

# Loss of flavor coherence in a dense neutrino gas

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# Neutrino flavor evolution

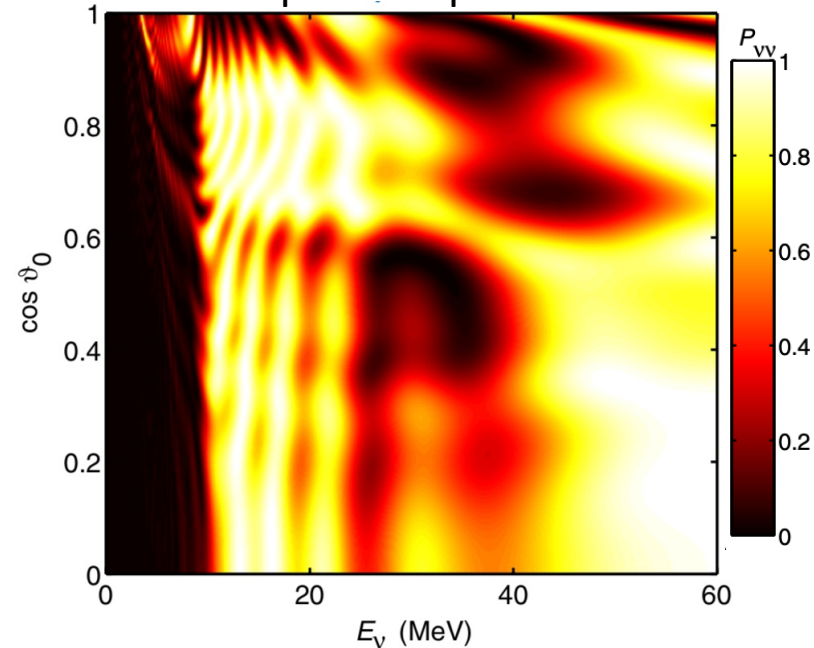
- In the MF approximation a rich set of phenomena is observed.
  - Collective oscillations
  - Spectral splits/swaps
    - Predictable in the spectrum
  - Extremely rapid flavor oscillations
    - Leads to average flavor scrambling/equipartition?
- Recent work in few neutrino many-body problem reveals substantial MB entanglement.

Patwardhan, Cervia, Balantekin  
*Phys.Rev.D* 100 (2019) 8,  
083001

Illa, Savage  
arXiv:2210.08656

$$\mu \rightarrow 0$$

Spectral Split



H. Duan, G. M. Fuller, J. Carlson, and Y.-Z. Qian. *Phys. Rev.*, D74:105014, 2006.

# Neutrino many body problem

- Two flavor approximation
- Need to retain  $\sim 2^N$  amplitudes for N neutrinos for quantum many body (MB) evolution
- Only need  $3N$  real numbers for N polarization vectors in the mean-field (MF)
- If MF works, lets do that!
  - No entanglement

$$\omega_i \approx \mathcal{O}(10) \text{ km}^{-1}$$

$$H = \sum_i \left[ \frac{\omega_i}{2} \mathbf{B} \cdot \boldsymbol{\sigma}_i \right] \quad \text{Vacuum Oscillations}$$

+

$$\frac{\mu}{2N} \sum_{i < j} (1 - \vec{v}_i \cdot \vec{v}_j) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \quad \text{Flavor Exchange}$$

$$100 < \frac{\mu(t)}{\omega_0} < 0$$

$$\text{U.C.} \implies \vec{v}_i = 0$$

$$\text{R.C.} \implies \vec{v}_i \neq 0$$

Y. Pehlivan, A. B. Balantekin,  
Toshitaka Kajino, and Takashi Yoshida  
*Phys. Rev. D* 84, 065008 (2011).

# Conserved quantities

$$H_{\text{vac}} = \sum_i \frac{\omega_i}{2} \mathbf{B} \cdot \boldsymbol{\sigma}_i$$

$$H_{\nu\nu} = \frac{\mu}{2N} \sum_{i < j} (1 - \vec{v}_i \cdot \vec{v}_j) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

- The uniform coupling case is integrable. There are as many conserved charges as there are dimensions of the Hilbert space.

$$\mathbf{J} = \frac{1}{2} \sum_i \boldsymbol{\sigma}_i$$

$$[H_{\text{vac}}, \mathbf{B} \cdot \mathbf{J}] = 0$$

$$[H_{\nu\nu}, J_\alpha] = 0 \quad [H_{\nu\nu}, J^2] = 0$$

$$[H_{\text{vac}} + H_{\nu\nu}, \mathbf{B} \cdot \mathbf{J}] = 0$$

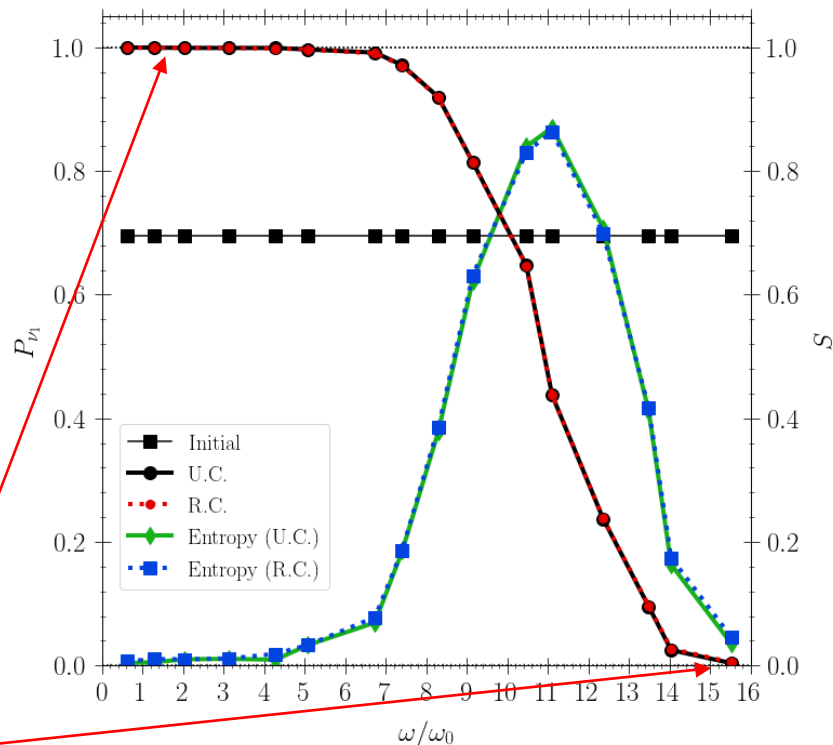
$$\left[ H_{\text{vac}} + H_{\nu\nu}^{(\text{U.C.})}, R_m \right] = 0$$

# Initially all equal states

- If all spins in the same state initially, this is an extreme state of the Hamiltonian – lives in max  $J^2$
- If adiabatic, we can predict the final state independently of the details of the 2-body couplings.
- Entropy is only large in the split
  - Does MF work for most states except those near the split?
  - Hybrid MB/MF method for solving?

Spectral Split!

$$|\Psi_i\rangle = |\nu_e\rangle^{\otimes N}$$



Patwardhan, Cervia, Balantekin  
Phys. Rev. D 104, 123035 (2021)

JDM, A. Roggero, H. Duan, J. Carlson  
arXiv:2301.07049

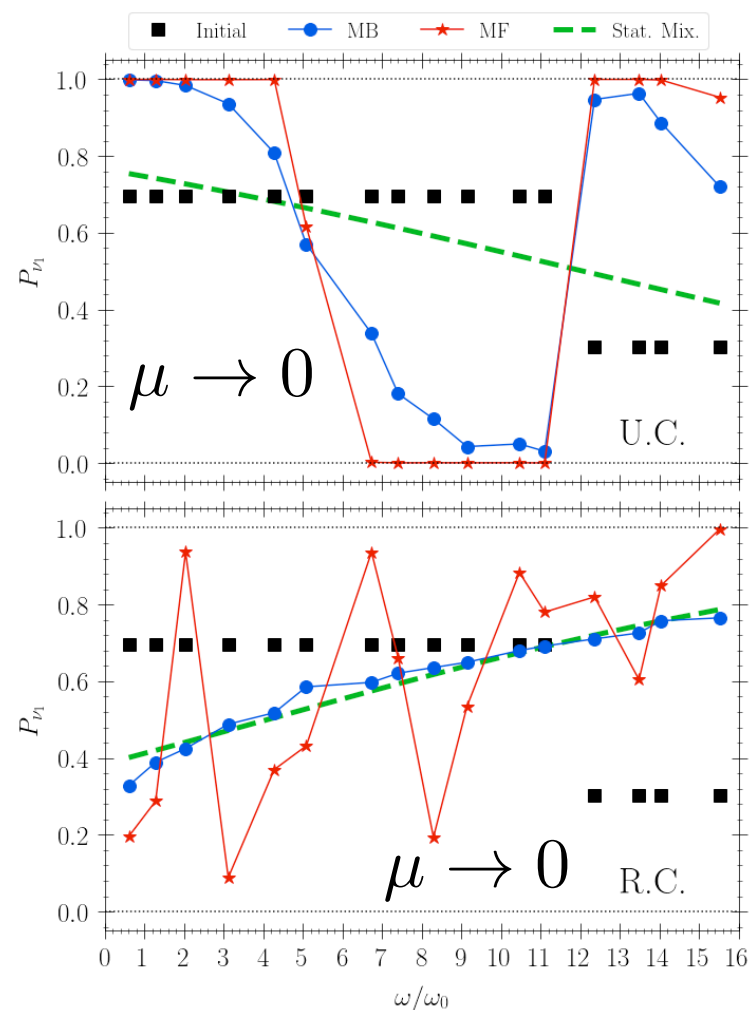
# Mixed flavor initial state

- The detailed evolution of more general states is sensitive to the choice of 2-body couplings for both formalisms

- Uniform 2-body couplings still show spectral split

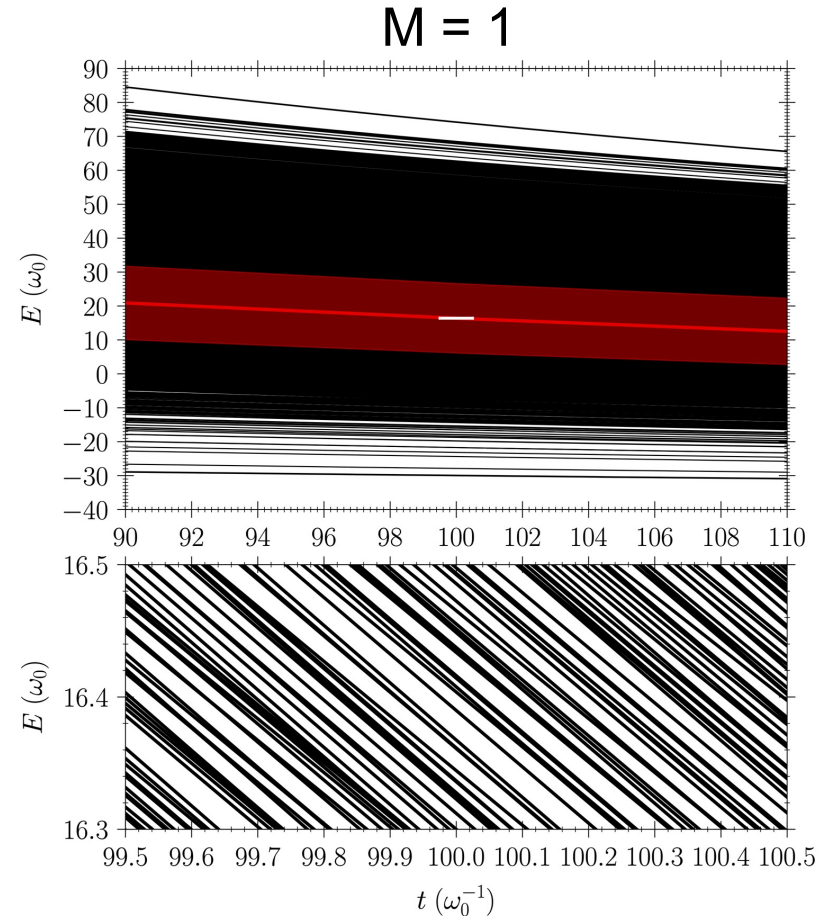
Patwardhan, Cervia, Balantekin  
Phys. Rev. D 104, 123035 (2021)

- Random couplings show poor agreement between MB and MF.



# Distribution in the spectrum

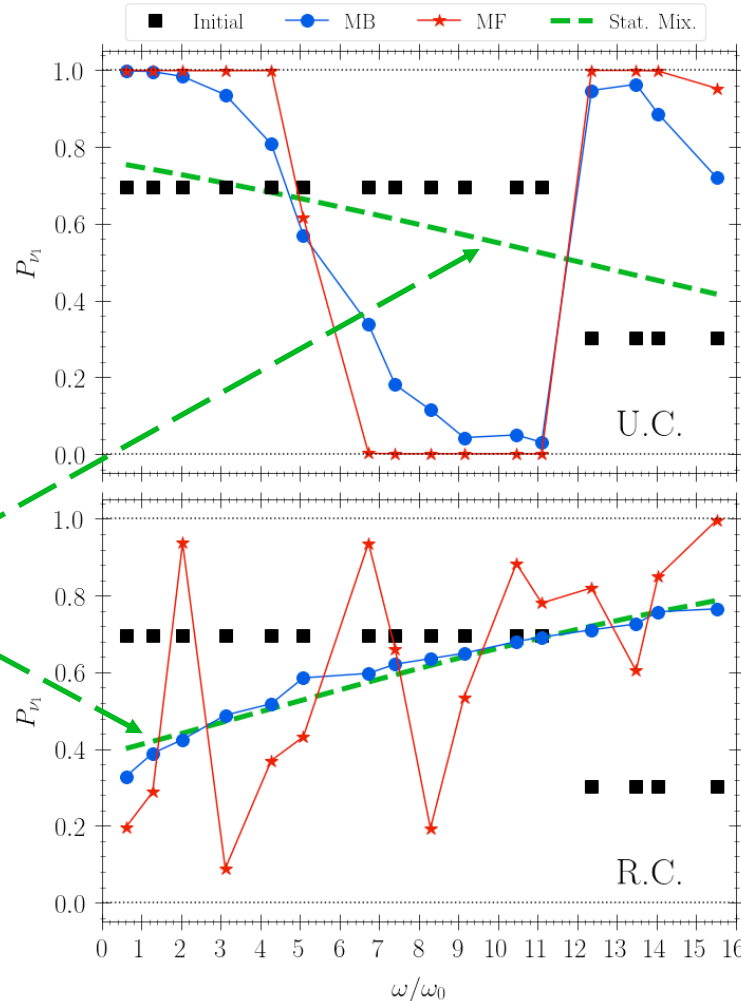
- $J_z$  is conserved, so each  $m$  subspace evolves independently.
- Initial state has overlap with lots of intermediate energy states
- States evolve in time – subsequently lots of avoided level crossings.
- Results in state having support on many more energy states than the initial state.
- Unless symmetry protected, trace reduced subsystems will likely display decoherence



# Mixed flavor initial state

- R.C.: Entanglement ends up in many-spin partitions of the system.
  - Few body subsystems look like statistical mixtures.
- Invites treatment as a classical statistical mixture with some temperature  $\beta$
- Stat. Mixture gives better agreement than MF
  - Obtaining this requires solving the MB system

$$\mathcal{P}_{\nu_1} \propto e^{-\beta(\omega - \mu)}$$





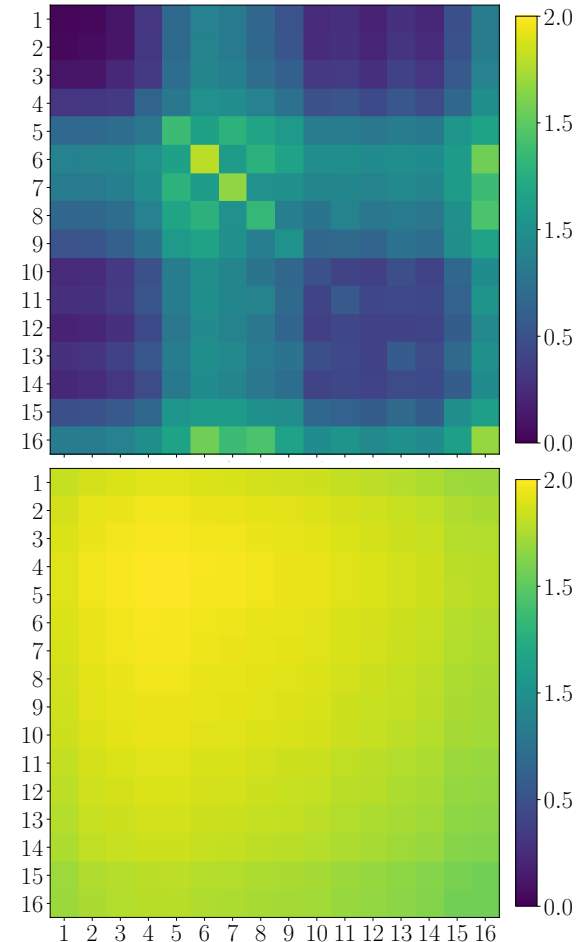
# Two body entanglement

- Symmetry protects against development of significant entanglement for the uniform coupling case.
  - Entropy largest in the spectral split as seen by Patwardhan, Cervia, & Balantekin
- Nearly maximal entanglement in both one- and two-body RDMs

$$S_{ij} = -\text{Tr} [\rho_{ij} \log_2 (\rho_{ij})]$$

$$\text{U.C.} \implies \vec{v}_i = 0$$

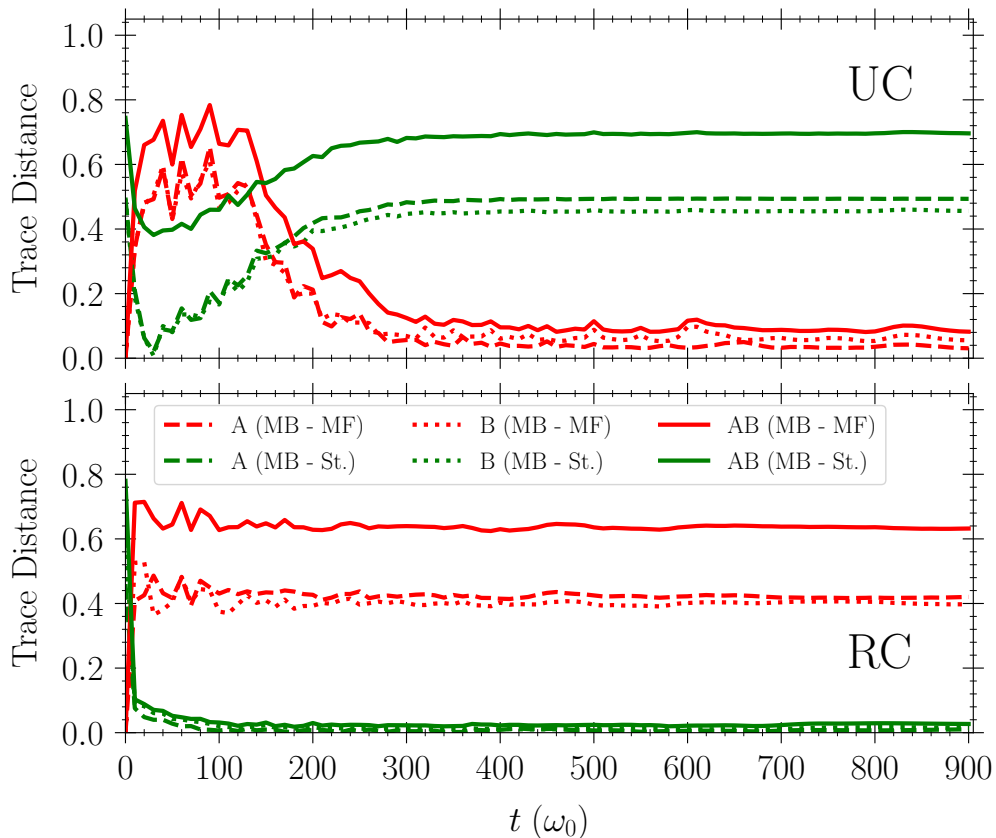
$$\text{R.C.} \implies \vec{v}_i \neq 0$$



# Loss of Coherence

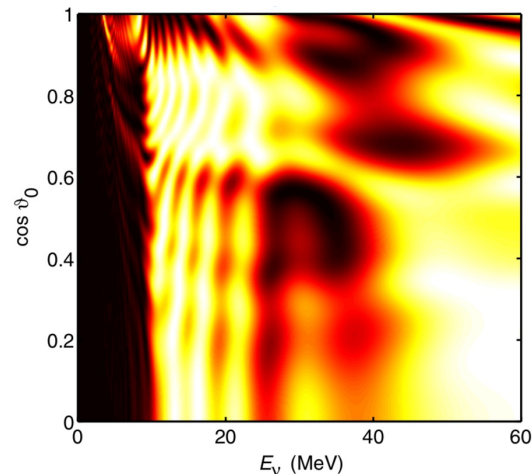
- Trace distance characterizes how distinguishable two quantum states (RDM's  $\rho, \rho'$ ) are.
- A = neutrino 12
- B = neutrino 13

$$T(\rho, \rho') = \frac{1}{2} \text{Tr} \left[ \sqrt{(\rho - \rho')^\dagger (\rho - \rho')} \right]$$



## Some next thoughts

- Is it possible to predict stat. params. without doing MB calculation?
- We probably can do upwards of 30 flavor-spins on HPC platforms. Can we find how these decohered one-body states depend on Hamiltonian parameters?
- What can we learn from quantum computing?



???

# Thanks!

- Collaborators:
  - Joe Carlson, Huaiyu Duan, Alessandro Roggero, Duff Neill
- You!

