Chiral magnetic waves in quark matter inside neutron stars and resulting gravitational waves

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Outline

- Introduction
- Chiral transport phenomena and chiral waves
- Chiral magnetic waves in quark matter
- Gravitational waves of chiral magnetic mode
- Summary & outlook

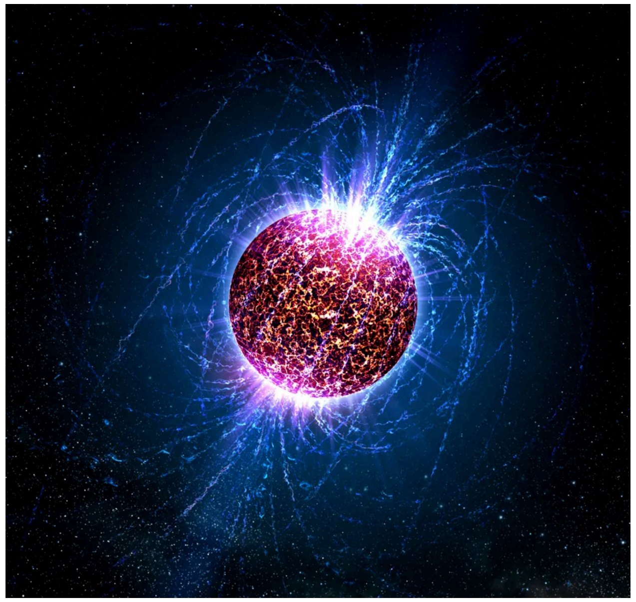
Introduction

Neutron stars

- Supernova explosion occurs when nuclear fuel runs out
- Star left in the core of supernovae：Neutron star
- Typical quantities
  - mass $\sim$ solar mass
  - radius $\sim 10\,\mathrm{km}$

  density $\sim 10^{15}\,\mathrm{g/cm^3}$

  - magnetic field (surface) $\sim 10^{12}$-$10^{15}\,\mathrm{Gauss}$
  - temperature $\sim 10^{6}$-$10^{9}\,\mathrm{K}$

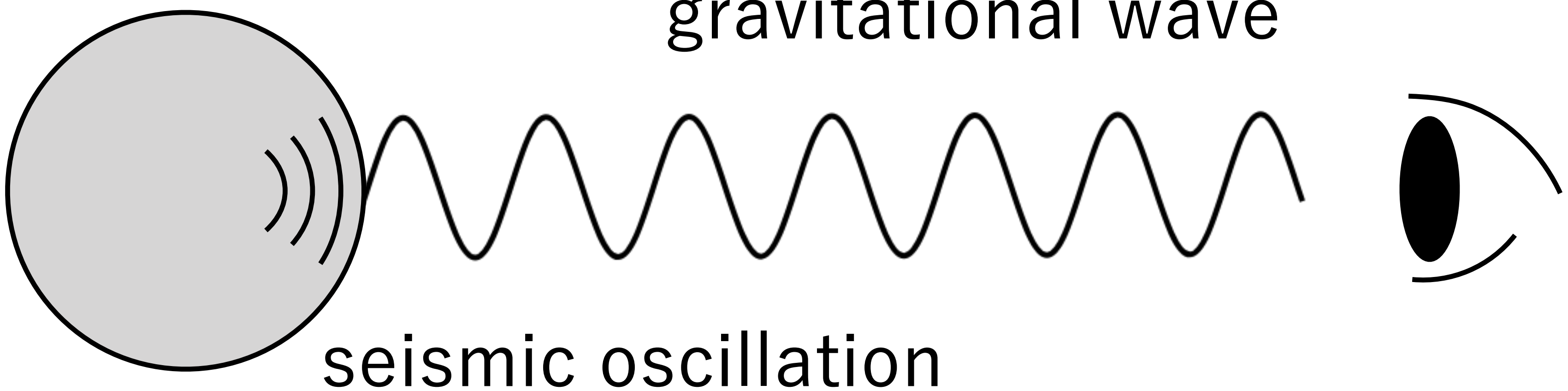


Credit: Casey Reed/ Penn State University

Internal structure of neutron stars ? → **Asteroseismology**

Asteroseismology of neutron stars

- Seismic oscillations inform us about the internal structure

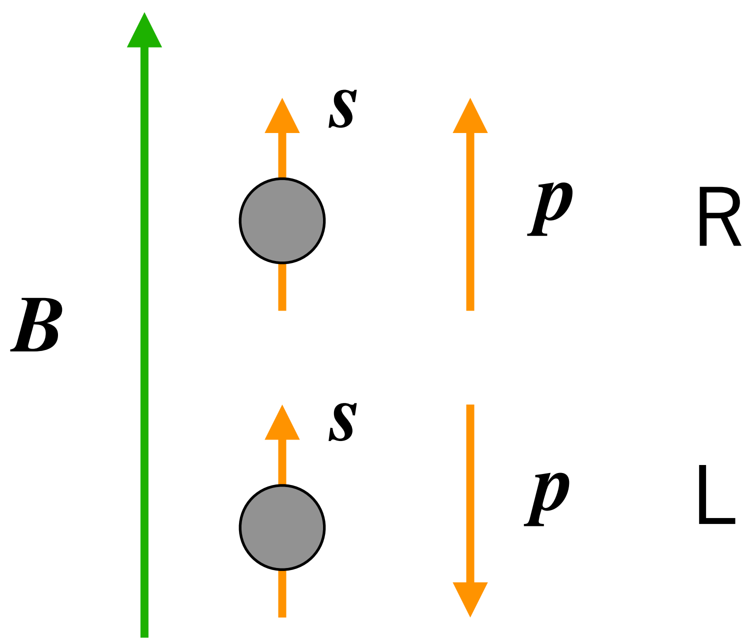


- Oscillation modes depending on physical origins
  - p-mode, g-mode, r-mode, ⋯

The **chirality** of quarks leads to a new type of seismic oscillation and gravitational wave

Chiral transport phenomena and chiral waves

Chiral magnetic/separation effect



- right-handed/left-handed current

$$\boldsymbol{j}_{\mathrm{R}} = \frac{e\mu_{\mathrm{R}}}{4\pi^2}\boldsymbol{B}, \qquad \boldsymbol{j}_{\mathrm{L}} = -\frac{e\mu_{\mathrm{L}}}{4\pi^2}\boldsymbol{B}$$

- vector/chiral chemical potential

$$\mu = \frac{\mu_{\mathrm{R}} + \mu_{\mathrm{L}}}{2}, \qquad \mu_5 = \frac{\mu_{\mathrm{R}} - \mu_{\mathrm{L}}}{2}$$

$$\boldsymbol{j} = \boldsymbol{j}_{\mathrm{R}} + \boldsymbol{j}_{\mathrm{L}} = \frac{e\mu_5}{2\pi^2}\boldsymbol{B}, \qquad \boldsymbol{j}_5 = \boldsymbol{j}_{\mathrm{R}} - \boldsymbol{j}_{\mathrm{L}} = \frac{e\mu}{2\pi^2}\boldsymbol{B}$$

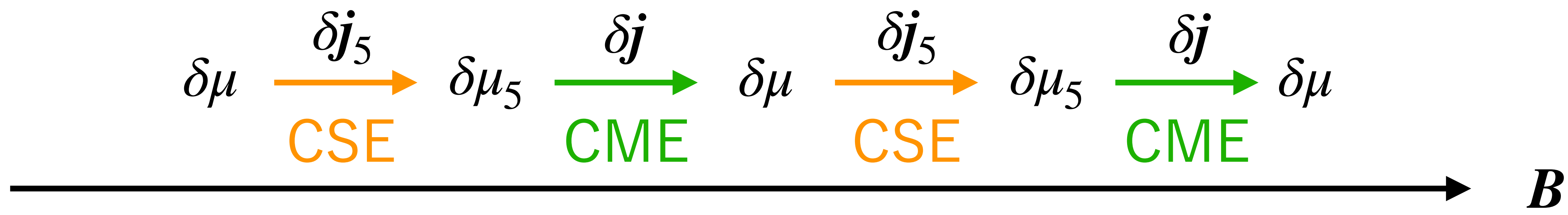
CME | CSE

Vilenkin (1980); Nilsen, Ninomiya (1983);
Fukushima, Kharzeev, Warringa (2008); ⋯

Son, Zhitnitsky (2004);
Metlitsky, Zhitnitsky (2005); ⋯

Chiral magnetic wave(CMW)

- CMW is a density wave by CME & CSE



- Wave equation & dispersion relation

$$\left[\partial_t^2 - \left(\frac{e\boldsymbol{B}}{2\pi^2\chi}\cdot\nabla\right)^2\right]\delta n = 0 \quad \Rightarrow \quad \omega_{\mathrm{CMW}} = \frac{e\boldsymbol{B}}{2\pi^2\chi}\cdot\boldsymbol{k}$$

- CMW can propagate **without** chirality imbalance at the equilibrium

- Discussed in the context of quark-gluon plasma

Chiral magnetic waves in quark matter

Hanai, Yamamoto (2022)

Chiral magnetic mode

- Quarks in neutron stars are relativistic
- CMW can propagate without chirality imbalance at the equilibrium
- Quark number density：no damping by electric conductivity

$\sim \mathrm{e}^{-\sigma t}$

- Possible strong magnetic field in neutron stars $(\sim 10^{18}\ \mathrm{Gauss})$

Lai, Shapiro (1991); Cardall, Prakash, Lattimer (2001); Ferrer, et al. (2010); ⋯

New oscillation mode (**CM-mode**) can appear
in the quark matter inside neutron stars

Chiral magnetic wave (CMW)

- Quark mass term

$$\mathscr{L}_{\mathrm{mass}} = -m_{\mathrm{q}} \underline{\bar{\psi}_{\mathrm{L}} \psi_{\mathrm{R}}} + \mathrm{h.c.} \Rightarrow \text{chirality flipping}$$

mixing

- Chirality flipping rate

$$\Gamma_{\mathrm{flip}} \sim \frac{\alpha_{\mathrm{s}}^2 m_{\mathrm{q}}^2}{\bar{\mu}_{\mathrm{q}}^2 q_{\mathrm{D}}} T^2$$

(The similar expression is valid for QED process)

CMW is damped by the chirality flipping

Gravitational waves of chiral magnetic mode

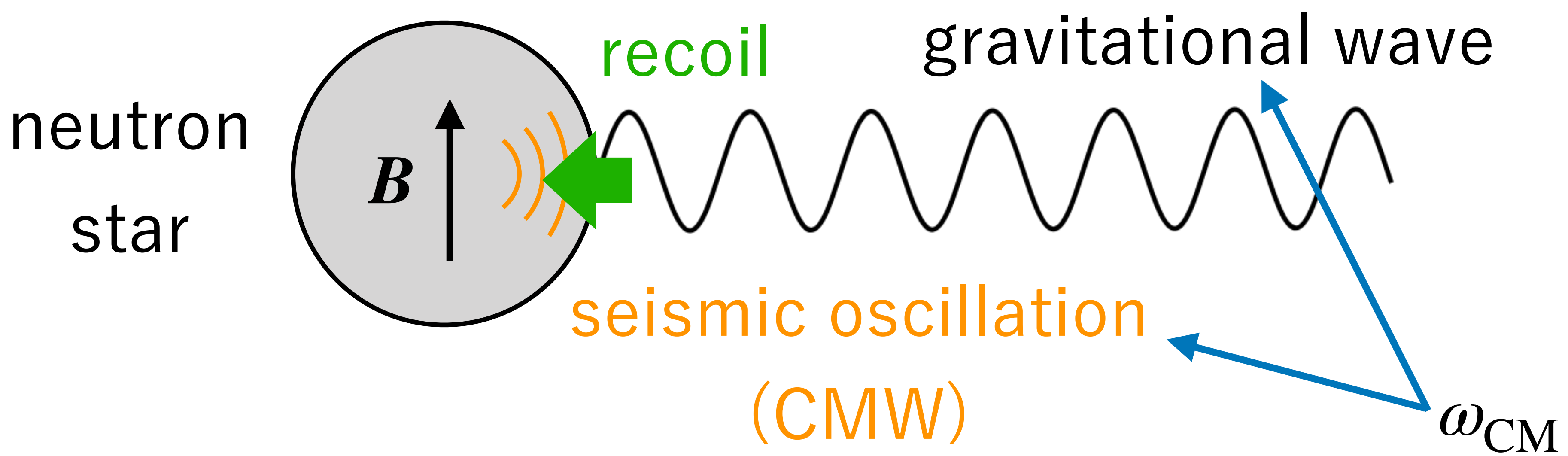
Hanai, Yamamoto (2022)

Cowling approximation

- Newton gravity：ignore the recoil by gravitational potential

Cowling (1941)

- General relativity：ignore the recoil by gravitational waves

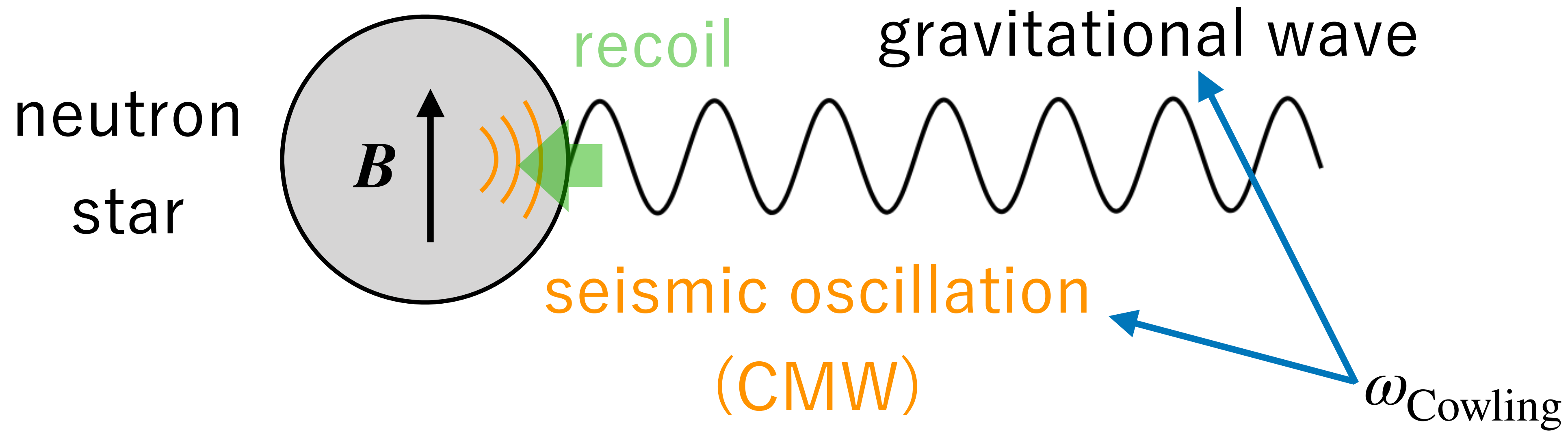


Cowling approximation

- Newton gravity : ignore the recoil by gravitational potential

Cowling (1941)

- General relativity : ignore the recoil by gravitational waves



Cowling approximation

$$\omega_{\mathrm{Cowling}} \simeq \omega_{\mathrm{CM}}$$

Frequency of CM-mode

- Dispersion relation of the CM-mode

$$\omega_{\rm CM} \simeq V_{\rm CM}|k_z| - \mathrm{i}\frac{\Gamma_{\rm flip}}{2} - \mathrm{i}e^{\lambda-\rho}Dk_z^2 \qquad V_{\rm CM} \equiv e^{\lambda-\rho}\frac{N_c eB}{6\pi^2\chi}$$

- The range of the frequency

chirality flipping $$\frac{\Gamma_{\rm flip}}{4\pi} \ll f_{\rm CM} \ll \frac{V_{\rm CM}^2}{2\pi D} = \frac{3V_{\rm CM}^2}{2\pi\tau}$$ diffusion

- Relaxation time

$$\frac{1}{\tau} \sim \#\alpha_s^2\frac{T^2}{q_D} + \#\alpha_s^2\frac{T^{5/3}}{q_D^{2/3}}$$

Heiselberg, Pethick (1993)

usual Fermi liquid — Landau damping

Estimate of the frequency

- Settings： $\alpha_s \simeq 0.5,\ \bar{\mu} \simeq 500\ \mathrm{MeV},\ m_{\mathrm{u,d}} \sim 3\text{-}5\ \mathrm{MeV}$, 2SC

- The range of the frequency

$$10\ \mathrm{Hz}\left(\frac{T}{10^6\ \mathrm{K}}\right)^2 \ll f_{\mathrm{CM}} \ll 10^3\ \mathrm{Hz}\left(\frac{B}{10^{18}\ \mathrm{Gauss}}\right)^2\left(\frac{T}{10^6\ \mathrm{K}}\right)^{5/3}$$

- Dependence of the physical parameters

$$f_{\mathrm{CM}} \sim 10^2\ \mathrm{Hz}\left(\frac{B}{10^{18}\ \mathrm{Gauss}}\right)\left(\frac{\bar{\mu}}{500\ \mathrm{MeV}}\right)^{-2}\left(\frac{k}{10^{-5}\ /\mathrm{cm}}\right)$$

CM-mode can be a new probe of the magnetic field
& quark matter in neutron stars

Amplitude of the gravitational wave

- Effective amplitude of gravitational waves

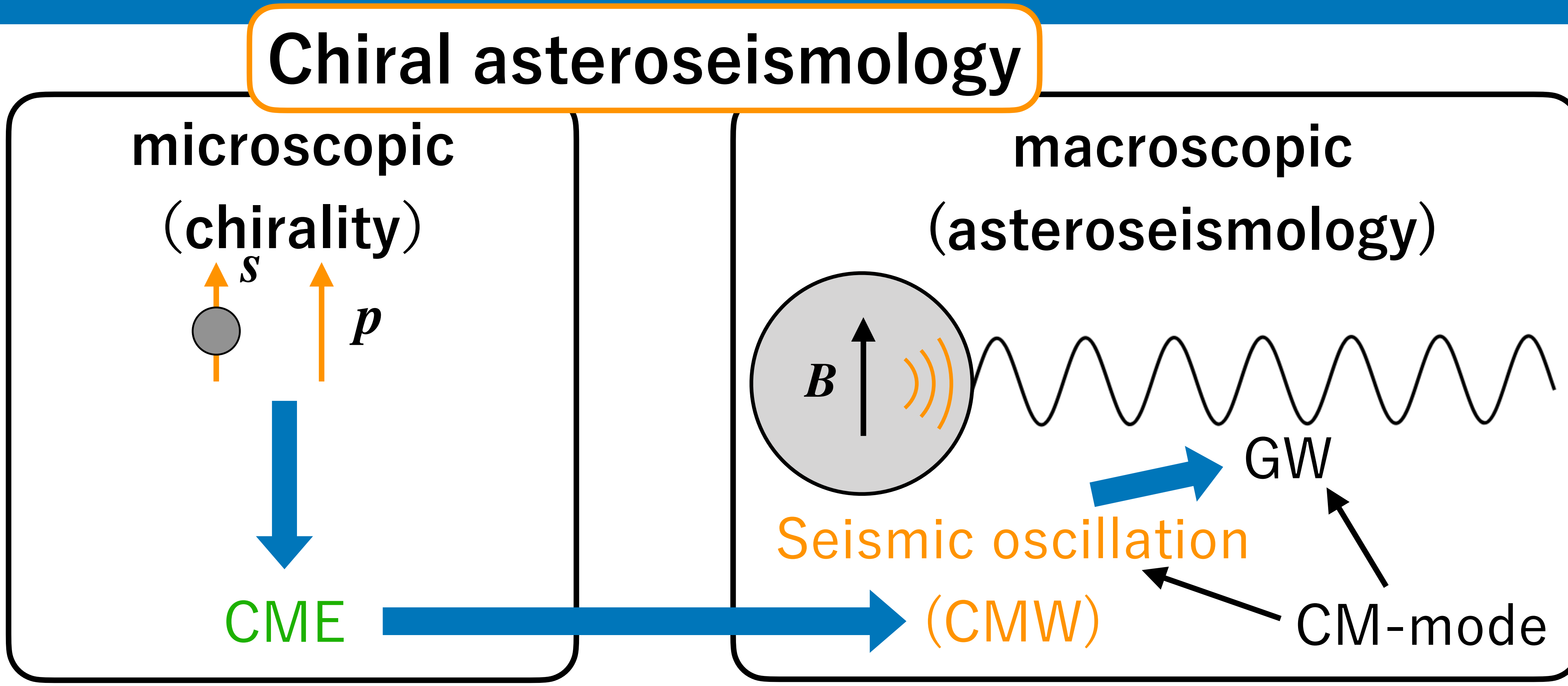
$$h \sim \frac{1}{d}\sqrt{\frac{GE_{\mathrm{GW}}}{f}}$$

- The released energy E_{GW} depends on events

- The amplitude of the gravitational waves
  - cf. giant flare $\sim 10^{46}$ erg, radius of our galaxy ~ 10 kpc

$$h_{\mathrm{CM}} \sim 10^{-21}\left(\frac{E_{\mathrm{GW}}}{10^{44}\ \mathrm{erg}}\right)^{1/2}\left(\frac{f_{\mathrm{CM}}}{10^{2}\ \mathrm{Hz}}\right)^{-1/2}\left(\frac{d}{10\ \mathrm{kpc}}\right)^{-1}$$

Summary & outlook



- CM-mode：a new probe of magnetic fields & quark matter
- CV-mode：another mode caused by CVW in supernovae
- Contribution of magnetic fields to Γ_{flip} , D ?

Appendix

Parity of transport

- Electric fields cause electric currents（Ohm's law）

$$e\boldsymbol{j} = \sigma \boldsymbol{E} \longrightarrow e(-\boldsymbol{j}) = \sigma(-\boldsymbol{E})$$

parity transformation

- Do magnetic fields cause electric currents ?

$$e\boldsymbol{j} = \sigma_{\mathrm{m}} \boldsymbol{B} \longrightarrow e(-\boldsymbol{j}) = \sigma_{\mathrm{m}} \boldsymbol{B}$$

parity transformation

If $\sigma_{\mathrm{m}} \to -\sigma_{\mathrm{m}}$ is satisfied, the current can be generated.

Chiral magnetic wave (CMW)

- Density fluctuations:

$$\mu = \bar{\mu} + \delta\mu, \qquad \delta\mu = \frac{1}{\chi}\delta n$$

$$\mu_5 = \bar{\mu}_5 + \delta\mu_5, \qquad \delta\mu_5 = \frac{1}{\chi}\delta n_5$$

(χ : susceptibility)

- Current fluctuations caused by CME & CSE

$$\boldsymbol{j} = \frac{e\mu_5}{2\pi^2}\boldsymbol{B} \qquad \Rightarrow \qquad \delta\boldsymbol{j} = \frac{e\boldsymbol{B}}{2\pi^2\chi}\delta n_5$$

$$\boldsymbol{j}_5 = \frac{e\mu}{2\pi^2}\boldsymbol{B} \qquad\qquad \delta\boldsymbol{j}_5 = \frac{e\boldsymbol{B}}{2\pi^2\chi}\delta n$$

Chiral magnetic wave (CMW)

- Linearized continuity equations

$$\partial_t \delta n + \nabla \cdot \delta \boldsymbol{j} = 0, \qquad \partial_t \delta n_5 + \nabla \cdot \delta \boldsymbol{j}_5 = 0$$

- Wave equation of the CMW

$$\left[\partial_t^2 - \left(\frac{e\boldsymbol{B}}{2\pi^2\chi} \cdot \nabla\right)^2\right] \delta n = 0 \quad \Rightarrow \quad \omega_{\mathrm{CMW}} = \frac{e\boldsymbol{B}}{2\pi^2\chi} \cdot \boldsymbol{k}$$

- Dispersion relation is independent of $\bar{\mu}_5$

- Discussed in the context of quark-gluon plasma

CMW of electron matter

- Electrons in neutron stars and supernovae are relativistic
- For electrons, charge and number density are not independend
- CMW has a gap by the chiral anomaly
- Charge density：damped by conductivity Shovkovy, Rybalka, Dorbar (2018)

$\sim \mathrm{e}^{-\sigma t}$

$$\Gamma_{\mathrm{flip}} \ll \frac{eB}{2\pi^2 \chi_{\mathrm{e}}} \frac{1}{l_{\mathrm{mfp}}} \ll \sigma$$ ➡ CMW can not appear

Scattering in electron matter

- Candidates of scattering processes：
  - electron-electron scattering
  - electron-proton scattering
  - electron-photon scattering

- e: degenerate, p: degenerate (NS) or non-degenerate (SN)

- photon density $\sim T^3 \ll$ electron density $\sim \bar{\mu}_e^3$

- Typical energy of electron: $\bar{\mu}_e \ll m_p$

The e-p scattering (Rutherford scattering) is dominant.

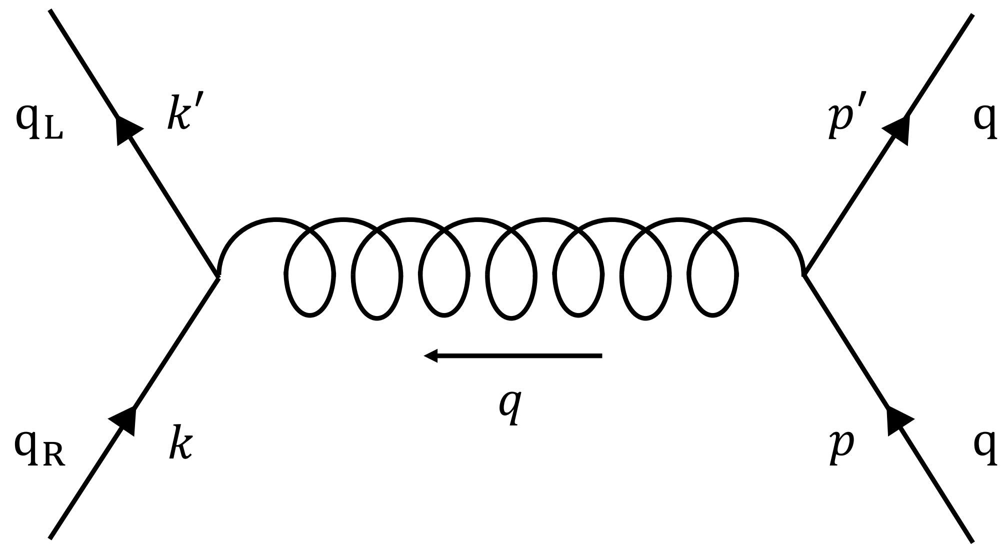
Chirality flipping

- Quark mass term

$$\mathscr{L}_{\text{mass}} = -m_{\text{q}}\bar{\psi}_{\text{L}}\psi_{\text{R}} + \text{h.c.}$$

mixing → chirality flipping

- Quark-quark scattering



$$|M_{\text{l}}|^2 \sim \left| \frac{j^0_{-+} j^0_h}{\boldsymbol{q}^2 + \Pi_{\text{l}}} \right|^2, \qquad |M_{\text{t}}|^2 \sim \left| \frac{\boldsymbol{j}_{-+,\text{t}} \cdot \boldsymbol{j}_{h,\text{t}}}{-(q^0)^2 + \boldsymbol{q}^2 + \Pi_{\text{t}}} \right|^2$$

longitudinal transverse

Chirality flipping

- Chirality flipping rate

$$\Gamma_{\rm flip} \equiv -\frac{\dot{n}_5}{n_5}$$

Boltzmann equation

$$\dot{f}_{\rm R/L} = \mp C(\boldsymbol{k}, t)$$

$$\sim \# \frac{\alpha_s^2 m_q^2}{\bar{\mu}_q^2 q_D} T^2 + \# \frac{\alpha_s^2 m_q^2}{\bar{\mu}_q^4 q_D^{2/3}} T^{11/3} - \# \frac{\alpha_s^2 m_q^2}{\bar{\mu}_q^3 q_D^2} T^4 + \# \frac{\alpha_s^2 m_q^2}{\bar{\mu}_q^2 q_D^{10/3}} T^{13/3}$$

longitudinal
(usual Fermi liquid)

transverse
(Landau damping)

CMW is damped by the chirality flipping

2-flavor color superconductivity(2SC)

- Quark condensation

One of colors is unpaired

$$\left\langle q^a_{\mathrm{L}i}\, q^b_{\mathrm{L}j} \right\rangle = \epsilon^{ab3} \epsilon_{ij} \Delta$$

(a, b : color, i, j : flavor)

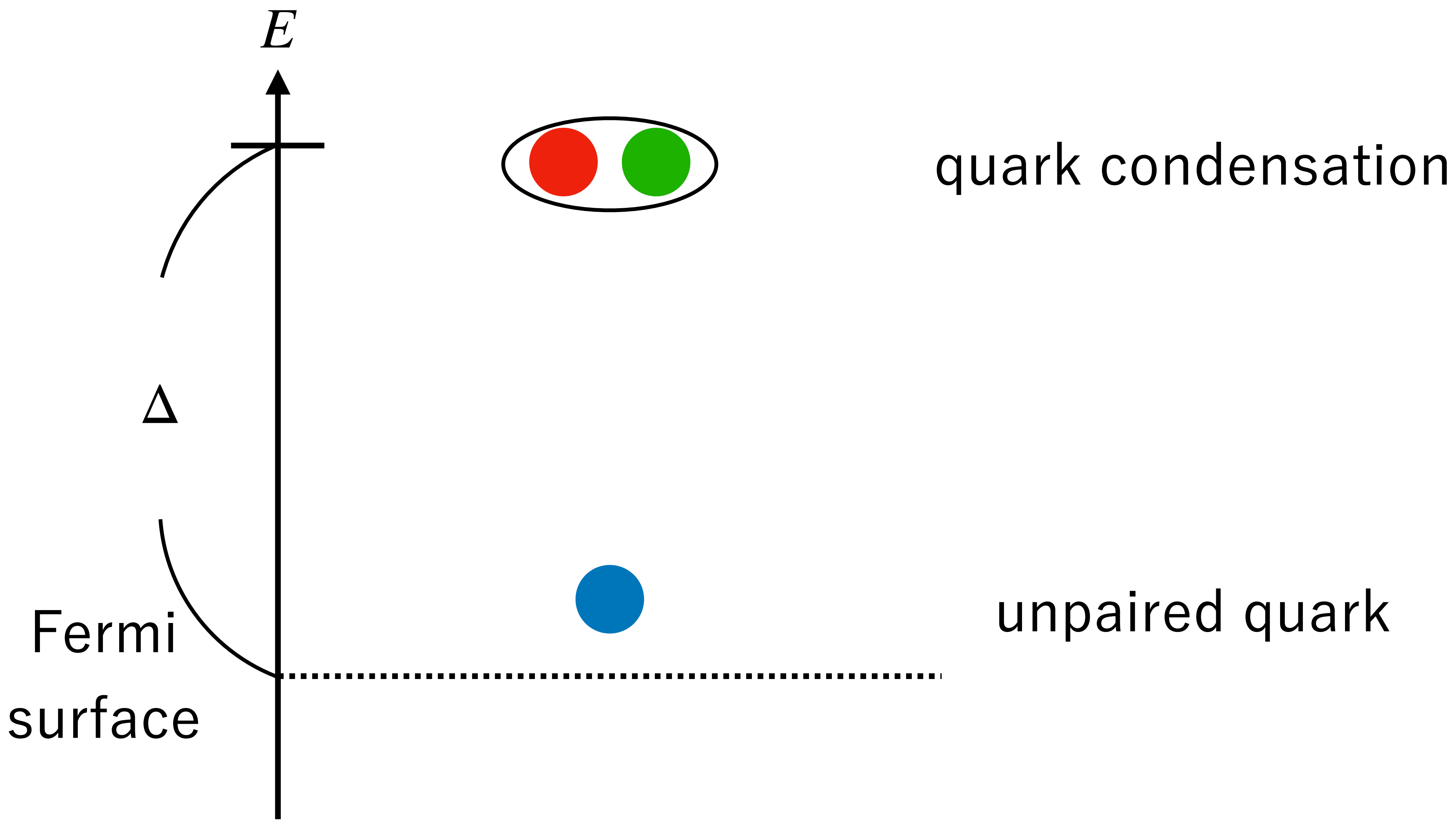
- Pattern of symmetry breaking

$$\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{U}(1)_{\mathrm{B}}$$
$$\to \mathrm{SU}(2)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{U}(1)_{\mathrm{Q}}$$

chiral symmetry is unbroken

2-flavor color superconductivity(2SC)

- Focusing on the energy smaller than the gap Δ



Analysis of gravitational waves

- Gravitational waves are the fluctuations of spacetime

- Metric in cylindrical coordinates

$$\boldsymbol{B} = B\boldsymbol{e}_z$$

$$g_{\alpha\beta} = \bar{g}_{\alpha\beta} + h_{\alpha\beta}$$

gravitational waves

$$\bar{g}_{\alpha\beta}\mathrm{d}x^{\alpha}\mathrm{d}x^{\beta} = -\mathrm{e}^{2\lambda}\mathrm{d}t^2 + \mathrm{e}^{2\nu}\mathrm{d}r^2 + r^2\mathrm{d}\phi^2 + \mathrm{e}^{2\rho}\mathrm{d}z^2$$

- It is hard to solve the linearized Einstein equations

→ Cowling approximation

Fundamental equations

- Metric in cylindrical coordinates

$$g_{\alpha\beta} = \bar{g}_{\alpha\beta} + h_{\alpha\beta}$$ Cowling approximation

$$\bar{g}_{\alpha\beta} \mathrm{d}x^\alpha \mathrm{d}x^\beta = -\mathrm{e}^{2\lambda}\mathrm{d}t^2 + \mathrm{e}^{2\nu}\mathrm{d}r^2 + r^2\mathrm{d}\phi^2 + \mathrm{e}^{2\rho}\mathrm{d}z^2$$

- Current fluctuations by CME & CSE with diffusion

$$\delta \boldsymbol{j} = \frac{e\boldsymbol{B}}{2\pi^2\chi}\delta n_5 - D\nabla\delta n, \qquad \delta \boldsymbol{j}_5 = \frac{e\boldsymbol{B}}{2\pi^2\chi}\delta n - D\nabla\delta n_5$$

- Linearized continuity equations

$$\nabla_\alpha \delta j^\alpha = 0, \qquad \nabla_\alpha \delta j_5^\alpha = -\Gamma_{\mathrm{flip}}\mathrm{e}^{-\lambda}\delta n_5$$

chirality flipping

Relaxation time of quark matter

- Medium effects
  - longitudinal: Debye screening (usual Fermi liquid like)
  - transverse: Landau damping

$$\frac{1}{\tau} \sim \#\alpha_s^2 \frac{T^2}{q_D} + \#\alpha_s^2 \frac{T^{5/3}}{q_D^{2/3}}$$

Heiselberg, Pethick (1993)

usual Fermi liquid

$(q_{IR} \sim q_D)$

Landau damping

$(q_{IR} \sim (q_D^2 T)^{1/3})$

CME & chiral anomaly

Nielsen, Ninomiya (1983)

- Energy necessary for the chirality imbalance

$$\mu_5 \frac{\mathrm{d}Q_5}{\mathrm{d}t} = \mu_5 \int \mathrm{d}^3\boldsymbol{x} \frac{e^2}{2\pi^2} \boldsymbol{E} \cdot \boldsymbol{B}$$

chiral anomaly

- Energy is suppled by electric currents

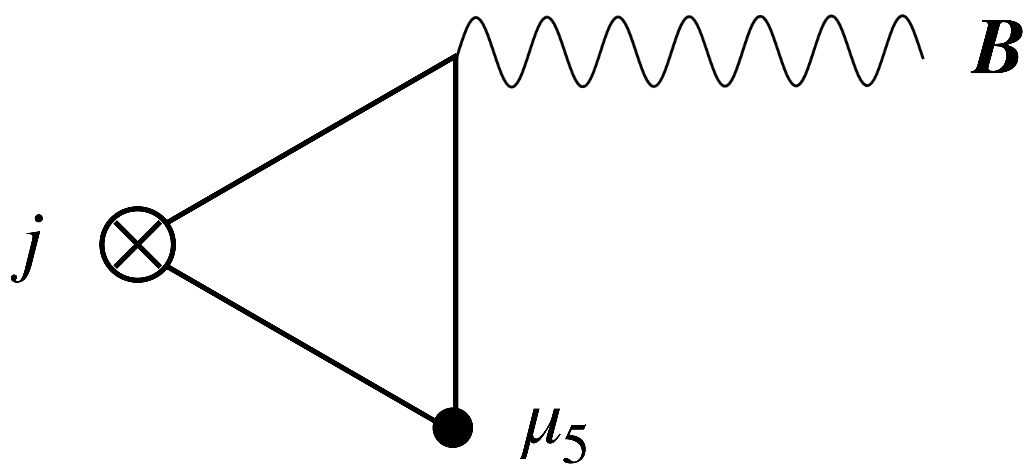
$$\mu_5 \frac{\mathrm{d}Q_5}{\mathrm{d}t} = \int \mathrm{d}^3 e \boldsymbol{j} \cdot \boldsymbol{E}$$

- For arbitrary electric fields,

$$\Rightarrow \quad \boldsymbol{j} = \frac{e\mu_5}{2\pi^2} \boldsymbol{B}$$

2 flavor VS 3 flavor

- Coefficient of CME



$$\boldsymbol{j} \propto \mathrm{tr}(VAQ)\mu_5 \boldsymbol{B}$$

$$V = \mathbf{1}_{N_\mathrm{f}}, \qquad A = \mathbf{1}_{N_\mathrm{f}}, \qquad Q = \begin{cases} \mathrm{diag}\left(+\frac{2}{3}, -\frac{1}{3}\right) & (N_\mathrm{f} = 2) \\ \mathrm{diag}\left(+\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right) & (N_\mathrm{f} = 3) \end{cases}$$

Since $\boldsymbol{j} \propto \mathrm{tr}(Q) = 0 \quad (N_\mathrm{f} = 3)$,

the CMW exists only for two-flavor case.

Kharzeev, Son (2011)

Generation of gravitational waves

- The CMW is the fluctuation of quark number density

number density → energy density → gravitational wave

$\delta n \propto \mathrm{e}^{\mathrm{i}\omega t}$ $\delta \varepsilon \propto \mathrm{e}^{\mathrm{i}\omega t}$ $h \propto \mathrm{e}^{\mathrm{i}\omega t}$

CM-mode CM-mode

Contribution of metric functions

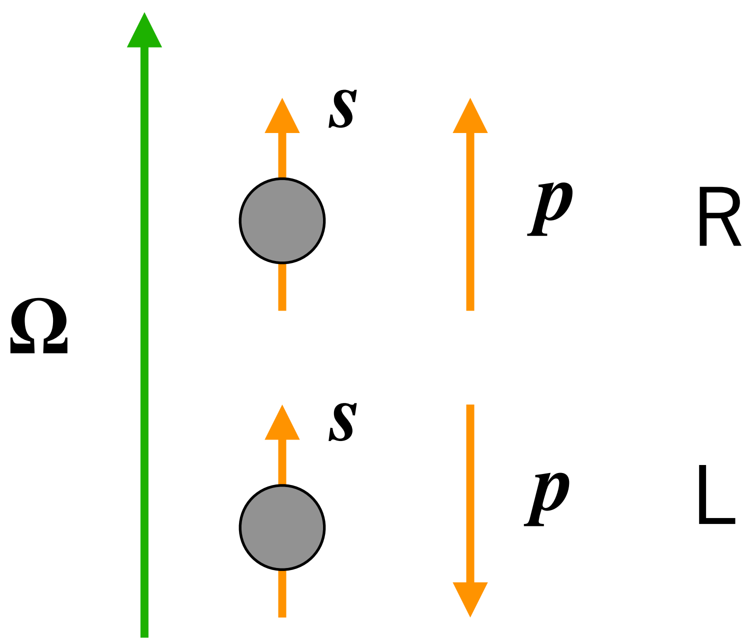
- The metric functions include the correction of general relativity

- Typical metric functions in the case of neutron stars

$$\lambda,\, \rho \sim \frac{GM}{R} \sim 10^{-1}$$

- It is reasonable to set the metric functions to be 0

Chiral vortical effect



- Correspondence between $\boldsymbol{B}$ and $\boldsymbol{\Omega}$

$$\underbrace{e\boldsymbol{v}\times\boldsymbol{B}}_{\text{Lorentz Force}} \leftrightarrow \underbrace{2m\boldsymbol{v}\times\boldsymbol{\Omega}}_{\text{Coriolis force}}$$

Lorentz Force   Coriolis force

- Relativistic case

$$m \to E = p$$

CVE

$$j_{\mathrm{R}} = \left(\frac{\mu_{\mathrm{R}}^2}{4\pi^2} + \frac{T^2}{12}\right)\boldsymbol{\Omega}$$

$$j_{\mathrm{L}} = \left(\frac{\mu_{\mathrm{R}}^2}{4\pi^2} + \frac{T^2}{12}\right)\boldsymbol{\Omega}$$

Vilenkin (1979); Son, Surowka (2000); Landsteiner (2011); ⋯

Chiral vortical wave

- Wave equation

$$\left(\partial_t \pm \frac{\bar{\mu}_{\mathrm{R/L}}\boldsymbol{\Omega}}{2\pi^2\chi}\cdot\nabla\right)\delta n_{\mathrm{R/L}} = 0$$

- Dispersion relation

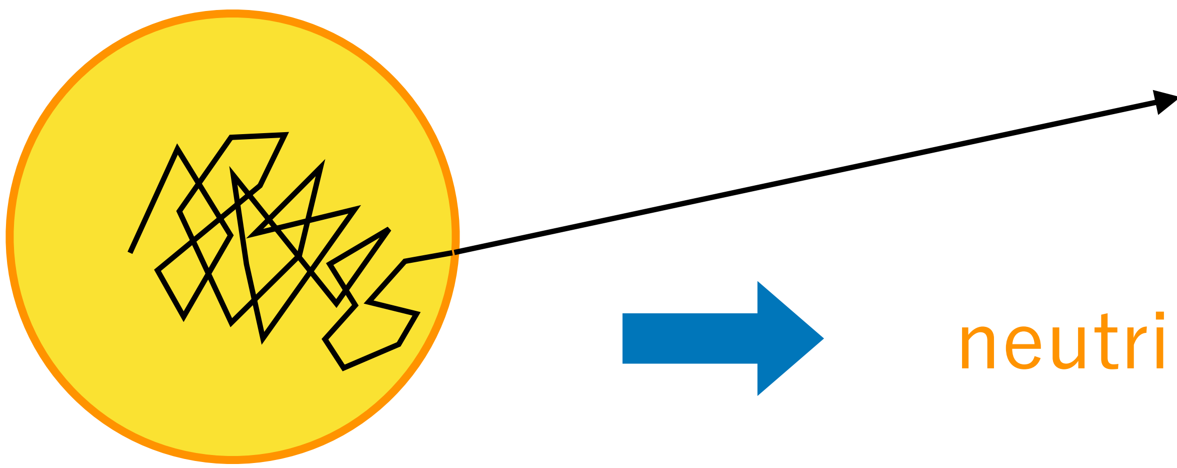
$$\omega_{\mathrm{R/L}} = \pm\frac{\bar{\mu}_{\mathrm{R/L}}\boldsymbol{\Omega}}{2\pi^2\chi}\cdot\boldsymbol{k}$$

- CVW needs the chirality imbalance to propagate

Neutrino matter

- Electron capture: $\mathrm{p} + \mathrm{e}_\mathrm{L}^- \rightarrow \mathrm{n} + \nu_{\mathrm{e,L}}$

- Timescale: gravitational collapse $<$ neutrino diffusion



neutrino

neutrino trapping

proto neutron star

- Matter with **only** left-handed neutrinos：chiral matter

Frequency of CV-mode

- Dispersion relation

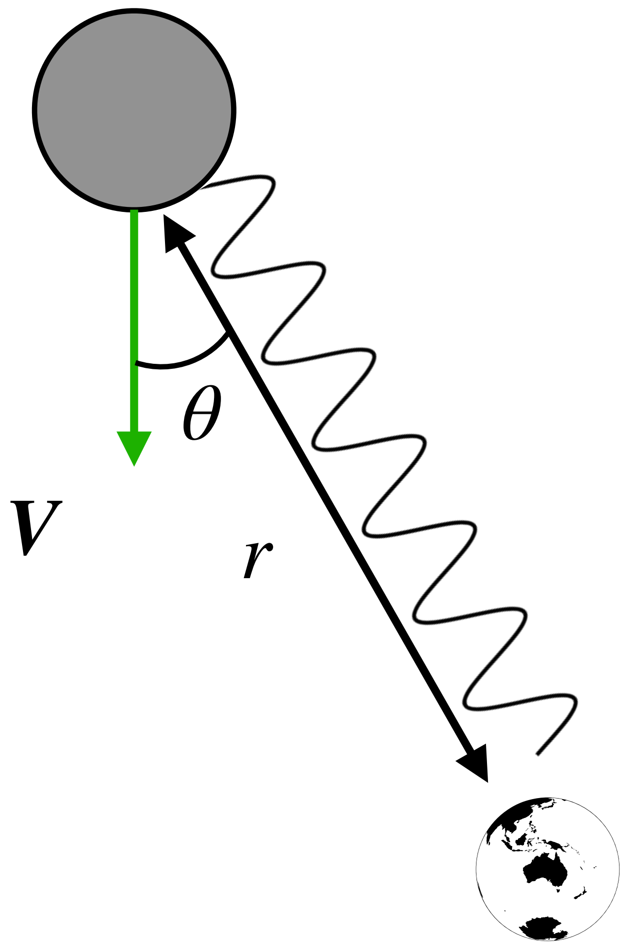
$$\omega_{\rm CM} \simeq -V_{\rm CV}k_z - {\rm i}{\rm e}^{\lambda-\rho}Dk_z^2 \qquad V_{\rm CV} \equiv {\rm e}^{\lambda-\rho}\frac{\bar{\mu}_\nu \Omega}{2\pi^2\chi_\nu}$$

- Energy scale of rotation is small

➡ speed of CV-mode < diffusion effect

$$\frac{V_{\rm CV}}{D} \simeq \frac{3\Omega}{\bar{\mu}_\nu l_{\rm mfp}} \sim 10^{-33}\ {\rm MeV}\left(\frac{\Omega/2\pi}{1\ {\rm Hz}}\right)$$

Angular dependence of GW



- Can assume that the source runs in constant velocity

- Quadrupole moment formula

$$\tilde{h}_{ab} = \frac{2G}{r}\frac{\partial^2}{\partial t^2} I_{ab}(t-r), \qquad I_{ab} = \int \mathrm{d}^3\boldsymbol{x}' T^{00}(t,\boldsymbol{x}) x'_a x'_b$$

- Angular dependence

$$\tilde{h}_{\theta\theta} \propto \sin^2\theta$$

Gravitational waves are emitted strongly at $\theta = \pi/2$