Stability and Boundary Constraints Characterizing Asymptotic Behaviors of Fast Flavor Conversion

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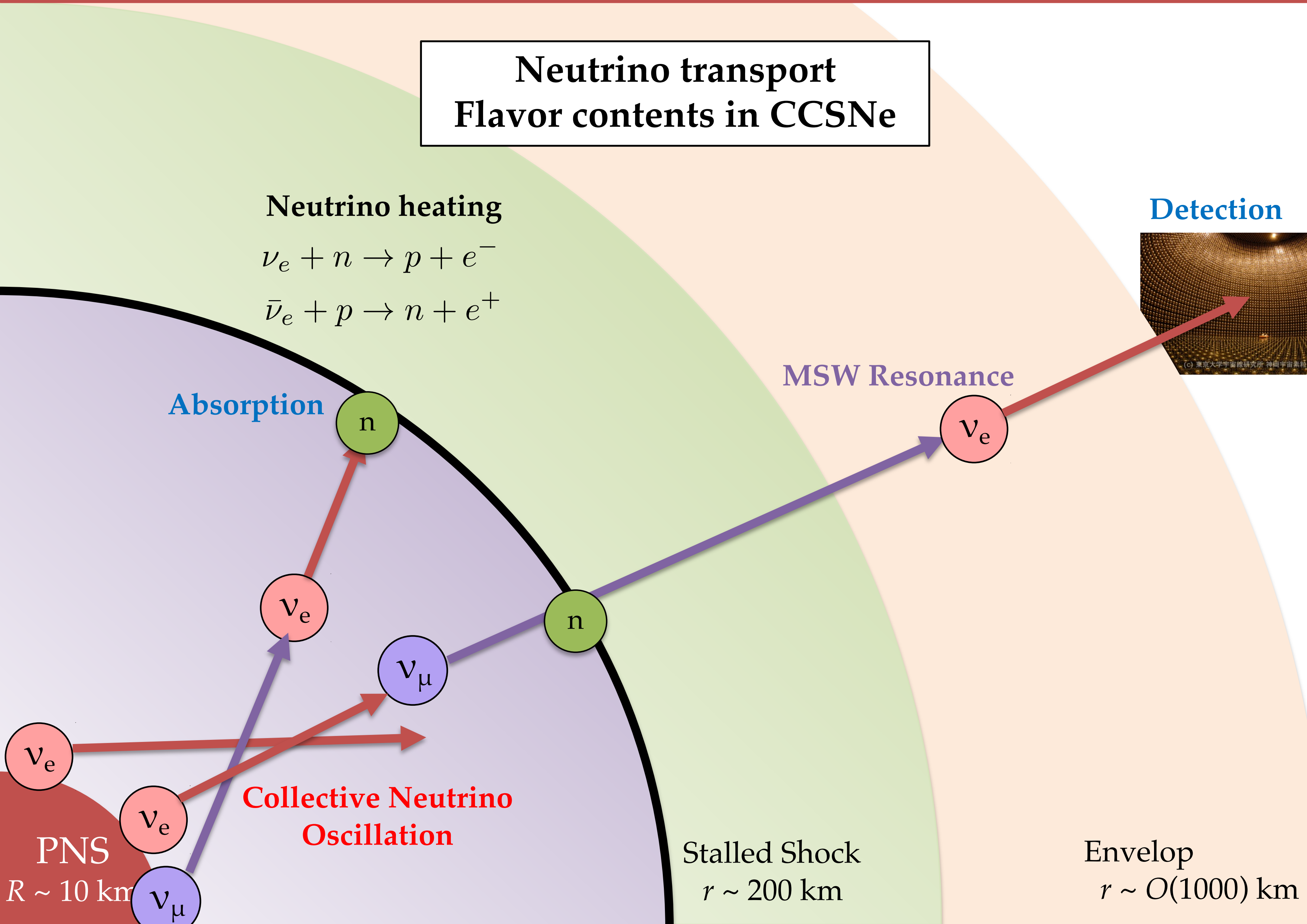
Neutrino Oscillations vs. CCSNe



Fast Flavor Conversion

``Fast flavor conversions" (FFC)

- Short scale of ~ $(G_F n_\nu)^{-1} \lesssim O(\mathrm{cm})$ or $O(\mathrm{ns})$. << stellar scale-height
- Triggered by ``angular crossings" in neutrino lepton number.

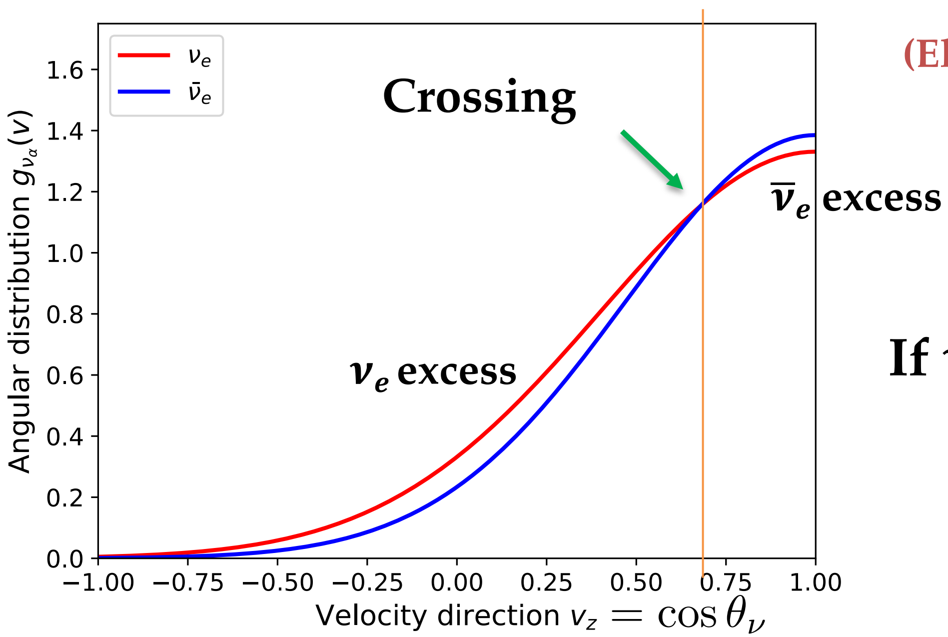
Izaguirre+ '17

Neutrino-flavor lepton number (NFLN) angular distribution.

$$G_{\boldsymbol{v}}^{ex} = \sqrt{2} G_{\mathrm{F}} \int \frac{E^2 \mathrm{d}E}{2\pi^2} \left[(f_{\nu_e} - f_{\bar{\nu}_e}) - (f_{\nu_x} - f_{\bar{\nu}_x}) \right]$$

= **ELN** − **XLN**

(Electron Lepton Number) (Heavy-leptonic one)



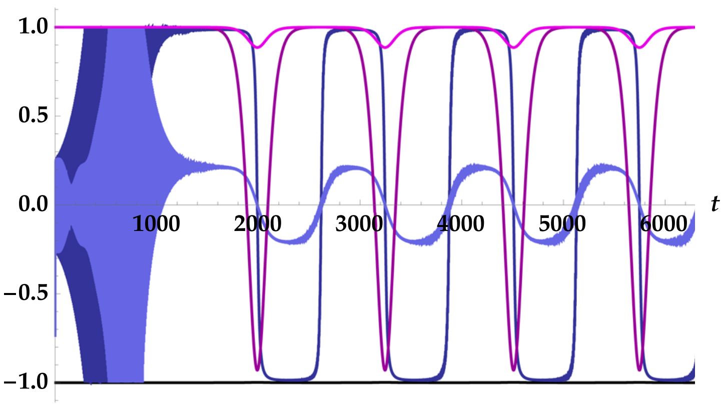
If $\nu_x = \bar{\nu}_x$, it is reduced to be only ELN.

(reasonable in CCSNe)

Asymptotic Behaviors

Homogeneous FFC
w/o collisions

$$\partial_t \rho_\nu = -\mathrm{i}\,[\mathcal{H}, \rho_\nu]$$



Johns+ '20

(Quasi-)periodic oscillation
Fast pendulum

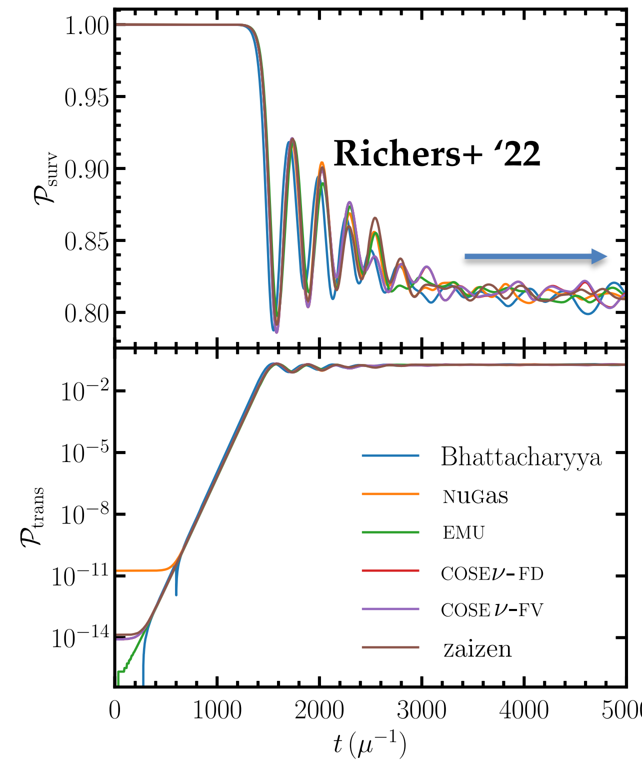
$$\boldsymbol{v} \cdot \nabla \rho_\nu$$

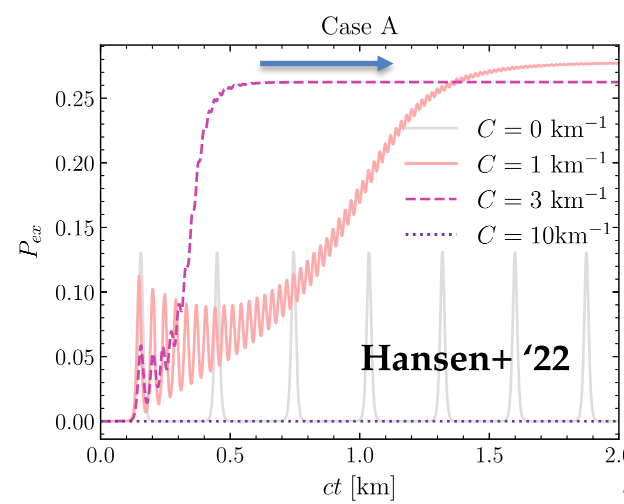
Cascade due to
advection term.

Break periodicity.
Asymptotic behaviors.

Matter-induced
decoherence.

$$\mathcal{C}[\rho_\nu, \bar{\rho}_\nu]$$





Stability of FFC

Governing equation for the off-diagonal components.

$$\mathrm{i}v^{\mu}\partial_{\mu}S_{p} = -\omega_{\mathrm{V}}S_{p} + S_{p}v^{\mu}\int\frac{\mathrm{d}^{3}p'}{(2\pi)^{3}}v'_{\mu}\,g_{p'}\,s_{p'}$$

$$\underline{- s_{p}v^{\mu}\int\frac{\mathrm{d}^{3}p'}{(2\pi)^{3}}v'_{\mu}\,g_{p'}\,S_{p'}} - \mathrm{i}\Gamma_{ex}S_{p}$$

Assumptions:

- $|S| << s \sim 1$.
- Ignore mode couplings.

= Linearization

$$\rho_{\nu} = \frac{\mathrm{Tr}\rho_{\nu}}{2}I_{2} + \frac{g_{p}}{2}\begin{pmatrix} s_{p} & S_{p} \\ S_{p}^{*} & -s_{p}\end{pmatrix}$$

Convolution between s_v & S_v in **nonlinear** regime.

$$\sum_{K'} v^{\mu}\tilde{D}_{\mu}^{K-K'}\tilde{s}_{p}^{K'}$$

$$\tilde{D}_{\mu}^{K} \equiv \int\frac{\mathrm{d}^{3}p}{(2\pi)^{3}}v_{\mu}\,g_{p}\tilde{S}_{p}^{K}$$

Main obstacles in fully nonlinear stability analysis.

Stability of FFC

Governing equation for the off-diagonal components.

$$\mathrm{i}v^\mu \partial_\mu S_p = -\omega_\mathrm{V} S_p + S_p v^\mu \int \frac{\mathrm{d}^3 p'}{(2\pi)^3} v'_\mu \, g_{p'} \, s_{p'}$$

$$- s_p v^\mu \int \frac{\mathrm{d}^3 p'}{(2\pi)^3} v'_\mu \, g_{p'} \, S_{p'} - \mathrm{i}\Gamma_{ex} S_p$$

Spatial- or time-averaged *$s_v(t,x)$* .
Capture the overall trends of
- nonlinear saturation
- quasi-steady state

Characterize a nonlinear saturation:
- Absence of **ELN-XLN** spectral crossings.

$$D(\omega, \boldsymbol{k}) \equiv \det\left[\Pi^{\mu\nu}(k)\right] = 0,$$

$$\Pi^{\mu\nu} = \eta^{\mu\nu} + \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \langle g_p(t,x) \rangle \frac{v^\mu v^\nu}{v^\lambda k_\lambda + \omega_\mathrm{V} + \mathrm{i}\Gamma_{ex}}$$

Conservative Forms

Quantum Kinetic Equation:

$$\mathrm{i}(\partial_t + v_z\partial_z)\rho_v = [H_{\nu\nu},\, \rho_v]$$
$$= [H_E,\, \rho_v] - [v_z H_F,\, \rho_v]$$

Angular-integration

$$H_E = \sqrt{2}G_\mathrm{F} \int \mathrm{d}\Gamma'\, \rho_{v'}$$
$$H_F = \sqrt{2}G_\mathrm{F} \int \mathrm{d}\Gamma'\, v_z'\rho_{v'}\,.$$

Conservative form:

$$\partial_t H_E + \partial_z H_F = 0$$

Flux term

Spatial-integration

For periodic boundary.

Conservation law:

$$\partial_t \langle H_E \rangle = 0$$

ELN & XLN.

Spatial-integration

For Dirichlet boundary.

Injecting & escaping components from inner/outer boundary.

More generally need to evaluate the flux term.

Asymptotic Behaviors

Key ingredients to characterize a quasi-steady state at a nonlinear saturation:

1. **Stability**
   - Disappearance of spectral crossing (in averaged-domain)
     = Establishment of flavor equipartition

2. **Boundary constraints** for ELN (XLN)
   - Conservation laws in periodic case
   - Compensates for flavor equipartition in one side.

↓ **Predictable**

FFC proceeds mainly in the shallow side of ELN angular distributions and works to eliminate the crossings.

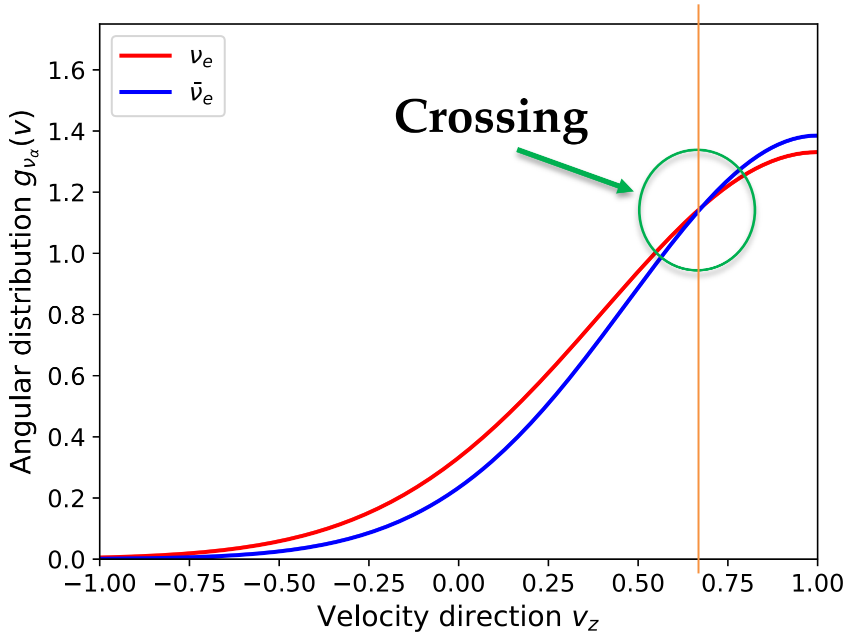
ELN Model

QKE:

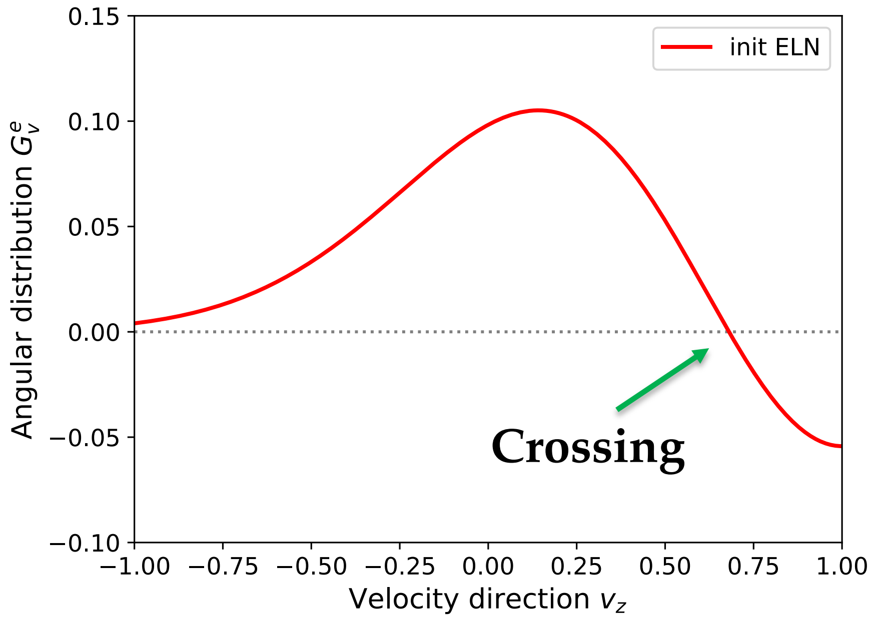
$$i(\partial_t + v_z \partial_z)\rho = [\mathcal{H}, \rho]$$

$$\mathcal{H} = \mu \int \mathrm{d}v'(1 - vv')G^{ex}_{v'}\rho_{v'}$$

Initially, pure electron state.
XLN is zero.

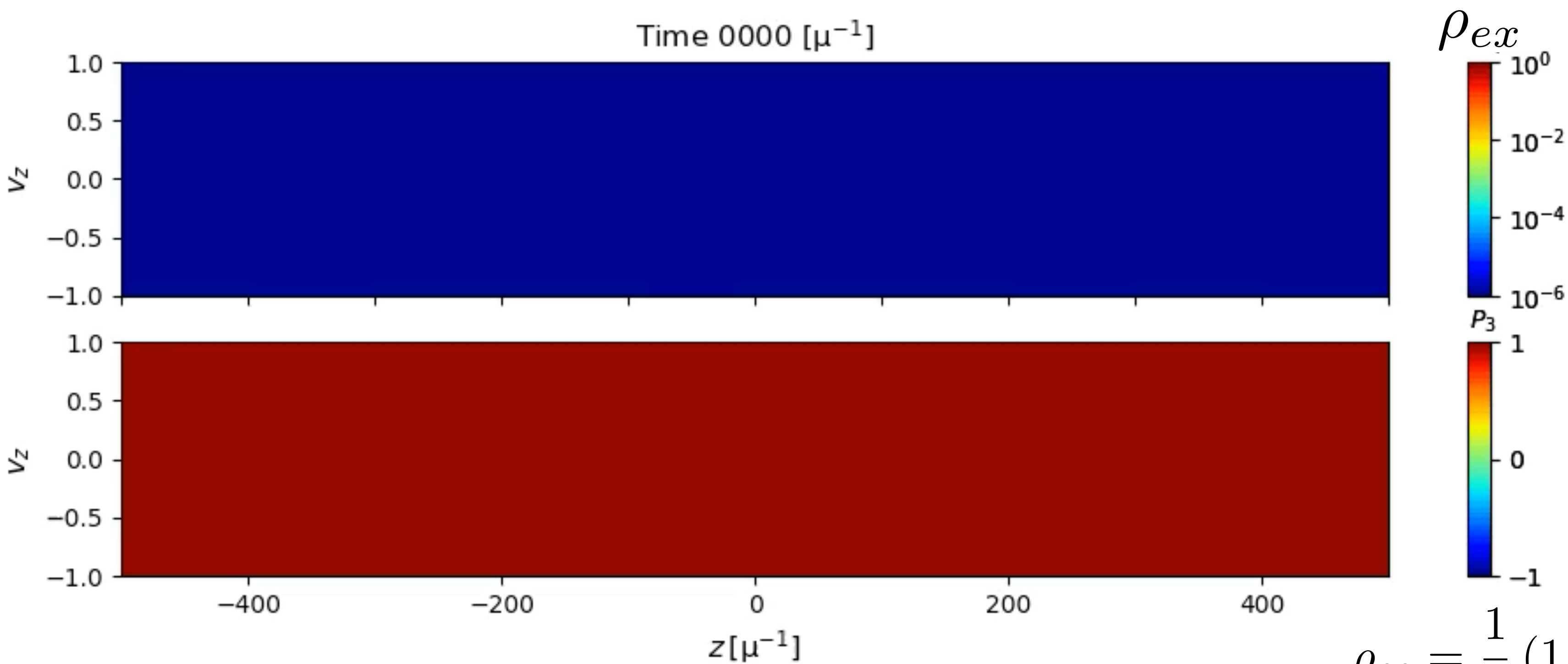


ELN angular distribution, G_{4b}



Flavor Simulation

Time evolution of neutrino density matrix for each component.



$$\rho_{ee} = \frac{1}{2}(1 + P_3)$$

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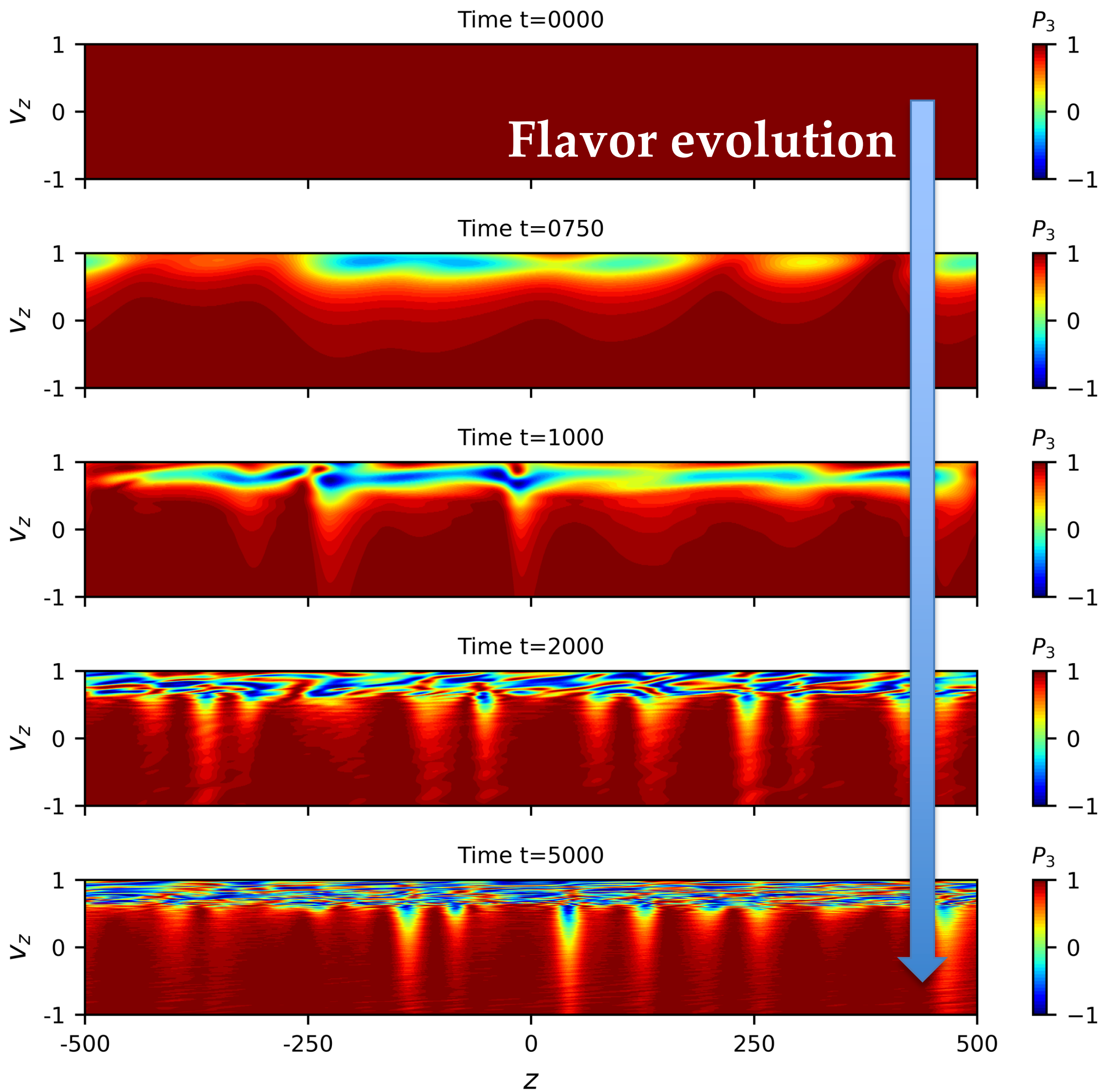
Spatial Structure

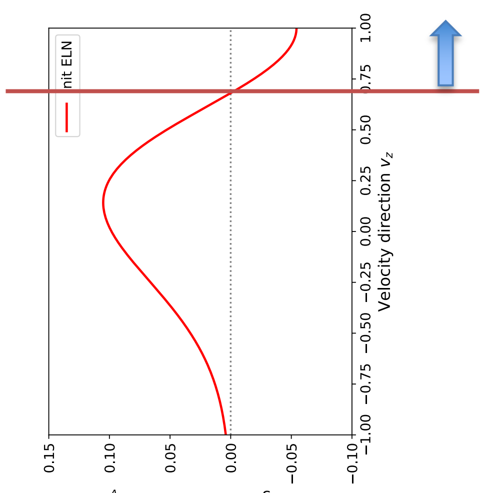


Generate smaller-scale structure in time.

As the overall trend,

- FFC occurs only within a crossing.
- FFC reaches nonlinear saturation at t ~ 2000.

Shallower side



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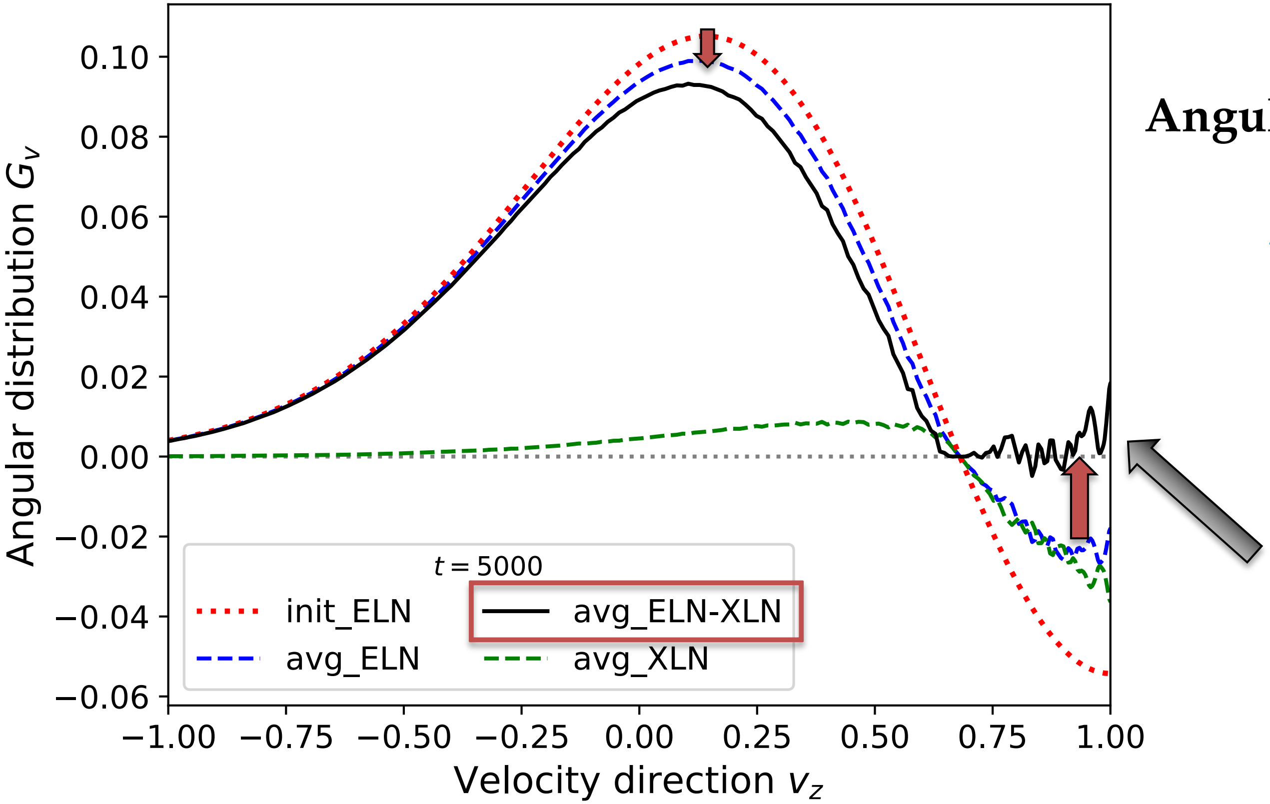
Spatial-Averaged Structure



Angular dist. at final state.

ELN still has a crossing.

No Crossing in **ELN-XLN** !!

Stability at a nonlinear saturation:

Absence of Angular Crossing in ELN-XLN ⇄ Establishment of quasi-steady state of FFC

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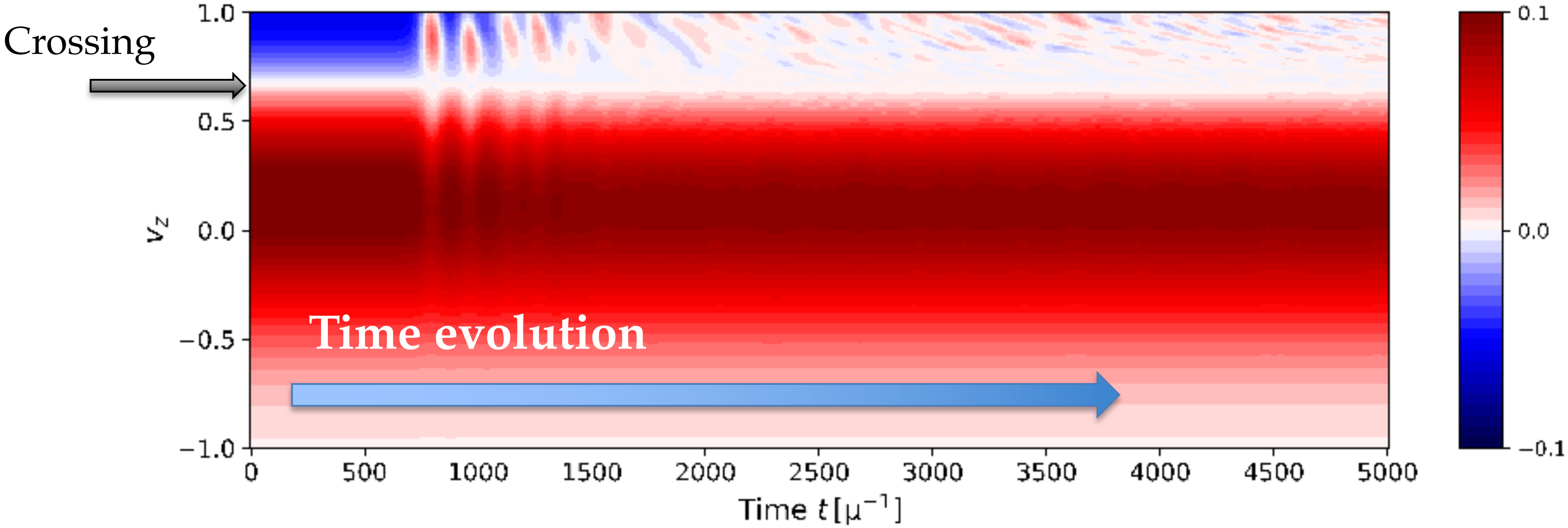
Spatial-Averaged Structure

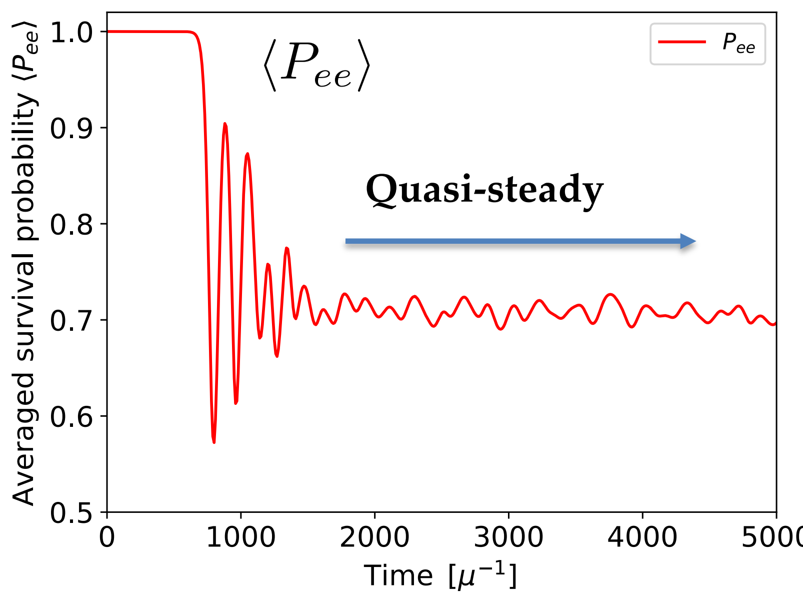
Spatial-averaged ELN-XLN.



Disappearance of crossing

Nonlinear saturation

FFC works to eliminate the crossings.



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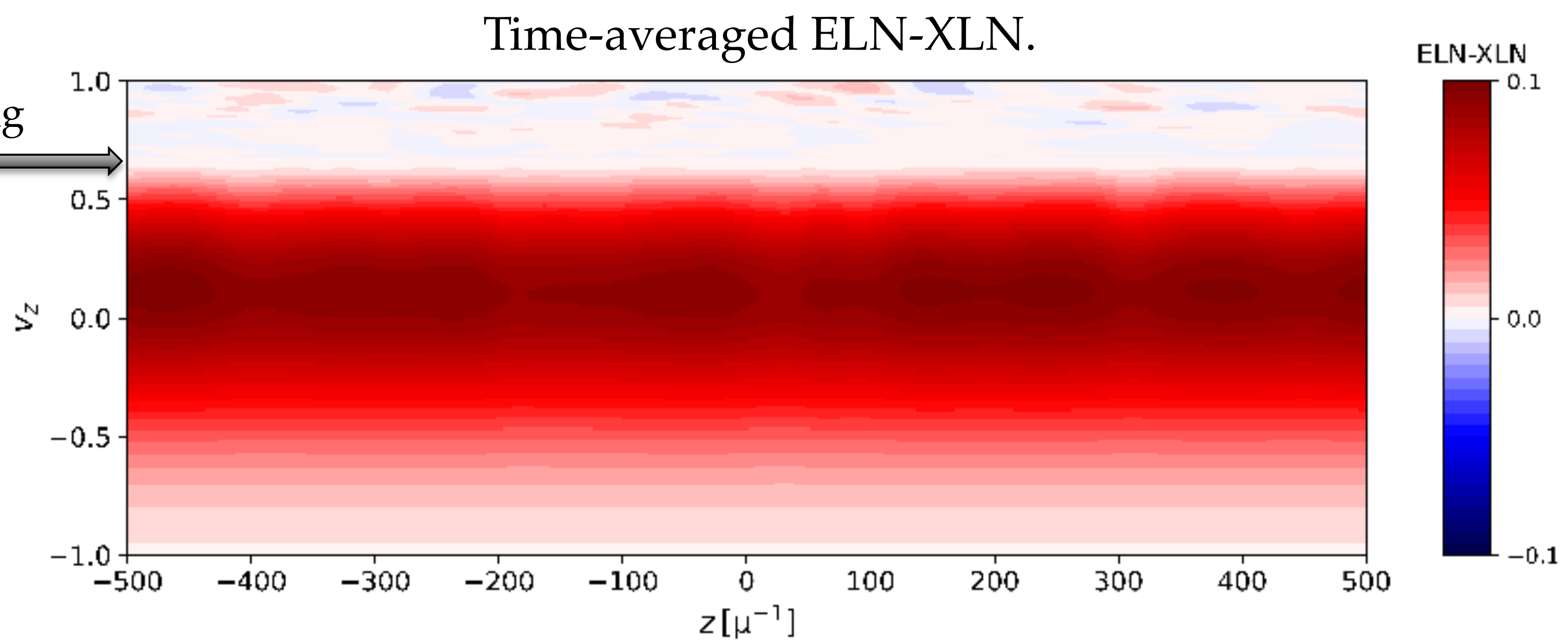
Time-Averaged Structure

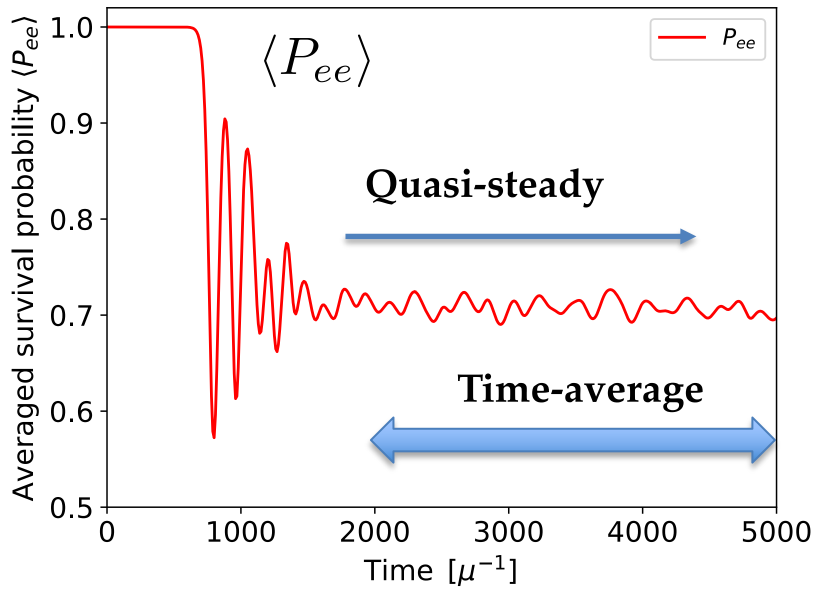
Crossing



Disappearance of crossing

Nonlinear saturation

FFC works to eliminate the crossings.



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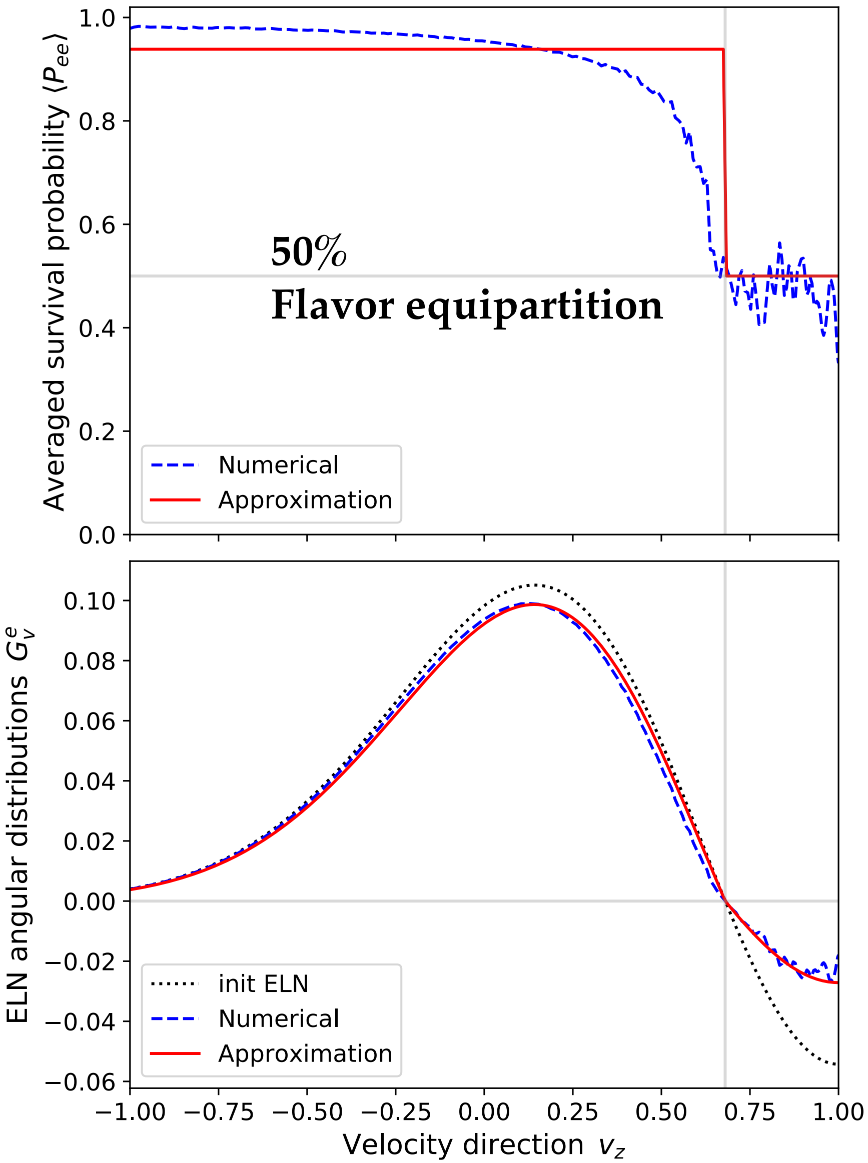
Modeling of Asymptotic State



Asymptotic behaviors:

1. **Stability**
   - Disappearance of crossing
2. **Conservation of ELN (XLN)**
   - Periodic boundary

Simplest model

$$P_{ee} = \begin{cases} p & \text{for } G_v < 0 \\ 1 - (1-p)\,\dfrac{A}{B} & \text{for } G_v > 0 \end{cases}$$

$$p = 1/2 \quad (\text{for 2 flavor case})$$

$$A \equiv \left| \int_{G^{ex}_{\boldsymbol{v}} < 0} \frac{d\boldsymbol{v}}{4\pi} G^{ex}_{\boldsymbol{v}} \right|$$

$$B \equiv \left| \int_{G^{ex}_{\boldsymbol{v}} > 0} \frac{d\boldsymbol{v}}{4\pi} G^{ex}_{\boldsymbol{v}} \right| .$$

For $B > A$

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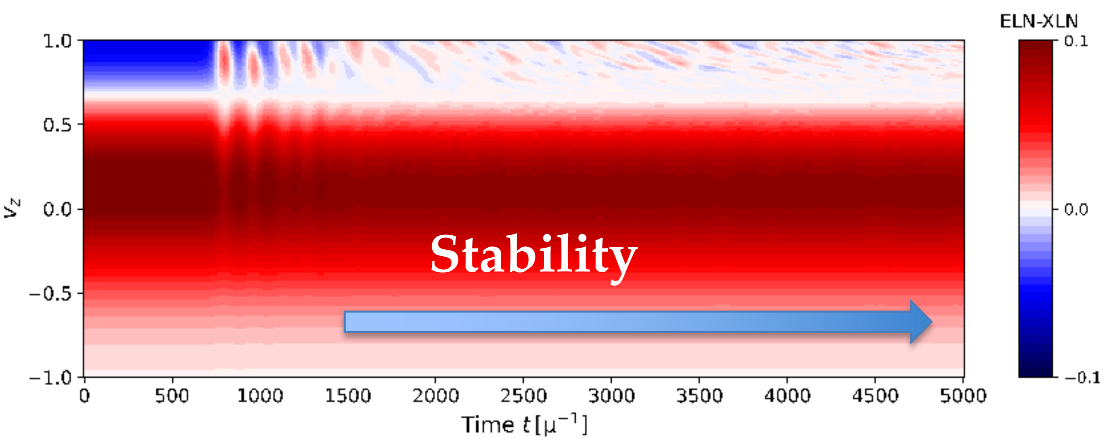
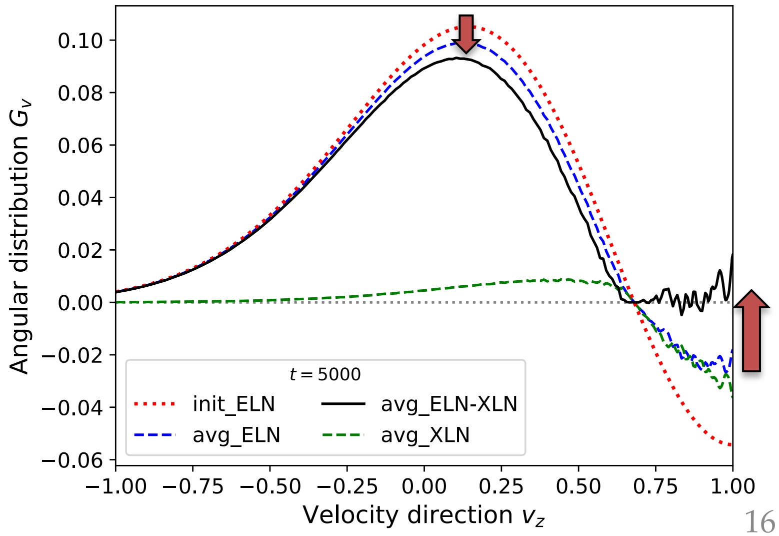
Asymptotic Behaviors

Key ingredients to characterize a quasi-steady state at a nonlinear saturation:

1. **Stability**
   - Disappearance of spectral crossing (in averaged-domain)
     = Establishment of flavor equipartition

2. **Boundary constraints** for ELN (XLN)
   - Conservation laws in periodic case
   - In the shallower side, more converted.





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Asymptotic Behaviors

Key ingredients to characterize a quasi-steady state at a nonlinear saturation:

1. **Stability**
   - Disappearance of spectral crossing (in averaged-domain)
     = Establishment of flavor equipartition

Effects of collisions?
→ Collision-dominated phase.
***c.f.*, Kato & Nagakura '22**

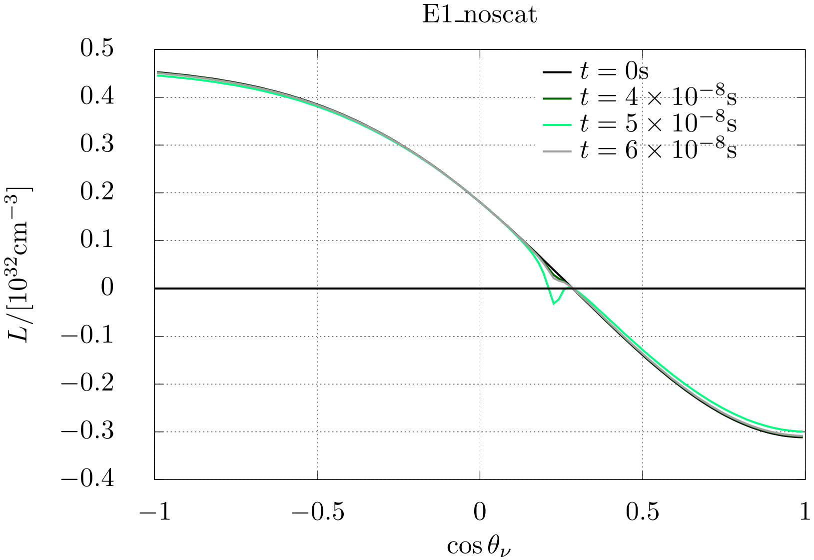
2. **Boundary constraints** for ELN (XLN)
   - Conservation laws in periodic case

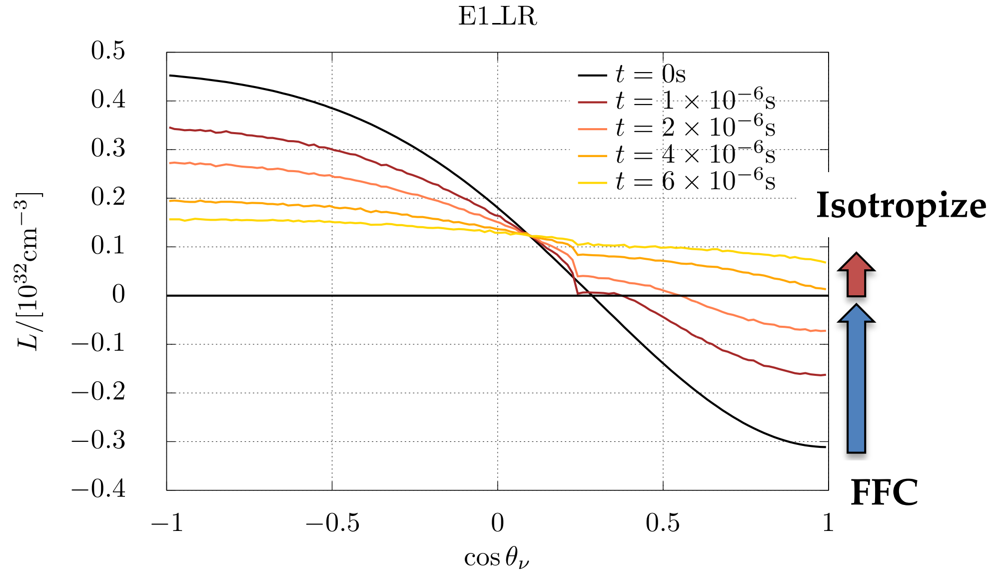
Other boundary conditions?
***e.g.*, Dirichlet case?**
→ Different asymptotic states
***c.f.*, Nagakura & Zaizen '22**

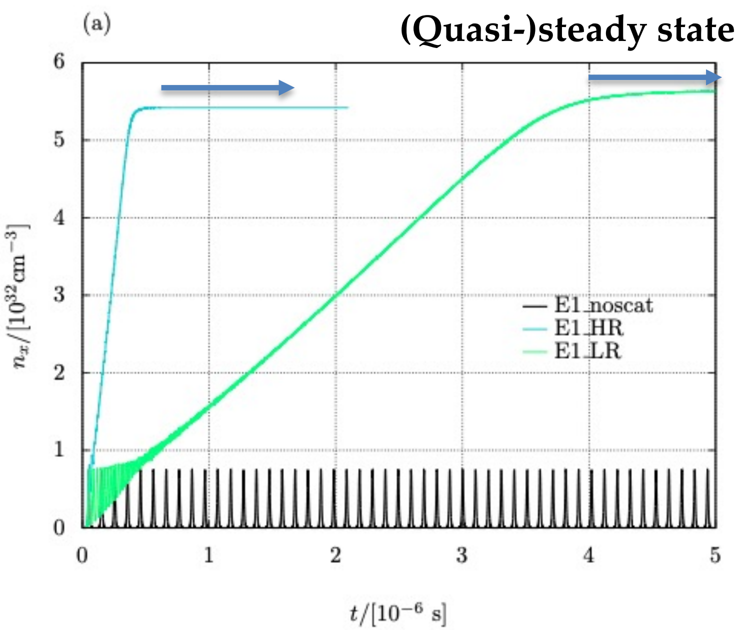
Stability w/ Collisions

L = **ELN-XLN** angular distributions.

Kato & Nagakura '22





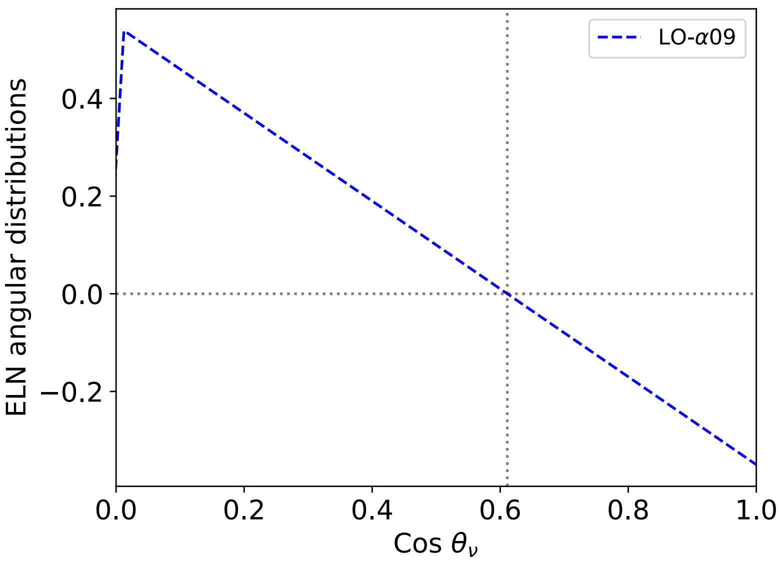


FFC + Scatterings (homogeneous)

FFC establishes asymptotic states through matter-induced decoherence.
After FFC ceases, scatterings only isotropize the angular distributions.

FFC-driven & Collision-dominated phase.

Boundary Constraints for Dirichlet



Nagakura & Zaizen '22

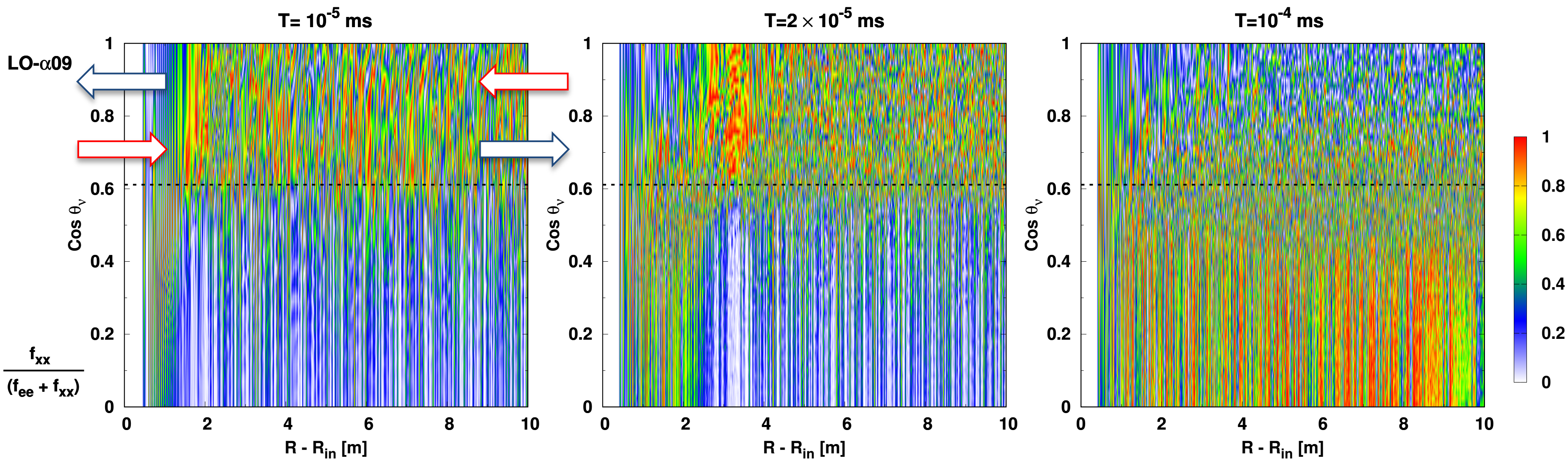
Dirichlet:

Much locally, same as periodic at early phase.

Reaches different quasi-steady states.

Conservative form:

$$\partial_t H_E + \partial_z H_F = 0$$

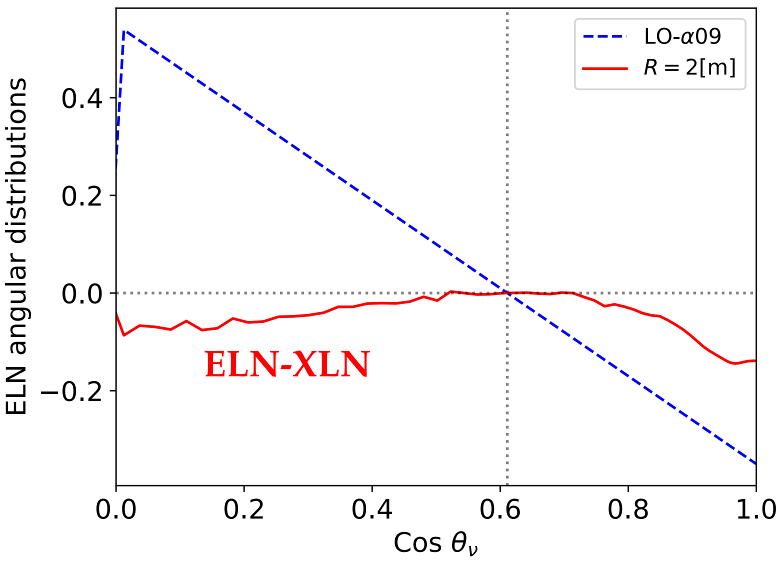


Similar to periodic case.

Different quasi-steady state.

Boundary Constraints for Dirichlet

Nagakura & Zaizen '22



Dirichlet:

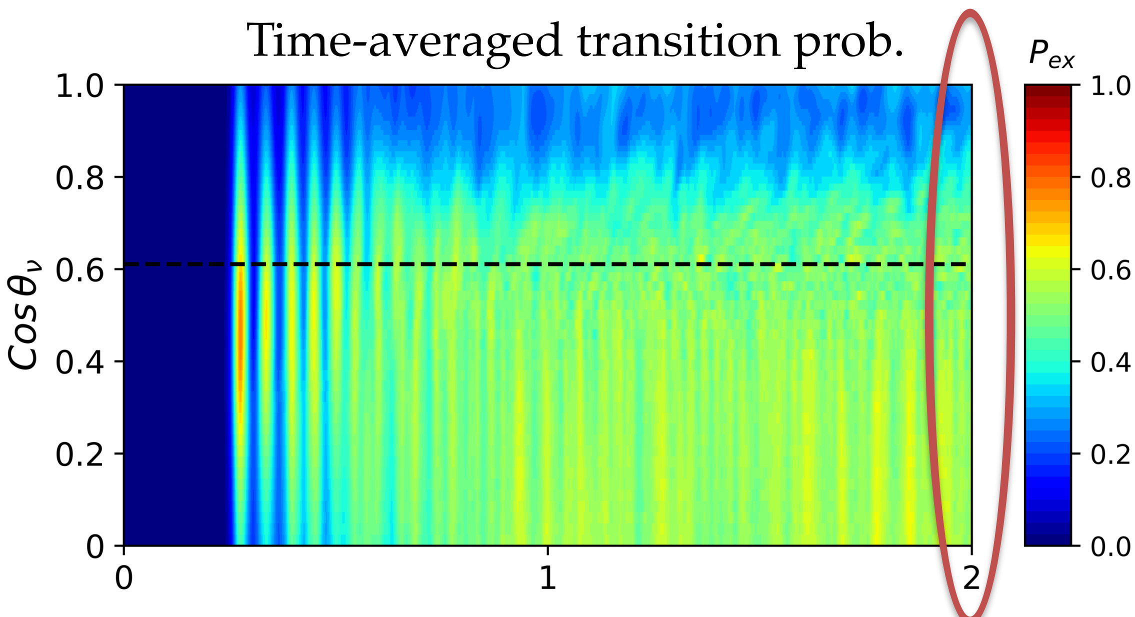
Spatial-average is inadequate.
→ Time-average during quasi-steady states.

ELN-XLN becomes negative in all directions.
(positive in periodic case.)

Conservative form:

$$\partial_t H_E + \partial_z H_F = 0$$

Time-averaged transition prob.

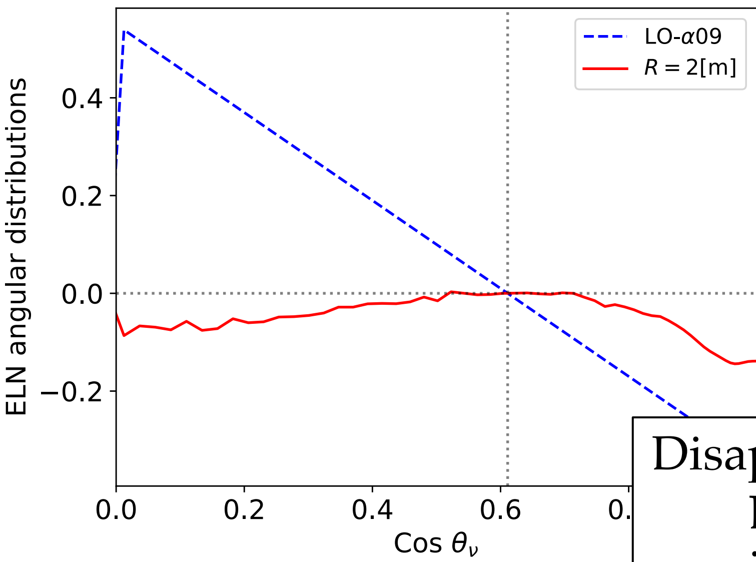


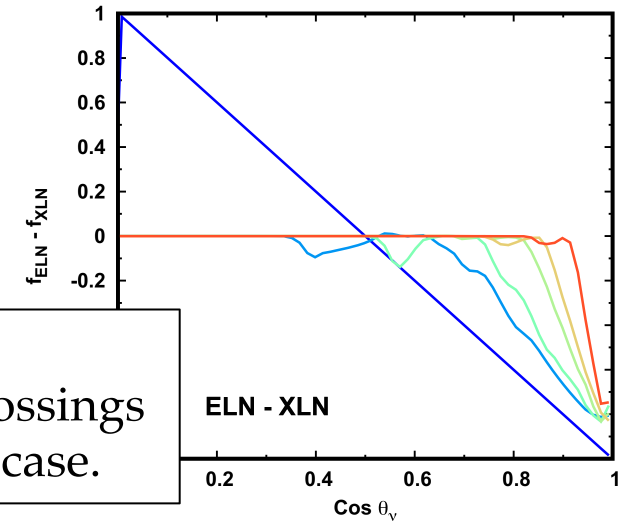
Small-scale fluctuations are smoothed.

nearly Flavor Equipartition

Boundary Constraints for Dirichlet

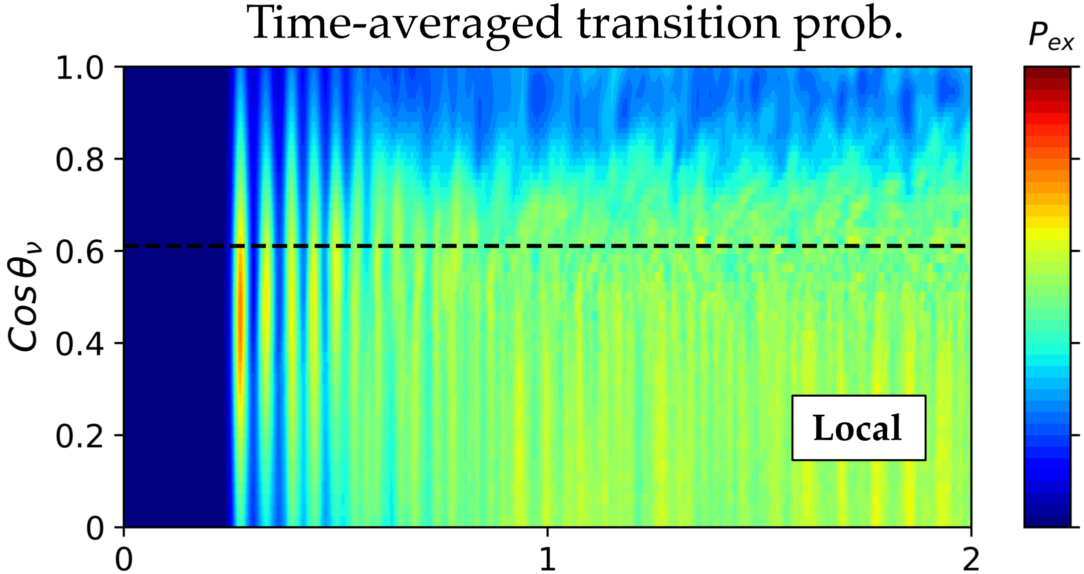
Nagakura & Zaizen '22

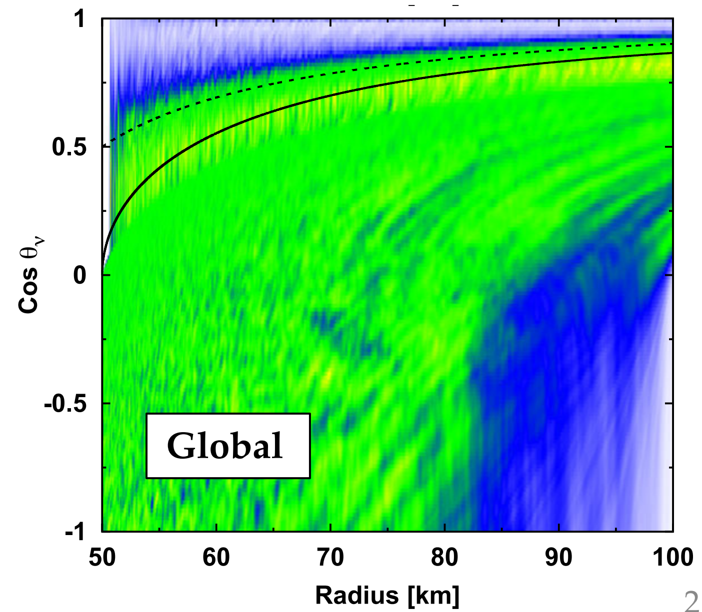




Disappearance of
ELN-XLN angular crossings
in both **local** / **global** case.

Time-averaged transition prob.





Summary

Key ingredients to characterize a quasi-steady state at a nonlinear saturation:

1. **Stability**
   - Disappearance of spectral crossing (in averaged-domain)
     (In FFC case, absence of angular crossings.)
   - Collisions are distinguished from FFC-stability.

Nonlinear saturation:

$$\Pi^{\mu\nu} = \eta^{\mu\nu} + \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \langle g_p(t,x) \rangle \frac{v^\mu v^\nu}{v^\lambda k_\lambda + \omega_{\mathrm{V}} + \mathrm{i}\Gamma_{ex}}$$

2. **Boundary Constraints** for ELN (XLN)
   - In periodic, conservation laws.
   - In Dirichlet, different quasi-steady state.
     - Need to evaluate the flux term.

Conservative form:

$$\partial_t H_E + \partial_z H_F = 0$$