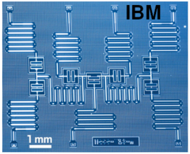
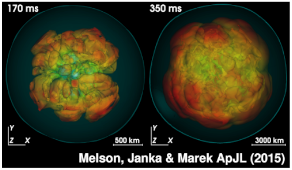
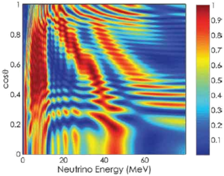
Simulation of Collective Neutrino Oscillations beyond the Mean Field Approximation

Alessandro Roggero







Duan et al. (2006)

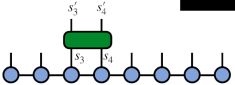
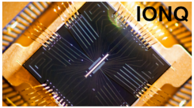
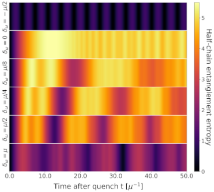


image from itensor.org



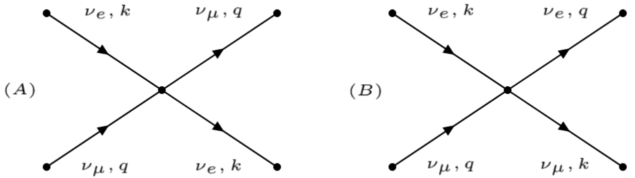


Focus Workshop on Collective Oscillations and Chiral Transport of Neutrinos

Taipei – 16 Mar, 2023

Neutrino-neutrino forward scattering

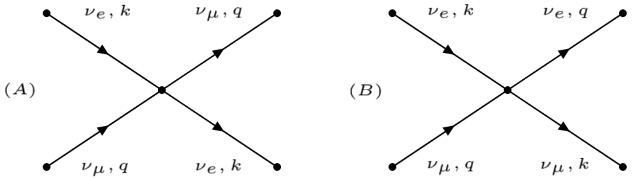
Fuller, Qian, Pantaleone, Sigl, Raffelt, Sawyer, Carlson, Duan, . . .



- diagonal contribution (A) does not impact flavor mixing
- off-diagonal term (B) equivalent to flavor/momentum exchange between two neutrinos
  - total flavor is conserved

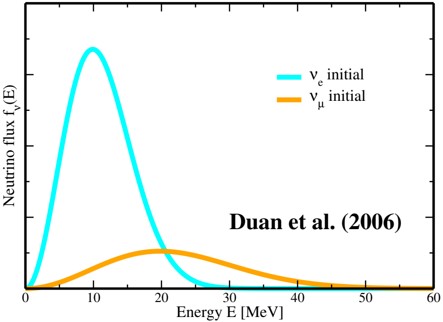
Neutrino-neutrino forward scattering

Fuller, Qian, Pantaleone, Sigl, Raffelt, Sawyer, Carlson, Duan, . . .



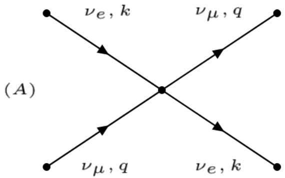
- diagonal contribution (A) does not impact flavor mixing
- off-diagonal term (B) equivalent to flavor/momentum exchange between two neutrinos
  - total flavor is conserved

Important effect if initial distributions are strongly flavor dependent



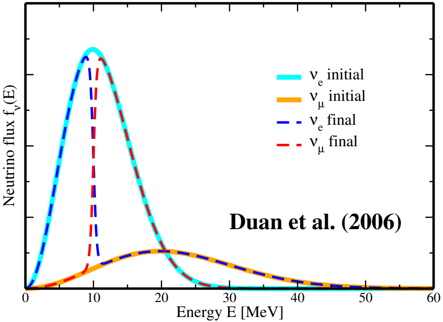
Neutrino-neutrino forward scattering

Fuller, Qian, Pantaleone, Sigl, Raffelt, Sawyer, Carlson, Duan, ...



- diagonal contribution (A) does not impact flavor mixing
- off-diagonal term (B) equivalent to flavor/momentum exchange between two neutrinos
    - total flavor is conserved

Important effect if initial distributions are strongly flavor dependent



Two-flavor approximation and the iso-spin Hamiltonian

Consider two active flavors (ν_e,ν_x) and encode flavor amplitudes for a neutrino with momentum p_i into an $SU(2)$ iso-spin:

$$|\Phi_i\rangle = \cos(\eta_i)|\nu_e\rangle + \sin(\eta_i)|\nu_x\rangle \equiv \cos(\eta_i)|\uparrow\rangle + \sin(\eta_i)|\downarrow\rangle$$

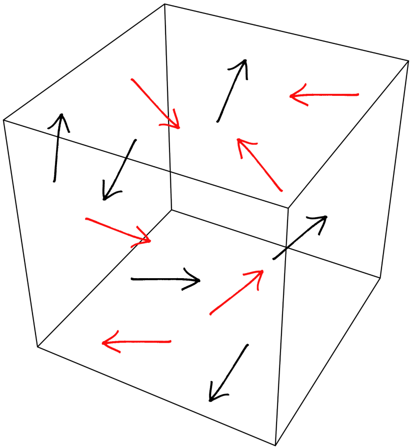
A system of N interacting neutrinos is then described by the Hamiltonian

$$H = \sum_i \frac{\Delta m^2}{4E_i}\vec{B}\cdot\vec{\sigma}_i + \lambda\sum_i \sigma_i^z + \frac{\mu}{2N}\sum_{i<j}\left(1-\cos(\phi_{ij})\right)\vec{\sigma}_i\cdot\vec{\sigma}_j$$

- vacuum oscillations: $\vec{B} = (\sin(2\theta_{mix}), 0, -\cos(2\theta_{mix}))$
- interaction with matter: $\lambda = \sqrt{2}G_F\rho_e$
- neutrino-neutrino interaction: $\mu = \sqrt{2}G_F\rho_\nu$
  - dependence on momentum direction: $\cos(\phi_{ij}) = \frac{\vec{p}_i}{\|\vec{p}_i\|}\cdot\frac{\vec{p}_j}{\|\vec{p}_j\|}$

for a full derivation, see e.g. Pehlivan et al. PRD(2011)

Finite size effects and thermodynamic limit



$$H = \sum_{i=1}^{N} \vec{B}_i \cdot \vec{\sigma}_i + \frac{\mu}{2N} \sum_{i<j}^{N} v_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

- the quantum system is defined in some finite volume V
- we have a finite number N of neutrinos within the box
- the neutrino density ρ_ν (and thus μ) is given by N/V

For astrophysically relevant predictions need to understand how the system behaves when $V \to \infty$ and $N \to \infty$ while keeping $\rho_\nu = N/V$ constant

The Mean Field approximation

The equations of motion for the flavor polarization $\langle\vec{\sigma}_i\rangle$ are

$$\frac{d}{dt}\langle\vec{\sigma}_i\rangle = \vec{B}_i \times \langle\vec{\sigma}_i\rangle + \frac{\mu}{2N}\sum_{j\neq i} v_{ij}\langle\vec{\sigma}_j \times \vec{\sigma}_i\rangle$$

The Mean Field approximation

The equations of motion for the flavor polarization $\langle\vec{\sigma}_i\rangle$ are

$$\frac{d}{dt}\langle\vec{\sigma}_i\rangle = \vec{B}_i \times \langle\vec{\sigma}_i\rangle + \frac{\mu}{2N}\sum_{j\neq i} v_{ij}\langle\vec{\sigma}_j \times \vec{\sigma}_i\rangle$$

The Mean Field approximation replaces $\langle\vec{\sigma}_j \times \vec{\sigma}_i\rangle$ with $\langle\vec{\sigma}_j\rangle \times \langle\vec{\sigma}_i\rangle$ so that

$$\frac{d}{dt}\langle\vec{\sigma}_i\rangle = \left(\vec{B}_i + \frac{\mu}{2N}\sum_{j\neq i} v_{ij}\langle\vec{\sigma}_j\rangle\right) \times \langle\vec{\sigma}_i\rangle$$

The Mean Field approximation

The equations of motion for the flavor polarization $\langle\vec{\sigma}_i\rangle$ are

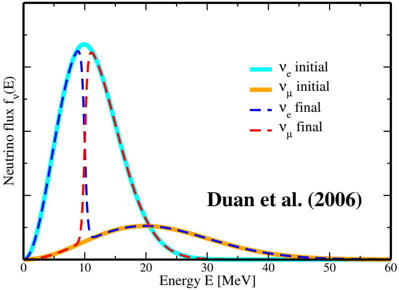
$$\frac{d}{dt}\langle\vec{\sigma}_i\rangle = \vec{B}_i \times \langle\vec{\sigma}_i\rangle + \frac{\mu}{2N}\sum_{j\neq i} v_{ij}\langle\vec{\sigma}_j \times \vec{\sigma}_i\rangle$$

The Mean Field approximation replaces $\langle\vec{\sigma}_j \times \vec{\sigma}_i\rangle$ with $\langle\vec{\sigma}_j\rangle \times \langle\vec{\sigma}_i\rangle$ so that

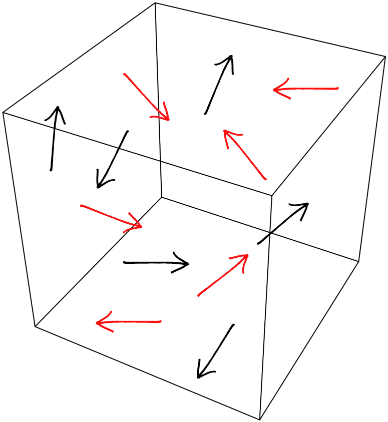
$$\frac{d}{dt}\langle\vec{\sigma}_i\rangle = \left(\vec{B}_i + \frac{\mu}{2N}\sum_{j\neq i} v_{ij}\langle\vec{\sigma}_j\rangle\right) \times \langle\vec{\sigma}_i\rangle$$

In this way we obtain a closed system of $3N$ coupled differential equations

- efficient solutions for systems containing $N \approx \mathcal{O}(10^{4-5})$ neutrino amplitudes [$\approx \mathcal{O}(100)$ energies and $\approx \mathcal{O}(100)$ angles]



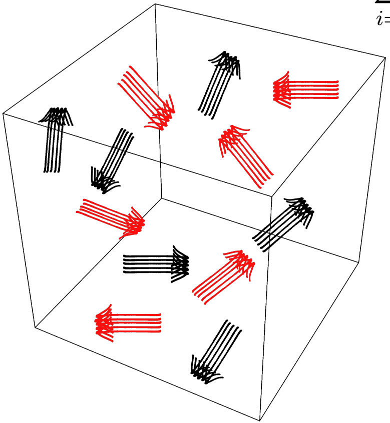
Why a Mean Field approximation?



$$H = \sum_{i=1}^{N} \vec{B}_i \cdot \vec{\sigma}_i + \frac{\mu}{2N} \sum_{i<j}^{N} v_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

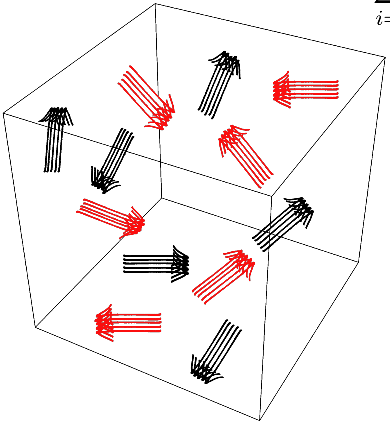
Why a Mean Field approximation?

$$H = \sum_{i=1}^{N}\sum_{a=1}^{M}\vec{B}_i\cdot\vec{\sigma}_{ia} + \frac{\mu}{2MN}\sum_{i<j}^{N}\sum_{a,b=1}^{M} v_{ij}\vec{\sigma}_{ia}\cdot\vec{\sigma}_{jb}$$



Why a Mean Field approximation?

$$H = \sum_{i=1}^{N}\sum_{a=1}^{M} \vec{B}_i \cdot \vec{\sigma}_{ia} + \frac{\mu}{2MN}\sum_{i<j}^{N}\sum_{a,b=1}^{M} v_{ij}\vec{\sigma}_{ia}\cdot\vec{\sigma}_{jb}$$

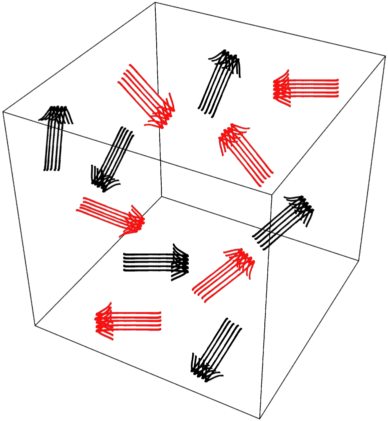


- group similar neutrinos into beams with M neutrinos each

$$\vec{P}_i = \frac{1}{M}\sum_{a=1}^{M}\vec{\sigma}_{ia}$$

- we have a finite number N of beams within the box
- the neutrino density ρ_ν (and thus μ) is given by $(NM)/V$

Why a Mean Field approximation?



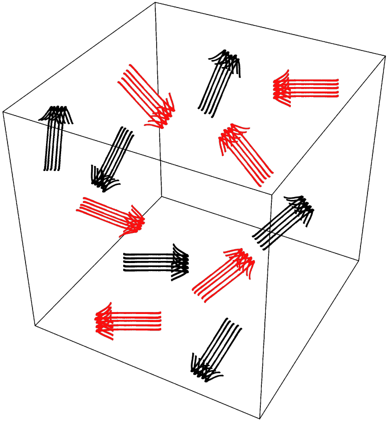
$$\frac{H}{M} = \sum_{i=1}^{N} \vec{B}_i \cdot \vec{P}_i + \frac{\mu}{2N} \sum_{i<j}^{N} v_{ij} \vec{P}_i \cdot \vec{P}_j$$

- group similar neutrinos into beams with M neutrinos each

$$\vec{P}_i = \frac{1}{M} \sum_{a=1}^{M} \vec{\sigma}_{ia}$$

- we have a finite number N of beams within the box
- the neutrino density ρ_ν (and thus μ) is given by $(NM)/V$

Why a Mean Field approximation?



$$\frac{H}{M} = \sum_{i=1}^{N} \vec{B}_i \cdot \vec{P}_i + \frac{\mu}{2N} \sum_{i<j}^{N} v_{ij} \vec{P}_i \cdot \vec{P}_j$$

- group similar neutrinos into beams with M neutrinos each

$$\vec{P}_i = \frac{1}{M} \sum_{a=1}^{M} \vec{\sigma}_{ia}$$

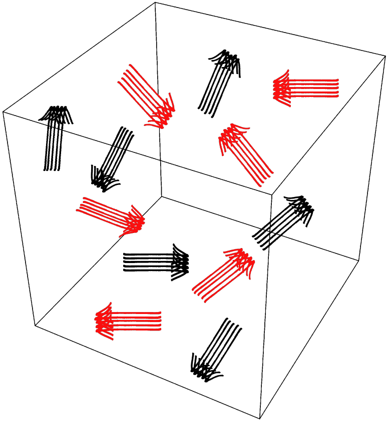
- we have a finite number N of beams within the box
- the neutrino density ρ_ν (and thus μ) is given by $(NM)/V$

Size of quantum fluctuations

$$\langle \vec{P}_i^2 \rangle - \langle \vec{P}_i \rangle^2 = \frac{1}{M} + \left(1 - \langle \vec{P}_i \rangle^2\right)$$

Why a Mean Field approximation?

$$\frac{H}{M} = \sum_{i=1}^{N} \vec{B}_i \cdot \vec{P}_i + \frac{\mu}{2N} \sum_{i<j}^{N} v_{ij} \vec{P}_i \cdot \vec{P}_j$$



- group similar neutrinos into beams with M neutrinos each

$$\vec{P}_i = \frac{1}{M} \sum_{a=1}^{M} \vec{\sigma}_{ia}$$

- we have a finite number N of beams within the box
- the neutrino density ρ_ν (and thus μ) is given by $(NM)/V$

Size of quantum fluctuations

$$\langle \vec{P}_i^2 \rangle - \langle \vec{P}_i \rangle^2 = \frac{1}{M} + \left(1 - \langle \vec{P}_i \rangle^2\right)$$

For initial flavor states $\langle \vec{P}_i \rangle^2 = 1$

MF approx $\equiv M \to \infty$, N fixed

Beyond Mean Field effects: a quick history

Apr '03 speedup through entanglement $\tau \sim \mu^{-1}$ Bell et al. PLB (2003)

Jul '03 in a highly symmetric limit the MF prediction is qualitatively correct up to times $\tau \propto \mu^{-1}\sqrt{N} \rightarrow \infty$ Friedland&Lunardini JHEP (2003)

Aug '04 neutrino-like models seem to produce $\tau \propto \mu^{-1}\log(N)$ Sawyer (2004)

2019 exact simulations for systems with small N show substantial entanglement buildup $(\langle \vec{P}_i \rangle^2 < 1)$ Cervia et al. PRD(2019), Rrapaj PRC(2020)

Beyond Mean Field effects: a quick history

Apr '03	speedup through entanglement $\tau \sim \mu^{-1}$	Bell et al. PLB (2003)
Jul '03	in a highly symmetric limit the MF prediction is qualitatively correct up to times $\tau \propto \mu^{-1}\sqrt{N} \rightarrow \infty$	Friedland&Lunardini JHEP (2003)
Aug '04	neutrino-like models seem to produce $\tau \propto \mu^{-1}\log(N)$	Sawyer (2004)
2019	exact simulations for systems with small N show substantial entanglement buildup $(\langle \vec{P}_i \rangle^2 < 1)$	Cervia et al. PRD(2019), Rrapaj PRC(2020)

- increasing effort in tackling the problem using a variety of methods: diagonalization, tensor networks and semiclassical approaches

Cervia et al. (2021), Patwardhan et al. (2021), AR $(2021)^2$, Xiong (2022), Martin, AR, et al. (2022), AR, Rrapaj, Xiong (2022), Lacroix et al. (2022), . . .

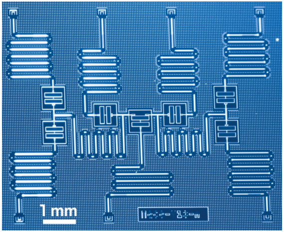
- Great potential for many-body simulations on quantum devices

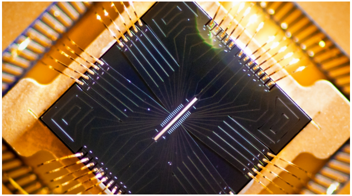
Hall, AR, et al. (2021), Yeter-Aydeniz et al. (2022), Illa & Savage (2022), Amitrano, AR, et al. (2023)

Beyond Mean Field effects: a quick history

- **Apr '03** speedup through entanglement $\tau \sim \mu^{-1}$ — Bell et al. PLB (2003)
- **Jul '03** in a highly symmetric limit the MF prediction is qualitatively correct up to times $\tau \propto \mu^{-1}\sqrt{N} \rightarrow \infty$ — Friedland&Lunardini JHEP (2003)
- **Aug '04** neutrino-like models seem to produce $\tau \propto \mu^{-1}\log(N)$ — Sawyer (2004)
- **2019** exact simulations for systems with small N show substantial entanglement buildup $(\langle \vec{P}_i \rangle^2 < 1)$ — Cervia et al. PRD(2019), Rrapaj PRC(2020)

Great potential for many-body simulations on quantum devices





Beyond Mean Field effects: a quick history

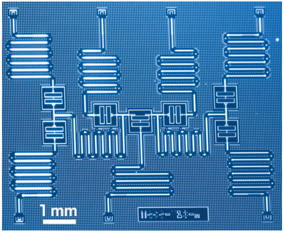
Apr '03 speedup through entanglement $\tau \sim \mu^{-1}$ — Bell et al. PLB (2003)

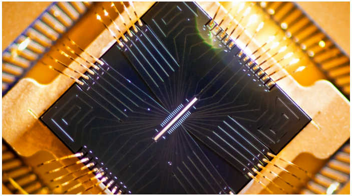
Jul '03 in a highly symmetric limit the MF prediction is qualitatively correct up to times $\tau \propto \mu^{-1}\sqrt{M} \rightarrow \infty$ — Friedland&Lunardini JHEP (2003)

Aug '04 neutrino-like models seem to produce $\tau \propto \mu^{-1}\log(N)$ — Sawyer (2004)

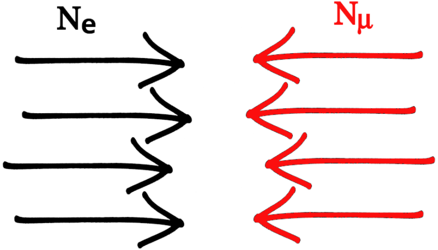
2019 exact simulations for systems with small N show substantial entanglement buildup $(\langle\vec{P}_i\rangle^2 < 1)$ — Cervia et al. PRD(2019), Rrapaj PRC(2020)

Great potential for many-body simulations on quantum devices





Beyond Mean Field effects: a simple example



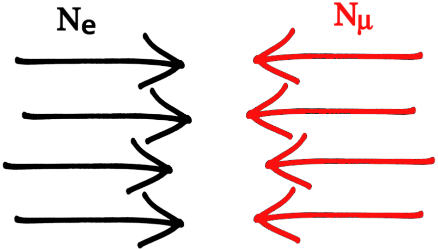
$$\frac{H}{M} = \vec{B}_a \cdot \vec{P}_a + \vec{B}_b \cdot \vec{P}_b + \frac{\mu}{4}\vec{P}_a \cdot \vec{P}_b$$

The initial state has $\langle \vec{P}_a \rangle \times \langle \vec{P}_b \rangle = 0$

If we choose $\vec{B}_a = \vec{B}_b$ parallel to $\langle \vec{P}_b \rangle$, no flavor evolution in MF approx.

- analytical solution evolves over $\tau \approx \mu^{-1}\sqrt{M}$ Friedland&Lunardini (2003)

Beyond Mean Field effects: a simple example



$$\frac{H}{M} = \vec{B}_a \cdot \vec{P}_a + \vec{B}_b \cdot \vec{P}_b + \frac{\mu}{4}\vec{P}_a \cdot \vec{P}_b$$

The initial state has $\langle \vec{P}_a \rangle \times \langle \vec{P}_b \rangle = 0$

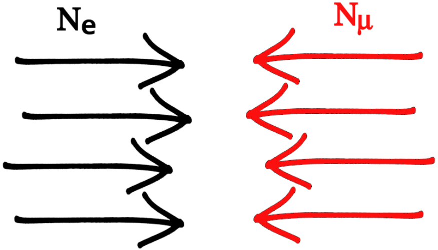
If we choose $\vec{B}_a = \vec{B}_b$ parallel to $\langle \vec{P}_b \rangle$, no flavor evolution in MF approx.

- analytical solution evolves over $\tau \approx \mu^{-1}\sqrt{M}$ Friedland&Lunardini (2003)

If we choose $\vec{B}_a = -\vec{B}_b$ parallel to $\langle \vec{P}_b \rangle$, no flavor evolution in MF approx.

- numerical solution evolves over $\tau \approx (B_a\mu)^{-\frac{1}{2}} \log M$ AR (2021), Xiong (2022)
- happens only if $B_a \lesssim \mu$ (bipolar oscillations in perturbed initial state)

Beyond Mean Field effects: a simple example



$$\frac{H}{M} = \vec{B}_a \cdot \vec{P}_a + \vec{B}_b \cdot \vec{P}_b + \frac{\mu}{4}\vec{P}_a \cdot \vec{P}_b$$

The initial state has $\langle \vec{P}_a \rangle \times \langle \vec{P}_b \rangle = 0$

If we choose $\vec{B}_a = \vec{B}_b$ parallel to $\langle \vec{P}_b \rangle$, no flavor evolution in MF approx.

- analytical solution evolves over $\tau \approx \mu^{-1}\sqrt{M}$ Friedland&Lunardini (2003)

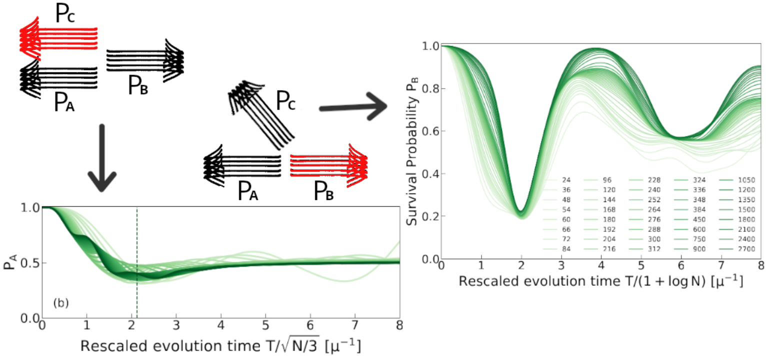
If we choose $\vec{B}_a = -\vec{B}_b$ parallel to $\langle \vec{P}_b \rangle$, no flavor evolution in MF approx.

- numerical solution evolves over $\tau \approx (B_a \mu)^{-\frac{1}{2}} \log M$ AR (2021), Xiong (2022)
- happens only if $B_a \lesssim \mu$ (bipolar oscillations in perturbed initial state)

Qualitative difference between these results can be explained in terms of Dynamical Phase Transitions leading to the bipolar instability AR (2021)

Beyond Mean Field effects: more simple examples

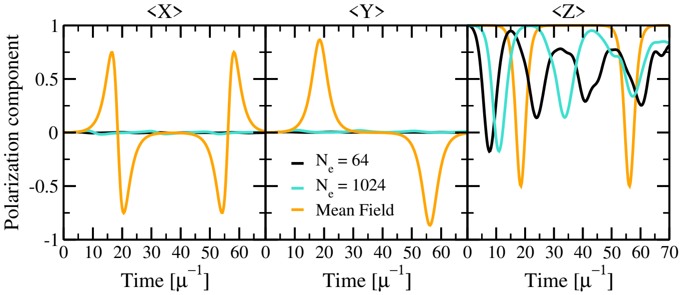
Same effect observed for $N = 3$, without vacuum oscillations, when perturbed system has unstable fast modes AR, E. Rrapaj, Z. Xiong PRD(2022)



- Dynamical Phase Transition appears equivalent to bipolar (slow) case
- fast/slow modes shown equivalent in MF D.Fiorillo & G.Raffelt PRD(2023)

Beyond Mean Field effects: even more simple examples

Similar effect observed for finite values of the mixing angle $\vec{B}_i \times \langle \vec{P} \rangle_i \neq 0$

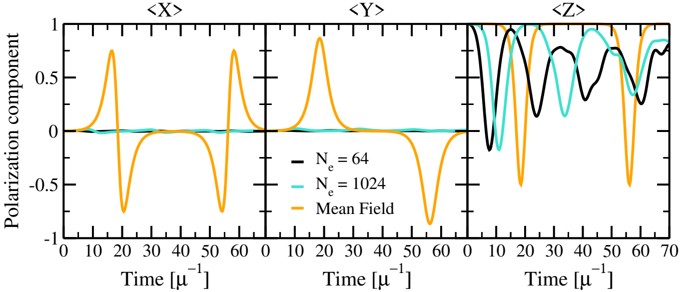


J. Martin, AR, H. Duan, J. Carlson, V. Cirigliano PRD(2022)

For small M oscillations scale as $\log(M)$ and convergence to the MF solution depends strongly on the mixing angle: $M \approx \frac{\mu^2}{\omega^2 \sin(2\theta)^2}$

Beyond Mean Field effects: even more simple examples

Similar effect observed for finite values of the mixing angle $\vec{B}_i \times \langle \vec{P} \rangle_i \neq 0$



J. Martin, AR, H. Duan, J. Carlson, V. Cirigliano PRD(2022)

For small M oscillations scale as $\log(M)$ and convergence to the MF solution depends strongly on the mixing angle: $M \approx \frac{\mu^2}{\omega^2 \sin(2\theta)^2}$

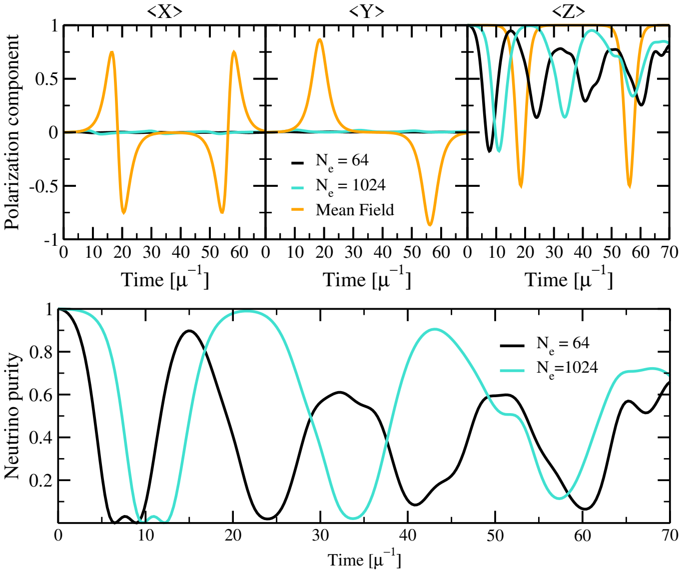
- Oscillation speedup is driven by quantum fluctuations

$$\delta P^2 = \frac{1}{M} + \left(1 - \langle \vec{P}_i \rangle^2\right)$$

- $\langle \vec{P}_i \rangle^2 = 1$ individual neutrino in pure state
- $\langle \vec{P}_i \rangle^2 < 1$ individual neutrino in mixed state

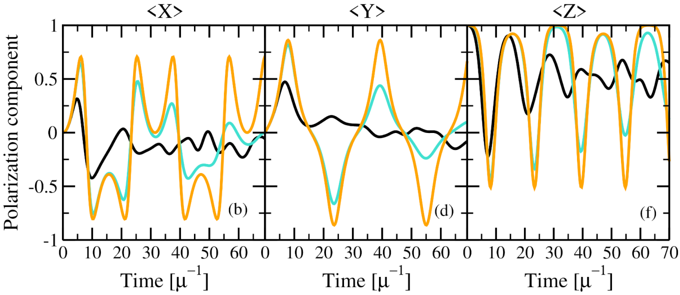
Beyond Mean Field effects: even more simple examples II

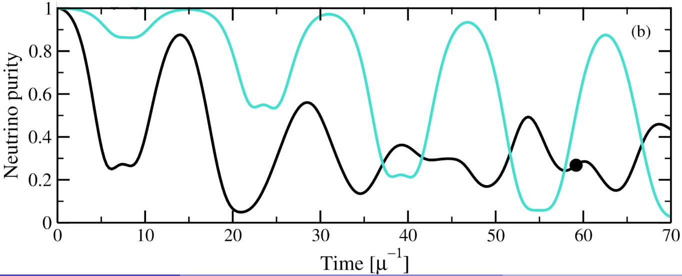
J. Martin, AR, H. Duan, J. Carlson, V. Cirigliano PRD(2022)



Beyond Mean Field effects: even more simple examples II

J. Martin, AR, H. Duan, J. Carlson, V. Cirigliano PRD(2022)





Quantum Computing and Quantum Simulations

R.Feynman(1982) we can use a controllable quantum system to simulate the behaviour of another quantum system

Quantum System we have control over

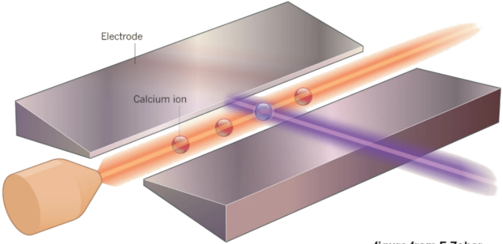
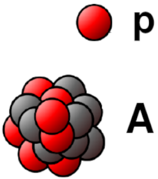


figure from E.Zohar

Quantum System we want to simulate



Quantum Computing and Quantum Simulations

R.Feynman(1982) we can use a controllable quantum system to simulate the behaviour of another quantum system

Quantum System we have control over

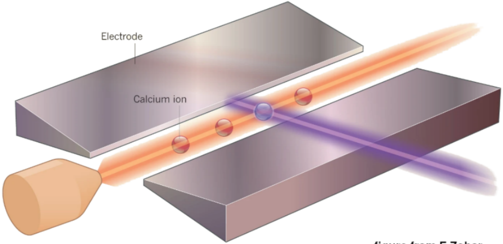
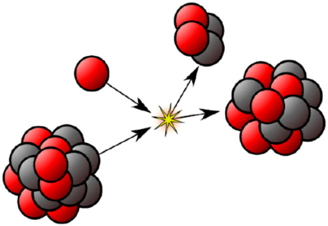


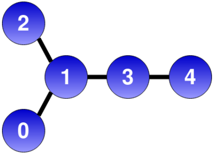
figure from E.Zohar

Quantum System we want to simulate



Quantum simulation of collective neutrino oscillations

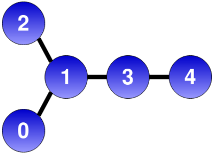
$$H = \sum_i \omega_i \vec{B} \cdot \vec{\sigma}_i + \frac{\mu}{2N} \sum_{i<j} J_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j$$



- with only 2 flavors direct map to spin 1/2 degrees of freedom (qubits)
- only one- and two-body interactions ⇒ only $\mathcal{O}(N^2)$ terms
- all-to-all interactions are difficult with reduced connectivity

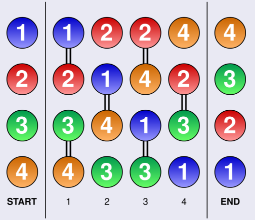
Quantum simulation of collective neutrino oscillations

$$H = \sum_i \omega_i \vec{B} \cdot \vec{\sigma}_i + \frac{\mu}{2N} \sum_{i<j} J_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j$$



- with only 2 flavors direct map to spin 1/2 degrees of freedom (qubits)
- only one- and two-body interactions $\Rightarrow$ only $\mathcal{O}(N^2)$ terms
- all-to-all interactions are difficult with reduced connectivity

SWAP network



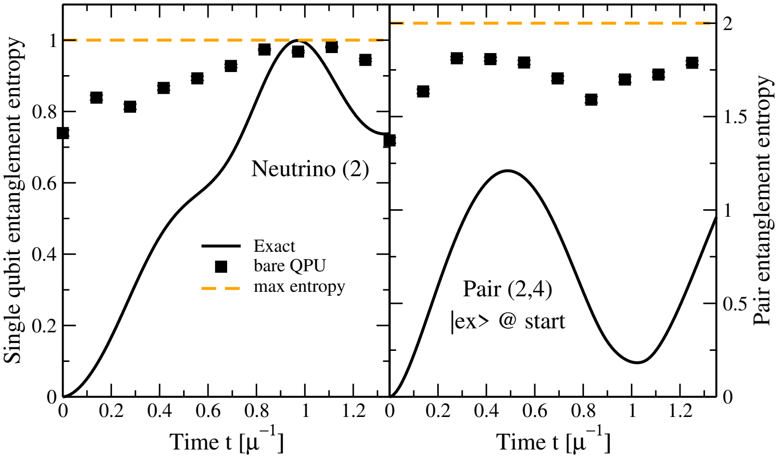
- SWAP qubits every time we apply time-evolution to neighboring terms
- in N steps we perform full evolution using only $\binom{N}{2}$ two qubit gates
  - NOTE: final order will be reversed

Kivlichan et al. PRL (2018)

B.Hall, AR, A.Baroni, J.Carlson PRD(2021)

Entanglement evolution and error mitigation with $N = 4$

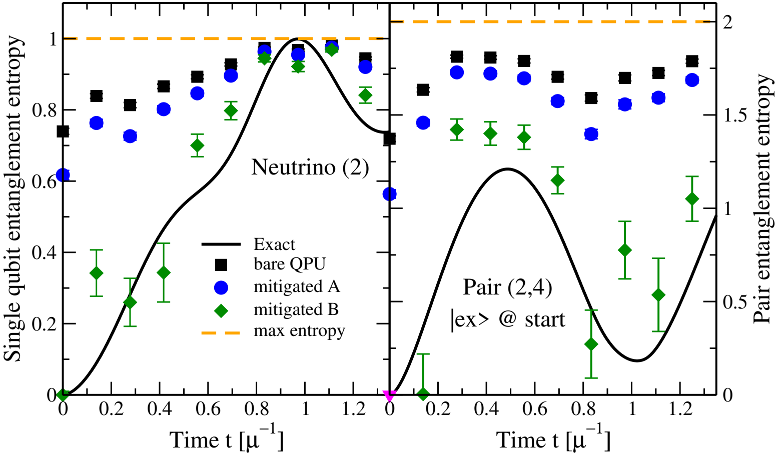
B.Hall, AR, A.Baroni, J.Carlson PRD(2021)



- Dechoerence with environment leads to increase in measured entropy

Entanglement evolution and error mitigation with $N = 4$

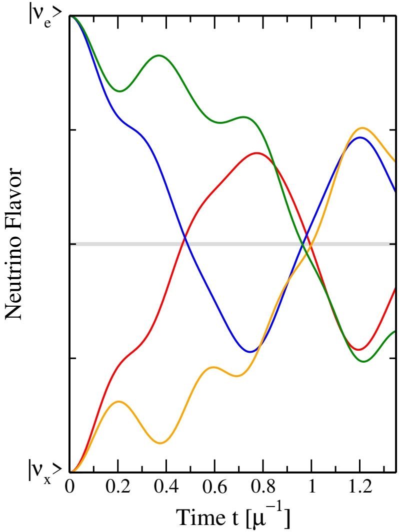
B.Hall, AR, A.Baroni, J.Carlson PRD(2021)



- Dechoerence with environment leads to increase in measured entropy
- Noise impact on observables can be modeled and effect mitigated

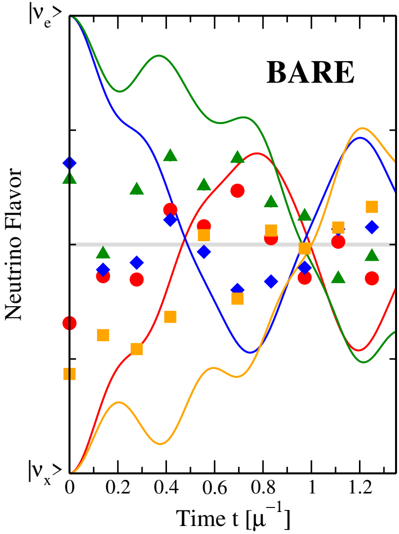
Accuracy in flavor evolution (≈ Fall 2020)

Entanglement is useful to understand collective oscillation mechanism but priority is to predict flavor evolution.



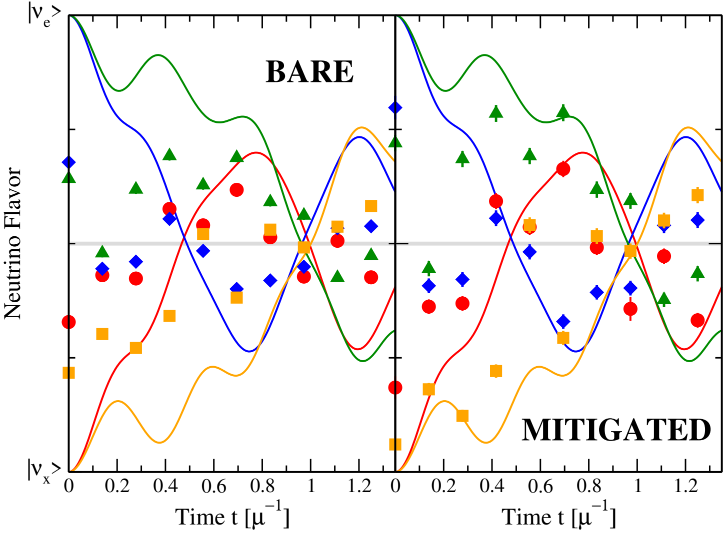
Accuracy in flavor evolution (≈ Fall 2020)

Entanglement is useful to understand collective oscillation mechanism but priority is to predict flavor evolution.



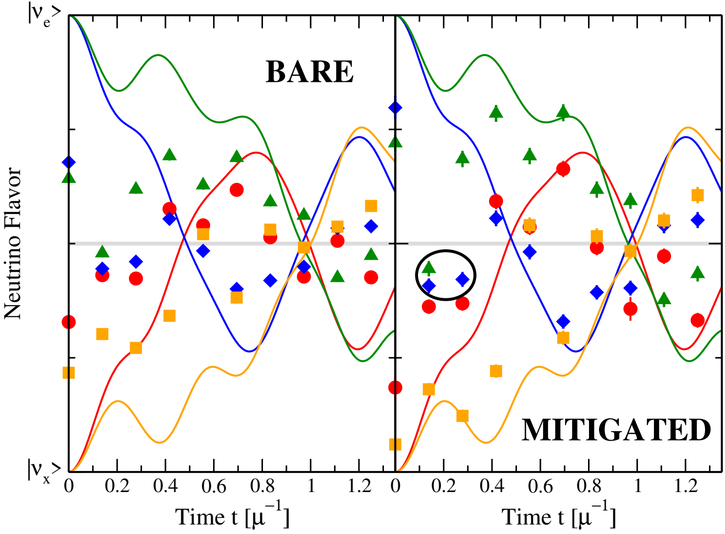
Accuracy in flavor evolution (≈ Fall 2020)

Entanglement is useful to understand collective oscillation mechanism but priority is to predict flavor evolution.



Accuracy in flavor evolution (≈ Fall 2020)

Entanglement is useful to understand collective oscillation mechanism but priority is to predict flavor evolution.



Fidelity of quantum hardware is improving fast

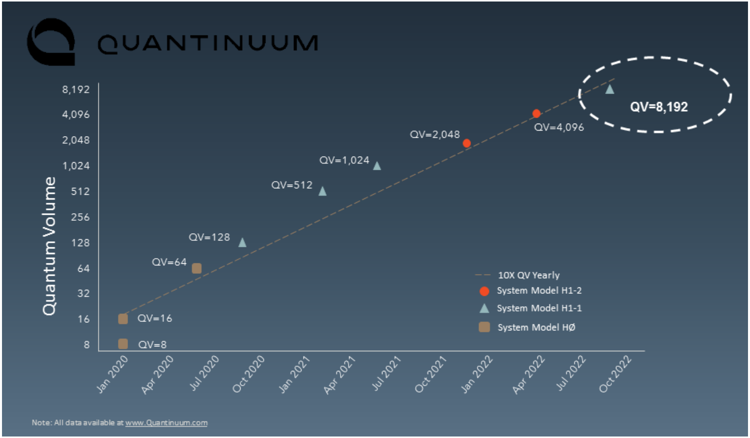
The device used for the previous results was Vigo with a QV of 16

$QV = 2^n \approx$ we can run n full layers on n qubits with fidelity $\geq 66\%$

Fidelity of quantum hardware is improving fast

The device used for the previous results was Vigo with a QV of 16

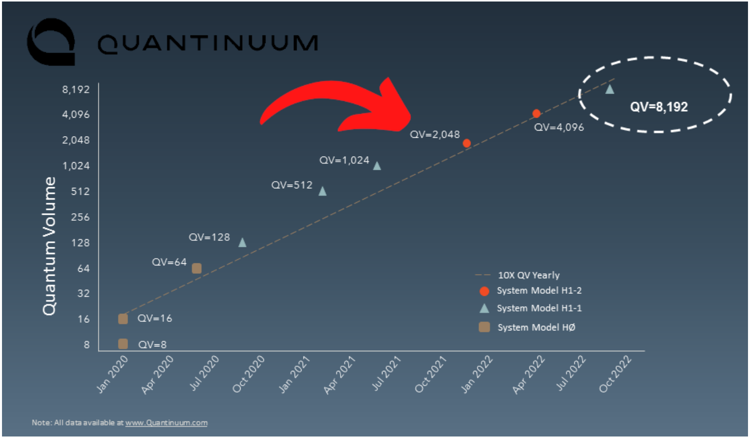
$QV = 2^n \approx$ we can run n full layers on n qubits with fidelity $\geq 66\%$



Fidelity of quantum hardware is improving fast

The device used for the previous results was Vigo with a QV of 16

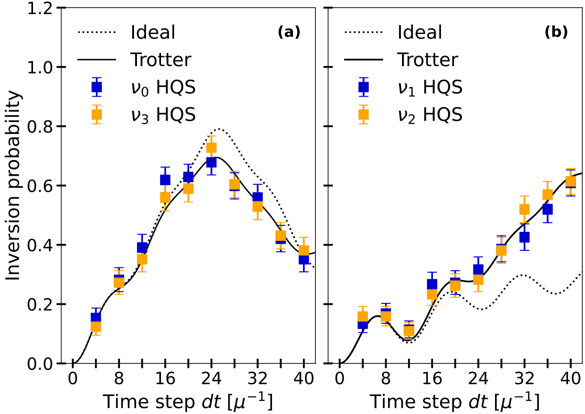
$QV = 2^n \approx$ we can run n full layers on n qubits with fidelity $\geq 66\%$



Recent progress in porting the scheme to trapped ions

V.Amitrano, AR, P.Luchi, F.Turro, L.Vespucci, F.Pederiva, PRD (2023)

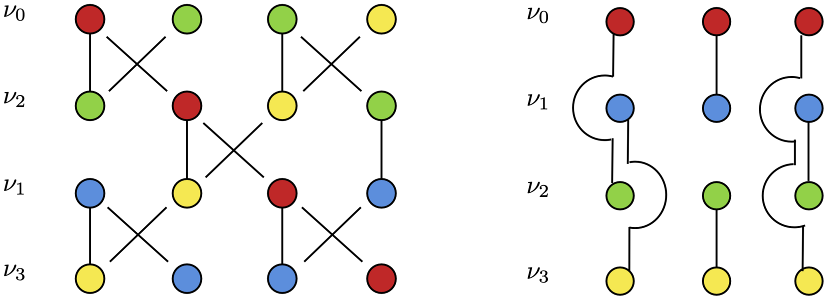
$N = 4$ neutrinos, one time step



Practical advantages of trapped ion devices

V.Amitrano, AR, P.Luchi, F.Turro, L.Vespucci, F.Pederiva, PRD (2023)

- all-to-all connectivity allows a reduction in circuit depth and the possibility of exploring different orderings for the decomposition

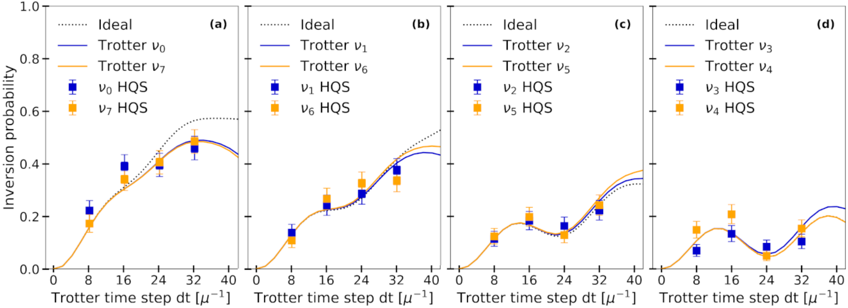


- removing SWAPs allows for a big **reduction in number of rotations**
- **very low infidelities**: $\approx 5 \times 10^{-5}$ one-qubit, $\approx 3 \times 10^{-3}$ two-qubit

Recent progress in porting the scheme to trapped ions II

V.Amitrano, AR, P.Luchi, F.Turro, L.Vespucci, F.Pederiva, PRD (2023)

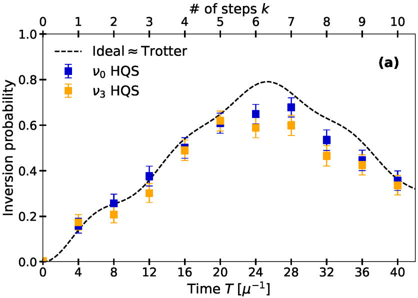
$N = 8$ neutrinos, one time step



Recent progress in porting the scheme to trapped ions III

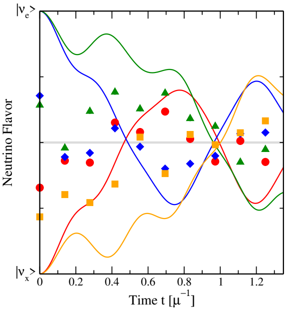
V.Amitrano, AR, P.Luchi, F.Turro, L.Vespucci, F.Pederiva, PRD (2023)

$N = 4$ neutrinos, multiple time steps

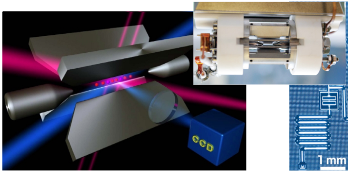


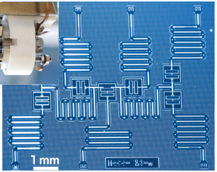
the the Last two points required: ≈ 350 two-qubit gates over 8 qubits

Current limitations of digital quantum simulations

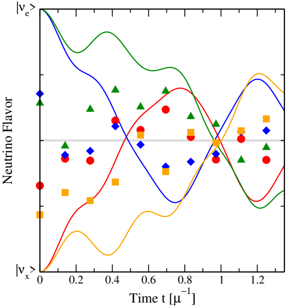


current and near term digital quantum devices have limited fidelity and might not scale much beyond $N = \mathcal{O}(10)$ neutrinos in next years

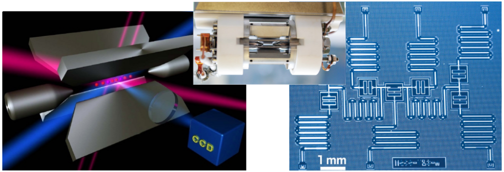




Current limitations of digital quantum simulations



current and near term digital quantum devices have limited fidelity and might not scale much beyond $N = \mathcal{O}(10)$ neutrinos in next years



Possible paths to scalability in the meantime

- Analog Quantum Simulators

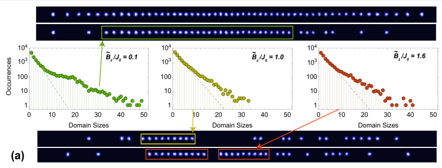


figure from Zhang et al Nature(2017)

- Describe low entanglement states with Tensor Networks

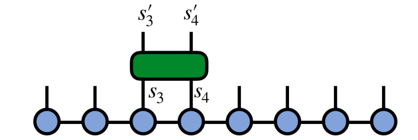
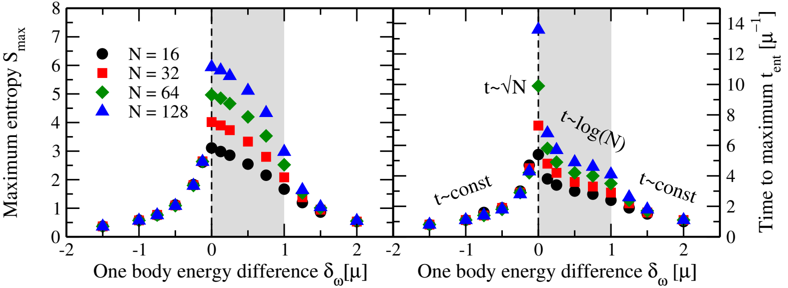


image from itensor.org

Collective oscillations and entanglement scaling

AR, PRD 104, 103016 (2021) & PRD 104, 123023 (2021)



Why is this interesting?

- entanglement scaling provides general criterion for appearance of collective modes in full many-body treatment
- entropy scaling as $\log(N) \Rightarrow$ large ab-initio simulations possible
- MPS method fails when entanglement too large $\Rightarrow$ we can use this to detect interesting regimes to study on quantum simulators!

Summary and perspectives

- beyond mean field effects in collective neutrino oscillations are an important systematic that needs to be better understood
  - small N large M behavior shows possibility of non-negligible effects when unstable modes are present and mixing angles are small
  - large N small M behavior computationally very demanding but some progress already now on classical computers See Josh Martin's talk
  - what thermodynamic limit is more suitable for supernovae?
- even the basic 2-flavor model for collective oscillations poses a challenging many-body problem well suited to quantum technologies
- first calculations on small scale digital devices show promise in studying flavor evolution and achievable fidelity is advancing at a rapid pace ($N = 12$ only few months ago [Illa & Savage arXiv:2210.08656])
- analog trapped ion devices are an ideal platform to study mid-size systems as the interactions can be embedded in a natural way
- tensor network methods can help push the boundary of classical simulations and identify interesting regimes to study with simulators

Thanks to my collaborators

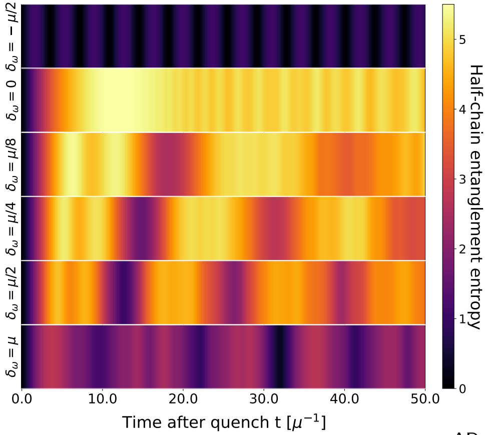
- Joseph Carlson (LANL)
- Vincenzo Cirigliano (LANL)
- Joshua Martin (LANL)
- Alessandro Baroni (LANL→ORNL)
- Huaiyu Duan (UNM)
- Benjamin Hall (MSU)

- Valentina Amitrano (UniTN/TIFPA)
- Piero Luchi (UniTN/TIFPA)
- Francesco Turro (UniTN/TIFPA)
- Luca Vespucci (UniTN/TIFPA)
- Francesco Pederiva (UniTN/TIFPA)

Collective oscillations with MPS

$$H = -\frac{\delta_\omega}{2}\left(\sum_{i\in\{1,\dots,N/2\}}\sigma_i^z - \sum_{i\in\{N/2+1,\dots,N\}}\sigma_i^z\right) + \frac{\mu}{2N}\sum_{i<j}\vec{\sigma}_i\cdot\vec{\sigma}_j\ ,$$

MF predicts no evolution, MPS has oscillations for $0 \leq \delta_\omega/\mu \lesssim 1$



AR, PRD 104, 123023 (2021)

Dynamical phase transitions

Heyl et al. PRL (2013), Heyl PRL (2015), Heyl RPP (2018)

Quantum quench protocols

1. the system starts as the ground-state of an initial Hamiltonian H_0
2. at time $t = 0$ we switch to a different Hamiltonian H and evolve

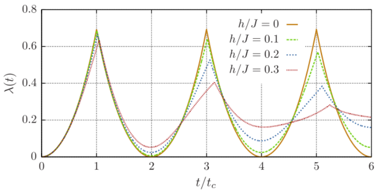
Dynamical critical behavior encoded in Loschmidt echo

$$\mathcal{L}(t) = \left|\langle \Psi_0 | e^{-iHt} | \Psi_0 \rangle\right|^2 \xrightarrow{N \gg 1} e^{-N\lambda(t)}$$

Loschmidt rate $\lambda(t)$ plays a similar role as the free energy in equilibrium.

$$H(h) = -\sum_{\langle ij \rangle} Z_i Z_j + h \sum_i X_i$$

- start in ground-state for $h \to \infty$
- quench across critical point at $h = 1$



Heyl PRL (2015)

Dynamical phase transitions

Heyl et al. PRL (2013), Heyl PRL (2015), Heyl RPP (2018)

Quantum quench protocols

1. the system starts as the ground-state of an initial Hamiltonian H_0
2. at time $t = 0$ we switch to a different Hamiltonian H and evolve

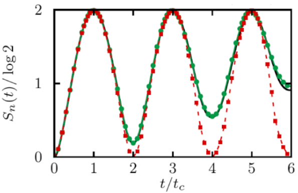
Dynamical critical behavior encoded in Loschmidt echo

$$\mathcal{L}(t) = \left|\langle\Psi_0|e^{-iHt}|\Psi_0\rangle\right|^2 \xrightarrow{N\gg 1} e^{-N\lambda(t)}$$

Loschmidt rate $\lambda(t)$ plays a similar role as the free energy in equilibrium.

$$H(h) = -\sum_{\langle ij\rangle} Z_i Z_j + h\sum_i X_i$$

- start in ground-state for $h \to \infty$
- quench across critical point at $h = 1$



Schmitt & Heyl SciPost Phys (2018)

DPT for systems with degenerate ground spaces

Heyl PRL (2014)

$$H_{XXZ} = J \sum_i \left[X_i X_{i+1} + Y_i Y_{i+1} + \Delta Z_i Z_{i+1} \right]$$

- disordered gapless phase for $\Delta < 1$
- anti-ferromagnetic phase for $\Delta > 1$
- critical point at $\Delta = 1$

$$|\Psi_0\rangle = |\uparrow\downarrow\uparrow\downarrow\cdots\rangle \qquad |\Psi_0'\rangle = |\downarrow\uparrow\downarrow\uparrow\cdots\rangle$$

Loschmidt Echo for degenerate ground-states

$$\mathcal{L}_0(t) = \left|\langle\Psi_0|e^{-itH}|\Psi_0\rangle\right|^2 \quad \mathcal{L}_1(t) = \left|\langle\Psi_0'|e^{-itH}|\Psi_0\rangle\right|^2 \ ,$$

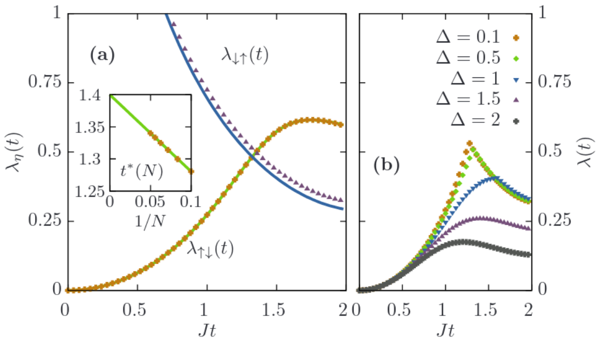
DPT $\Leftrightarrow$ non-analytic behavior of the total echo $\mathcal{L}(t) = \mathcal{L}_0(t) + \mathcal{L}_1(t)$

DPT for systems with degenerate ground spaces II

$$\mathcal{L}_0(t) = \left|\langle\Psi_0|e^{-itH}|\Psi_0\rangle\right|^2 \quad \mathcal{L}_1(t) = \left|\langle\Psi_0'|e^{-itH}|\Psi_0\rangle\right|^2 \ ,$$

DPT $\Leftrightarrow$ non-analytic behavior of the total echo $\mathcal{L}(t) = \mathcal{L}_0(t) + \mathcal{L}_1(t)$

Both scale exponentially in system size, but with different rates, there is a kink forming if the order between $\mathcal{L}_0(t)$ and $\mathcal{L}_1(t)$ changes at some $t = t^*$

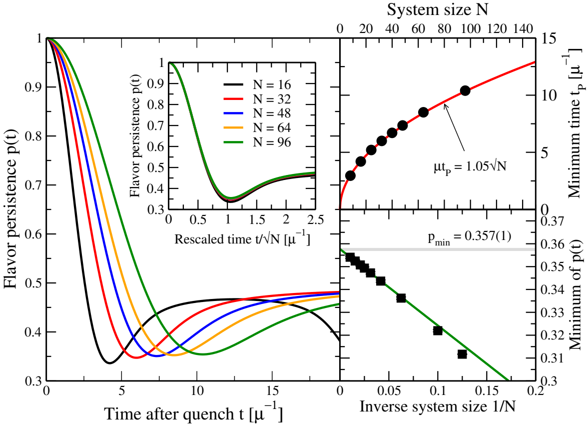


Simple neutrino model

Friedland & Lunardini (2003), AR (2021)

$$H = \frac{1}{2N}\sum_{i<j}\vec{\sigma}_i\cdot\vec{\sigma}_j = \frac{1}{N}S^2 + const..$$

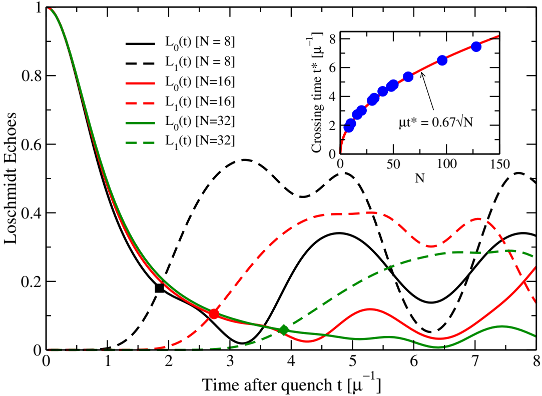
Initialize system in $|\Psi(0)\rangle = |\downarrow\rangle^{\otimes N/2}\otimes|\uparrow\rangle^{\otimes N/2}$ and compute the flavor persistence $p(t) = (1-\langle\Psi(t)|\sigma_1|\Psi(t)\rangle)/2$ for increasing system size



Simple neutrino model II

$$H(x) = \frac{x}{2N} S^2 + (1-x) \sum_{a \in \mathcal{A}} \sum_{b \in \mathcal{B}} Z_a Z_b \ ,$$

start at $x = 0$ and evolve with $x = 1$. State is $|\Psi_0\rangle = |\downarrow\rangle^{\otimes N/2} \otimes |\uparrow\rangle^{\otimes N/2}$.



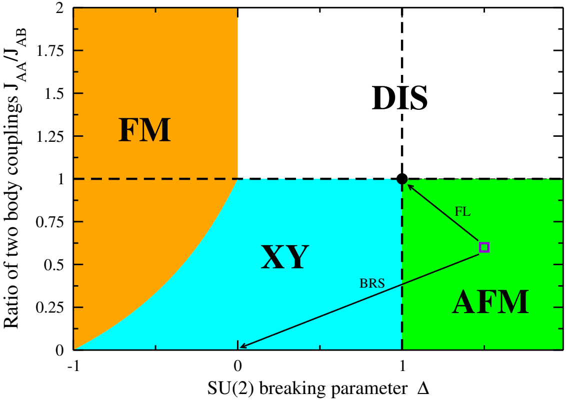
Crossing time t^* diverges as $\sqrt{N}$ $\Rightarrow$ no evolution for a large system!

Many-body speedup in unphysical model

Bell,Rawlinson,Sawyer PLB(2003), AR (2021)

$$H_{BRS} = \frac{1}{2N} \sum_{i<j} \mathcal{J}_{ij} \left(X_i X_j + Y_i Y_j + \Delta Z_i Z_j \right)$$

with $\mathcal{J}_{ij} = J_{AA}$ for (i,j) in $\mathcal{A}$ or $\mathcal{B}$ and $\mathcal{J}_{ij} = J_{AB}$ otherwise. Our initial state is (degenerate) gs of H_{BRS} in the limit $\Delta \gg 1$ and $J_{AA} < J_{AB}$

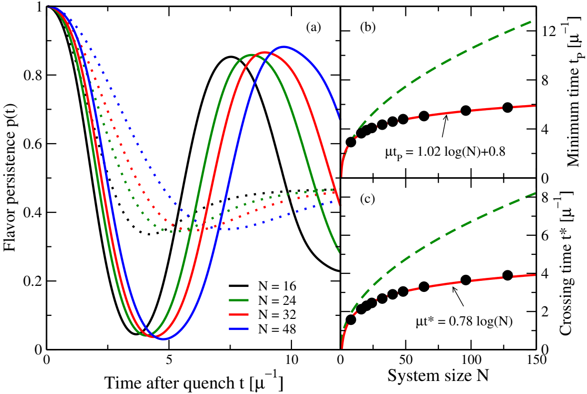


Many-body speedup in unphysical model II

Bell,Rawlinson,Sawyer PLB(2003), AR (2021)

$$H_{BRS} = \frac{1}{2N}\sum_{i<j}\mathcal{J}_{ij}\left(X_iX_j + Y_iY_j + \Delta Z_iZ_j\right)$$

with $\mathcal{J}_{ij} = J_{AA}$ for (i,j) in $\mathcal{A}$ or $\mathcal{B}$ and $\mathcal{J}_{ij} = J_{AB}$ otherwise. Our initial state is (degenerate) gs of H_{BRS} in the limit $\Delta \gg 1$ and $J_{AA} < J_{AB}$



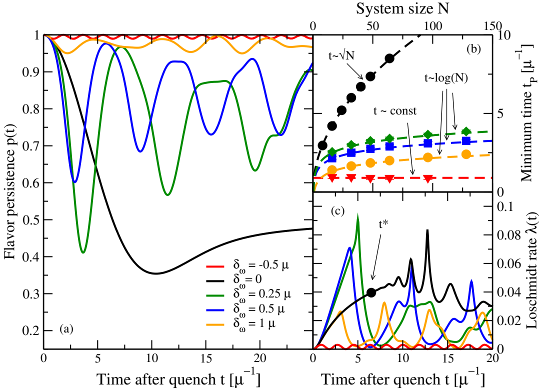
Many-body speedup in a physical model

To engineer a "DPT" we can ensure the system crosses a critical point

$$H = -\frac{\delta_\omega}{2}\left(\sum_{i\in\mathcal{A}}\sigma_i^z - \sum_{i\in\mathcal{B}}\sigma_i^z\right) + \frac{\mu}{2N}\sum_{i<j}\vec{\sigma}_i\cdot\vec{\sigma}_j \ ,$$

AFM ($\mu > 0$) transition at $\delta_\omega = 0$ between gapped phases

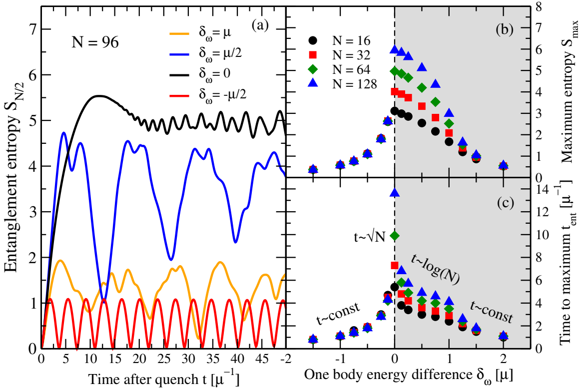
FM ($\mu < 0$) transitions at $\delta_\omega = \pm\mu$ between gapped and gapless phases



Many-body speedup in a physical model II

To engineer a "DPT" we can ensure the system crosses a critical point

$$H = -\frac{\delta_\omega}{2}\left(\sum_{i\in\mathcal{A}}\sigma_i^z - \sum_{i\in\mathcal{B}}\sigma_i^z\right) + \frac{\mu}{2N}\sum_{i<j}\vec{\sigma}_i\cdot\vec{\sigma}_j\ ,$$



Entanglement entropy for bipolar model