Role of helicities for amplification of the magnetic field

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Kei Kotake (Fukuoka University)

Outline

- Introduction
  - Core-collapse supernova
  - Key physics for magnetic field amplification in this talk

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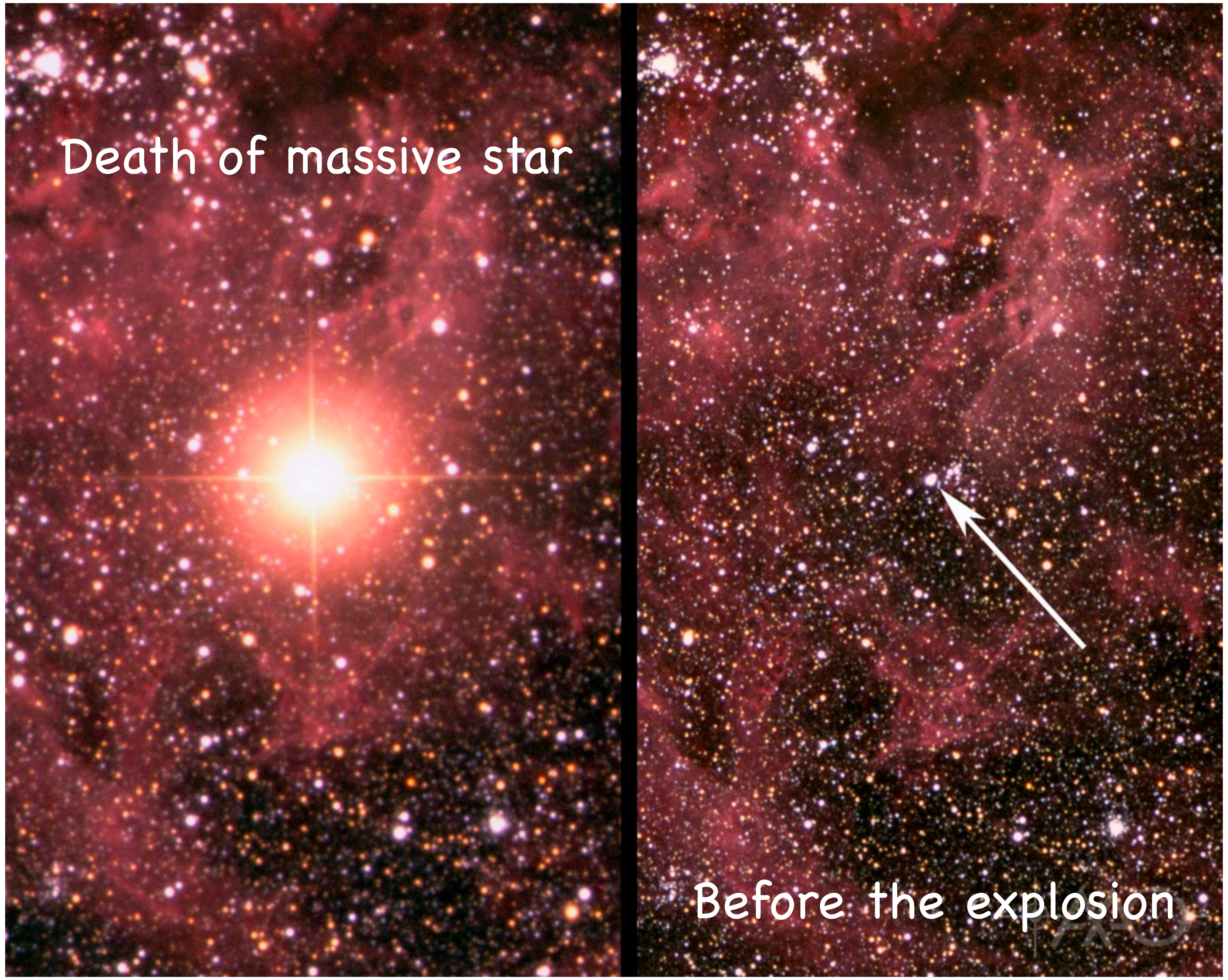
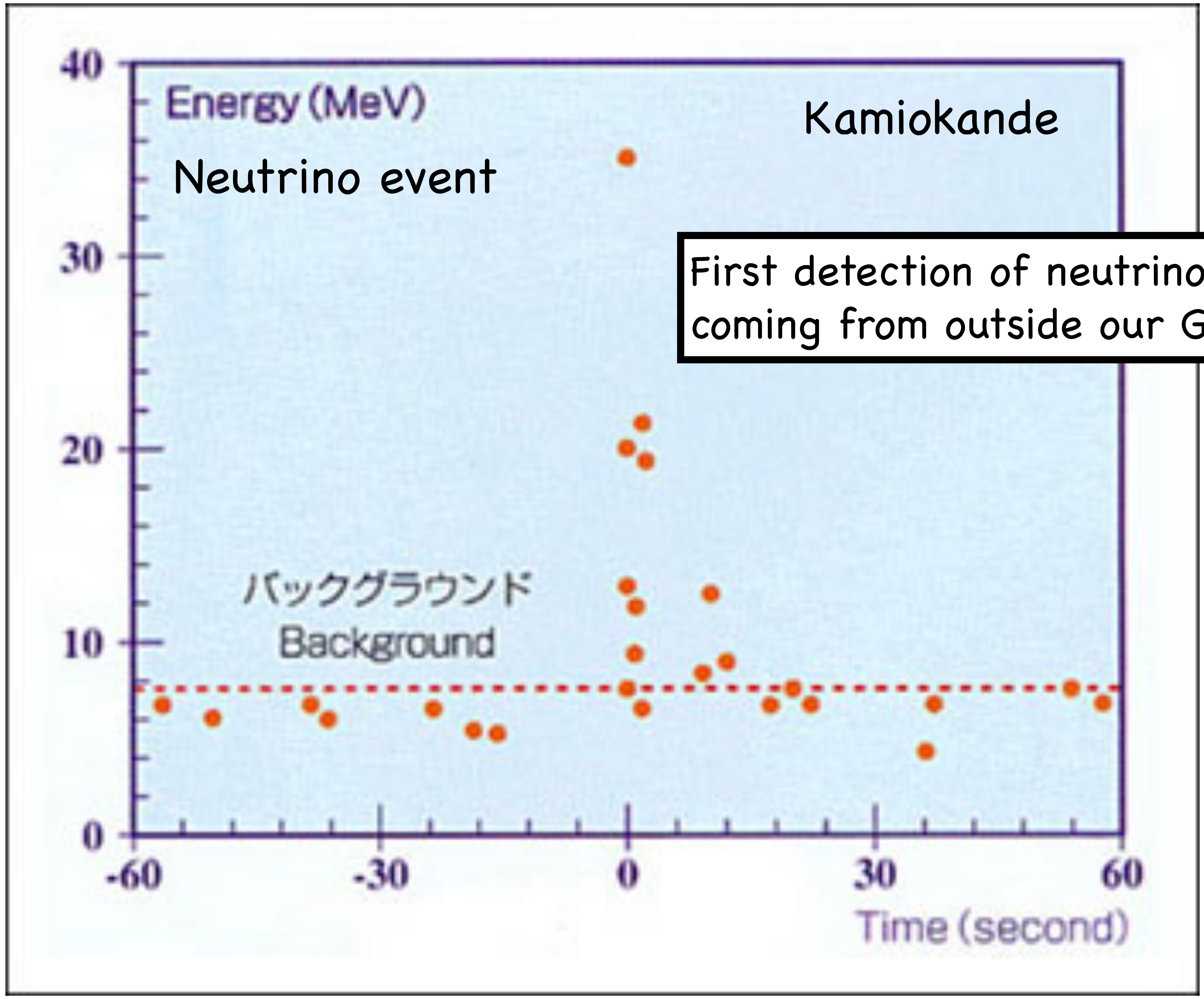
■ Chiral magnetohydrodynamic (MHD) simulations in local box

■ Global MHD simulations of core-collapse supernova

Core-collapse supernova



David Malin / Australian Astronomical Observatory



First detection of neutrinos coming from outside our Galaxy

http://www-sk.icrr.u-tokyo.ac.jp/sk/_images/photo/sk/shinsei_gazou02.jpg

- SN 1987A
  Large Magellanic Cloud (49 kpc ~ 16×10^4 light years)
- Explosion energy: ~ 10^{51} ergs

- Neutrino heating to explode massive star

Magnetic field may change explosion mechanism.

Tomoya's talk (yesterday) and Naoki's talk (this morning)

Key physics in this talk

If the magnetic field evolves following linear equation,

$$\partial_t \boldsymbol{B} = \nabla \times (\alpha \boldsymbol{B}) + \eta \Delta \boldsymbol{B}$$

the magnetic field is exponentially amplified.

α : just a coefficient

η : magnetic resistivity

Key physics in this talk

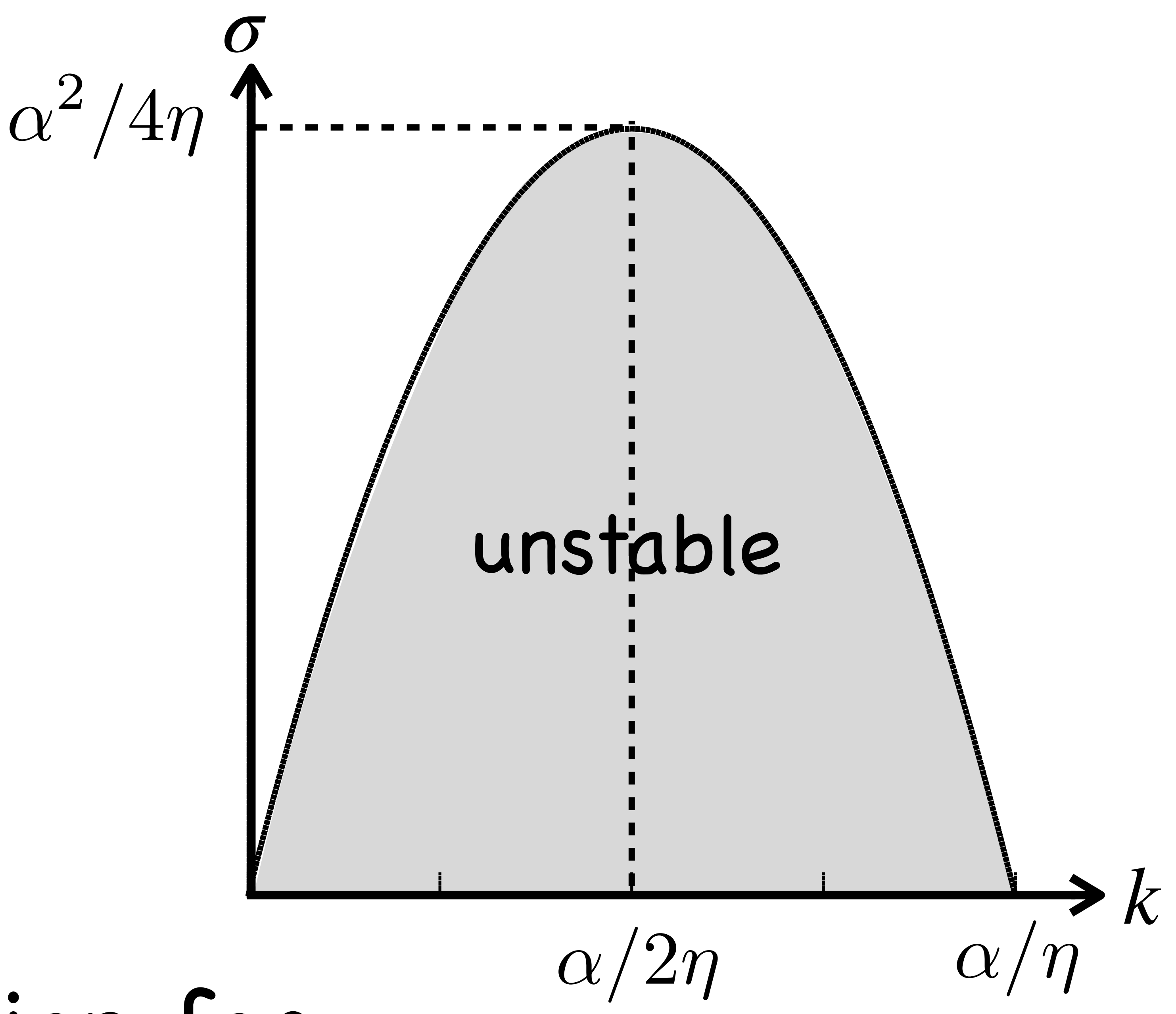
$$\partial_t \boldsymbol{B} = \nabla \times (\alpha \boldsymbol{B}) + \eta \Delta \boldsymbol{B}$$

linear analysis (WKB approximation): $\delta \boldsymbol{B} = \exp[i\boldsymbol{k} \cdot \boldsymbol{r} + \sigma t]$

- -> dispersion relation:

$$\sigma = \alpha k - \eta k^2$$
$$= \underline{-\eta[k - \alpha/(2\eta)]^2 + \alpha^2/(4\eta)}$$

parabolic equation of k



condition for
exponential growth: $\alpha/\eta > k$

Origin of α

- Chiral MHD equation (e.g., Brandenburg+17, Masada+18, Schober+22, JM+22)

induction equation: $\partial_t \boldsymbol{B} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \Delta \boldsymbol{B} + \underline{\eta \nabla \times (\xi_B \boldsymbol{B})}$

(effective) Chiral Magnetic effect
e.g., Vilenken 80, Nielsen & Ninomiya 83, Fukushima+08

- Mean-field theory of magnetic field (e.g., Brandenburg & Subramanian 05)

induction equation if we consider only turbulent component:

$$\frac{\partial \langle \boldsymbol{B} \rangle}{\partial t} = \nabla \times (\underline{\alpha} \langle \boldsymbol{B} \rangle) + \eta_t \Delta \langle \boldsymbol{B} \rangle$$

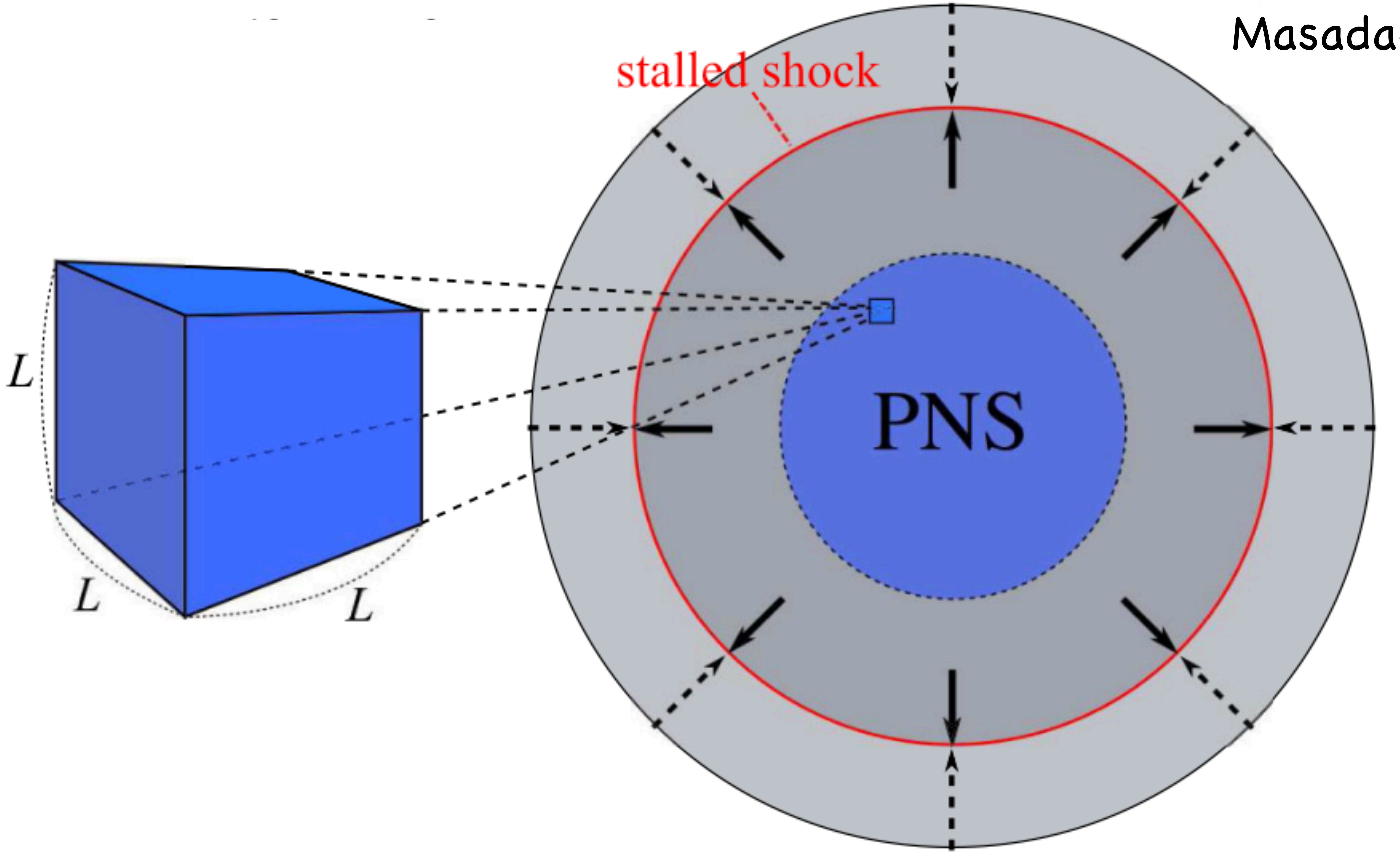
$$\boxed{\begin{aligned} \alpha &\equiv -\frac{1}{3}\tau_{\rm cor} h_{\rm K} \\ \eta_t &\equiv \frac{1}{3}\tau_{\rm cor}\langle v'^2 \rangle \end{aligned}}$$

kinetic helicity in rotating convection system

- Chiral magnetohydrodynamic (MHD) simulations in local box in the context of core-collapse supernova (CCSN)

Chiral MHD simulations

Chiral MHD simulations in the context of CCSN (Masada+18, JM22)



Masada+18

Basic equations for chiral MHD

conservative form

JM+22

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0 ,$$

$$\frac{\partial}{\partial t}(\rho \boldsymbol{v}) + \nabla \cdot \left[\rho \boldsymbol{v}\boldsymbol{v} - \boldsymbol{B}\boldsymbol{B} + \left(P + \frac{B^2}{2}\right)\mathbf{I}\right] = \boldsymbol{S} ,$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \frac{1}{\Gamma - 1}P + \frac{B^2}{2}\right) + \nabla \cdot \left[\left(\frac{1}{2}\rho v^2 + \frac{\Gamma}{\Gamma - 1}P\right)\boldsymbol{v} + \boldsymbol{E} \times \boldsymbol{B}\right] = \boldsymbol{S} \cdot \boldsymbol{v} - \boldsymbol{J}_{\mathrm{CME}} \cdot \boldsymbol{E} ,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \Delta \boldsymbol{B} + \eta \nabla \times (\xi_B \boldsymbol{B}) ,$$

$$\boxed{\xi_B = \frac{1}{4}\left(\frac{3}{\pi^4}\right)^{1/3}\left[(n_{\mathrm{e}} + n_{5,\mathrm{eff}})^{1/3} - (n_{\mathrm{e}} - n_{5,\mathrm{eff}})^{1/3}\right]}$$

$$\frac{\partial n_{5,\,\mathrm{eff}}}{\partial t} = \frac{1}{2\pi^2}\boldsymbol{E} \cdot \boldsymbol{B} ,$$

← effective chiral charge

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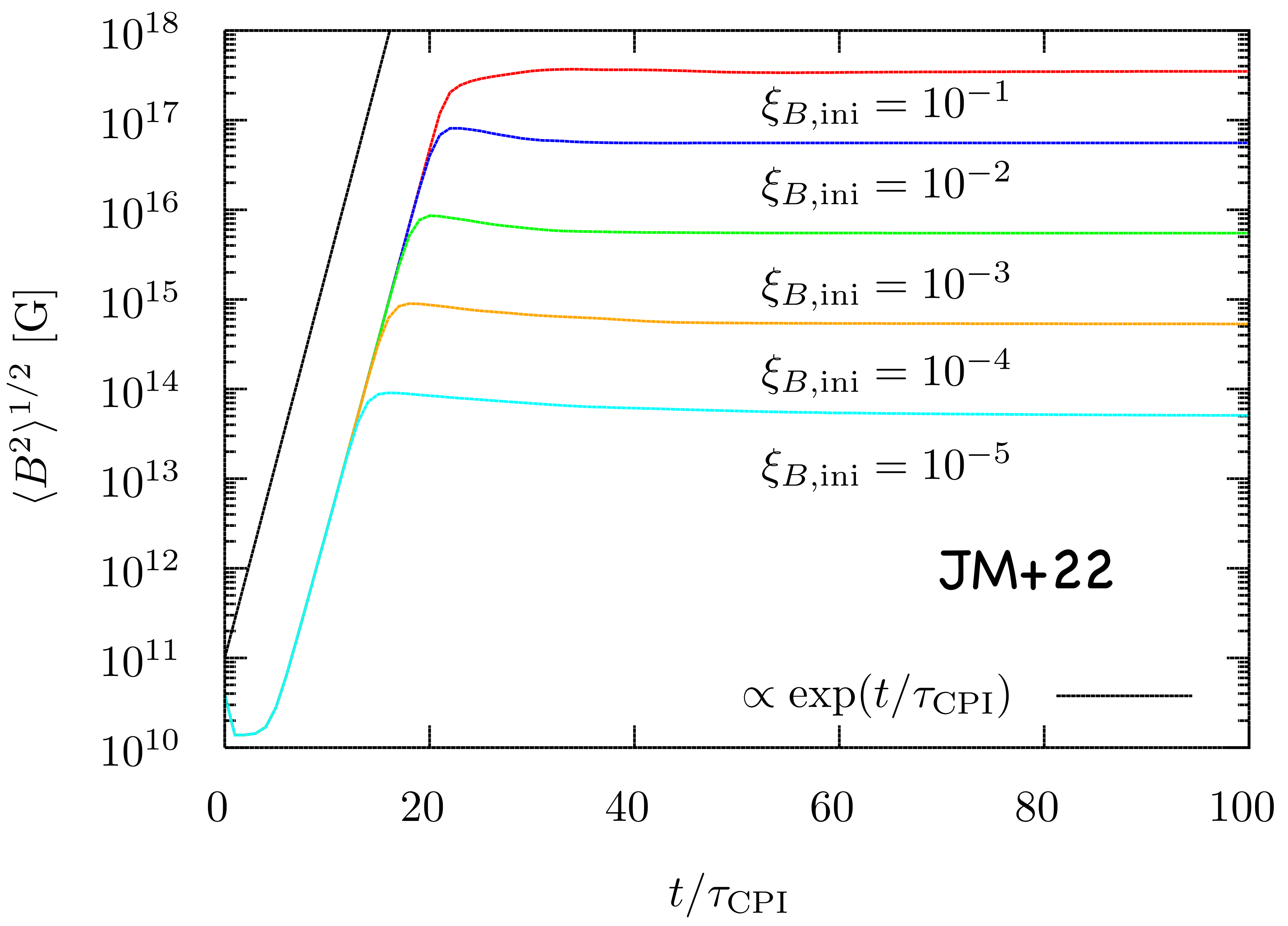
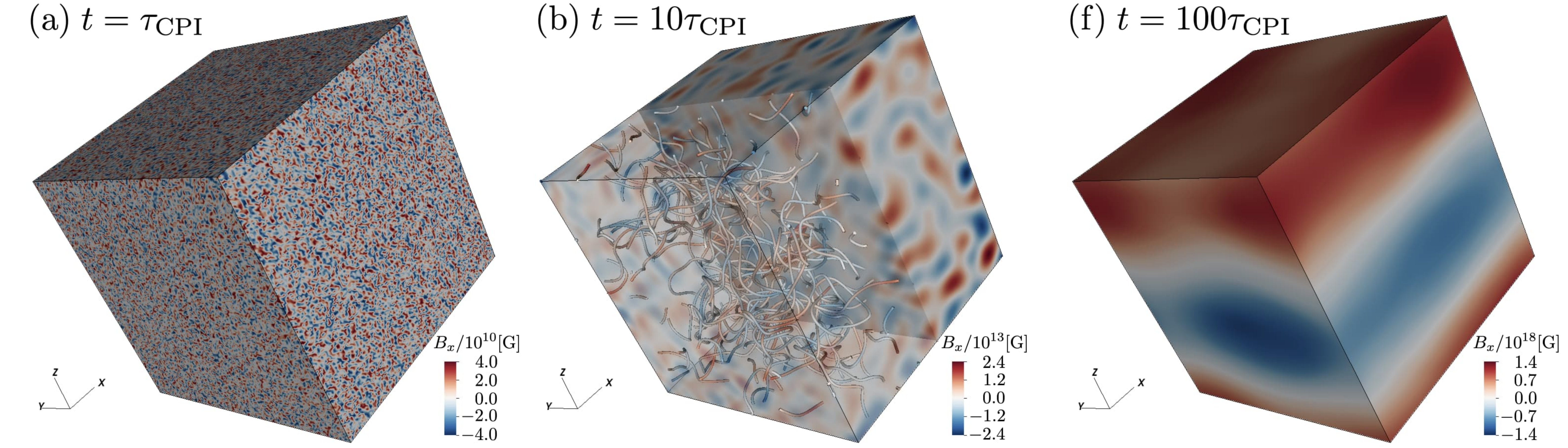
conservation of total helicity: $\frac{d}{dt}\left(Q_5 + \frac{H_{\mathrm{mag}}}{4\pi^2}\right) = 0$ $\quad H_{\mathrm{mag}} \equiv \int d^3\boldsymbol{x}\boldsymbol{A}\cdot\boldsymbol{B}$

Chiral MHD simulations

Chiral MHD simulations in the context of CCSN (Masada+18, JM22)



fm scale



Chiral plasma instability (CPI, Akamatsu+13):

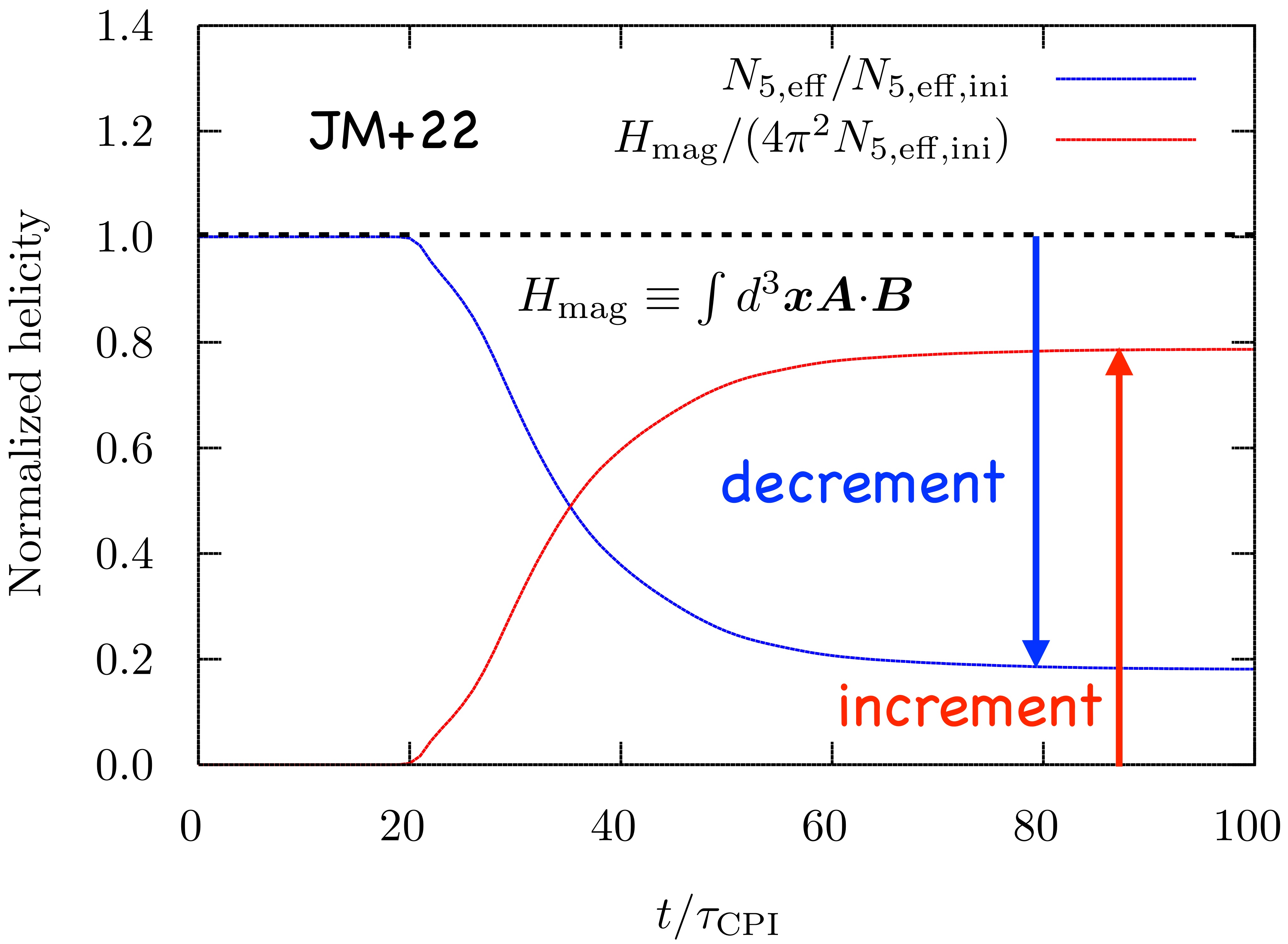
Exponential amplification of magnetic field

Correlation length of magnetic field becomes larger over time.

Important feature for CCSN but its mechanism is unclear.

Chiral MHD simulations

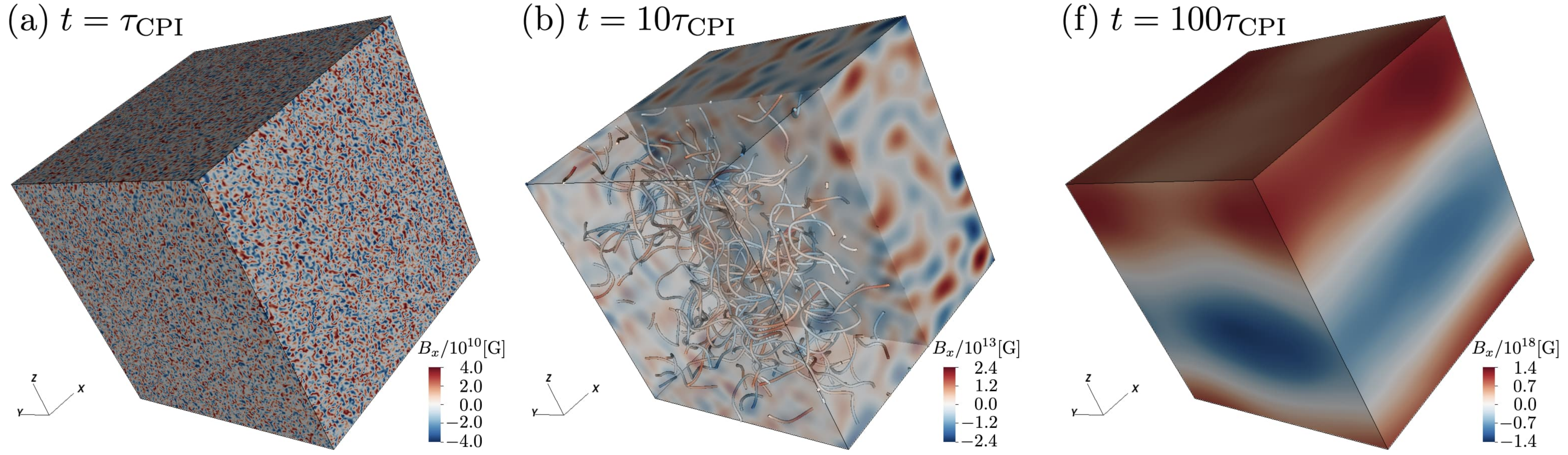
Chiral MHD simulations in the context of CCSN (Masada+18, JM22)



conservation of total helicity:

$$\frac{d}{dt}\left(Q_5 + \frac{H_{\mathrm{mag}}}{4\pi^2}\right) = 0$$

fm scale



Correlation length of magnetic field becomes larger over time.

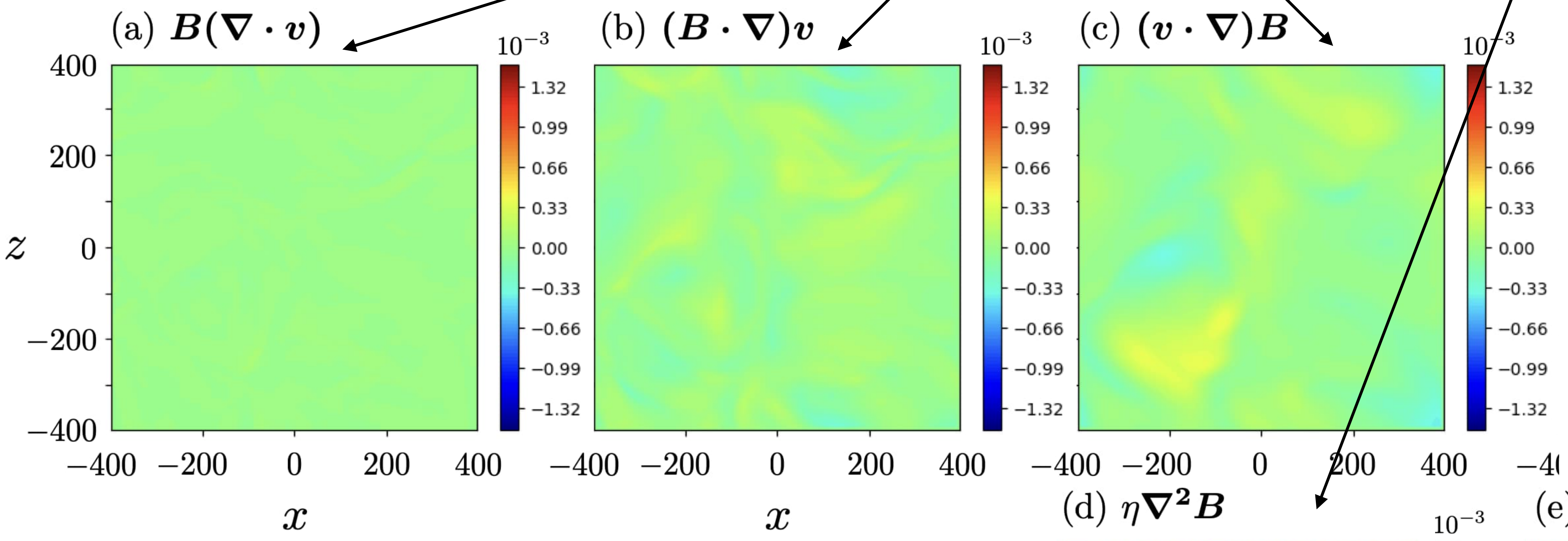
Important feature for CCSN but its mechanism is unclear.

Contribution for evolution of the magnetic field

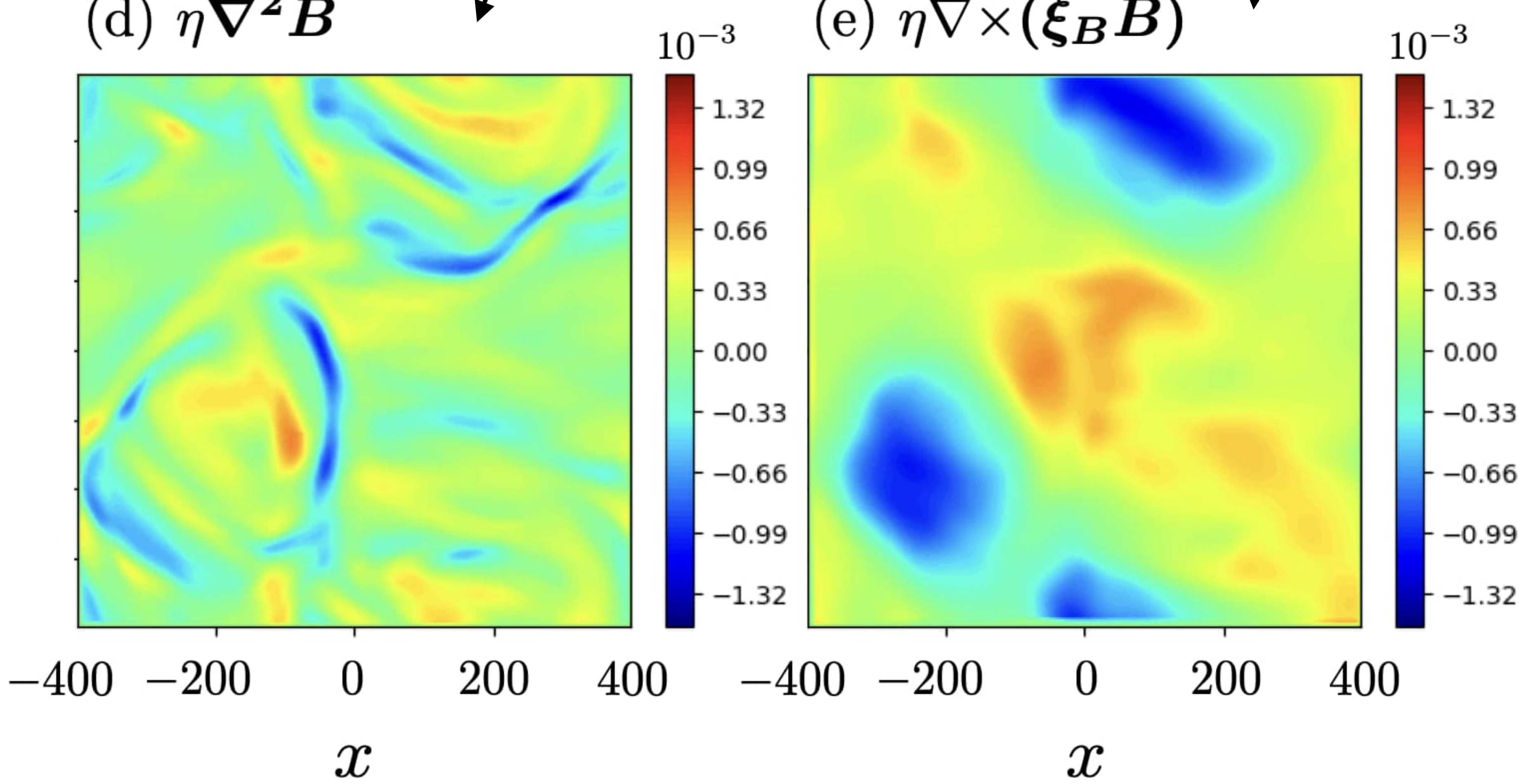
$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{v}(\cancel{\nabla\cdot}\boldsymbol{B}) - \boldsymbol{B}(\nabla\cdot\boldsymbol{v}) + (\boldsymbol{B}\cdot\nabla)\boldsymbol{v} - (\boldsymbol{v}\cdot\nabla)\boldsymbol{B} + \eta\Delta\boldsymbol{B} + \eta\,\mathrm{rot}(\xi_B\boldsymbol{B})$$

compression stretching advection diffusion CME

JM+22



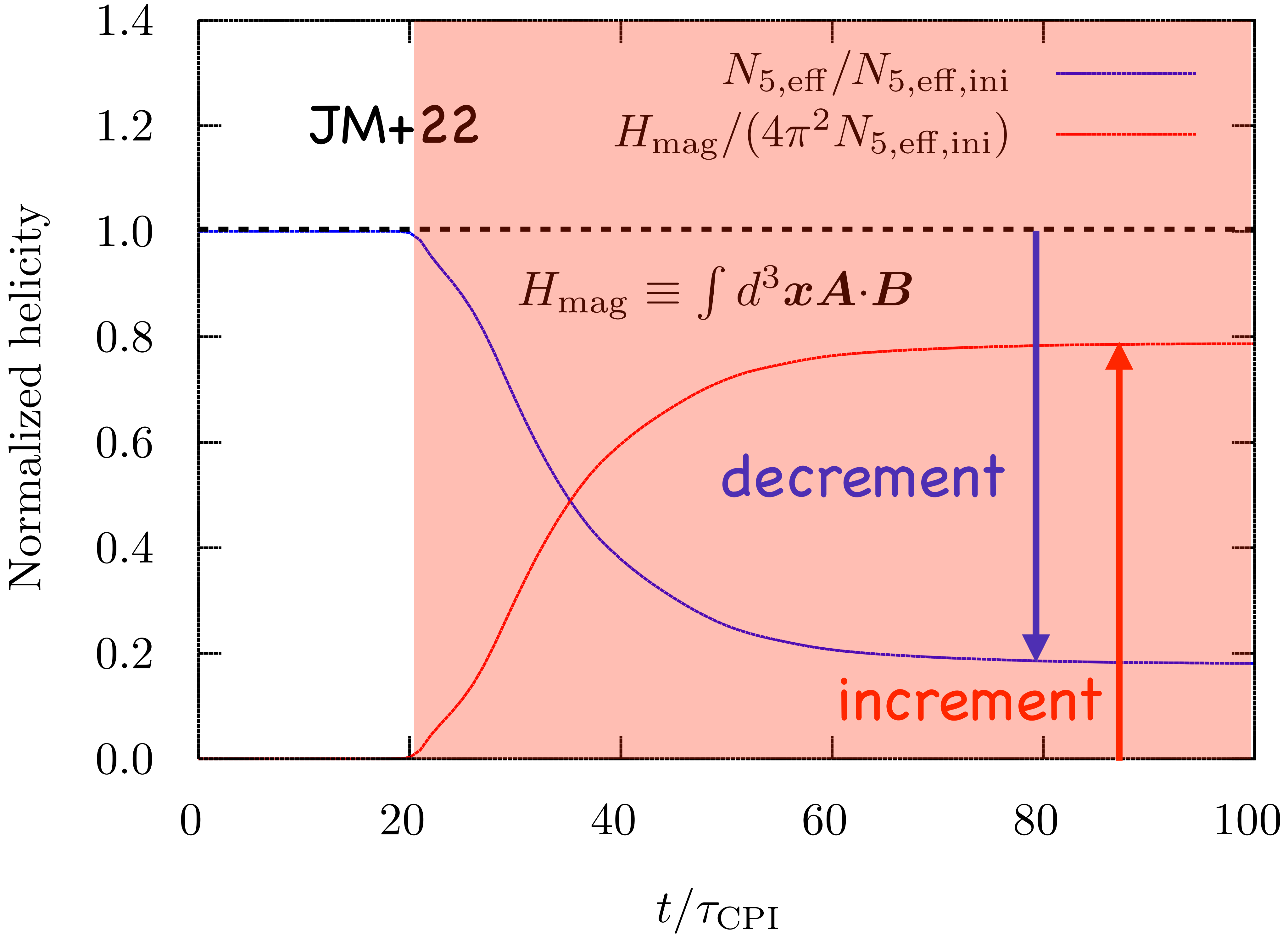
Contribution of nonlinear terms, $\nabla\times(\boldsymbol{v}\times\boldsymbol{B})$, for the evolution of the magnetic field is less than that of CME.



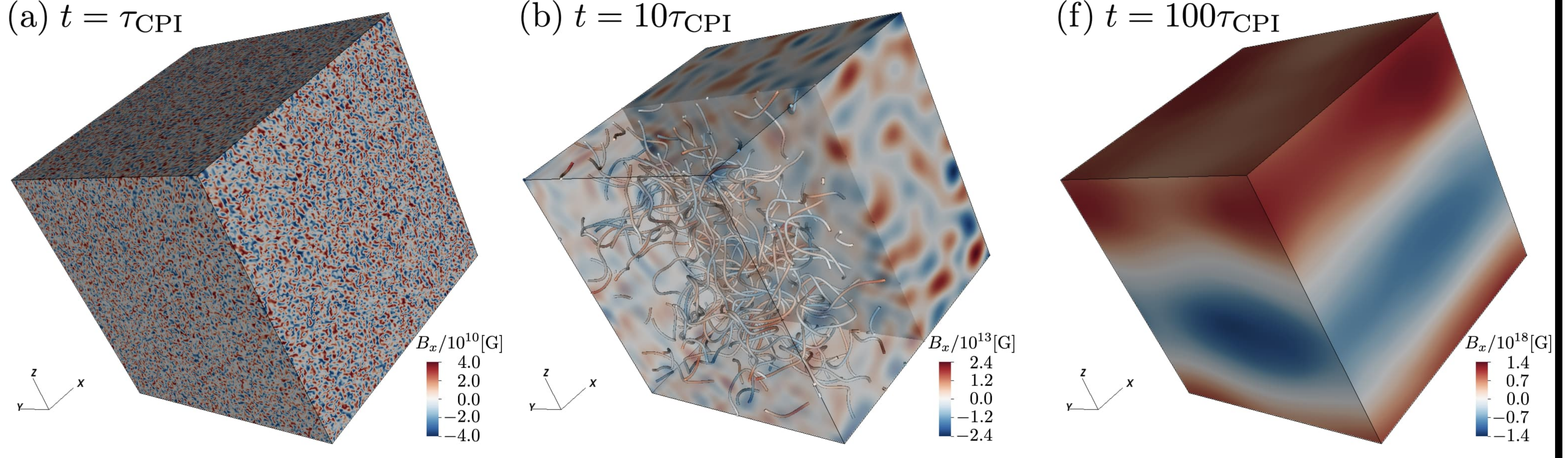
induction equation: $\partial_t B = \cancel{\nabla\times(v\times B)} + \eta\Delta\boldsymbol{B} + \eta\nabla\times(\xi_B\boldsymbol{B})$

Condition for inverse cascade of magnetic field

Chiral MHD simulations in the context of CCSN (Masada+18, JM22)



fm scale



Correlation length of magnetic field becomes larger over time.

Even in the nonlinear phase,

$$\partial_t \boldsymbol{B} = \cancel{\nabla\times(\boldsymbol{v}\times\boldsymbol{B})} + \eta\Delta\boldsymbol{B} + \eta\nabla\times(\xi_B\boldsymbol{B})$$

is governing equation.

Since ξ_B decreases as time passes, typical wavelength of CPI becomes large.

$$\lambda_{\mathrm{CPI}} \equiv 2\pi/k_{\mathrm{CPI}} = 4\pi/\xi_B$$

Key physics in this talk

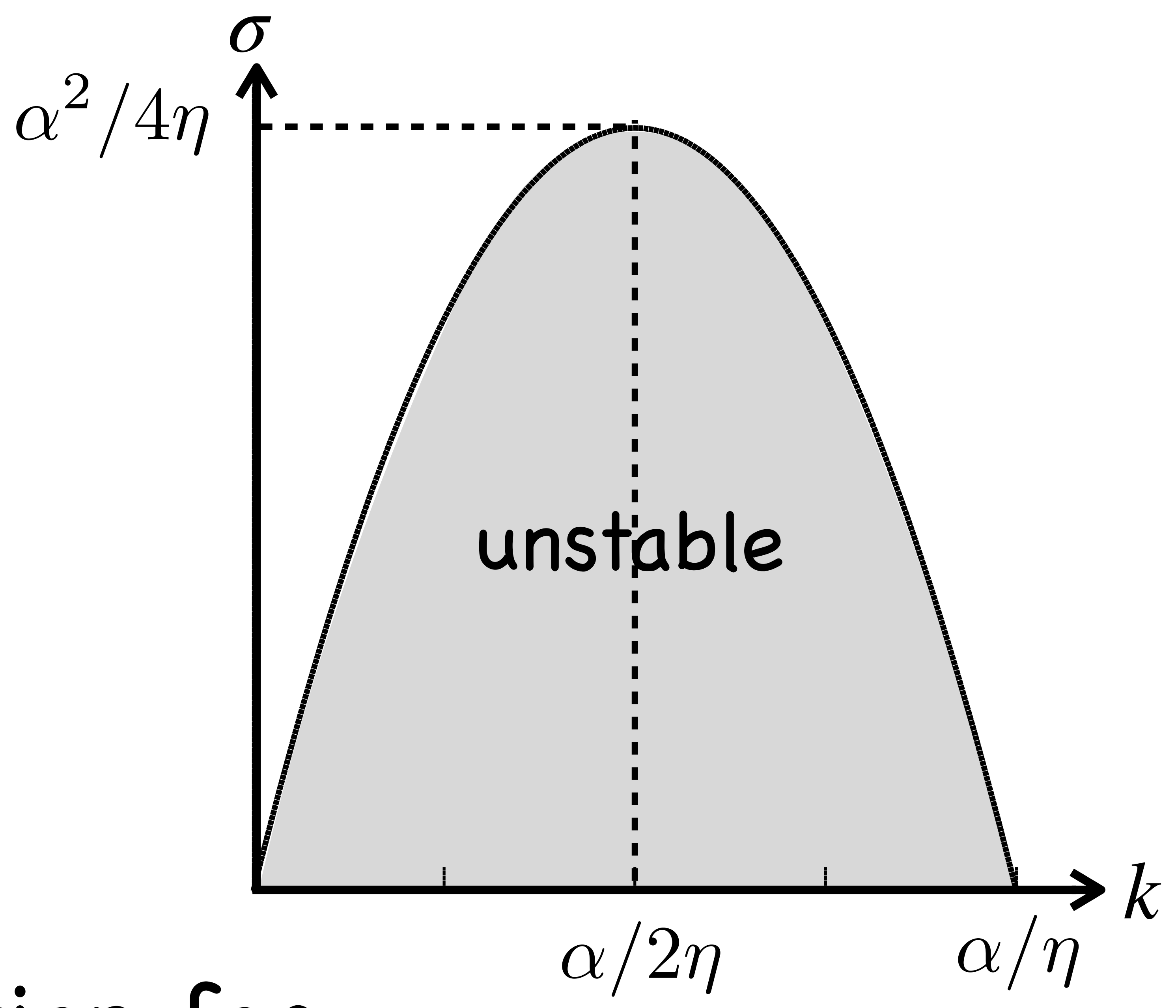
$$\partial_t \boldsymbol{B} = \nabla \times (\alpha \boldsymbol{B}) + \eta \Delta \boldsymbol{B}$$

linear analysis (WKB approximation): $\delta \boldsymbol{B} = \exp[i\boldsymbol{k} \cdot \boldsymbol{r} + \sigma t]$

- -> dispersion relation:

$$\sigma = \alpha k - \eta k^2$$
$$= \underline{-\eta[k - \alpha/(2\eta)]^2 + \alpha^2/(4\eta)}$$

parabolic equation of k



condition for
exponential growth: $\alpha/\eta > k$

Key physics in this talk

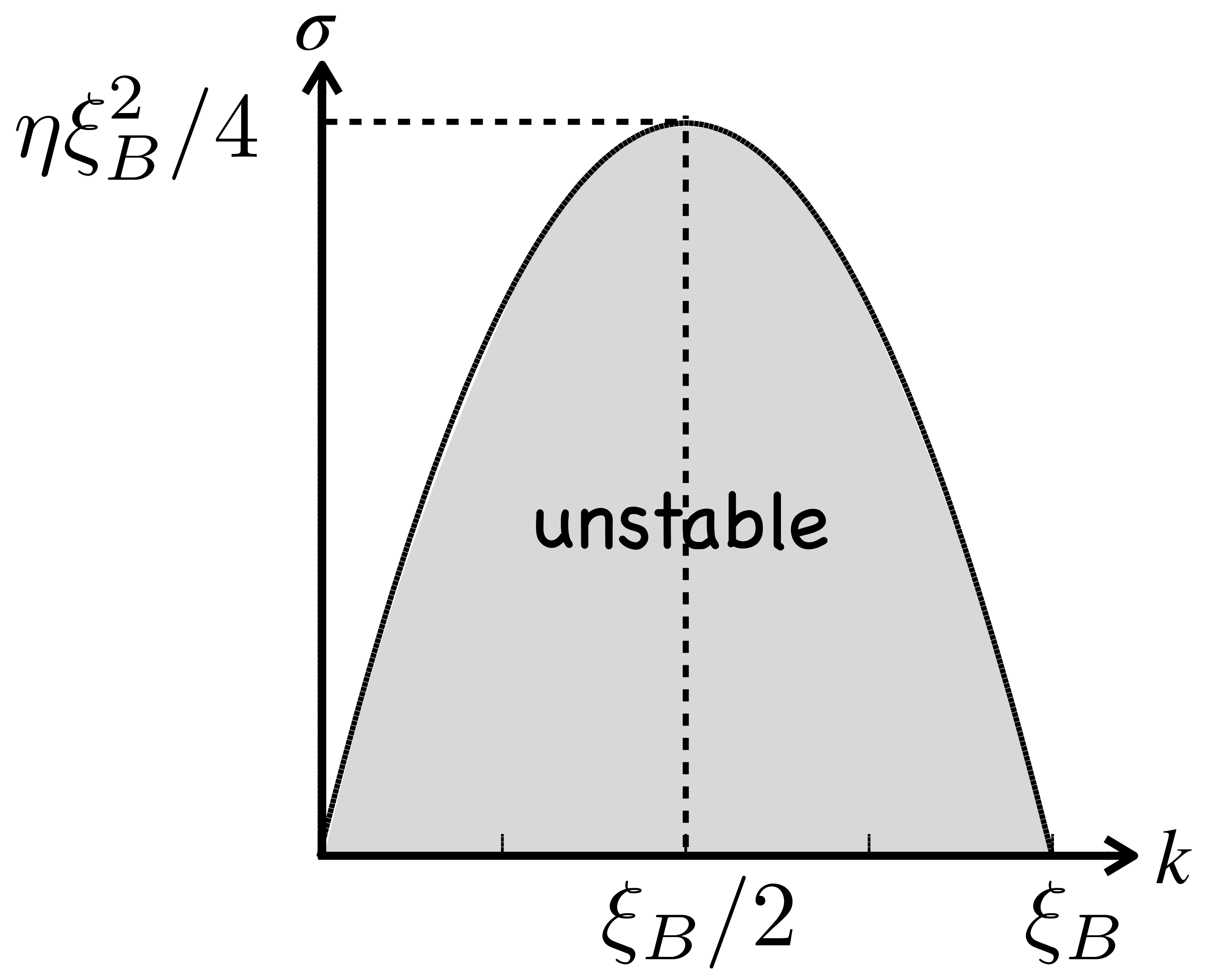
$$\partial_t \boldsymbol{B} = \eta \nabla \times (\xi_{\mathrm{B}} \boldsymbol{B}) + \eta \Delta \boldsymbol{B}$$

linear analysis (WKB approximation): $\delta \boldsymbol{B} = \exp[i\boldsymbol{k} \cdot \boldsymbol{r} + \sigma t]$

- -> dispersion relation:

$$\sigma = \eta \xi_B k - \eta k^2$$
$$= \underline{-\eta[k - \xi_B/2]^2 + \eta \xi_B^2/4}$$

parabolic equation of k

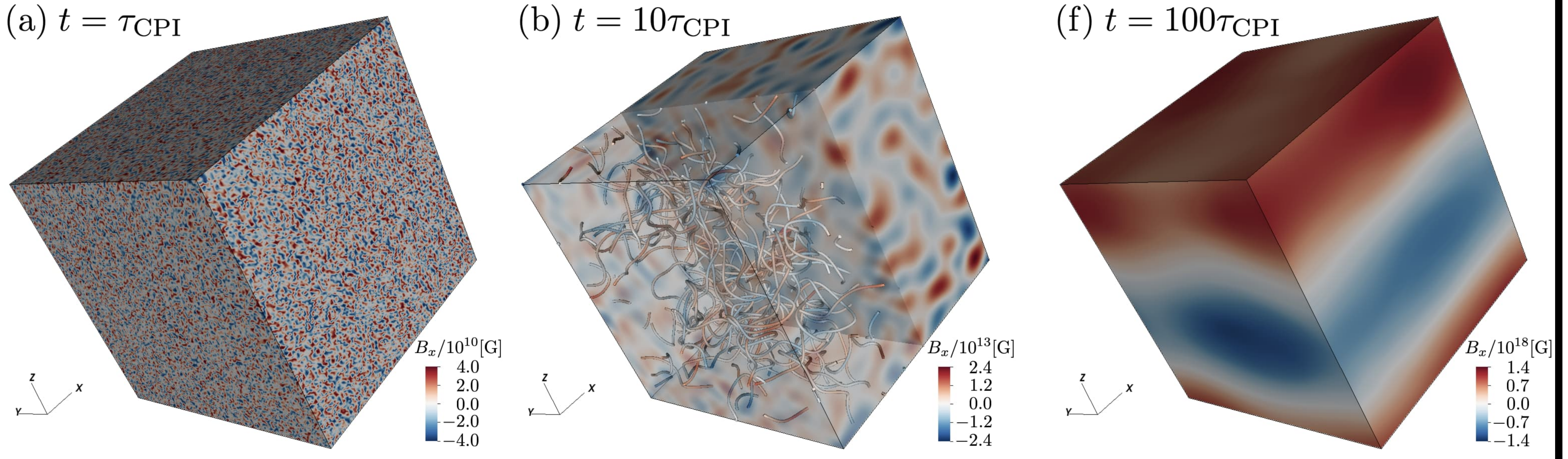


$$\lambda_{\mathrm{CPI}} \equiv 2\pi/k_{\mathrm{CPI}} = 4\pi/\xi_B$$

Condition for inverse cascade of magnetic field

Chiral MHD simulations in the context of CCSN (Masada+18, JM22)

fm scale



Correlation length of magnetic field becomes larger over time.

Since ξ_B decreases as time passes, typical wavelength of CPI becomes large.

$$\lambda_{\rm CPI} \equiv 2\pi/k_{\rm CPI} = 4\pi/\xi_B$$

Inverse cascade of magnetic field is important feature for CCSN.

The condition that the process of the CPI is dominant is

$$|v| < \eta|\xi_B|$$

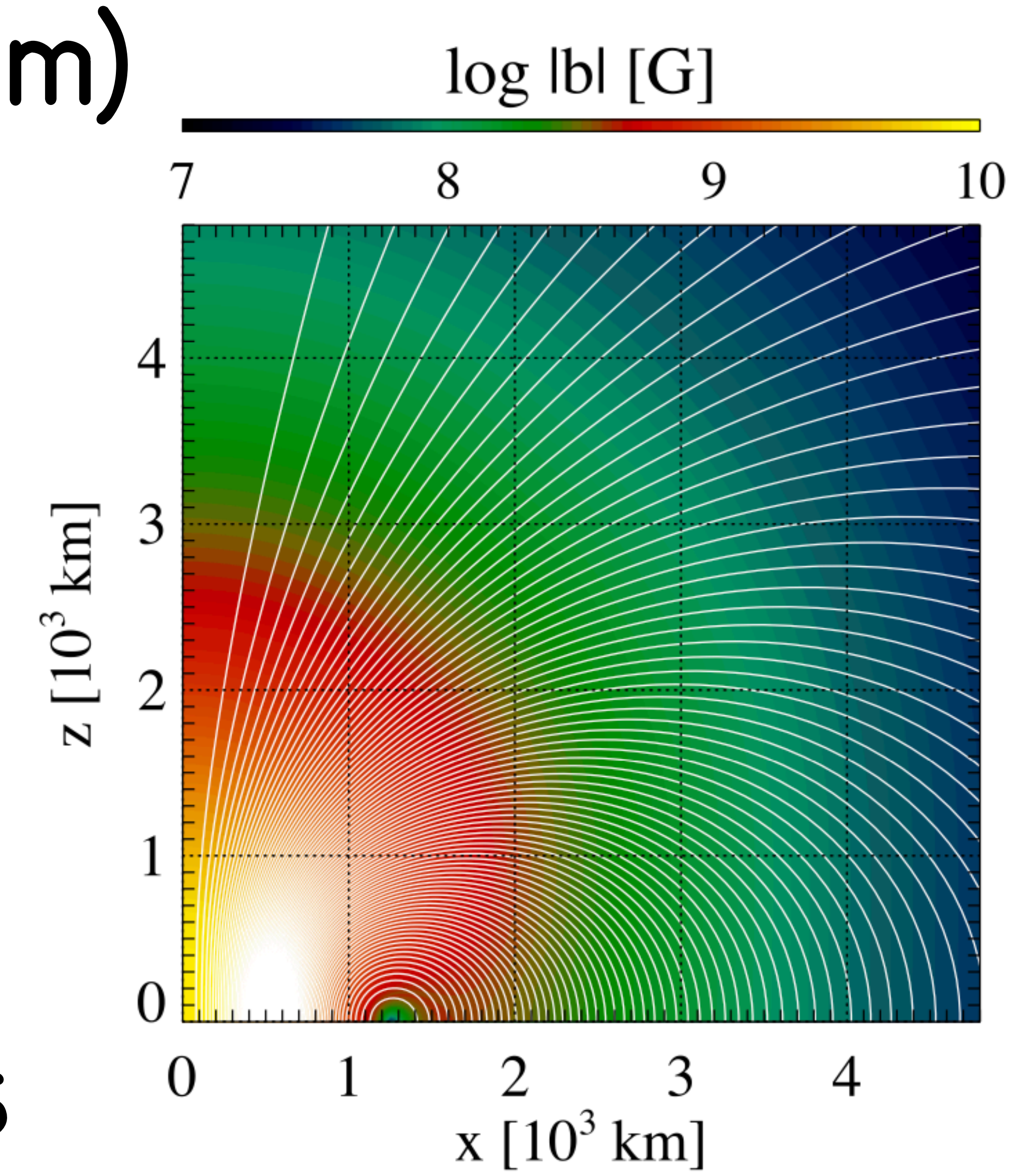
- Global MHD simulations of core-collapse supernova

Settings

- 3DnSNe code (Takiwaki+16) updated to MHD (See JM+20)
- approximate Riemann solver: HLLD (Miyoshi & Kusano 05)
- three-flavour neutrino transport based on Isotropic Diffusion Source Approximation (Kotake+18)
- EOS: Lattimer & Swesty (1991; incompressibility K=220 MeV)
- progenitor : s27.0 (Woosley+02)
- distribution of B-field: uniform (r < 1000km) + dipole (r > 1000km) (e.g., Suwa+07, Takiwaki+14, Obergaulinger+14)
- initial B-field: 10^{12} (strong field model) G
- vector potencial: $$A_\phi = \frac{B_0}{2}\frac{r_0^3}{r^3+r_0^3} r\sin\theta,$$
- grid spacing: 480 (r) x 64 (θ) x 128 (ϕ), 0 < r < 5000 km
- 2D run -> 3D run at t_{pb}=13 ms to save computational resources

- rigid rotation
  $\omega_0 = 0.3, 0.1, 0$ rad/s

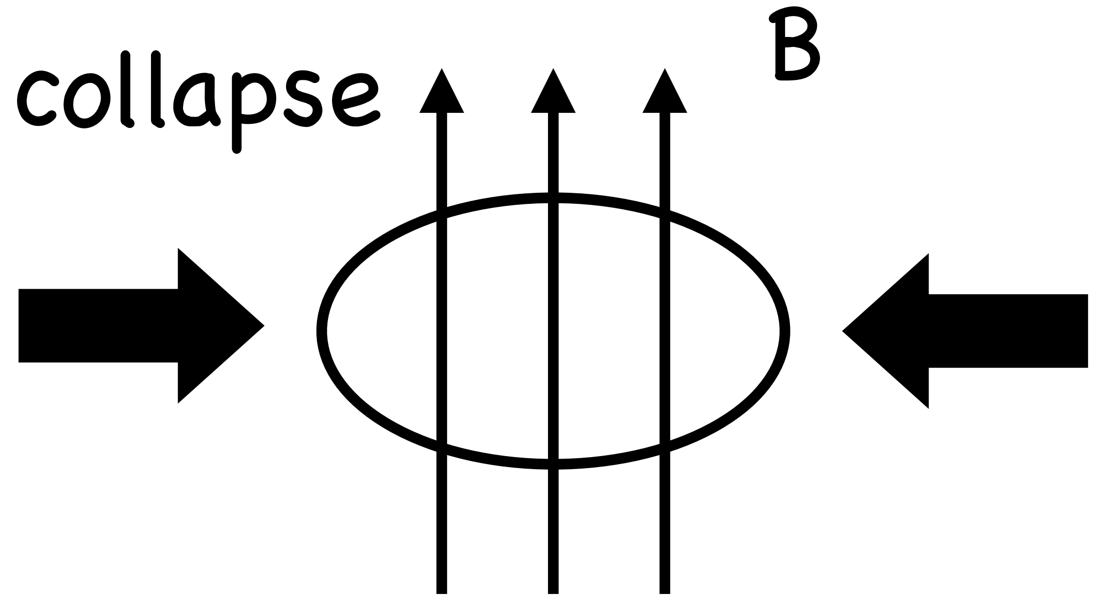
Obergaulinger+14



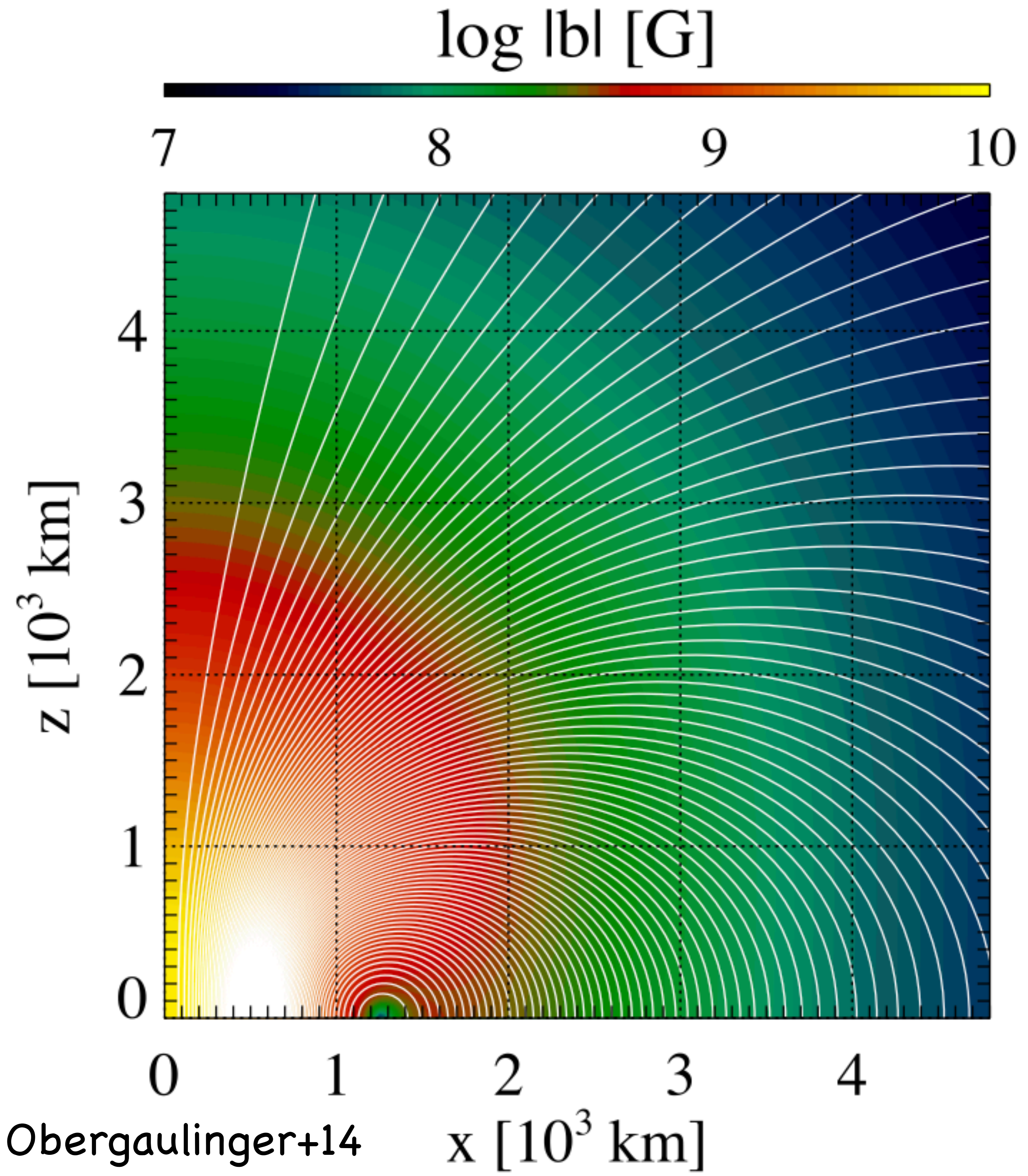
Initial condition of B-field

possible formation scenarios of magnetar

- turbulent dynamo amplification in a rapidly rotating PNS (Thompson+93)
- fossil field hypothesis (magnetic flux conservation) (Ferrario+06)



- initial B-field: 10^{12} (strong field model) G



Obergaulinger+14

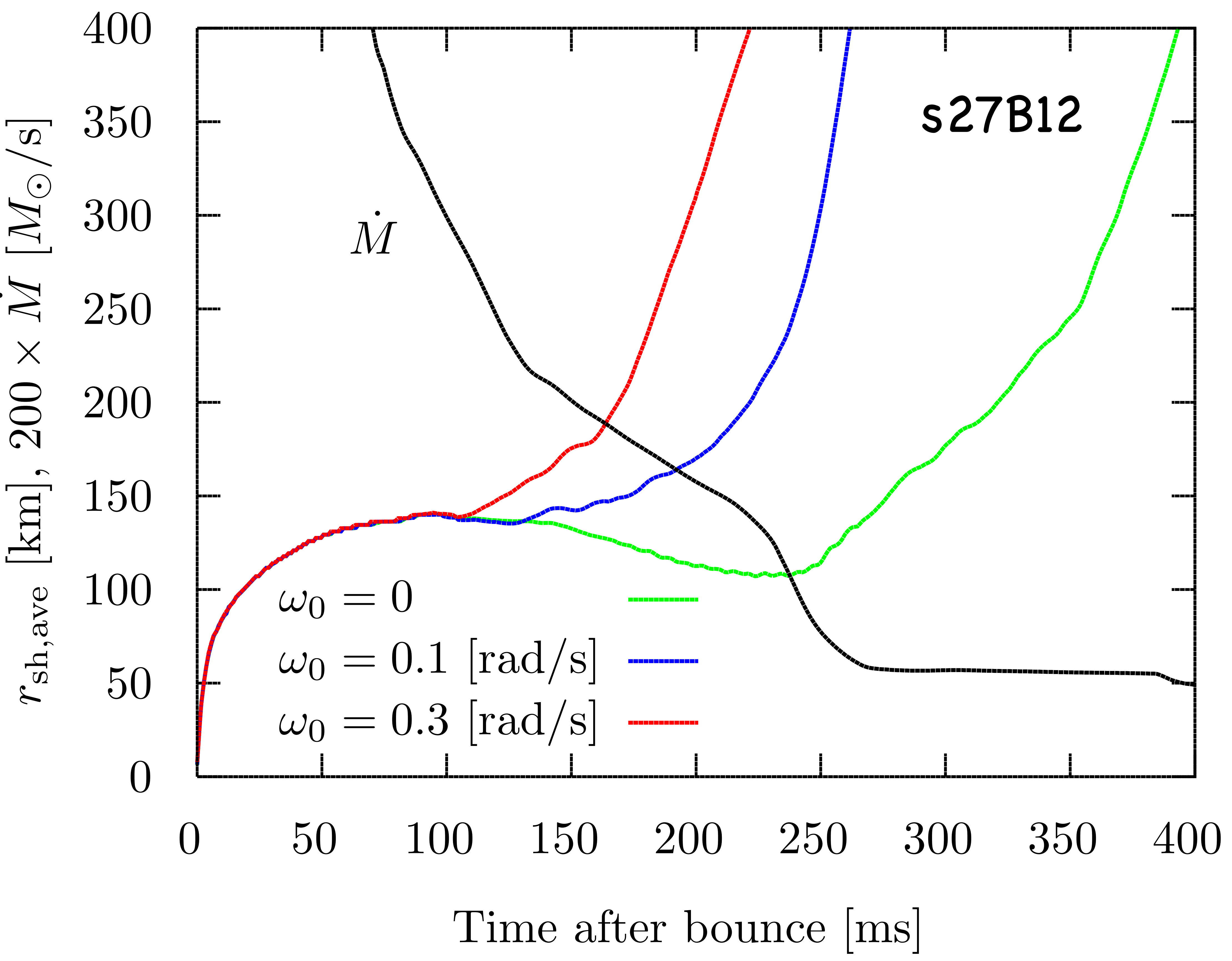
magnetic flux conservation: $B_{\rm PNS} \sim 10^{15}\ {\rm G} \left(\frac{B_{0,r=1000{\rm km}}}{10^{12}{\rm G}} \right) \left(\frac{30{\rm km}}{r_{\rm PNS}} \right)^2$

10^{12} G (strong field model): -> 10^{15} G (r < 30km) < - - magnetar class

10^{10} G (weak field model): -> 10^{13} G (r < 30km)

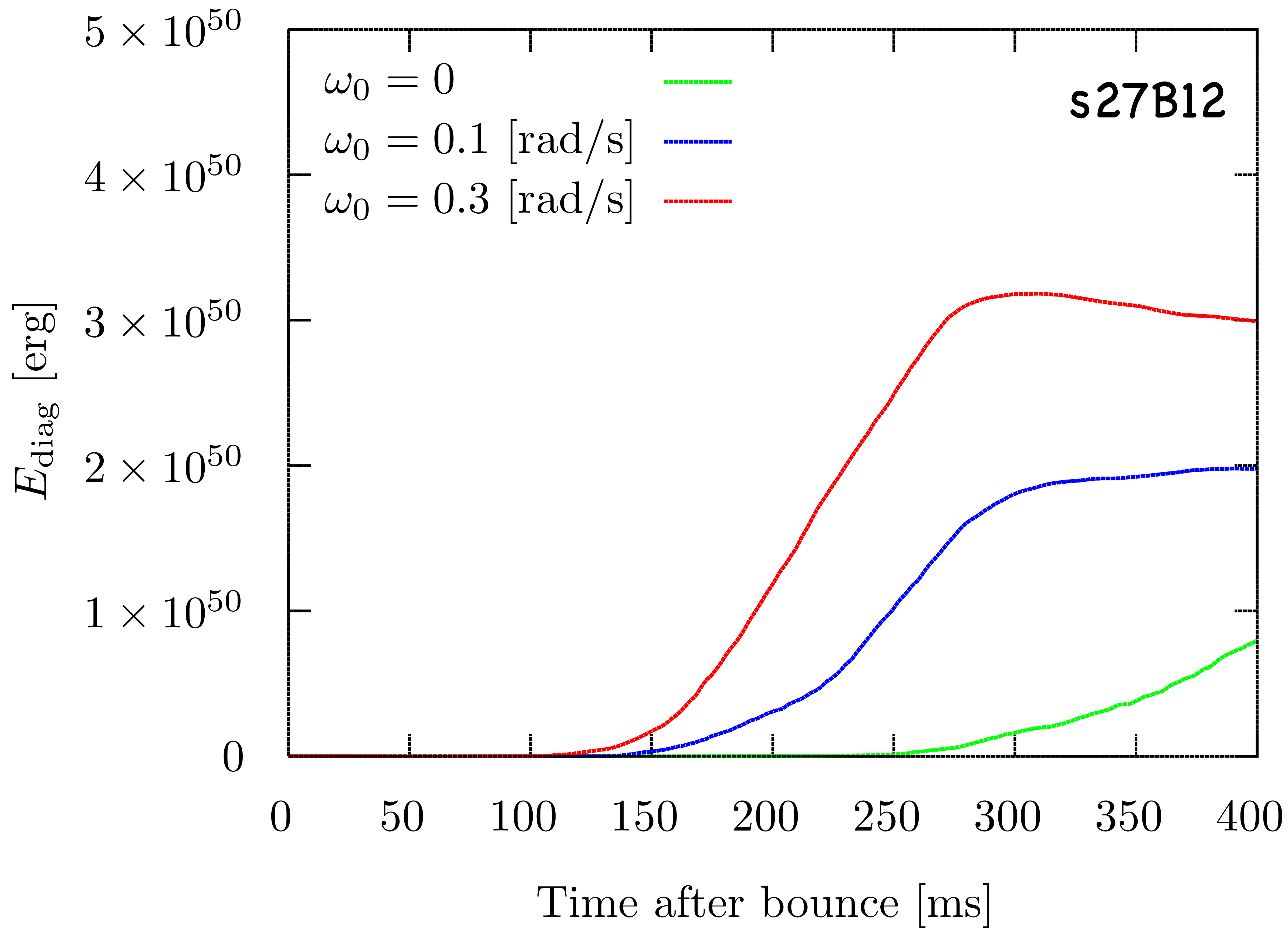
Dependence of the rotation

shock evolution



Magnetic pressure driven explosion occurs in rotating models. The magnetic field is fully amplified due to the effect of turbulence.

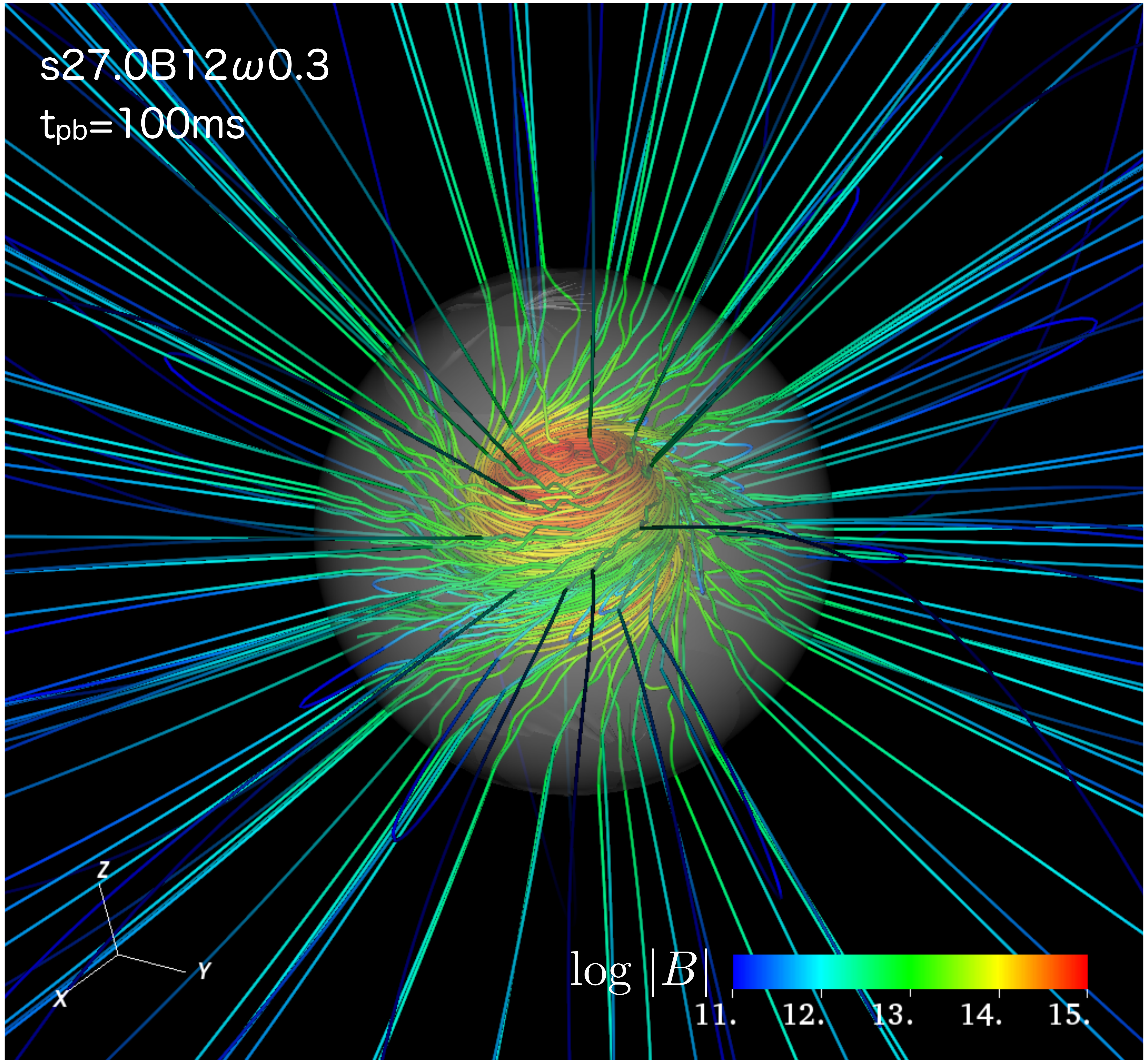
evolution of explosion energy



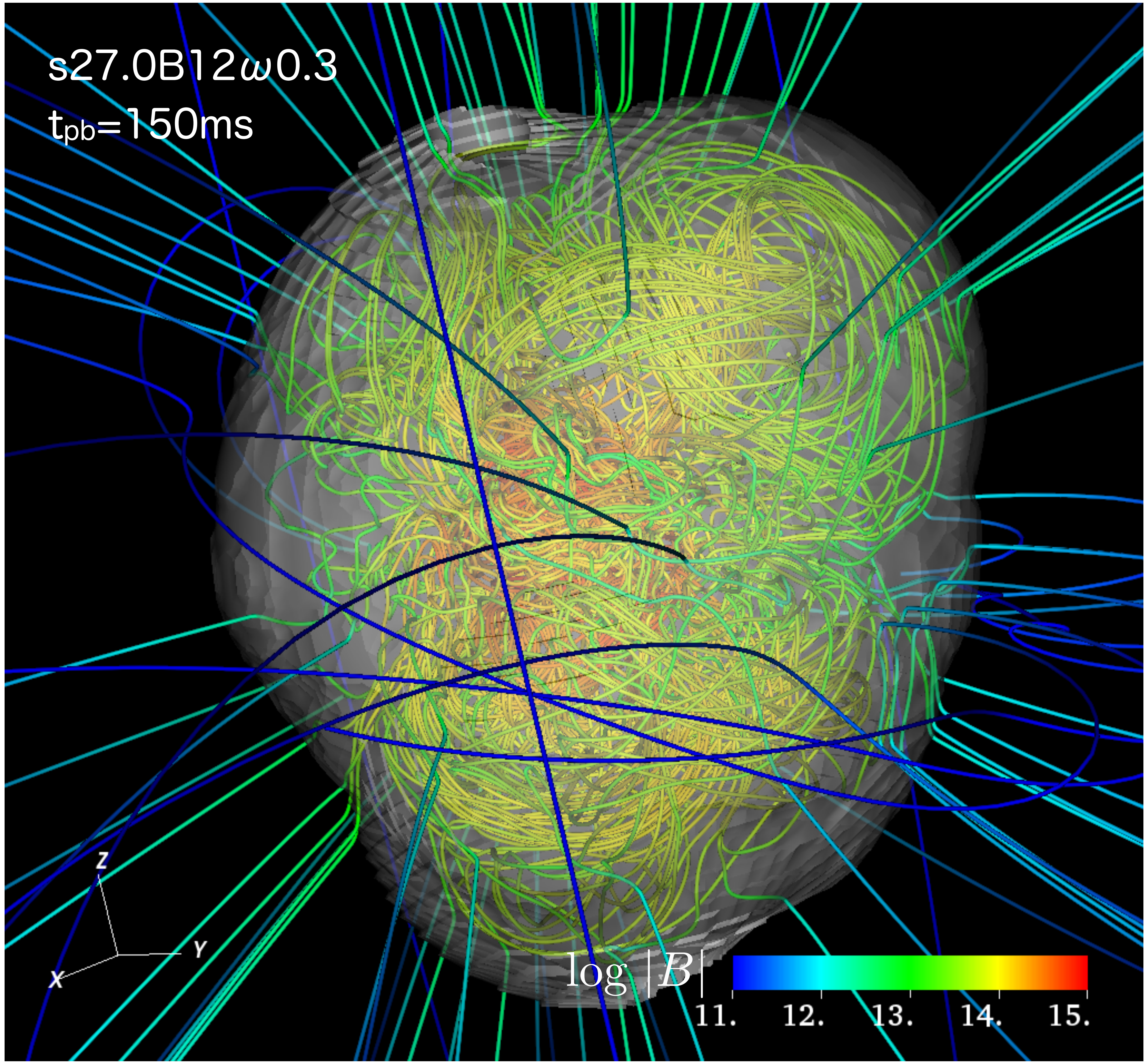
Explosion energy in faster explosion model is larger.

Distribution of B-field

onset of neutrino-driven convection

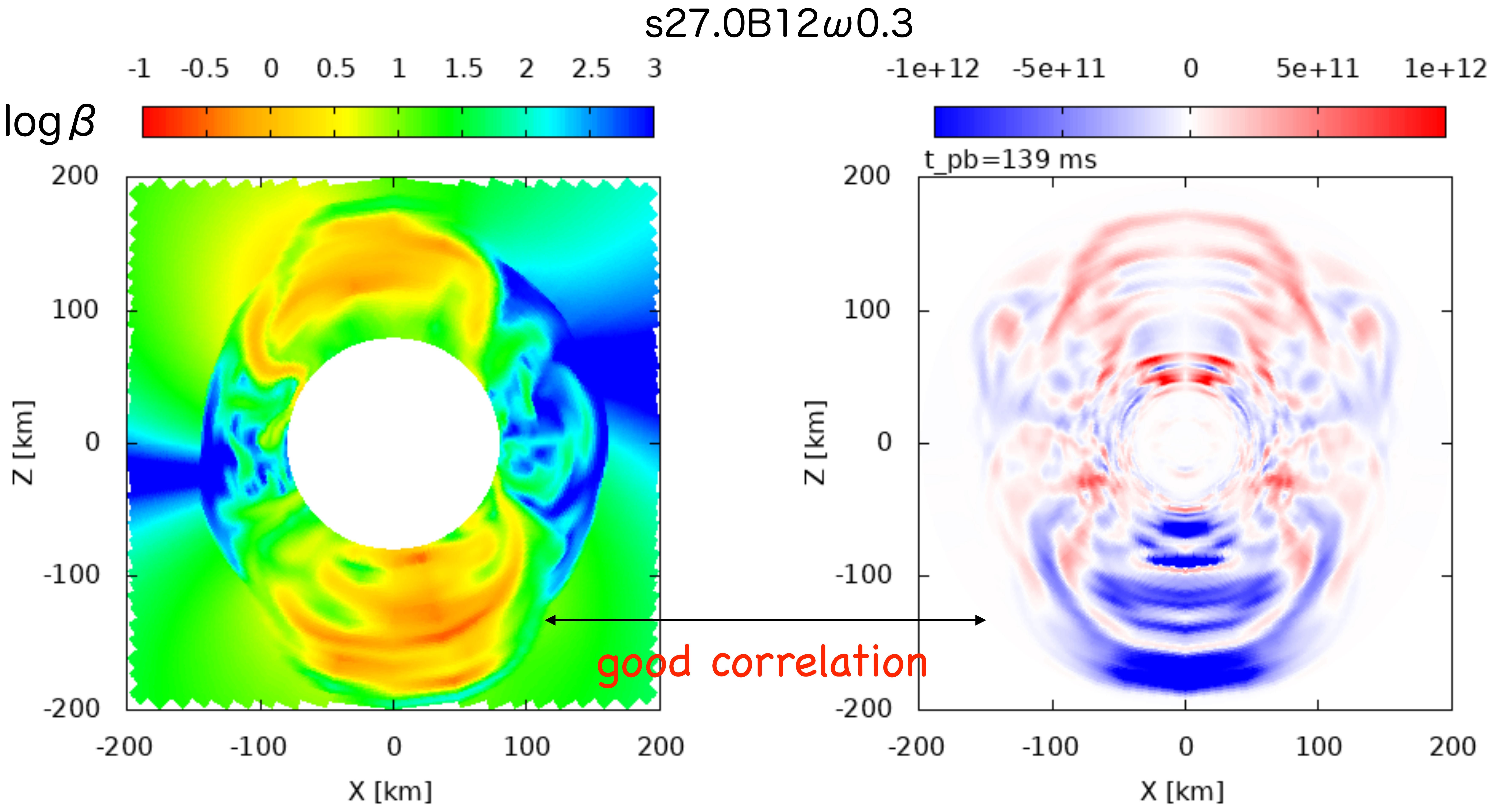


after shock revival



Amplification of the magnetic field

plasma $\beta \equiv P_{\rm gas}/P_{\rm mag}$ kinetic helicity $H_{\rm K} = \langle \boldsymbol{v}' \cdot \boldsymbol{\omega}' \rangle_\phi$



Magnetic pressure driven explosion

mean field theory

$$\boldsymbol{v}(r,\theta,\phi) = \langle \boldsymbol{v} \rangle(r,\theta) + \boldsymbol{v}'(r,\theta,\phi)\ ,$$
$$\boldsymbol{B}(r,\theta,\phi) = \langle \boldsymbol{B} \rangle(r,\theta) + \boldsymbol{B}'(r,\theta,\phi)\ .$$

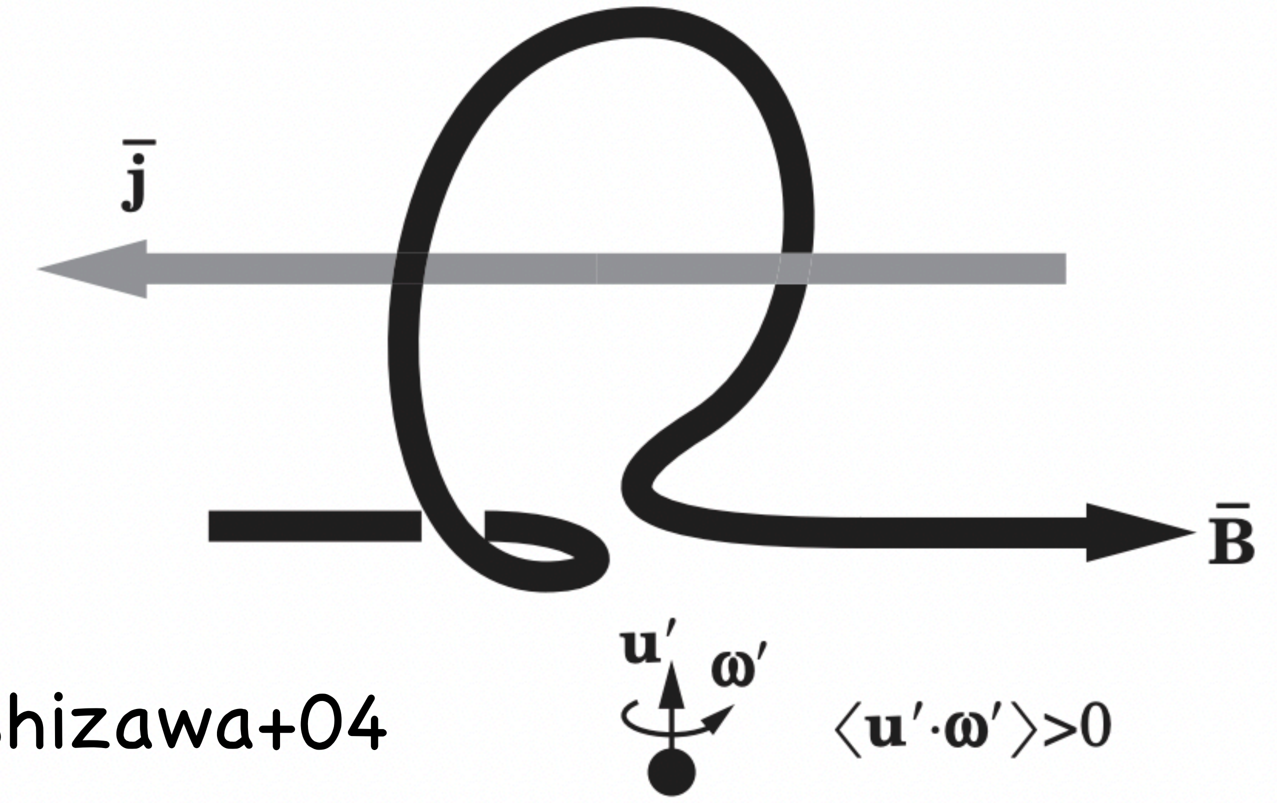
induction equation:

$$\frac{\partial \langle \boldsymbol{B} \rangle}{\partial t} = \nabla \times (\langle \boldsymbol{v} \rangle \times \langle \boldsymbol{B} \rangle - \eta \nabla \times \langle \boldsymbol{B} \rangle + \boldsymbol{\epsilon})$$

$$\boldsymbol{\epsilon} \equiv \alpha \langle \boldsymbol{B} \rangle - \eta_t \nabla \times \langle \boldsymbol{B} \rangle$$
$$\alpha \equiv -\frac{1}{3}\tau_{\rm cor} h_{\rm K}$$
$$\eta_t \equiv \frac{1}{3}\tau_{\rm cor}\langle v'^2 \rangle$$

Brandenburg+05

α -effect

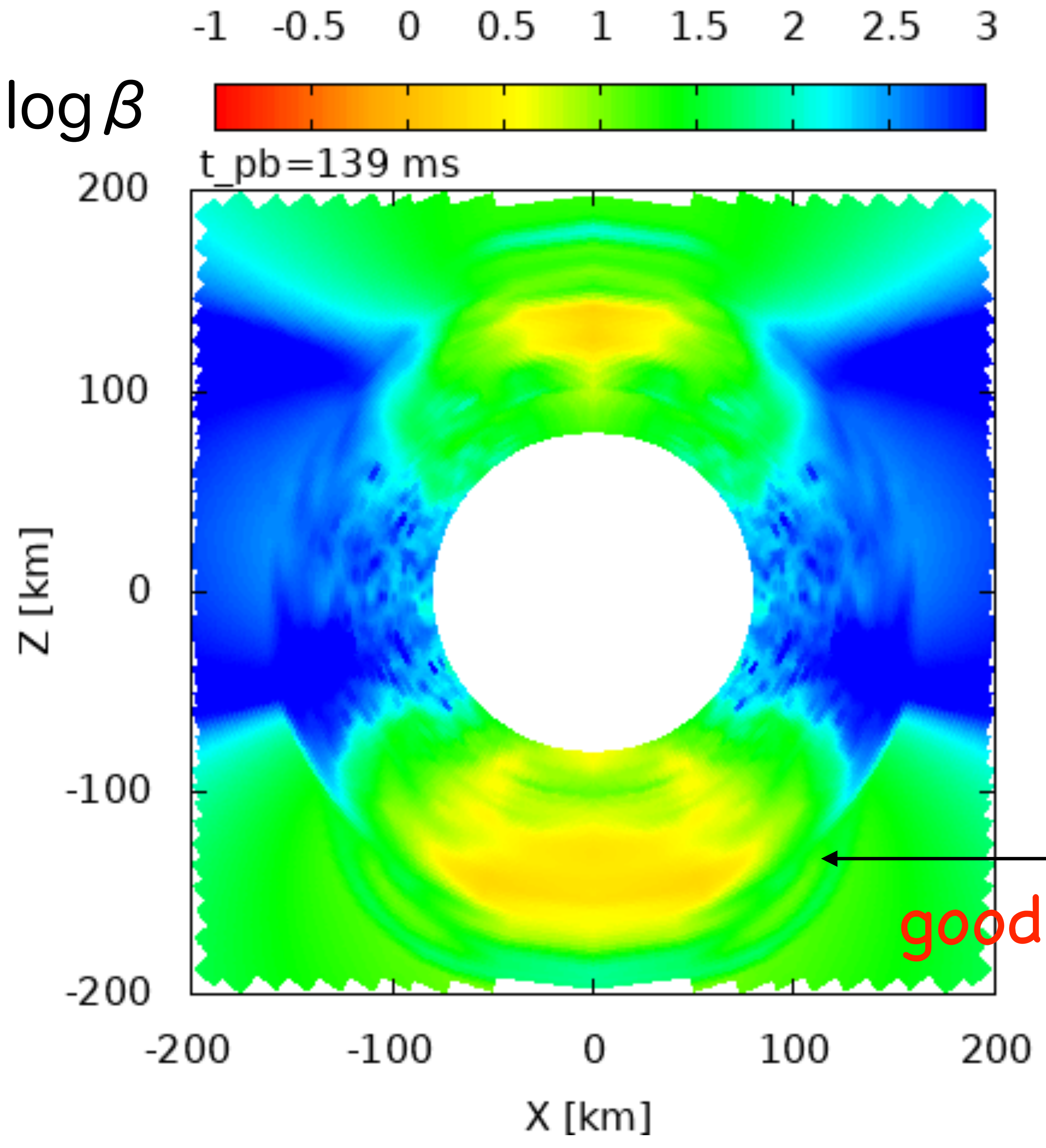


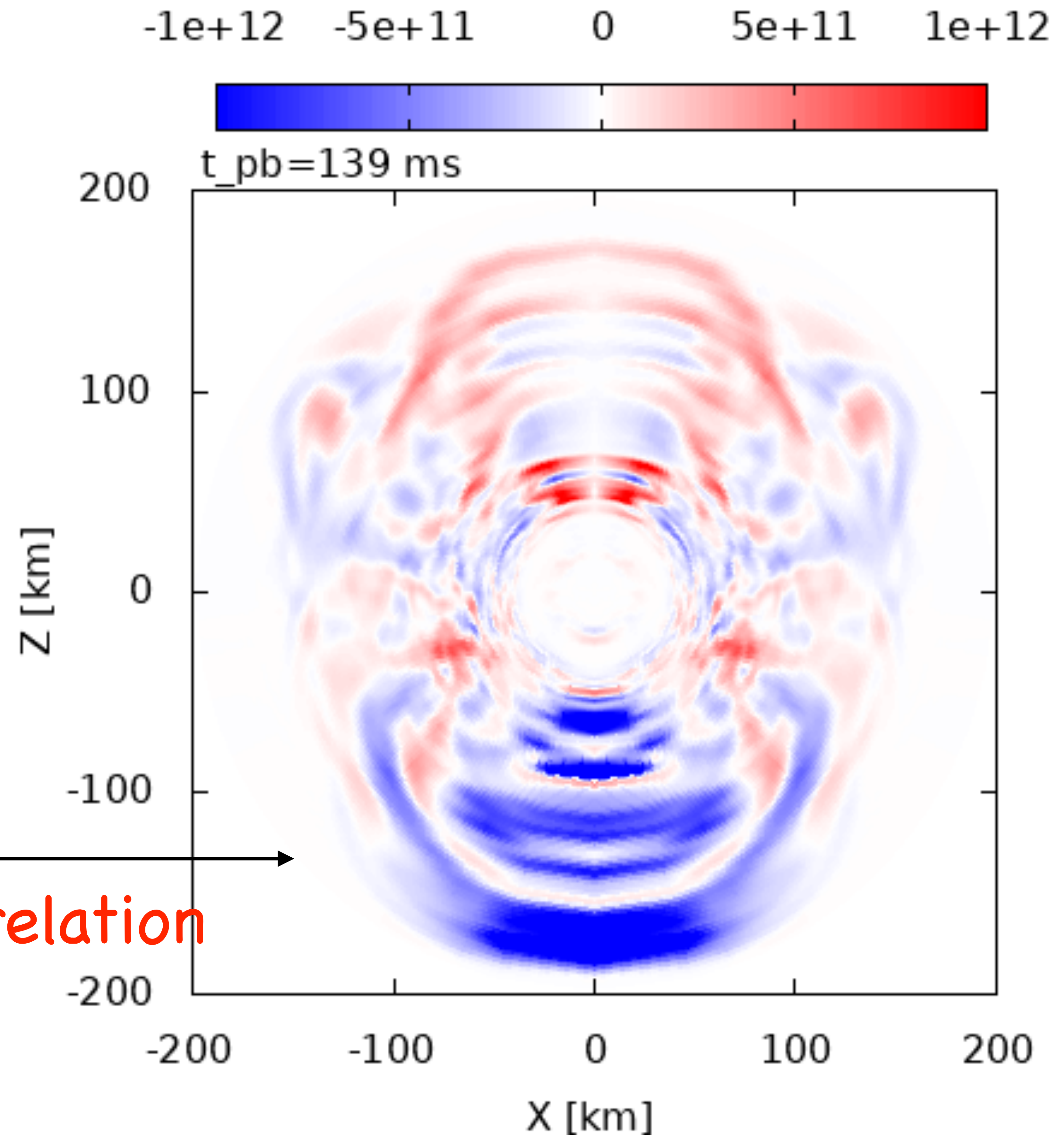
Yoshizawa+04

Amplification of the magnetic field

averaged plasma $\beta \equiv P_{\rm gas}/P_{\rm mag}$ kinetic helicity $H_{\rm K} = \langle \boldsymbol{v}' \cdot \boldsymbol{\omega}' \rangle_\phi$

s27.0B12ω0.3





good correlation

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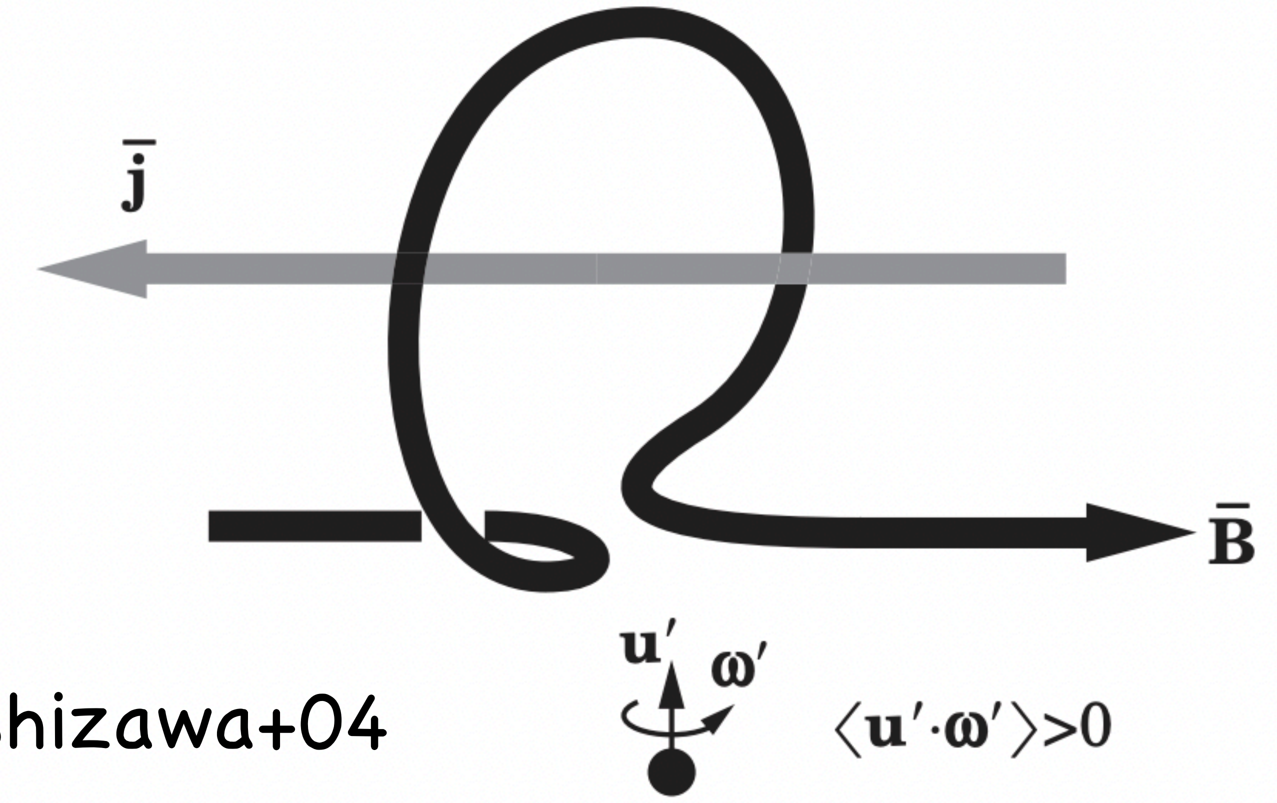

Yoshizawa+04

Amplification of the magnetic field

plasma $\beta \equiv P_{\rm gas}/P_{\rm mag}$

kinetic helicity $H_{\rm K} = \langle \boldsymbol{v}' \cdot \boldsymbol{\omega}' \rangle_\phi$

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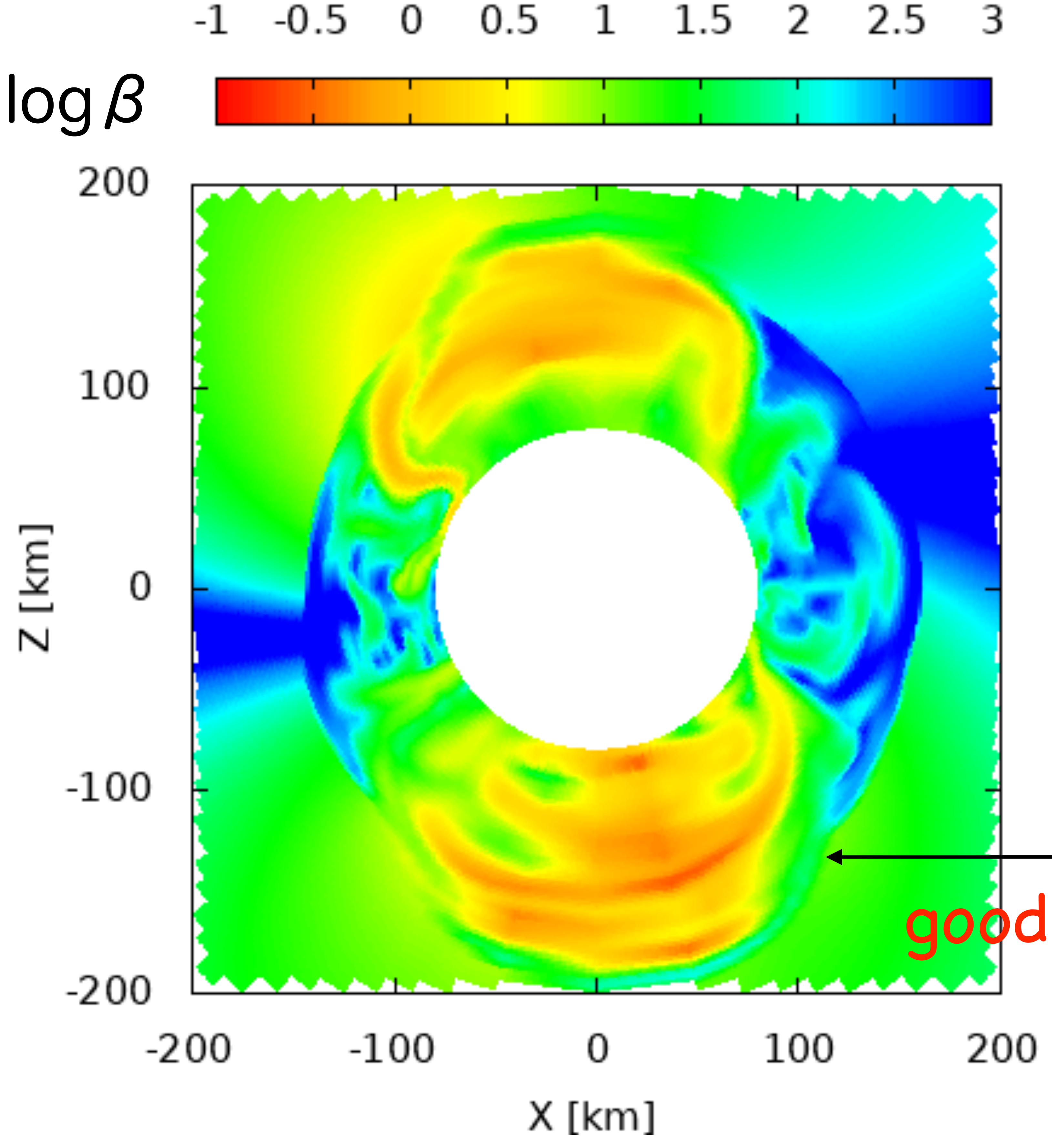

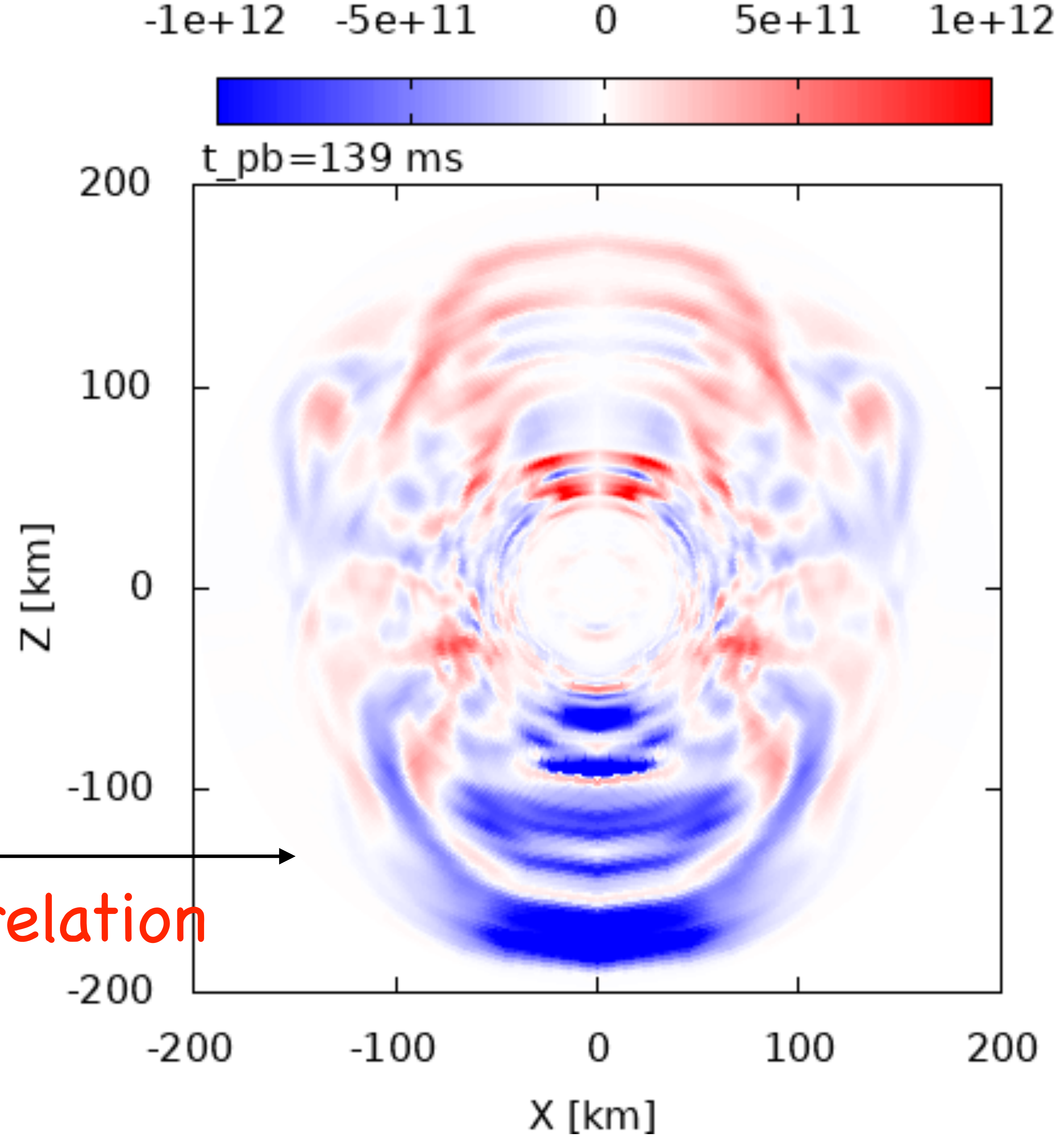

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Brandenburg+05

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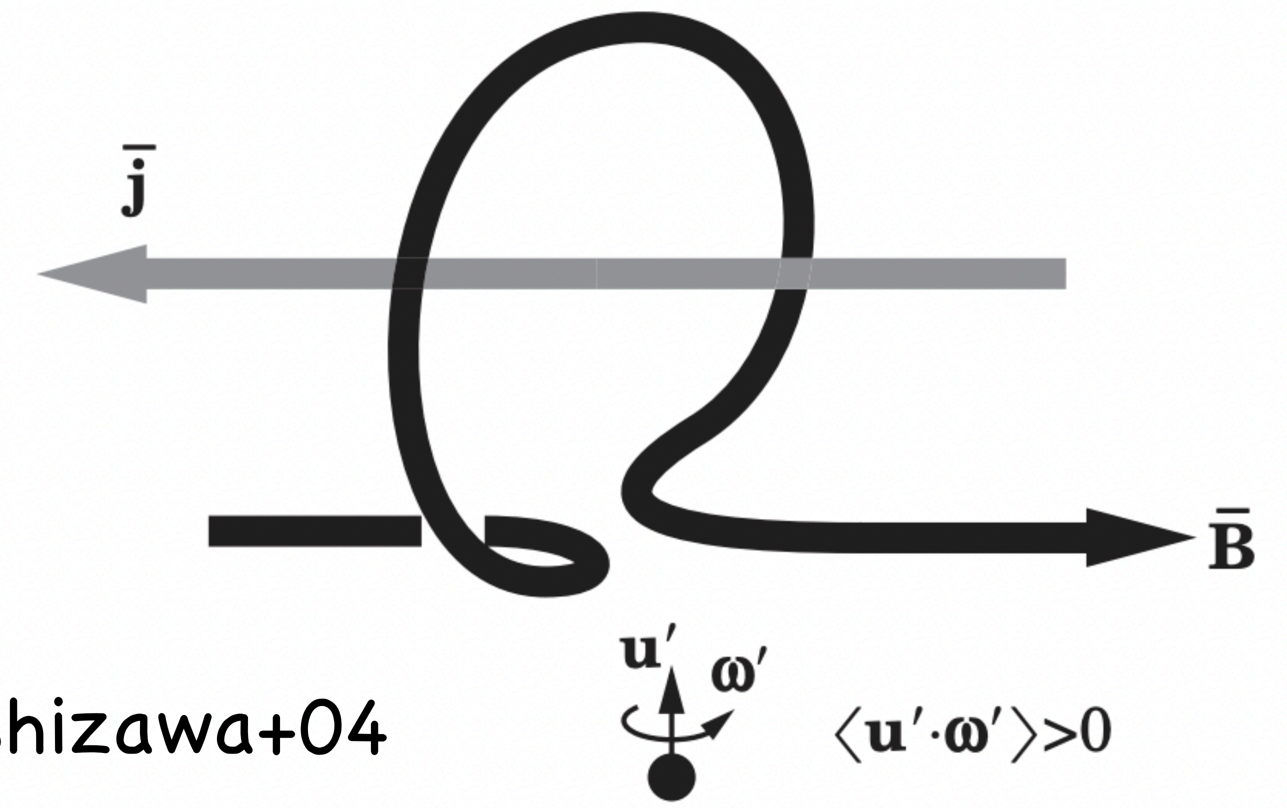
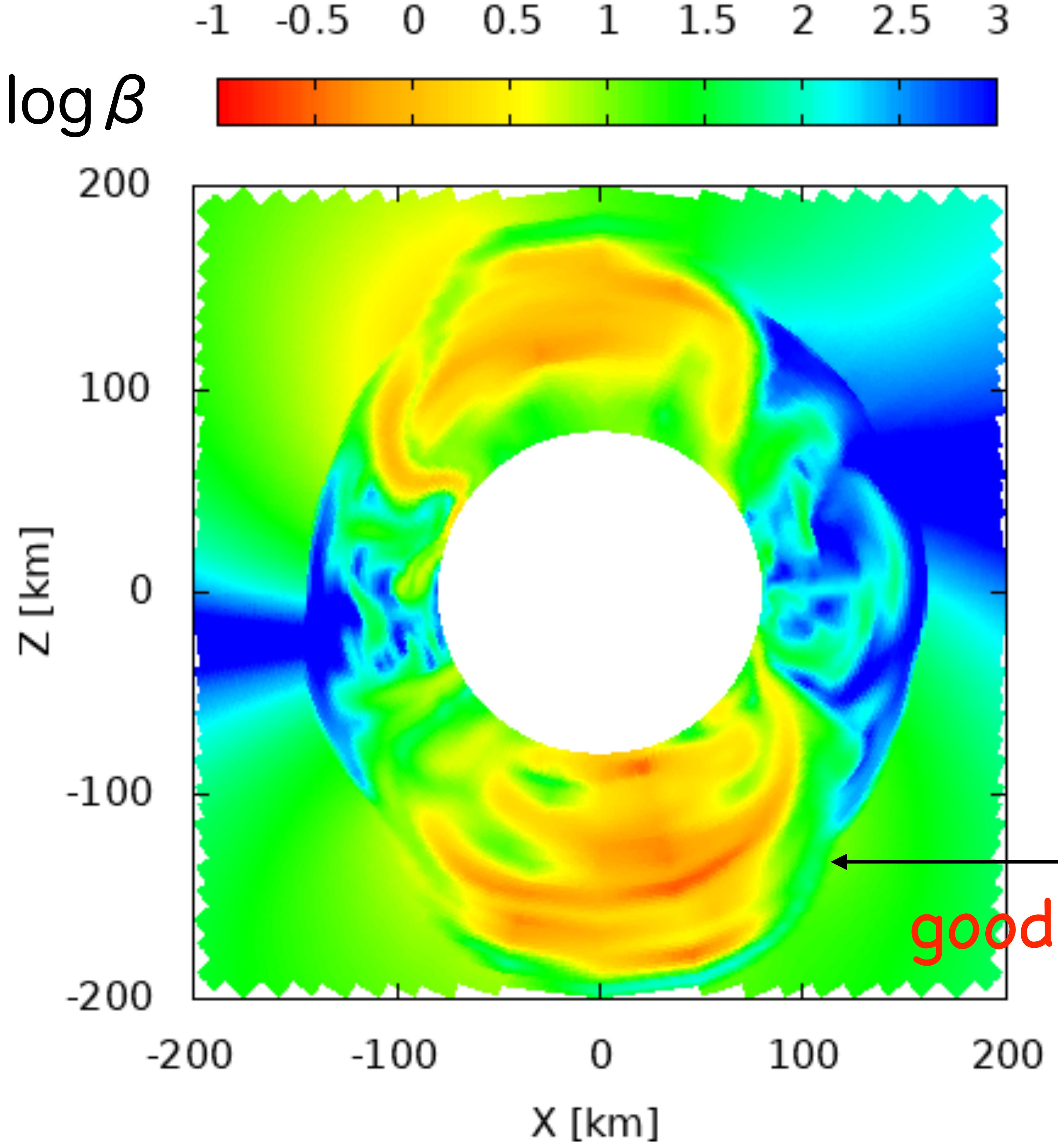


Yoshizawa+04

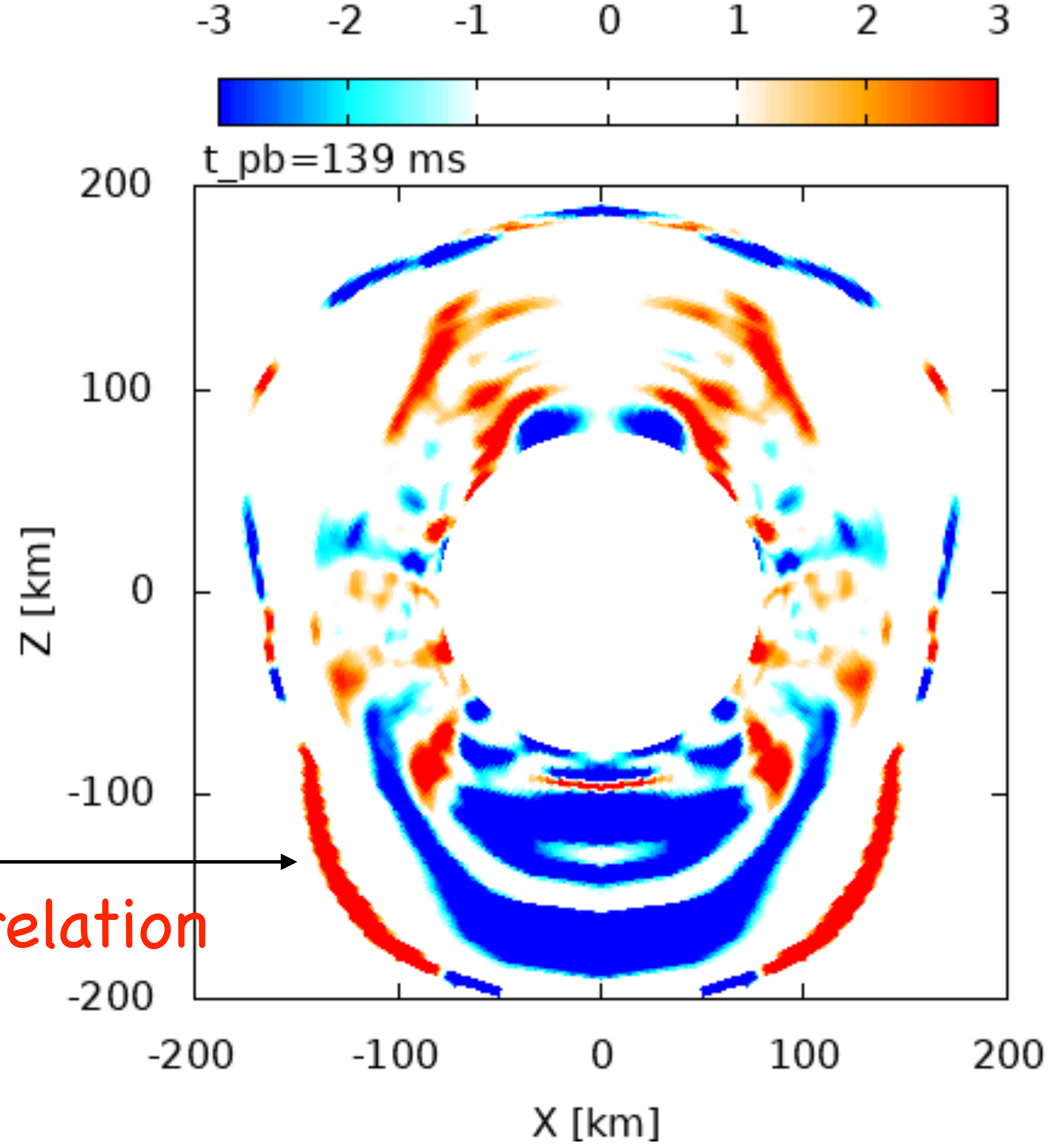
Amplification of the magnetic field

plasma $\beta \equiv P_{\rm gas}/P_{\rm mag}$

s27.0B12ω0.3



dynamo number
in gain region



good correlation

Magnetic pressure driven explosion

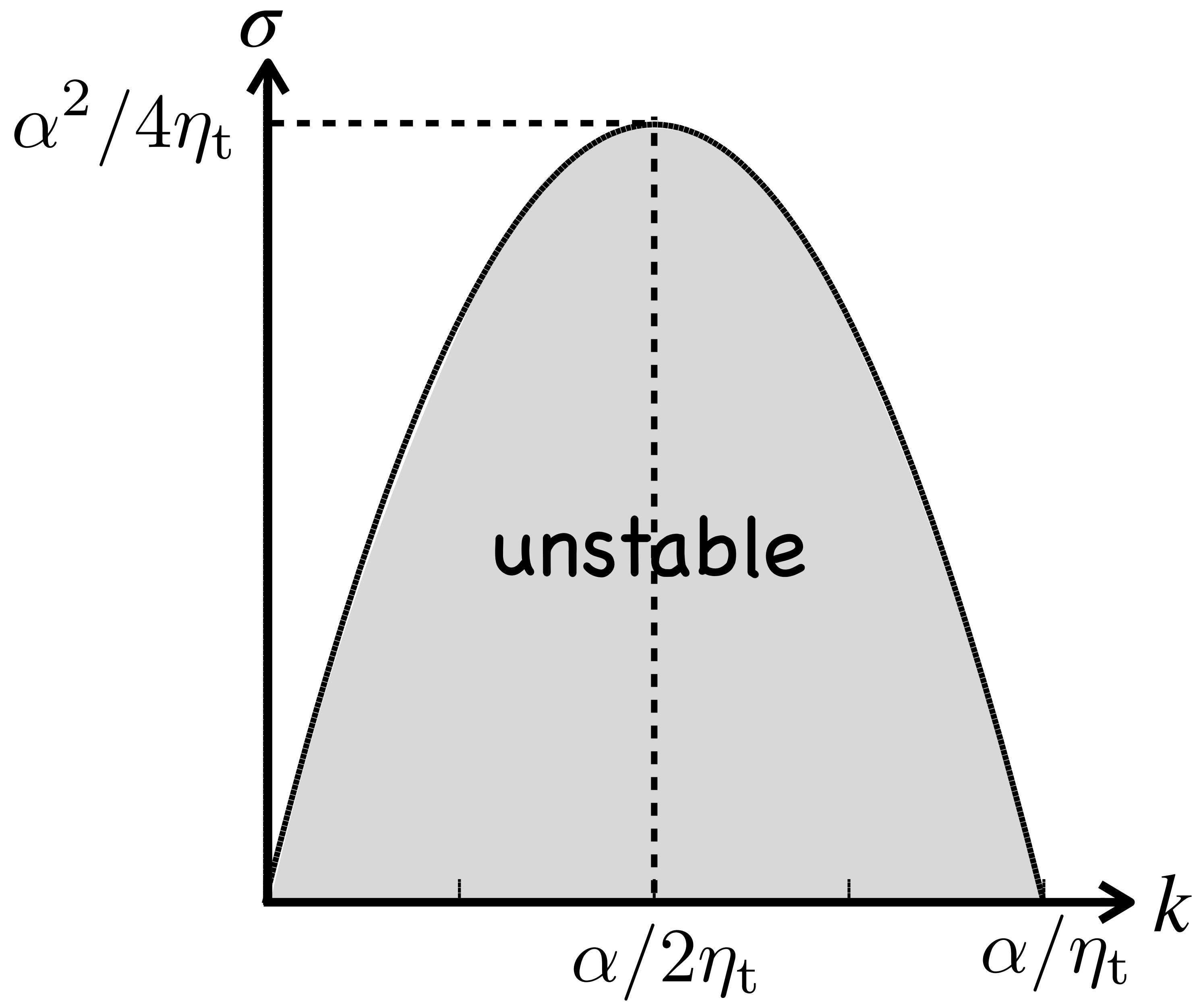
$|D| > 1$

α-dynamo occurs.

$$\frac{\partial \langle \boldsymbol{B} \rangle}{\partial t} = \nabla \times (\alpha \langle \boldsymbol{B} \rangle) + \eta_t \Delta \langle \boldsymbol{B} \rangle$$

-> linearization

dispersion relation:



condition for
exponential growth: $\alpha/\eta_{\rm t} > k$

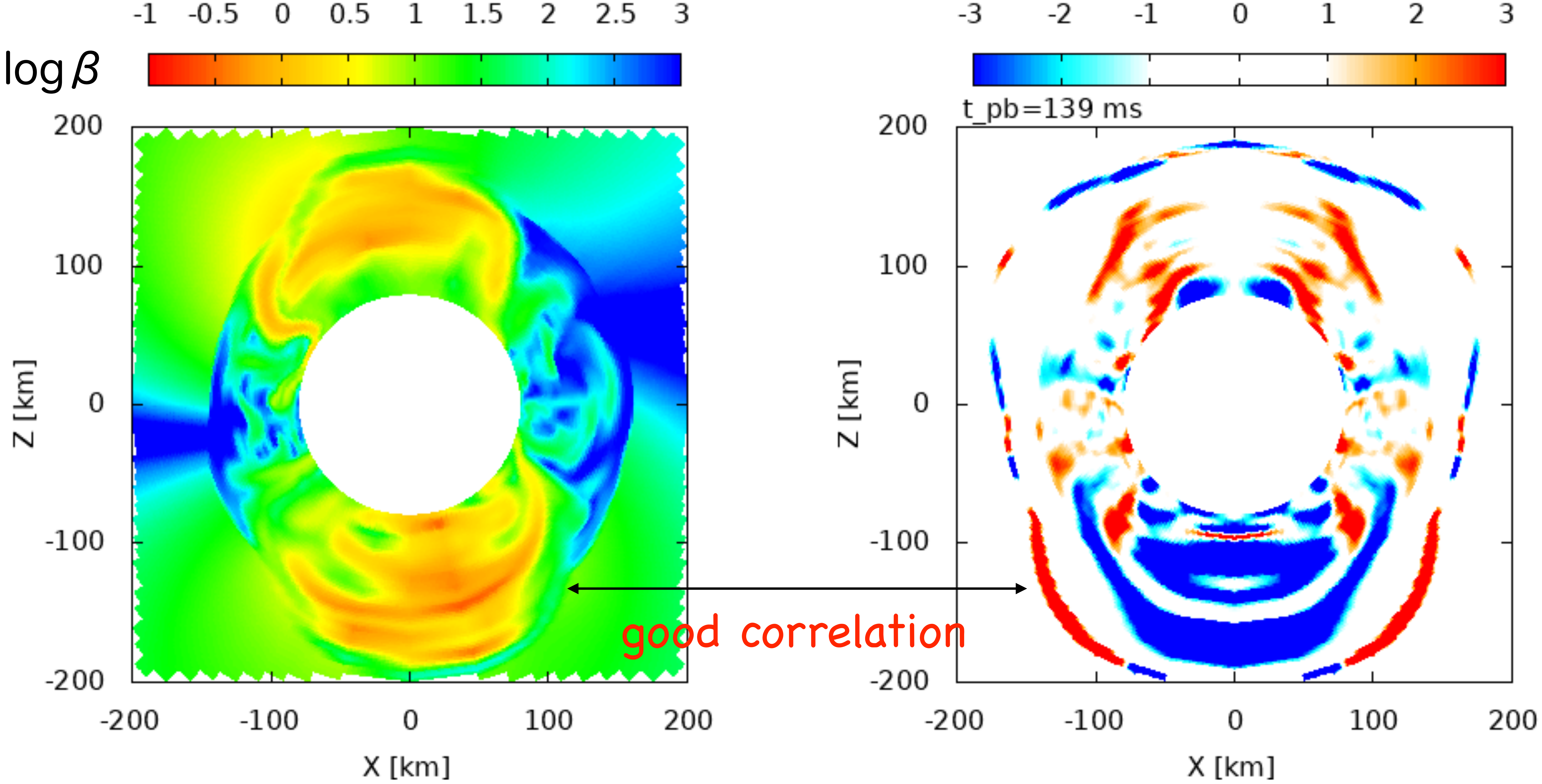
$\Rightarrow$ $D \equiv \alpha/k\eta_{\rm t} > 1$

Amplification of the magnetic field

plasma $\beta \equiv P_{\rm gas}/P_{\rm mag}$

s27.0B12ω0.3

dynamo number in gain region



Magnetic pressure driven explosion

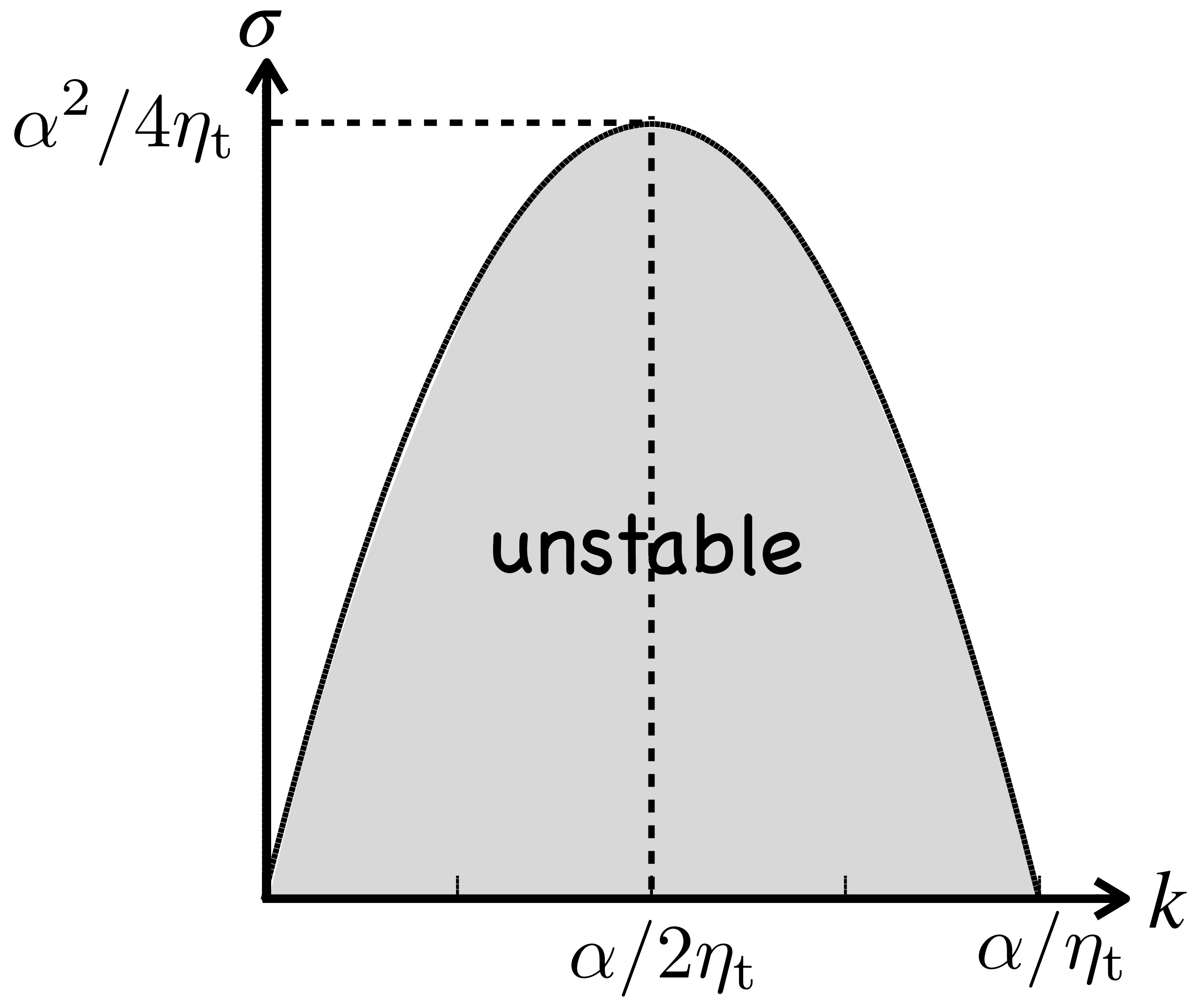
$|D| > 1$

α-dynamo occurs.

$$\frac{\partial \langle \boldsymbol{B} \rangle}{\partial t} = \nabla \times (\alpha \langle \boldsymbol{B} \rangle) + \eta_t \Delta \langle \boldsymbol{B} \rangle$$

-> linearization

dispersion relation:

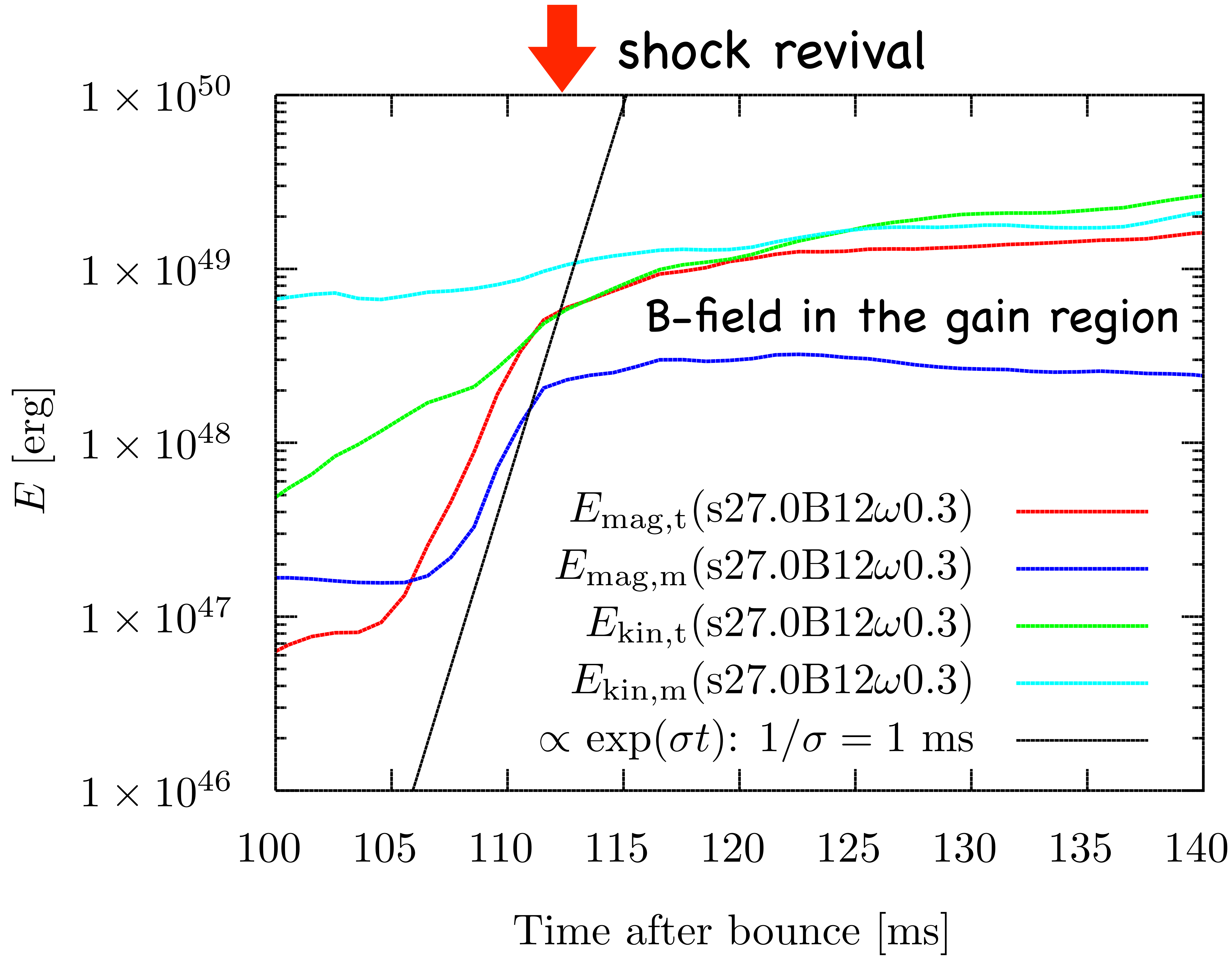


$$\sigma_{\rm max} = \alpha^2/4\eta_{\rm t} = Dk\alpha/4$$

$$\sim \frac{DH_{\rm K}}{v_{\rm turb}} \sim \frac{1 \cdot 10^{12}}{10^9} = \underline{10^3\ {\rm s}^{-1}}$$

growth rate

Growth rate of the magnetic energy



Mean magnetic field is amplified by α-effect.

In addition, turbulent magnetic field is also amplified via α-dynamo action of mean magnetic field.

Induction equation for turbulent magnetic field:

$$\frac{\partial \boldsymbol{B}'}{\partial t} = \nabla \times (\boldsymbol{v}' \times \underline{\langle \boldsymbol{B} \rangle})$$

mean magnetic field

Magnetic pressure amplified due to α-effect is responsible for fast explosion in our rotating model.

Summary

Key physics of the magnetic field amplification:

$$\partial_t \boldsymbol{B} = \nabla \times (\alpha \boldsymbol{B}) + \eta \Delta \boldsymbol{B}$$

Exponential amplification of the magnetic field

- Chiral MHD simulations in local box

  The condition that process of the CPI is dominant is $|v| < \eta|\xi_B|$.

- Global MHD simulations of core-collapse supernova

  α-dynamo (kinetic helicity) is responsible for the exponential amplification of the magnetic field in the gain region.

  Magnetic pressure driven explosion in rotating model