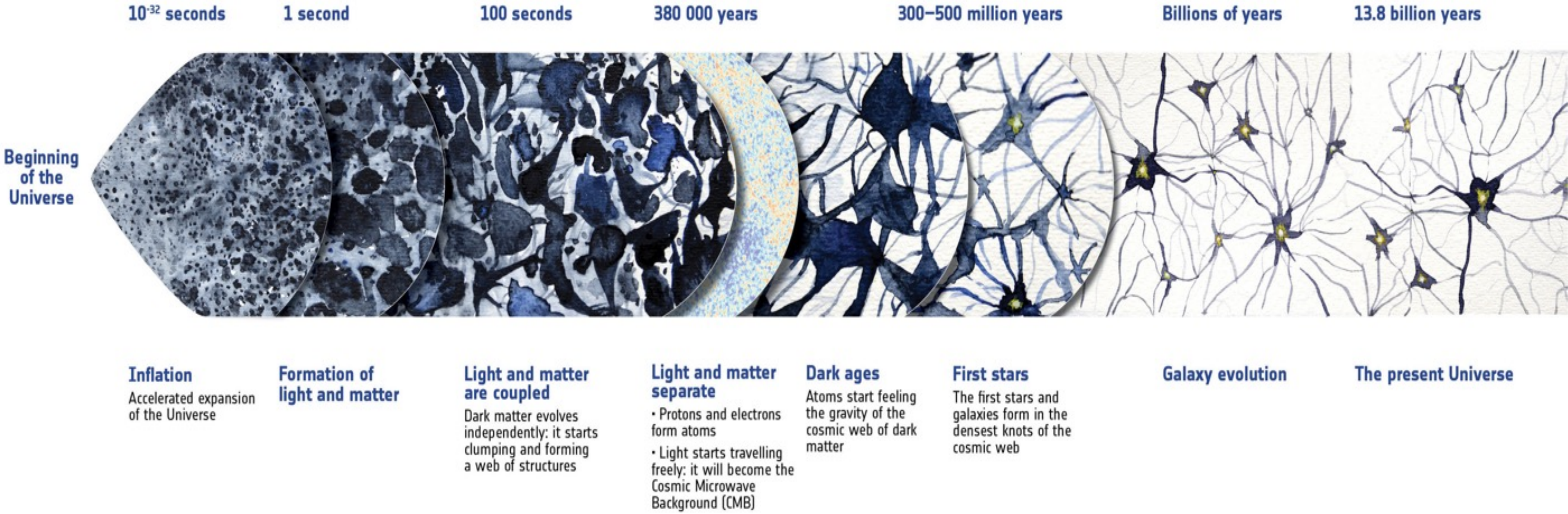
Cosmic history in a nutshell



Beginning of the Universe

10^{-32} seconds

Inflation

Accelerated expansion of the Universe

1 second

Formation of light and matter

100 seconds

Light and matter are coupled

Dark matter evolves independently: it starts clumping and forming a web of structures

380 000 years

Light and matter separate

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- Light starts travelling freely: it will become the Cosmic Microwave Background (CMB)

Dark ages

Atoms start feeling the gravity of the cosmic web of dark matter

300–500 million years

First stars

The first stars and galaxies form in the densest knots of the cosmic web

Billions of years

Galaxy evolution

13.8 billion years

The present Universe

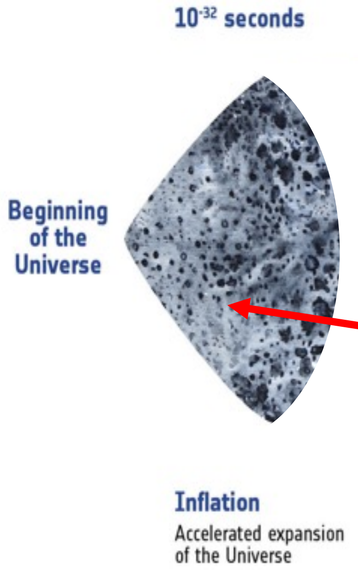
[credit: ESA]

Jennifer Schober

Cosmic history in a nutshell

Beginning
of the
Universe

Cosmic history in a nutshell



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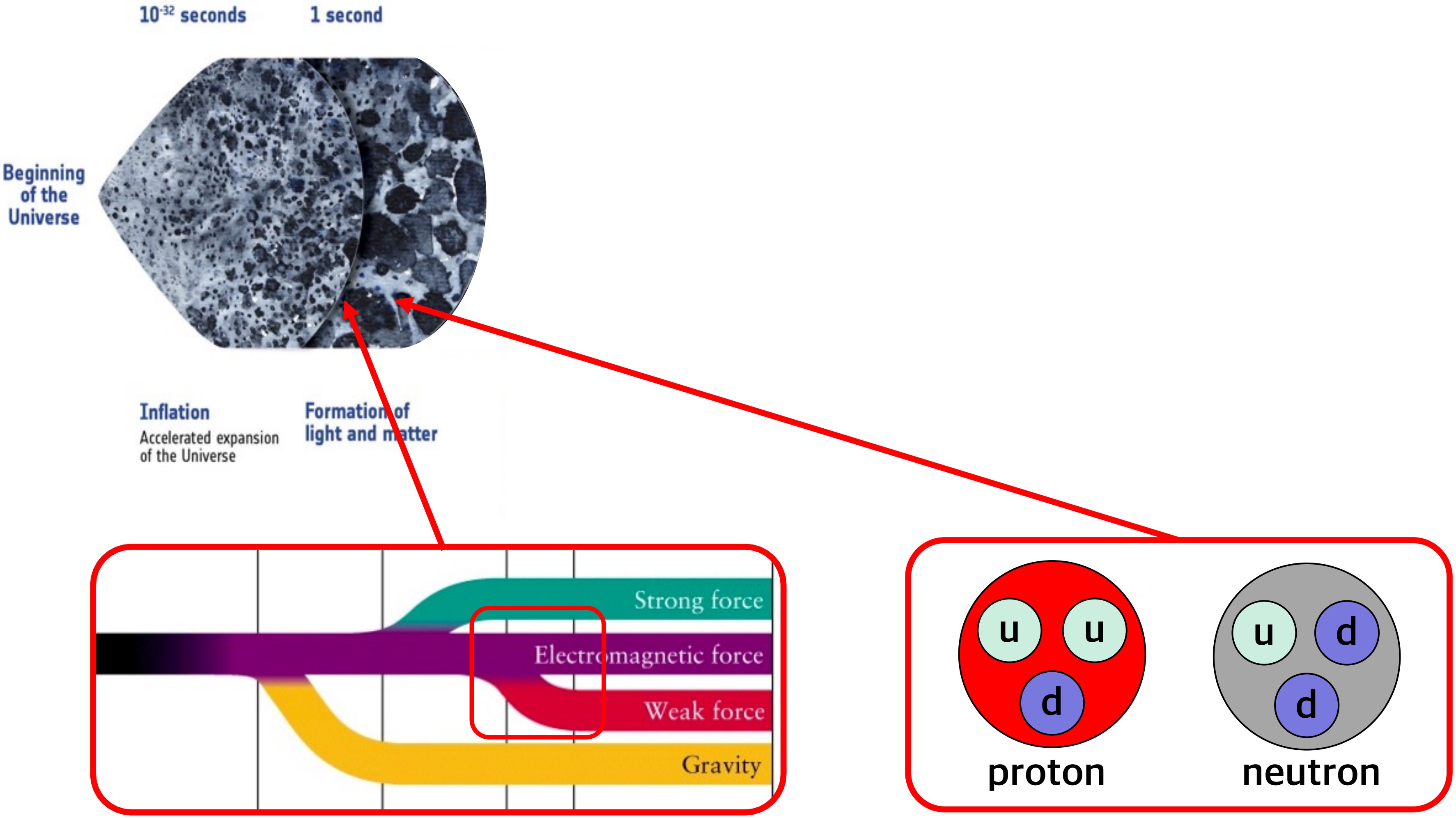
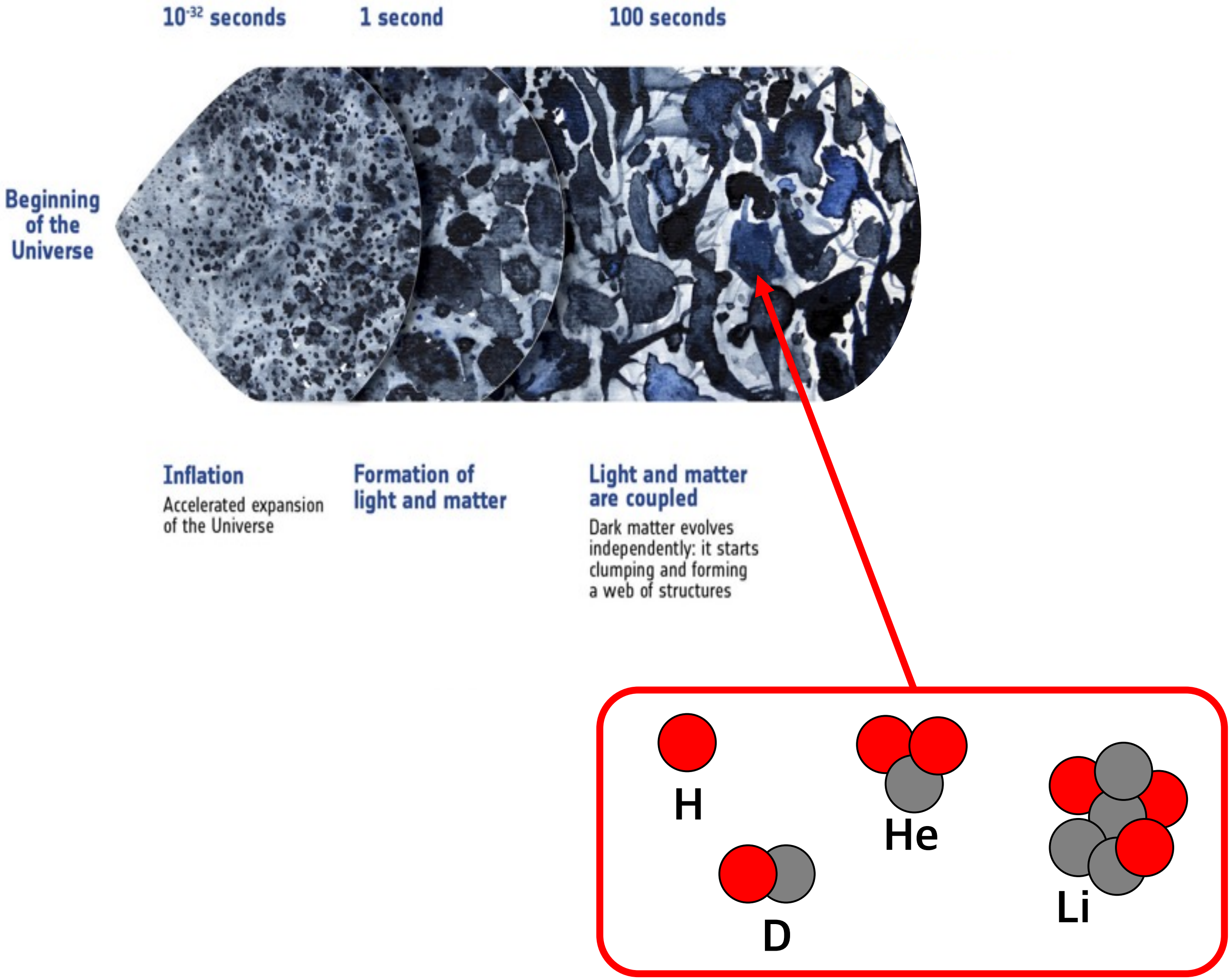


Electroweak phase transition

QCD phase transition

Cosmic history in a nutshell



Big Bang nucleosynthesis

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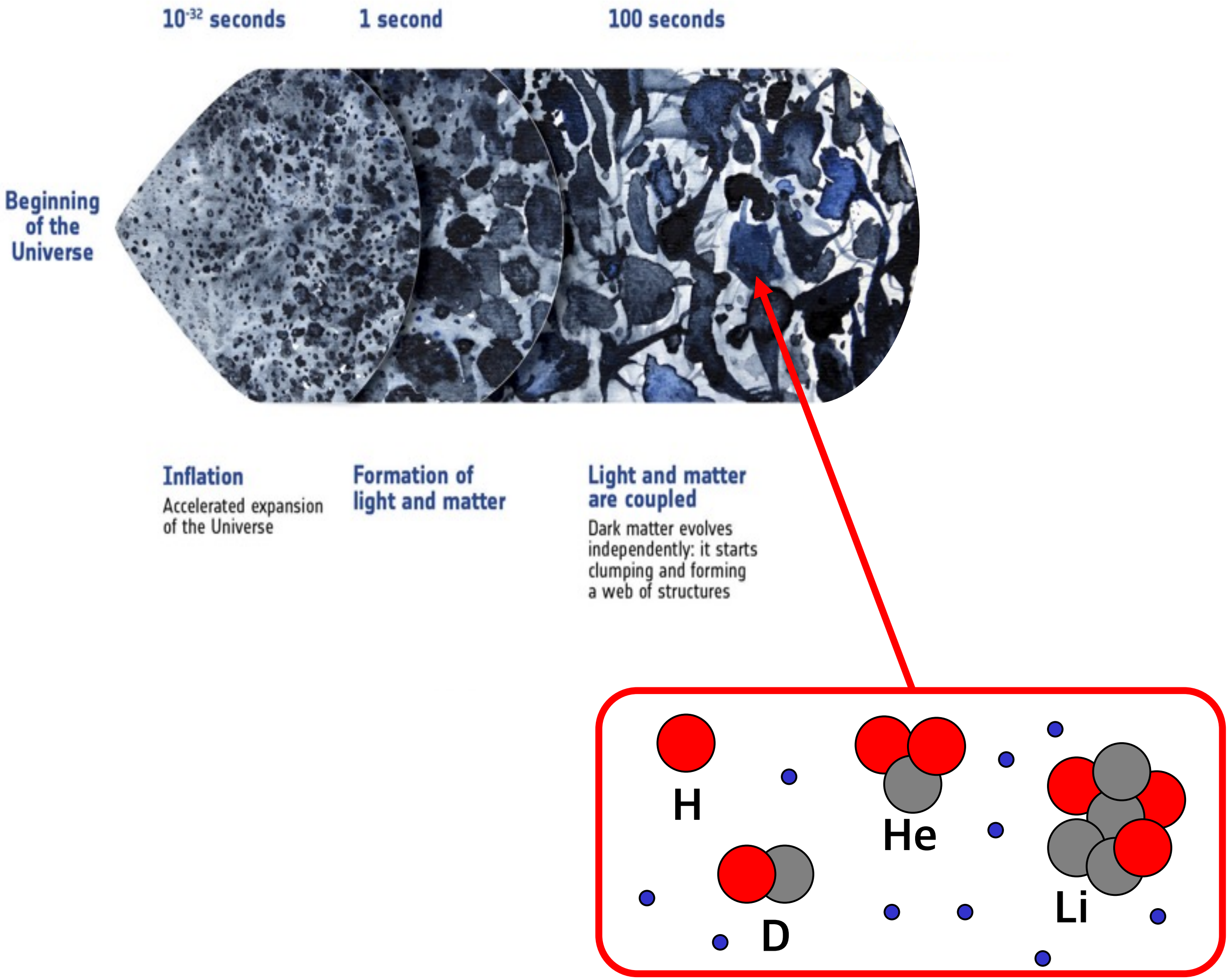

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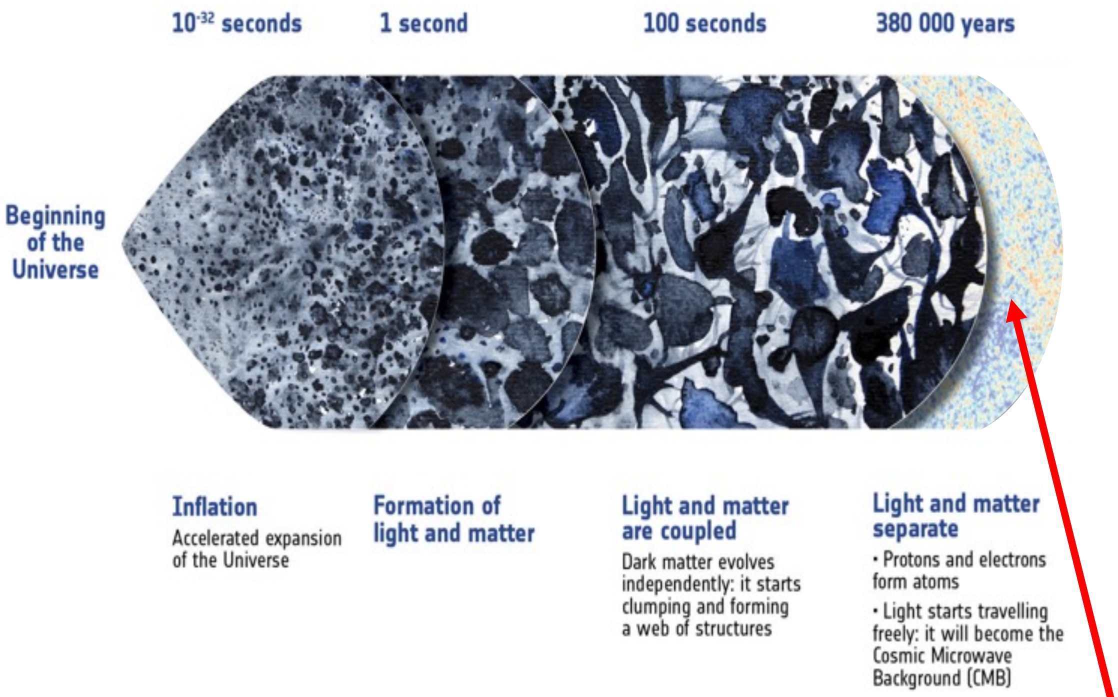

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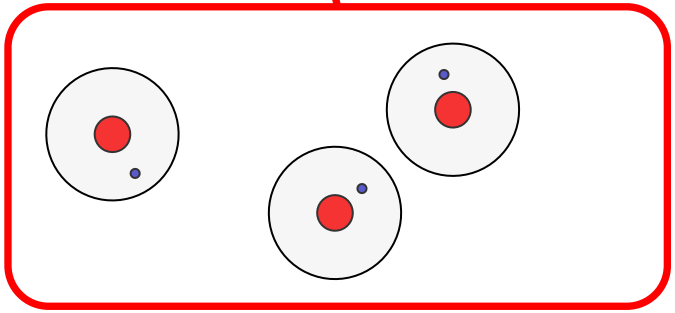
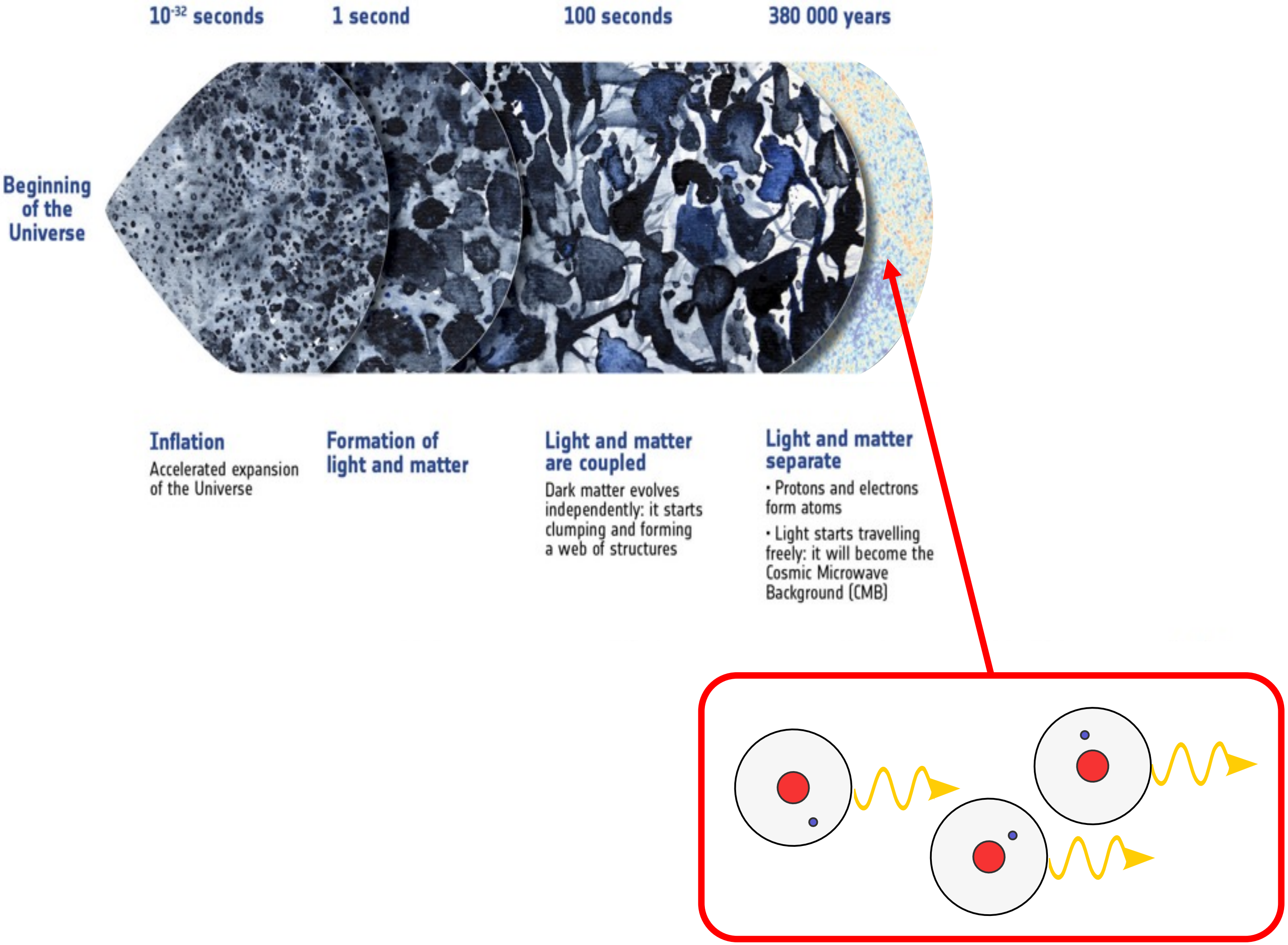


Recombination

Jennifer Schober

Cosmic history in a nutshell



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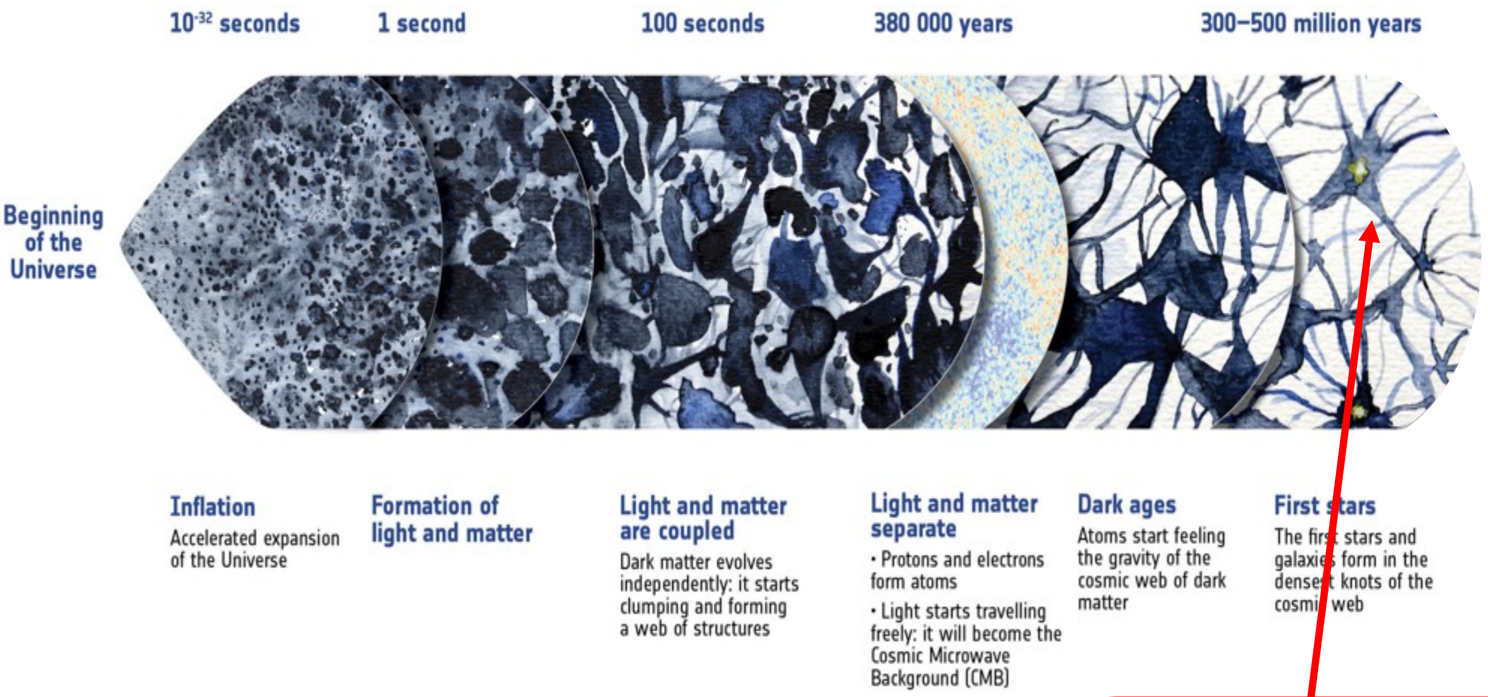

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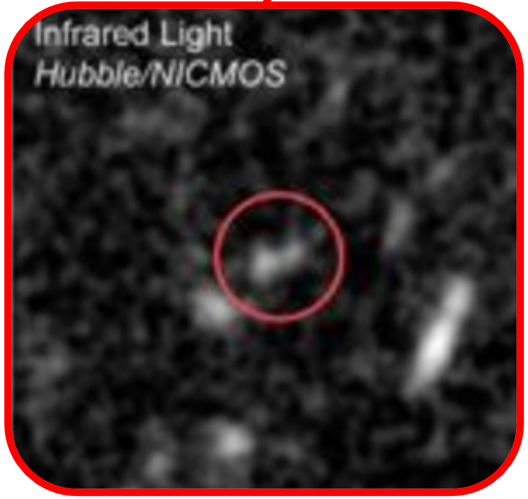

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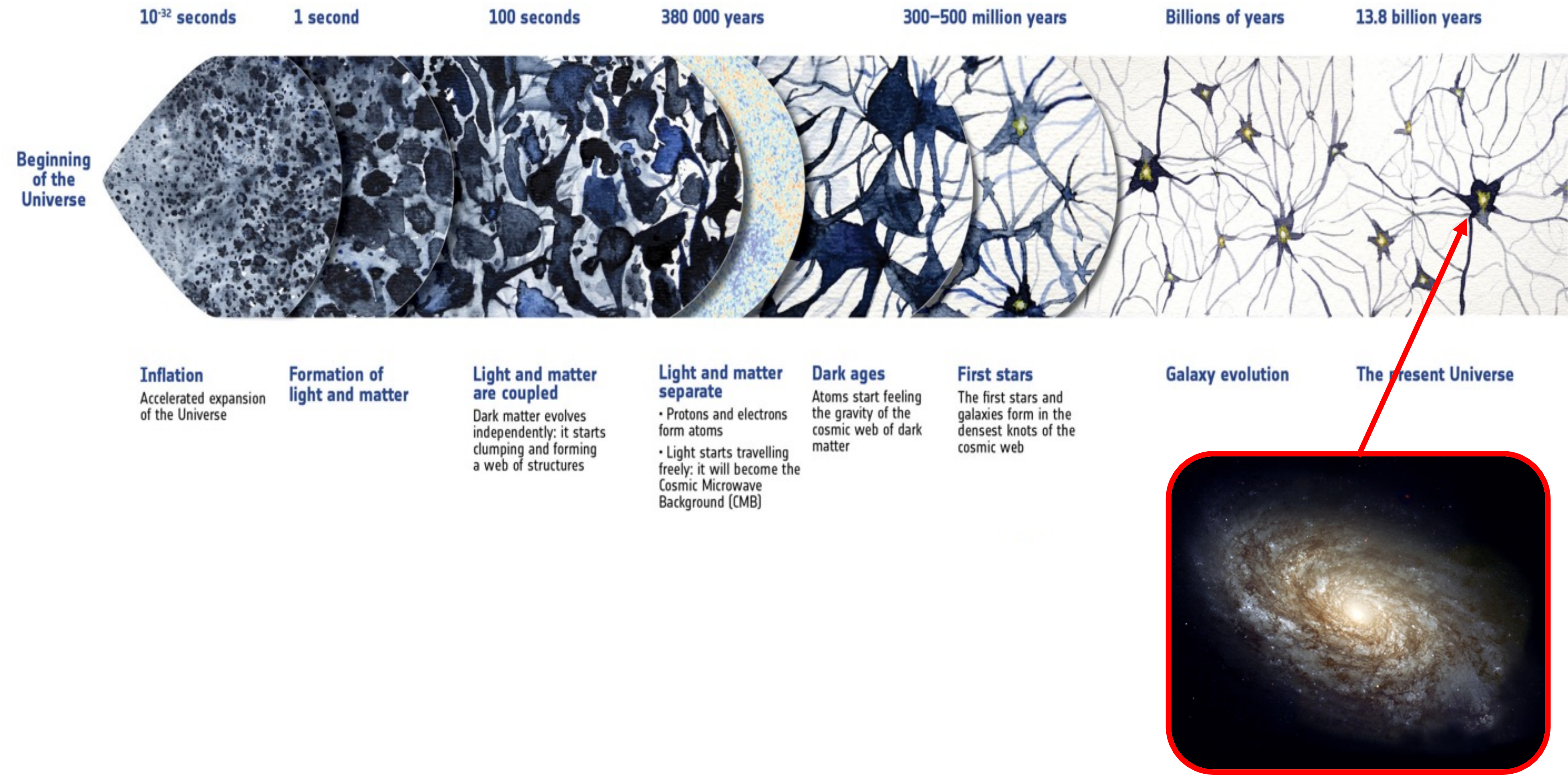

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Jennifer Schober

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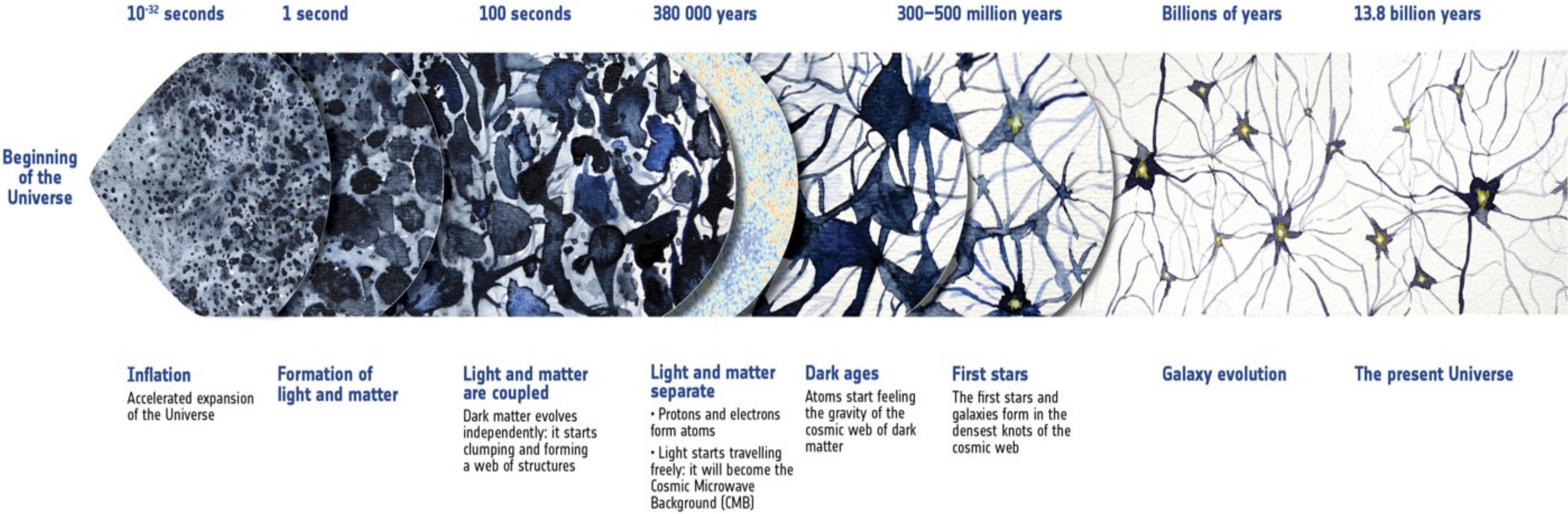

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The present Universe

What caused inflation?
Why matter/anti-matter asymmetry?
What is Dark Matter?
What is Dark Energy?
...

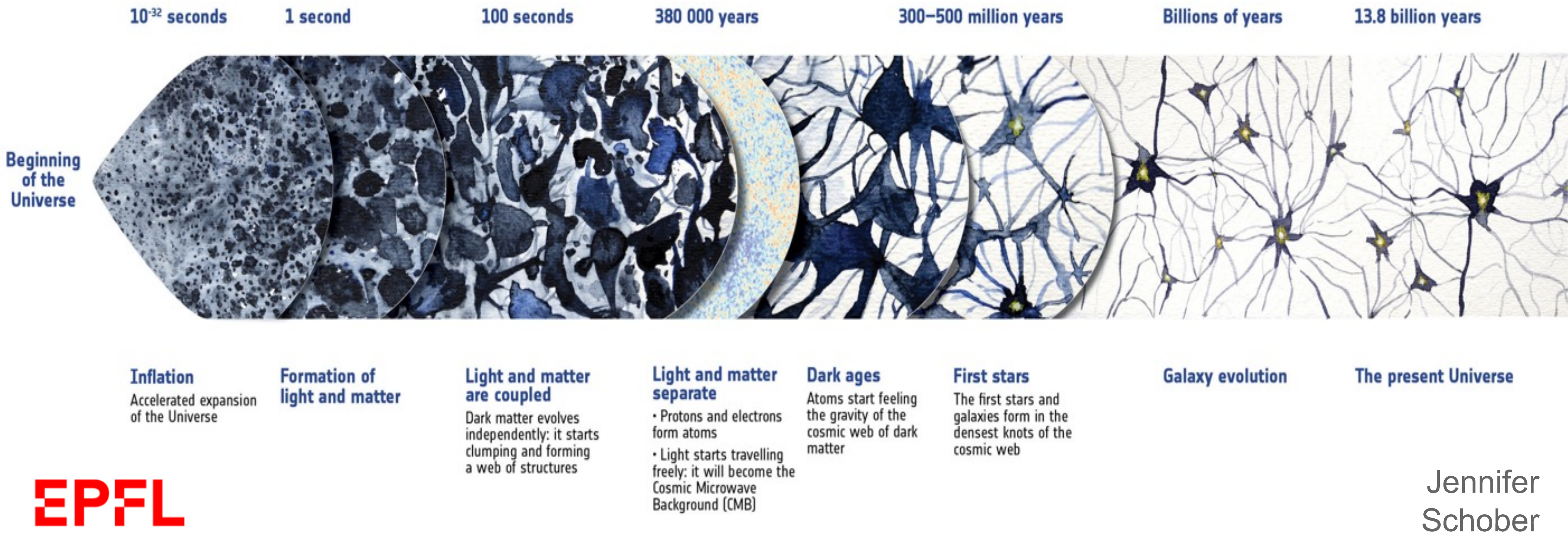
Observational accessibility

galaxies

[record: GNS-z11, $z = 11.9$, age $0.4\ \mathrm{Gyr}$, *Oesch et al. 2016*]

stars

[record: MACS J1149, $z = 1.49$, age $4.3\ \mathrm{Gyr}$, *Kelly et al. 2018*]



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The present Universe

EPFL

Jennifer Schober

Observational accessibility

cosmic microwave background (CMB)

[$z = 1089$, age $380000\ \mathrm{yr}$]

galaxies

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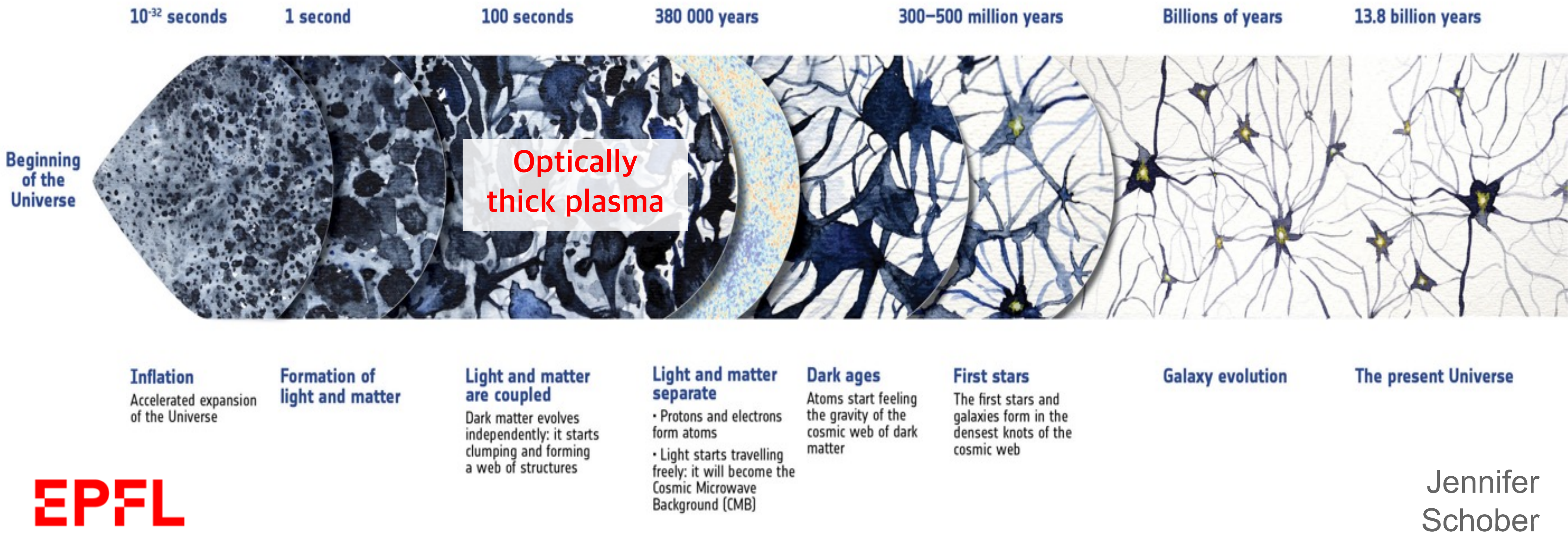

Optically thick plasma

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Jennifer Schober

Observational accessibility

cosmic neutrino background

[age $1\ \mathrm{s}$]

cosmic microwave background (CMB)

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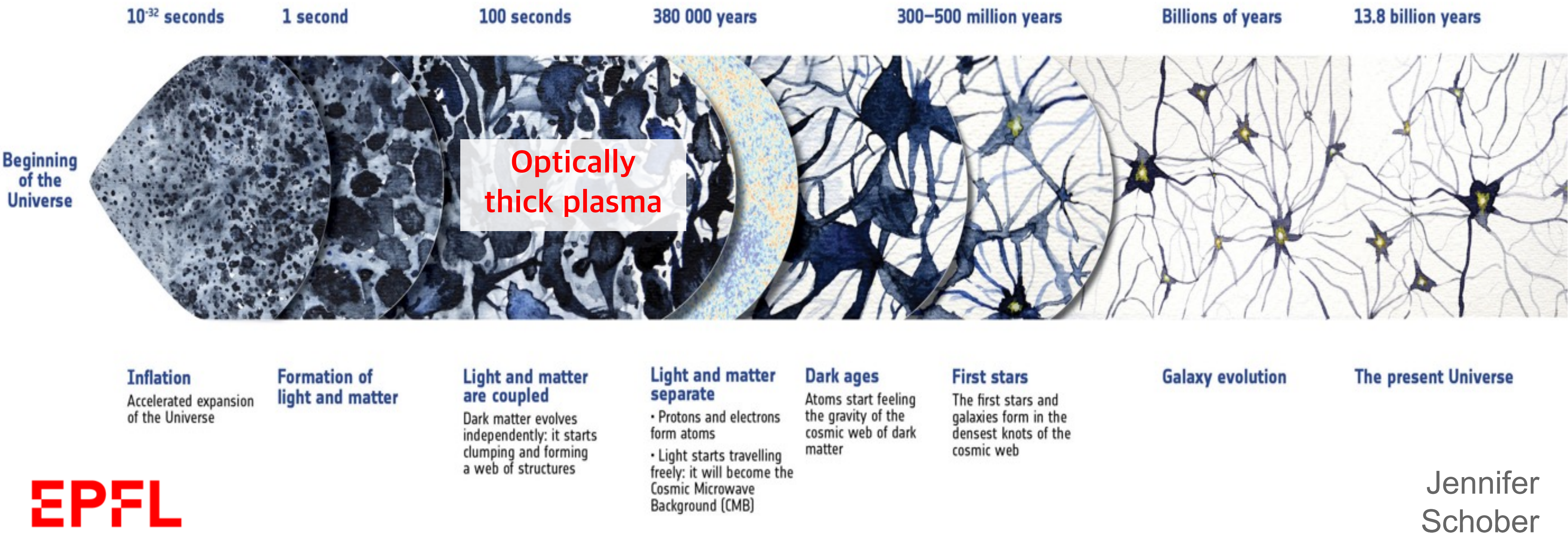

Optically thick plasma

Observational accessibility

primordial magnetic fields + gravitational waves

[age < 1 s]

cosmic neutrino background

[age 1 s]

cosmic microwave background (CMB)

[$z = 1089$, age 380000 yr]

galaxies

[record: GNS-z11, $z = 11.9$, age 0.4 Gyr , *Oesch et al. 2016*]

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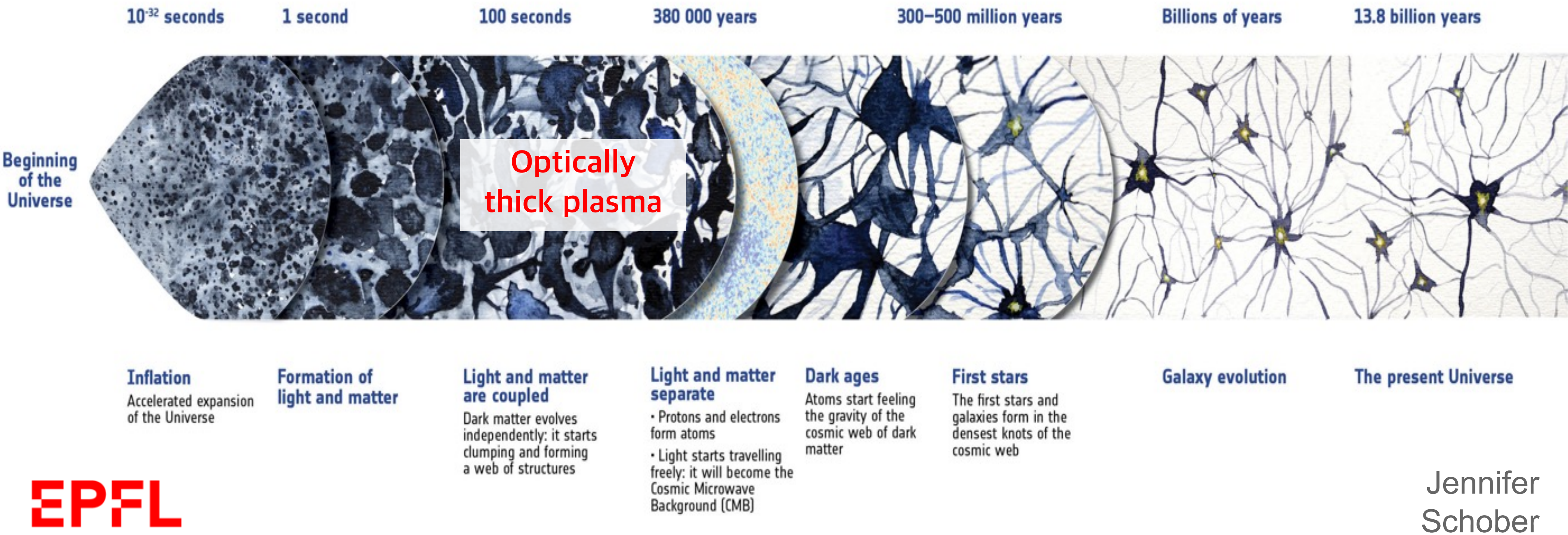

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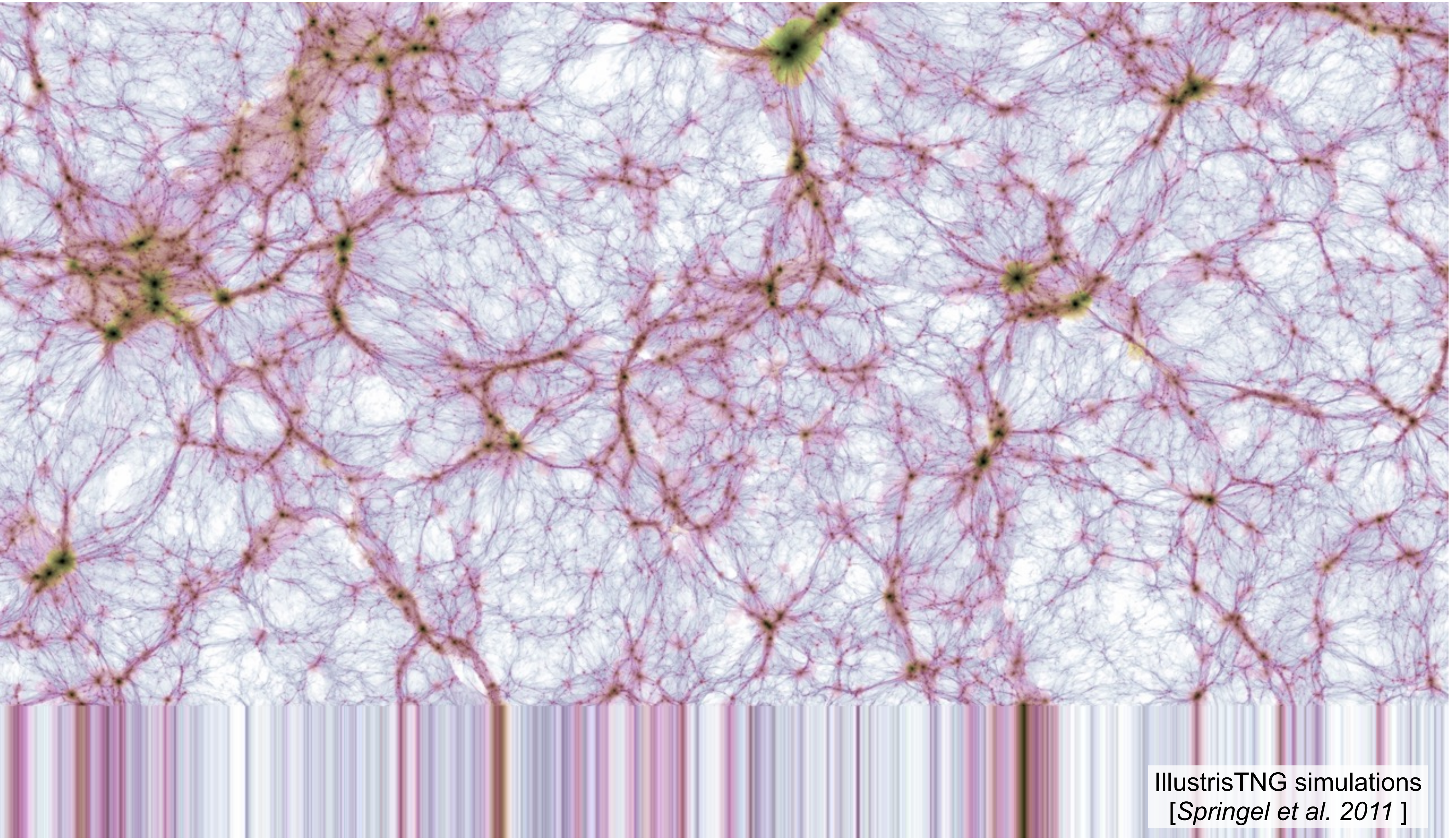
Jennifer Schober

Hints of primordial magnetic fields?



IllustrisTNG simulations
[*Springel et al. 2011*]

Jennifer
Schober

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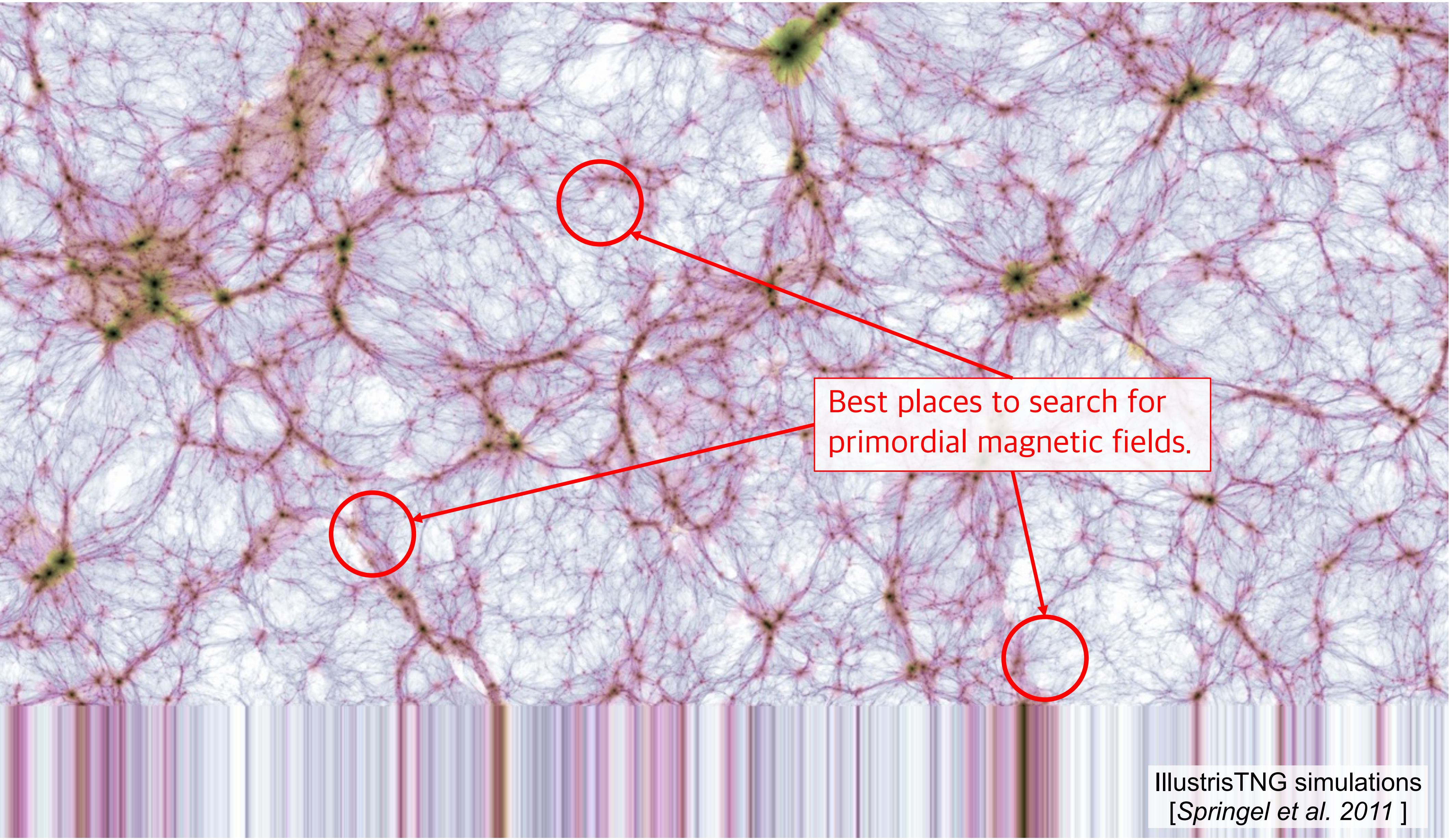
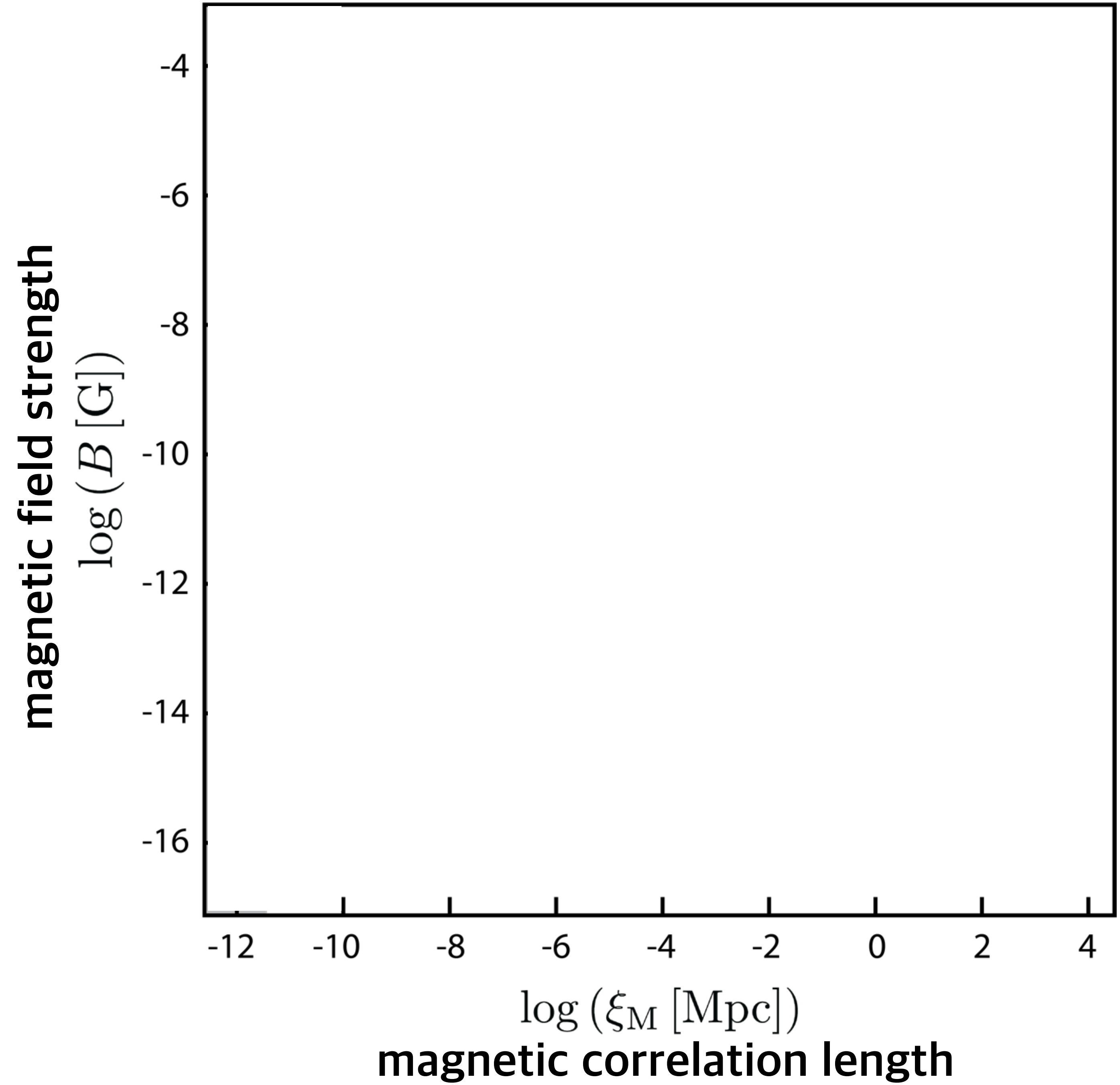


Best places to search for primordial magnetic fields.

IllustrisTNG simulations [*Springel et al. 2011*]

Jennifer Schober

Constraints on void magnetic field



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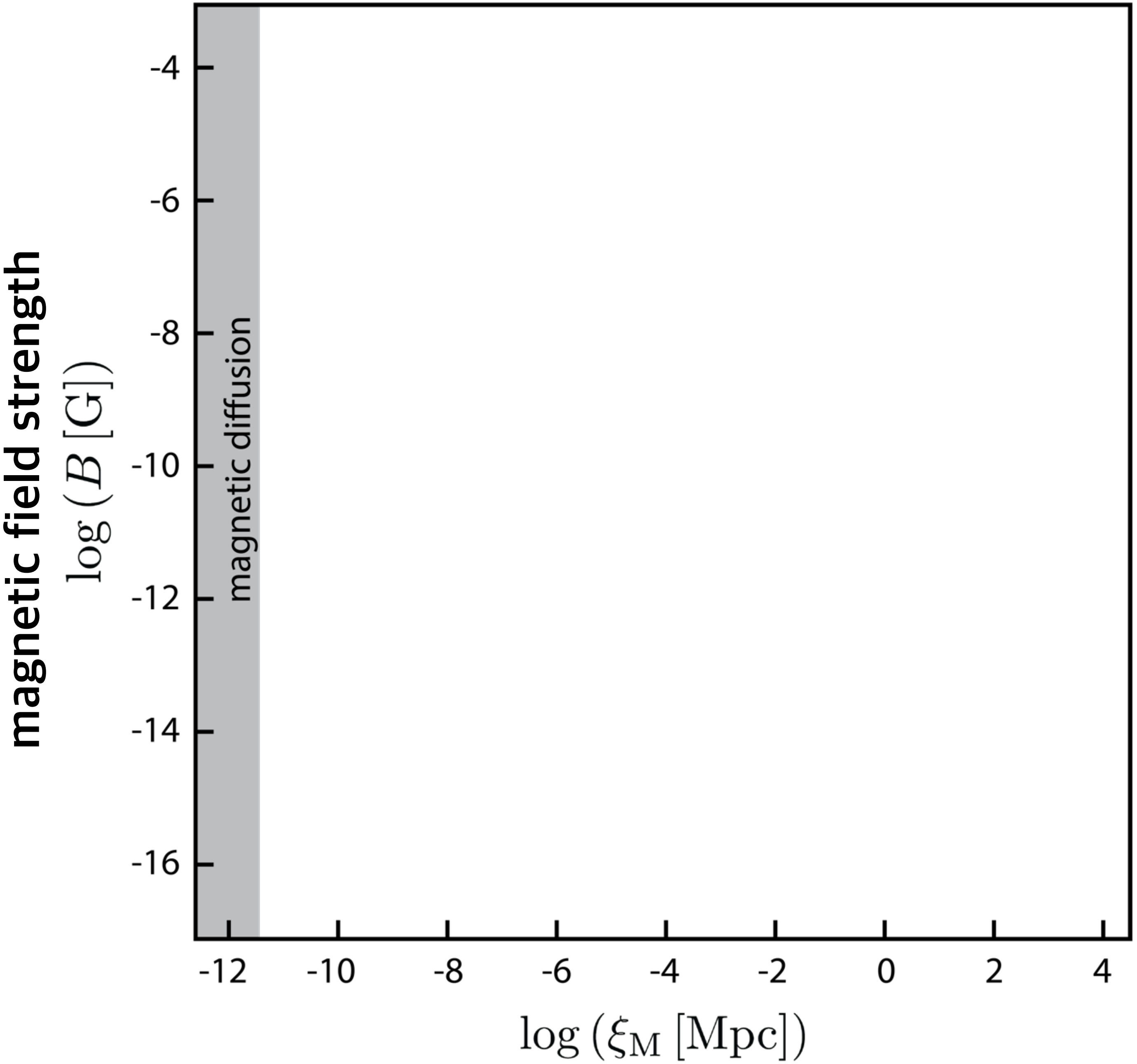
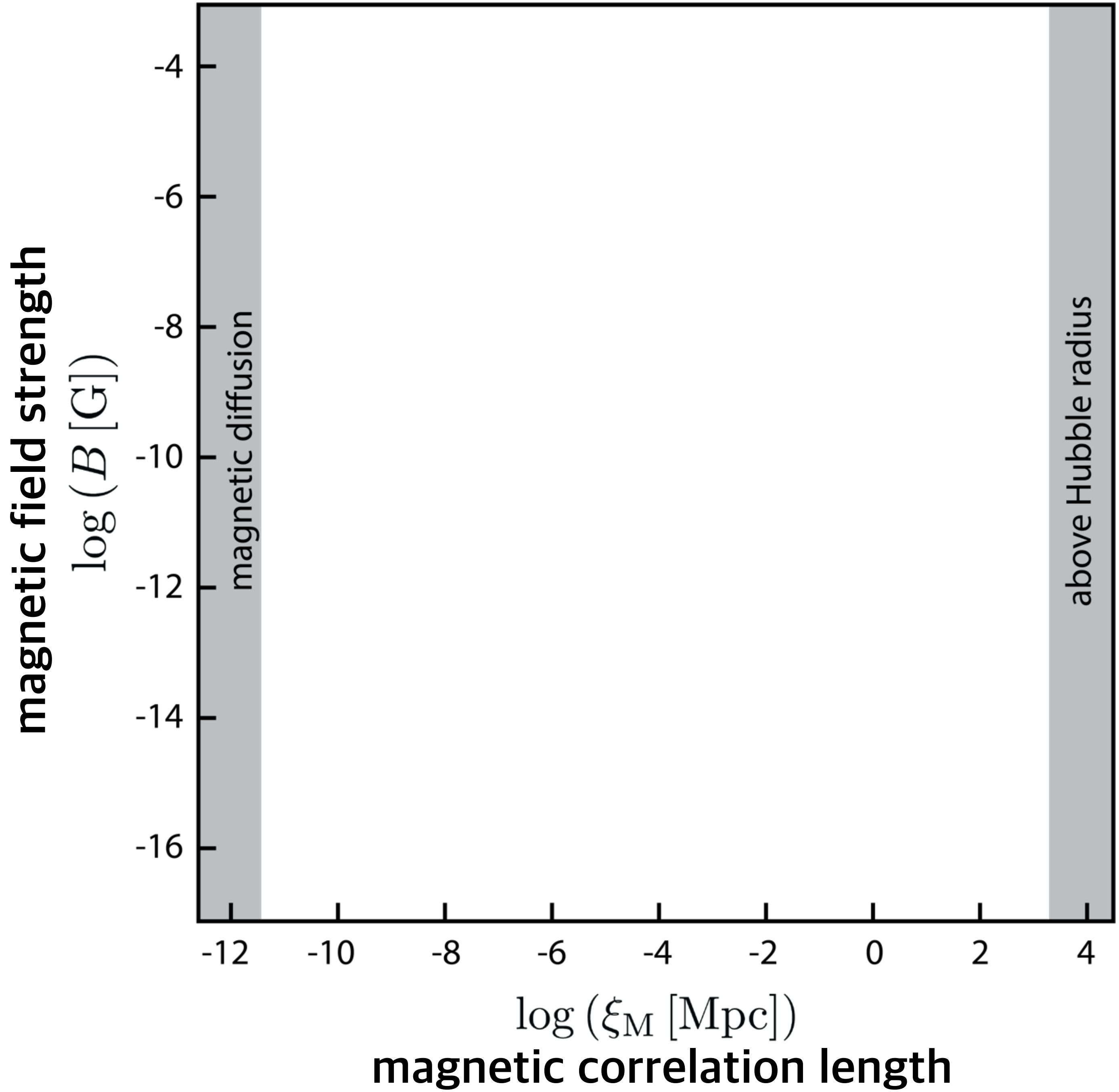


magnetic correlation length

Constraints on void magnetic field



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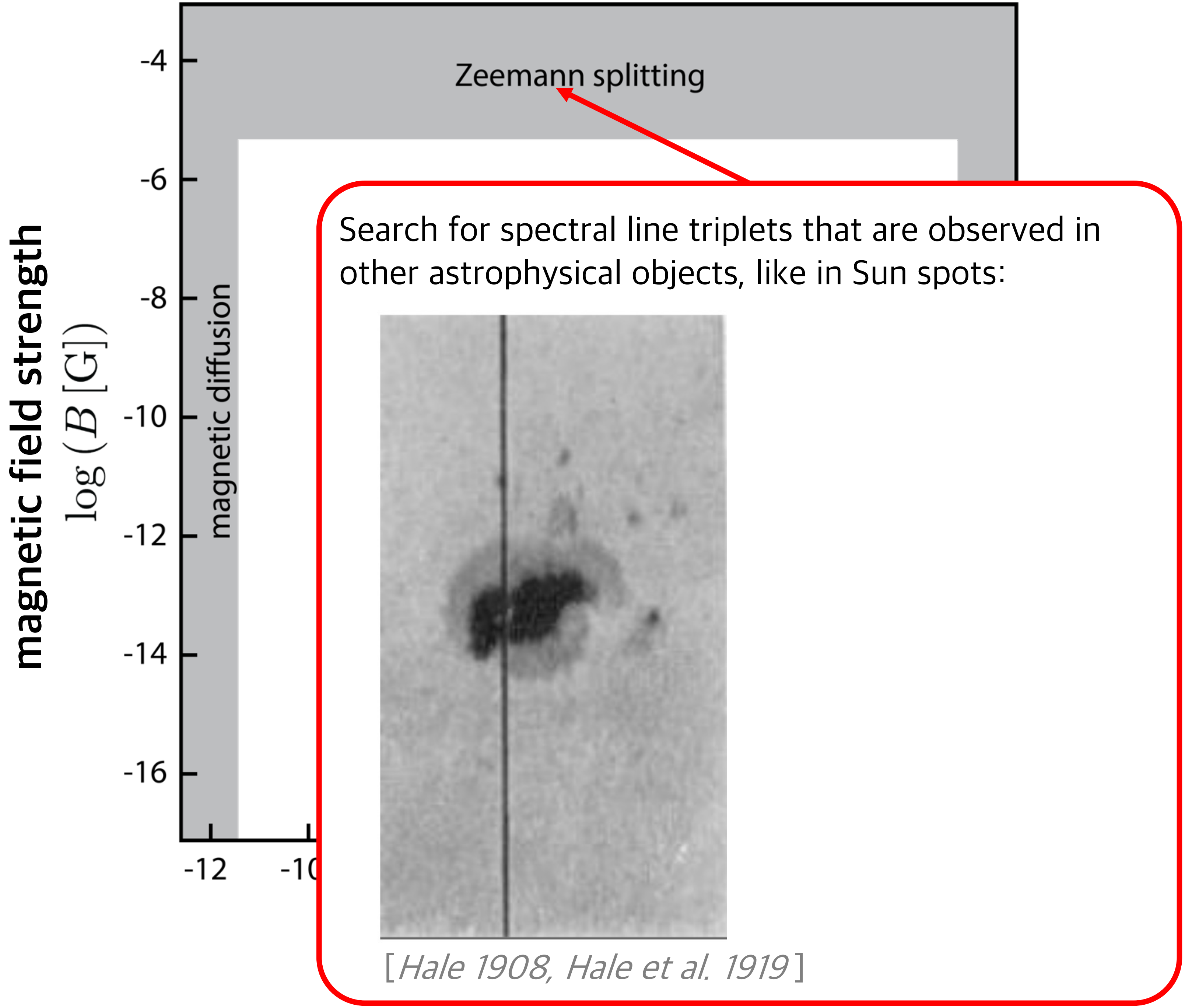
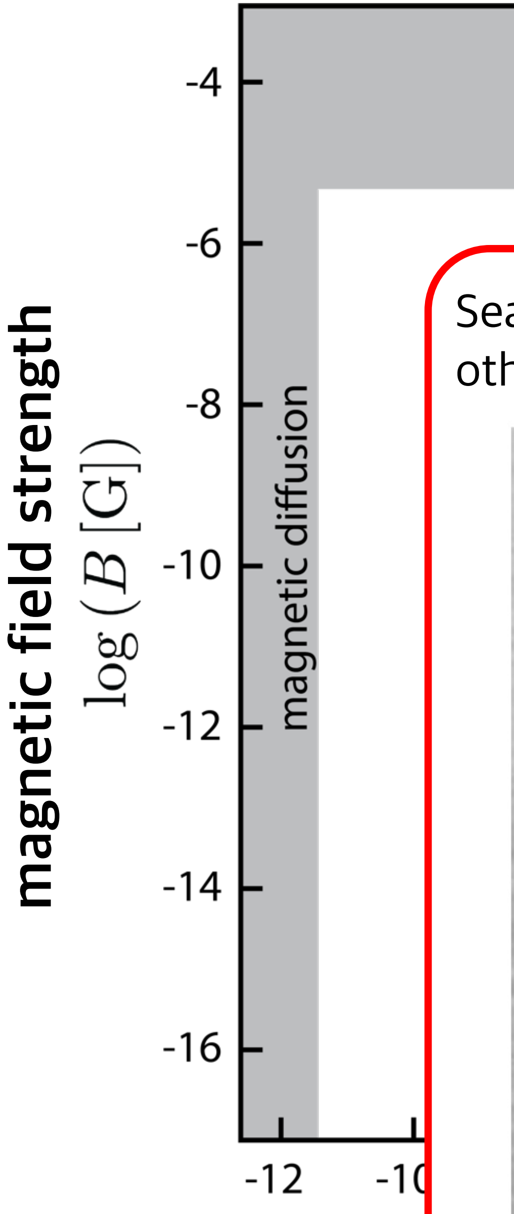


Search for spectral line triplets that are observed in other astrophysical objects, like in Sun spots:

[*Hale 1908, Hale et al. 1919*]

Jennifer Schober

Constraints on void magnetic field



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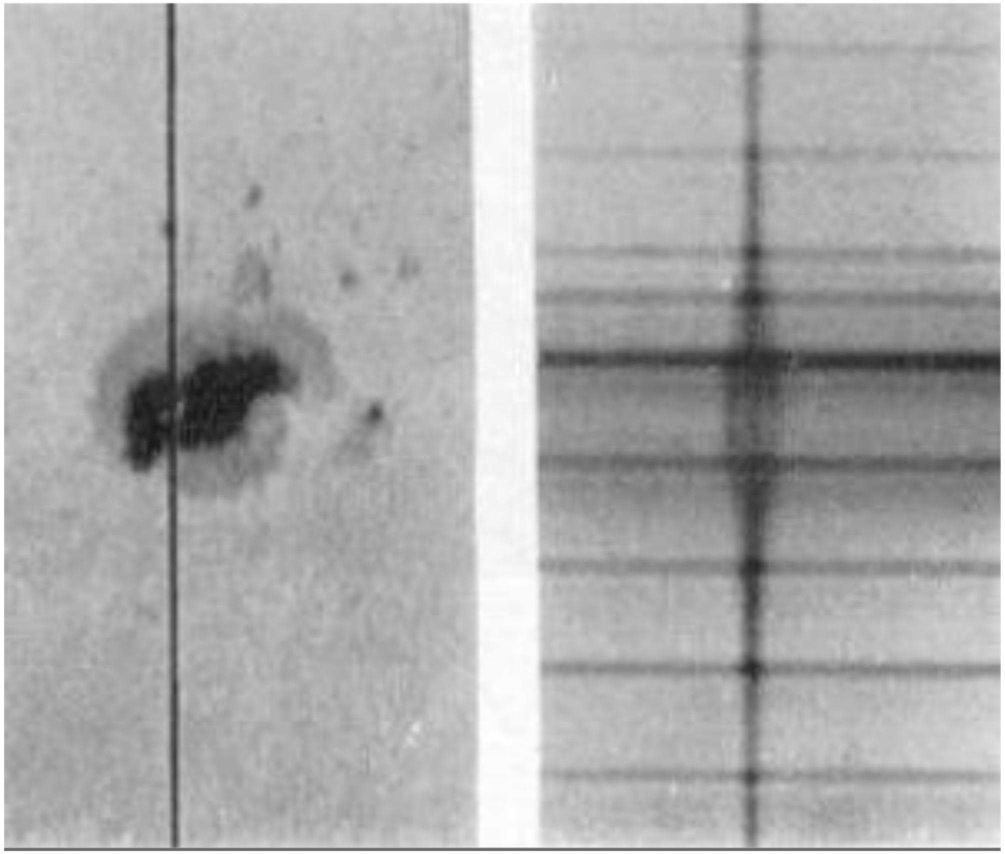

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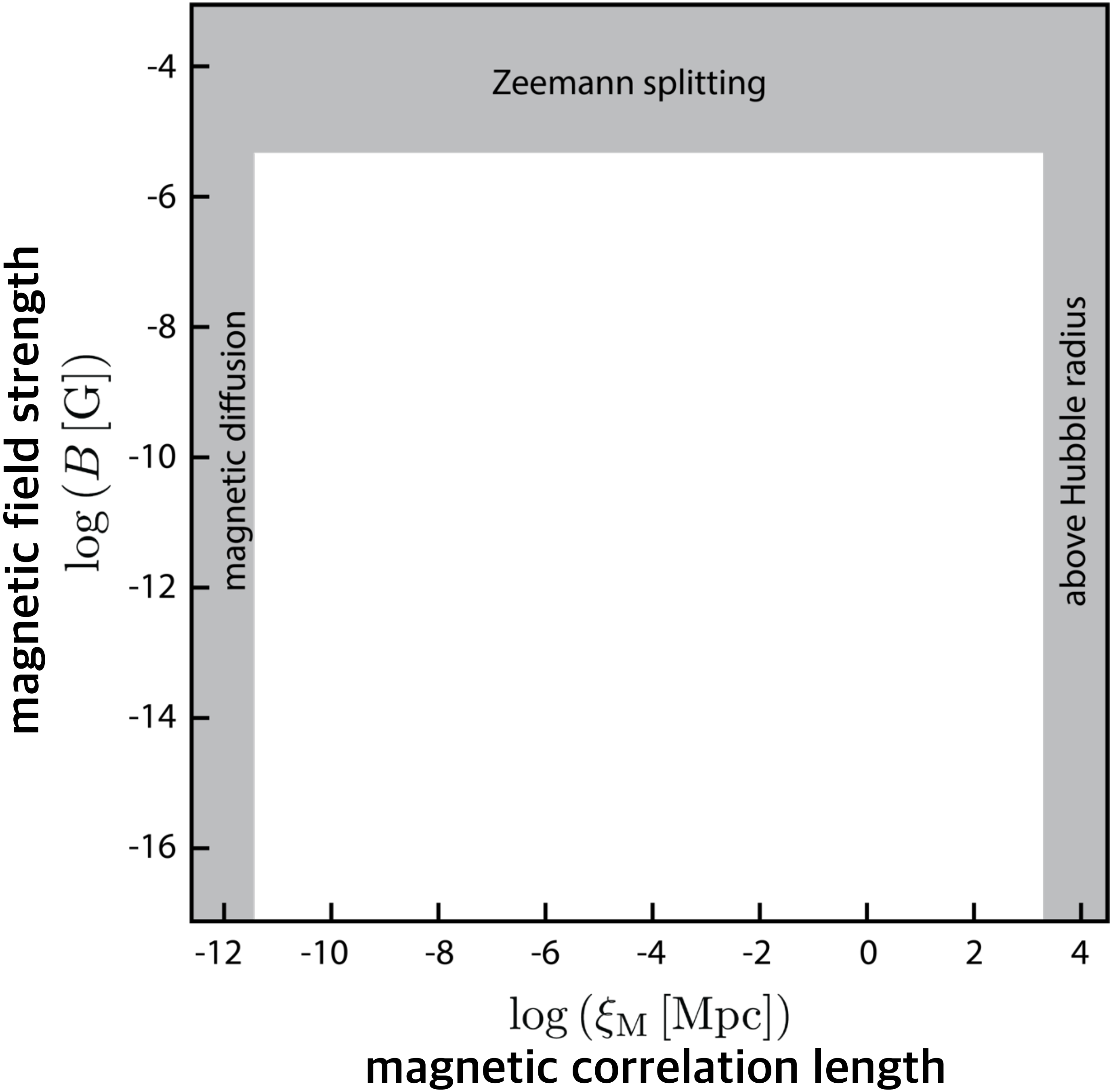
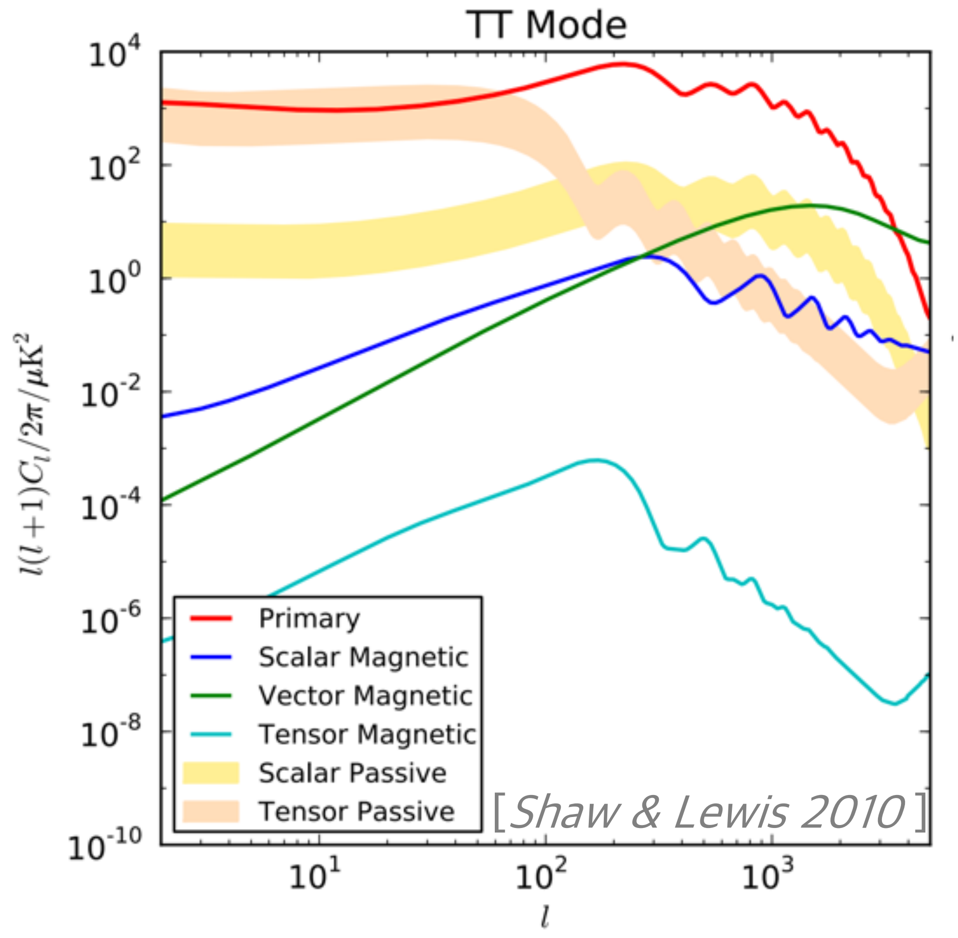
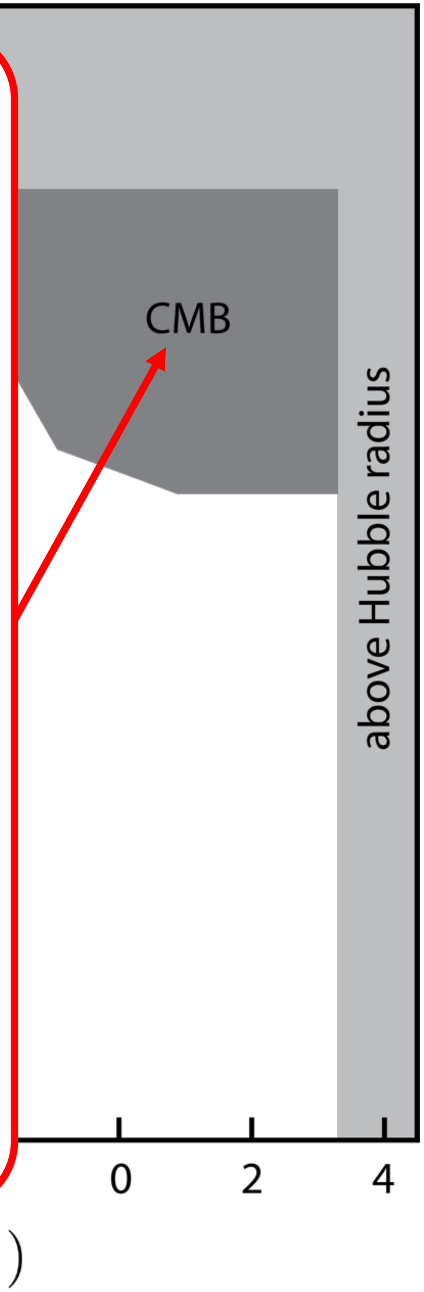


Constraints on void magnetic field

Direct effects:	changes in the baryon velocity field caused by the Lorentz force
Indirect effects:	heating via magnetic diffusion



[Shaw & Lewis 2010]



magnetic correlation length

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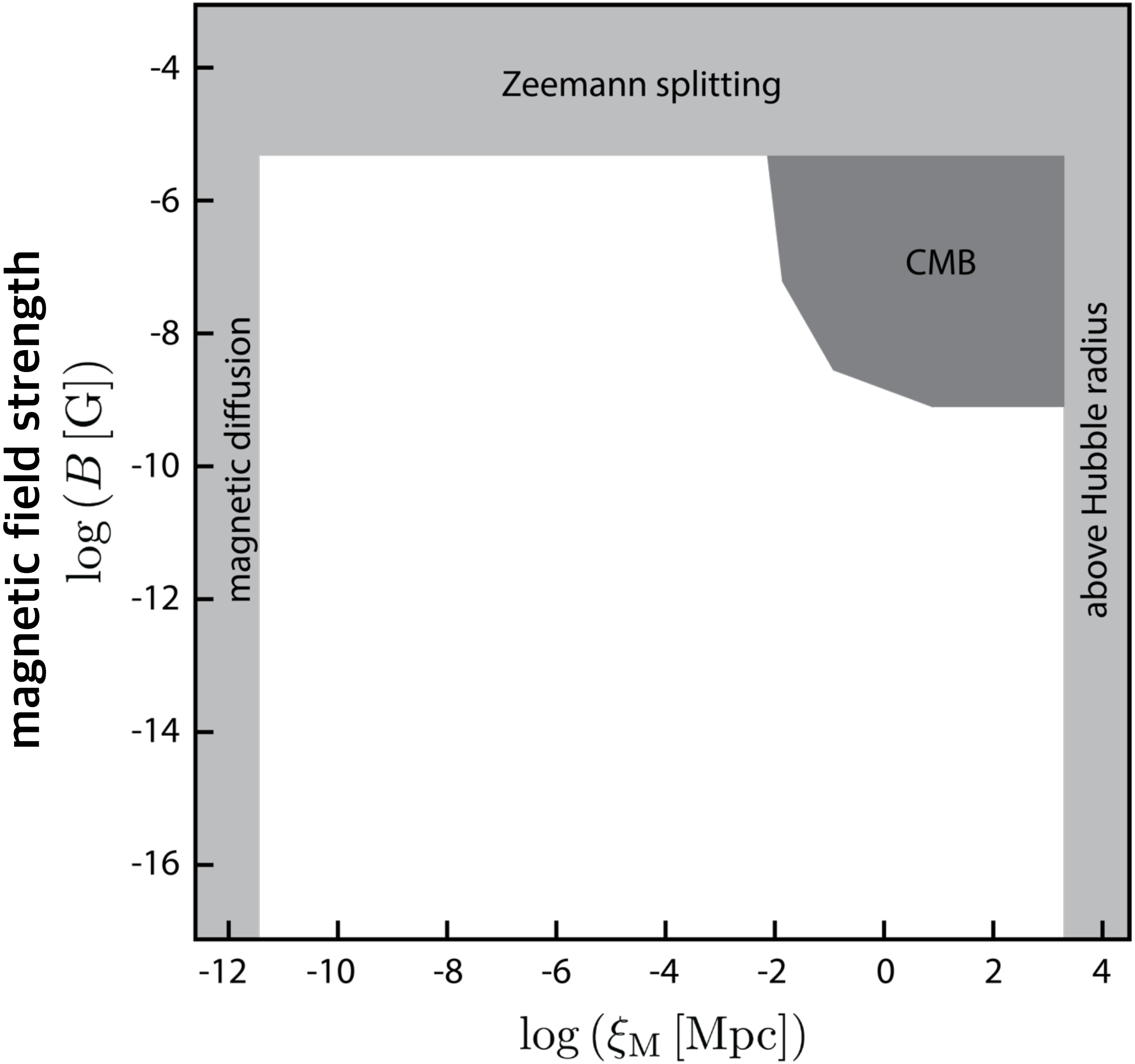

magnetic field strength

magnetic correlation length

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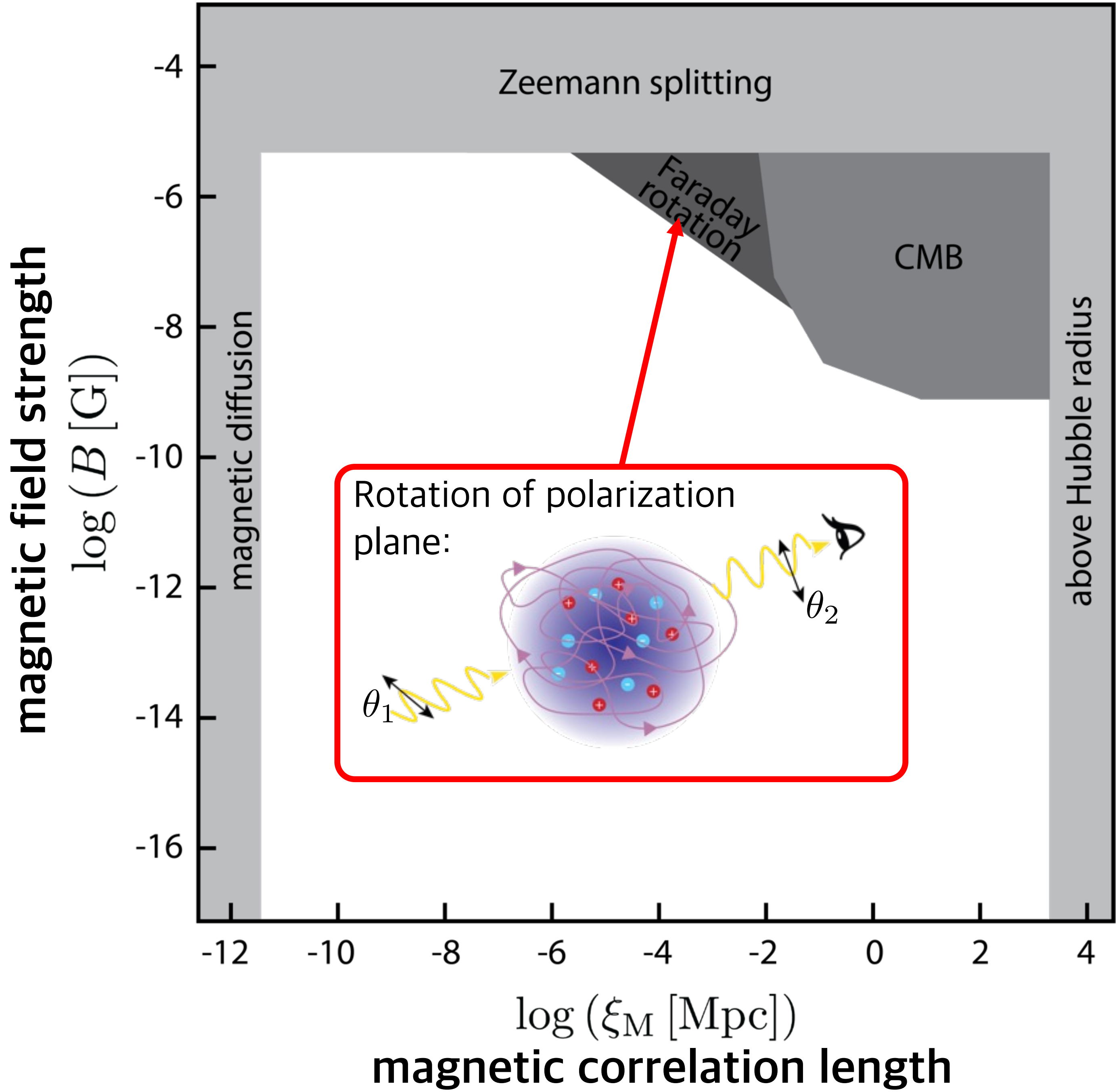
Schober

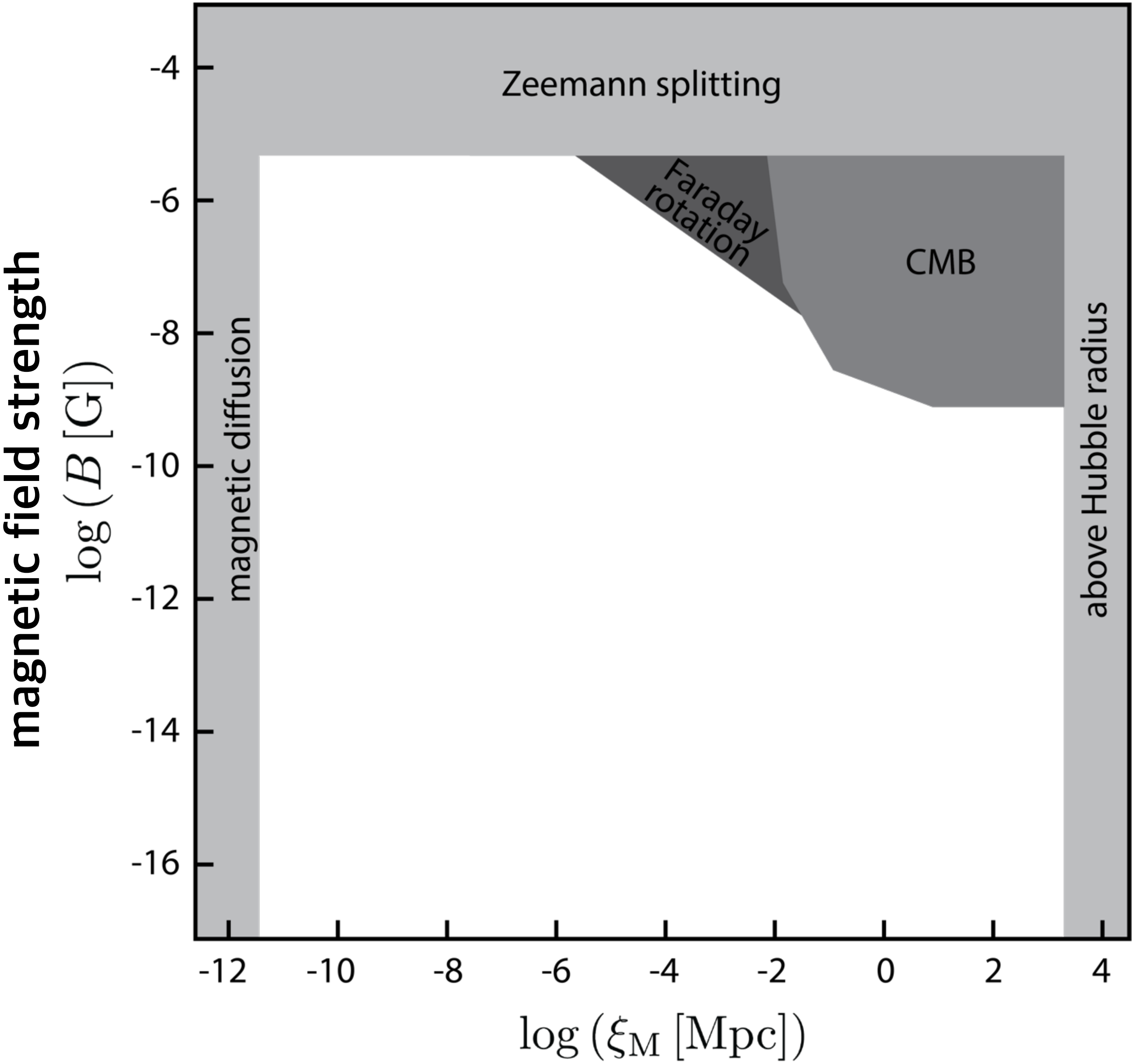
Constraints on void magnetic field



Rotation of polarization plane:

Jennifer Schober

Constraints on void magnetic field



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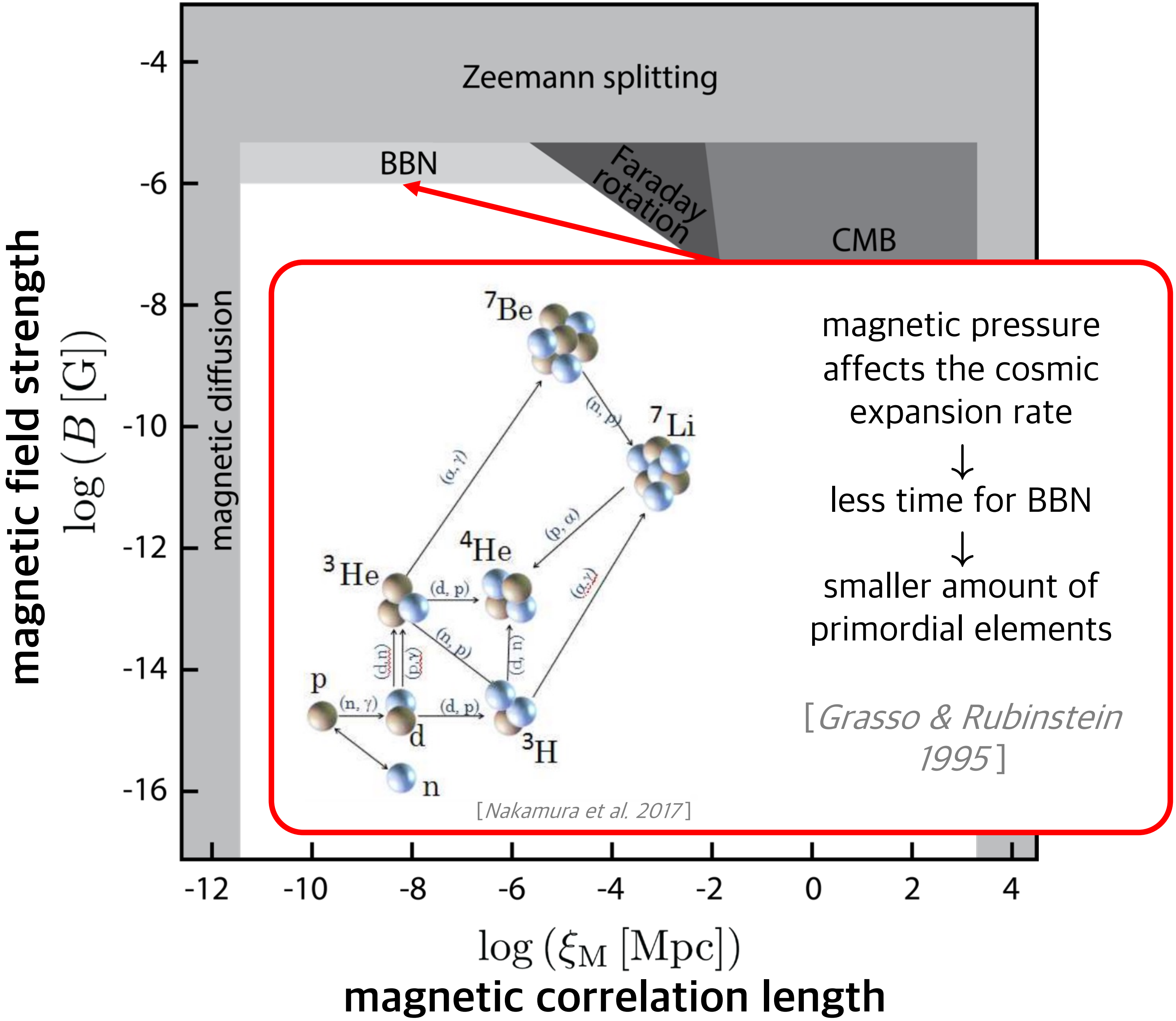


[Nakamura et al. 2017]

magnetic pressure affects the cosmic expansion rate

↓

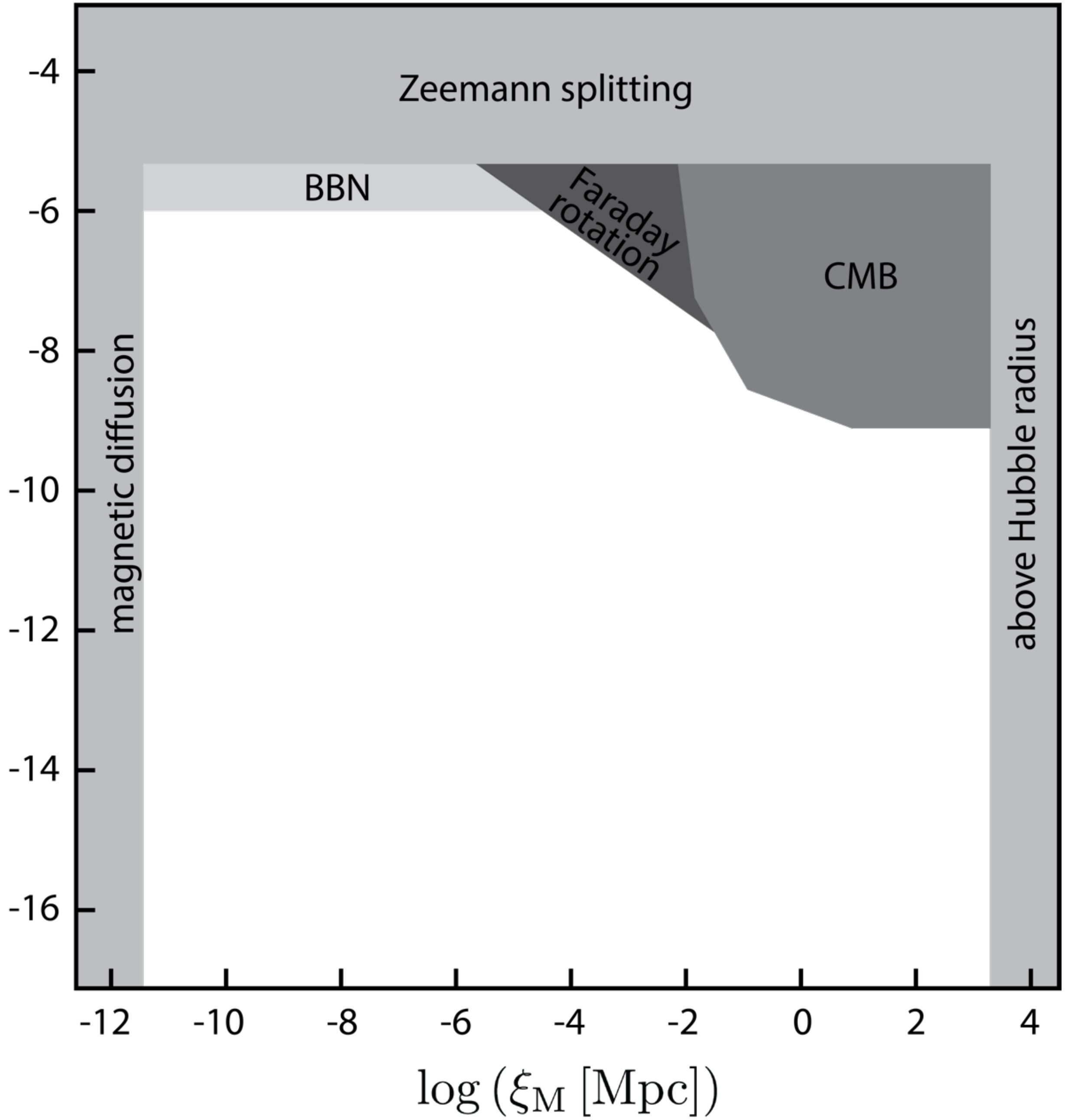
less time for BBN

↓

smaller amount of primordial elements

[*Grasso & Rubinstein 1995*]

Constraints on void magnetic field



magnetic field strength

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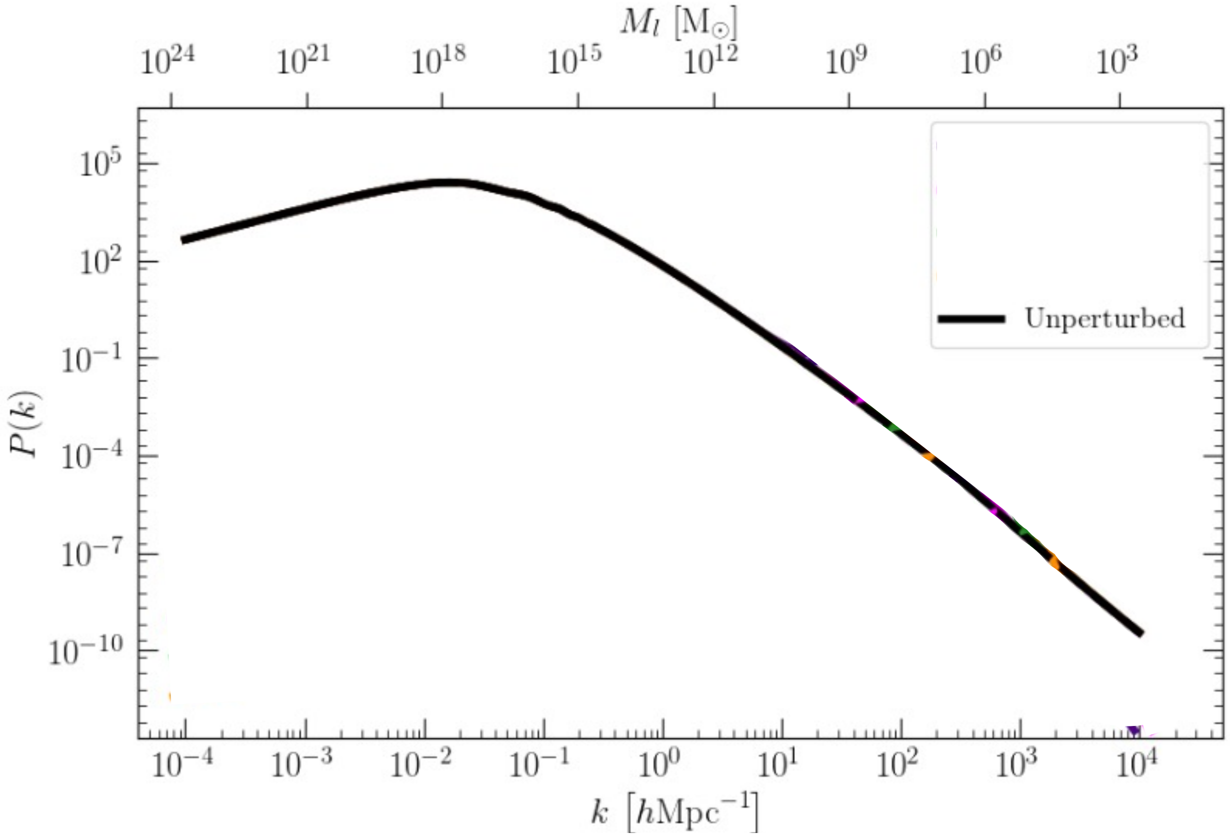
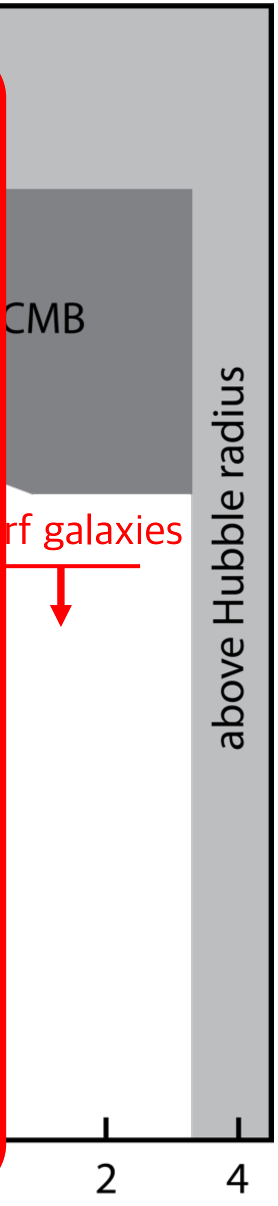
Constraints on void magnetic field

Cosmological simulations of dwarf galaxies
[*Sanati et al. 2020*]:

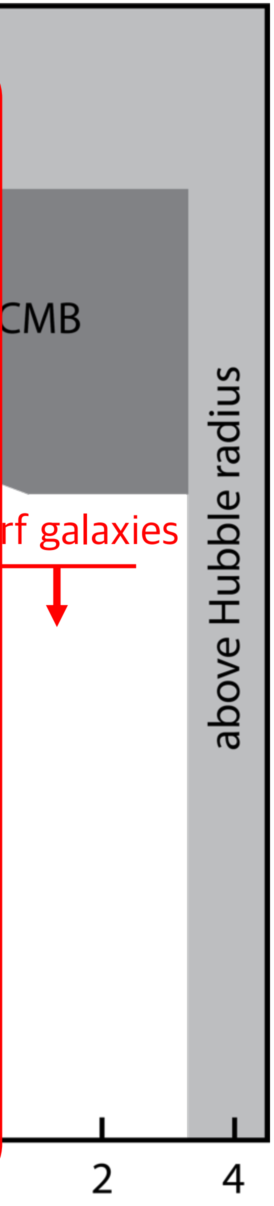
Primordial magnetic fields induce additional density fluctuations
⇨ modification of the primordial matter power spectrum





$\log\left(\xi_M\ [\mathrm{Mpc}]\right)$
magnetic correlation length

Constraints on void magnetic field



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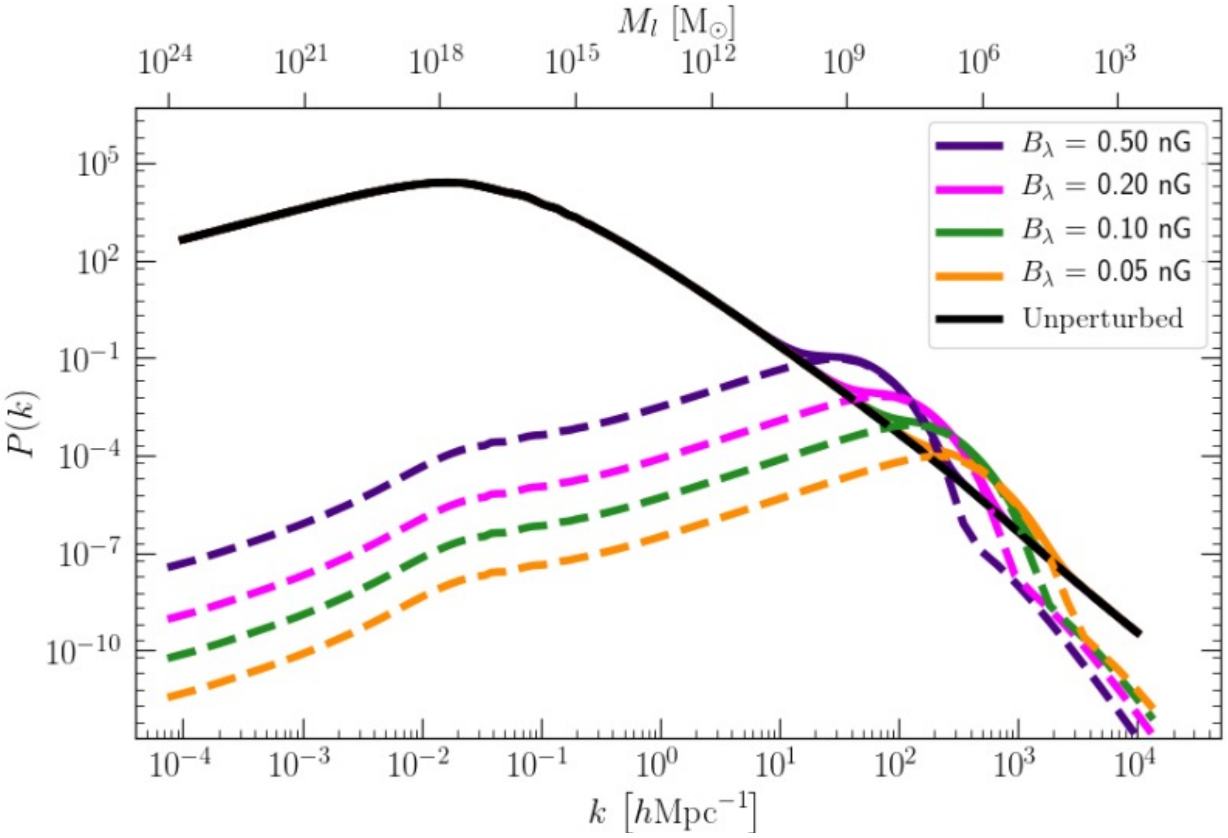

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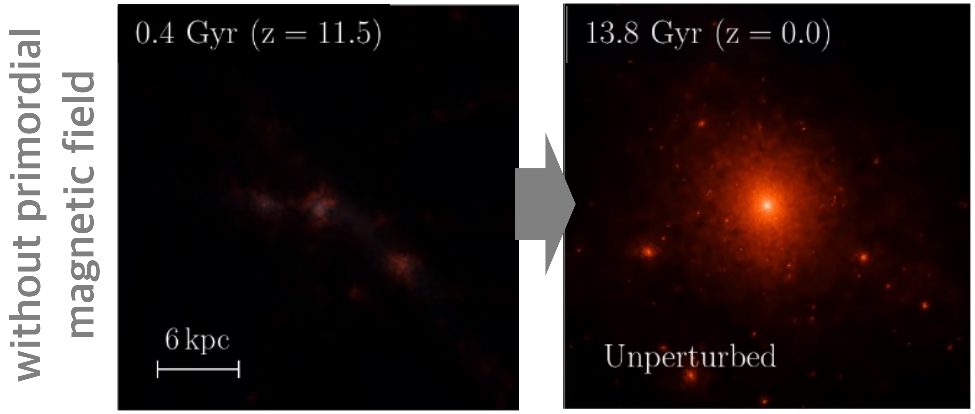

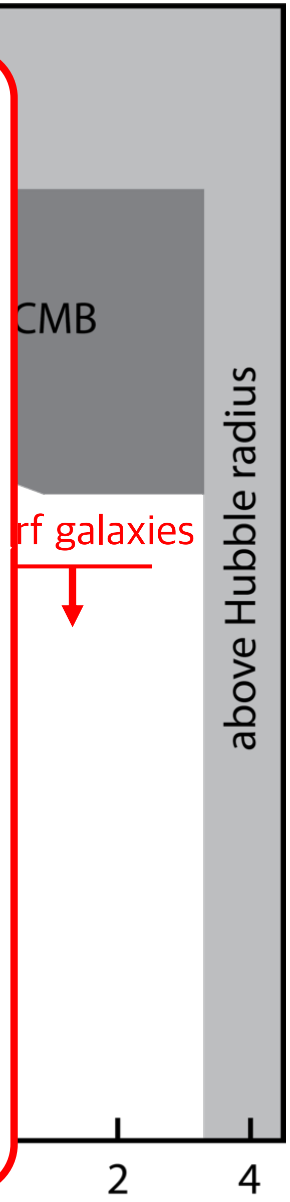

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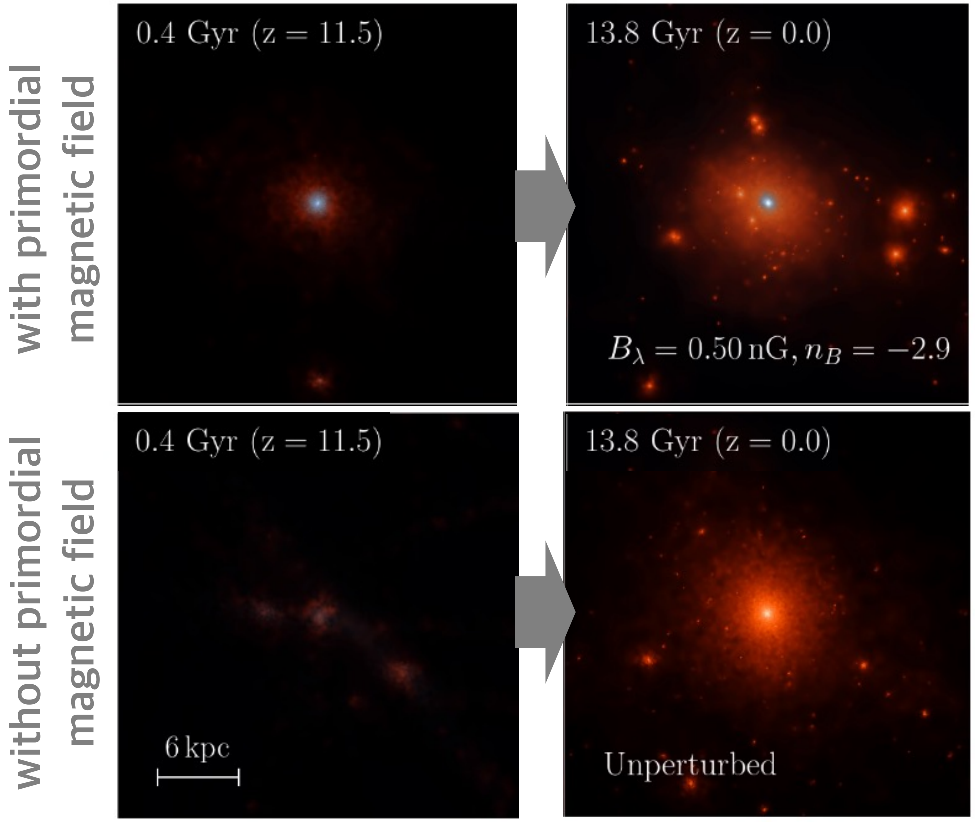

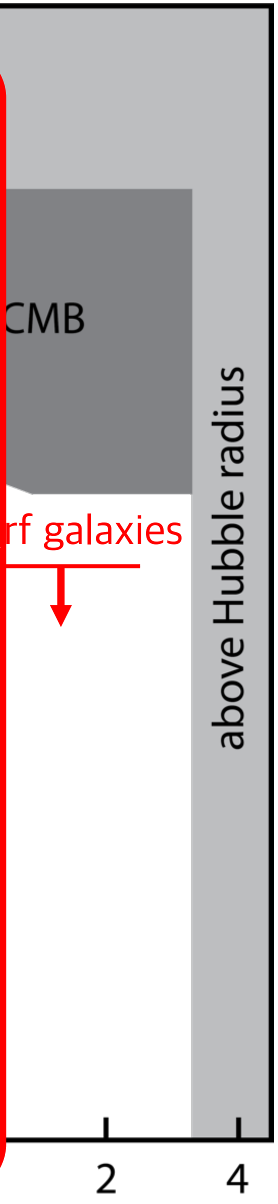

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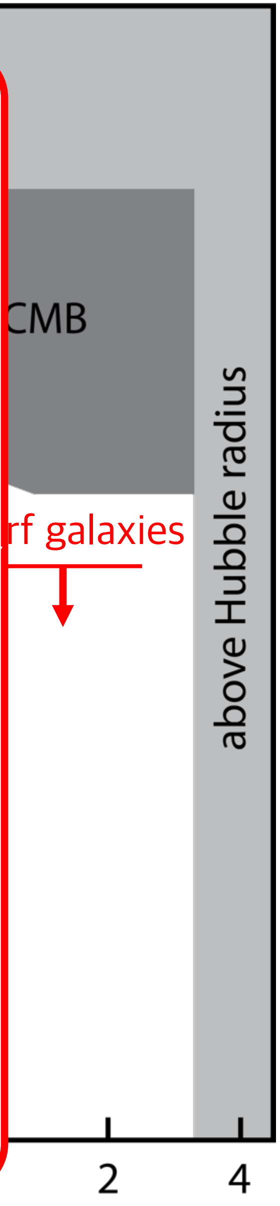

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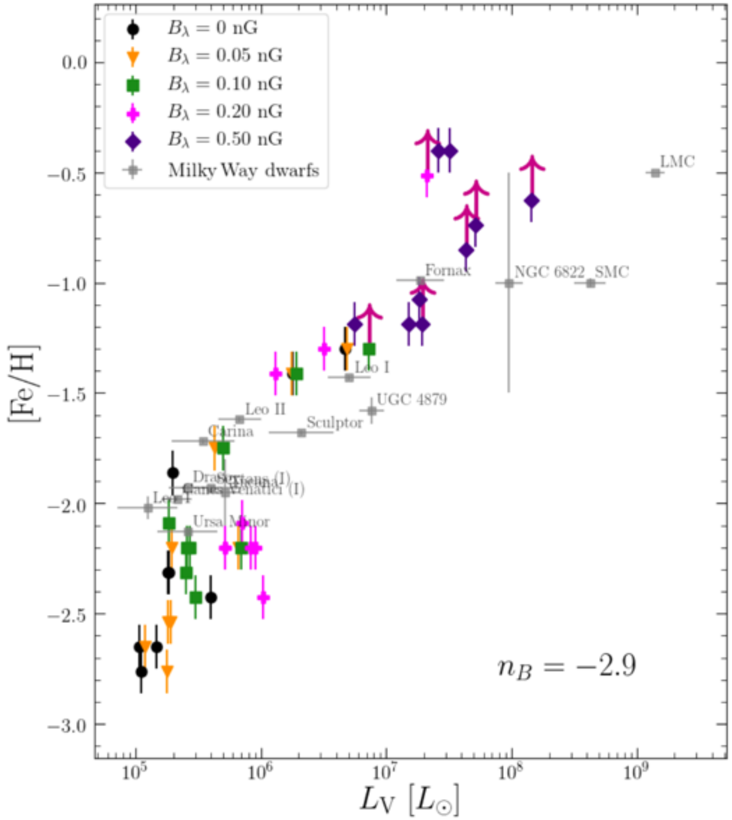

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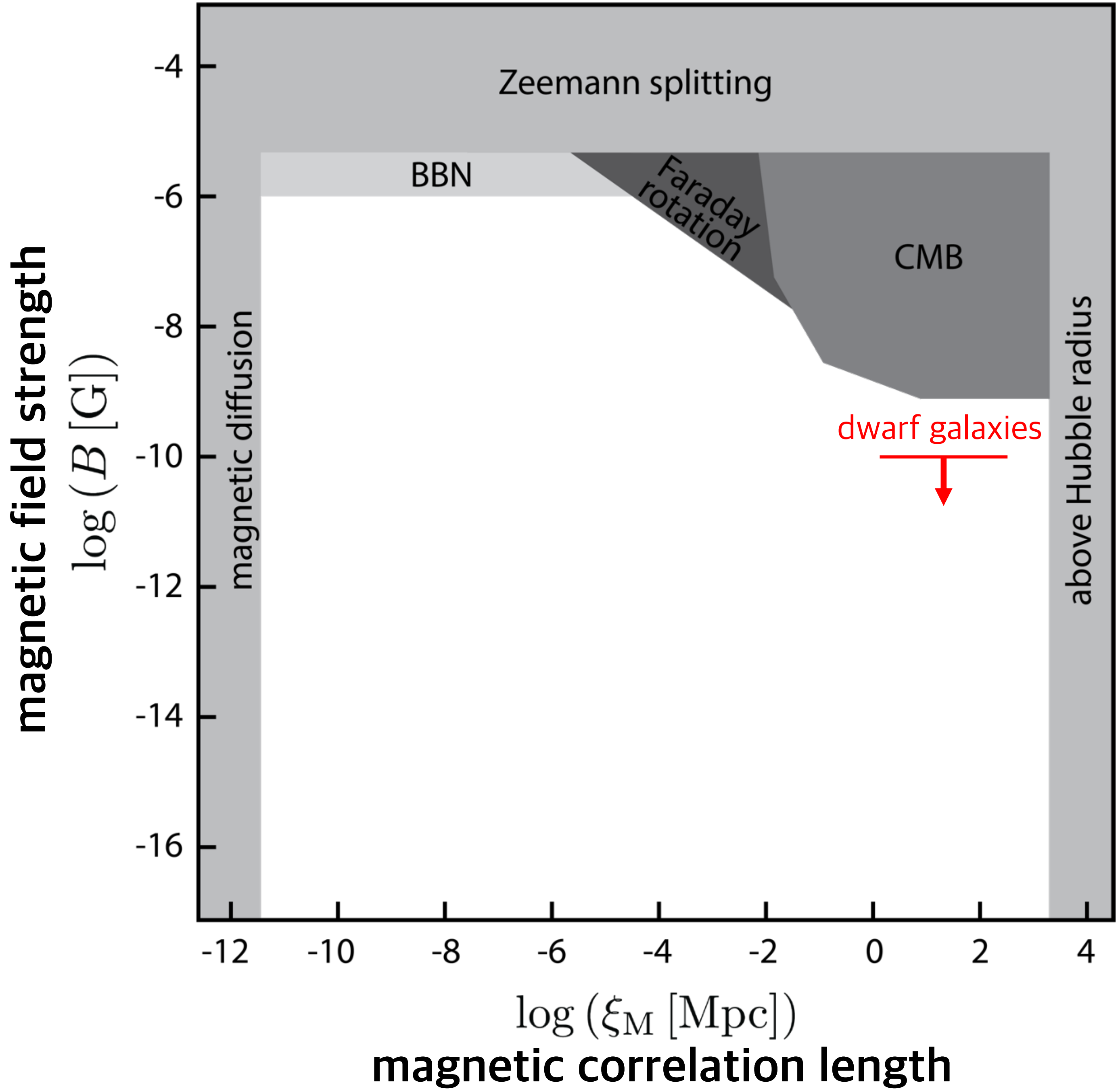

EPFL

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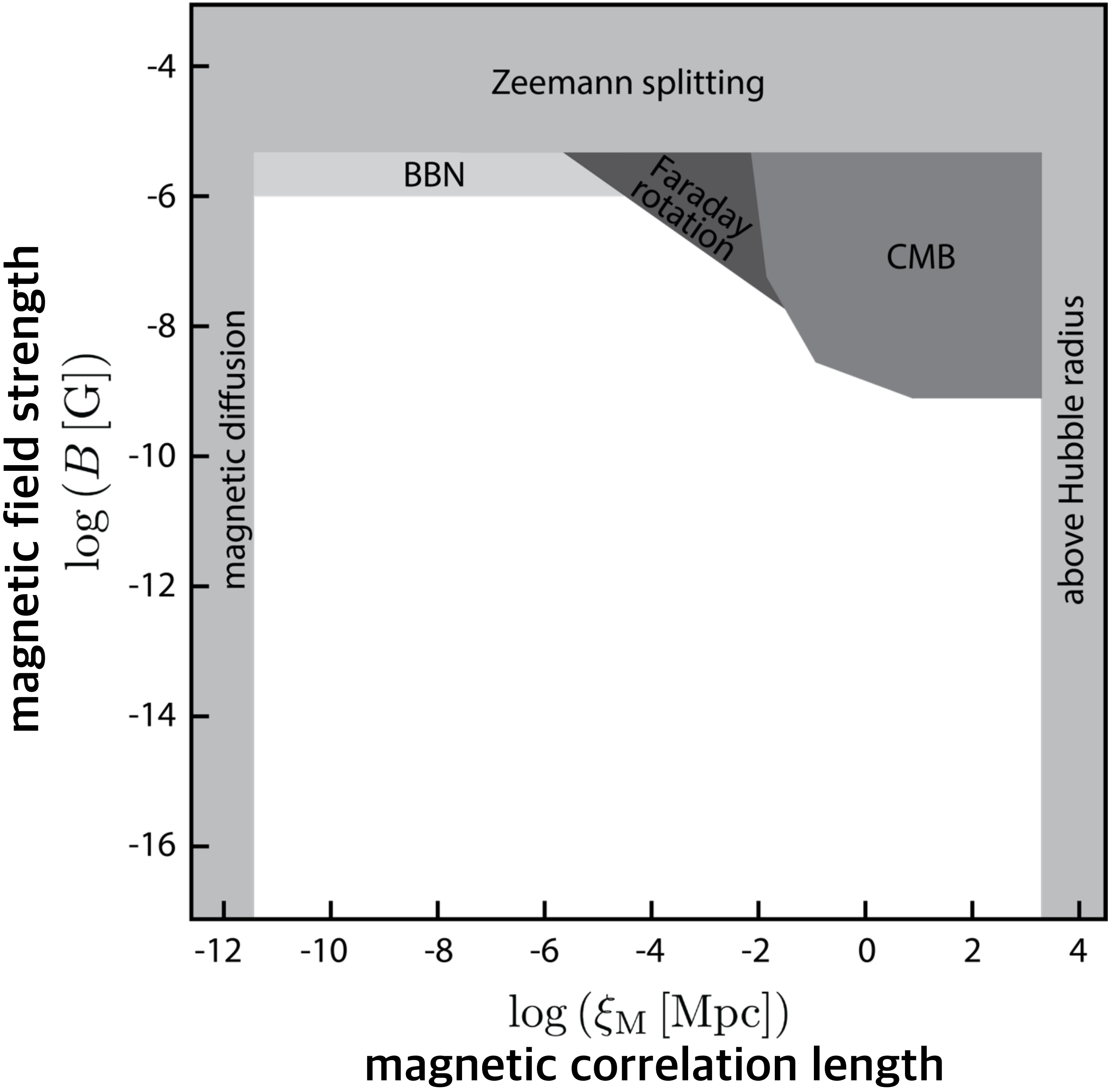
Constraints on void magnetic field



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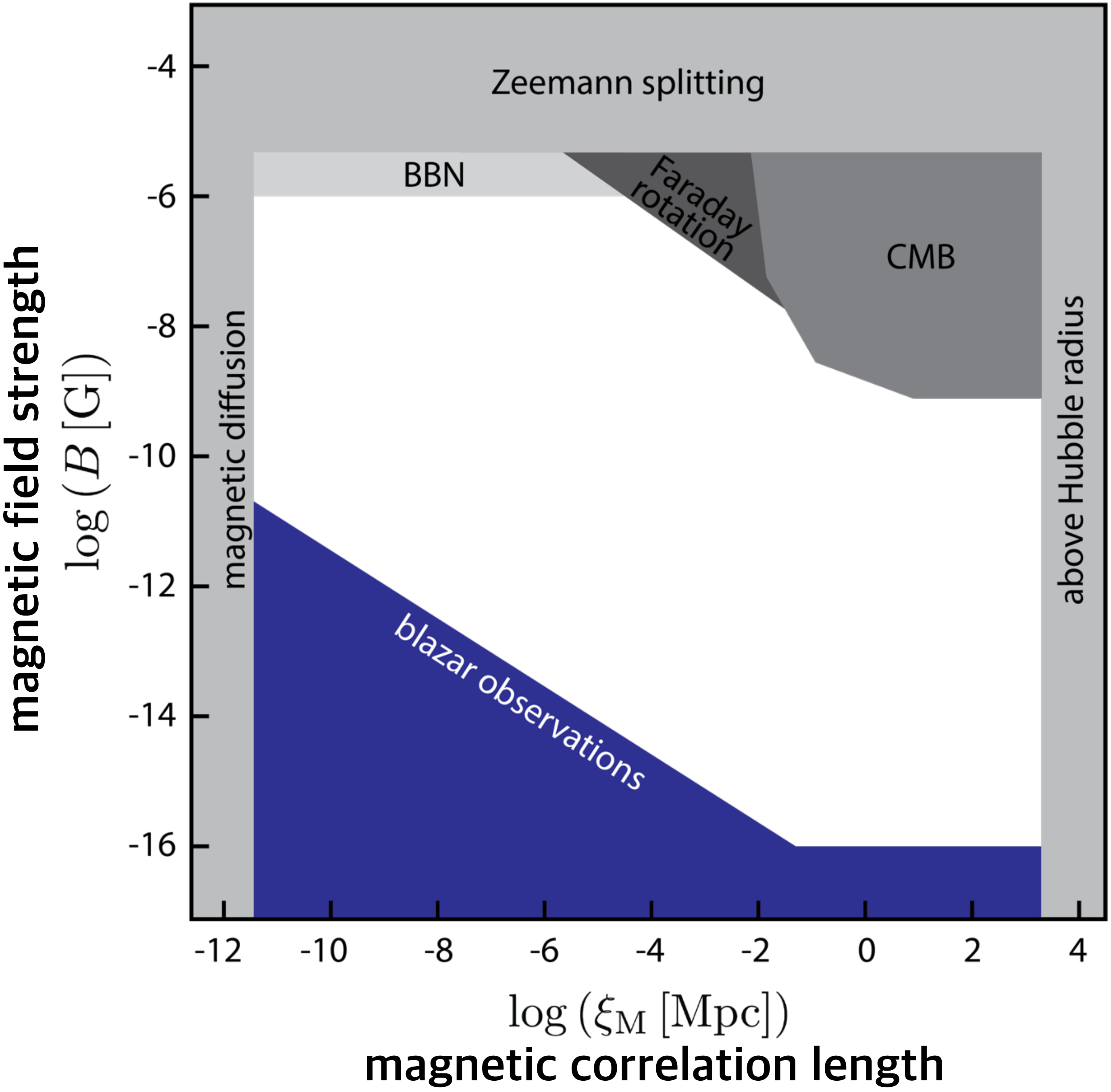

Neronov & Vovk (2010)

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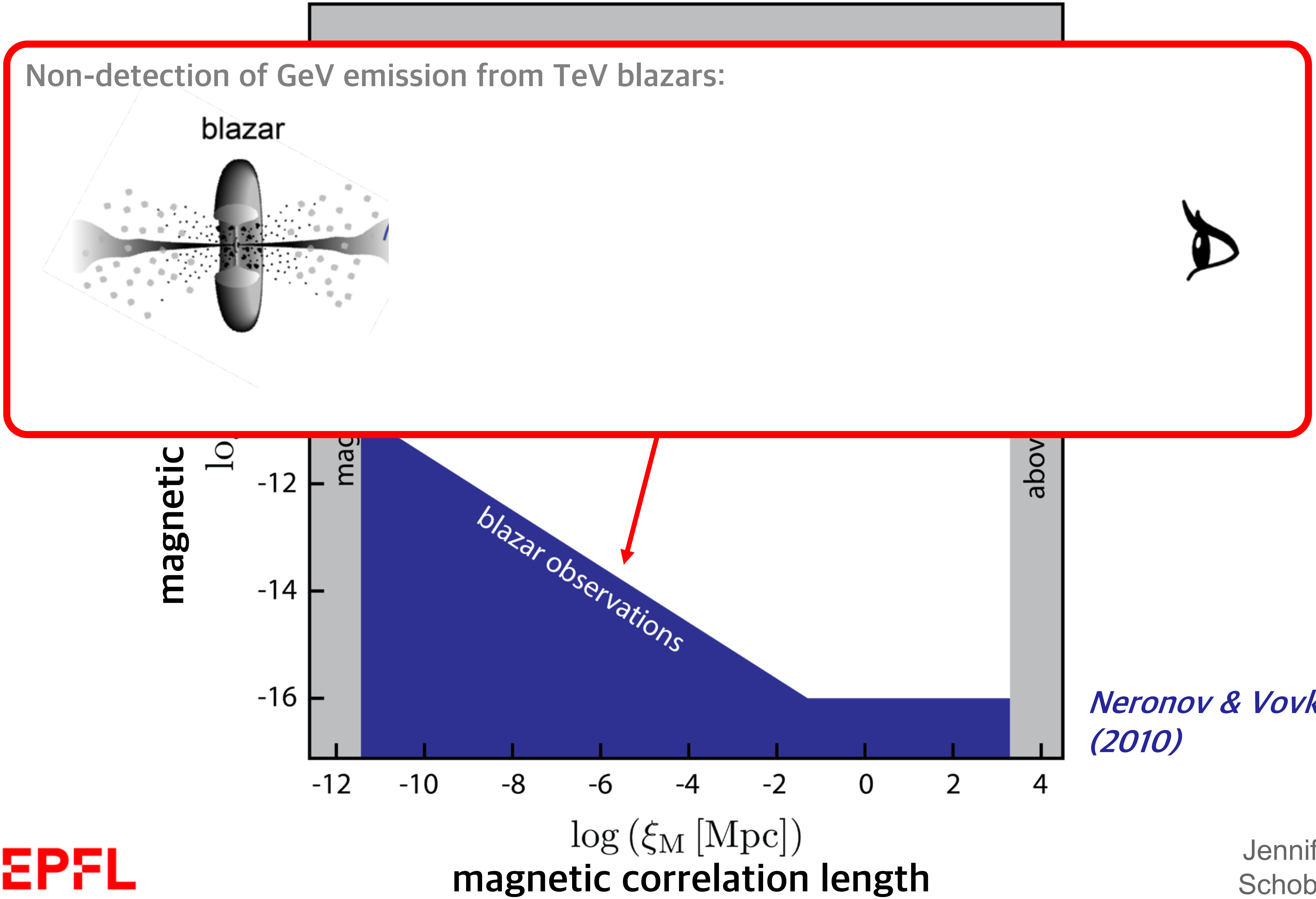
Constraints on void magnetic field



Non-detection of GeV emission from TeV blazars:

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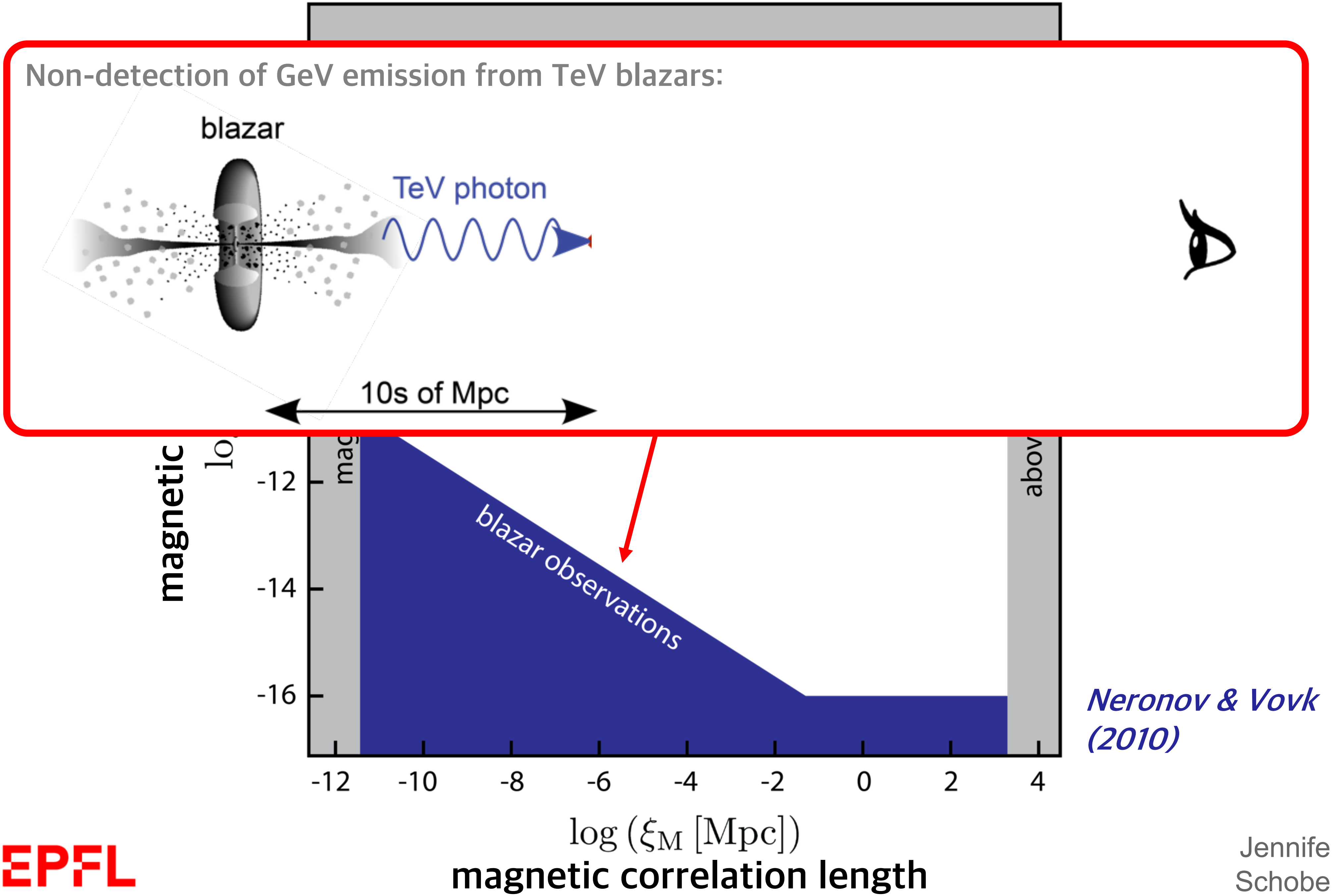

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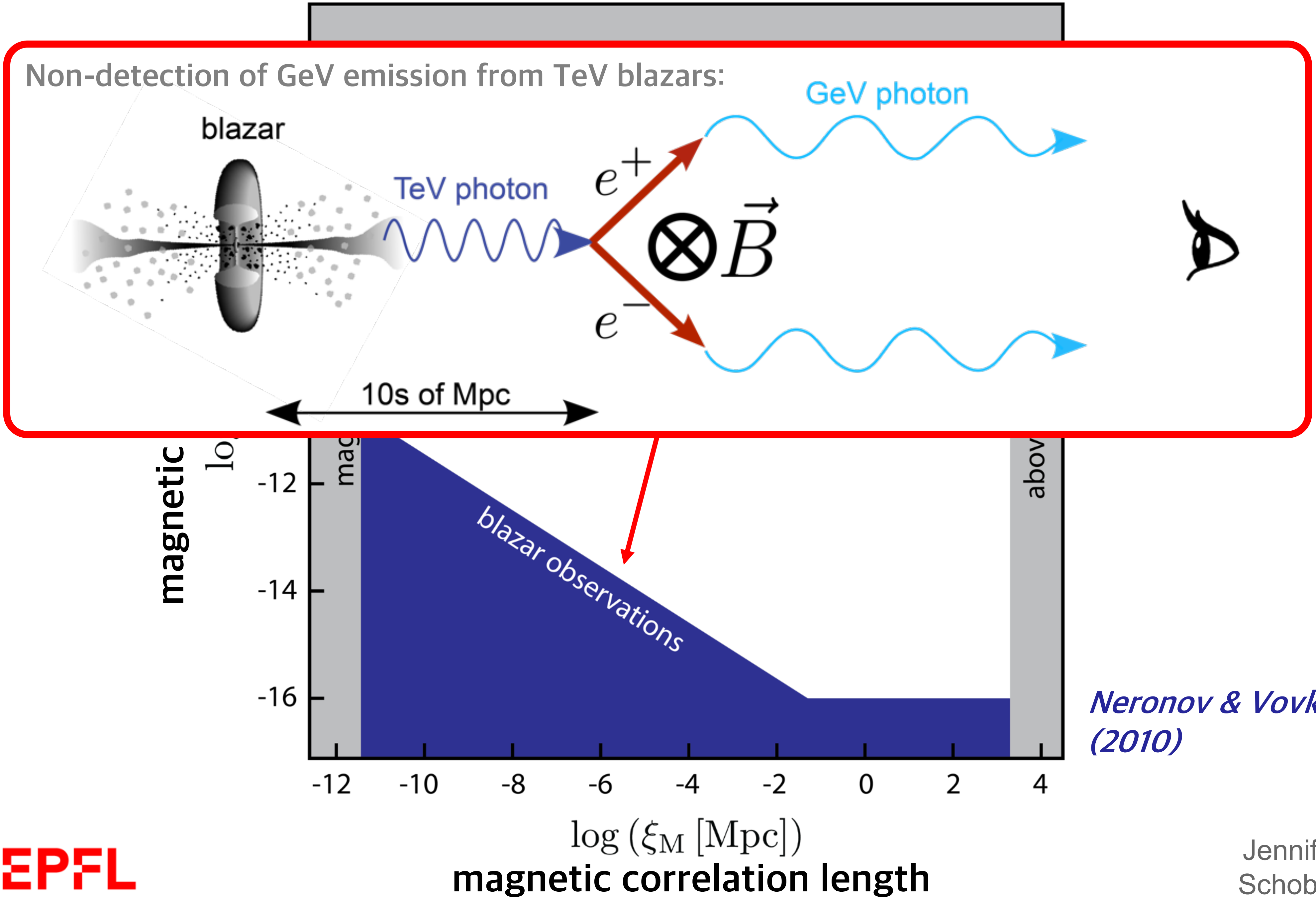

Neronov & Vovk (2010)

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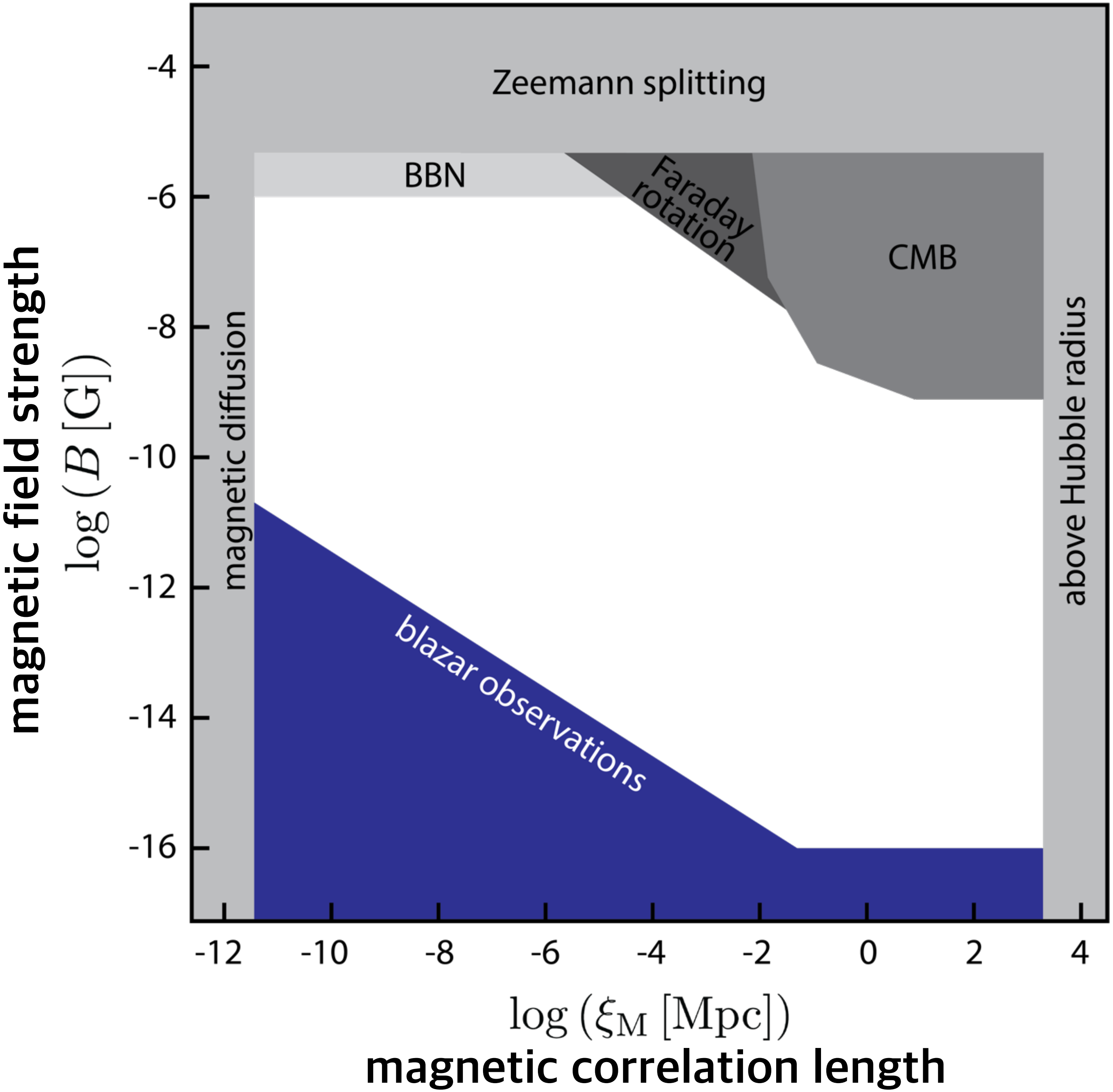

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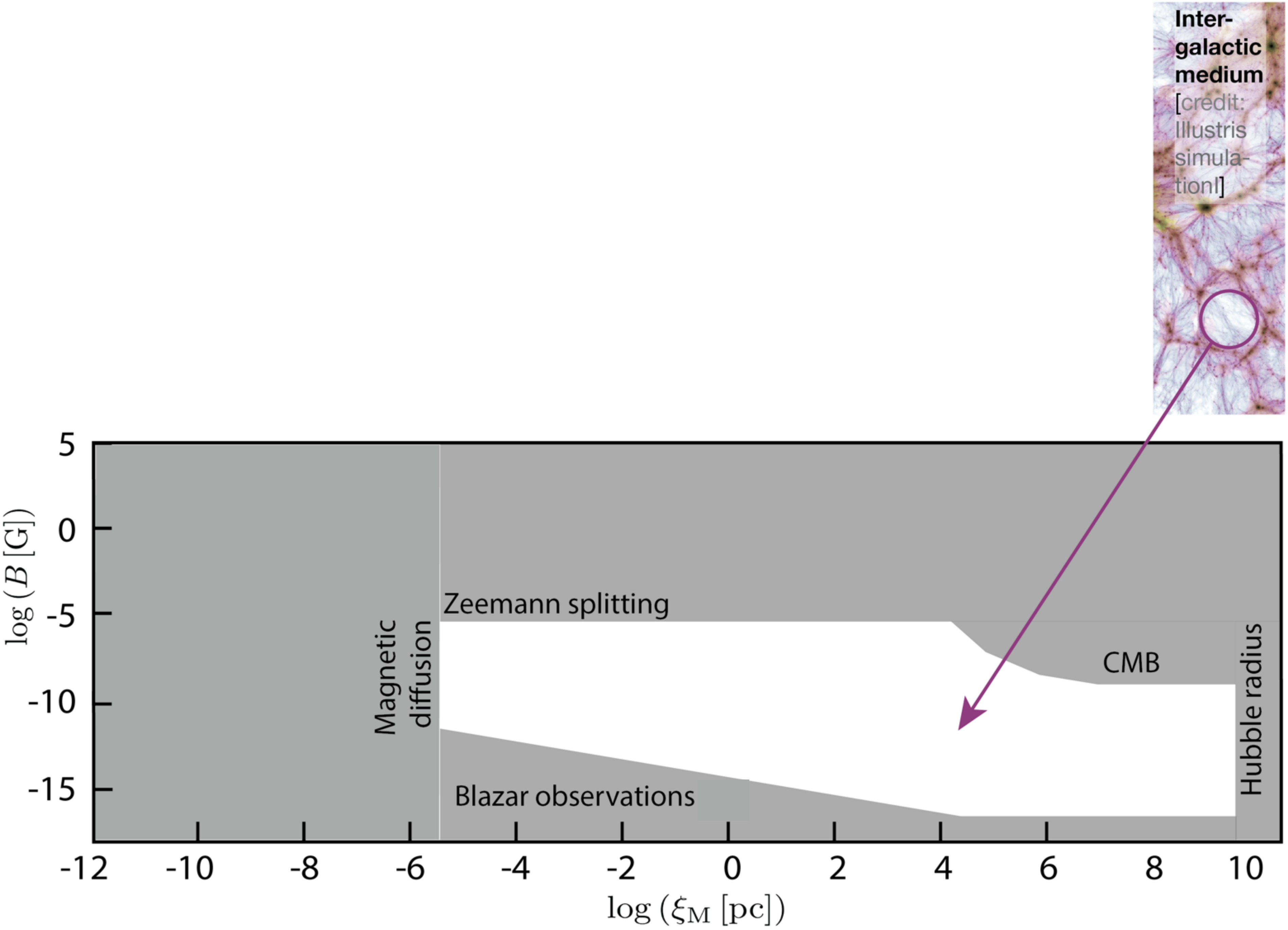
Magnetic fields across the Universe



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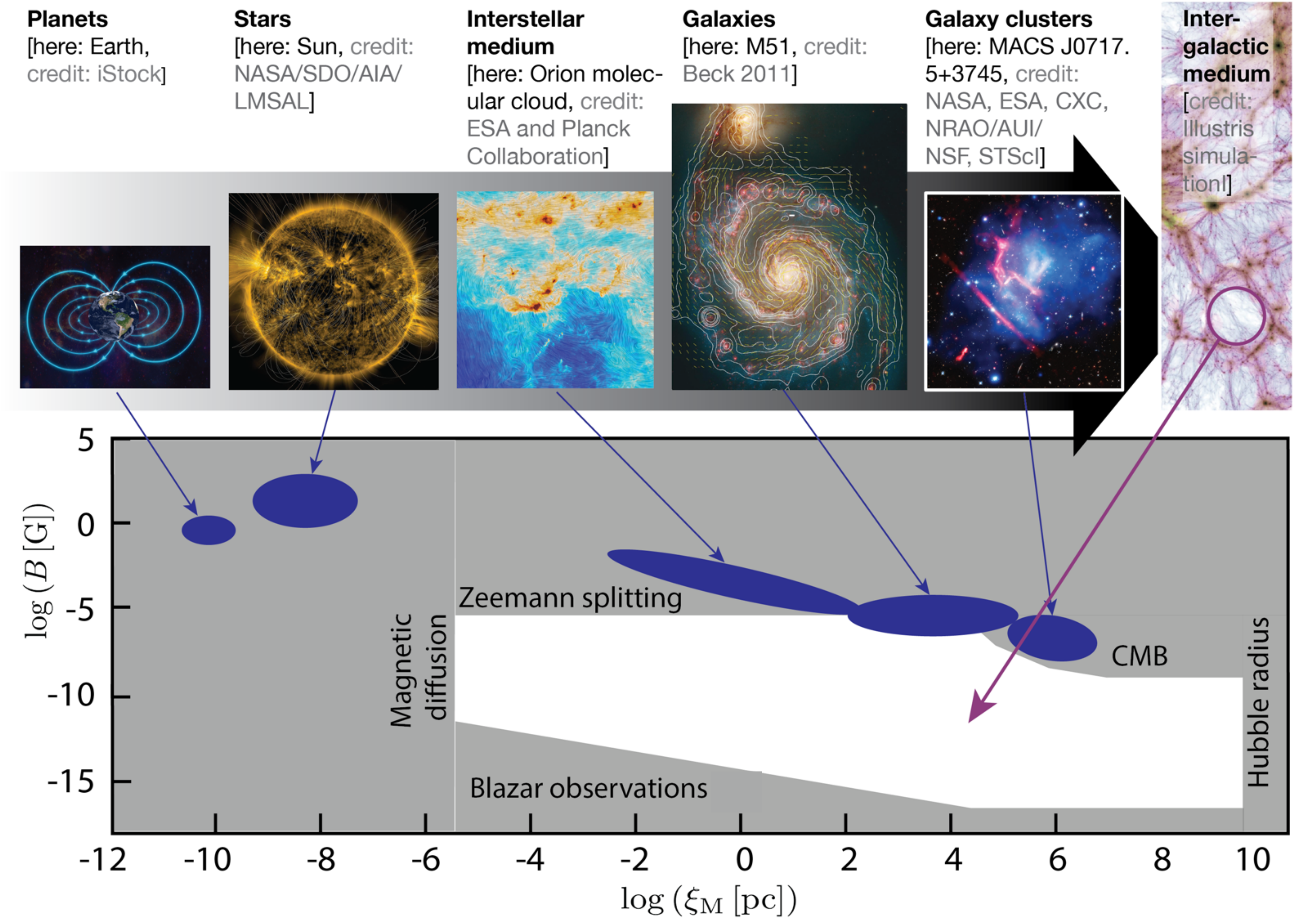


Planets
[here: Earth, credit: iStock]

Stars
[here: Sun, credit: NASA/SDO/AIA/LMSAL]

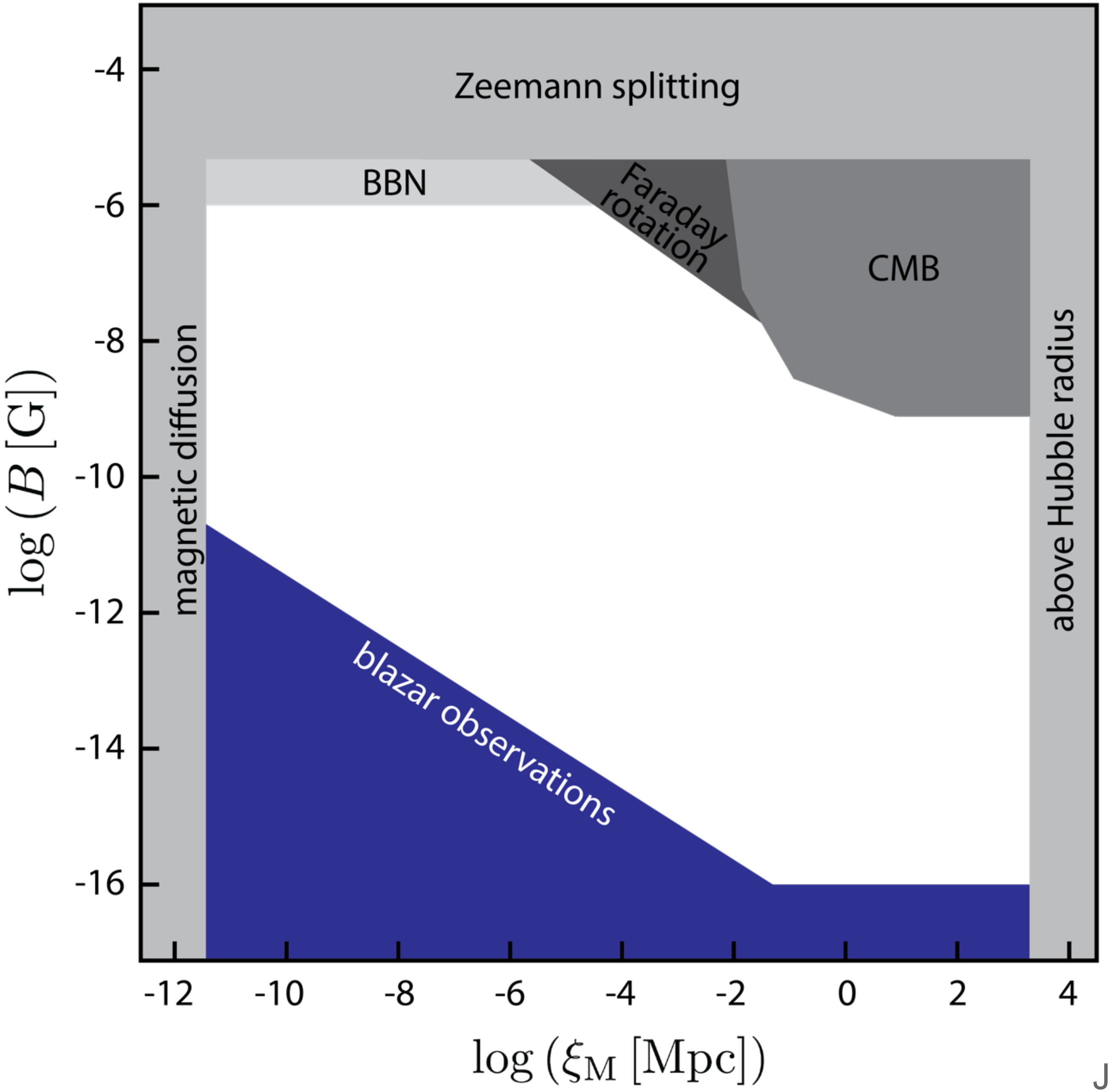
Interstellar medium
[here: Orion molecular cloud, credit: ESA and Planck Collaboration]

Galaxies
[here: M51, credit: Beck 2011]

Galaxy clusters
[here: MACS J0717.5+3745, credit: NASA, ESA, CXC, NRAO/AUI/NSF, STScI]

Inter-galactic medium
[credit: Illustris simulation]

Connecting observation with theory



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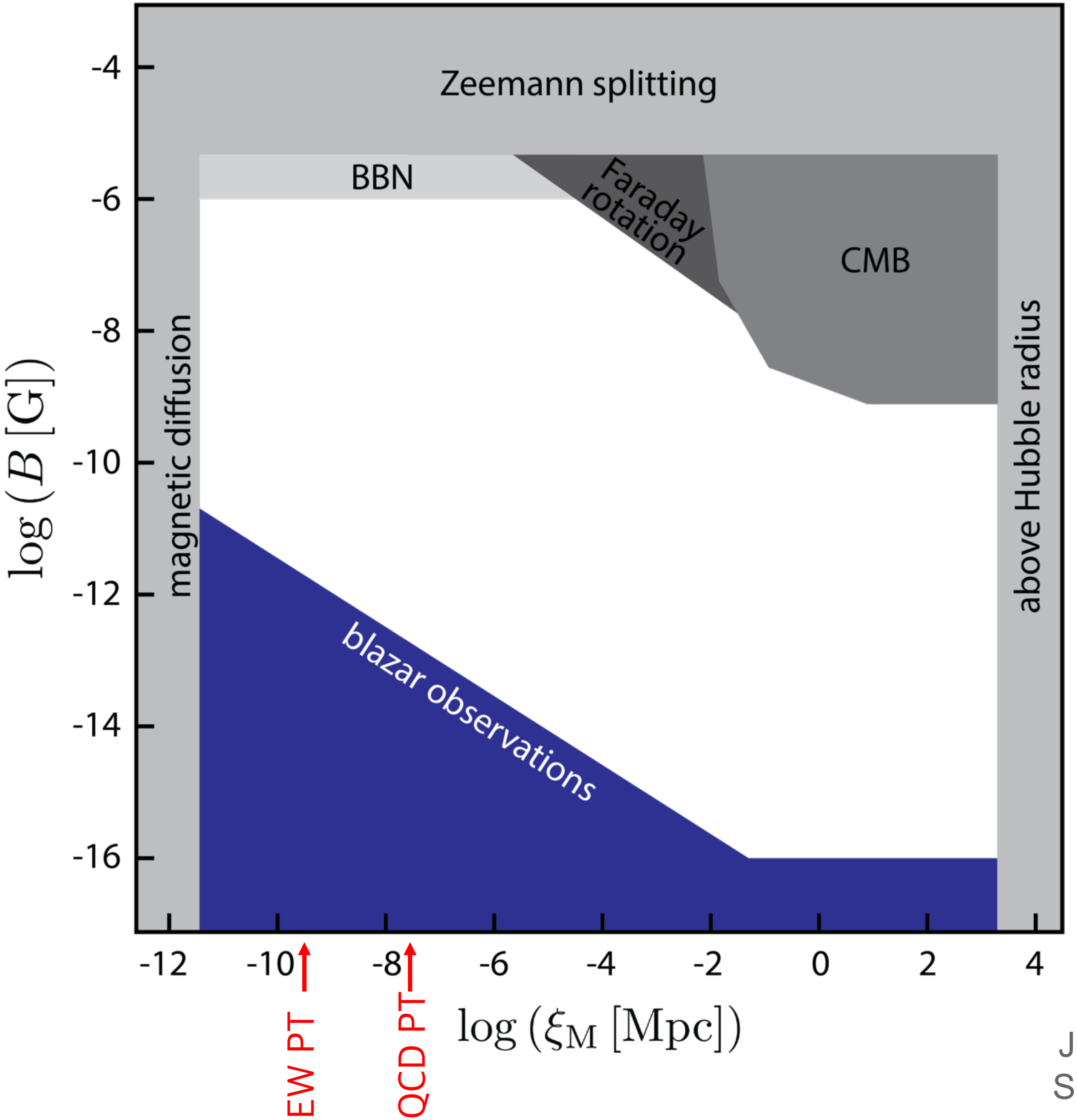

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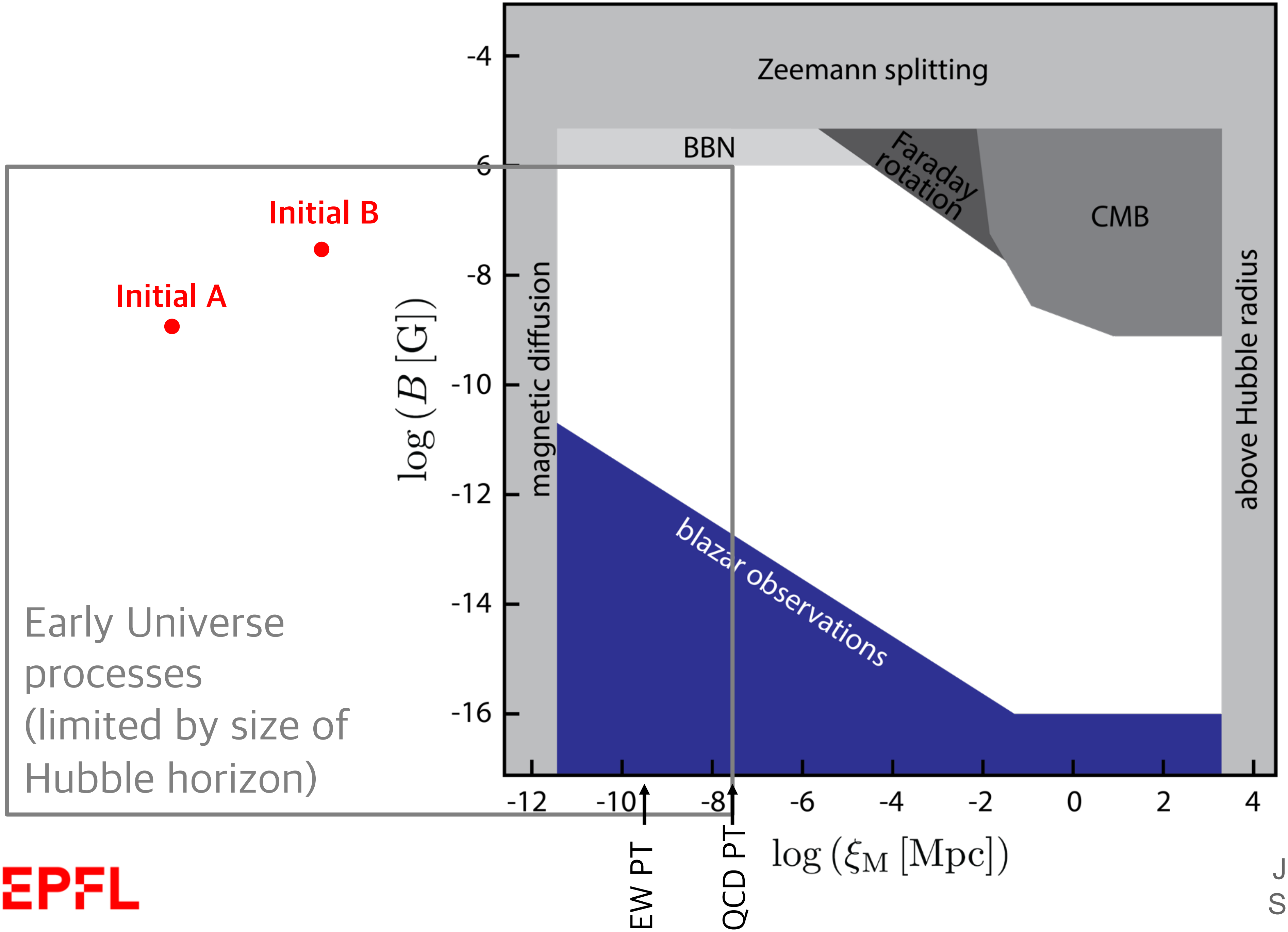

Early Universe processes
(limited by size of Hubble horizon)

Jennifer Schober

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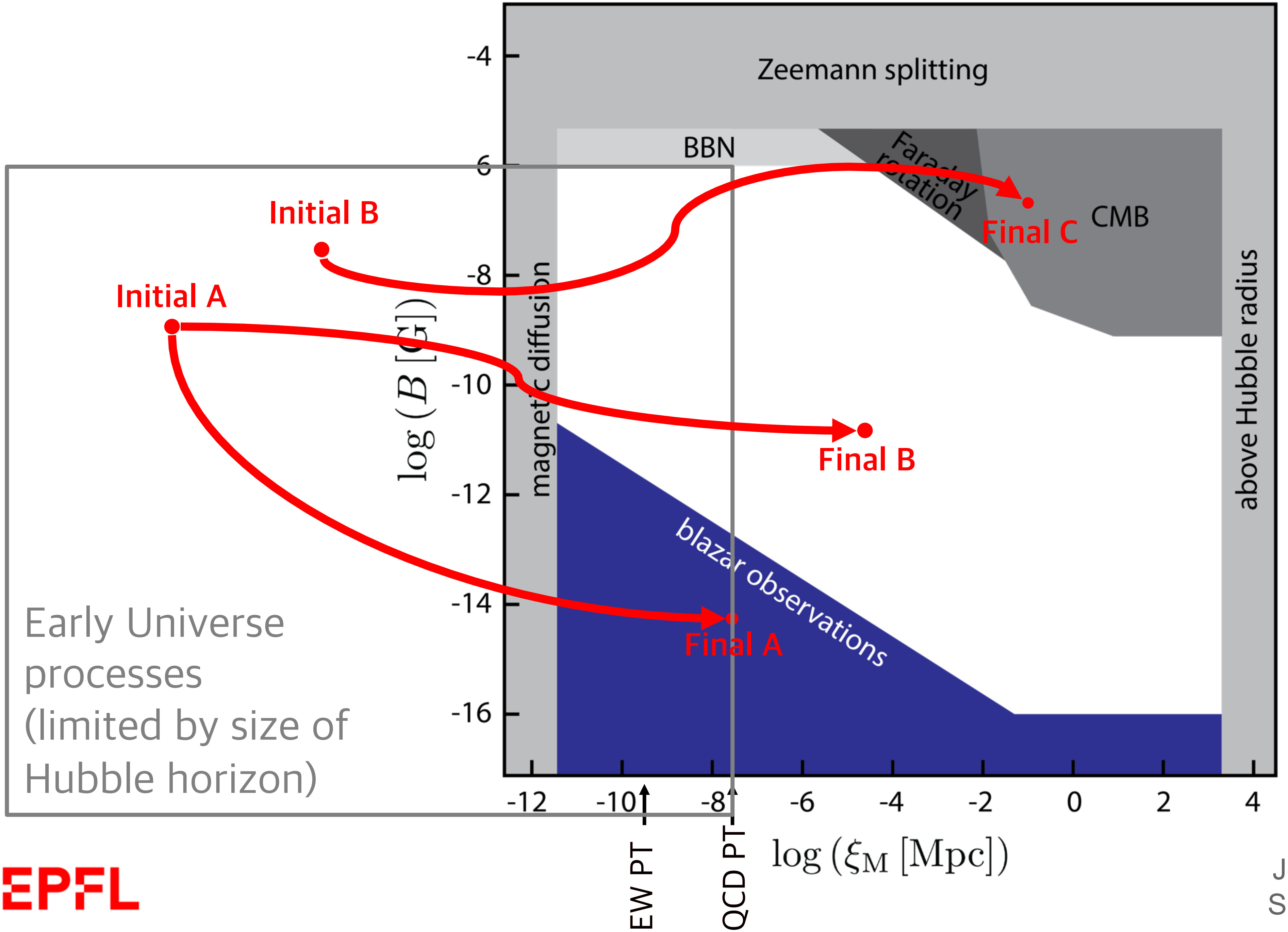

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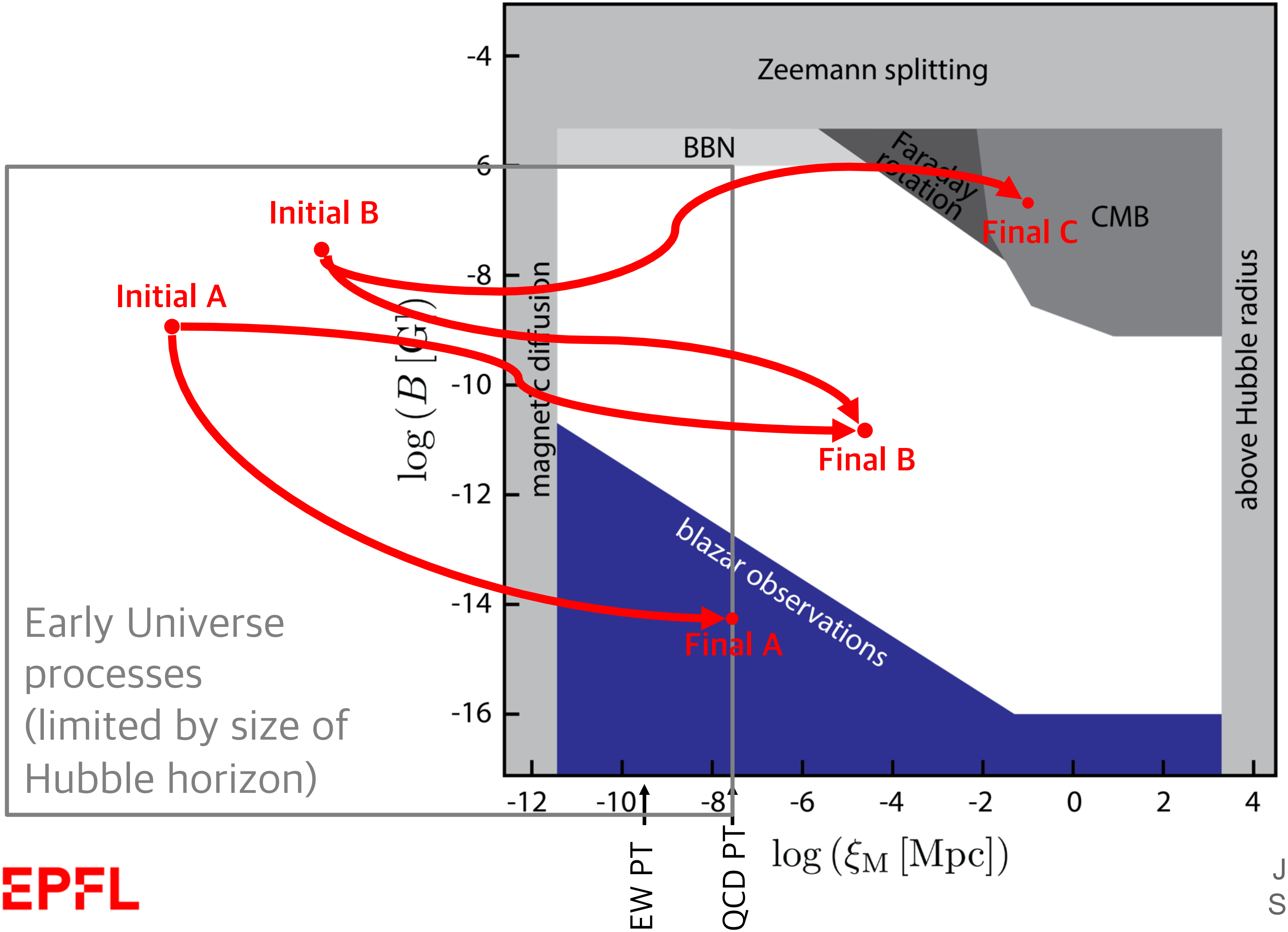

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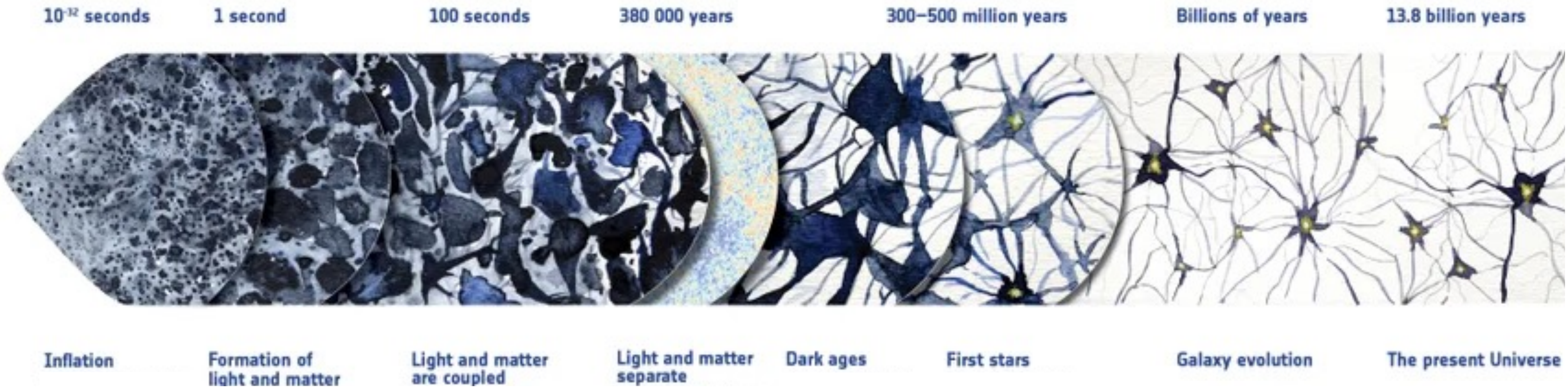
Magnetic history of the Universe



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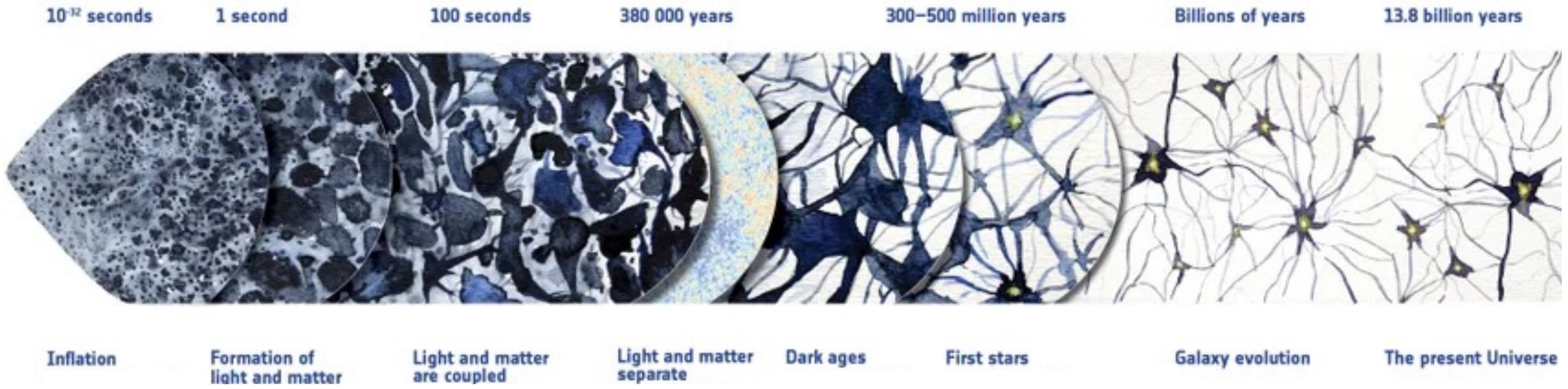
Magnetic history of the Universe

Seed magnetic fields:

- **Inflation:**
  fluctuations of the electromagnetic field are increased
  [*Turner & Widrow 1988, Ratra 1992*]

$$B_0 \approx 10^{-65} - 10^{-9}\ \mathrm{G}$$



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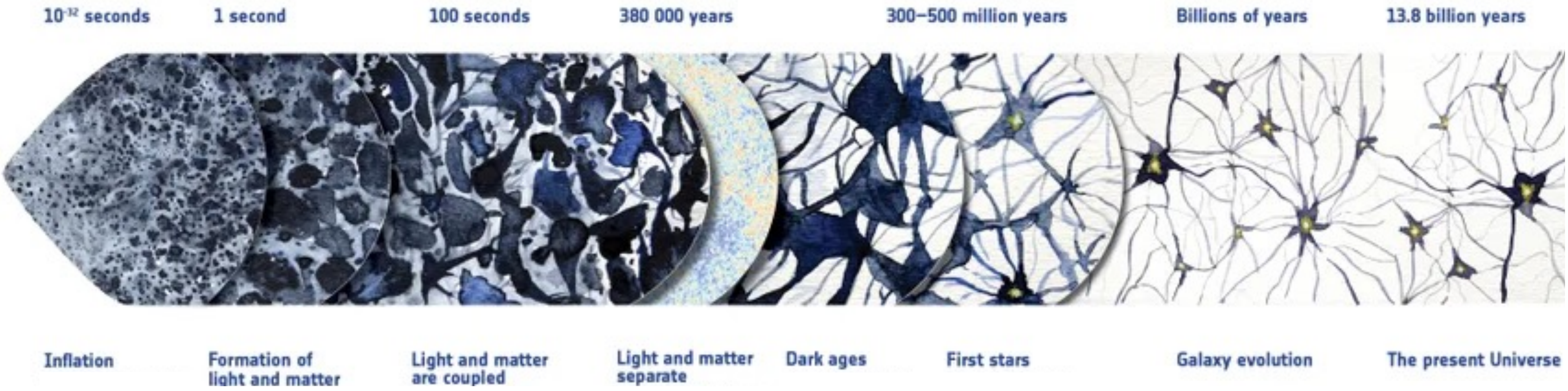
- **(First-order) phase transitions:**
  non-equilibrium conditions allow for battery processes
  [*Hogan 1983, Sigl et al. 1997*]

  $$B_0 \approx 10^{-29} - 10^{-20}\ \mathrm{G}$$



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Magnetic history of the Universe

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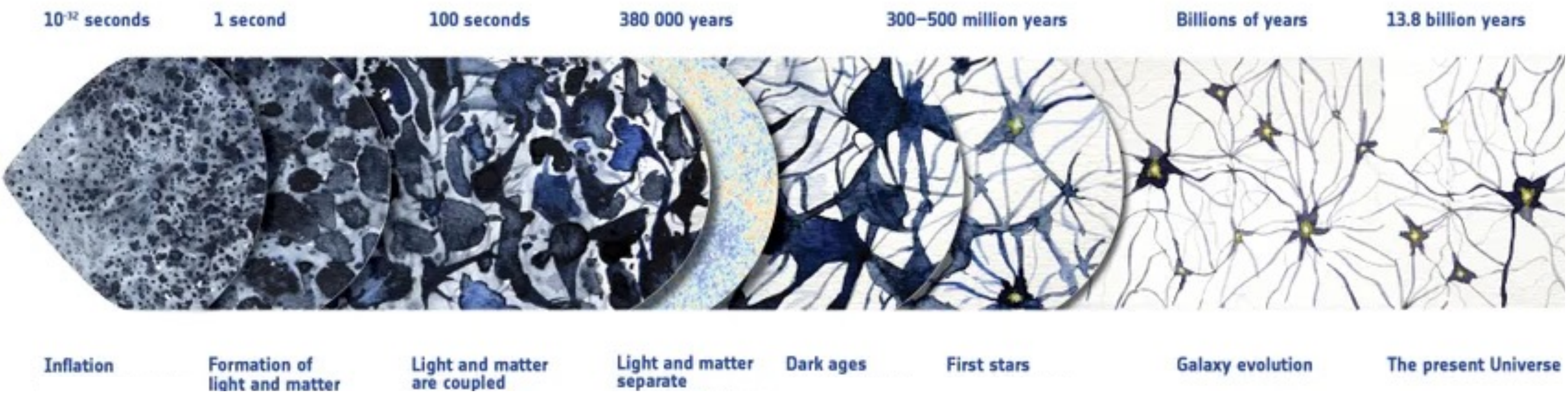

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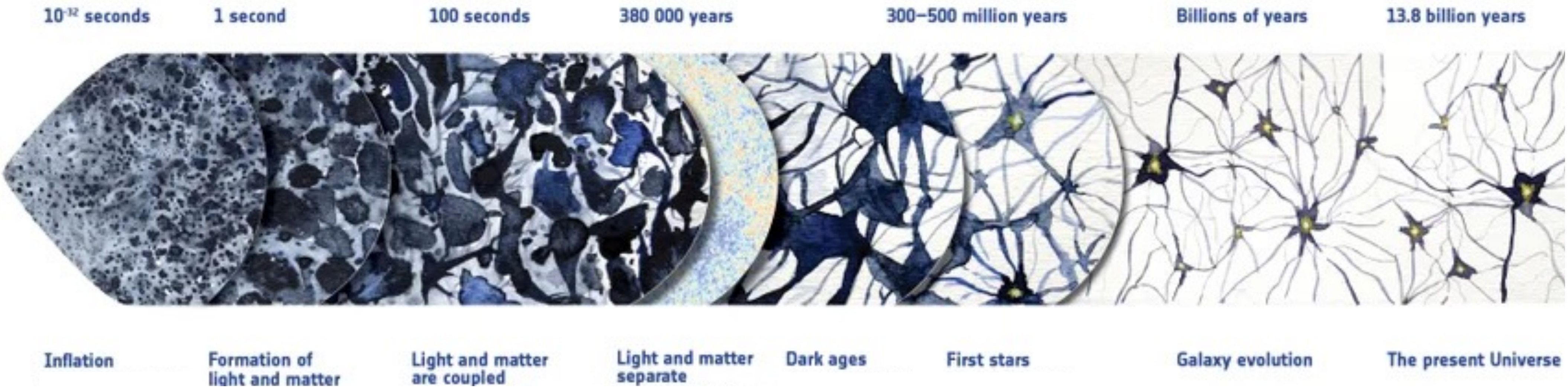
Magnetic history of the Universe

Modelling the primordial plasma

- Dynamical variables: magnetic field $\boldsymbol{B}$, velocity field $\boldsymbol{U}$, density ρ + equation of state

Seed magnetic fields



Magnetic history of the Universe

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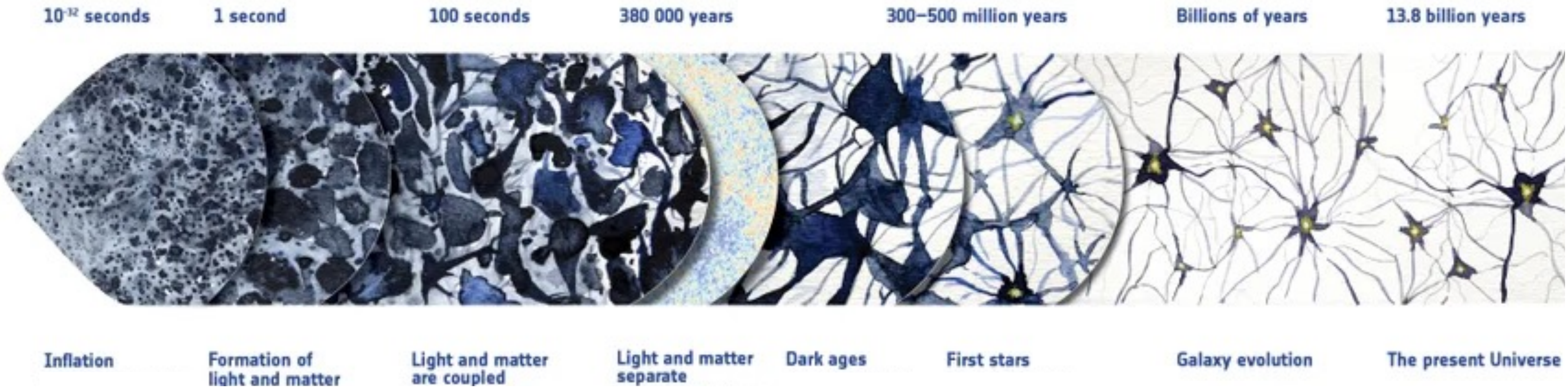
- Evolution equations:

Fully general relativistic MHD equations

Seed magnetic fields



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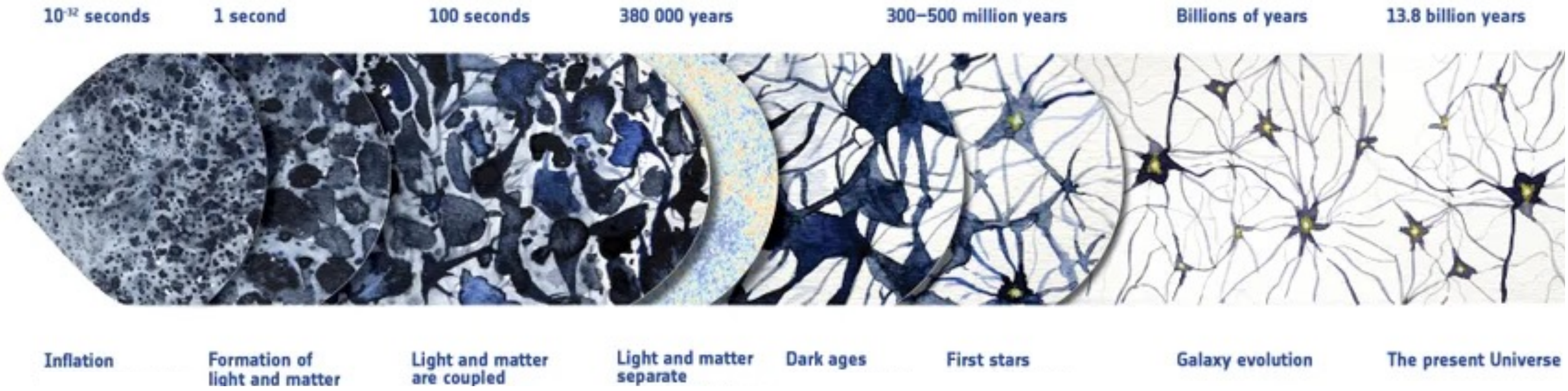
Use comoving quantities like $\tilde{\mathrm{B}} \equiv a^2\mathbf{B}$ and conformal time $\mathrm{d}\tilde{t} \equiv a^{-1}\mathrm{d}t$ $[\,\mathrm{d}s^2 = c^2\mathrm{d}t^2 - a(t)^2\mathrm{d}\Sigma\,]$.

MHD equations

[*Brandenburg et al. 1996*]

Seed magnetic fields



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Magnetic history of the Universe

Decaying magnetohydrodynamical (MHD) turbulence:

- The evolution of magnetic fields is governed by:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times [\boldsymbol{U} \times \boldsymbol{B} - \eta \nabla \times \boldsymbol{B}]$$

magnetic field

velocity

magnetic resistivity

Seed magnetic fields

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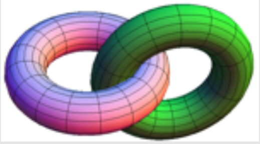
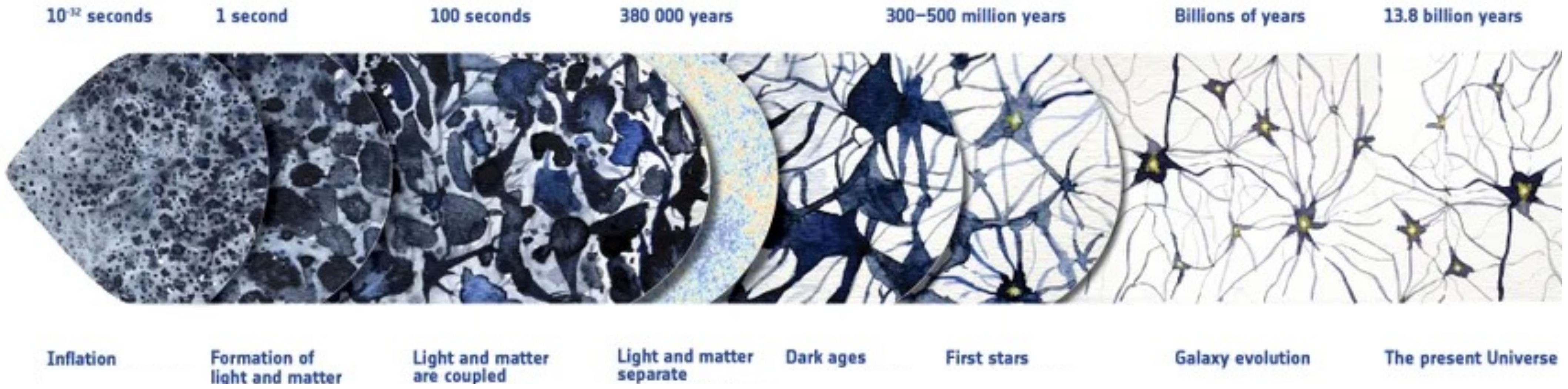
- It implies that magnetic helicity $\boldsymbol{A} \cdot \boldsymbol{B}$ (with the vector potential defined in $\boldsymbol{B} = \nabla \times \boldsymbol{A}$) is conserved in the limit $\eta \to 0$:

$$\frac{\partial (\boldsymbol{A} \cdot \boldsymbol{B})}{\partial t} = 0$$



Seed magnetic fields



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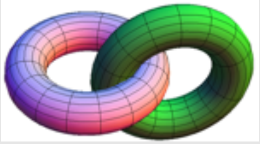
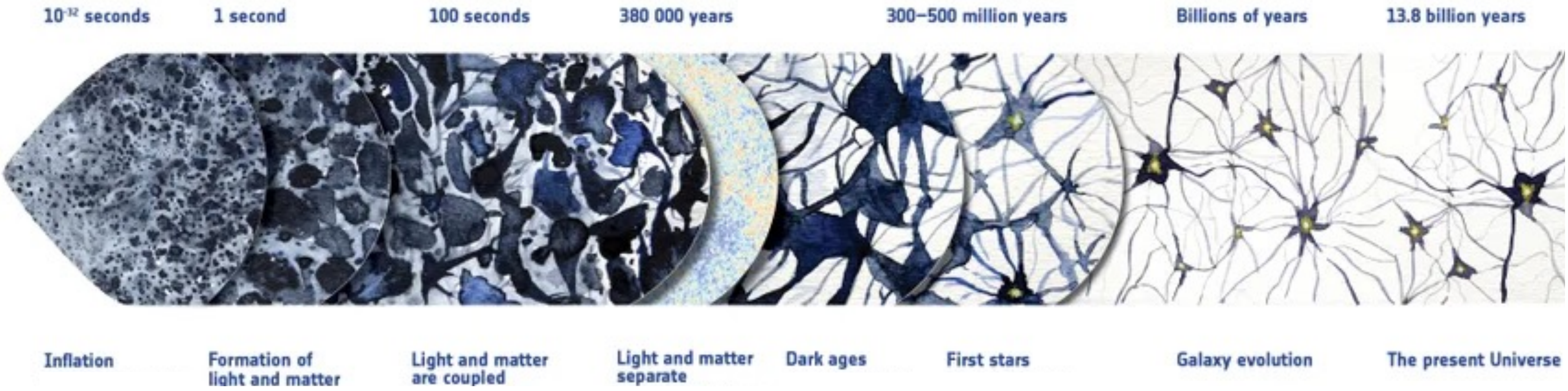


$$\frac{\partial (\boldsymbol{A} \cdot \boldsymbol{B})}{\partial t} = 0$$

- By using the correlation length $\xi_{\rm M}$ to write $\boldsymbol{A} \cdot \boldsymbol{B} \approx \xi_{\rm M} B^2$, we find

$$B \searrow \quad \to \quad \xi_{\rm M} \nearrow$$ ("inverse cascade")

Seed magnetic fields



Magnetic history of the Universe

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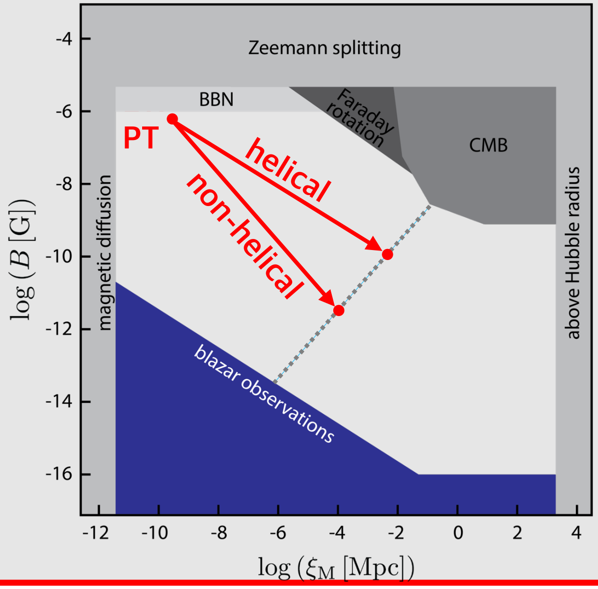

[*Kahniashvili et al. 2013 ,*
Brandenburg et al. 2017,
Hosking & Schekochihin 2021]

Seed magnetic fields

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Magnetic history of the Universe

Seed magnetic fields

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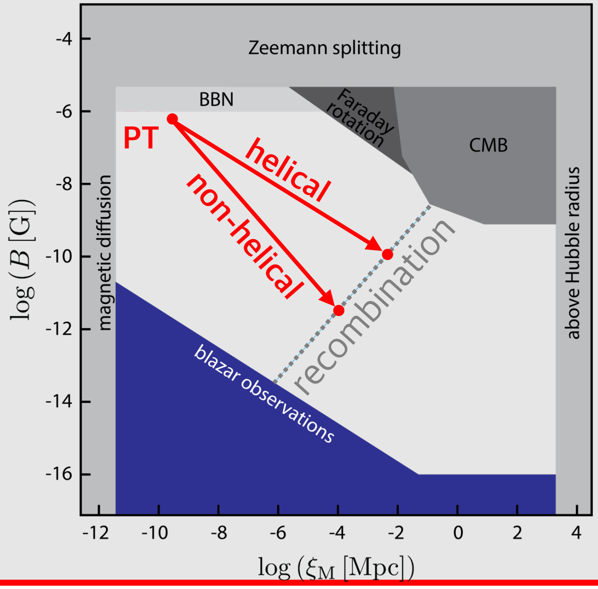
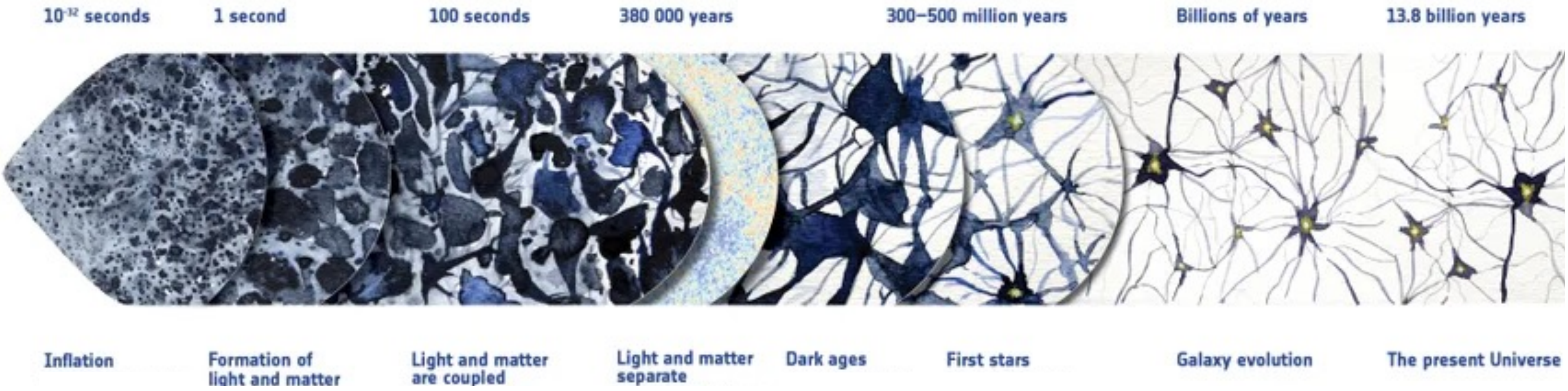


[*Banerjee & Jedamzik 2004*]

[*Kahniashvili et al. 2013*, *Brandenburg et al. 2017*, *Hosking & Schekochihin 2021*]



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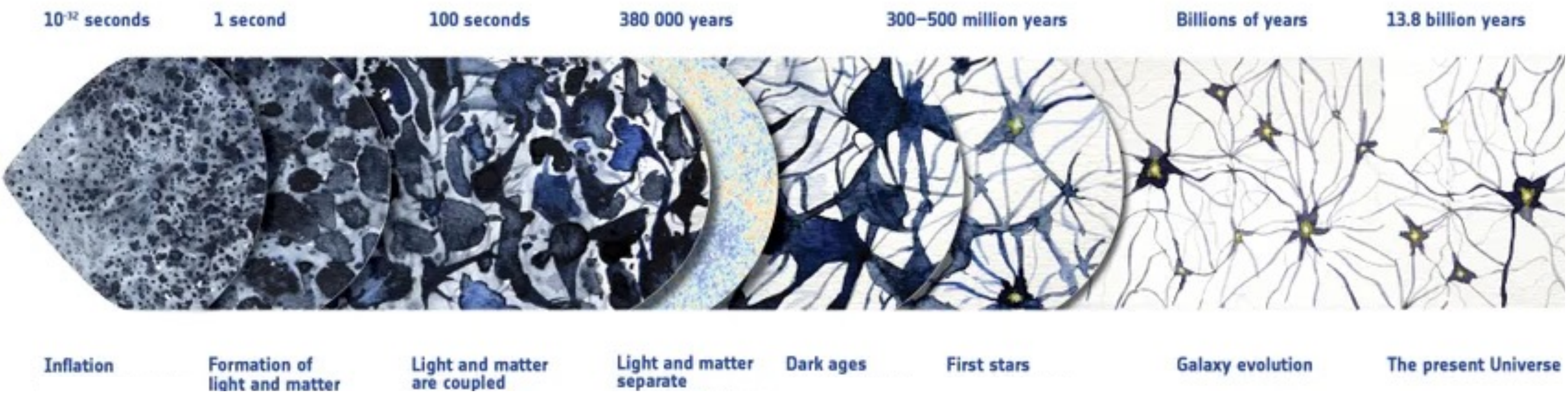
Magnetic history of the Universe

Seed magnetic fields

Decaying MHD turbulence



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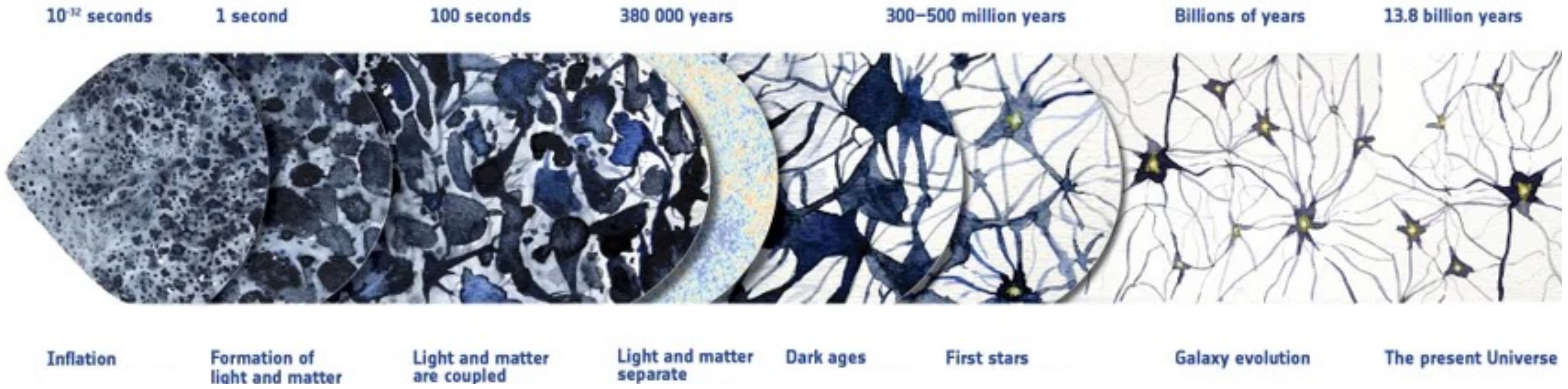
Magnetic history of the Universe

Seed magnetic fields

Decaying MHD turbulence

Passive evolution:

Magnetic field is frozen in Hubble expansion



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Magnetic history of the Universe

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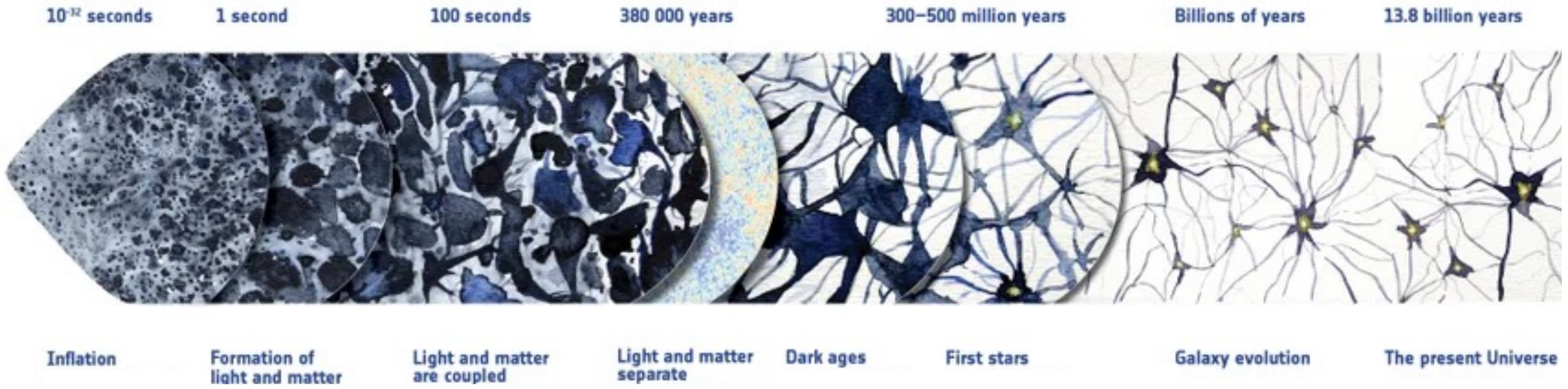

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Magnetic history of the Universe

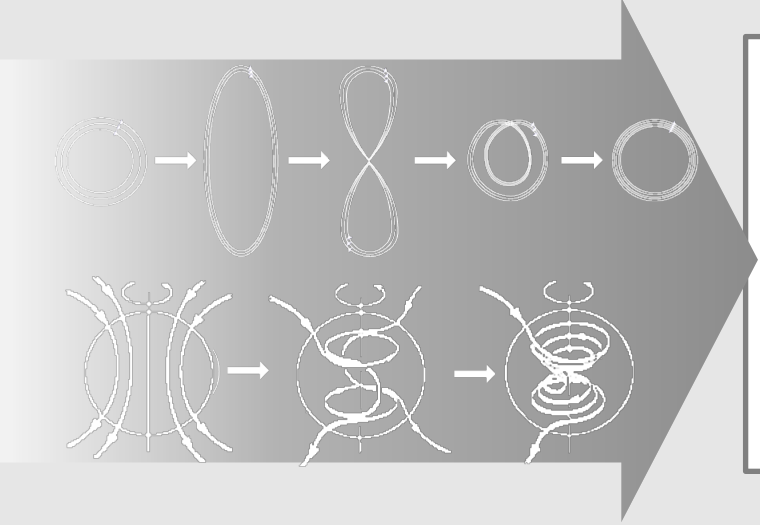
Seed magnetic fields

De

MHD dynamos [*Brandenburg & Subramanian 2005*]:

Kinetic energy:

- Turbulence (accretion, supernovae)
- Large-scale rotation



Magnetic energy

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Magnetic history of the Universe

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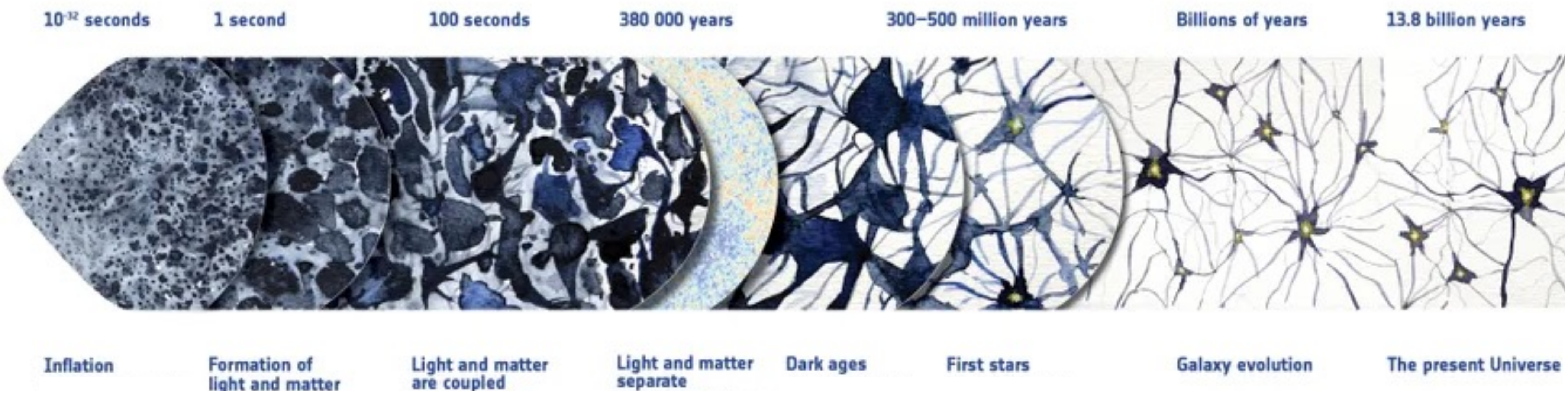

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Magnetic history of the Universe

Reviews:

Durrer & Neronov 2013

Subramanian 2016

Vachaspati 2020

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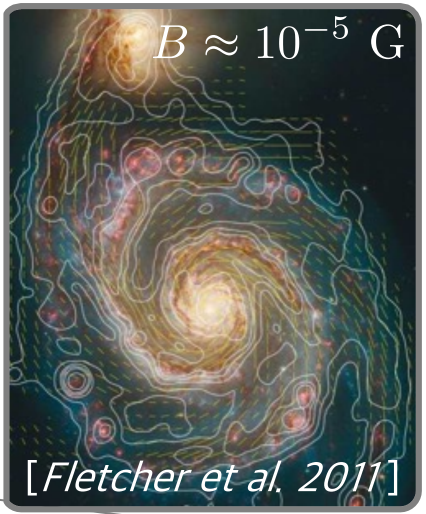
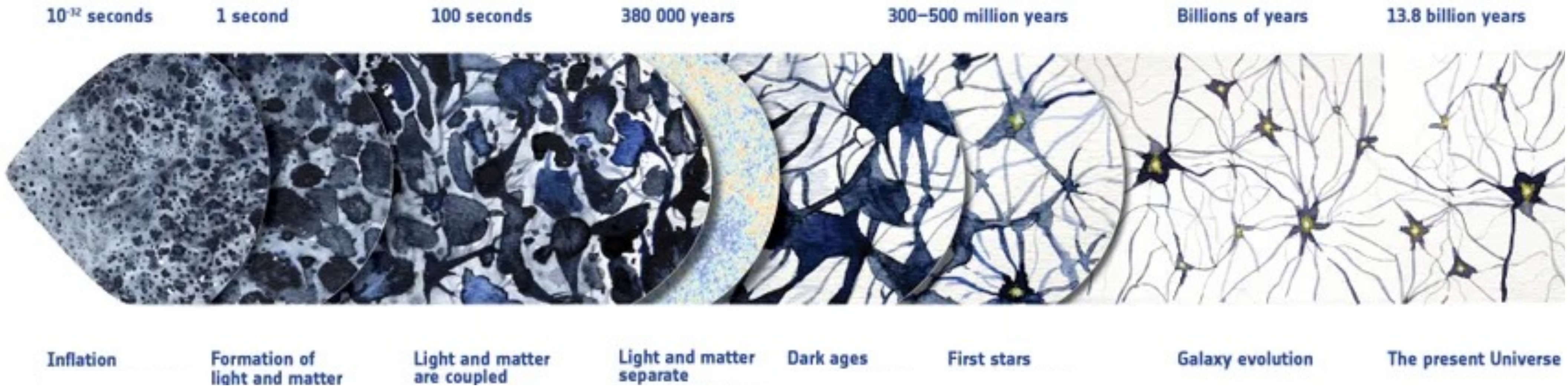


[*Fletcher et al. 2011*]



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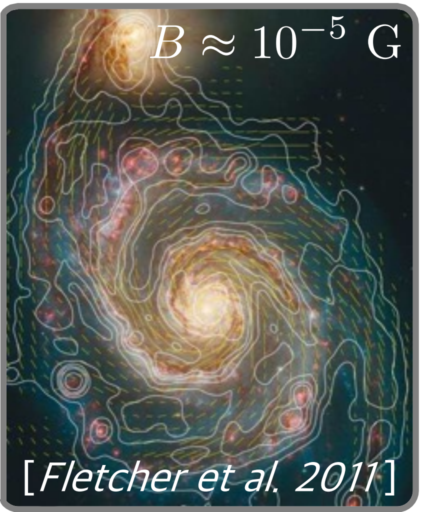
Evolution in non-linear structure formation:



$B \approx 10^{-5}$ G

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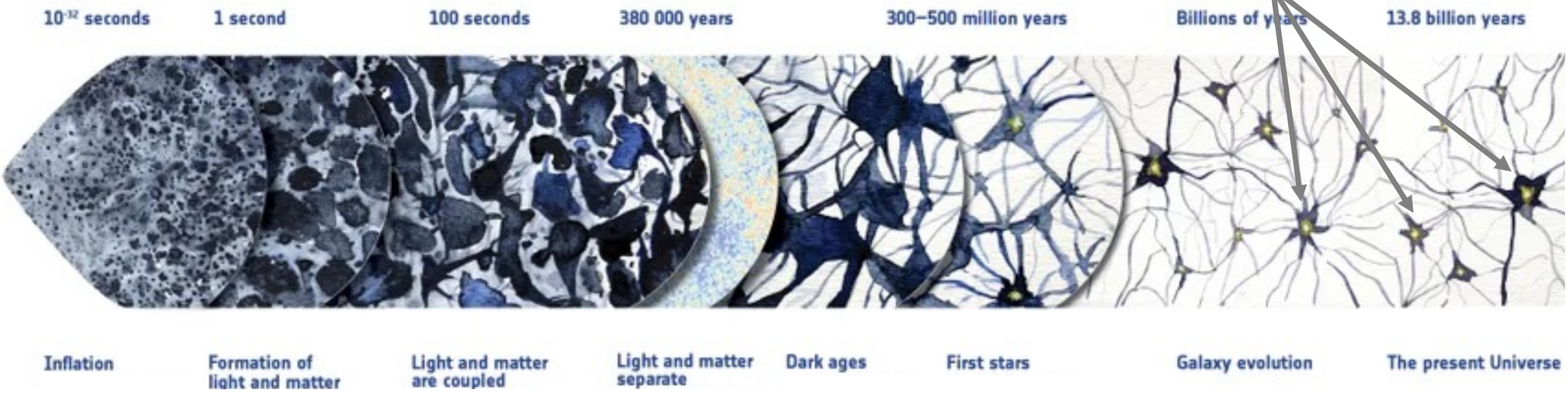

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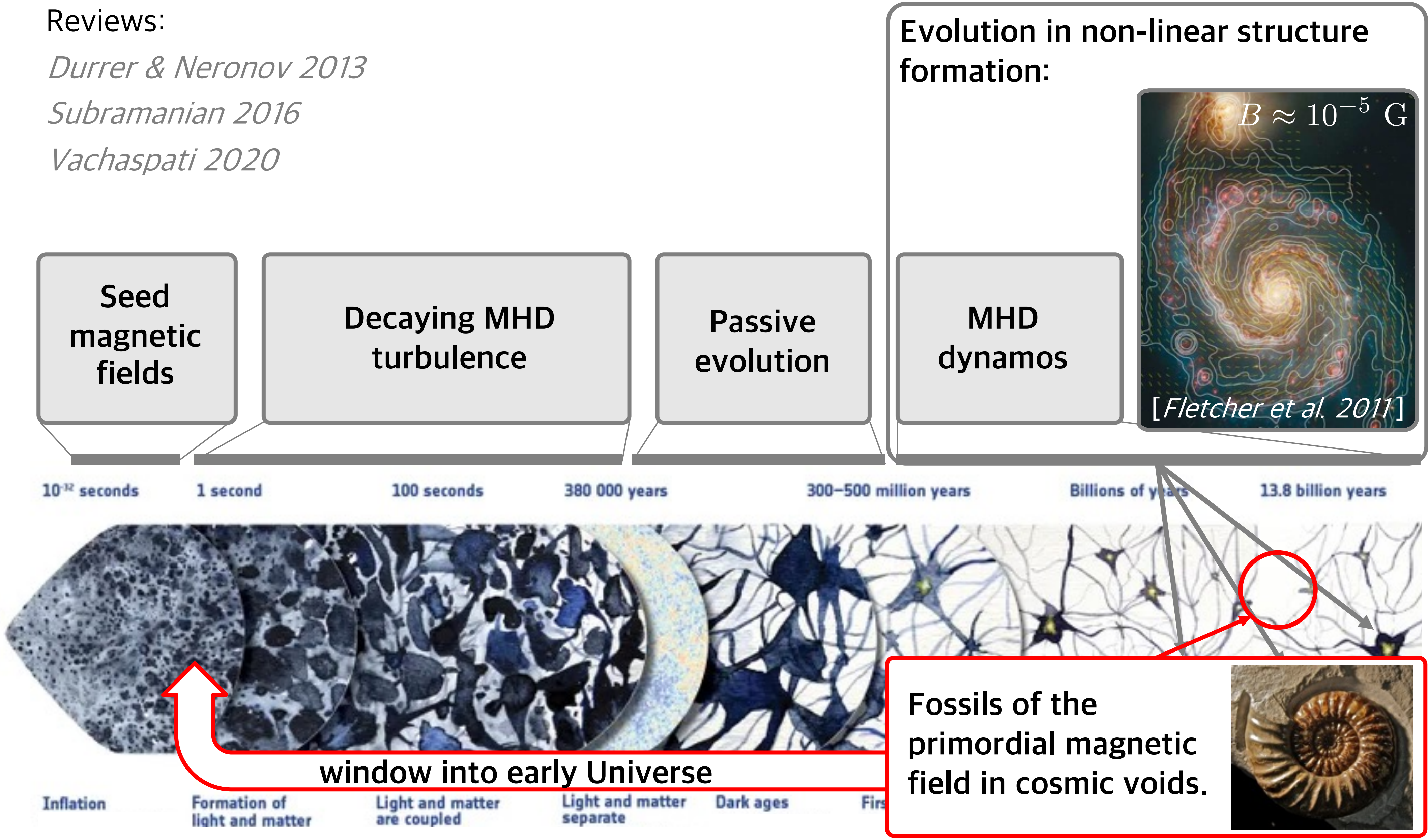

Evolution in non-linear structure formation:

Seed magnetic fields

Decaying MHD turbulence

Passive evolution

MHD dynamos

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window into early Universe

Fossils of the primordial magnetic field in cosmic voids.

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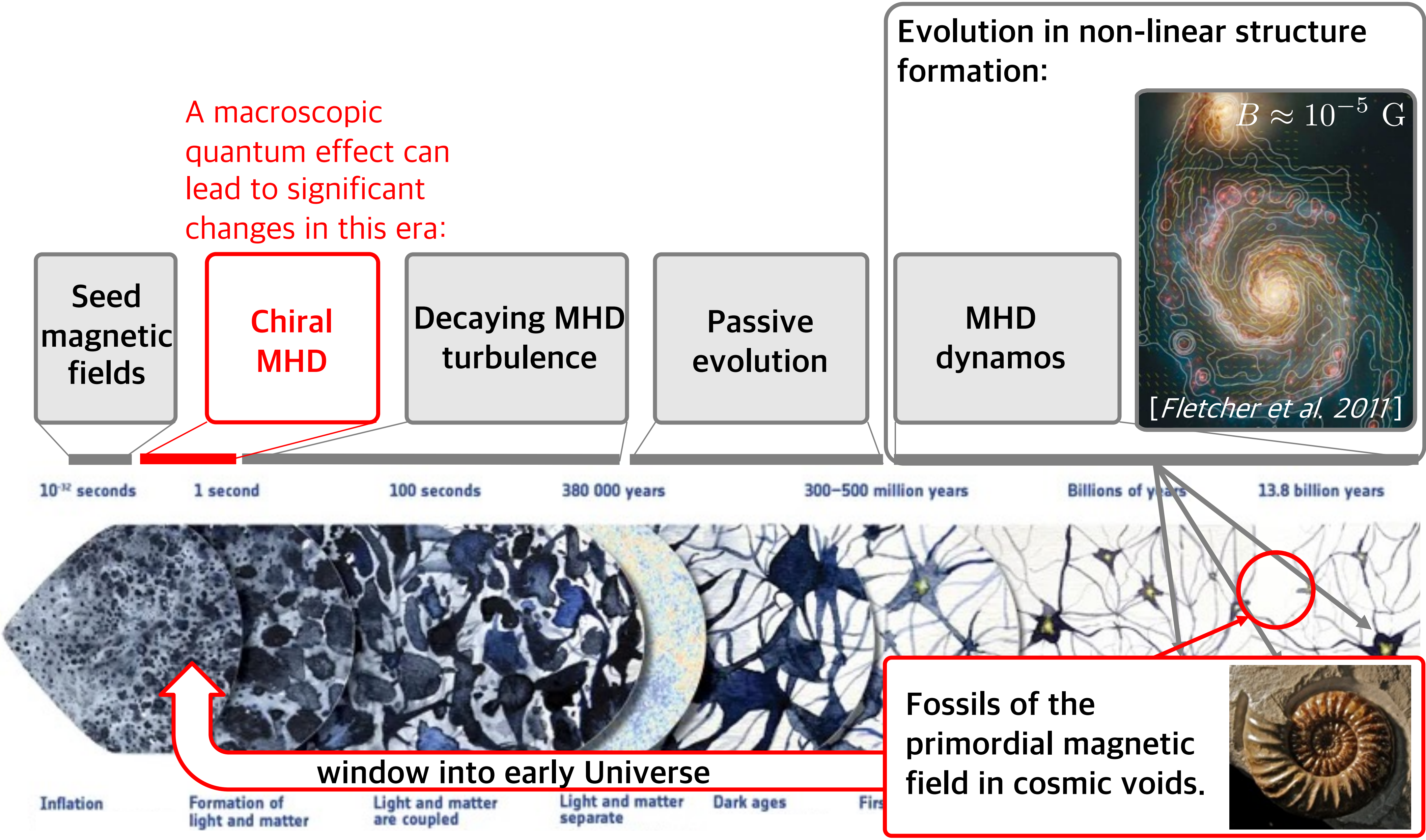
Magnetic history of the Universe



A macroscopic quantum effect can lead to significant changes in this era:

Seed magnetic fields

Chiral MHD

Decaying MHD turbulence

Passive evolution

MHD dynamos

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window into early Universe

EPFL

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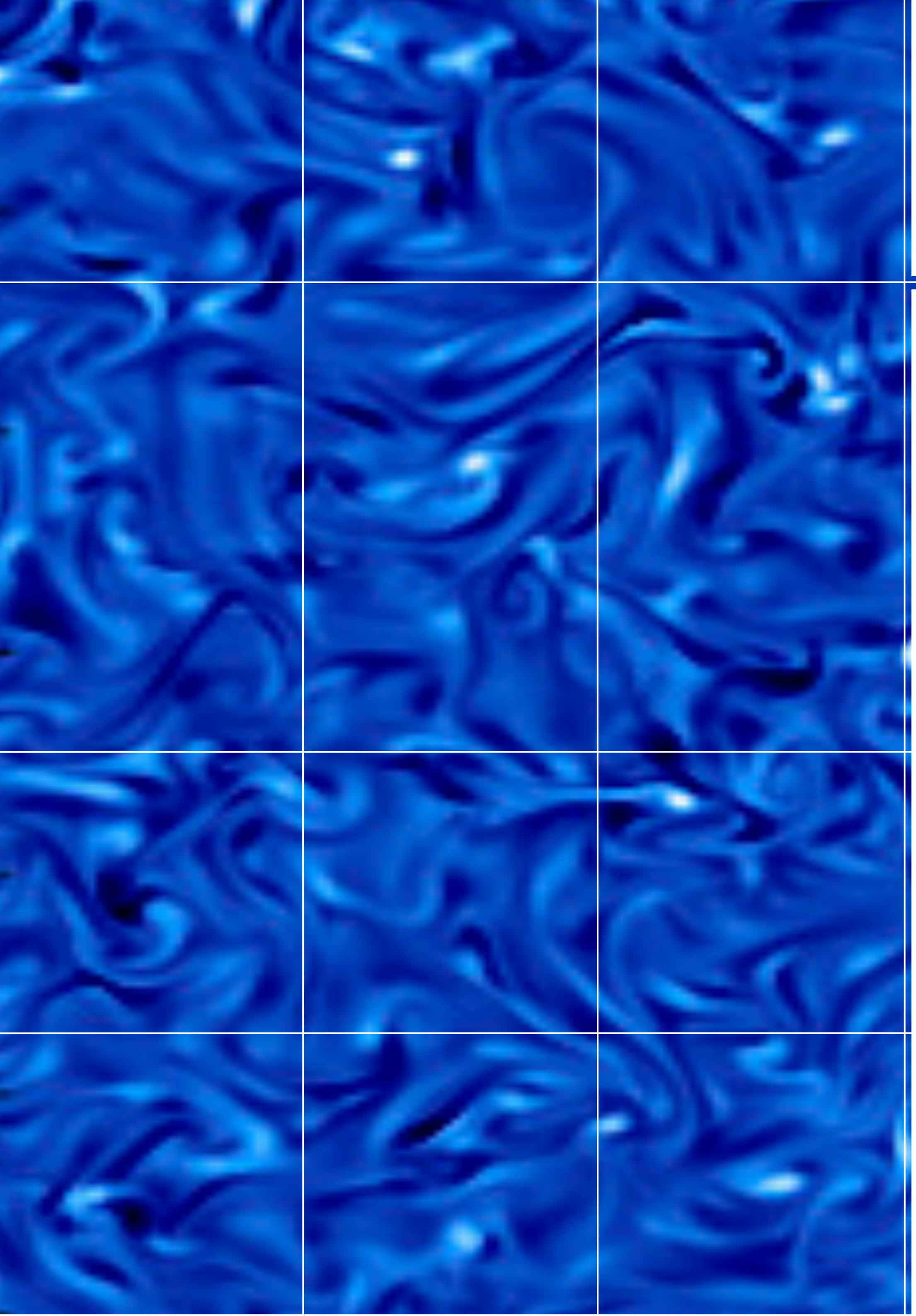


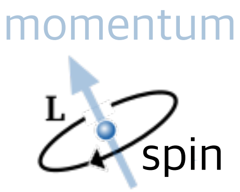
“Simulations of chiral magnetohydrodynamics”

-Outline-

1) Motivation: Observing the Big Bang through cosmic magnetic fields
2) **Chiral magnetohydrodynamics and numerical simulations of its nonlinear dynamics**
3) Insights from simulations
4) Conclusions

New currents at high energies

Chiral Magnetic Effect (CME)



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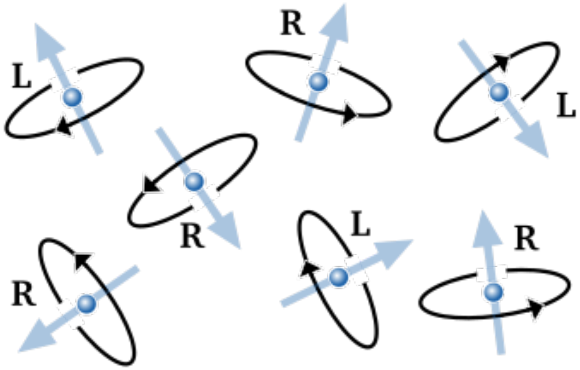

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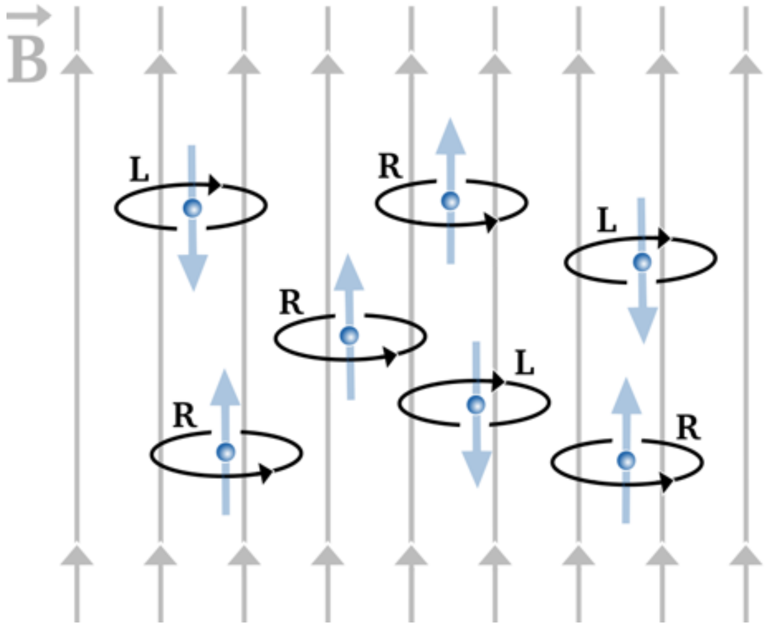

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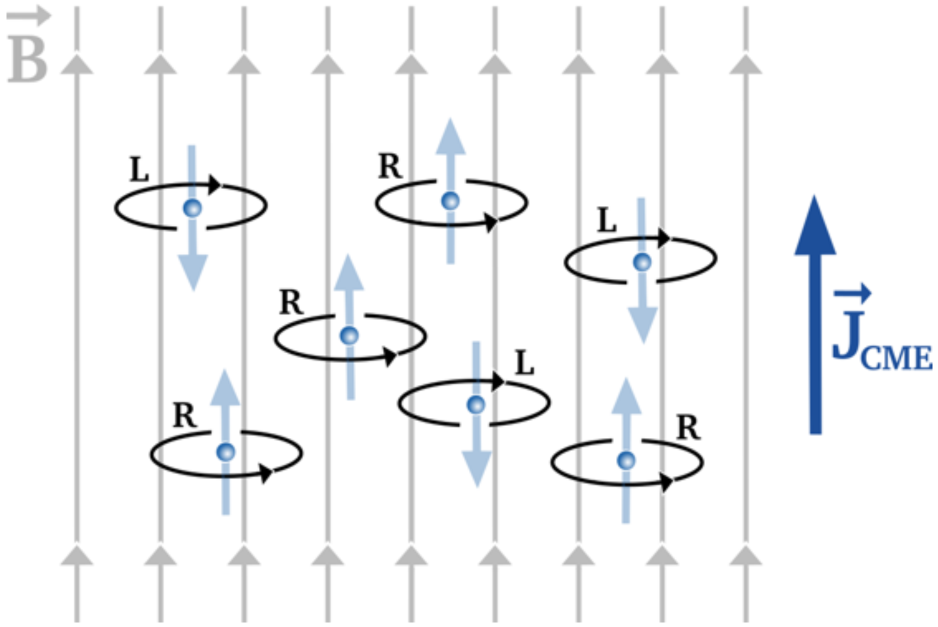

A chiral chemical potential

$$\mu_5 \equiv \mu_L - \mu_R$$

leads to an additional electric current

$$\mathbf{J}_{\rm CME} \propto \mu_5 \mathbf{B}$$

[*Vilenkin 1980, see talk by Naoki Yamamoto*].

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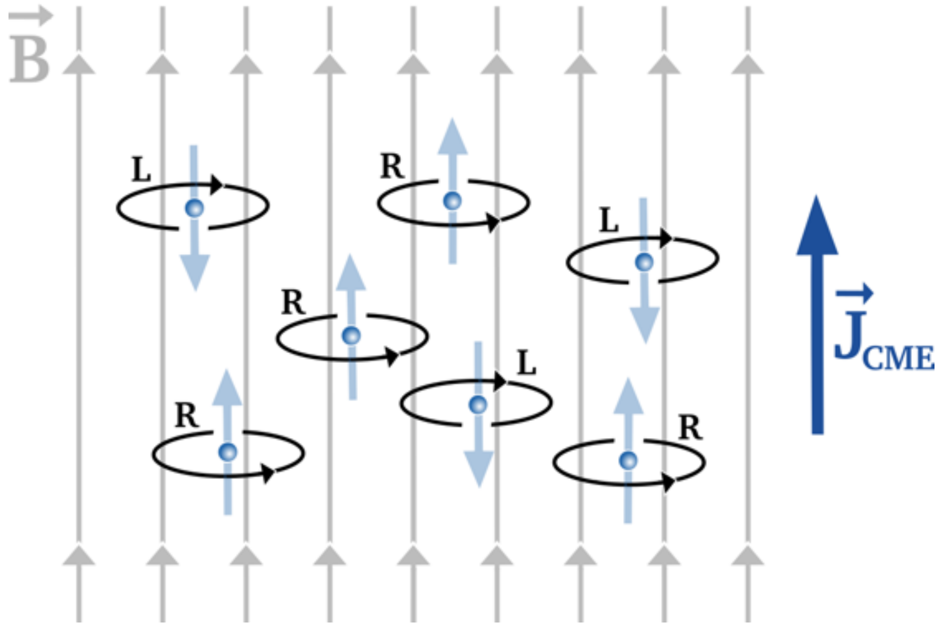

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Where/when is the CME important?

The CME is a Standard Model effect that occurs in a magnetized plasma, if chirality flipping reactions are supressed, i.e. at $T > 10$ MeV.

New currents at high energies

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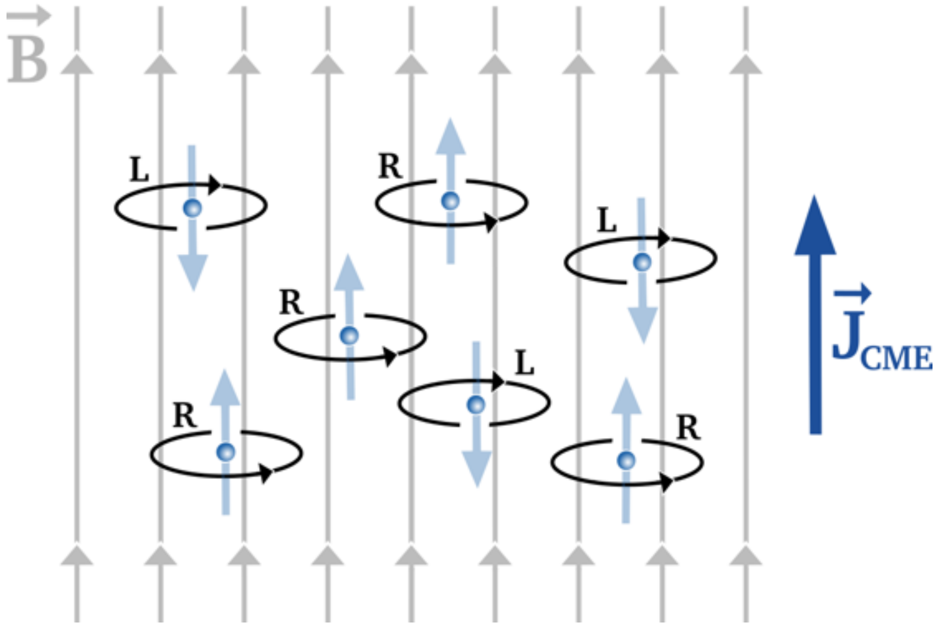

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  [*Joyce & Shaposhnikov 1997; Fröhlich & Pedrini 2000; Semikoz & Sokoloff 2004; Boyarsky et al. 2012; Pavlovic et al 2017*]

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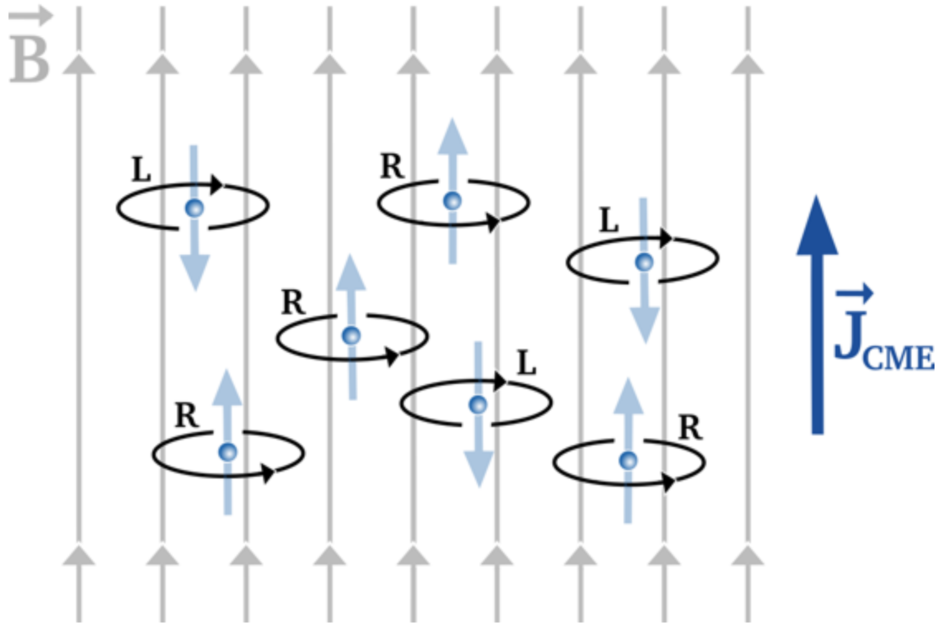

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- **(Proto-)neutron stars**
  [*Dvornikov & Semikoz 2015; Grabowska et al. 2015; Sigl & Leite 2016; Yamamoto 2016*]
- **Heavy-ion collisions**
  [*ALICE collaboration, 2013; Hirono, Hirano, & Kharzeev 2014*]
- **Condensed matter (Weyl semimetals)**
  [*Galitski, Kargarian, & Syzranov 2018*]

Extension of MHD to high energies

Classical MHD

Electric current

$$\boldsymbol{J} = \boldsymbol{J}_{\mathrm{Ohm}}$$

and Maxwell's equations yield the **induction equation**:

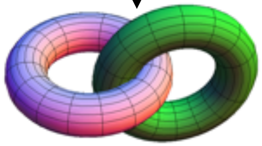
$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times [\boldsymbol{U} \times \boldsymbol{B} - \eta \nabla \times \boldsymbol{B}]$$

Conservation law

(valid for $\eta \rightarrow 0$):

$$\frac{\partial}{\partial t}\langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle = 0$$

$$\downarrow$$



EPFL

Jennifer Schober

Extension of MHD to high energies

Classical MHD

Electric current

$$\boldsymbol{J} = \boldsymbol{J}_{\mathrm{Ohm}}$$

and Maxwell's equations yield the **induction equation**:

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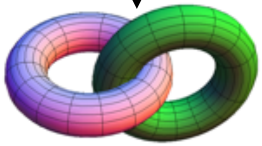

Transition at high temperature ($> 10\ \mathrm{MeV}$) *[Boyarsky, Froehlich, & Ruchayskiy 2012]*

Chiral MHD

Electric current with quantum effects

$$\boldsymbol{J} = \boldsymbol{J}_{\mathrm{Ohm}} + \boldsymbol{J}_{\mathrm{CME}}$$

and Maxwell's equations yield the **chiral induction equation**:

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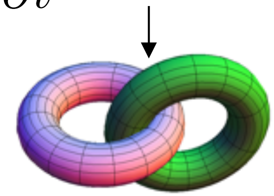

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$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times [\boldsymbol{U} \times \boldsymbol{B} - \eta\ (\nabla \times \boldsymbol{B} \boxed{- \mu_5 \boldsymbol{B})}]$$

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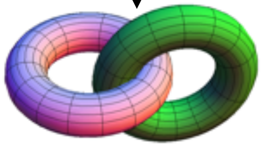

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$$\frac{D\mu_5}{Dt} = \lambda \eta \left[\boldsymbol{B}\boldsymbol{\cdot}(\boldsymbol{\nabla} \times \boldsymbol{B}) - \mu_5 \boldsymbol{B}^2\right]$$

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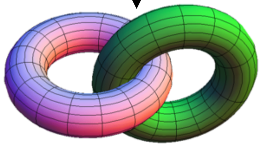

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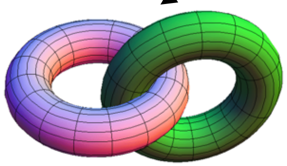
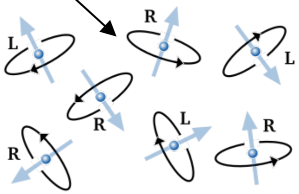
Conservation law

(valid for any η):

$$\frac{\partial}{\partial t} \left(\langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle + \frac{2 \langle \mu_5 \rangle}{\lambda} \right) = 0$$





Jennifer Schober

Chiral MHD

Full set of equations

$$\begin{aligned}
\frac{\partial \boldsymbol{B}}{\partial t} &= \boldsymbol{\nabla} \times \left[\boldsymbol{U} \times \boldsymbol{B} - \eta \left(\boldsymbol{\nabla} \times \boldsymbol{B} - \mu_5 \boldsymbol{B}\right)\right] \\
\rho \frac{D\boldsymbol{U}}{Dt} &= (\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B} - \boldsymbol{\nabla} p + \boldsymbol{\nabla} \cdot (2\nu\rho\mathbf{S}) \\
\frac{D\rho}{Dt} &= -\rho \boldsymbol{\nabla} \cdot \boldsymbol{U} \\
\frac{D\mu_5}{Dt} &= \mathcal{D}_5 \Delta\mu_5 + \lambda \eta \left[\boldsymbol{B} \cdot (\boldsymbol{\nabla} \times \boldsymbol{B}) - \mu_5 \boldsymbol{B}^2\right]
\end{aligned}$$

Exponential ansatz:

$B(t) \propto \exp(\gamma t)$

$$\boxed{\gamma = \eta\mu_5 k - \eta k^2}$$

[k: wavenumber]

Jennifer Schober

Chiral MHD

Full set of equations

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla}\times[\boldsymbol{U}\times\boldsymbol{B} - \eta\,(\boldsymbol{\nabla}\times\boldsymbol{B} - \mu_5\boldsymbol{B})]$$

$$\rho\frac{D\boldsymbol{U}}{Dt} = (\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B} - \boldsymbol{\nabla}p + \boldsymbol{\nabla}\cdot(2\nu\rho\mathbf{S})$$

$$\frac{D\rho}{Dt} = -\rho\,\boldsymbol{\nabla}\cdot\boldsymbol{U}$$

$$\frac{D\mu_5}{Dt} = \mathcal{D}_5\,\Delta\mu_5 + \lambda\,\eta\,[\boldsymbol{B}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B}) - \mu_5\boldsymbol{B}^2]$$

Exponential ansatz:

$B(t) \propto \exp(\gamma t)$

$$\boxed{\gamma = \eta\mu_5 k - \eta k^2}$$

[k : wavenumber]

→ The maximum growth rate is given by

$$\gamma_5 = \frac{\eta\mu_5^2}{4}$$

and attained on the scale:

$$k_5 = \frac{\mu_5}{2}$$

[*Joyce & Shaposhnikov 1997* , see also talk by Tomoya Takiwaki]

Chiral MHD

Full set of equations

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times \left[\boldsymbol{U} \times \boldsymbol{B} - \eta \left(\boldsymbol{\nabla} \times \boldsymbol{B} - \mu_5 \boldsymbol{B}\right)\right]$$

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$$\frac{D\mu_5}{Dt} = \mathcal{D}_5 \, \Delta\mu_5 + \lambda \eta \left[\boldsymbol{B} \cdot (\boldsymbol{\nabla} \times \boldsymbol{B}) - \mu_5 \boldsymbol{B}^2\right]$$

Exponential ansatz:

$B(t) \propto \exp(\gamma t)$

$$\boxed{\gamma = \eta \mu_5 k - \eta k^2}$$

Non-linearity determined by

$\lambda = 3\hbar c \frac{(8\alpha_{\mathrm{em}})^2}{(k_B T)^2}$

Saturation occurs when μ_5 vanishes according to the conservation law:

$$\boxed{\frac{\partial}{\partial t}\left(\langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle + \frac{2\langle \mu_5 \rangle}{\lambda}\right) = 0}$$

Chiral MHD

Full set of equations

$$
\begin{aligned}
\frac{\partial \boldsymbol{B}}{\partial t} &= \boldsymbol{\nabla} \times \left[\boldsymbol{U} \times \boldsymbol{B} - \eta \left(\boldsymbol{\nabla} \times \boldsymbol{B} - \mu_5 \boldsymbol{B}\right)\right] \\
\rho \frac{D\boldsymbol{U}}{Dt} &= (\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B} - \boldsymbol{\nabla} p + \boldsymbol{\nabla}\boldsymbol{\cdot}(2\nu\rho\mathbf{S}) \\
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\end{aligned}
$$

[*Rogachevskii et al. 2017*]

Chiral MHD

Full set of equations

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times [\boldsymbol{U} \times \boldsymbol{B} - \eta\,(\boldsymbol{\nabla} \times \boldsymbol{B} - \mu_5 \boldsymbol{B})]$$

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[*Rogachevskii et al. 2017*]

PENCIL CODE

[*pencil-code.nordita.org*]

Direct numerical simulations

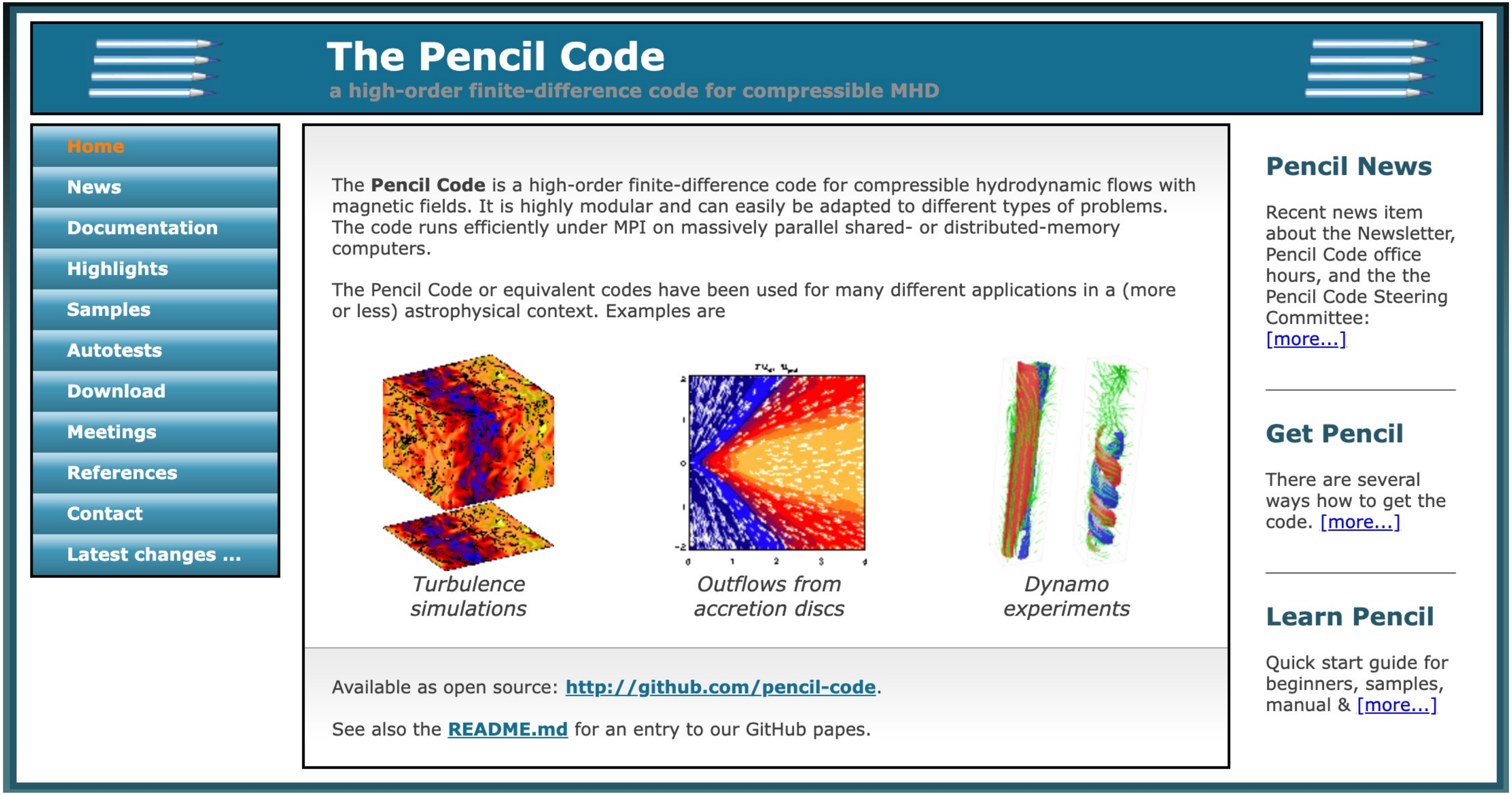
Code properties:

- Grid based
- 6th-order explicit finite difference method in space
    - 3rd-order accurate time-stepping method

Setup for chiral MHD:

- 3D box with periodic boundary conditions
- Resolution up to 1024^3 grid cells
- Parallelization up to 1024 cores
- Explicit viscosity and magnetic resistivity

The Pencil Code [*pencil-code.nordita.org*]



The Pencil Code

a high-order finite-difference code for compressible MHD

The **Pencil Code** is a high-order finite-difference code for compressible hydrodynamic flows with magnetic fields. It is highly modular and can easily be adapted to different types of problems. The code runs efficiently under MPI on massively parallel shared- or distributed-memory computers.

The Pencil Code or equivalent codes have been used for many different applications in a (more or less) astrophysical context. Examples are

Turbulence simulations

Outflows from accretion discs

Dynamo experiments

Available as open source: **http://github.com/pencil-code**.

See also the **README.md** for an entry to our GitHub papes.

Pencil News

Recent news item about the Newsletter, Pencil Code office hours, and the the Pencil Code Steering Committee: [more...]

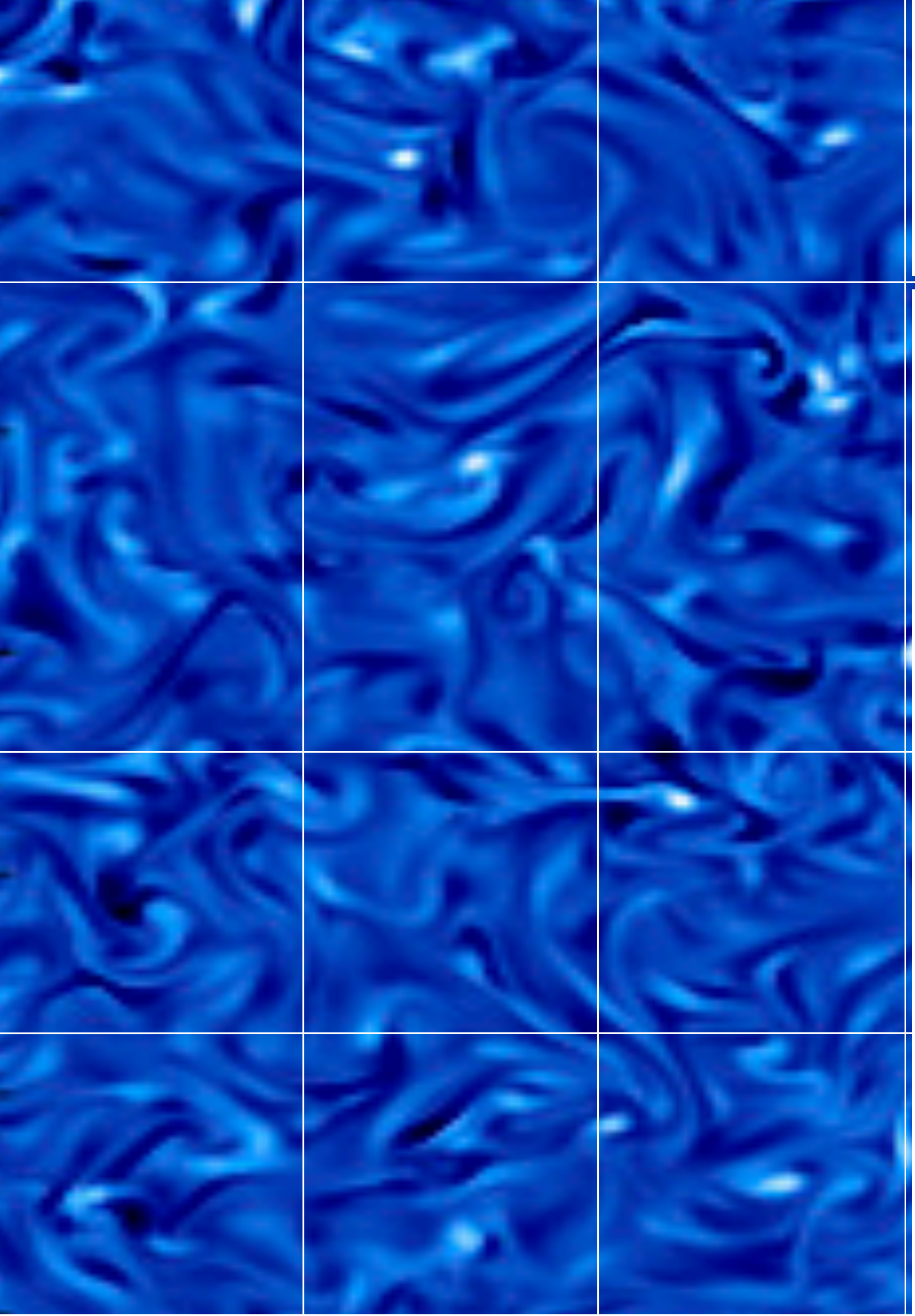
Get Pencil

There are several ways how to get the code. [more...]

Learn Pencil

Quick start guide for beginners, samples, manual & [more...]

Chiral MHD sample at: pencil-code/samples/2d-tests/chiral_dynamo



"Simulations of chiral magnetohydrodynamics"

-Outline-

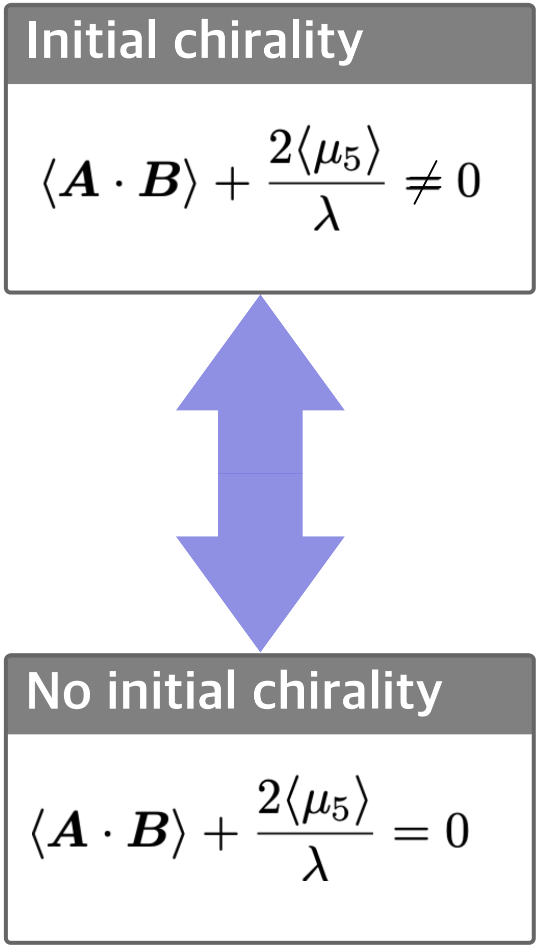
1) Motivation: Observing the Big Bang through cosmic magnetic fields

2) Chiral magnetohydrodynamics and numerical simulations of its nonlinear dynamics

3) **Insights from simulations**

4) Conclusions

Chiral MHD scenarios



Initial chirality

$$\langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle + \frac{2\langle \mu_5 \rangle}{\lambda} \neq 0$$

No initial chirality

$$\langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle + \frac{2\langle \mu_5 \rangle}{\lambda} = 0$$

Chiral MHD scenarios

Initial chirality

$$\langle \boldsymbol{A}\cdot\boldsymbol{B}\rangle + \frac{2\langle\mu_5\rangle}{\lambda} \neq 0$$

$$\underset{\approx 0}{\langle \cancel{\boldsymbol{A}\cdot\boldsymbol{B}}\rangle} + \underset{\gg 0}{\frac{2\langle\mu_5\rangle}{\lambda}} = \frac{2\langle\mu_5\rangle}{\lambda}$$

"Classical" chiral dynamo

Rogachevskii et al. 2017 &

Schober et al. 2018

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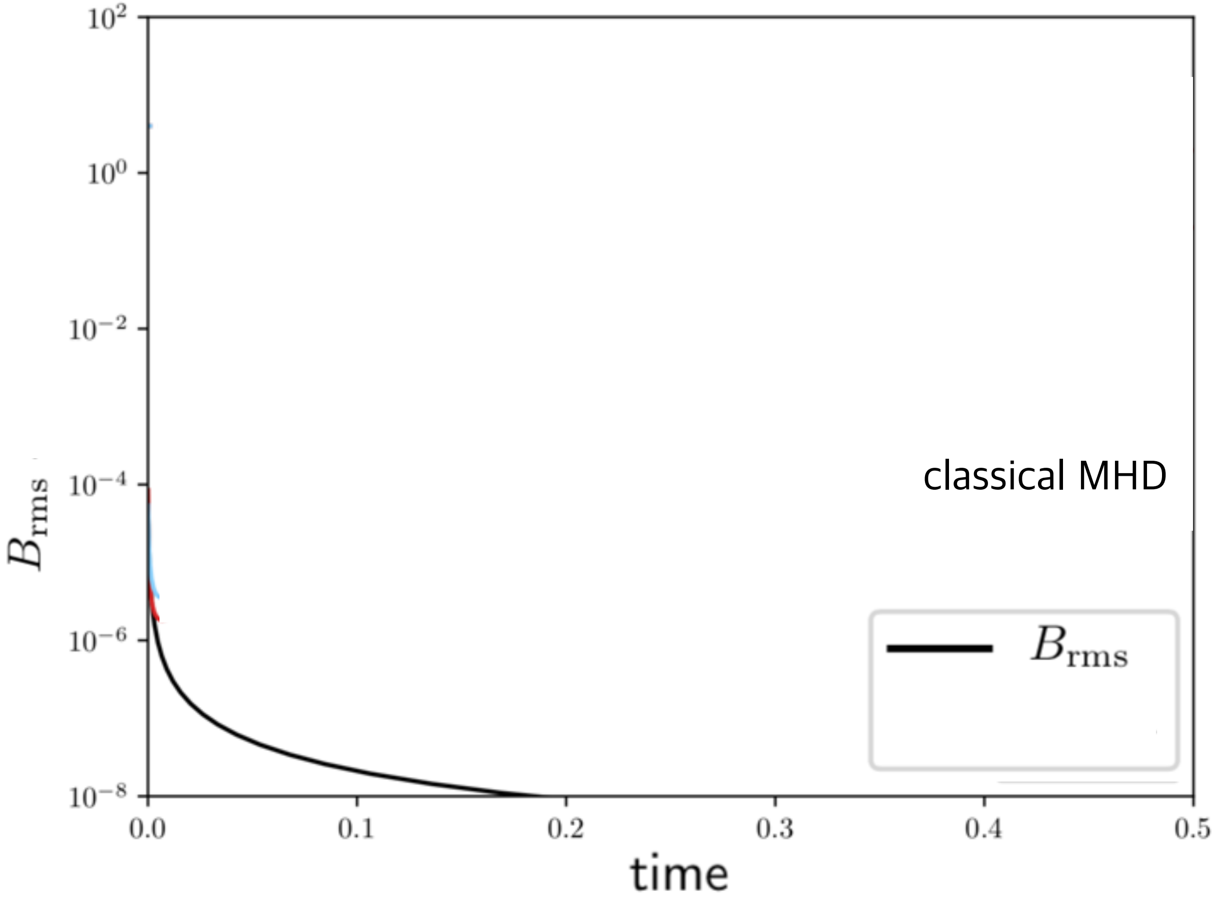
Chiral MHD dynamos

Initial condition

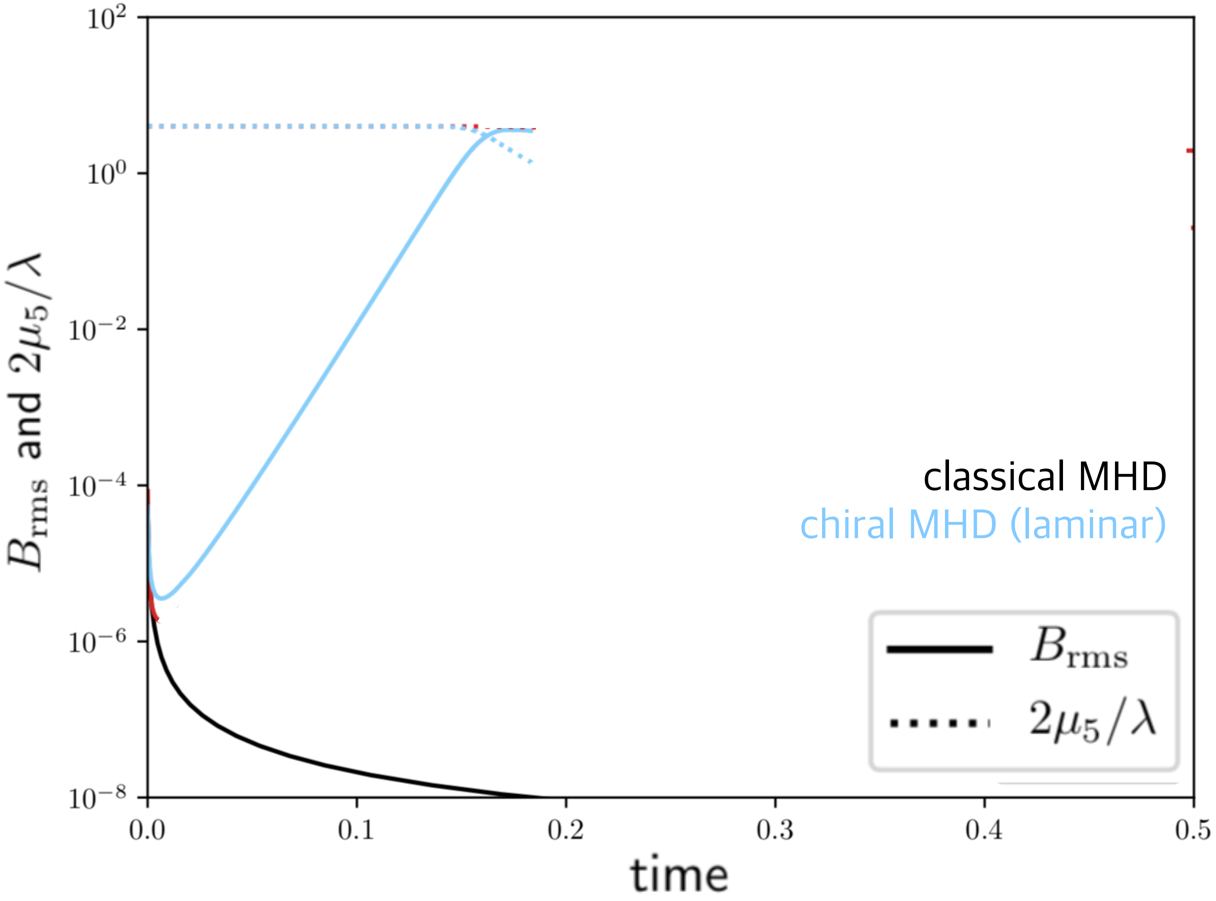
- week magnetic seed field B



Chiral MHD dynamos

Initial condition

- week magnetic seed field B
- uniform chiral asymmetry μ_5



Chiral MHD dynamos

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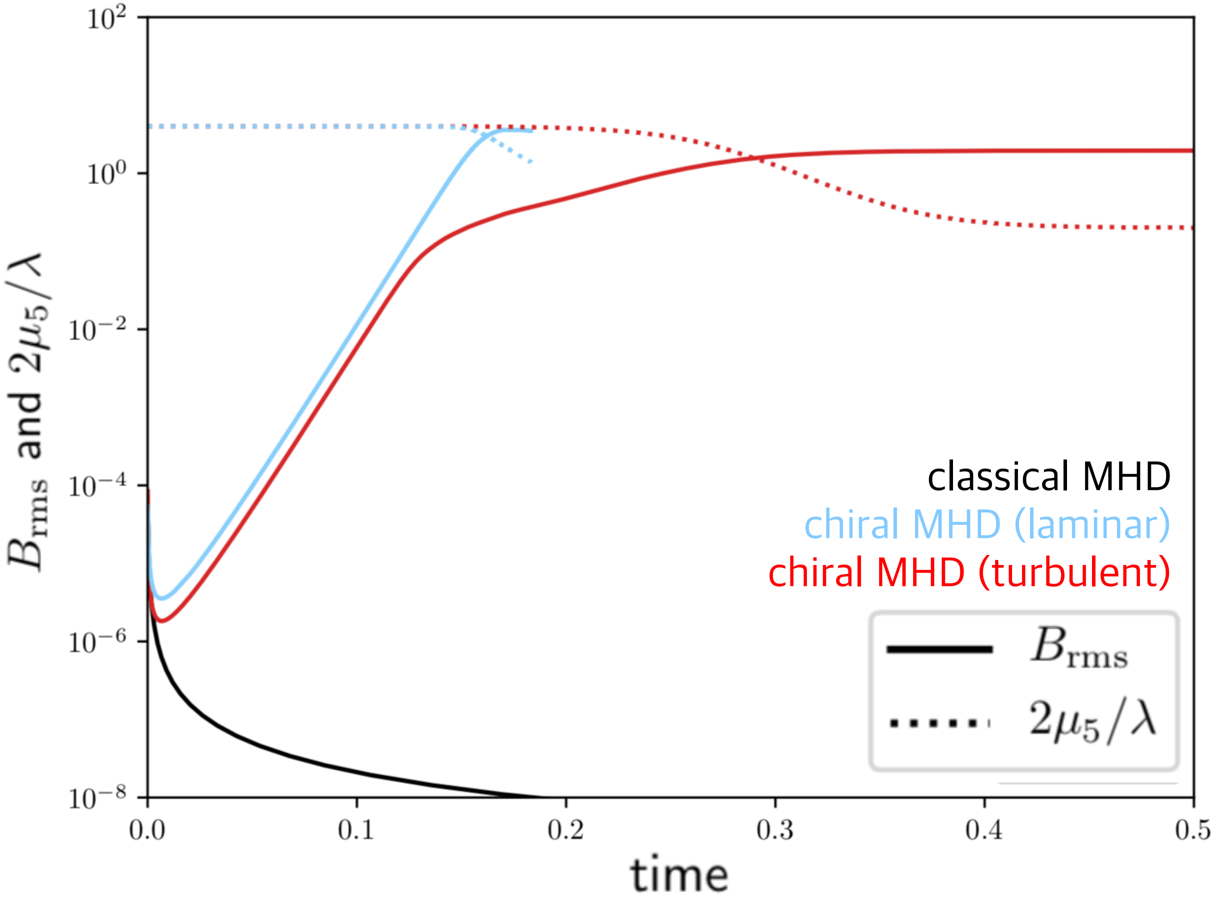

EPFL

Jennifer Schober

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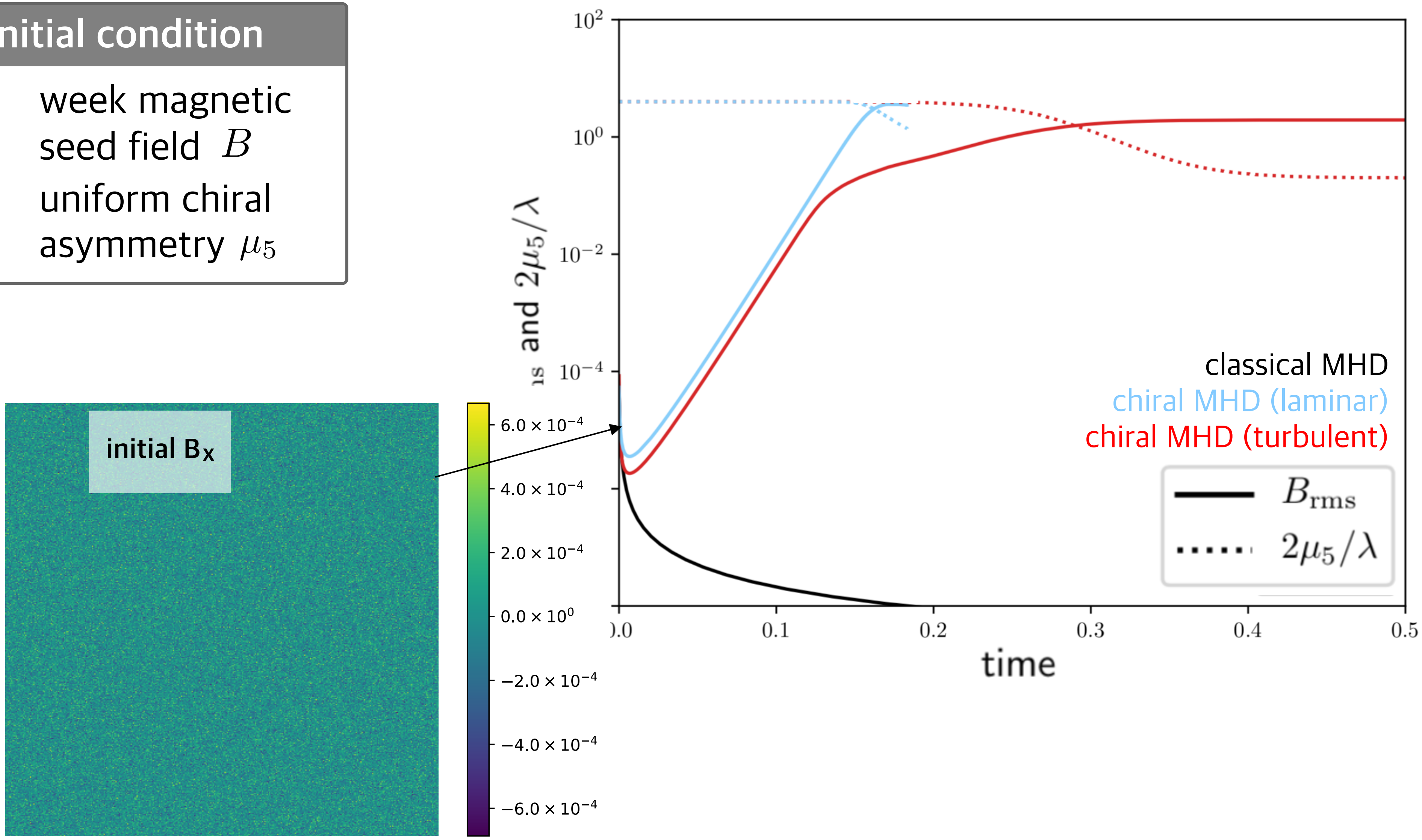

Jennifer Schober

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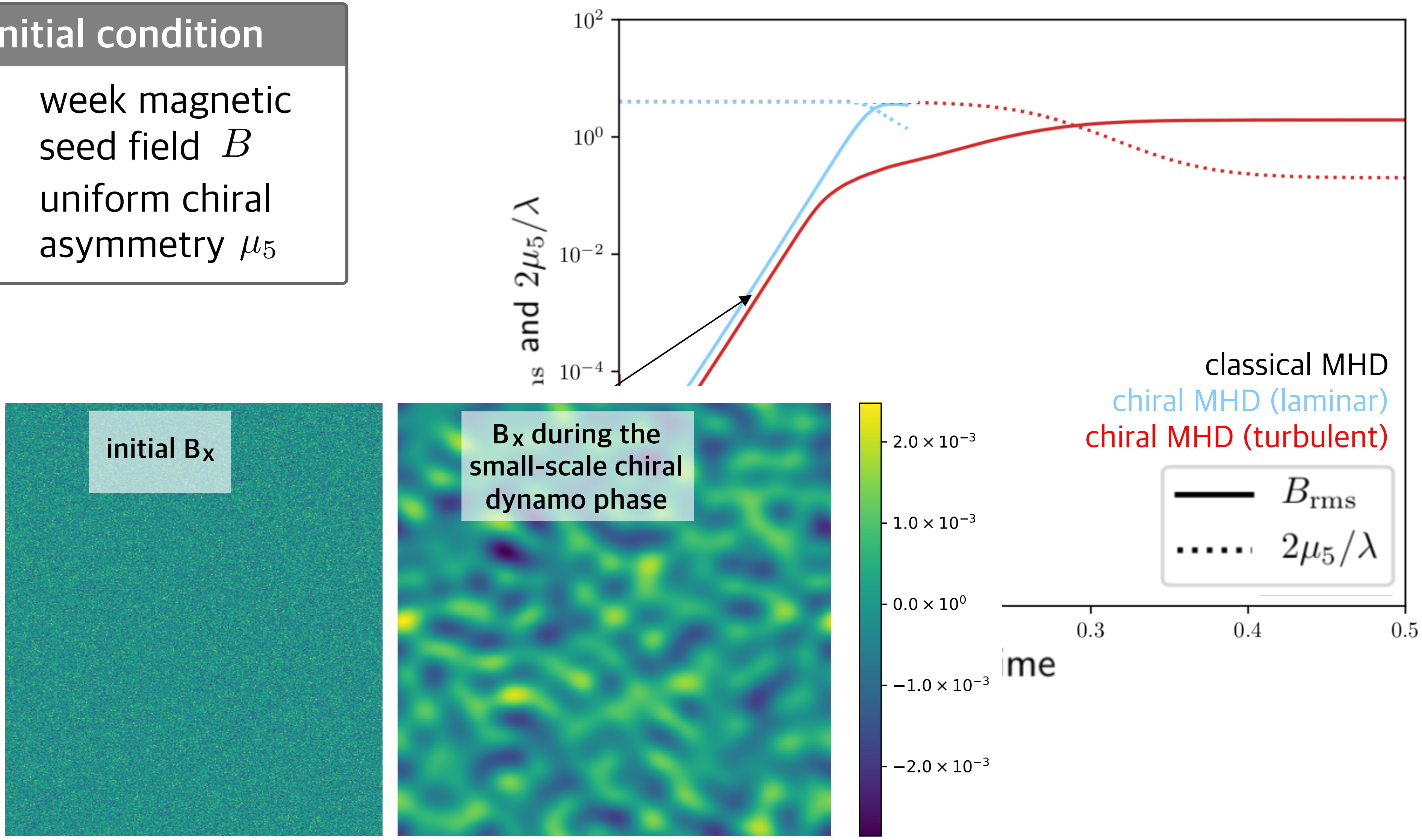

$$\gamma_5 = \frac{\eta\mu_5^2}{4}$$

$$k_5 = \frac{\mu_5}{2}$$

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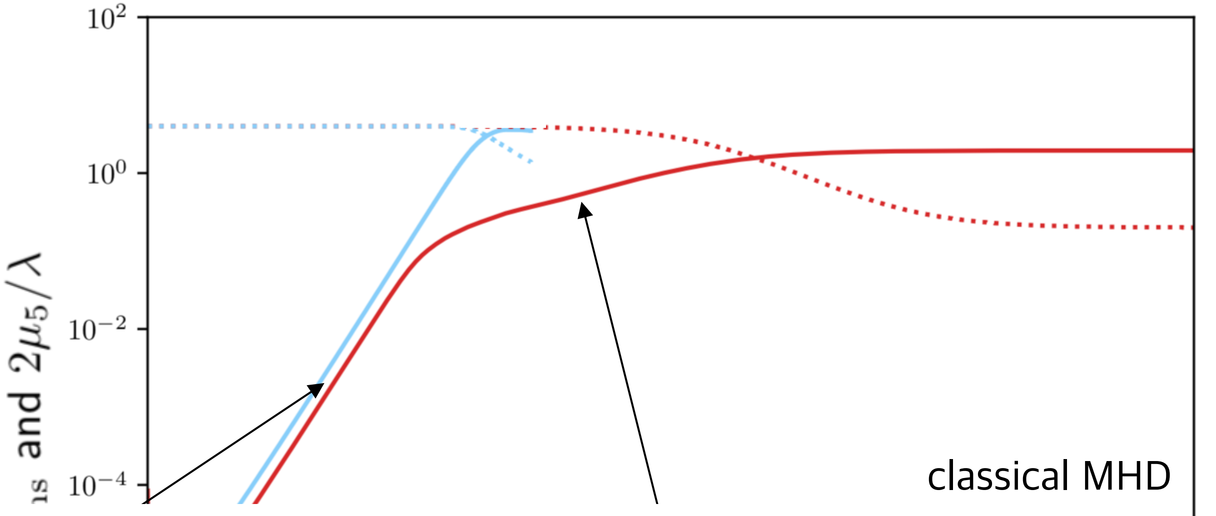

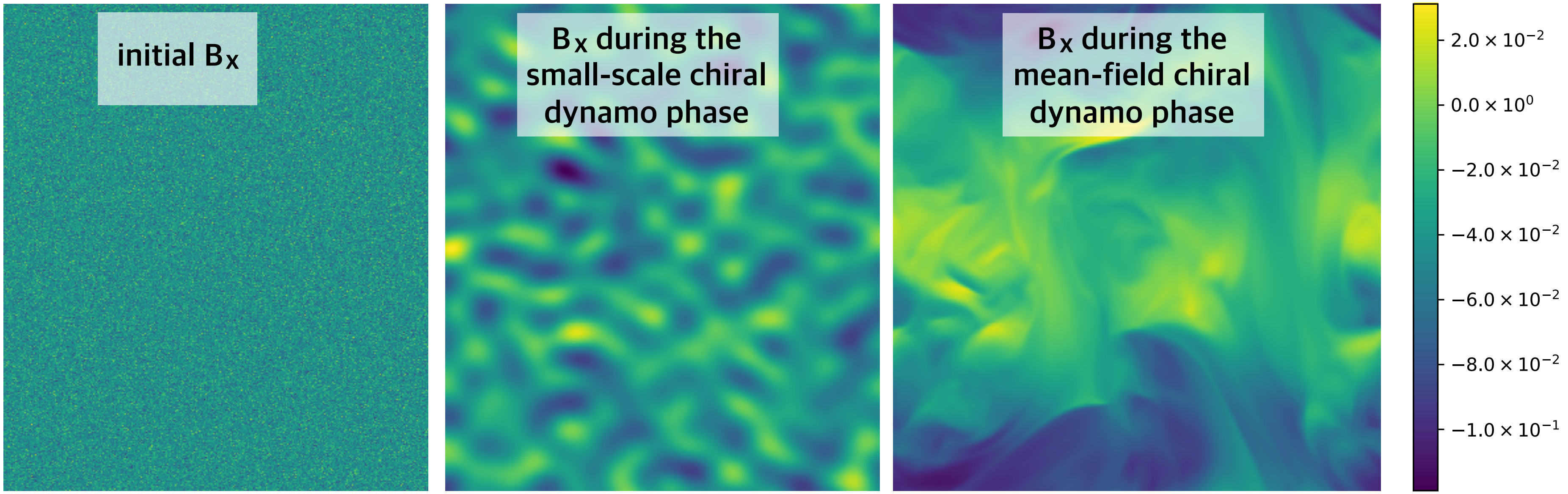

initial B_x

B_x during the small-scale chiral dynamo phase

B_x during the mean-field chiral dynamo phase

$$\gamma_5 = \frac{\eta \mu_5^2}{4}$$

$$k_5 = \frac{\mu_5}{2}$$

Jennifer Schober

Mean field theory

Mean-field theory developed by *Rogachevskii et al. (2017)*:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \left[\boldsymbol{U} \times \boldsymbol{B} - \eta \left(\nabla \times \boldsymbol{B} - \mu_5 \boldsymbol{B}\right)\right]$$

Separation into mean and fluctuations:

$$\mu_5 = \overline{\mu_5} + \delta\mu_5$$

$$\boldsymbol{B} = \overline{\boldsymbol{B}} + \delta\boldsymbol{B}$$

$$\boldsymbol{U} = \overline{\boldsymbol{U}} + \delta\boldsymbol{U}$$

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$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \nabla \times \left[\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + (\eta\overline{\mu_5} + \alpha_\mu)\overline{\boldsymbol{B}} - (\eta + \eta_{\mathrm{T}})\nabla \times \overline{\boldsymbol{B}}\right]$$

Mean field theory

Mean-field theory developed by *Rogachevskii et al. (2017)*:

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Ansatz:

$$\overline{\boldsymbol{B}}(\boldsymbol{x}, t) \propto \exp(\gamma t + i\boldsymbol{k}\cdot\boldsymbol{x})$$

$$\gamma = (\eta\bar{\mu}_5 + \alpha_\mu)k - (\eta + \eta_{\mathrm{T}})k^2$$

Mean field theory

Mean-field theory developed by *Rogachevskii et al. (2017)*:

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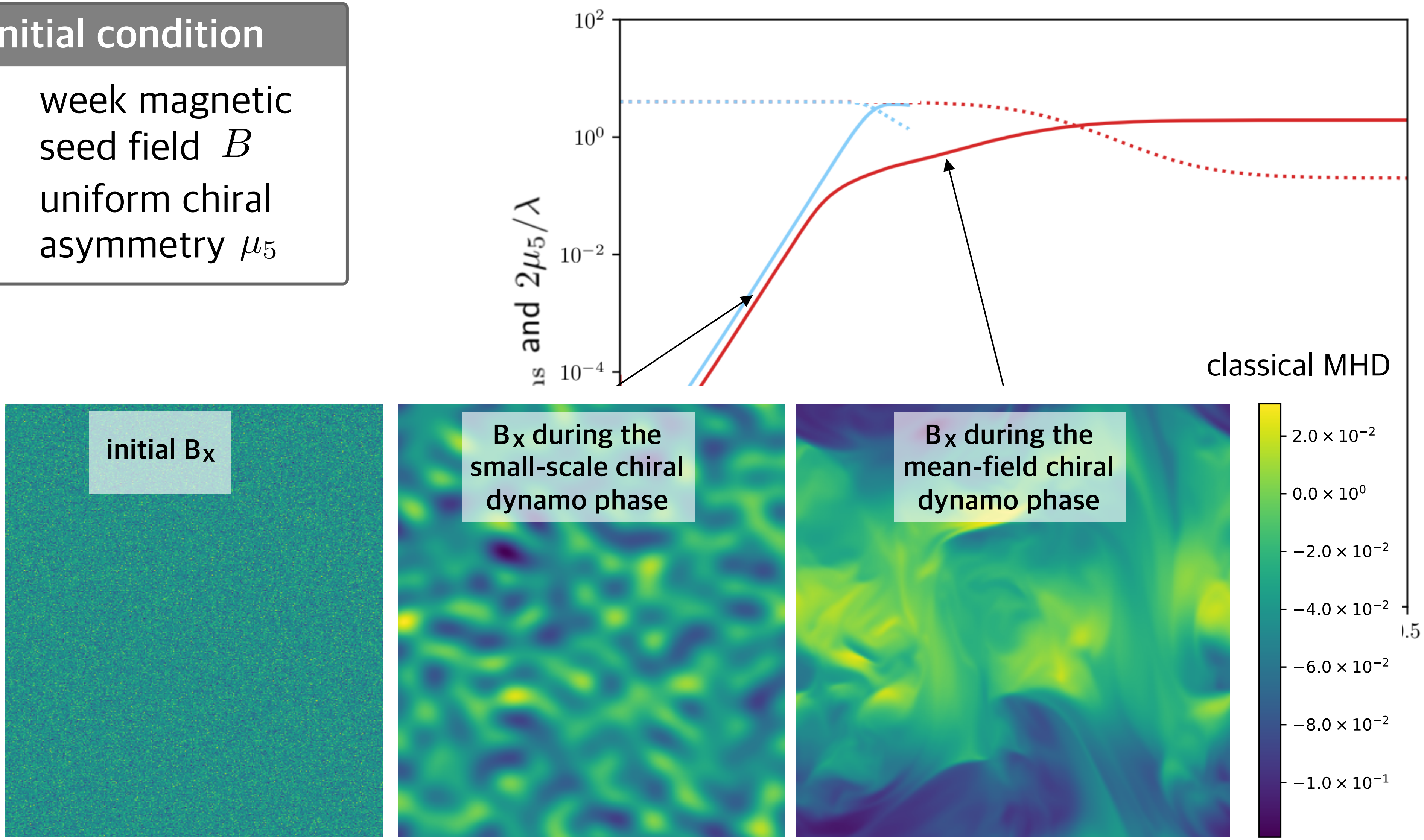
$$\gamma = (\eta\bar{\mu}_5 + \boxed{\alpha_\mu})k - (\eta + \boxed{\eta_{\mathrm{T}}})k^2$$

$$\alpha_\mu = -\frac{2}{3}\eta\overline{\mu_5}\log(\mathrm{Re_M}) \qquad \eta_{\mathrm{T}} = \frac{\mathrm{Re_M}}{3\eta}$$

Chiral MHD dynamos

Initial condition

- week magnetic seed field B
- uniform chiral asymmetry μ_5



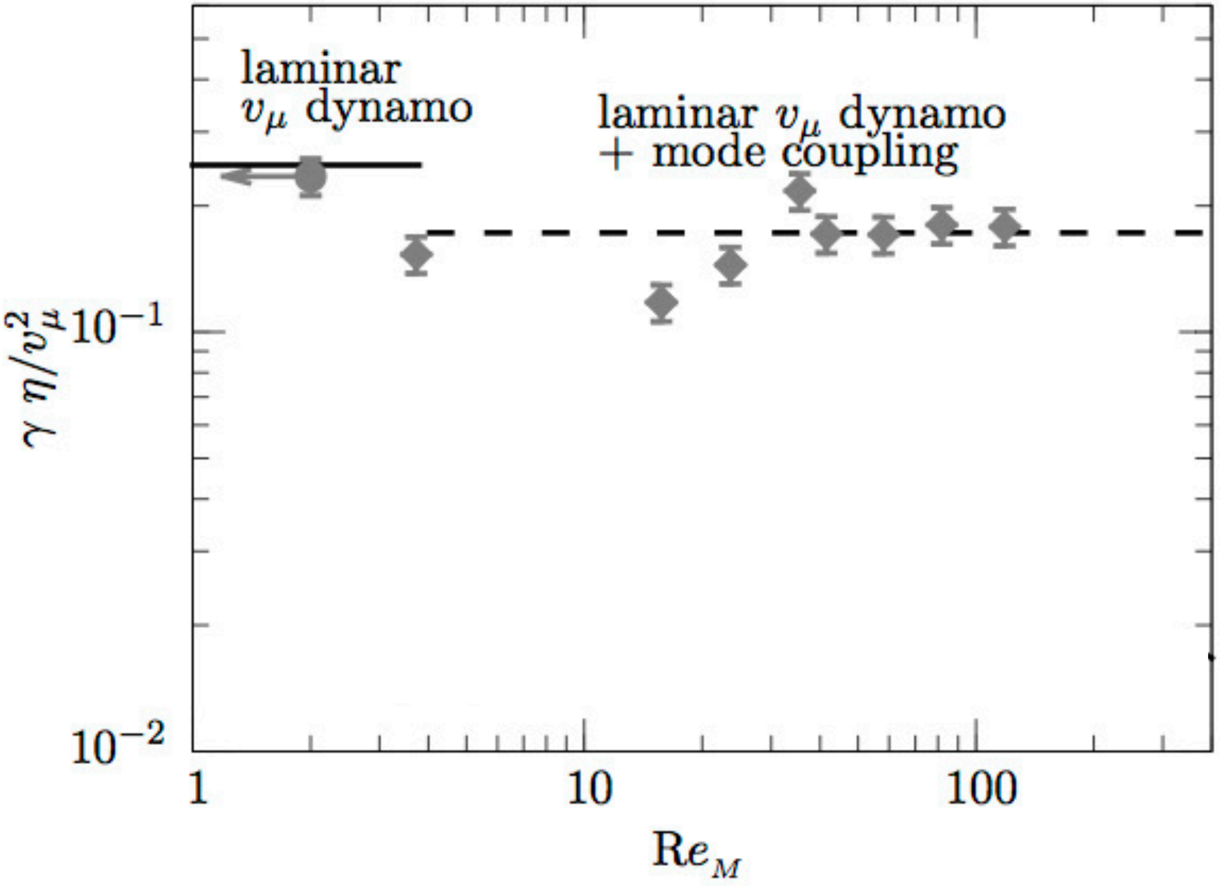
$$\gamma_5 = \frac{\eta\mu_5^2}{4} \qquad \gg \qquad \gamma_\alpha = \frac{(\eta\overline{\mu_5}+\alpha_\mu)^2}{4(\eta+\eta_\mathrm{T})}$$

$$k_5 = \frac{\mu_5}{2} \qquad \gg \qquad k_\alpha = \frac{\eta\overline{\mu_5}+\alpha_\mu}{2(\eta+\eta_\mathrm{T})}$$

Scenario 1: Chiral MHD dynamos

Initial condition

- week magnetic seed field B
- large chiral asymmetry μ_5 (uniform)



Jennifer Schober

Scenario 1: Chiral MHD dynamos

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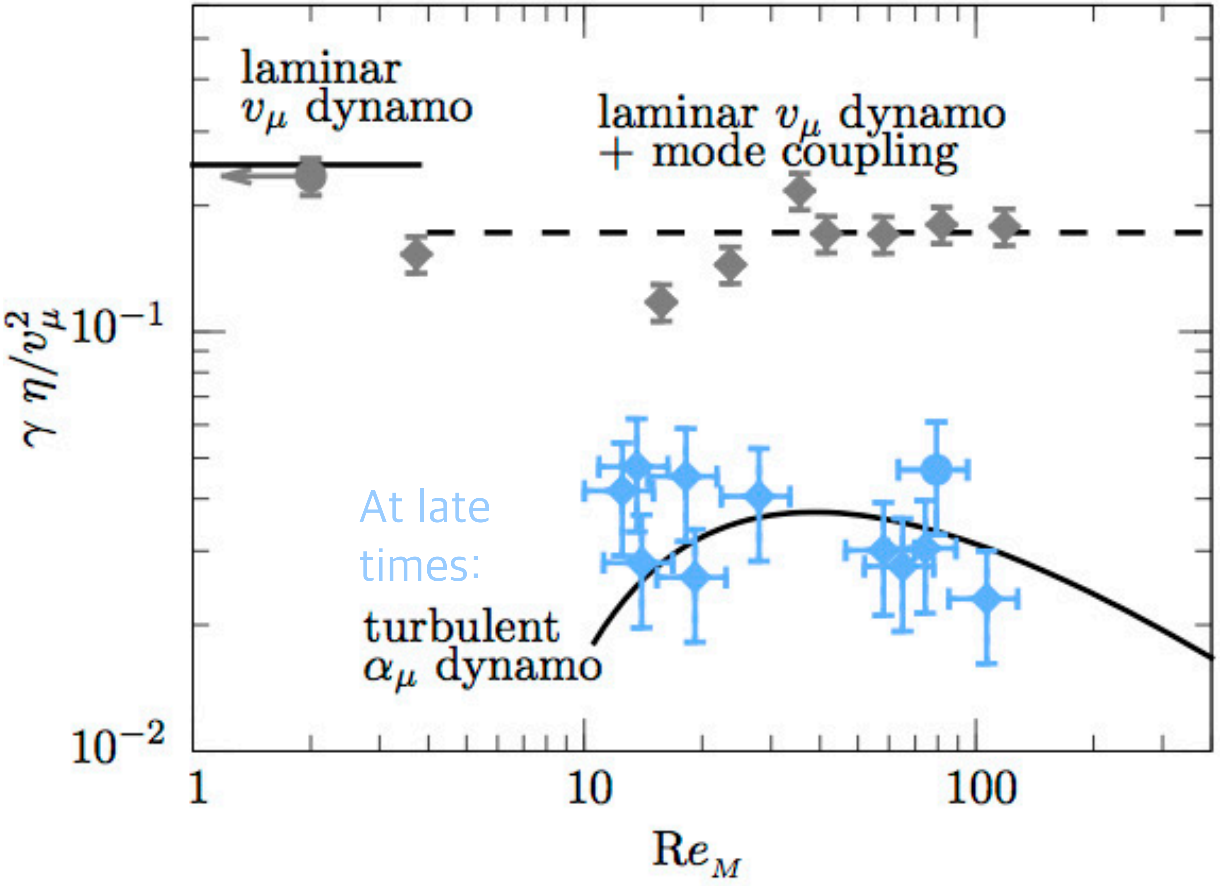

Scenario 1: Chiral MHD dynamos

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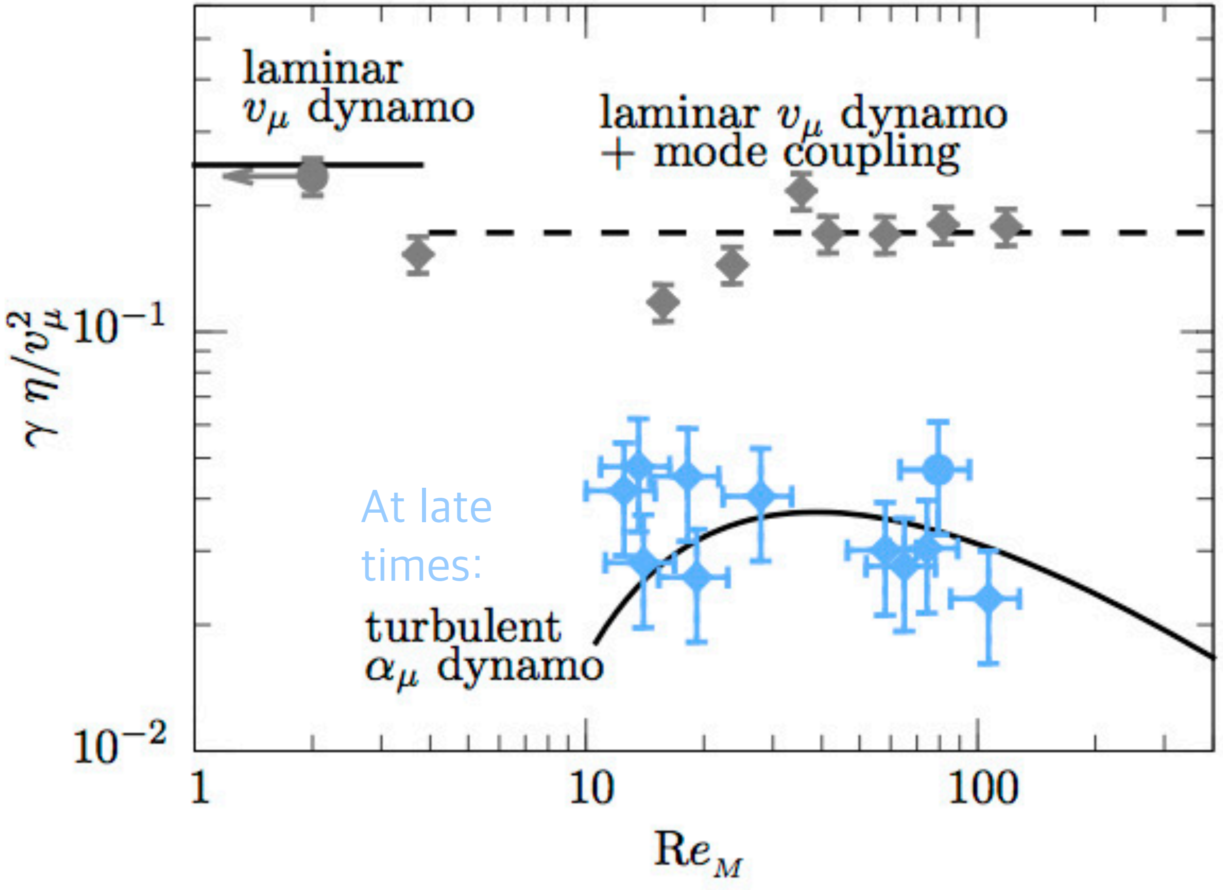
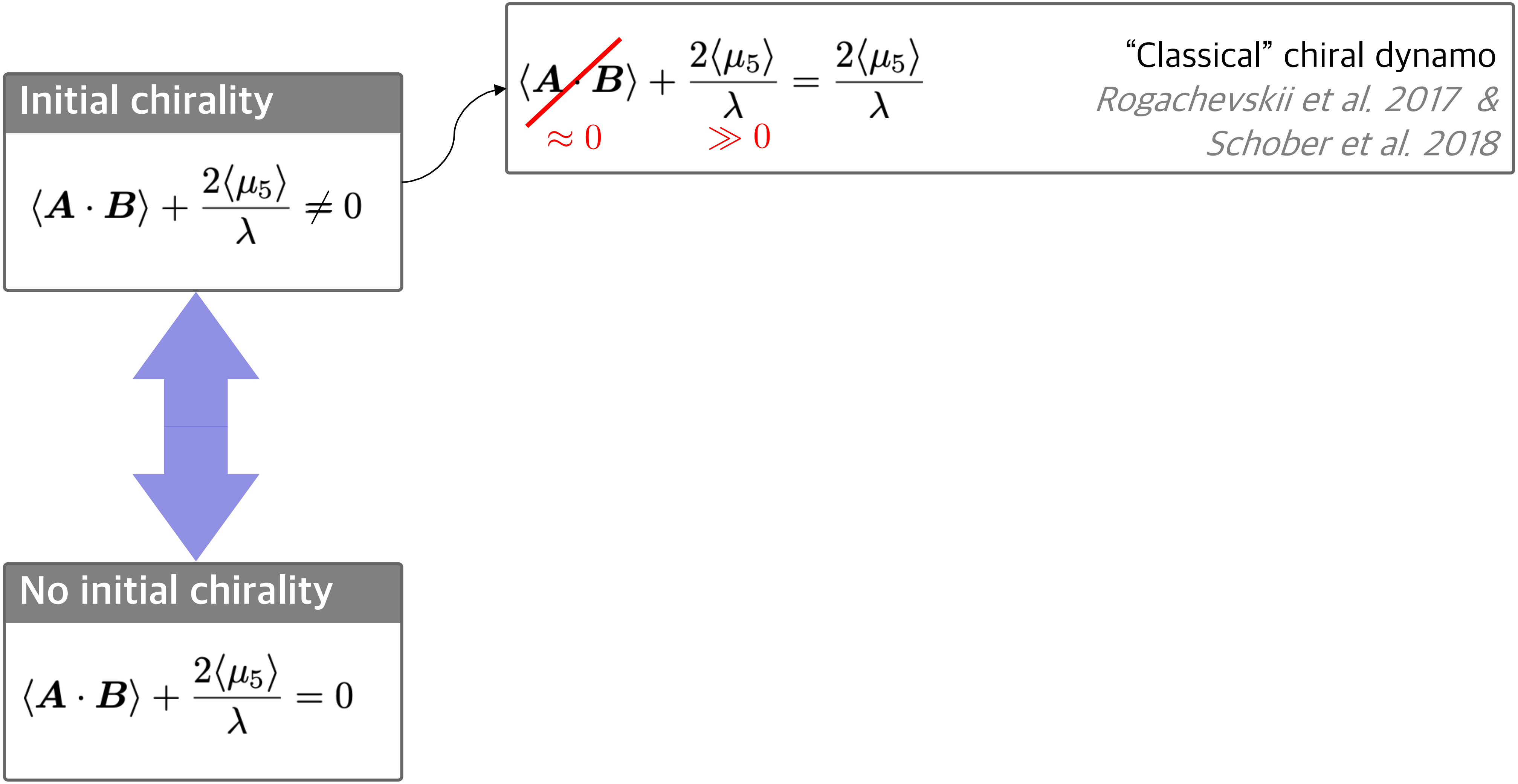


Results of *Rogachevskii et al. (2017)* & *Schober et al. (2018)*:

- The small-scale chiral dynamo instability drives turbulence.
- Mean-field dynamos generate magnetic fields on large spatial scales.

Chiral MHD scenarios



Initial chirality

$$\langle \boldsymbol{A}\cdot\boldsymbol{B}\rangle + \frac{2\langle\mu_5\rangle}{\lambda} \neq 0$$

$$\underset{\approx 0}{\cancel{\langle \boldsymbol{A}\cdot\boldsymbol{B}\rangle}} + \underset{\gg 0}{\frac{2\langle\mu_5\rangle}{\lambda}} = \frac{2\langle\mu_5\rangle}{\lambda}$$

“Classical” chiral dynamo

Rogachevskii et al. 2017 &
Schober et al. 2018

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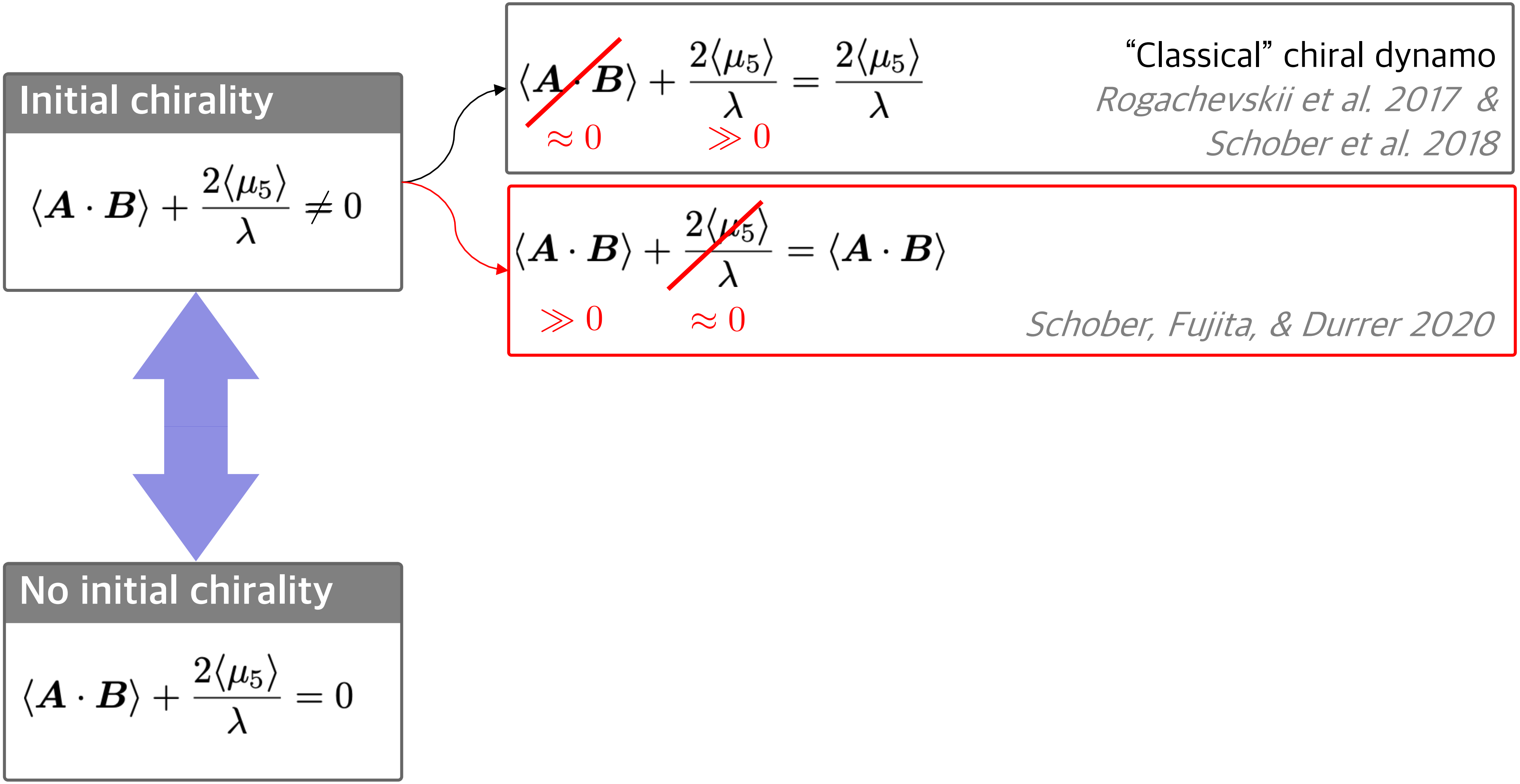
Chiral MHD scenarios



Initial chirality

$$\langle \boldsymbol{A}\cdot\boldsymbol{B}\rangle + \frac{2\langle\mu_5\rangle}{\lambda} \neq 0$$

$$\underbrace{\cancel{\langle \boldsymbol{A}\cdot\boldsymbol{B}\rangle}}_{\approx 0} + \underbrace{\frac{2\langle\mu_5\rangle}{\lambda}}_{\gg 0} = \frac{2\langle\mu_5\rangle}{\lambda}$$

“Classical” chiral dynamo

Rogachevskii et al. 2017 & Schober et al. 2018

$$\underbrace{\langle \boldsymbol{A}\cdot\boldsymbol{B}\rangle}_{\gg 0} + \underbrace{\cancel{\frac{2\langle\mu_5\rangle}{\lambda}}}_{\approx 0} = \langle \boldsymbol{A}\cdot\boldsymbol{B}\rangle$$

Schober, Fujita, & Durrer 2020

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Generation of chiral asymmetry

Initial condition

- strong helical magnetic field
- no chiral asymmetry

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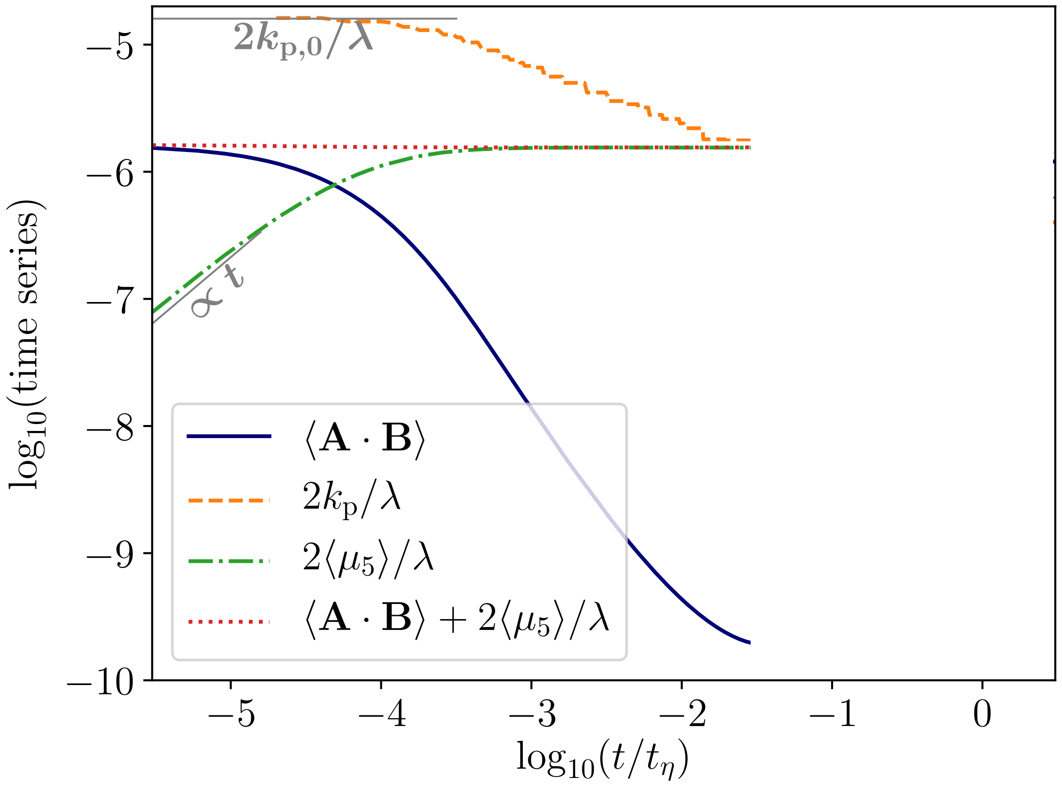

Schober, Fujita, & Durrer 2020

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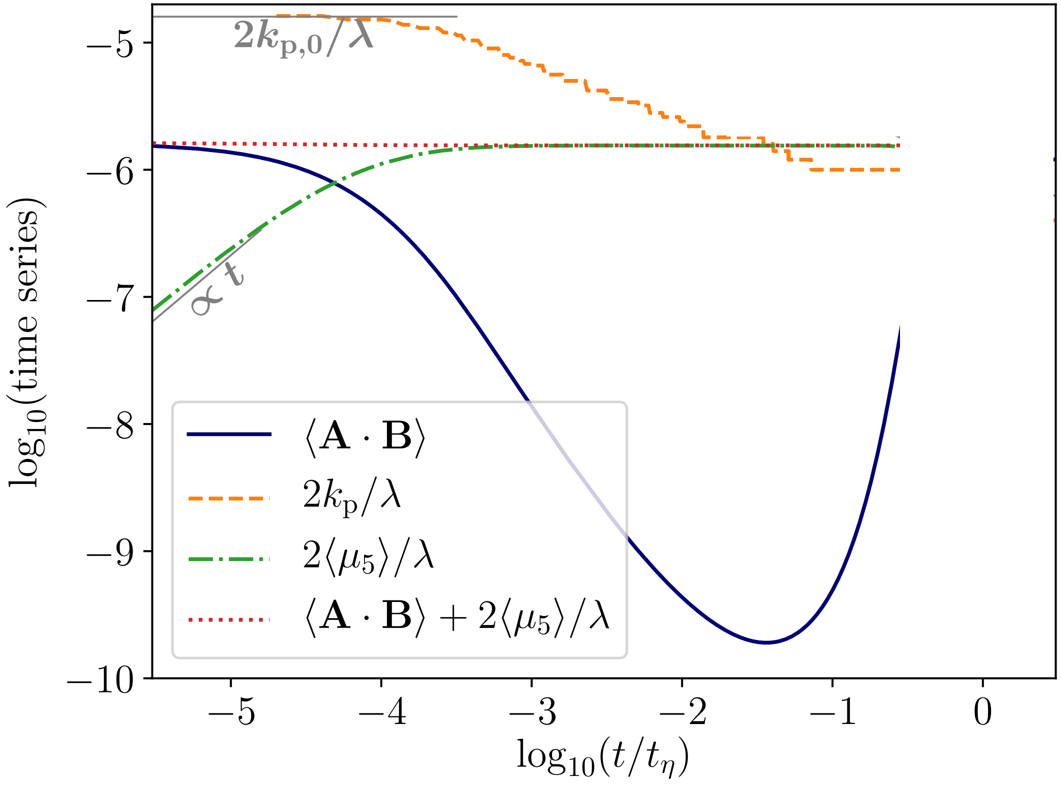

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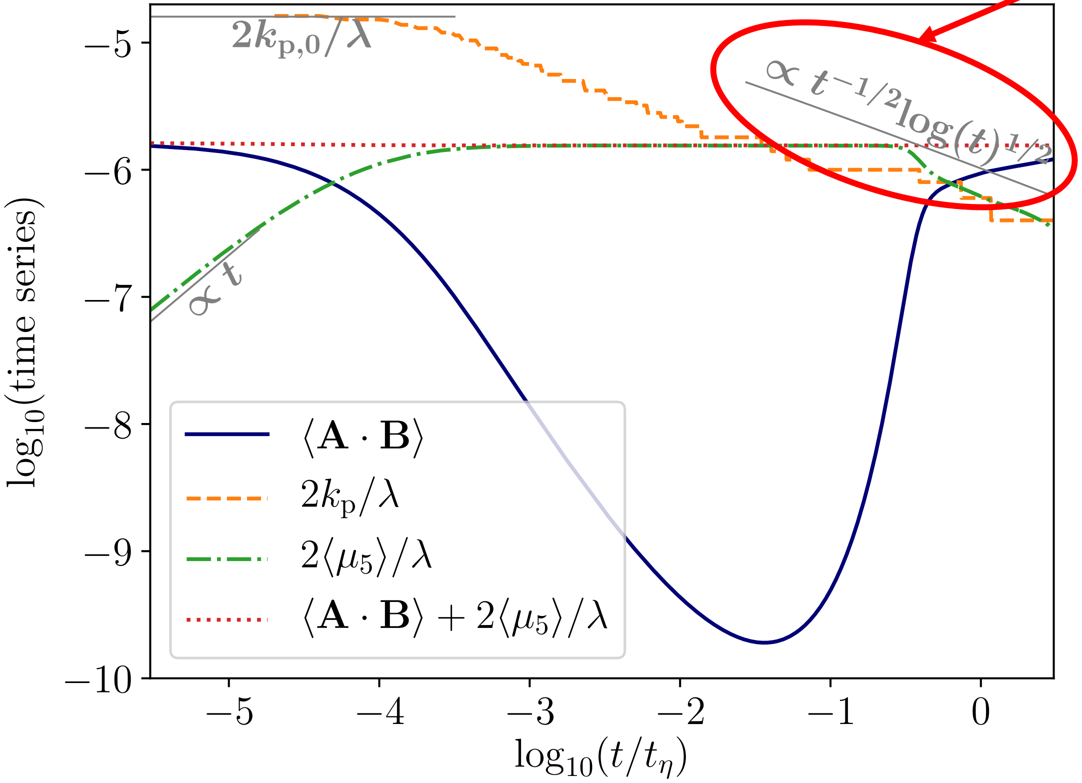

Schober, Fujita, & Durrer 2020

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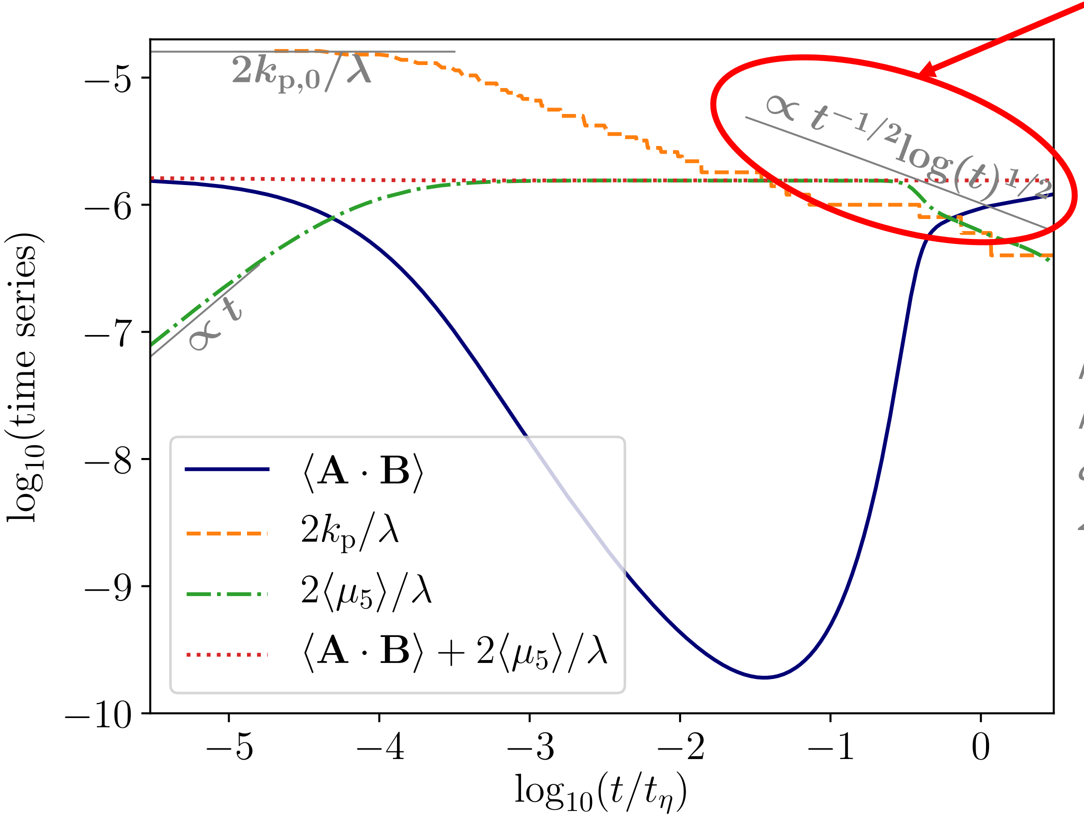

[predicted analytically but without log-correction, *Hirono, Kharzeev, & Yin 2016*]

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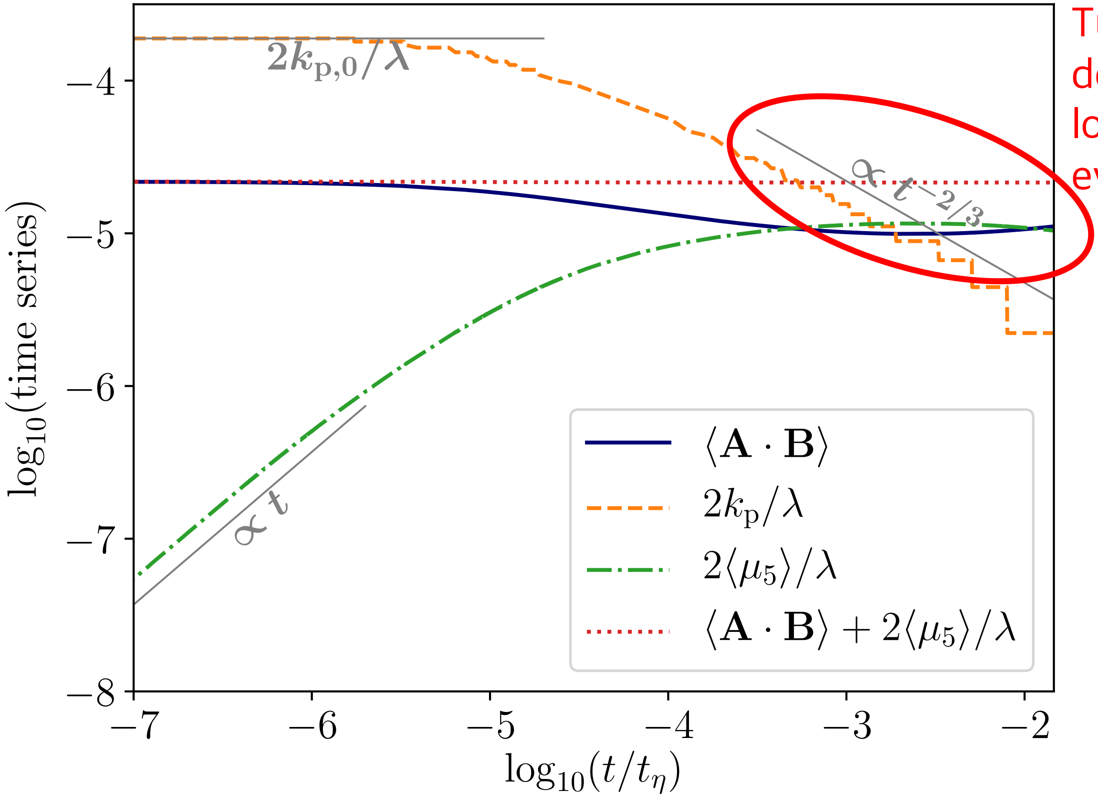

Turbulence determines long-term evolution.

Schober, Fujita, & Durrer 2020

EPFL

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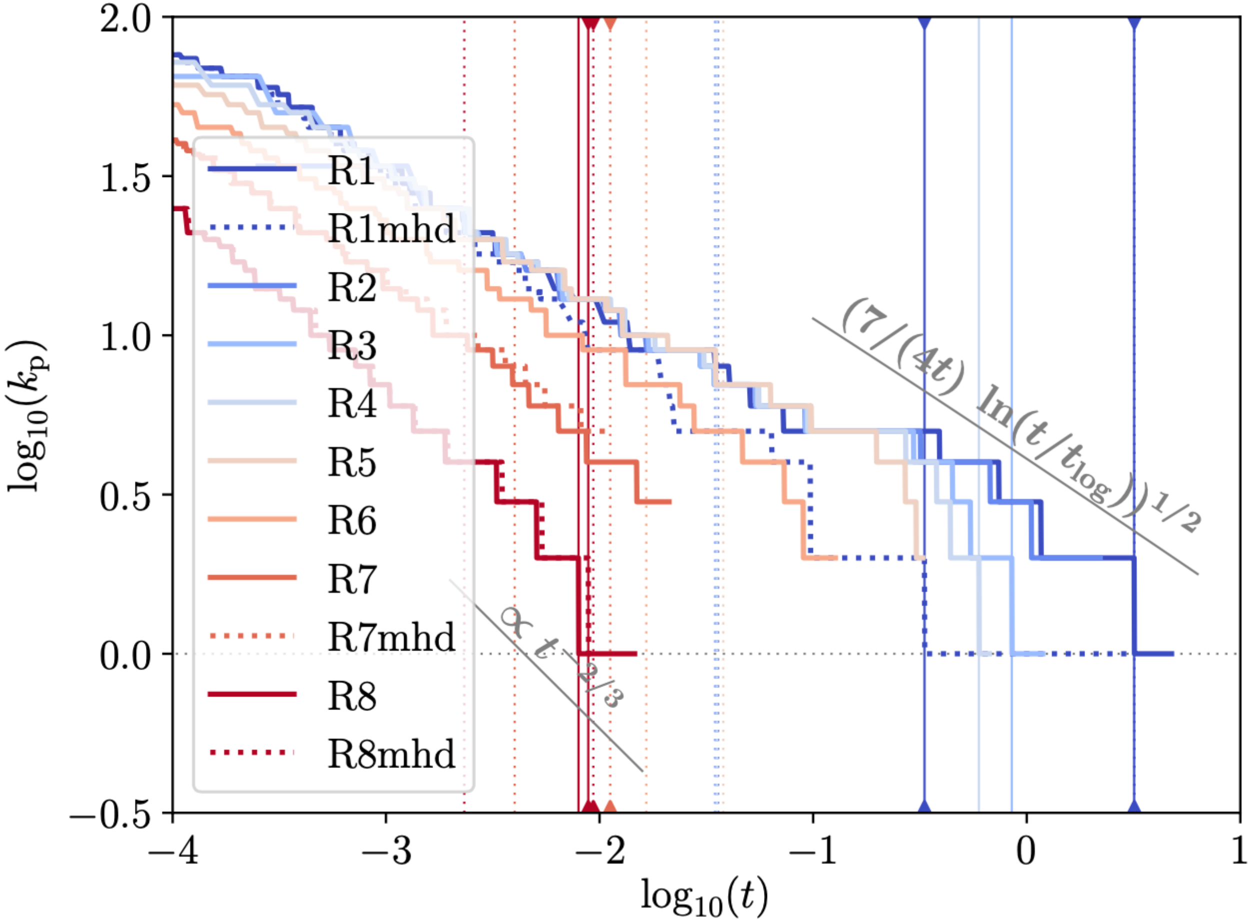

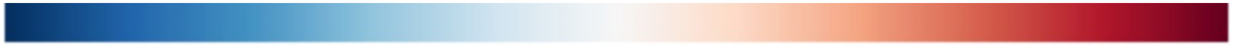

Schober, Fujita, & Durrer 2020

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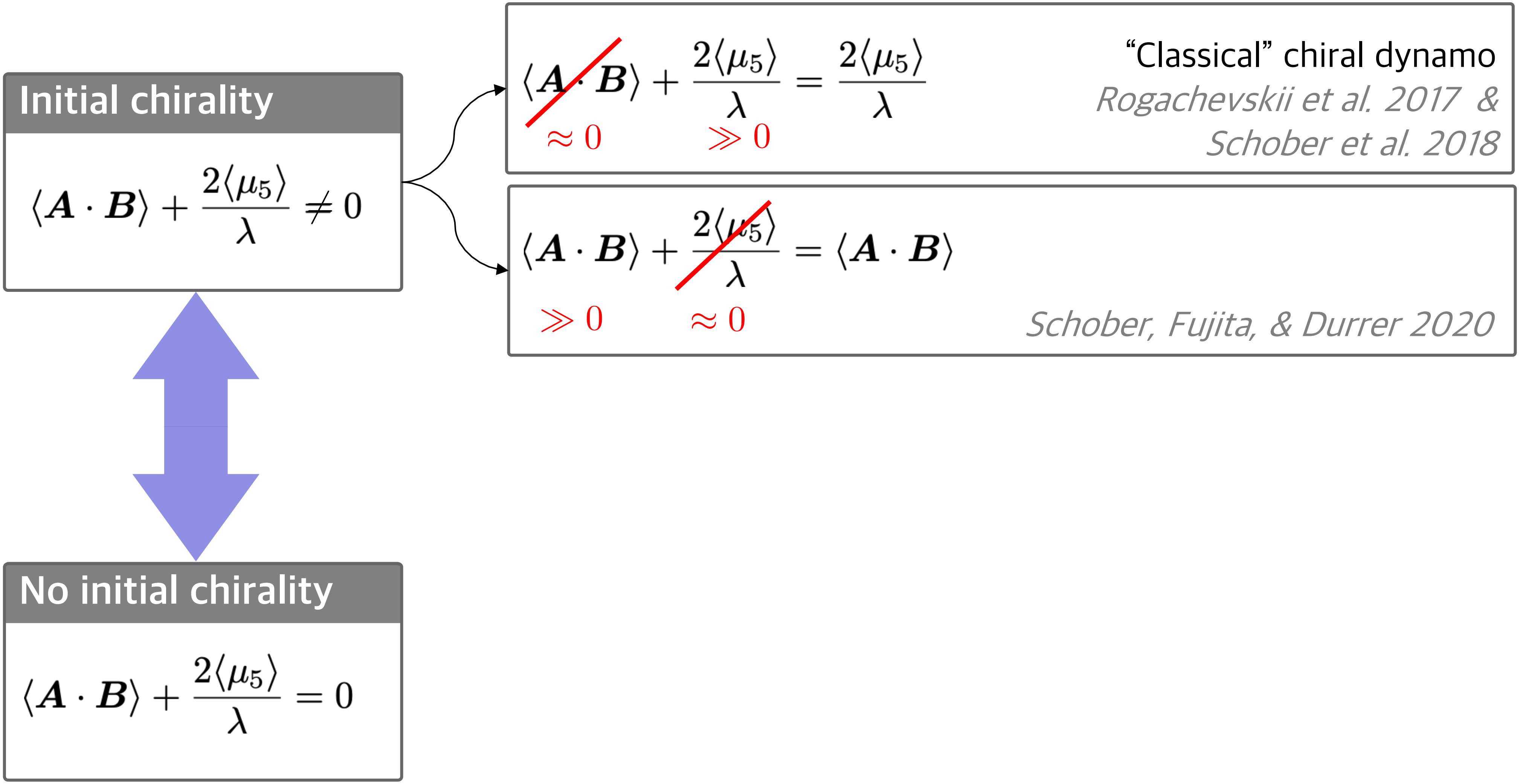
Chiral MHD scenarios



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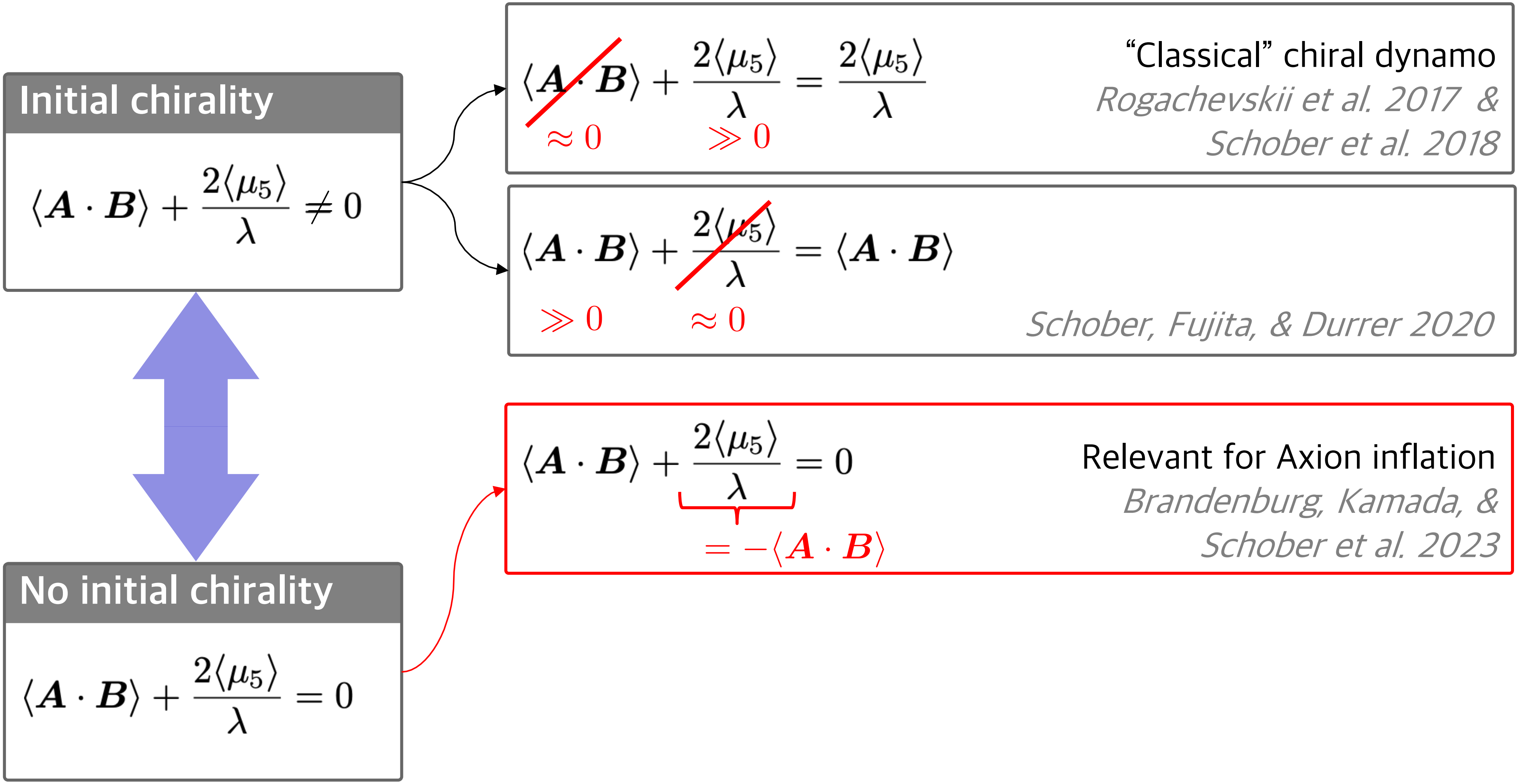

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Relevant for Axion inflation

Brandenburg, Kamada, &
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Decay of helicity balanced by chirality

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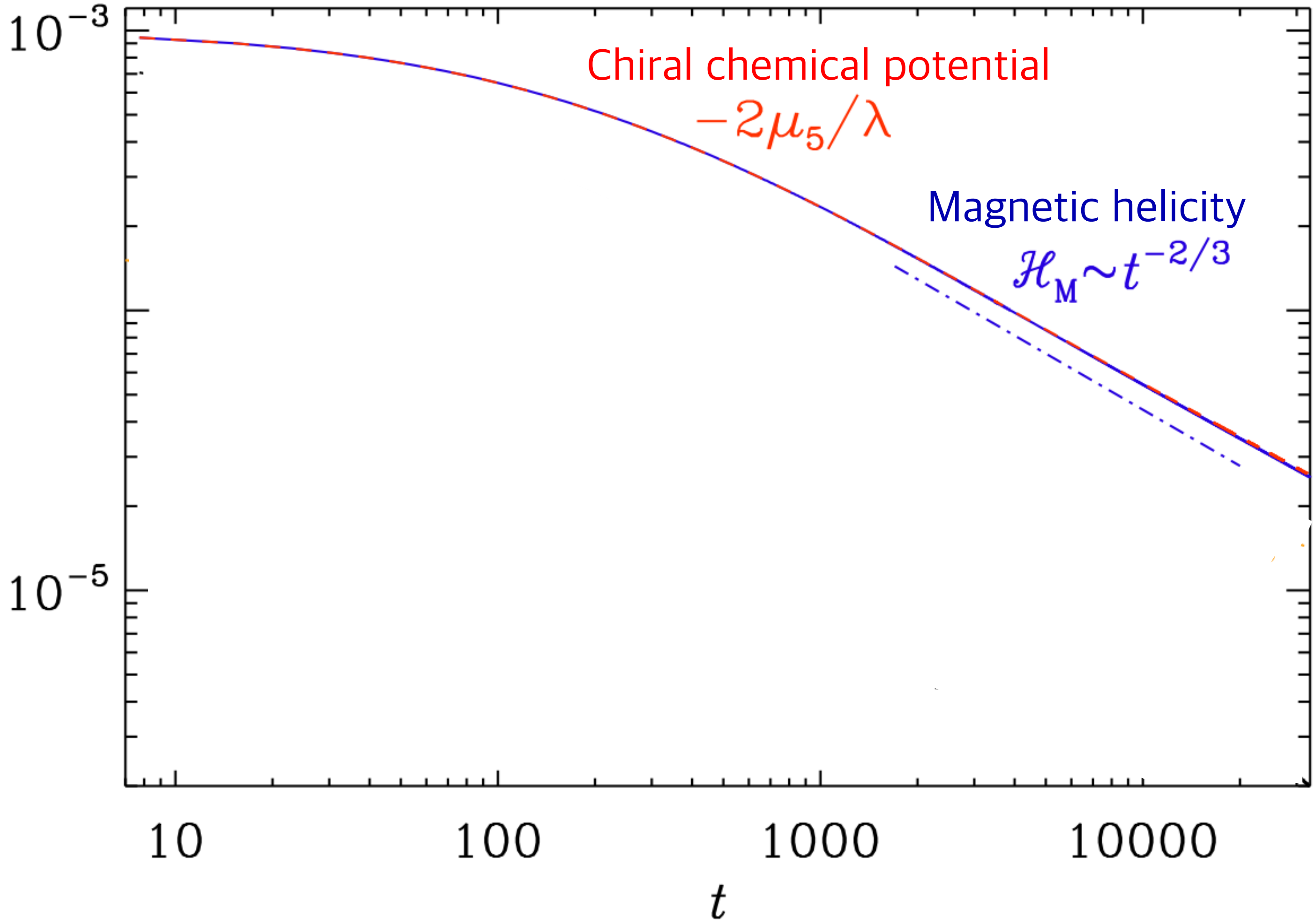
Brandenburg, Kamada, & Schober et al. 2023



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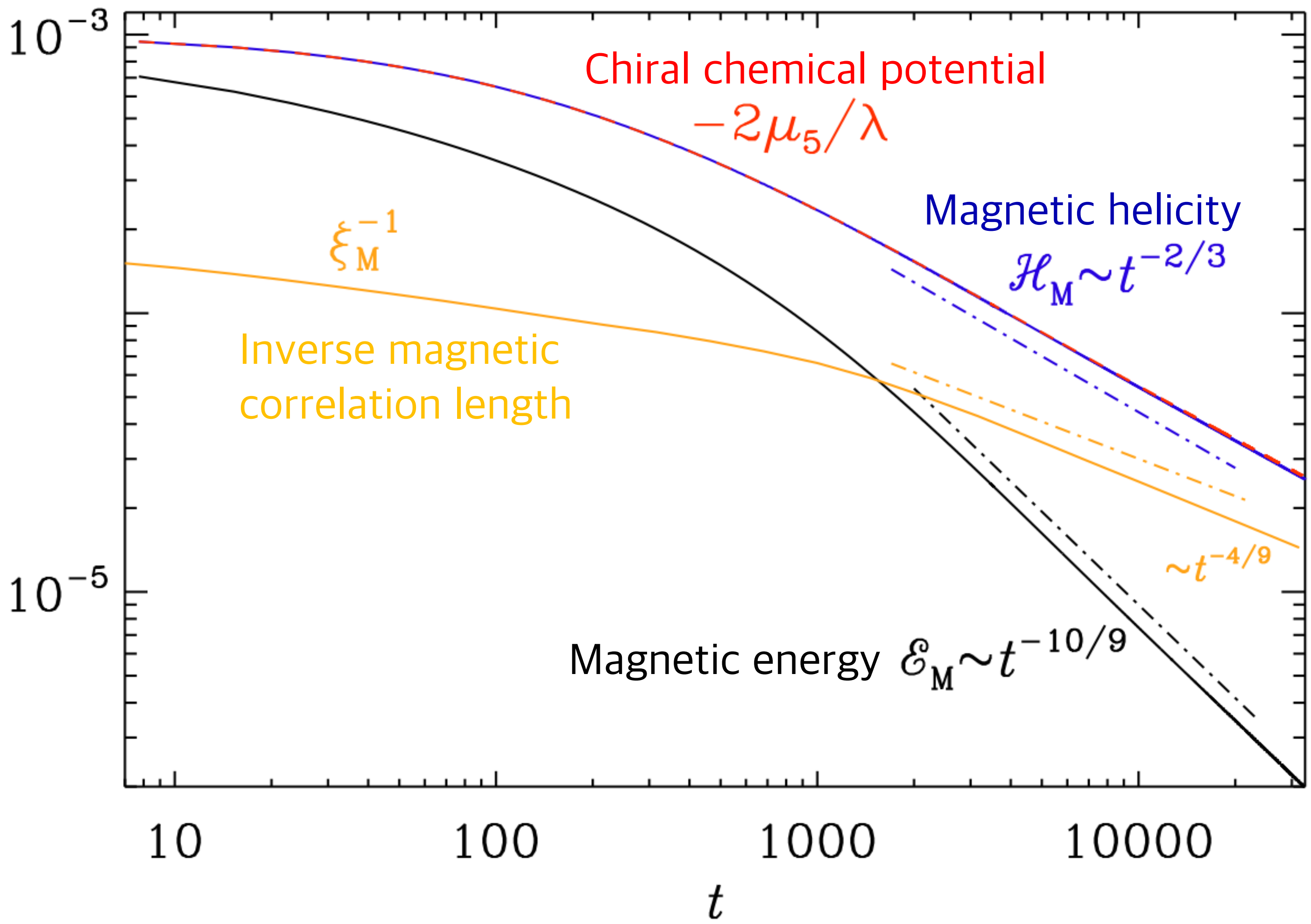

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Decay of helicity balanced by chirality

Explanation of decay laws with the
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$$\mathcal{I}_{\mathrm{H}}(R) = \int_{V_R} \langle h(\boldsymbol{x}) h(\boldsymbol{x}+\boldsymbol{r}) \rangle \, \mathrm{d}^3 r$$

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$$h = \boldsymbol{A} \cdot \boldsymbol{B}$$

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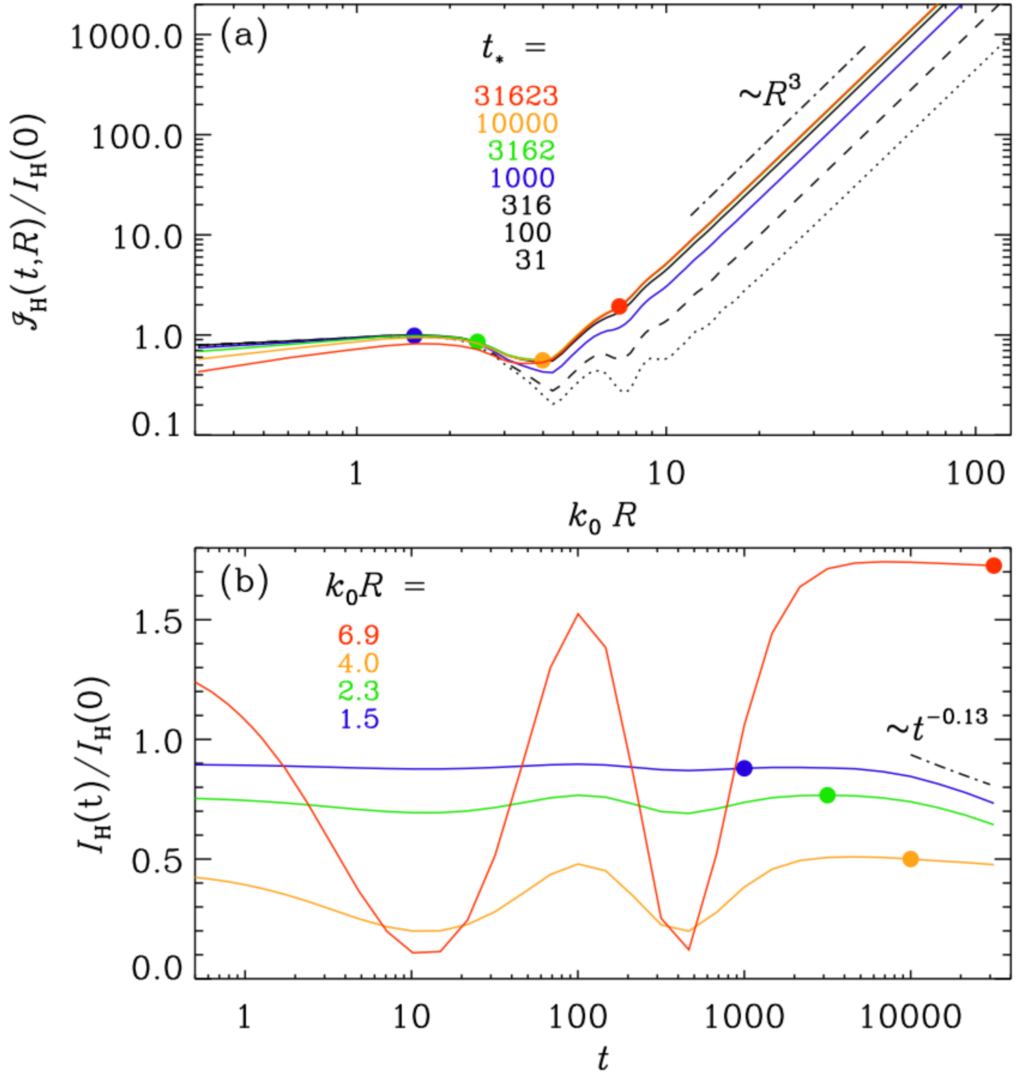

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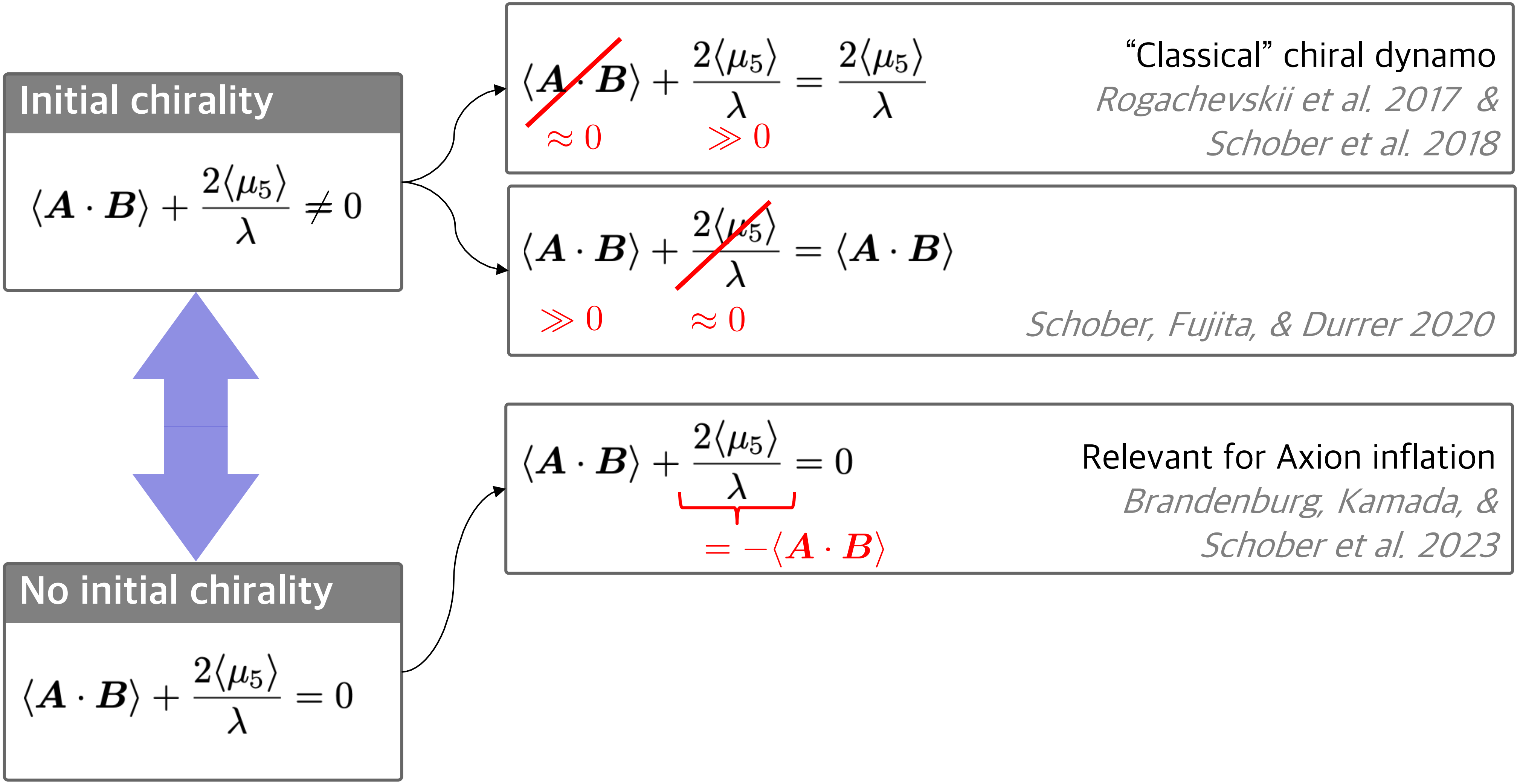

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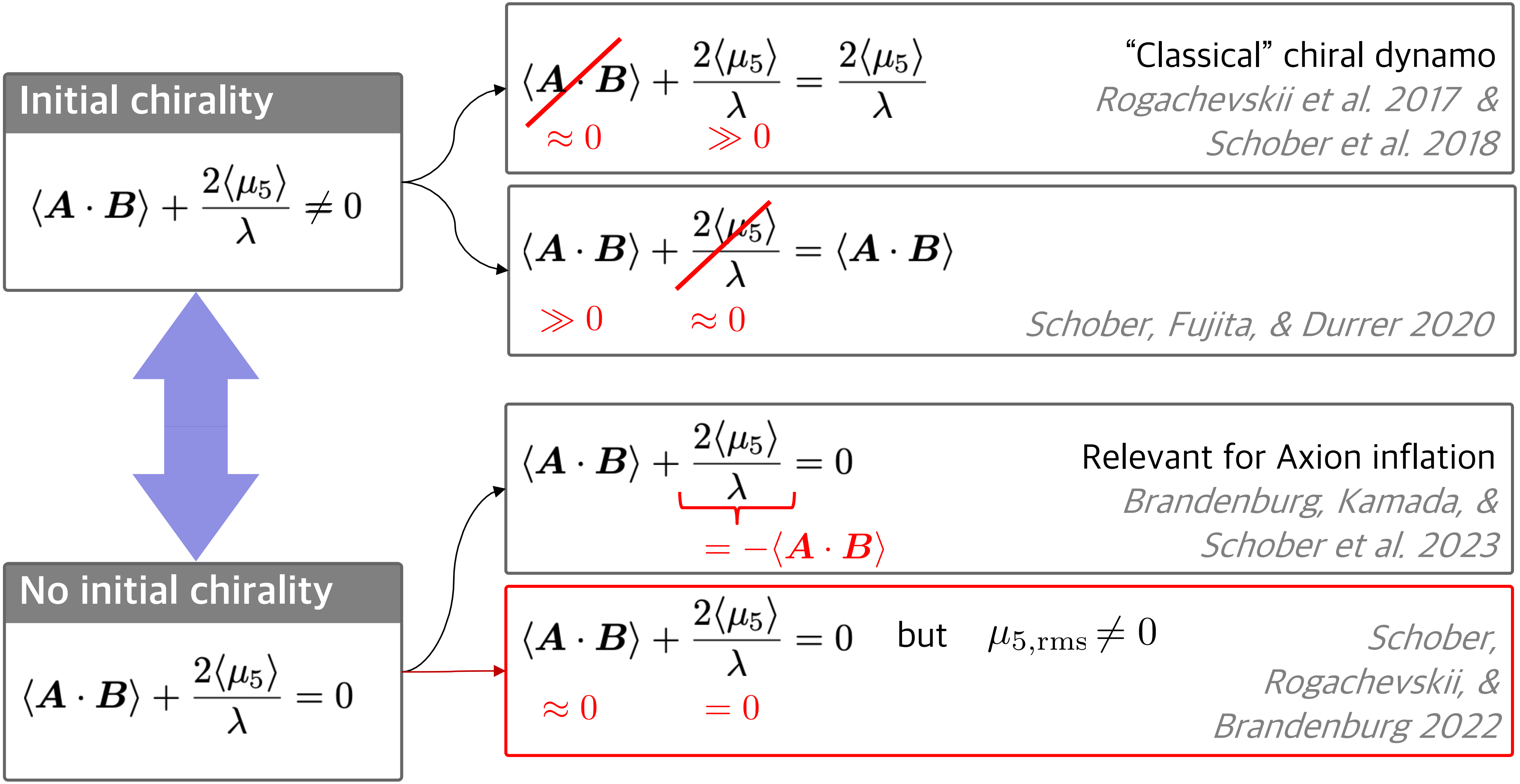

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Schober,
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Brandenburg 2022

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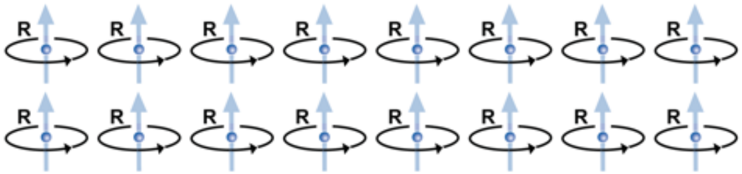

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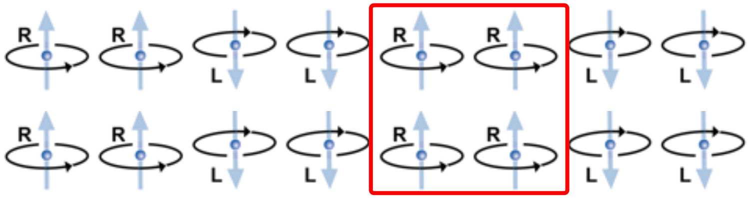

'Islands with non-vanishing average

Jennifer Schober

Inhomogeneous chiral MHD dynamos

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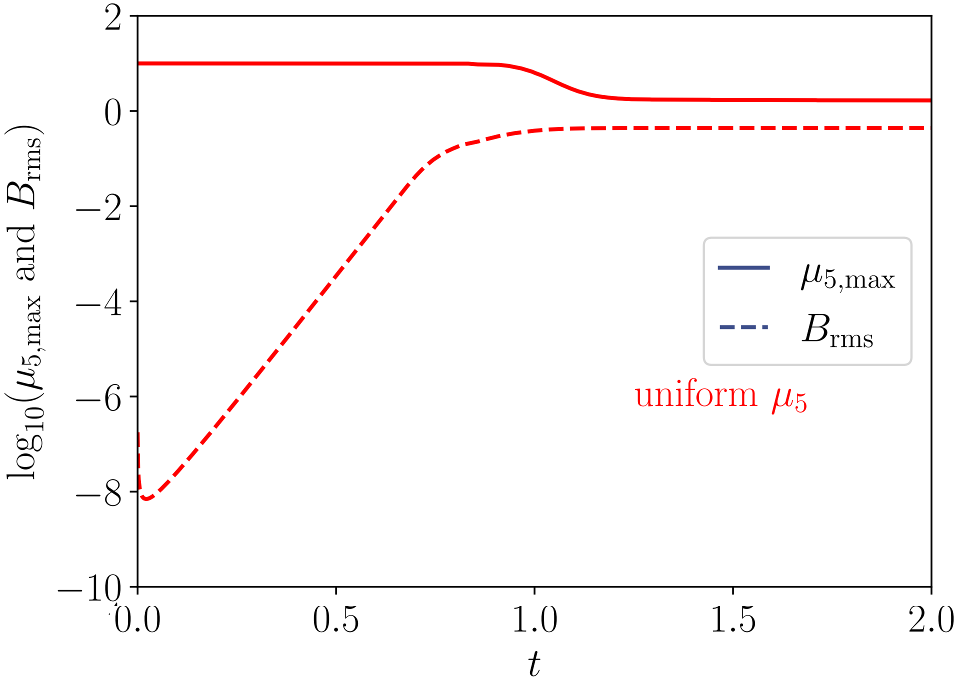
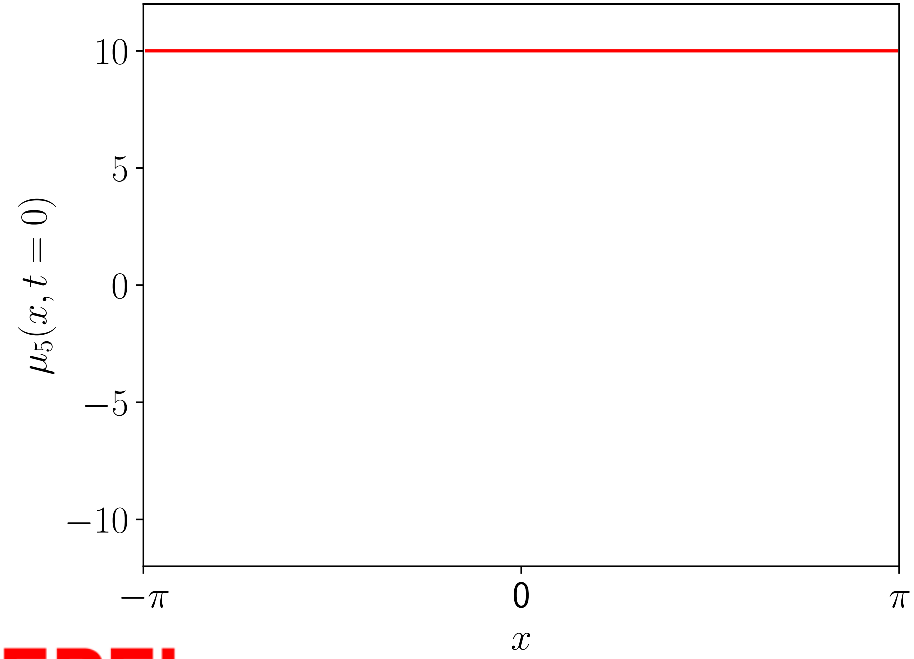


Toy models at initial time:



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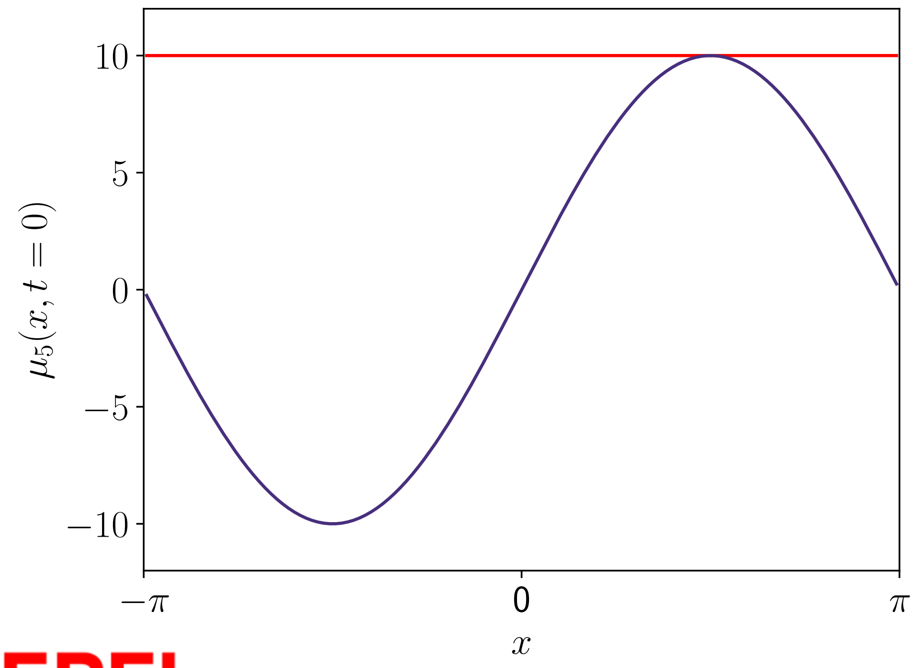

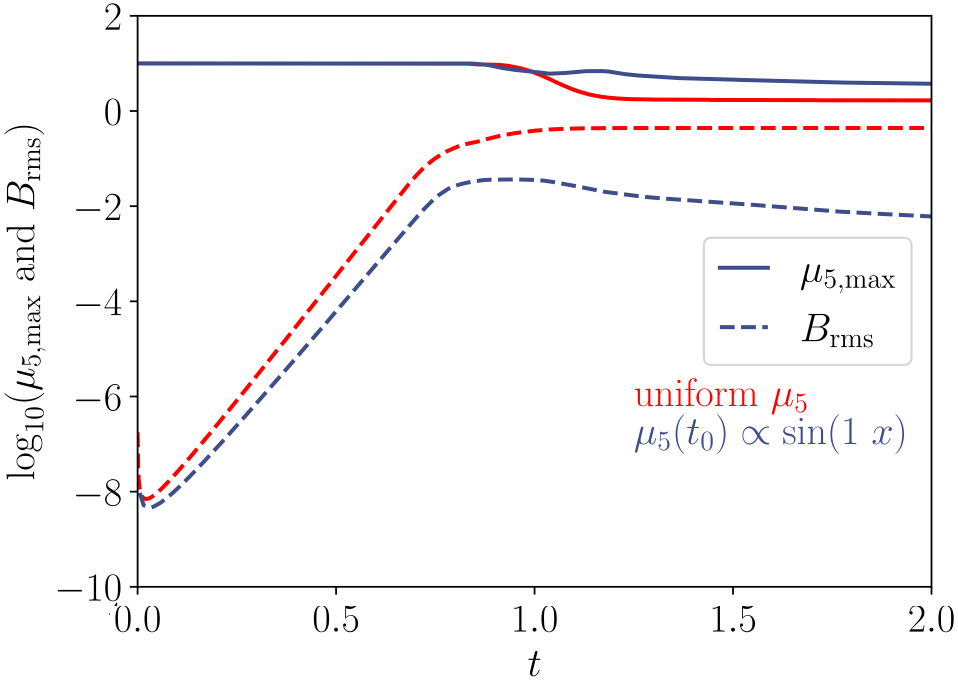

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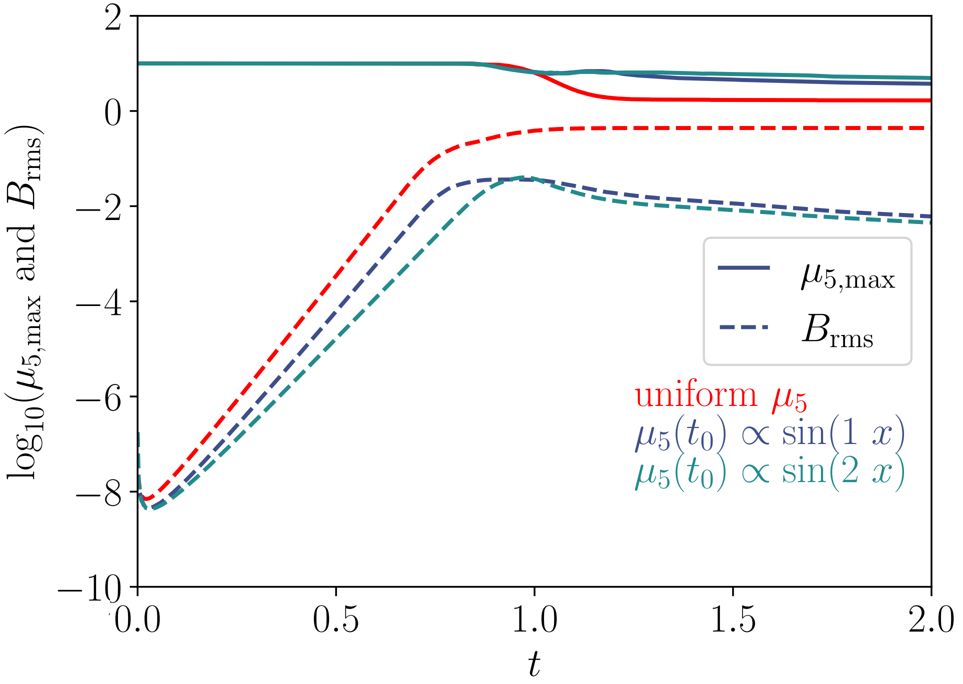

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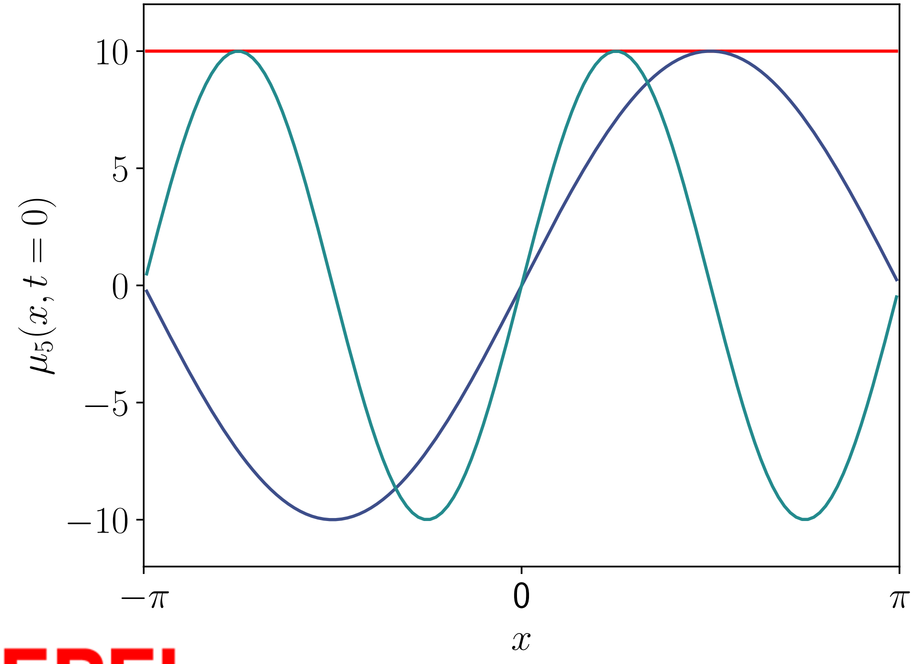

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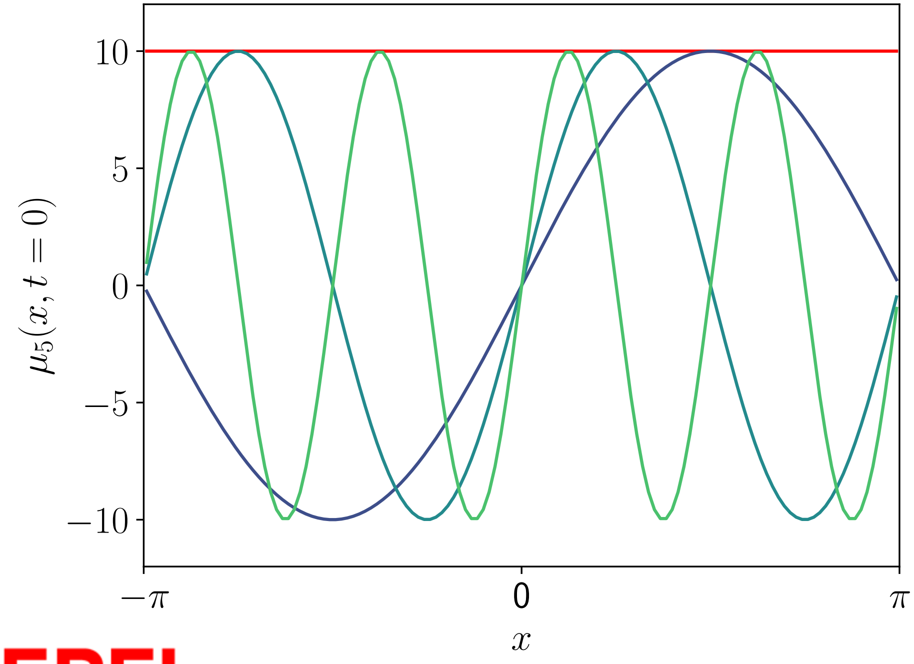

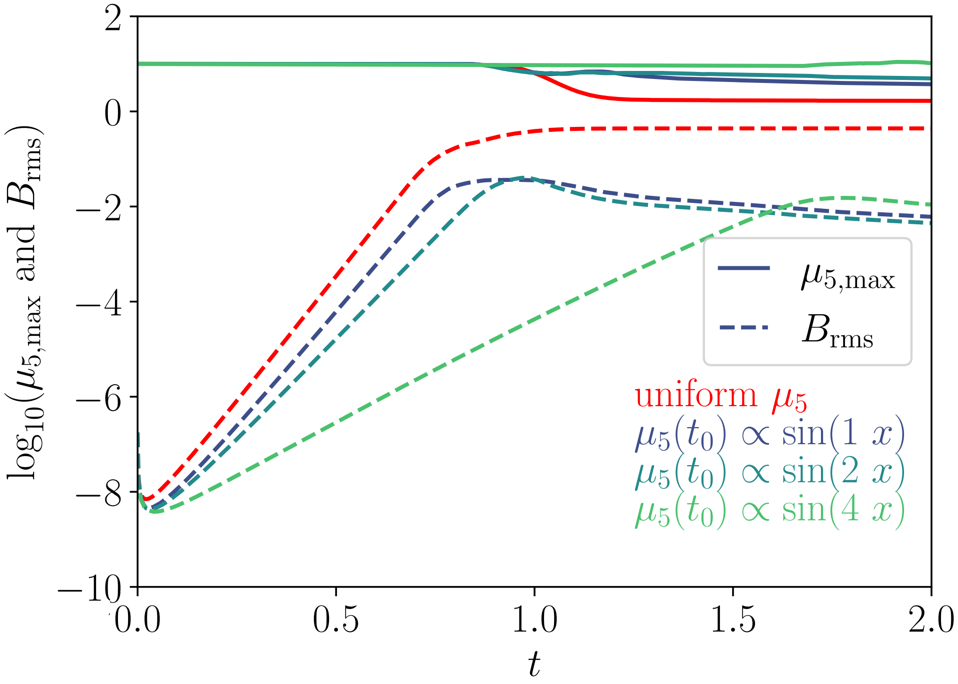

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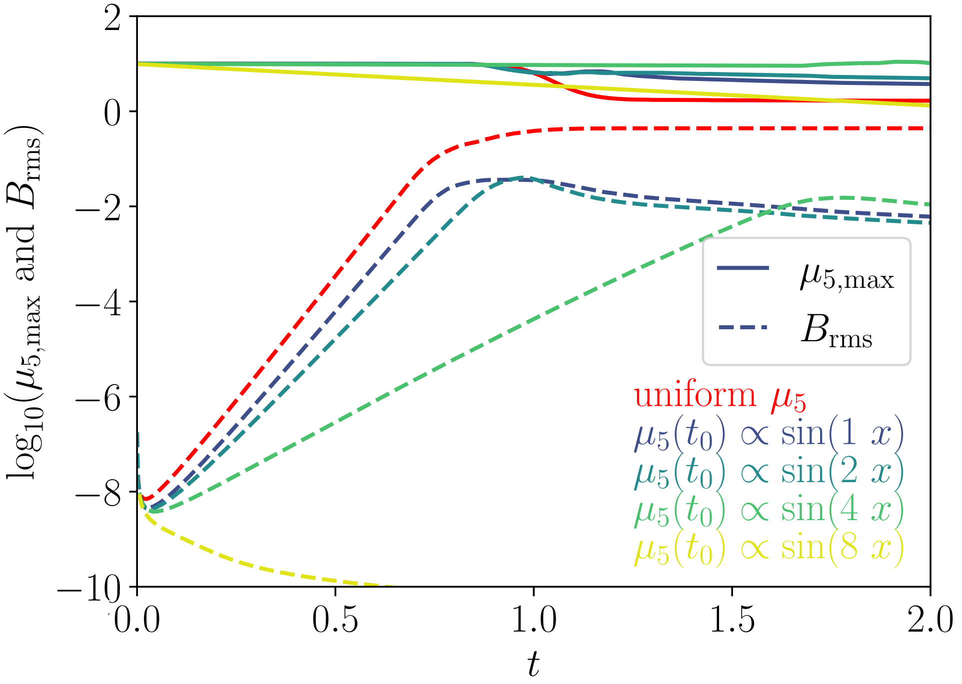

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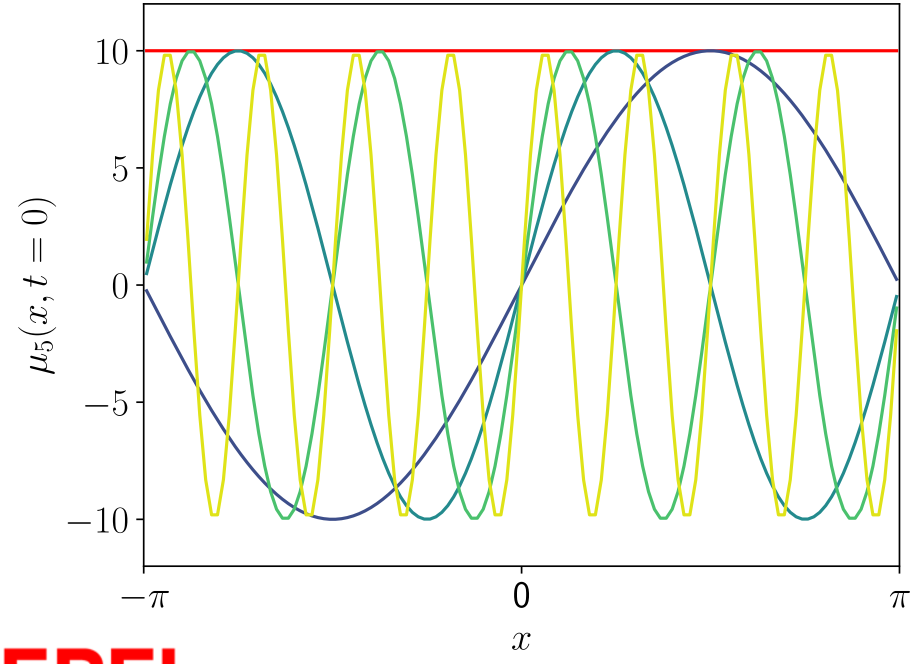

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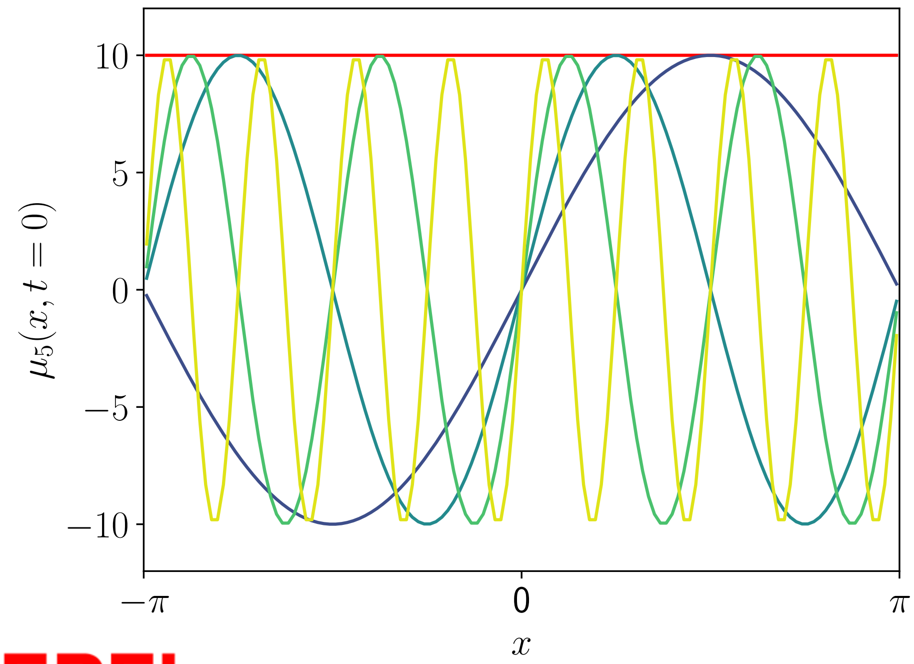

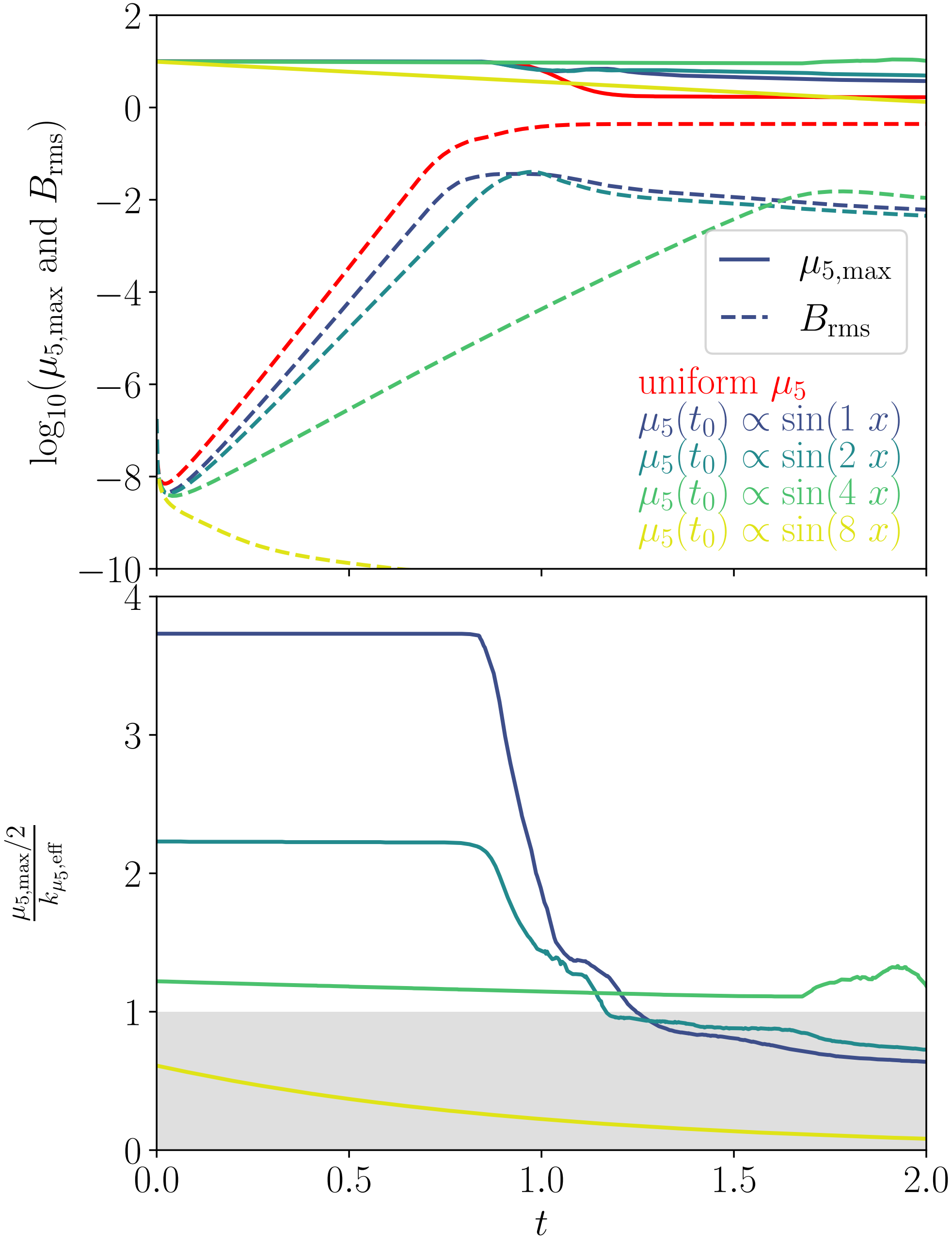

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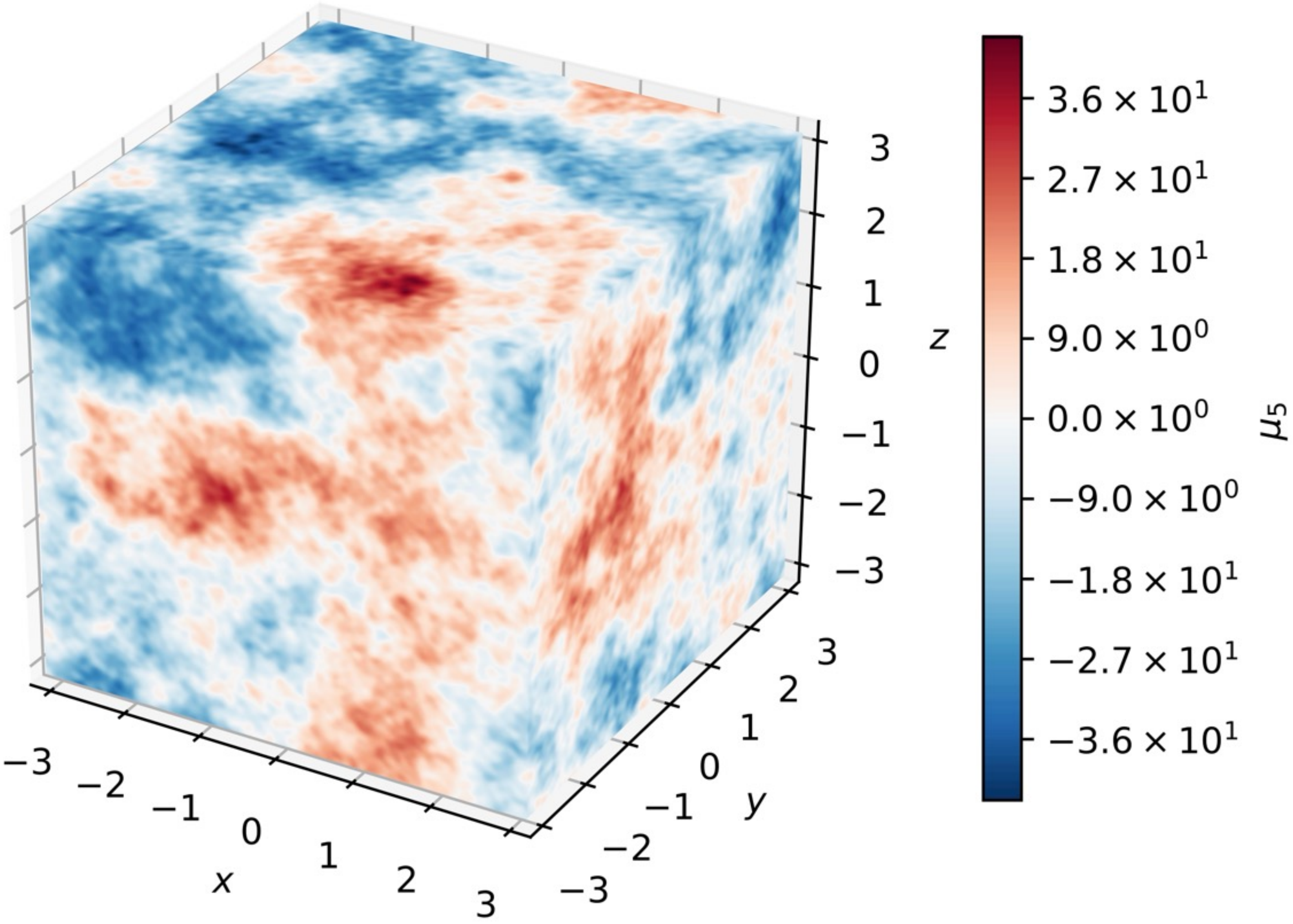

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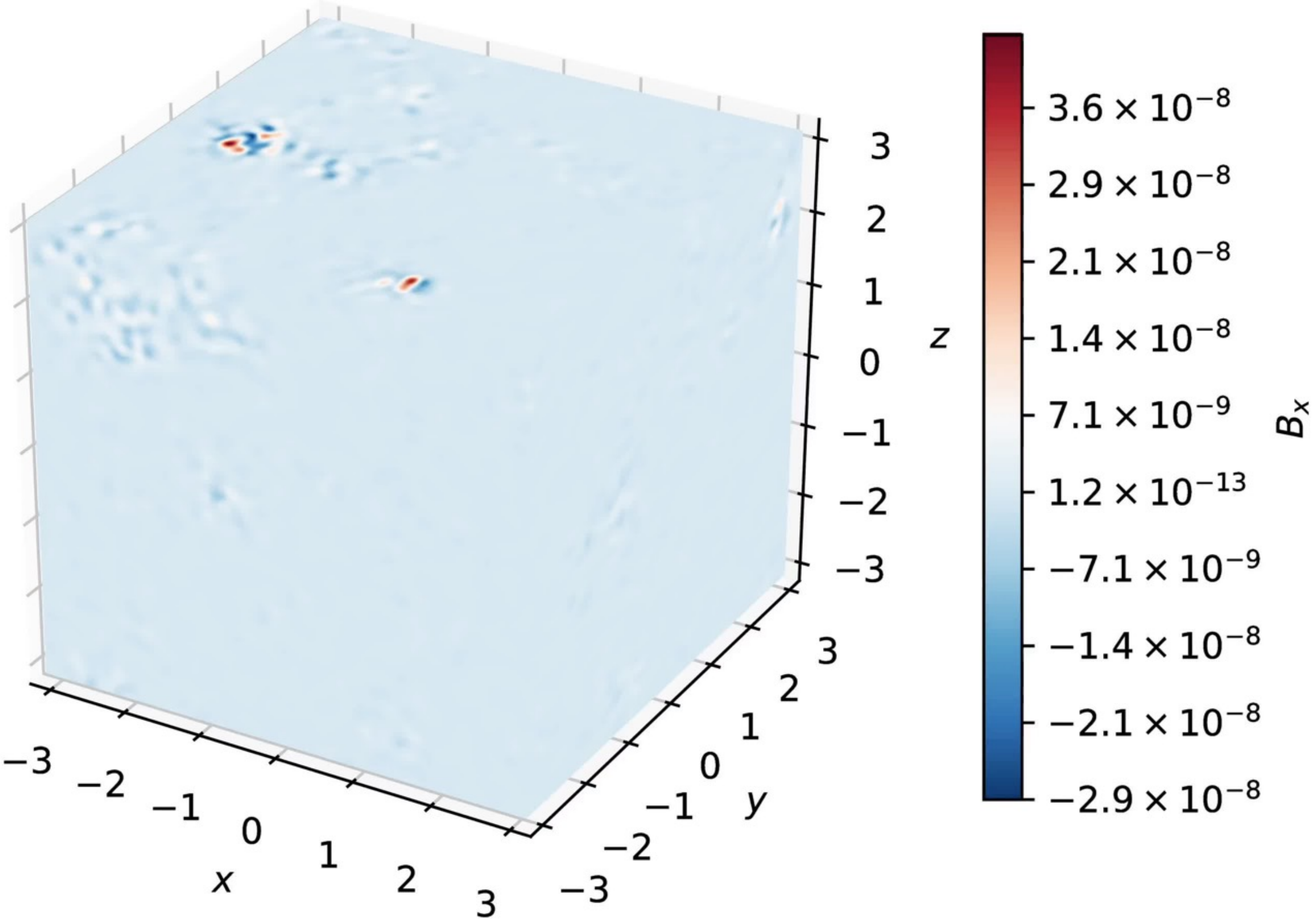

EPFL

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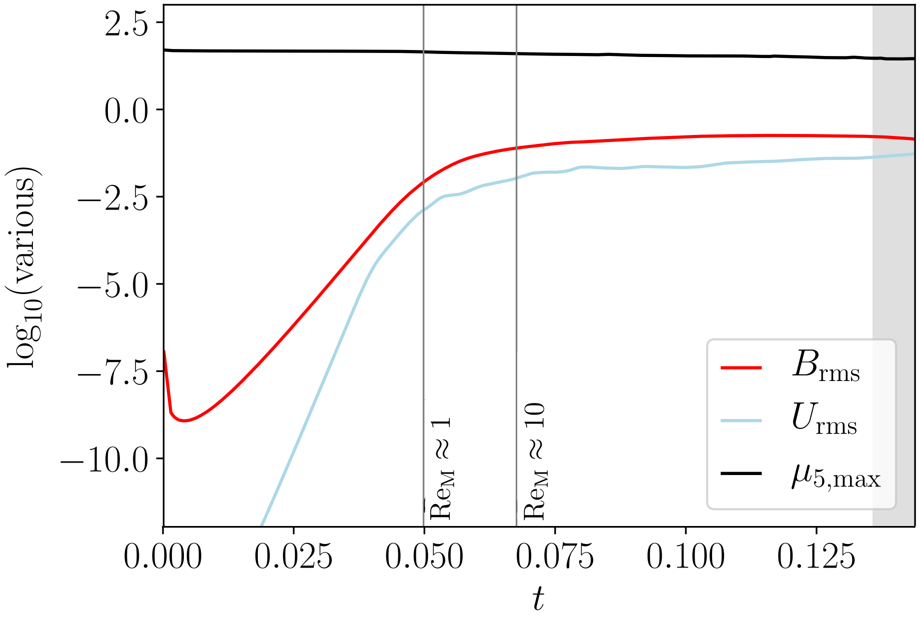

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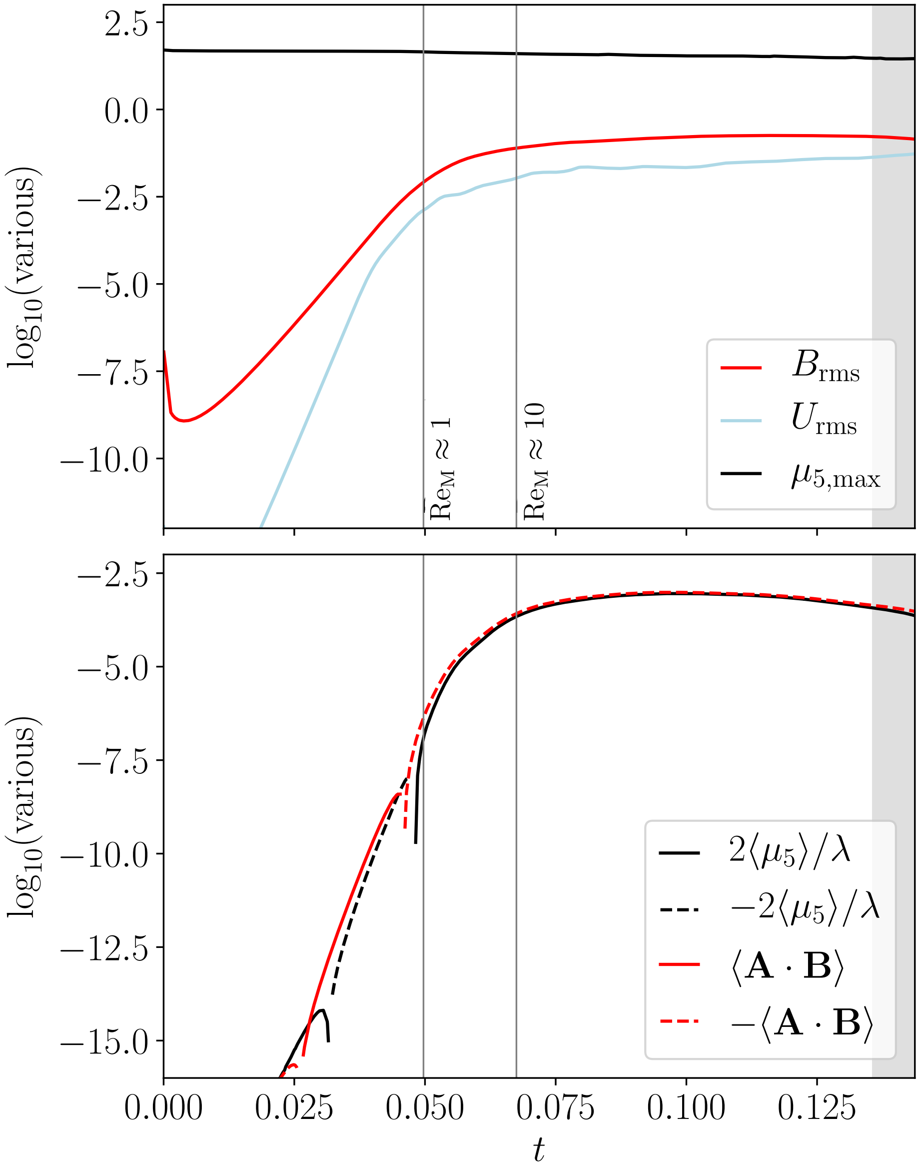

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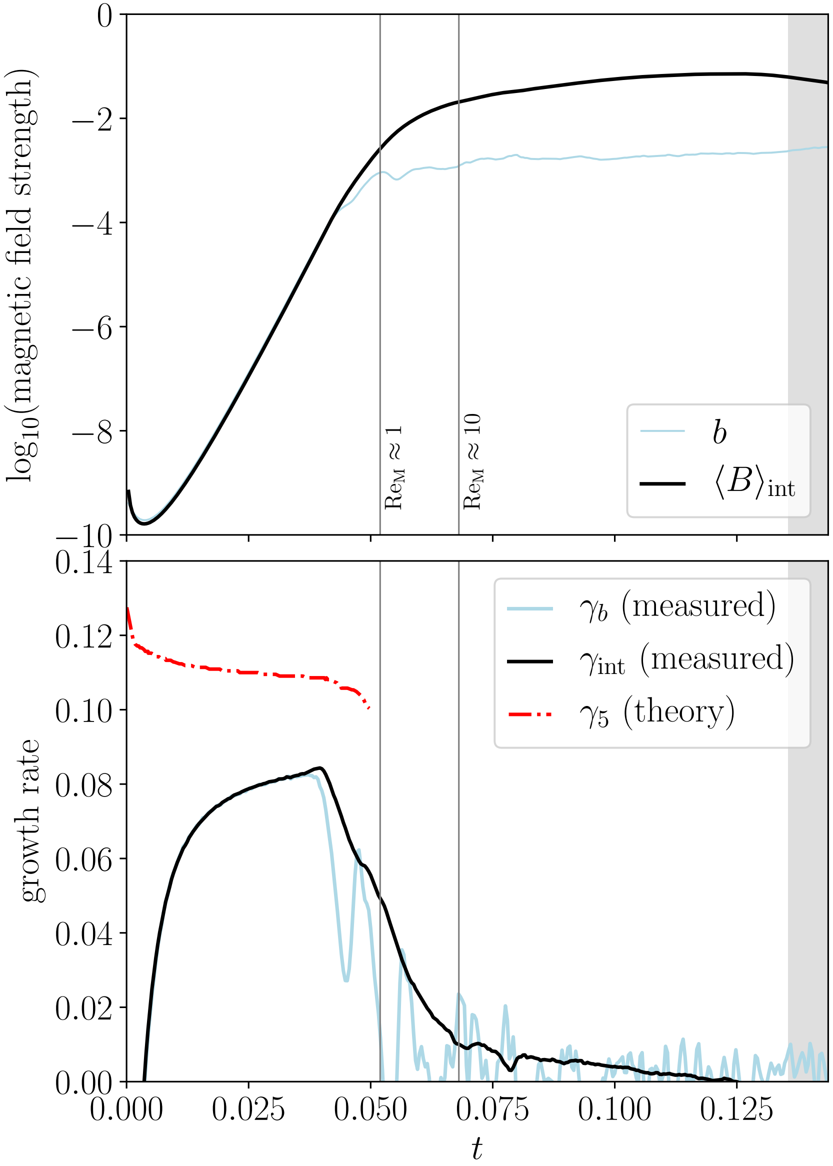

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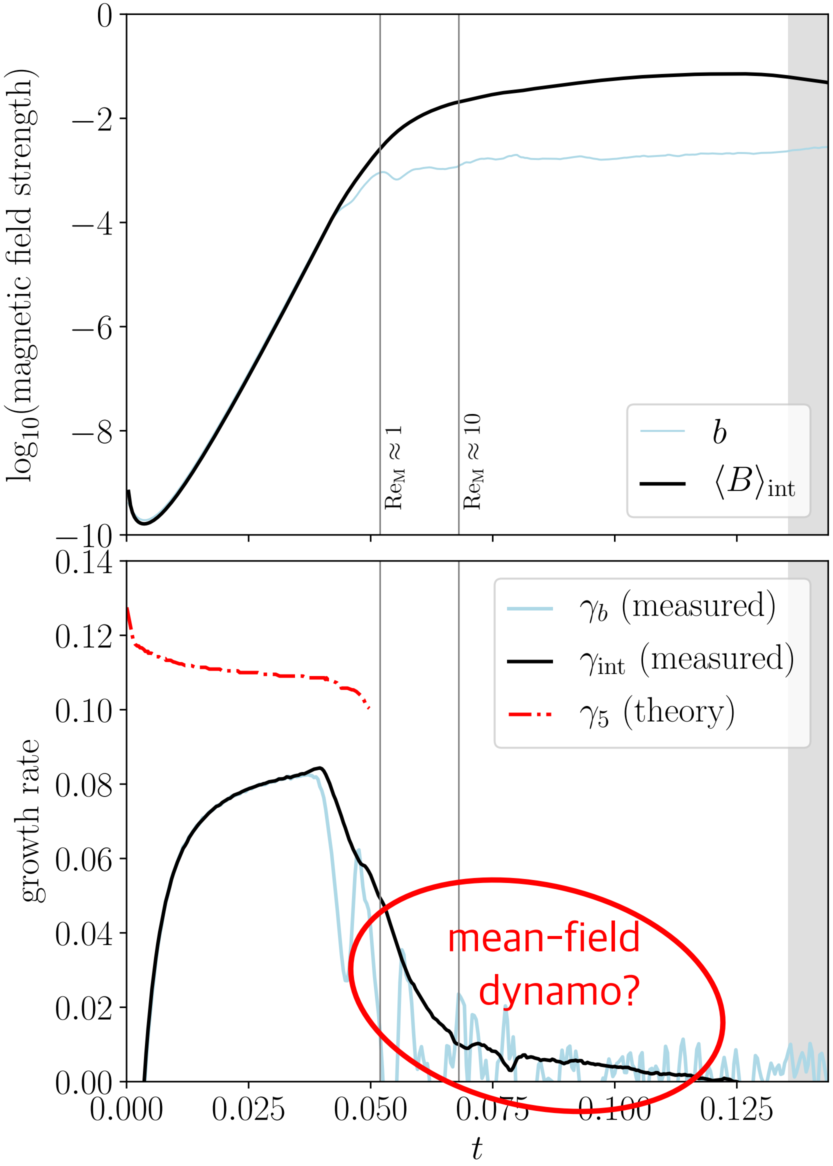

mean-field dynamo?

Inhomogeneous chiral MHD dynamos

Initial condition

- week magnetic seed field B
- large chiral asymmetry μ_5 (non-uniform, $\langle\mu_5\rangle(t_0) = 0$)

Random fluctuations spectrum:

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Mean-field dynamo analysis:

Budget equation for magnetic helicity:

$$\frac{\partial}{\partial t}\overline{\boldsymbol{a}\cdot\boldsymbol{b}} + \nabla\cdot\boldsymbol{F} = 2\eta\overline{\mu_5}\overline{\boldsymbol{b}^2} - 2\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\boldsymbol{B}} - 2\eta\,\overline{\boldsymbol{b}\,(\boldsymbol{\nabla}\times\boldsymbol{b})}$$

with $\overline{\boldsymbol{\mathcal{E}}} = \overline{\boldsymbol{u}\times\boldsymbol{b}} = \alpha_{\rm M}\overline{\boldsymbol{B}} - \eta_{\rm T}(\nabla\times\overline{\boldsymbol{B}})$.

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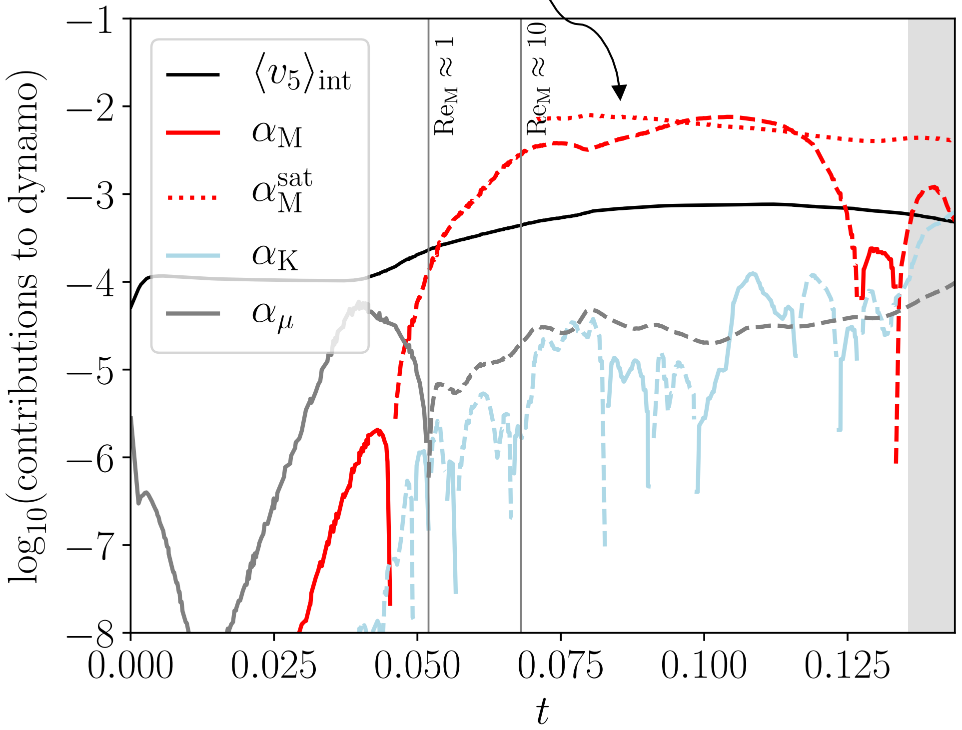

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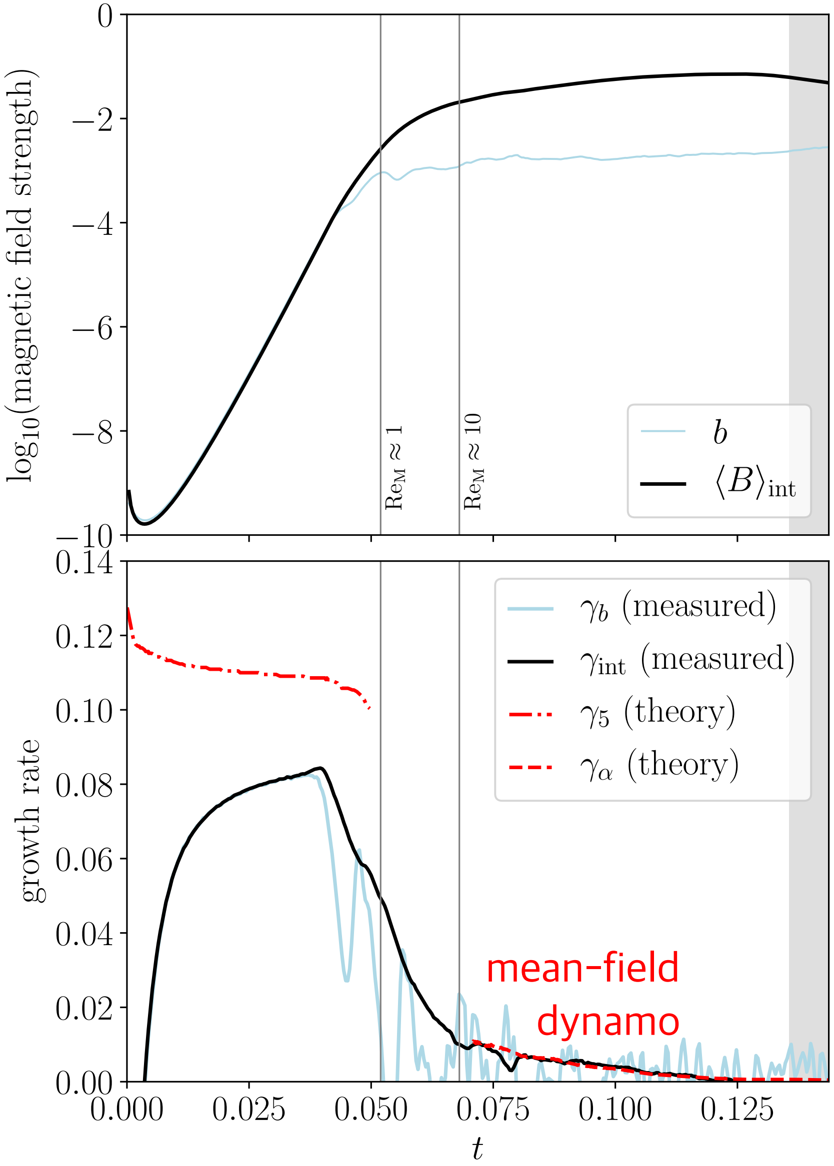

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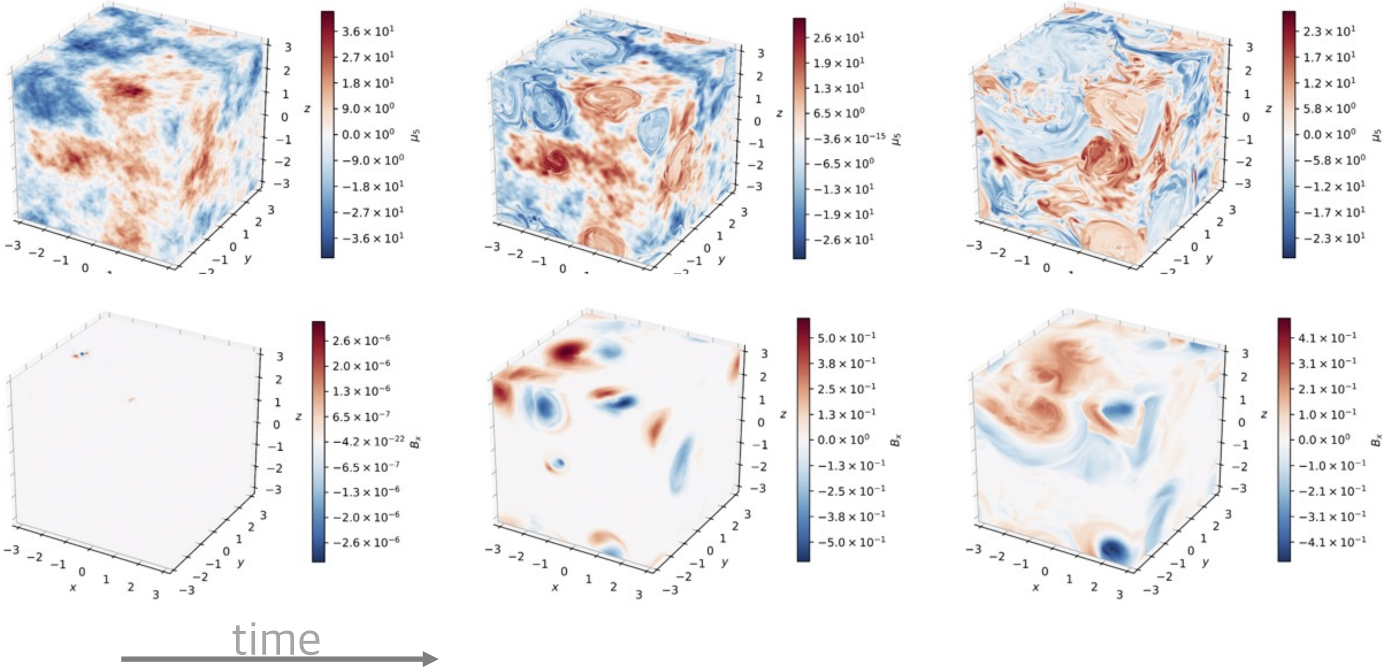

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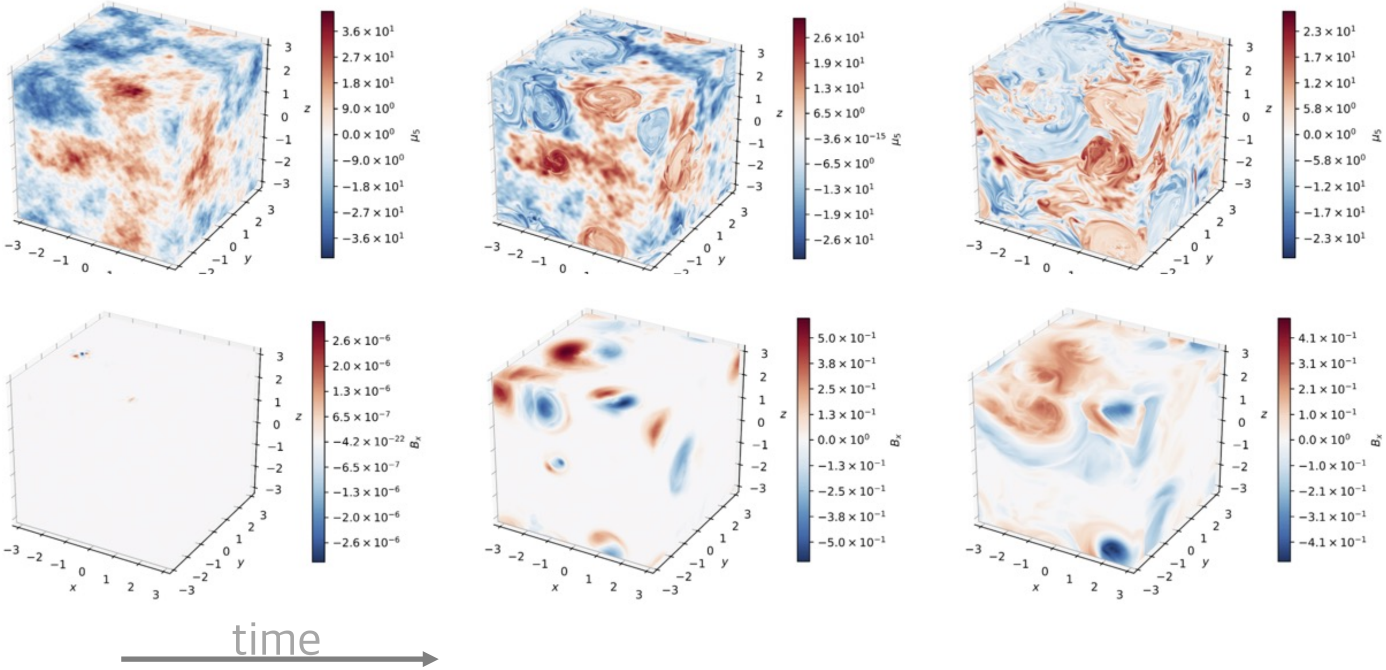

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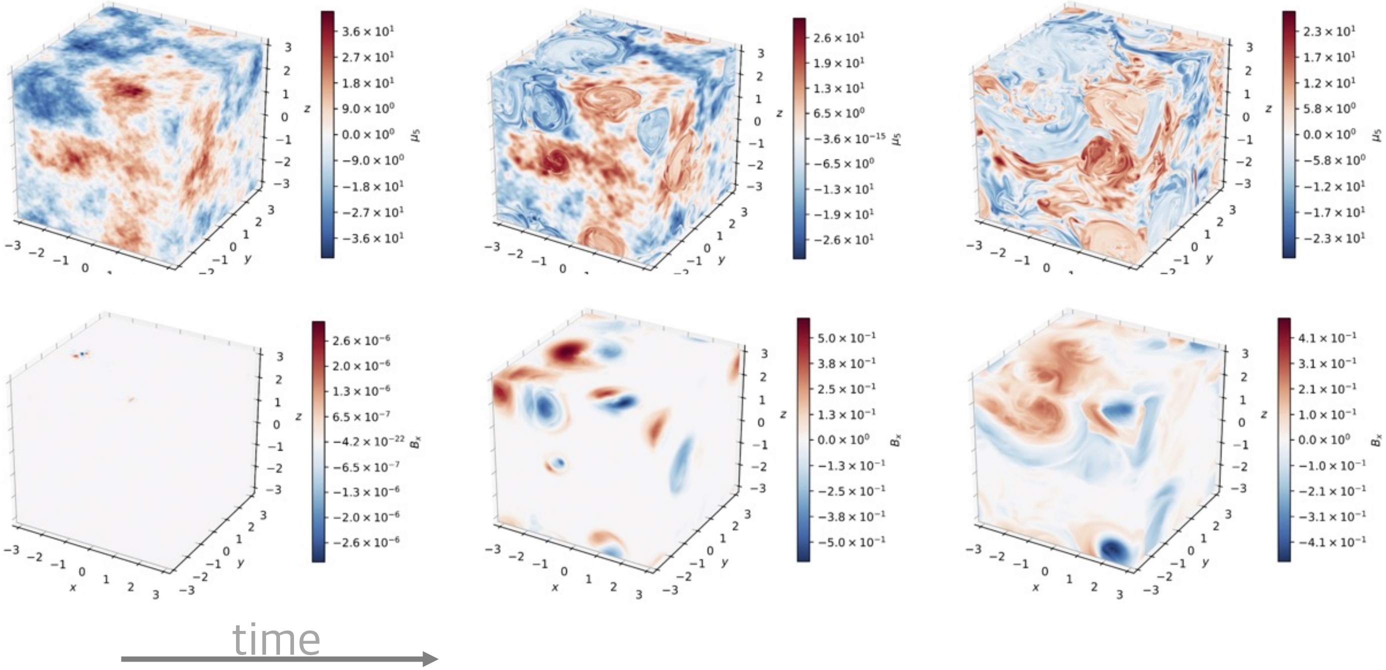

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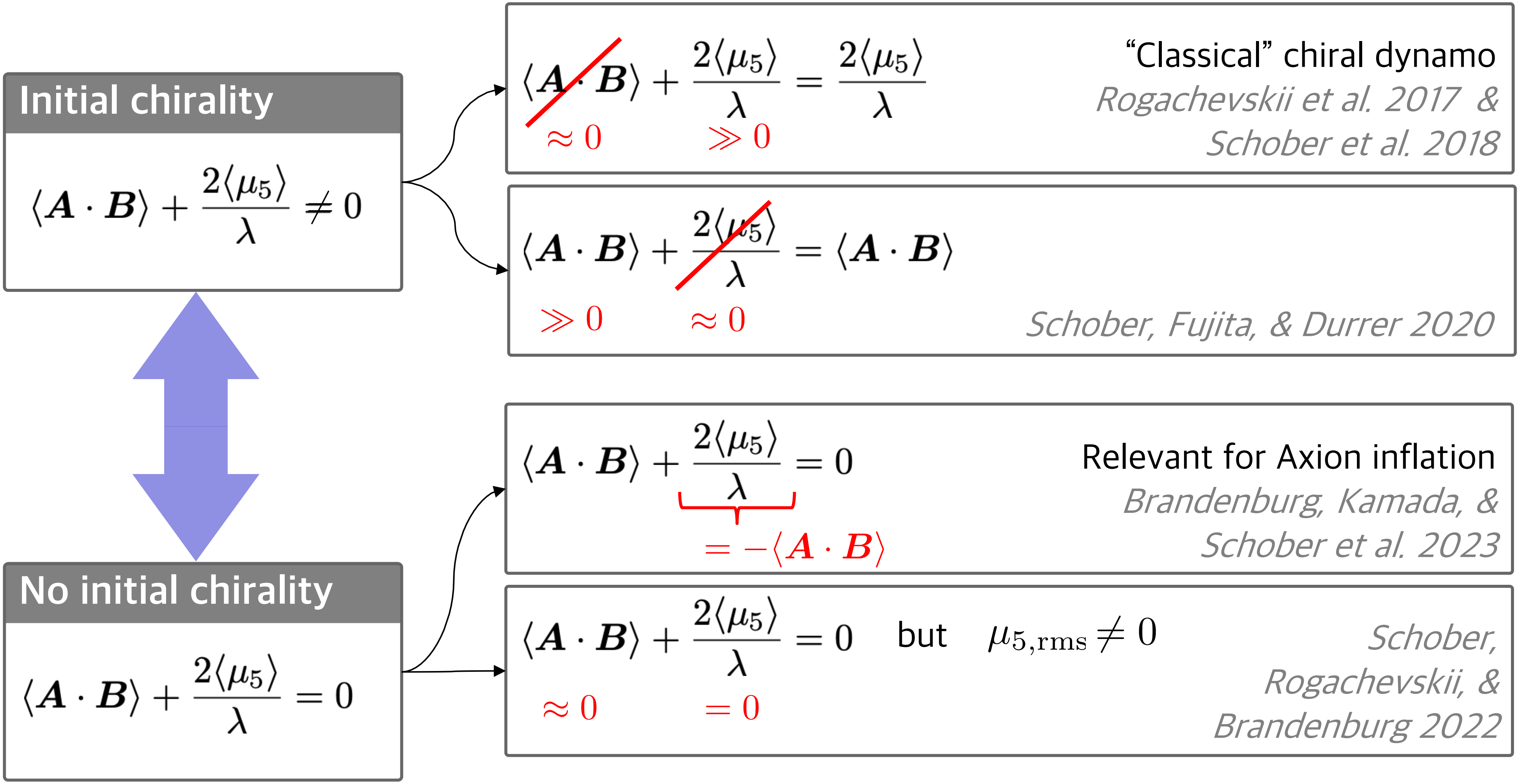
Chiral MHD scenarios



Initial chirality

$$\langle \boldsymbol{A}\cdot\boldsymbol{B}\rangle + \frac{2\langle\mu_5\rangle}{\lambda} \neq 0$$

$$\underset{\approx 0}{\langle \cancel{\boldsymbol{A}\cdot\boldsymbol{B}}\rangle} + \underset{\gg 0}{\frac{2\langle\mu_5\rangle}{\lambda}} = \frac{2\langle\mu_5\rangle}{\lambda}$$

“Classical” chiral dynamo

Rogachevskii et al. 2017 &
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Schober, Fujita, & Durrer 2020

No initial chirality

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$$\underset{\approx 0}{\langle \boldsymbol{A}\cdot\boldsymbol{B}\rangle} + \underset{= 0}{\frac{2\langle\mu_5\rangle}{\lambda}} = 0 \quad \text{but} \quad \mu_{5,\mathrm{rms}} \neq 0$$

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Chiral magnetic waves?

in prep.

Chiral magnetic waves

The chiral separation effect (CSE) describes the interaction between the chiral chemical potential μ_5 and the chemical potential

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Chiral magnetic waves

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$$\mu = \mu_{\rm R} + \mu_{\rm L}\,.$$

Full system of equations, including the CME and the CSE:

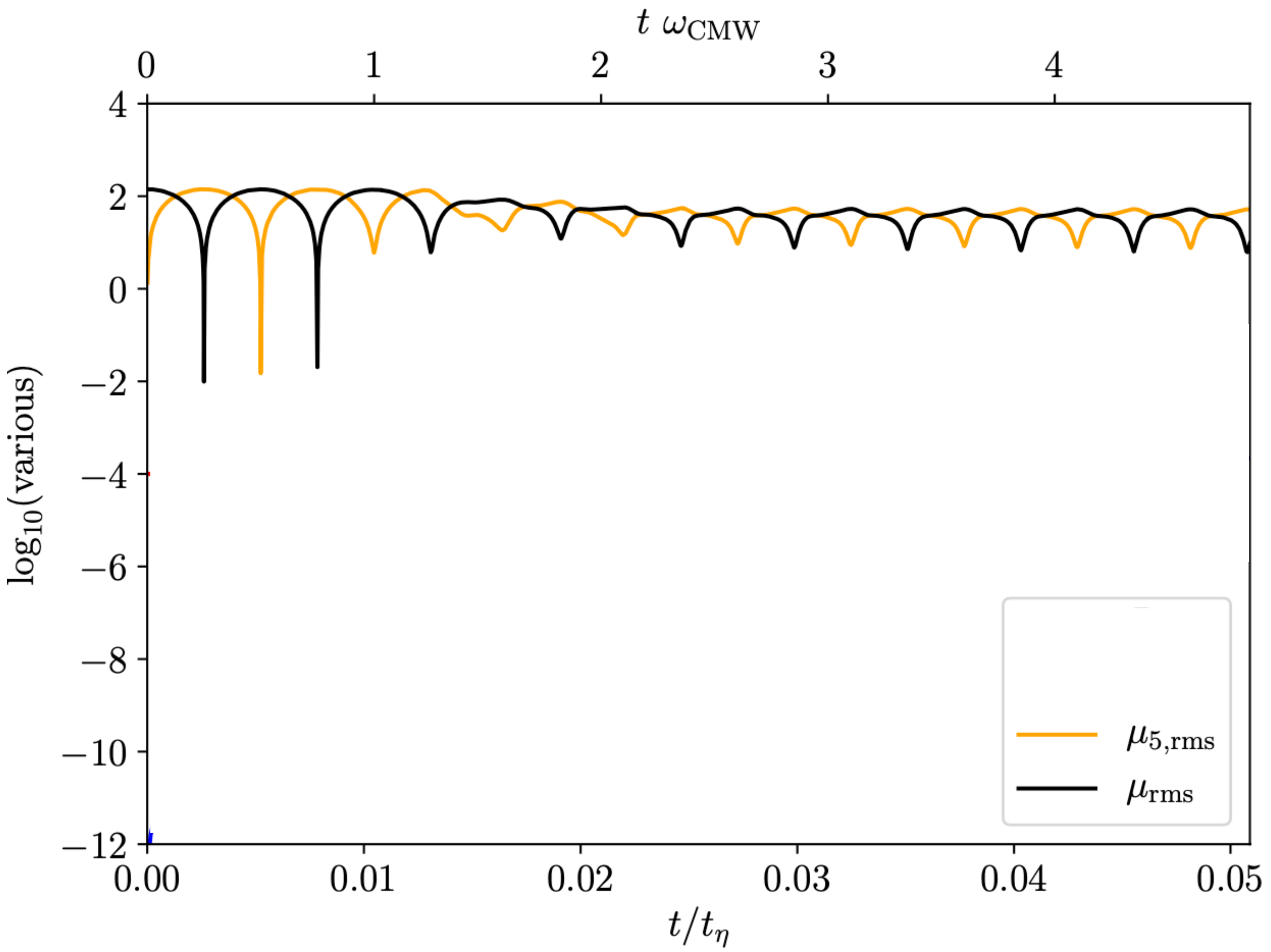
$$\begin{aligned}
\frac{\partial \boldsymbol{B}}{\partial t} &= \nabla \times \left[\boldsymbol{U} \times \boldsymbol{B} - \eta\left(\nabla \times \boldsymbol{B} - \mu_5 \boldsymbol{B}\right)\right] \\
\rho \frac{D\boldsymbol{U}}{Dt} &= (\nabla \times \boldsymbol{B}) \times \boldsymbol{B} - \nabla p + \nabla \cdot (2\nu\rho S) \\
\frac{D\rho}{Dt} &= -\rho \nabla \cdot \boldsymbol{U} \\
\frac{D\mu_5}{Dt} &= -\mathcal{D}_5 \nabla^4 \mu_5 + \lambda\eta \left[\boldsymbol{B} \cdot (\nabla \times \boldsymbol{B}) - \mu_5 \boldsymbol{B}^2\right] \boxed{- C_5 (\boldsymbol{B} \cdot \nabla)\mu} \\
&\boxed{\frac{D\mu}{Dt} = -\mathcal{D}_\mu \nabla^4 \mu - C_\mu (\boldsymbol{B} \cdot \nabla)\mu_5}
\end{aligned}$$

Kharzeev & Yee (2011)

Chiral magnetic waves

Preliminary simulations:

time normalized by CMW frequency



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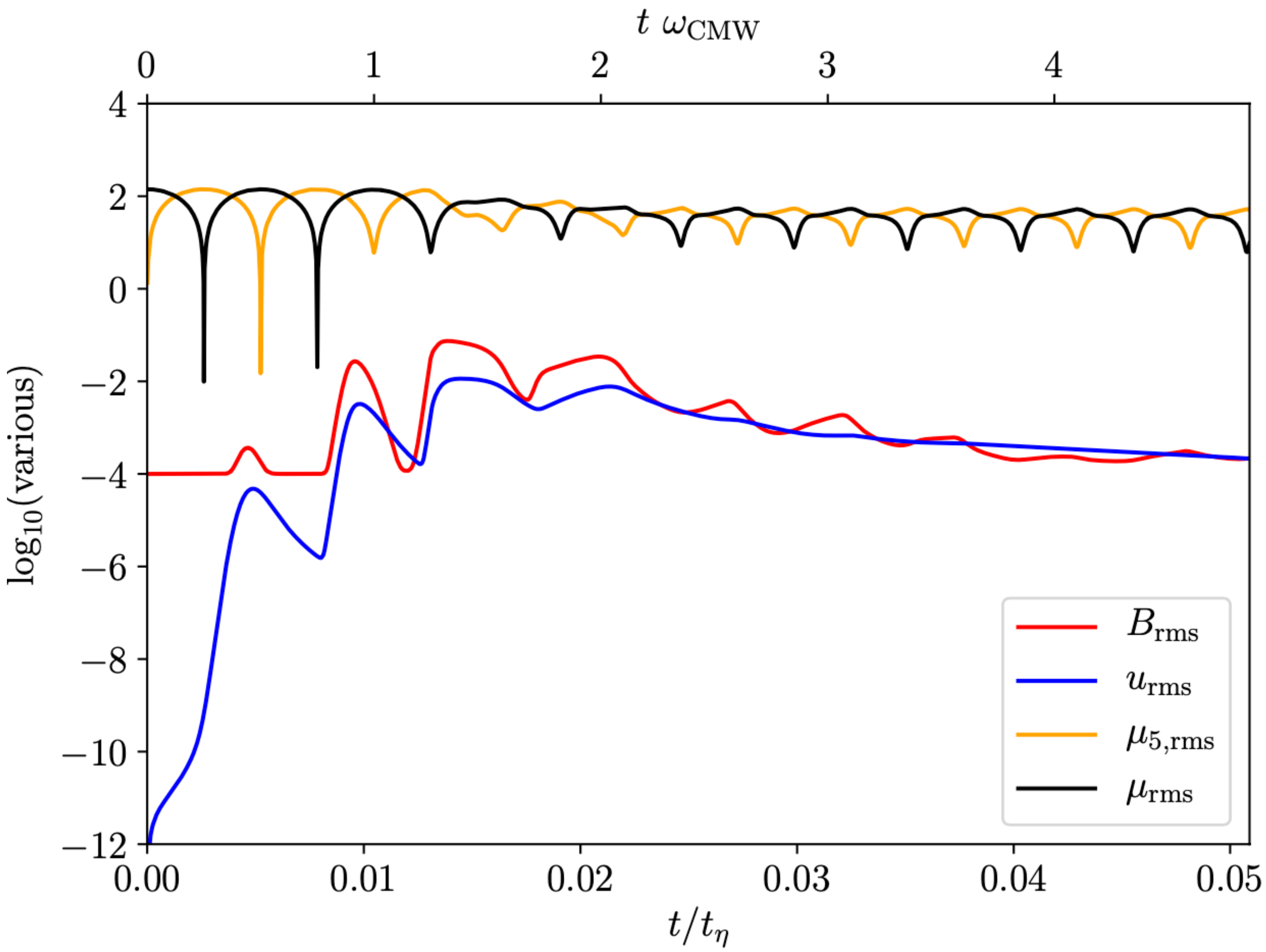

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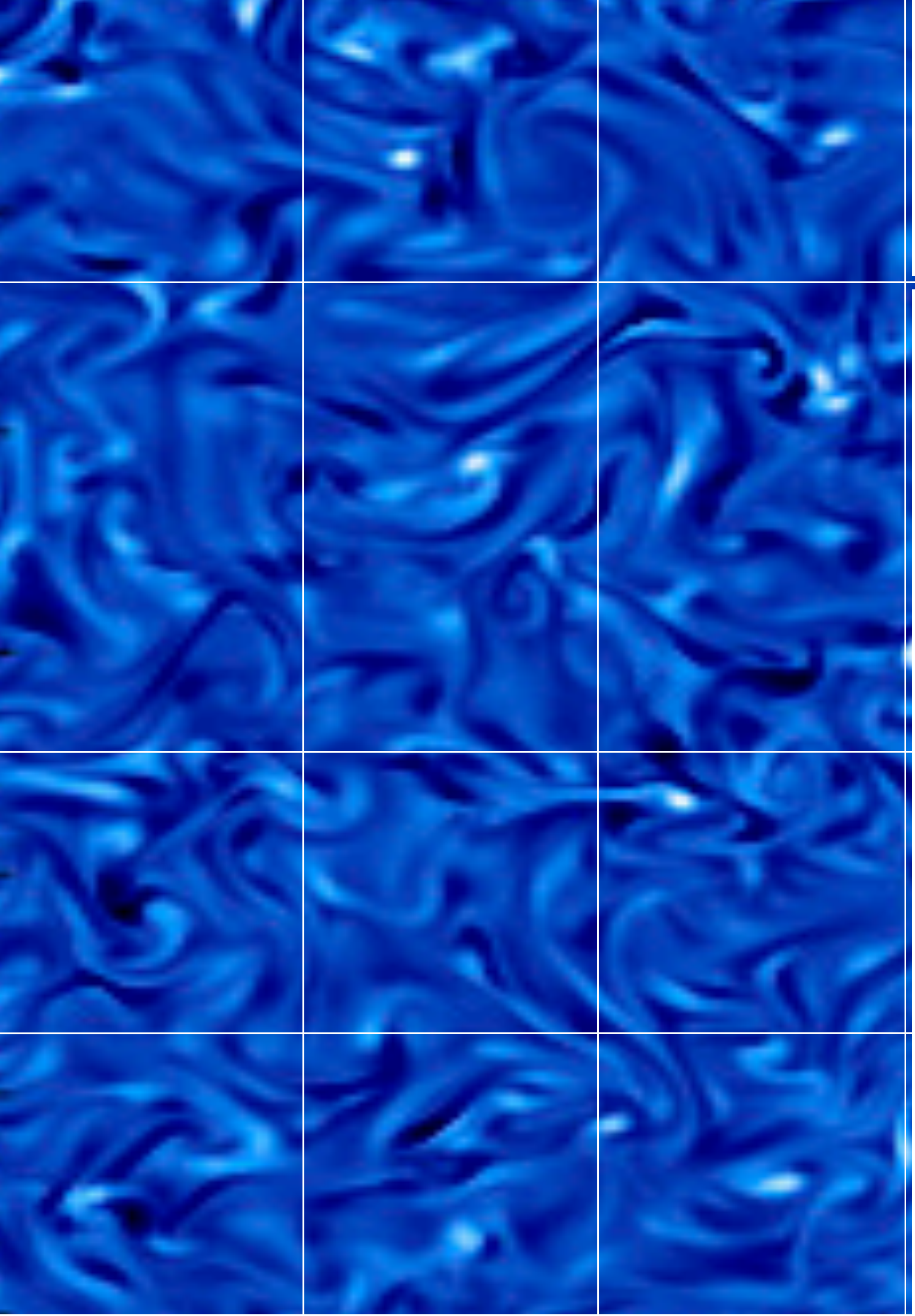


“Simulations of chiral magnetohydrodynamics”

-Outline-

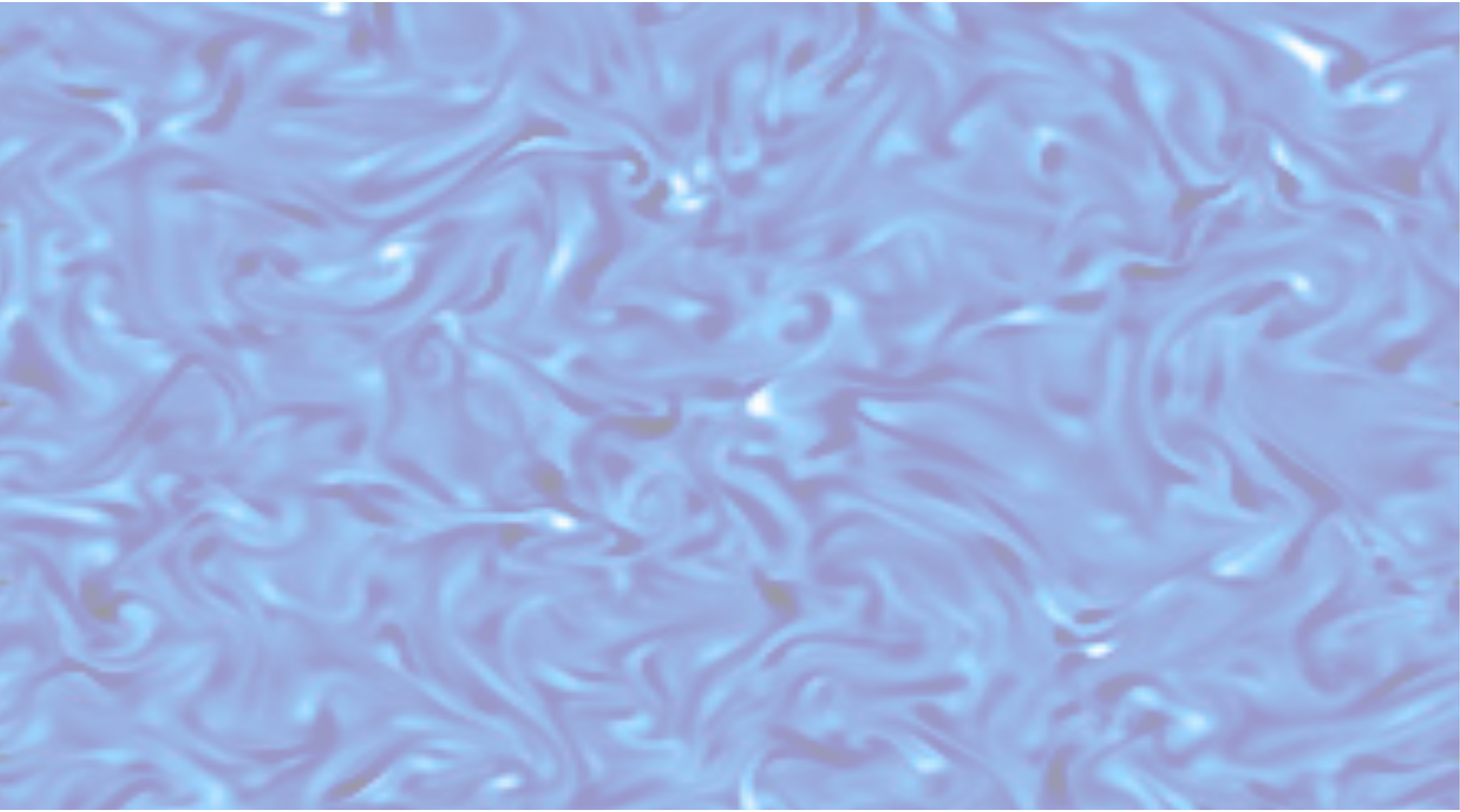
1) Motivation: Observing the Big Bang through cosmic magnetic fields

2) Chiral magnetohydrodynamics and numerical simulations of its nonlinear dynamics

3) Insights from simulations

4) **Conclusions**

Essence of this talk



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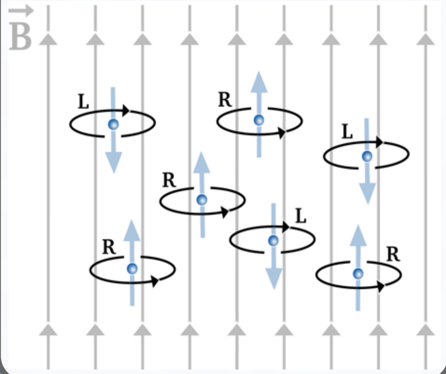
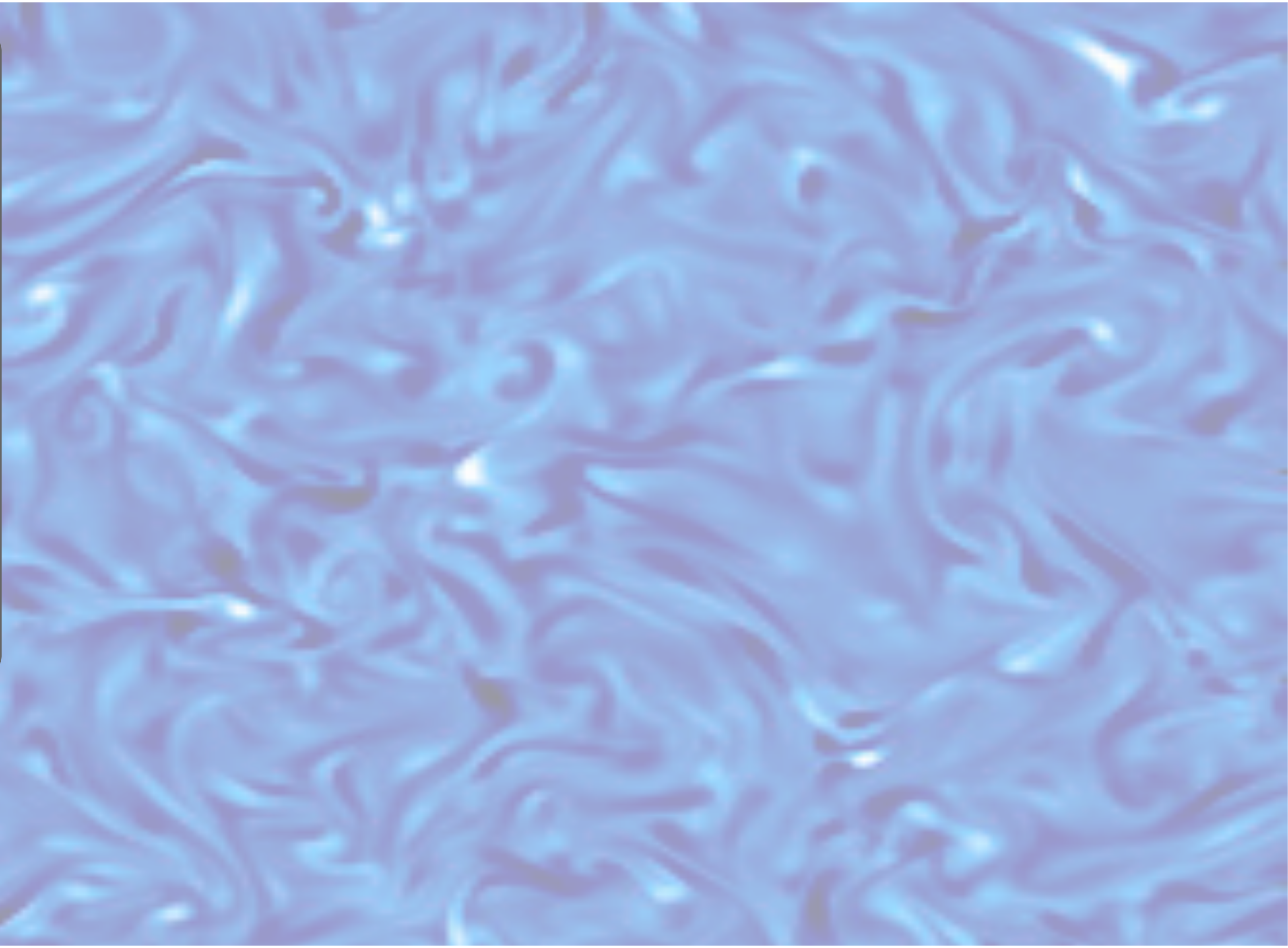
The evolution of **primordial magnetic fields** is coupled to the one of the **chiral asymmetry.**





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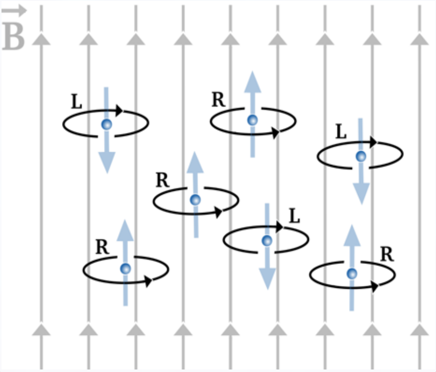

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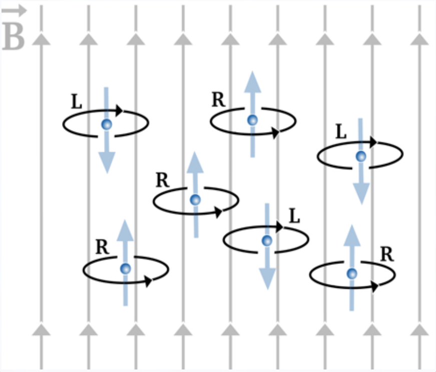

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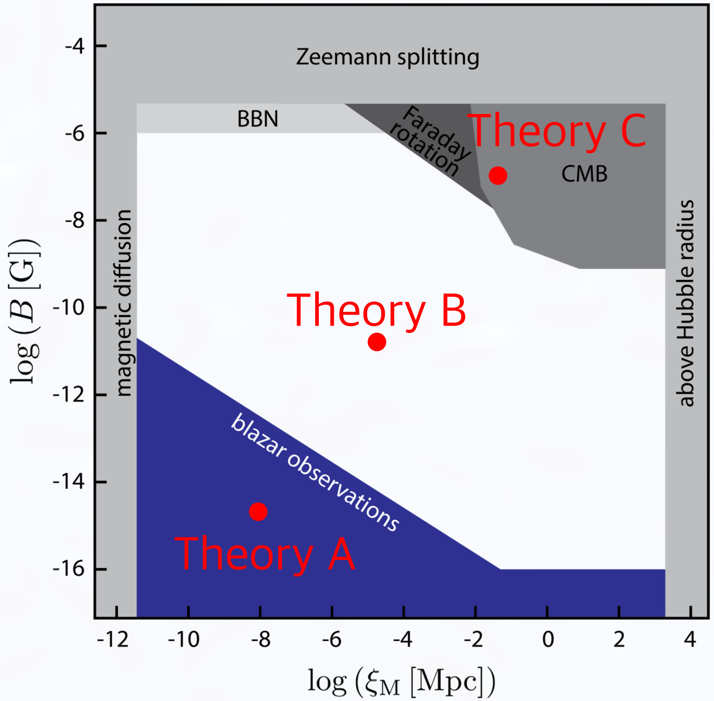

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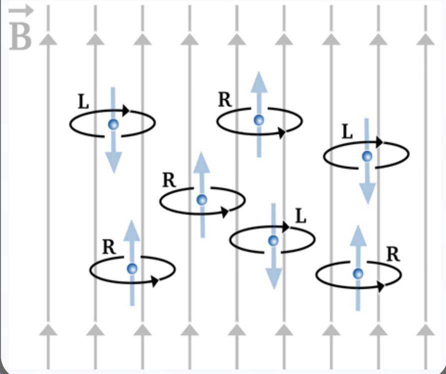

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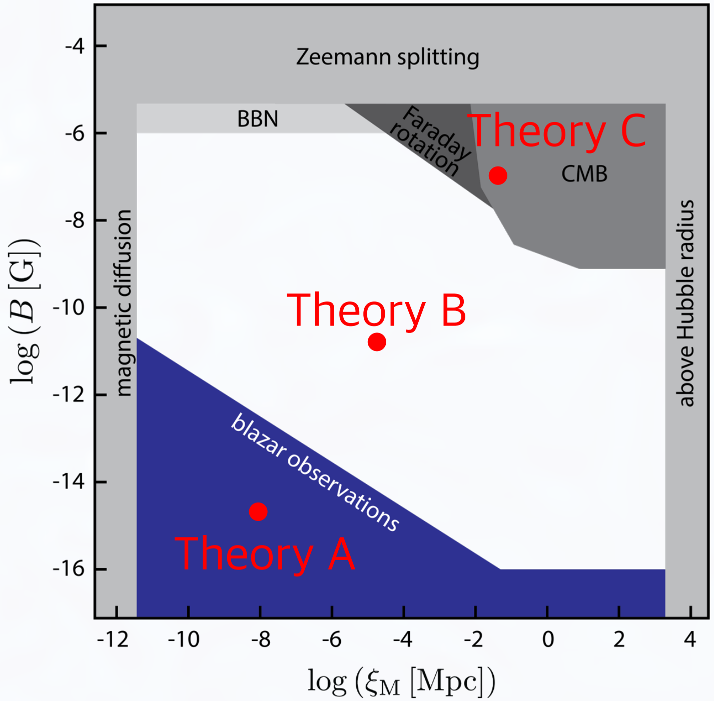

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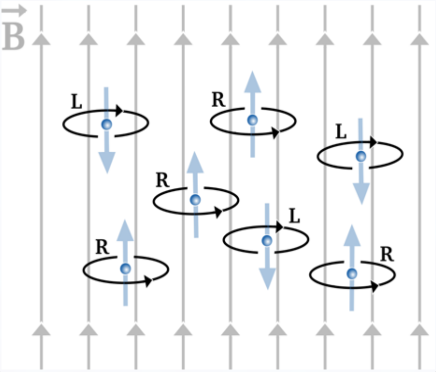

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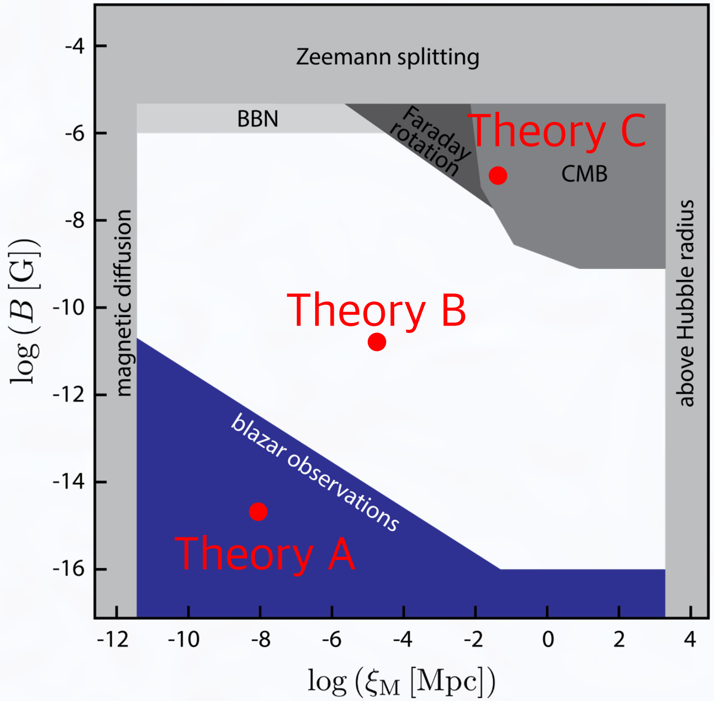

More details: Schober, Rogachevskii, & Brandenburg, Geophysical & Astrophysical Fluid Dynamics, 2020 → Technical aspects on chiral MHD simulations

Schober, Rogachevskii, & Brandenburg, PRL, 2022
Brandenburg, Kamada, & Schober, arXiv: 2302.00512
} Latest applications

Thanks for your attention!

Jennifer Schober