Towards self-consistent modeling of supernova and binary neutron star merger with quantum kinetic neutrino transport

Hiroki Nagakura
(National Astronomical Observatory of Japan)

Focus workshop on collective oscillations and chiral transport of neutrinos:
March 14-17, 2023, @ Institute of Physics, Academia Sinica, Taiwan

Multi-physics elements in CCSN and BNSM theories

General Relativity

Numerical Relativity

(M)Hydro dynamics

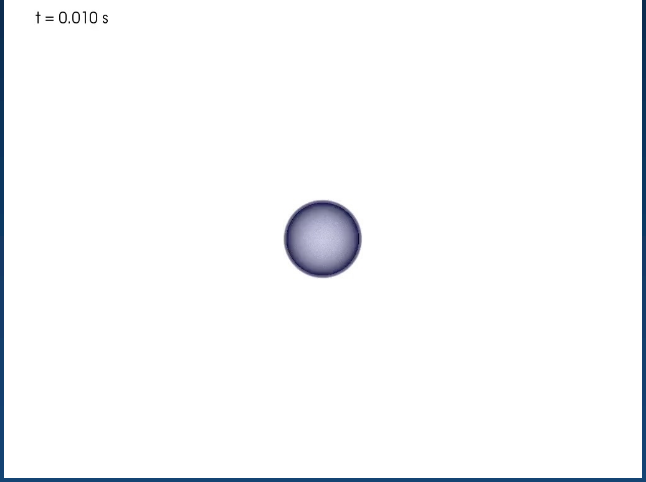
High-resolution
Shock-capturing Scheme

Fluid instabilities
(Convection, SASI, etc...)

Neutrino Transport

Boltzmann Transport
or alternative method
(e.g., M1 scheme)

Neutrino Quantum Kinetics
(Neutrino oscillation)



Simulations

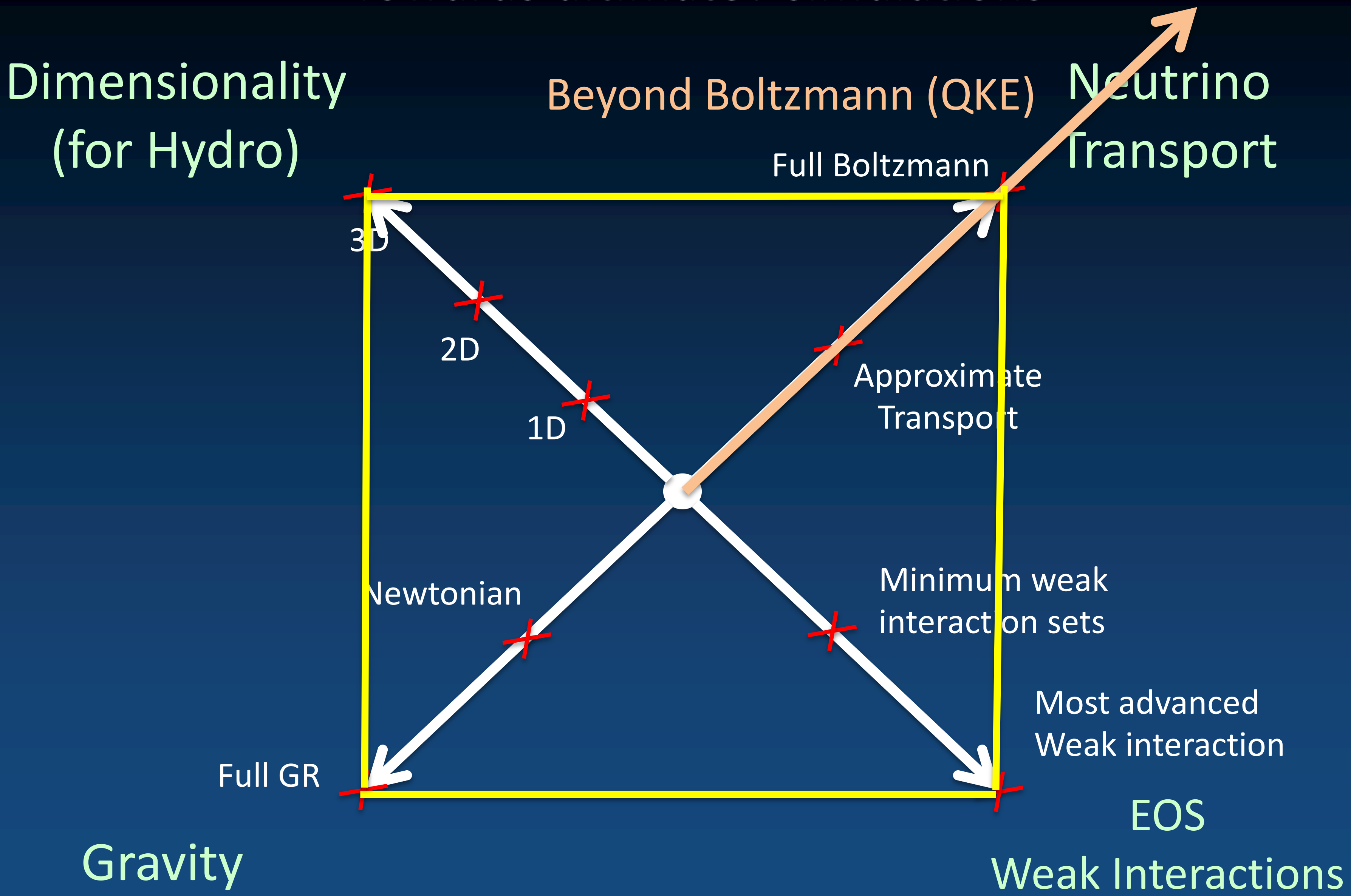
Nuclear Network

NSE @ high ρ and T
Nucleosynthesis

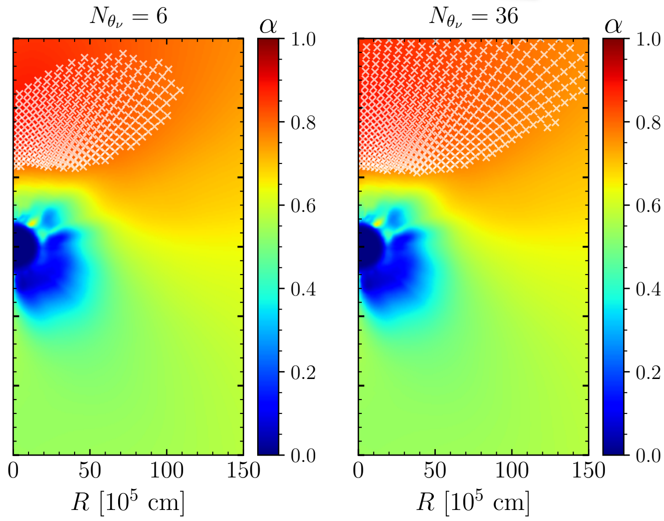
Weak Interaction

Emission/absorption
Scatterings
Many-body corrections

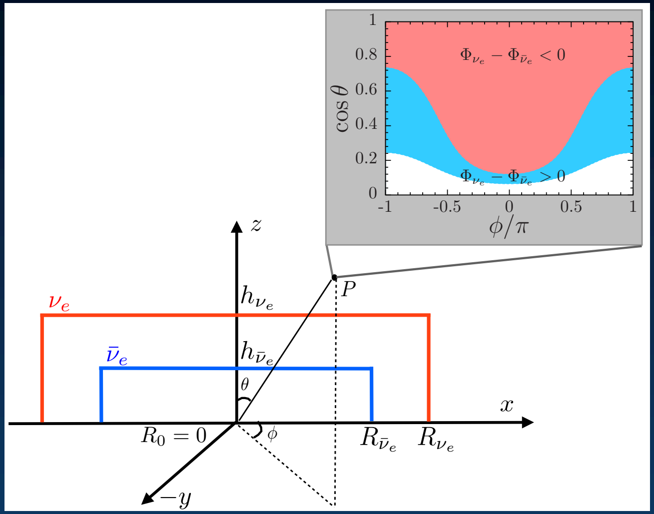
- Towards ultimate? simulations-



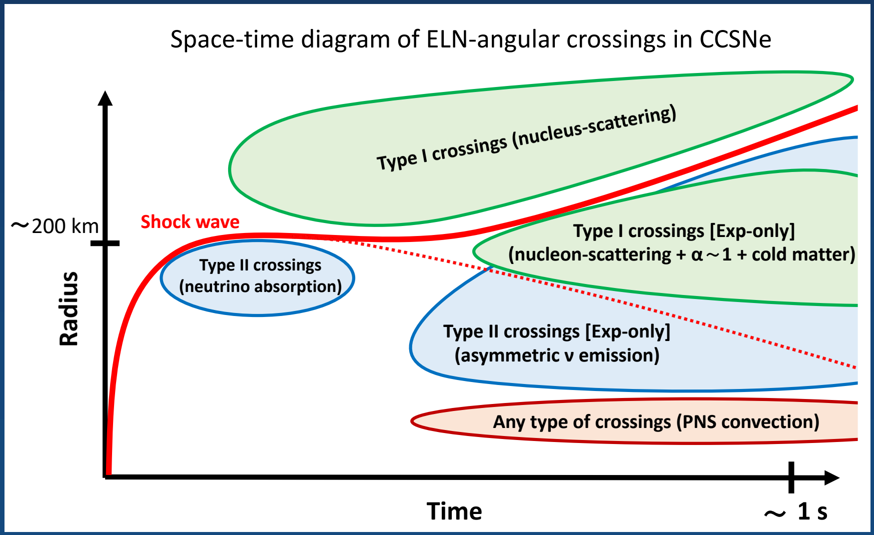
Fast neutrino-flavor conversion may ubiquitously occur in CCSN and BNSM



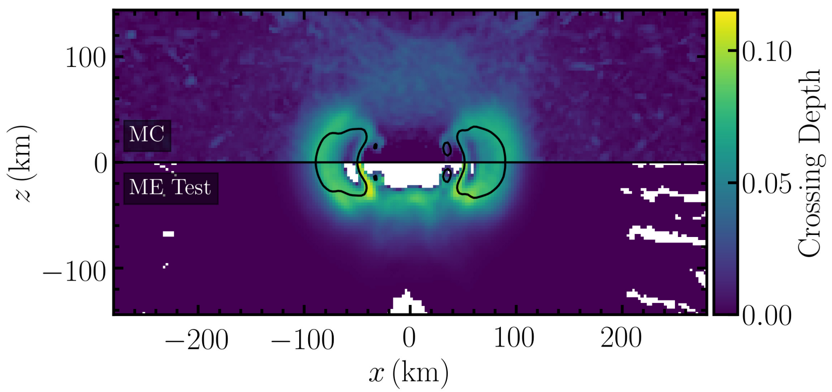
Abbar et al. 2019



Wu and Tamborra 2017



Nagakura et al. 2021



Richers 2022

Quantum Kinetic neutrino transport:

Vlasenko et al. 2014, Volpe 2015,
Blaschke et al. 2016, Richers et al. 2019

$$p^\mu \frac{\partial \overset{(-)}{f}}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial \overset{(-)}{f}}{\partial p^i} = -p^\mu u_\mu \overset{(-)}{S}_{\rm col} + \underbrace{ip^\mu n_\mu [\overset{(-)}{H}, \overset{(-)}{f}],}_{\text{Oscillation term}}$$

Density matrix

$$\overset{(-)}{f} = \begin{bmatrix} \overset{(-)}{f}_{ee} & \overset{(-)}{f}_{e\mu} & \overset{(-)}{f}_{e\tau} \\ \overset{(-)}{f}_{\mu e} & \overset{(-)}{f}_{\mu\mu} & \overset{(-)}{f}_{\mu\tau} \\ \overset{(-)}{f}_{\tau e} & \overset{(-)}{f}_{\tau\mu} & \overset{(-)}{f}_{\tau\tau} \end{bmatrix}$$

Hamiltonian

$$\overset{(-)}{H} = \overset{(-)}{H}_{\rm vac} + \overset{(-)}{H}_{\rm mat} + \overset{(-)}{H}_{\nu\nu},$$

Self-interaction

$$H_{\nu\nu} = \sqrt{2} G_F \int \frac{d^3 q'}{(2\pi)^3} \left(1 - \sum_{i=1}^{3} \ell'_{(i)} \ell_{(i)}\right) \left(f(q') - \bar{f}^*(q')\right),$$

Challenge: Huge disparity in scales between neutrino oscillations and CCSN/BNSM

Scale of fast collective-mode	$\mathrm{T}_{\mathrm{n}_\nu} \equiv \left(\sqrt{2} G_F n_\nu\right)^{-1} = 7.84 \times 10^{-12}\ \mathrm{s} \left(\frac{L_\nu}{4 \times 10^{52} \mathrm{erg/s}}\right)^{-1} \left(\frac{E_{\rm ave}}{12\mathrm{MeV}}\right) \left(\frac{R}{50\mathrm{km}}\right)^2 \left(\frac{\kappa}{1/3}\right)$	$\ell_{\mathrm{n}_\nu} \equiv c\,\mathrm{T}_{\mathrm{n}_\nu} = 0.235\ \mathrm{cm} \left(\frac{L_\nu}{4 \times 10^{52} \mathrm{erg/s}}\right)^{-1} \left(\frac{E_{\rm ave}}{12\mathrm{MeV}}\right) \left(\frac{R}{50\mathrm{km}}\right)^2 \left(\frac{\kappa}{1/3}\right)$
CCSN/BNSM scale	> 100 ms	> 1000 km

Phenomenological approach: Philosophy

Li and Siegel 2021, Just et al. 2022, Fernandez et al. 2022, Jacob et al. 2023

Radiation-hydrodynamic simulations with classical neutrino transport

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = \left(\frac{\delta f}{\delta \tau}\right)_{\rm col},$$

Boltzmann transport

or

$$\begin{aligned}
&\partial_t(\sqrt{\gamma}E) + \partial_j[\sqrt{\gamma}(\alpha F^j - \beta^j E)] \\
&\qquad = \alpha\sqrt{\gamma}[P^{ij}K_{ij} - F^j\partial_j \ln\alpha - S^\alpha n_\alpha], \\
&\partial_t(\sqrt{\gamma}F_i) + \partial_j[\sqrt{\gamma}(\alpha P_i{}^j - \beta^j F_i)] \\
&\qquad = \sqrt{\gamma}\Big[-E\partial_i\alpha + F_k\partial_i\beta^k + \frac{\alpha}{2}P^{jk}\partial_i\gamma_{jk} + \alpha S^\alpha \gamma_{i\alpha}\Big],
\end{aligned}$$

Approximate transport (e.g., moment method)

+

Assessing instabilities of flavor conversions

Linear stability analysis

or

approximate ones

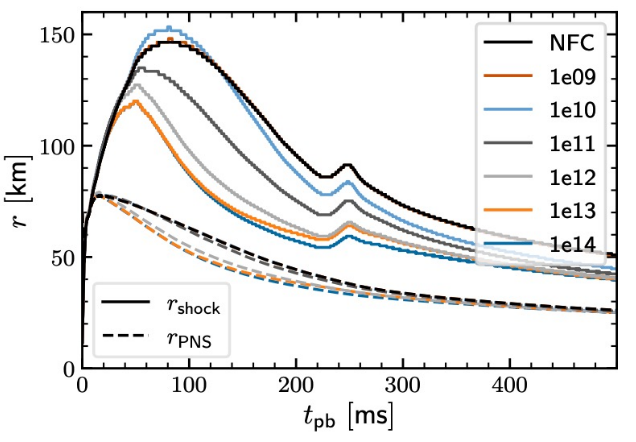
Mixing-scheme

with a parametric manner

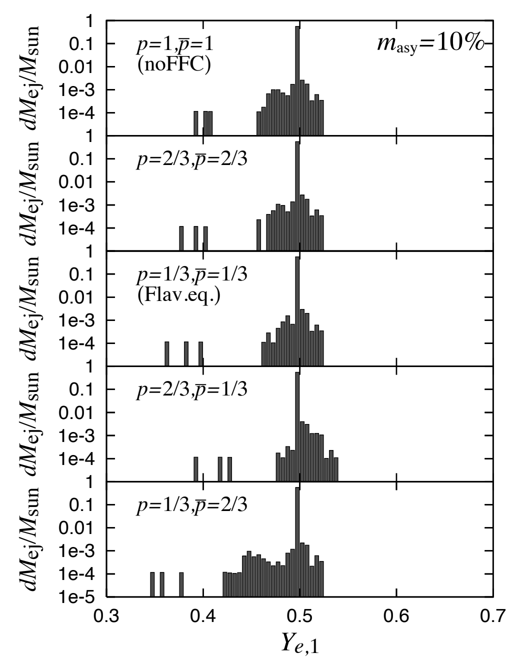
$$\begin{aligned}
L^{\rm osc}_{\nu_e} &= (1 - a_{\rm osc})L^*_{\nu_e} + a_{\rm osc}L_{\nu_x} \\
L^{\rm osc}_{\bar\nu_e} &= (1 - b_{\rm osc})L^*_{\bar\nu_e} + b_{\rm osc}L_{\bar\nu_x}.
\end{aligned}$$

Fernandez et al. 2022

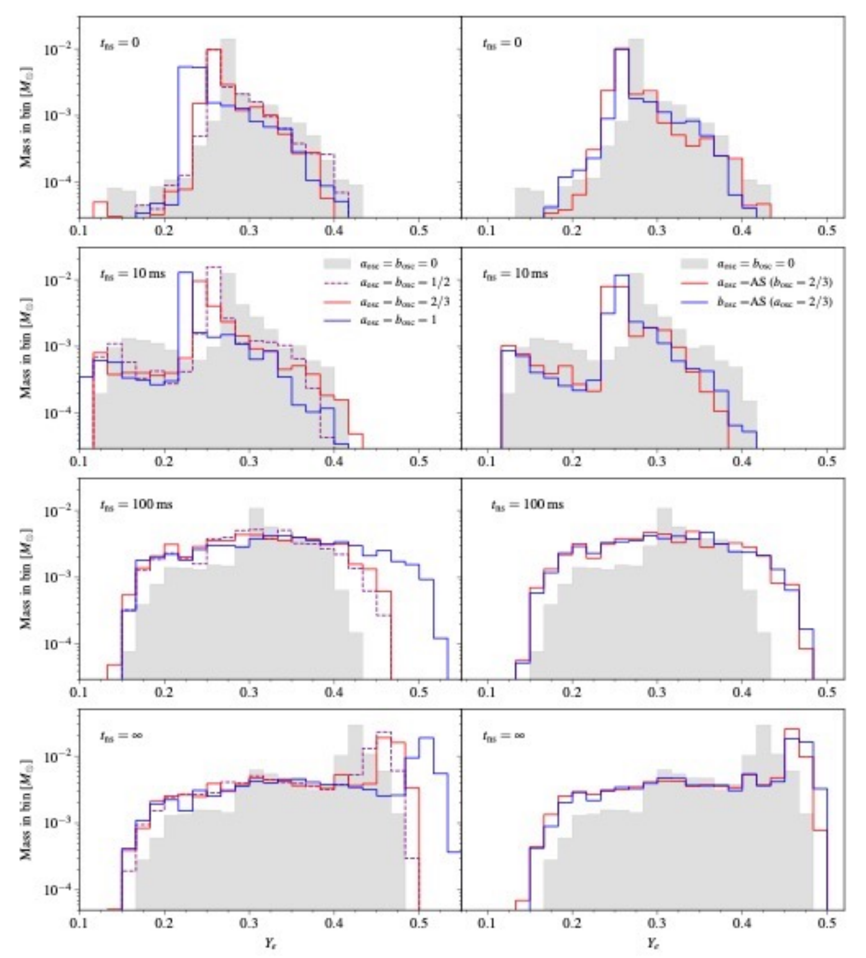
Phenomenological approach: Demonstrations



Jacob et al. 2023



Fujimoto and H.N 2023



Fernandez et al. 2022

See also Li and Siegel 2021, Just et al. 2022

Phenomenological approach: Uncertainties

- Degree of flavor mixing can not be determined.

  It is a parameter in phenomenological models

- No reliable approximate neutrino transport have been established.

  Requirements of quantum closure relations for angular moments

- Systematic errors are involved due to collision term (neutrino-matter interactions).

  Non-linear evolution of flavor conversions strongly hinge on collision term

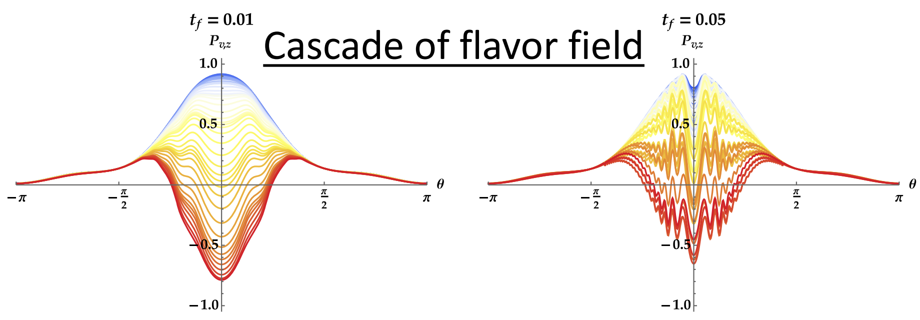
  See Chinami Kato’s talk

⇩

These issues can be addressed only by solving quantum kinetic neutrino transport

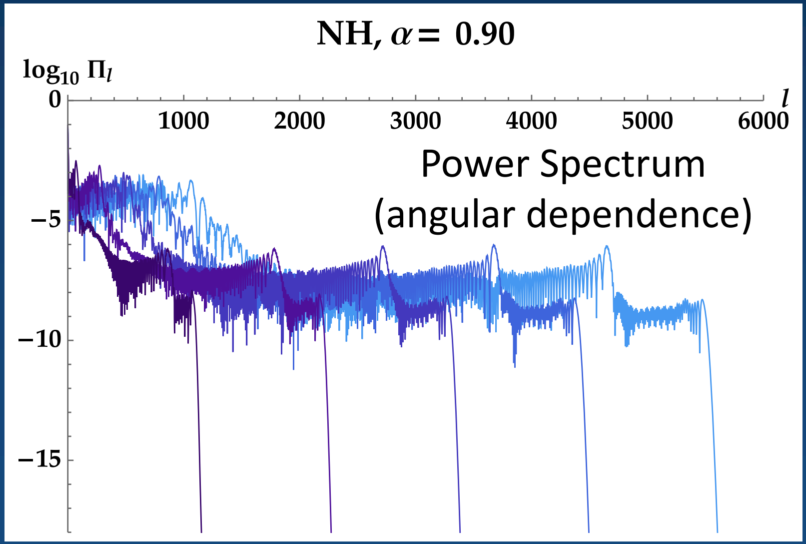
- Homogeneous Simulations (FFCs)

$$\varepsilon \frac{df}{dt} = -p^{\mu} u_{\mu} \overset{(-)}{S}_{\mathrm{col}} + i p^{\mu} n_{\mu} [\overset{(-)}{H}, \overset{(-)}{f}],$$



Cascade of flavor field

Johns and H.N et al. 2020

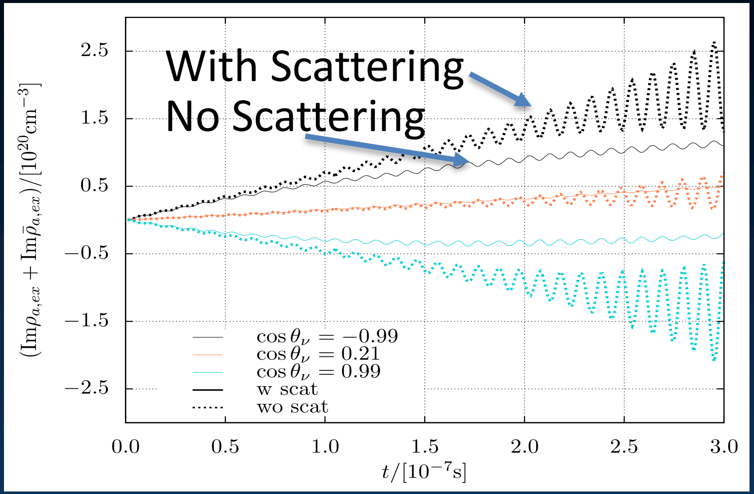


Power Spectrum (angular dependence)

Johns and H.N et al. 2020

Effects of iso-energetic scatterings on FFC

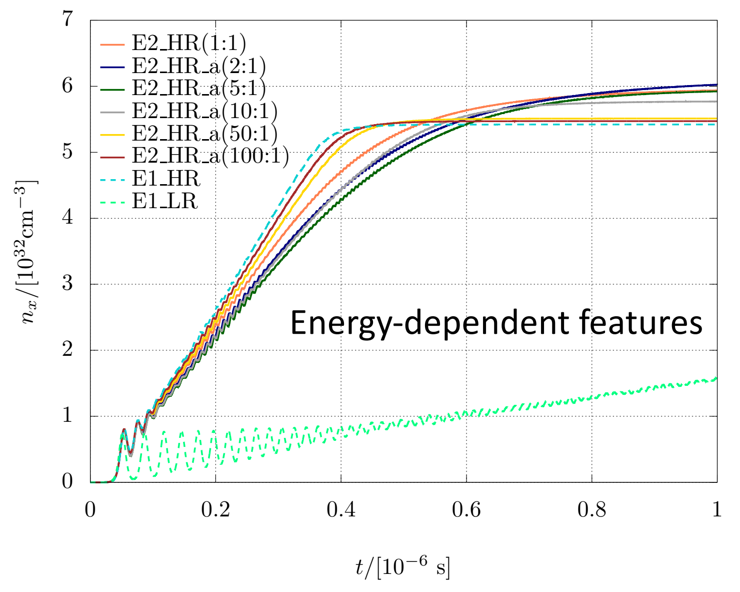
Kato and H.N 2021



With Scattering

No Scattering

But see also Johns and H.N 2021 for consistency issues.



Energy-dependent features

Kato and H.N 2022

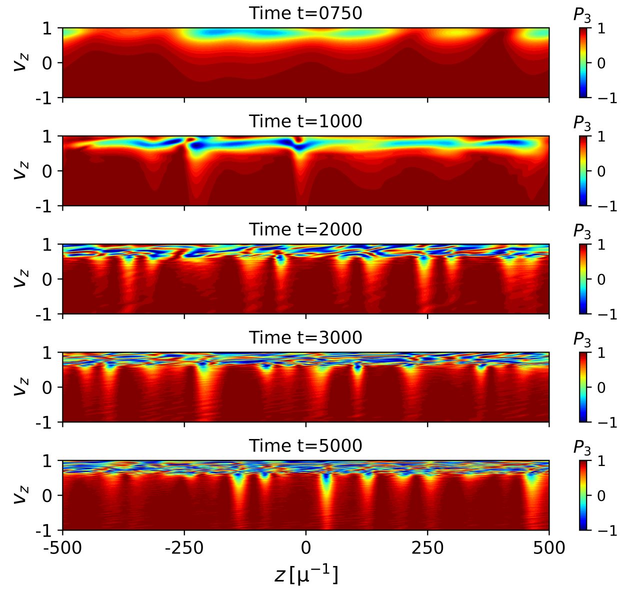
- Inhomogeneous Simulations (local)

$$\frac{\partial f_{ab}}{\partial t} + \underline{c\boldsymbol{\Omega}\cdot\nabla f_{ab}} = \mathcal{C}_{ab} - \frac{i}{\hbar}\left[\mathcal{H}, f\right]_{ab}$$

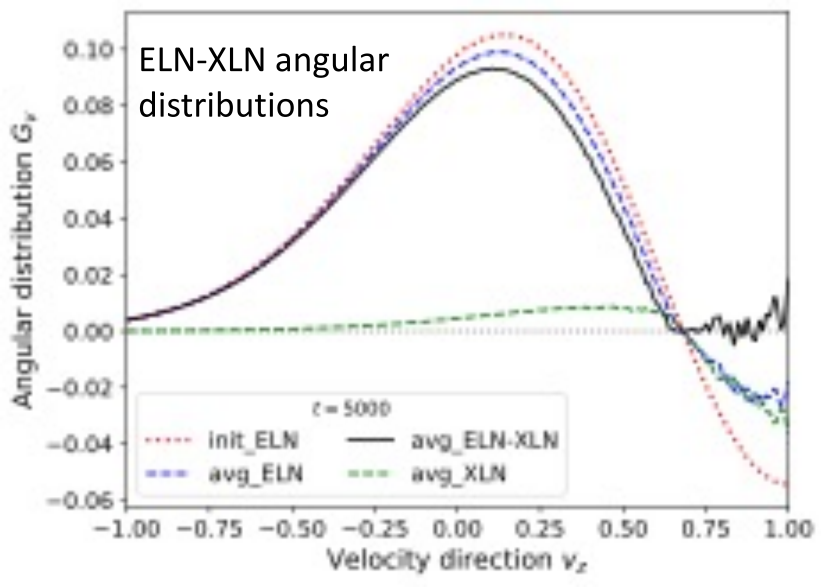
Advection term (flat + cartesian-coordinate)

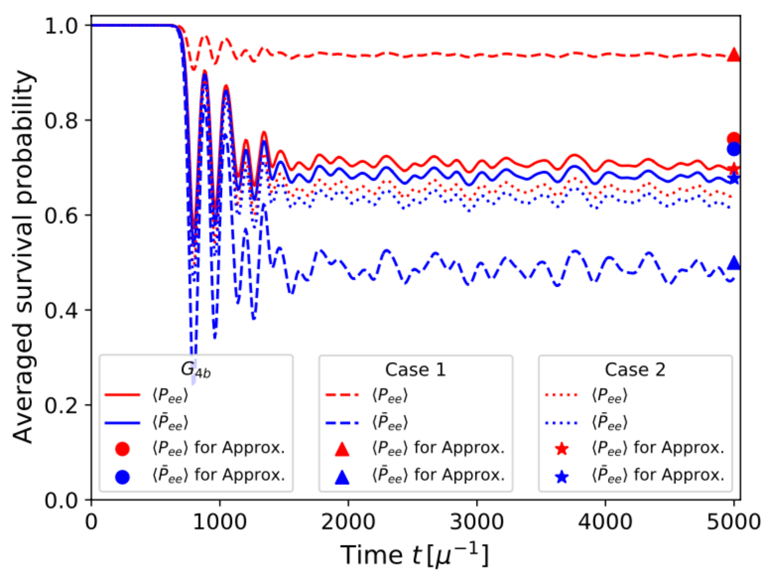
1D simulation with periodic boundary condition

Zaizen and H.N (arXiv:2211.0934)



Asymptotic states in FFCs

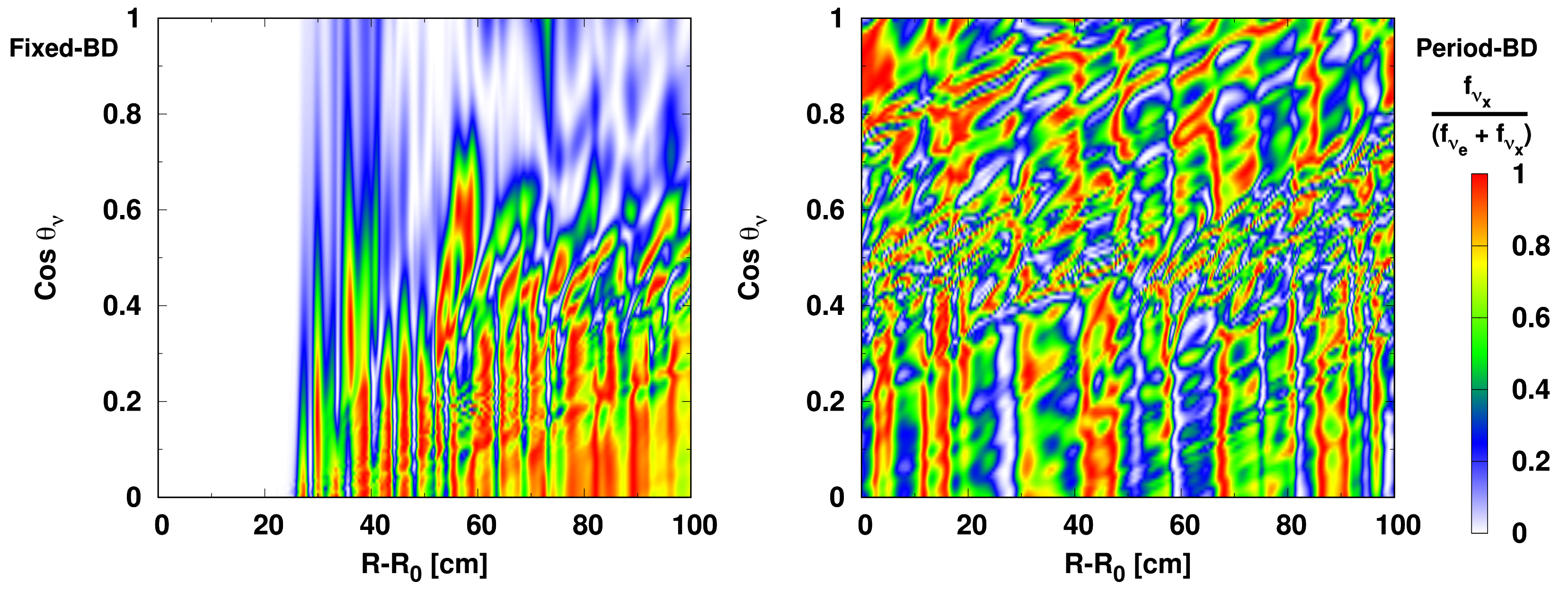




See also Bhattacharyya et al. 2021, Wu et al. 2021

- Inhomogeneous Simulations (local)

Asymptotic states of FFC depend on boundary conditions in space.



Nagakura 2022

Lepton number conservation in each flavor of neutrinos accounts for the difference

Zaizen and H.N (arXiv:2211.0934)

$$H_E = \sqrt{2}G_\mathrm{F}\int \mathrm{d}\Gamma'\,\rho_{v'}$$

$$H_F = \sqrt{2}G_\mathrm{F}\int \mathrm{d}\Gamma'\,v'_z\rho_{v'}.\qquad \partial_t H_E + \partial_z H_F = 0,$$

See Masamichi Zaizen's talk

- Global Simulations: code development

General-relativistic quantum-kinetic neutrino transport (GRQKNT)

Nagakura 2022

$$p^{\mu}\frac{\partial \overset{(-)}{f}}{\partial x^{\mu}} + \frac{dp^{i}}{d\tau}\frac{\partial \overset{(-)}{f}}{\partial p^{i}} = -p^{\mu}u_{\mu}\overset{(-)}{S}_{\rm col} + ip^{\mu}n_{\mu}[\overset{(-)}{H}, \overset{(-)}{f}],$$

- ✓ Fully general relativistic (3+1 formalism) neutrino transport
- ✓ Multi-Dimension (6-dimensional phase space)
- ✓ Neutrino matter interactions (emission, absorption, and scatterings)
- ✓ Neutrino Hamiltonian potential of vacuum, matter, and self-interaction
- ✓ 3 flavors + their anti-neutrinos
- ✓ Solving the equation with Sn method (explicit evolution: WENO-5th order)
- ✓ Hybrid OpenMP/MPI parallelization

- Global Simulations: demonstrations

Nagakura and Zaizen 2022 (PRL and PRD)

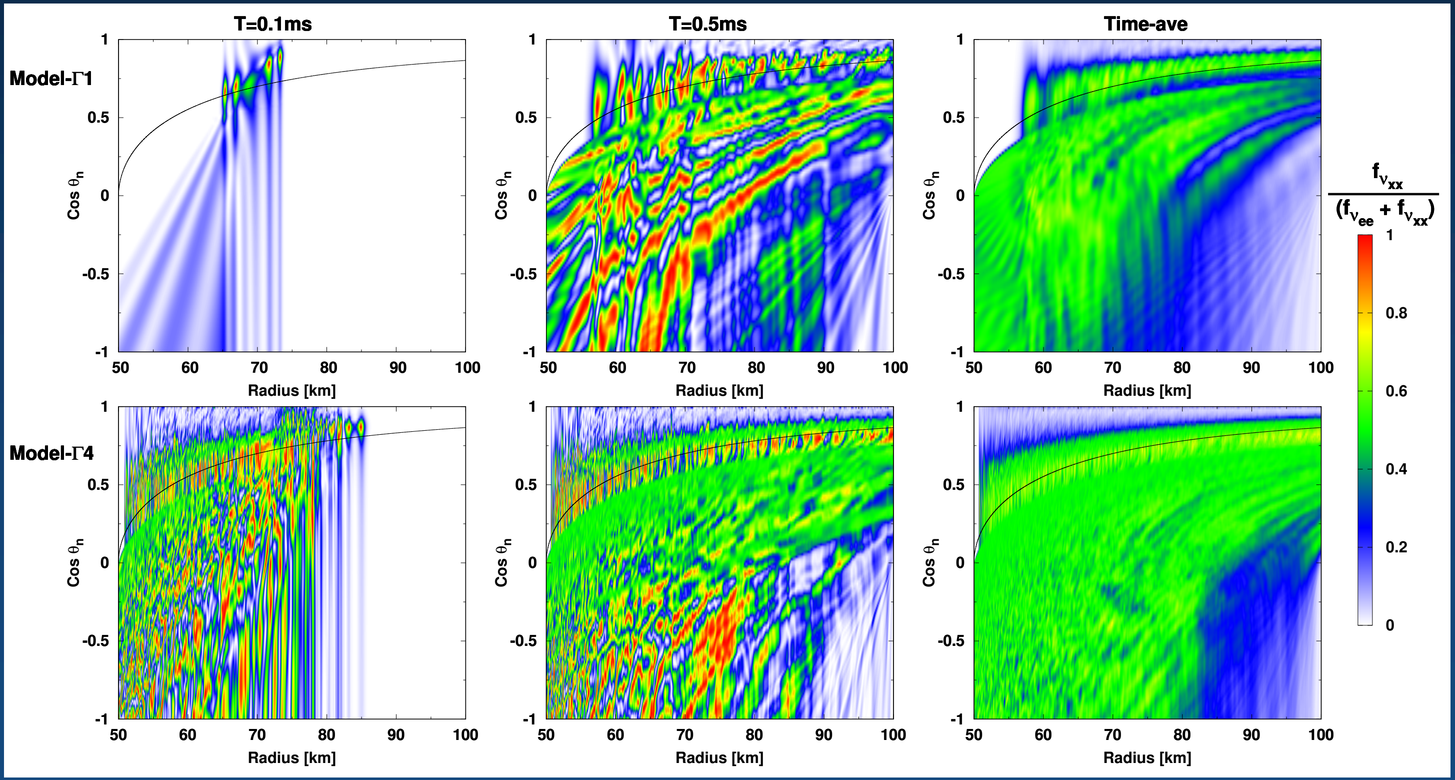
Large-scale (50km – 100km) FFC simulations

$$\frac{\partial \overset{(-)}{f}}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cos\theta_\nu \overset{(-)}{f}\right) - \frac{1}{r\sin\theta_\nu}\frac{\partial}{\partial \theta_\nu}\left(\sin^2\theta_\nu \overset{(-)}{f}\right)$$

$$= -i\xi\,[\overset{(-)}{H}, \overset{(-)}{f}],$$

Attenuating Hamiltonian

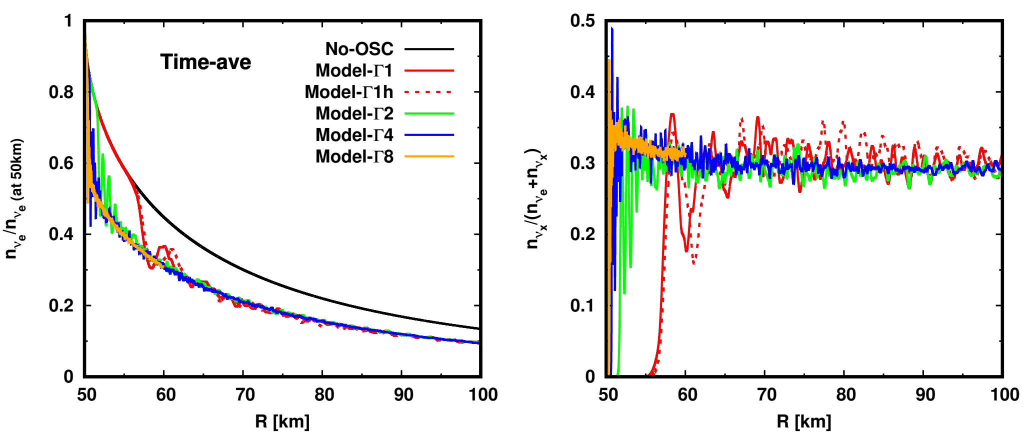
See also Xiong et al. arXiv:2210.08254



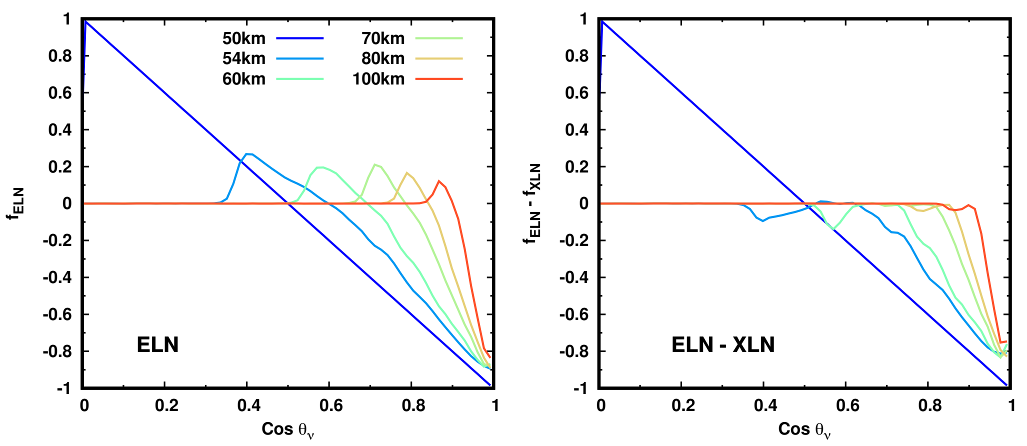
- Global Simulations: <u>key takeaways</u>

Nagakura and Zaizen 2022 (PRL and PRD)

- Time-averaged structures are insensitive to attenuation of Hamiltonian.



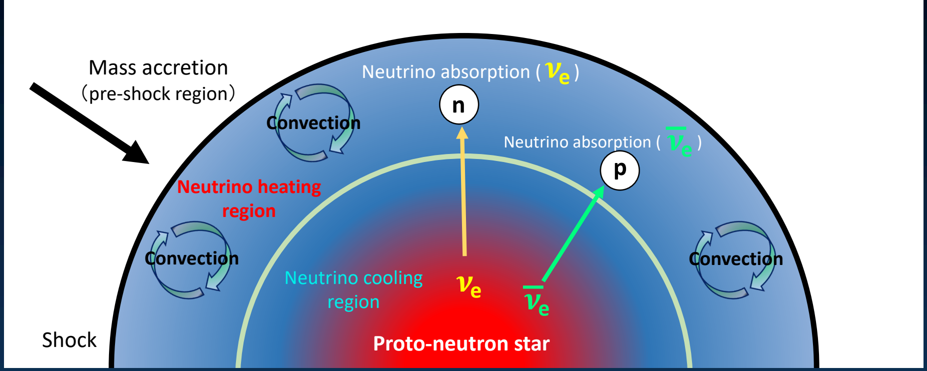
- Disappearance of ELN-XLN angular crossings is a key ingredient to characterize asymptotic states of FFC in non-linear phase.



- Global Simulations: Impacts on CCSN explosion

Nagakura (arXiv:2301.10785)

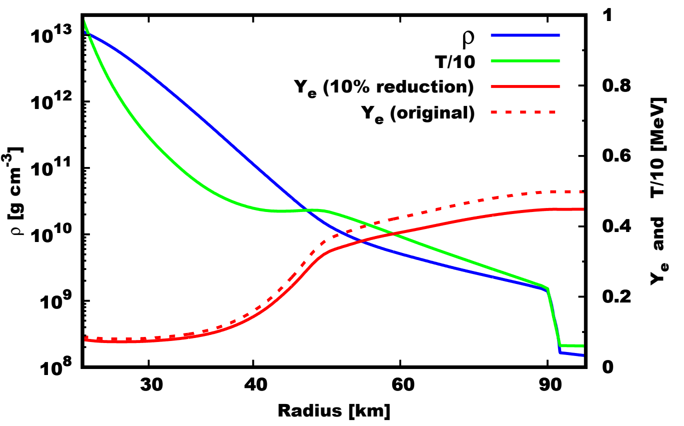
Neutrino heating mechanism

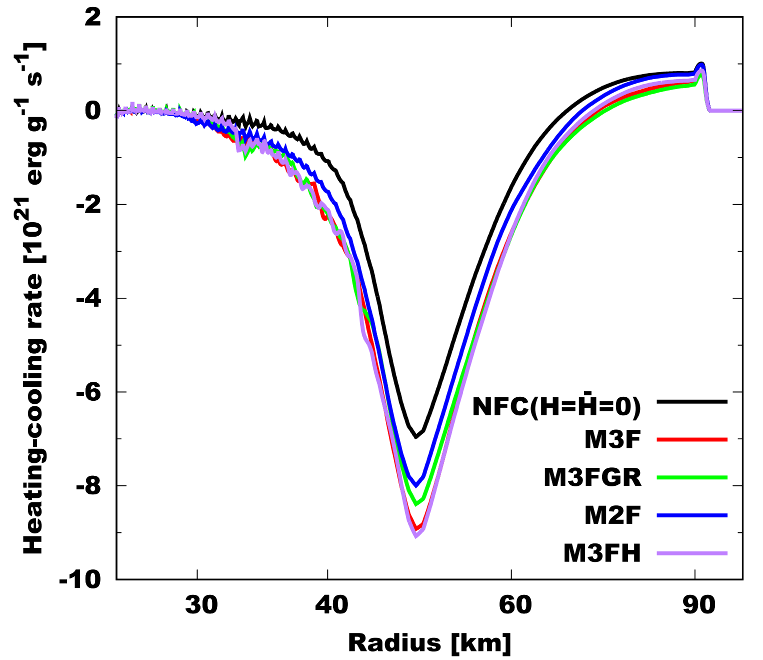


$$
\begin{aligned}
&\frac{\partial}{\partial t}\left[\left(1-\frac{2M}{r}\right)^{-1/2}\overset{(-)}{f}\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\cos\theta_\nu\left(1-\frac{2M}{r}\right)^{1/2}\overset{(-)}{f}\right] \\
&-\frac{1}{\nu^2}\frac{\partial}{\partial \nu}\left[\frac{M}{r^2}\left(1-\frac{2M}{r}\right)^{-1/2}\nu^3\cos\theta_\nu\overset{(-)}{f}\right] \\
&-\frac{1}{\sin\theta_\nu}\frac{\partial}{\partial \theta_\nu}\left[\sin^2\theta_\nu\frac{r-3M}{r^2}\left(1-\frac{2M}{r}\right)^{-1/2}\overset{(-)}{f}\right] \\
&= \overset{(-)}{S} - i\xi[\overset{(-)}{H}, \overset{(-)}{f}],
\end{aligned}
$$

- ✓ Covering a wide post-shock region
- ✓ Solving GRQKNT (Schwarzschild spacetimes)
- ✓ Collision terms are taken into account.

Neutrino-heating is suppressed by FFCs
Neutrino-cooling is accelerated by FFCs





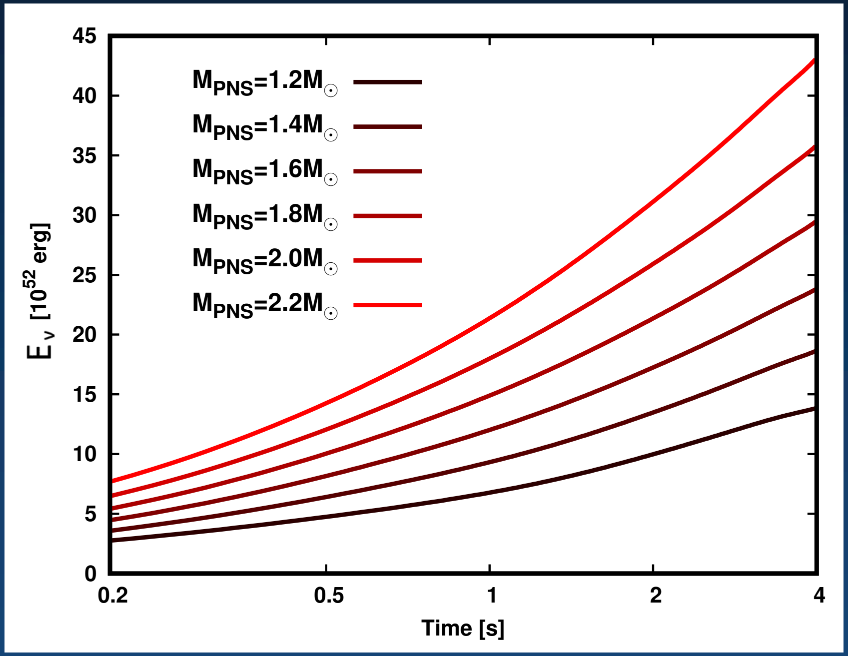
Can we observationally place a constraint on neutrino flavor conversion in CCSNe?

Correlation analysis with gravitational waves is of great use.

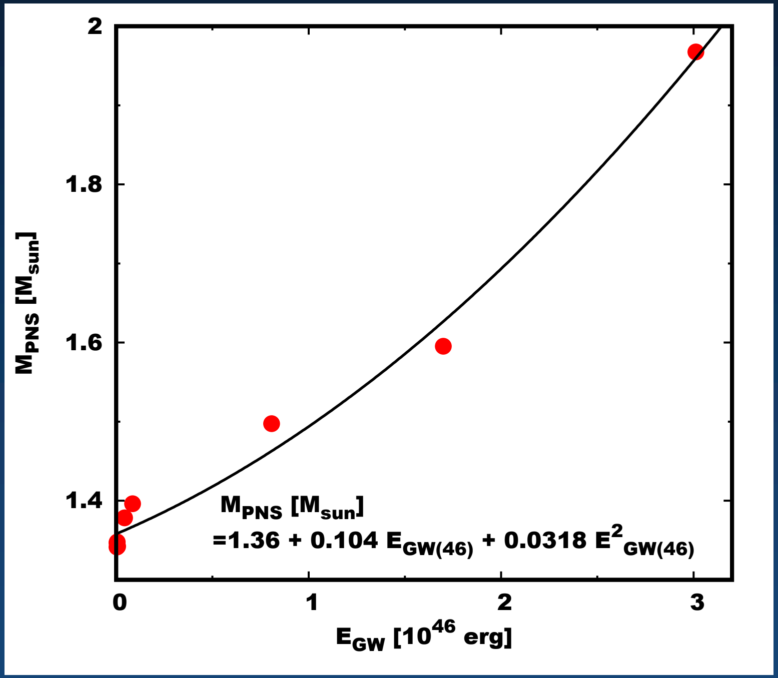
Nagakura and Vartanyan in prep

Proto neutron star (PNS) mass is a key ingredient to characterize GW and neutrino signal

Irradiated neutrino energy versus time

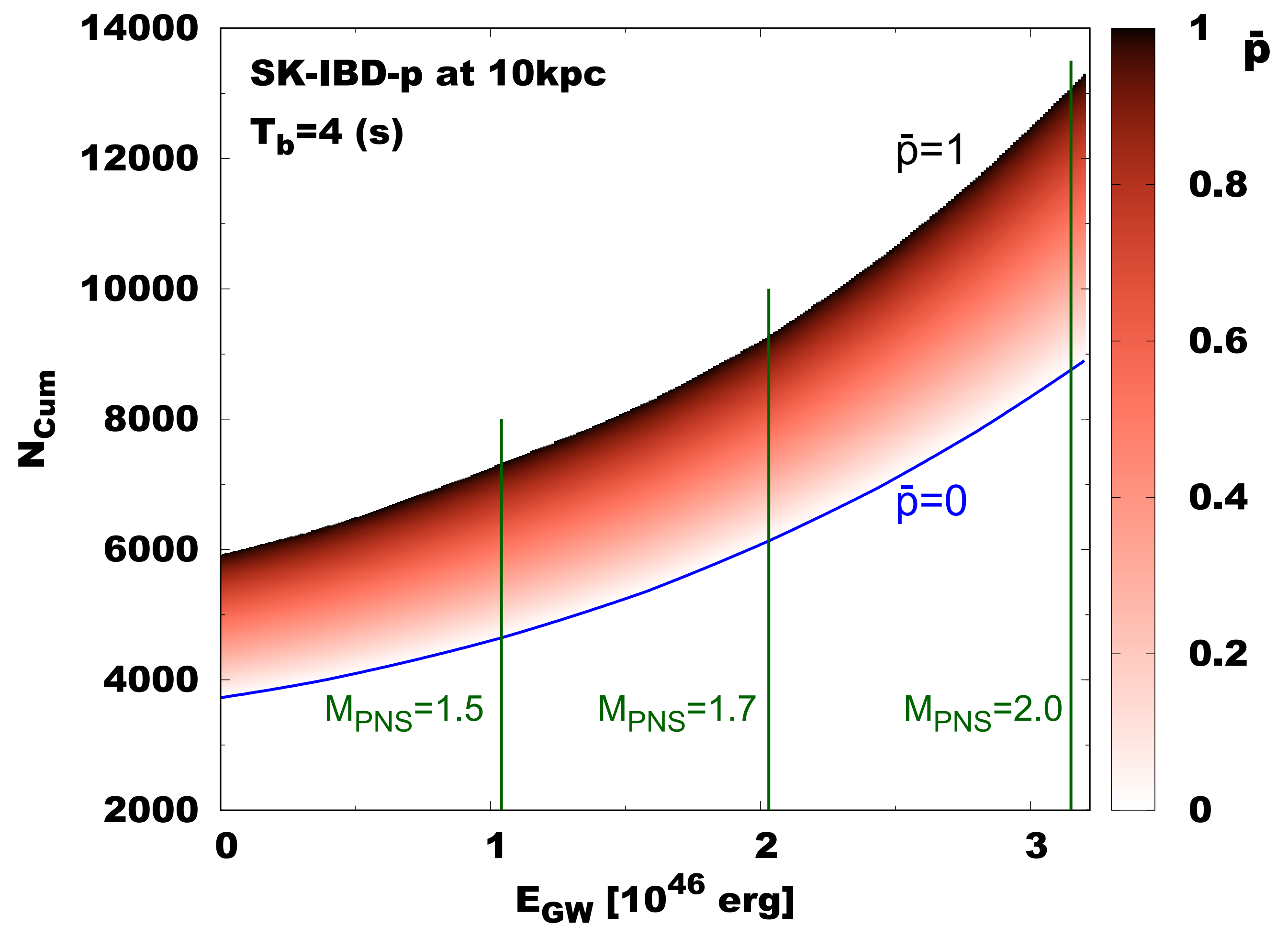


Irradiated GW energy vs. PNS mass



Nagakura and Vartanyan 2022, MNRAS

Constraining survival probability of neutrinos from correlation analysis of GWs and neutrino signals



Summary:

- Neutrino flavor conversion induced by self-interactions is one of the greatest uncertainties in CCSN/BNSM theories.
- The research field is rapidly evolving owing to both phenomenological and first-principles approaches of quantum kinetic neutrino transport.
- ELN-XLN crossings characterize not only the FFC instability but also their asymptotic states.
- Attenuation of Hamiltonian allows us to carry out global simulations.
- We demonstrate in large-scale QKE simulations that FFCs give large impact on CCSN explosion.
- The correlation analysis of GWs and neutrinos in realistic theoretical models can break the degeneracy between detection counts and flavor conversion.