Collective neutrino oscillations in 1 and 2 dimensions: what can we learn from simulations

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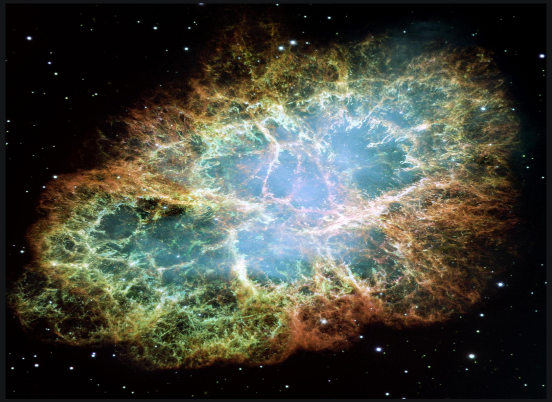
INTRODUCTION



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- $\sim 10^{53}$ ergs of energy us carried out by $\sim 10^{58}\nu$
- supernova shock revival
- The N_p/N_n ratio is determined by the ν_e and $\overline{\nu}_e$ density in the environment

$$n + \nu_e \rightarrow p + e^-$$

$$p + \overline{\nu}_e \rightarrow n + e^+$$

- Influence nucleosynthesis outcome

νFFC: Formalism

- Flavor content of the neutrinos described using density matrix ρ

$$\rho(t, \boldsymbol{r}, \boldsymbol{p}) = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{ee}* & \rho_{xx} \end{pmatrix}$$

  - $\rho_{ee/xx}$ is the $\nu_{e/x}$ destribution
  - ρ_{ex} is the correlation between ν_e and ν_x

- $i(\partial_t + v.\nabla)\rho_{\boldsymbol{p}} = [H_{\boldsymbol{p}}, \rho_{\boldsymbol{p}}] + iC$

$$H_{\boldsymbol{p}} = H_{\mathrm{vac}} + H_{\mathrm{matter}} + H_{\nu\nu}$$

$$H_{\nu\nu} = \sqrt{2} G_F \int \frac{d^3 \boldsymbol{p}'}{(2\boldsymbol{\pi})^3} (1 - \boldsymbol{v}.\boldsymbol{v}')(\rho(t, r, p') - \overline{\rho}^*(t, r, p'))$$

- $\mathbf{l}_{\nu\nu} \sim (\sqrt{2} G_F n_{\nu_e})^{-1} \sim cm, \quad \mathbf{l}_{\mathbf{vac}} \sim (\frac{|\Delta m^2|}{2E})^{-1} \sim km$

νFFC: Formalism

- *Assumptions:*
  - Contribution from Vacuum oscillation is negligible
  - Matter distribution is homogeneous
  - Neutrino beams with different energies behave identically

$$H_{\nu\nu} = \mu \int \frac{d^3 \boldsymbol{v}'}{2\boldsymbol{\pi}} (1 - \boldsymbol{v}.\boldsymbol{v}')(g_{\nu_e} \varrho(t, r, v') - \alpha g_{\overline{\nu}_e} \overline{\varrho}^*(t, r, v'))$$

$$g_{\nu_\beta} = \frac{1}{4\boldsymbol{\pi}^2 n_{\nu_\beta}} \int dE\, E^2 f_{\nu_\beta}(\boldsymbol{p}), \quad \alpha = n_{\nu_e}/n_{\overline{\nu}_e}$$

1+1+1 D simulation

[MRW, M. G, C-Y. Lin, Z. Xiong]

- *Reduction from 1+3+2 → 1+1+1:*
  - System is homogeneous in both x and y directions all times
  - Angular distrubution is azimuthally symmetric
  - Both initial condition and the solution respect the above assumptions. In other words we neglect the symmetry breaking solutions

- ϱ depends only on t, z and v_z

$$i(\partial_t + v_z \partial_z)\varrho(t, z, v_z) = [H_{\nu\nu}(t, z, v_z, \varrho(t, z, v_z]$$

$$H_{\nu\nu} = \mu \int dv_z' \; (1 - v_z v_z')(g_{\nu_e}\varrho(t, z, v_z') - \alpha g_{\overline{\nu}_e}\overline{\varrho}^*(t, z, v_z'))$$

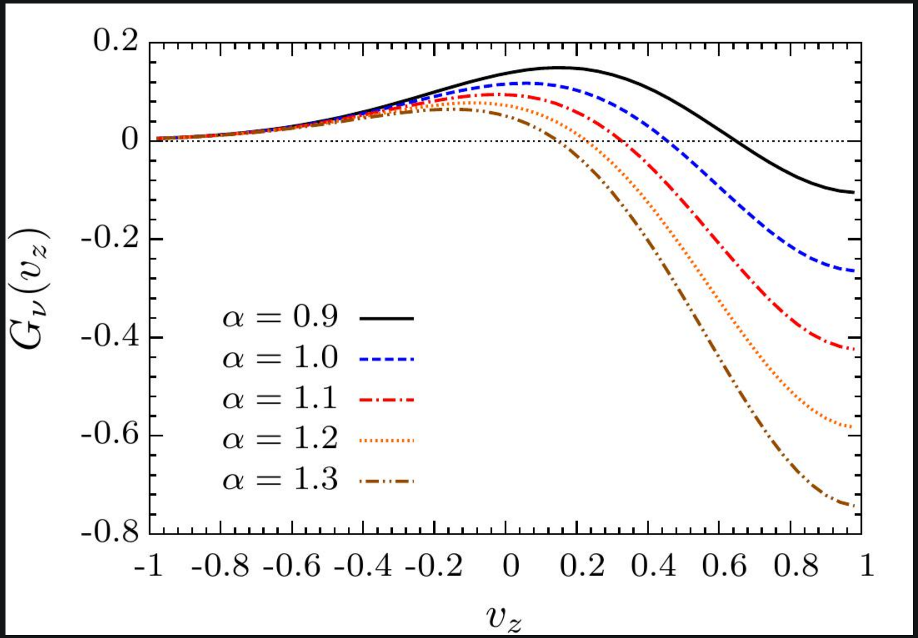
1+1+1 D simulation: *Initial conditions*

- We use following form for the angular distribution for both ν_e and $\overline{\nu}_e$

$$g_{\sigma_{\nu/\overline{\nu}}} = N\,exp[-(\frac{(1 - v_z)^2}{2\sigma_{\nu/\overline{\nu}}^2})]$$

$$\sigma_{\nu_e} = 0.6,\ \sigma_{\overline{\nu}_e} = 0.5$$

$$G_\nu = g_{\nu_e} - \alpha\, g_{\overline{\nu}_e}$$



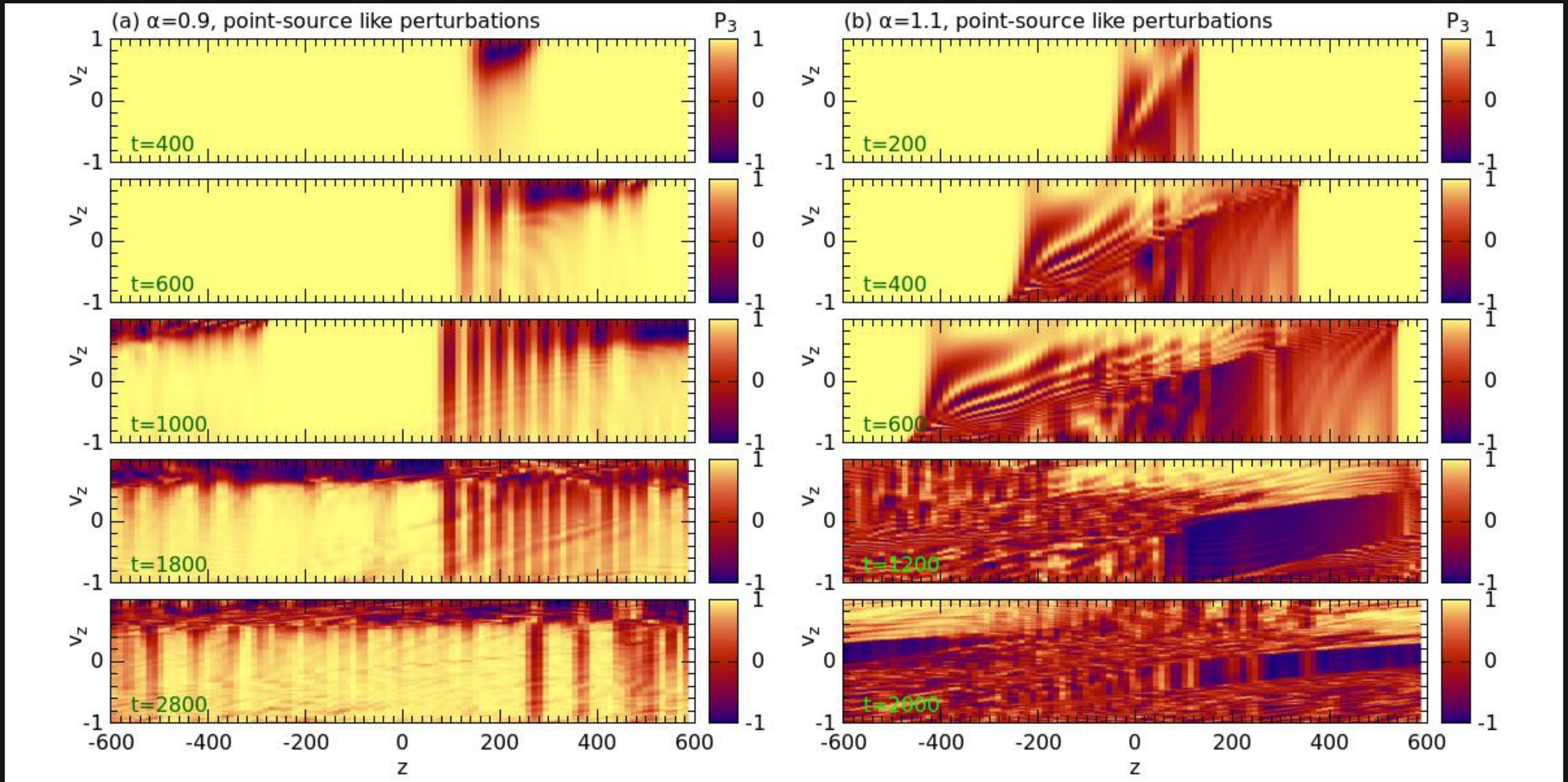
- We discretize the spatial domain into N_z grid points and angular domain into N_{v_z} bins. $z \in$ [-600, 600], $v_z \in$ [-1, 1].

- We also used two methods to carry out the simulation

  – Finite diffeterence with Kreiss – Oliger error suppresson

  – Finite volume method with 7^{th} order WENO scheme for flux reconstruction

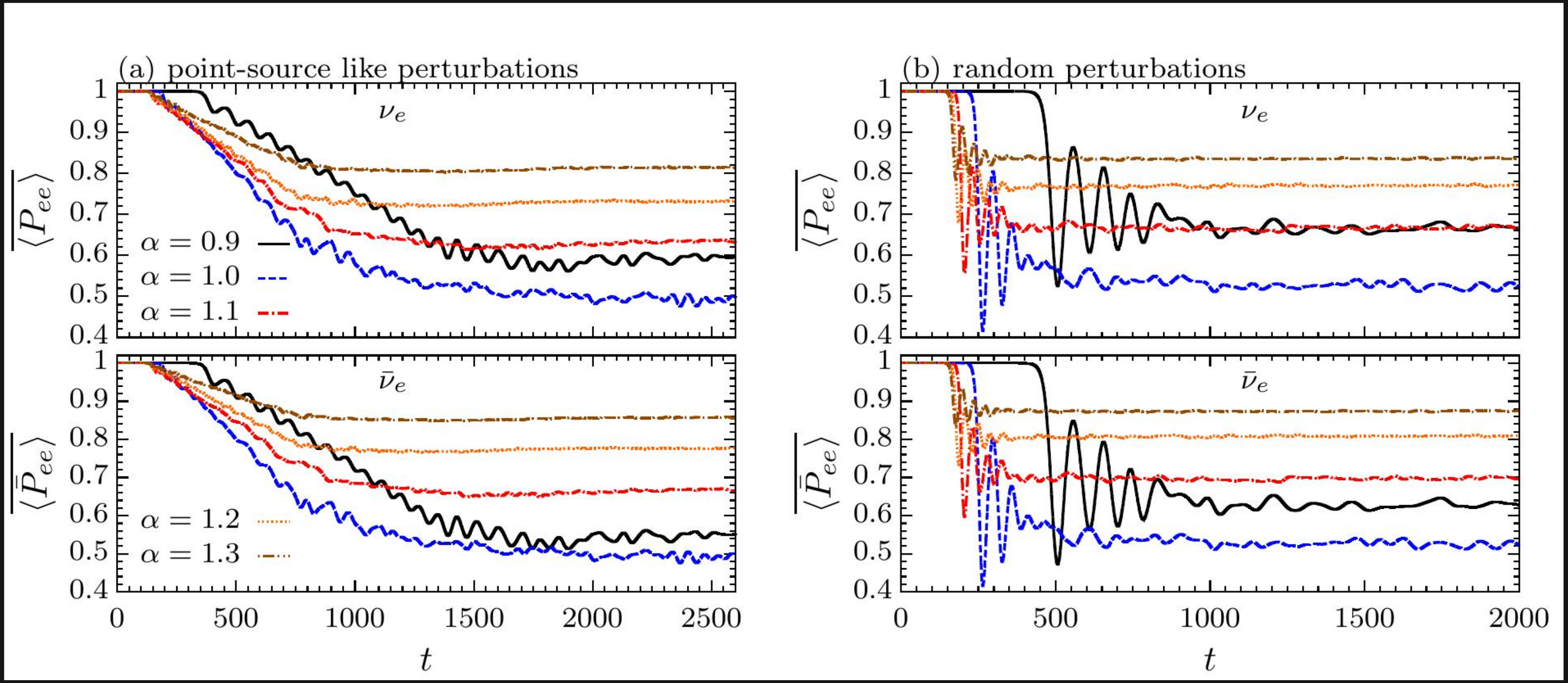
1+1+1 D simulation: Flavor wave (FW) evolution



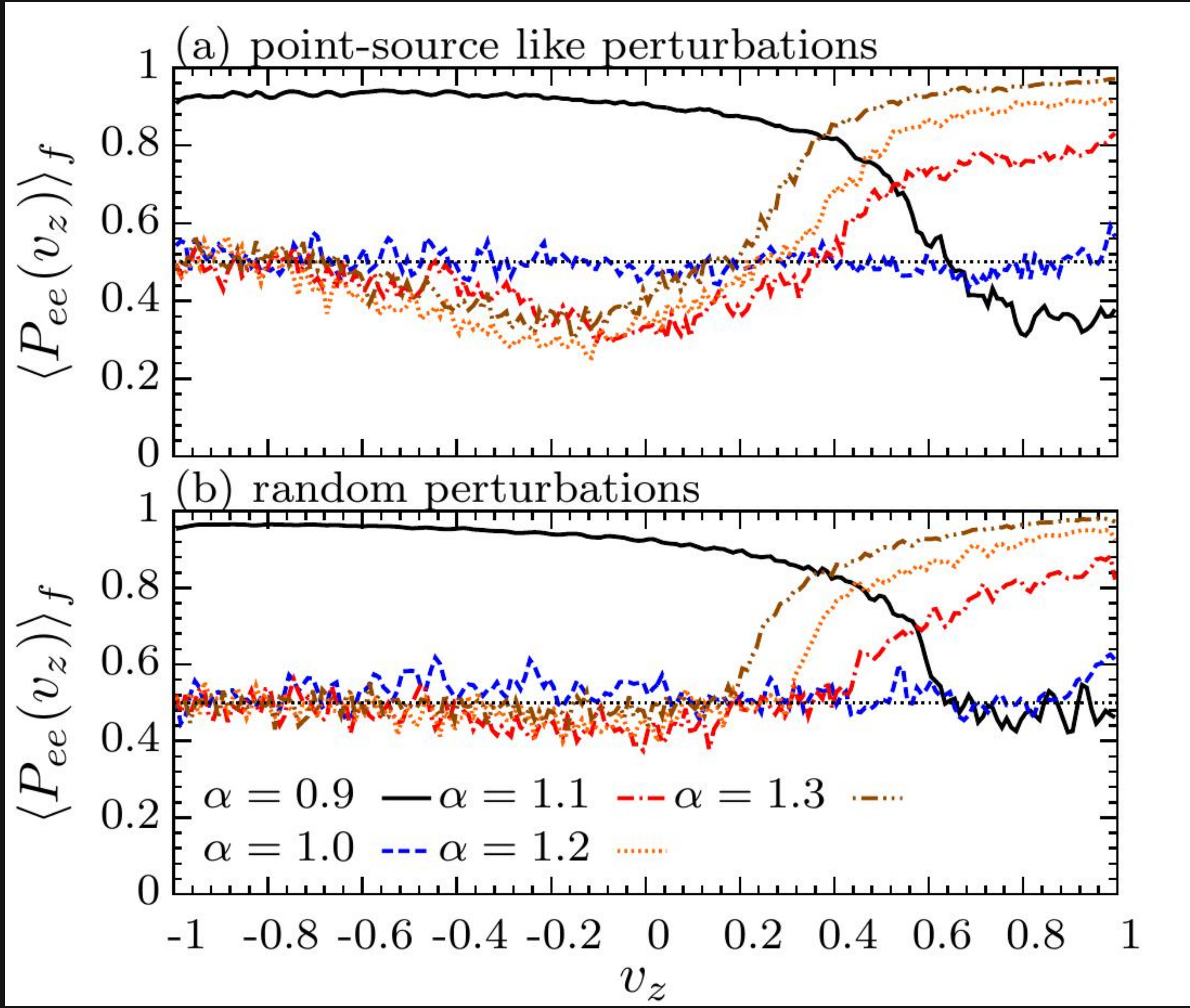
- FW propagates to the right
- Initially all v_z affected, diminishes later to $v_z < v_{z,c} \sim 0.65$
- Wave collision pushes entire pattern to right, small scale structures appear.

- FW propagates to either sides
- FW Collision gives rise to smaller structures,
- Major part of $v_z \leq v_{z,c} \sim 0.45$) approaches flavor depolarization,

1+1+1 D simulation: Survival probabilities



1+1+1 D simulation: Survival probabilities



– FW For $\alpha = 1$, entire νELN reaches ~ full flavor depolarization

– For $\alpha \neq 1$, only shallow part of the νELN reaches full depolarization to respect e-x lepton number conservation

$$L_{e-x} = \int dz\, dv_z\, G_\nu(v_z)\, P_3(z, v_z)$$

1+1+1 D simulation: Takeaways

1. The system evolve towards a final sate in which at least some sub spaces of ν_z achieve almost full depolarization.

2. The evolution of the flavor state respects the conservation of νELN.

3. For $\nu\mathrm{ELN} = 0$, entire ELN spectrum get depolarized. $\nu\mathrm{ELN} \neq 0$, modes in the shallower part of the ELN get depolarized.

4. Although the rate of depolarization depend on the initial perturbation, the differences in the late time behaviour is rather negligible.

A few questions:

? What happens to the final state in multi-dimension.

? What happens when collisions are involved.

? Is there a way to have information about the final steady state without going through the simulations.

1+2+2 D simulation

[In progress]

- *Assumptions:*

—The neutrino angular distributions have reflection symmetry about ϕ=0, where ϕ is the azimuthal angle, so that there is no net flux in the y direction.

—Both the initial conditions and the solutions respect the above symmetry.

—The effects of vacuum and matter terms can be neglected as did in the 1D case.

$$i(\partial_t + \sqrt{1-v_z^2}\cos(\phi)\partial_x + v_z\partial_z)\varrho(t, z, v_z, \phi) = [H_{\nu\nu}(t, z, v_z, \phi), \varrho(t, z, v_z, \phi)]$$

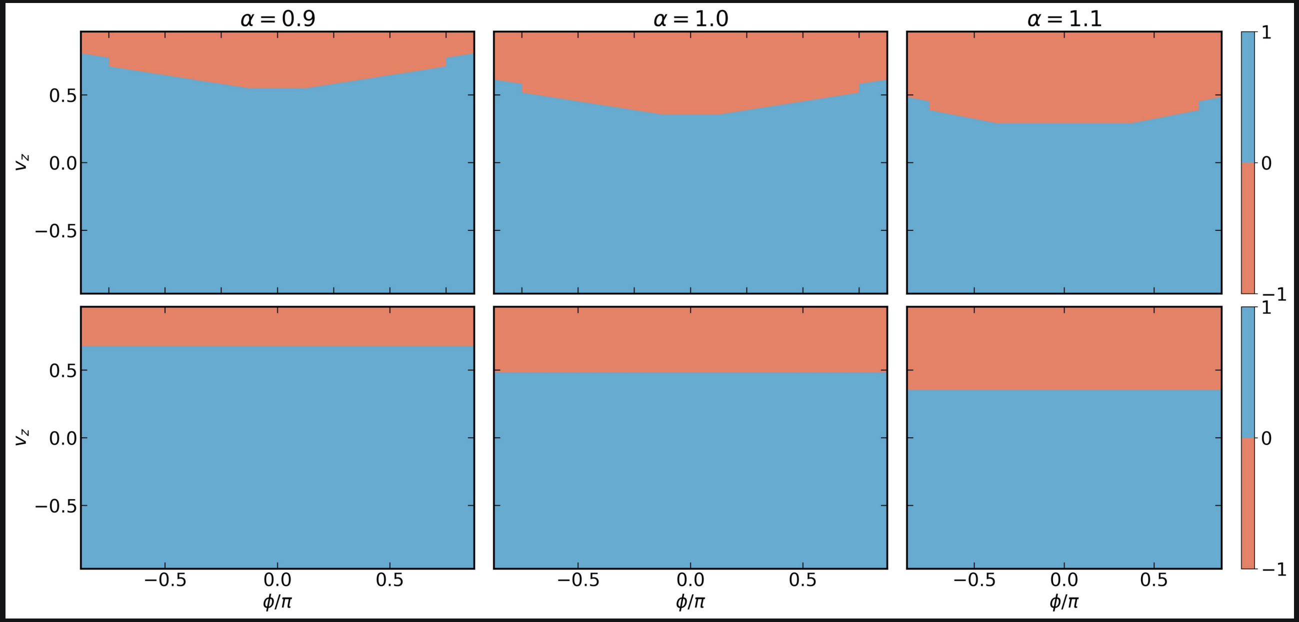
$$H_{\nu\nu} = \mu\int\frac{\mathbf{d}v_z'\mathbf{d}\phi}{2\pi}\left(1 - \sqrt{(1-v_z^2)(1-v'^2_z)}cos(\phi-\phi') - v_z v_z'\right) \times (g_{\nu_e}\varrho(t, x, z, v_z', \phi') - \alpha g_{\overline{\nu}_e}\overline{\varrho}^*(t, x, z, v_z', \phi'))$$

1+2+2D Simulation: Initial condition

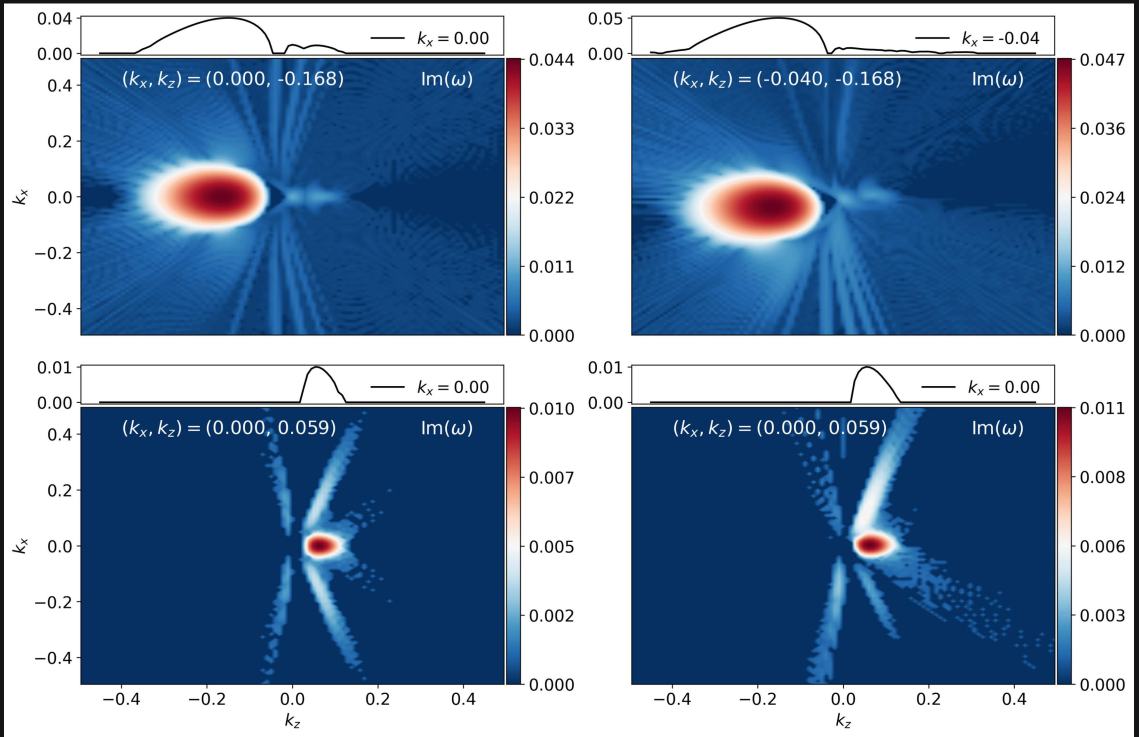
- For the 2D case we use,

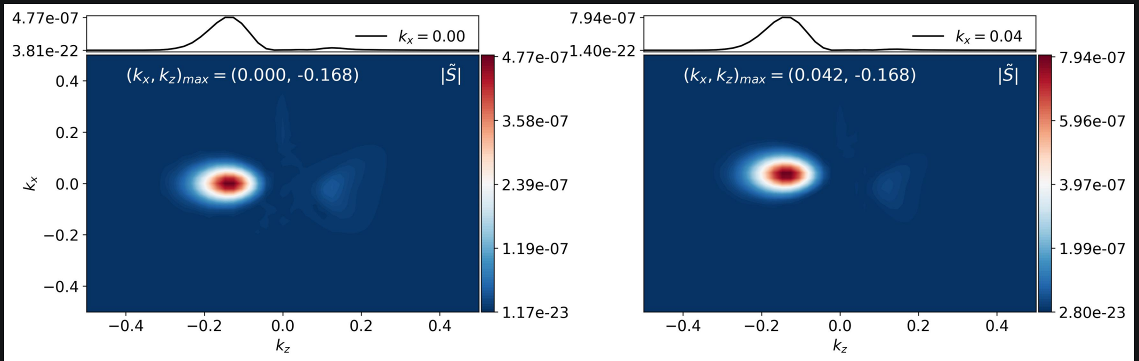
$$g_{\sigma_{\nu/\bar{\nu}}} = N\exp\left[-\left(\frac{(1-v_z)^2}{2{\sigma^z_{\nu/\bar{\nu}}}^2} + \frac{(1-v_x)^2}{2{\sigma^x_{\nu/\bar{\nu}}}^2}\right)\right]$$

	P-1	P-2
$(\sigma^x{}_\nu, \sigma^x{}_{\bar{\nu}})$	(∞, ∞)	(2.5, 2.0)
$(\sigma^z{}_\nu, \sigma^z{}_{\bar{\nu}})$	(0.6, 0.5)	(0.6, 0.5)



2D Simulation: Comparison with linear analysis

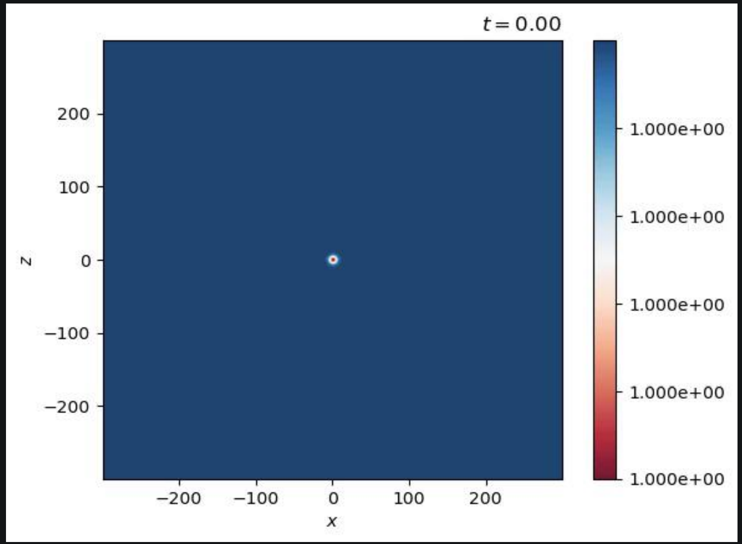




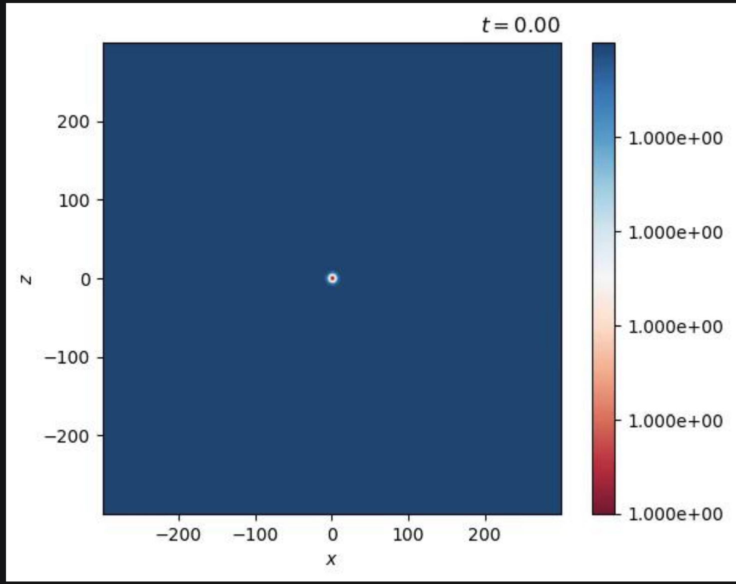
- Im(ω) depends on both k_x and k_z, max[Im(ω)] occurs at $k_x = 0$ for azimuthally symmetric profile
- The dispersion branch with max[Im(ω)] is identical to that in the 1D case.
- Location of the maximum Fourier power from the simulation is same as the location of for max[Im(ω)] from the linear analysis

1+2+2D Simulation: Results

$v_z = 0.97, \ \phi = -157.5$

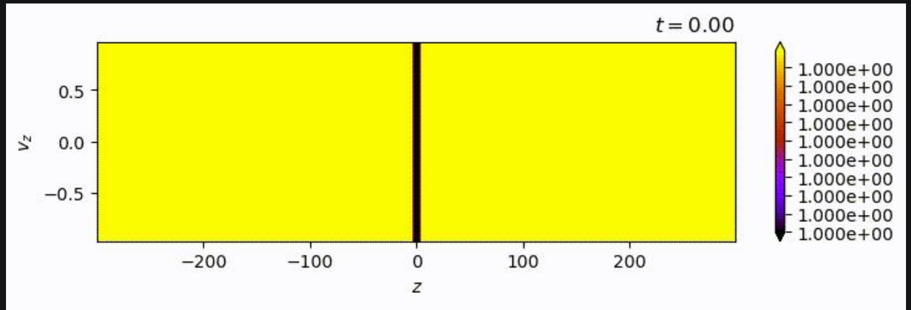


$P - 1$

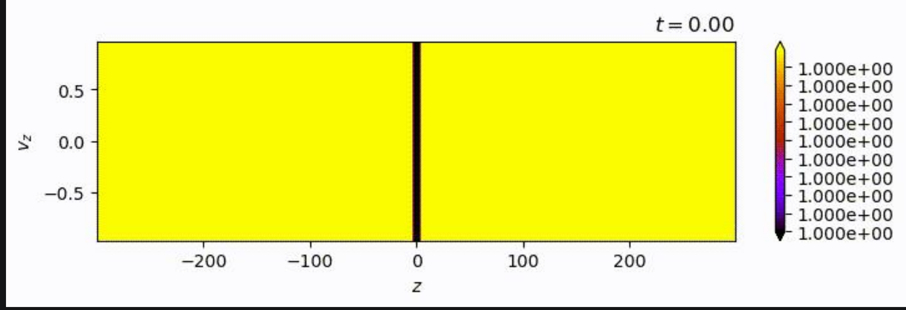


$P - 2$

$\alpha = 0.9$

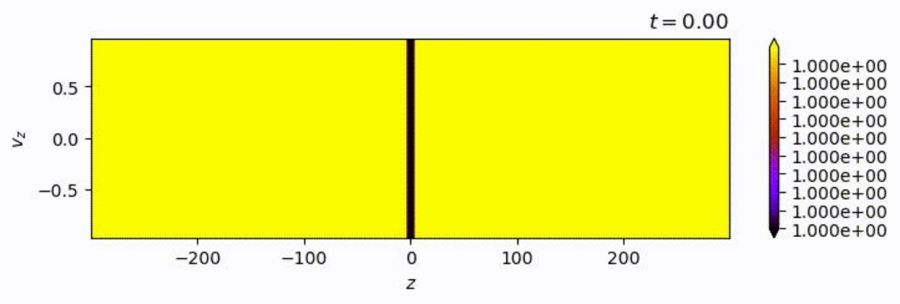


$P - 1$

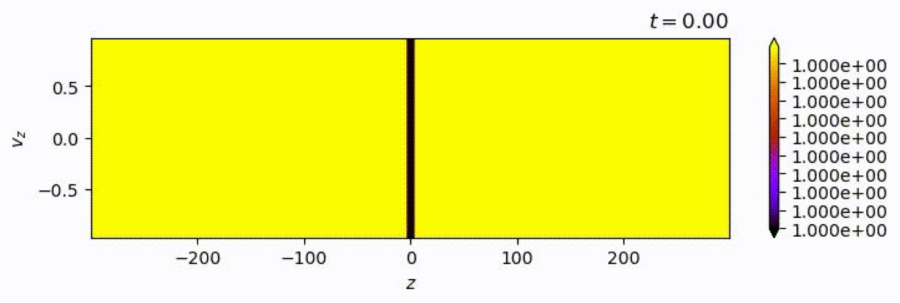


$P - 2$

$\alpha = 1.0$



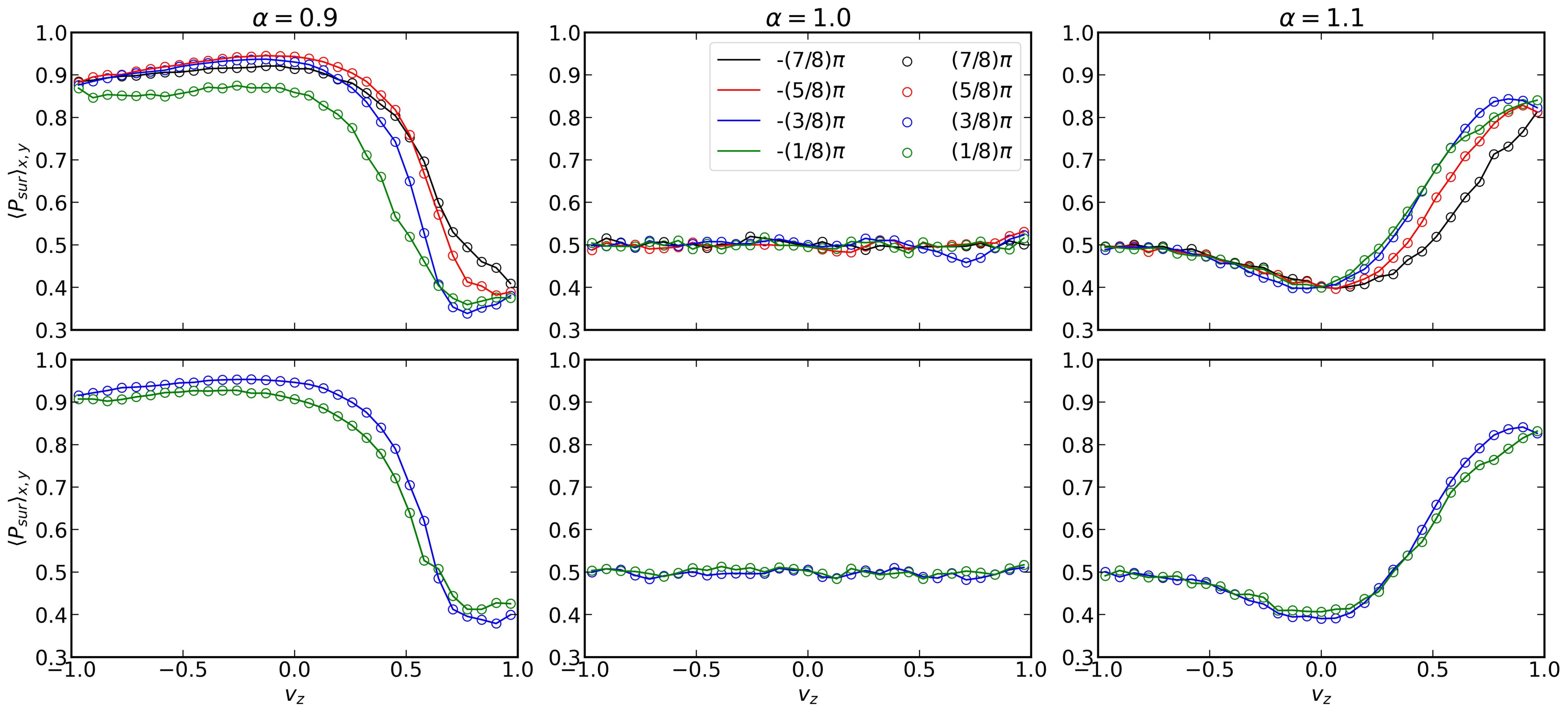
$P - 1$



$P - 2$

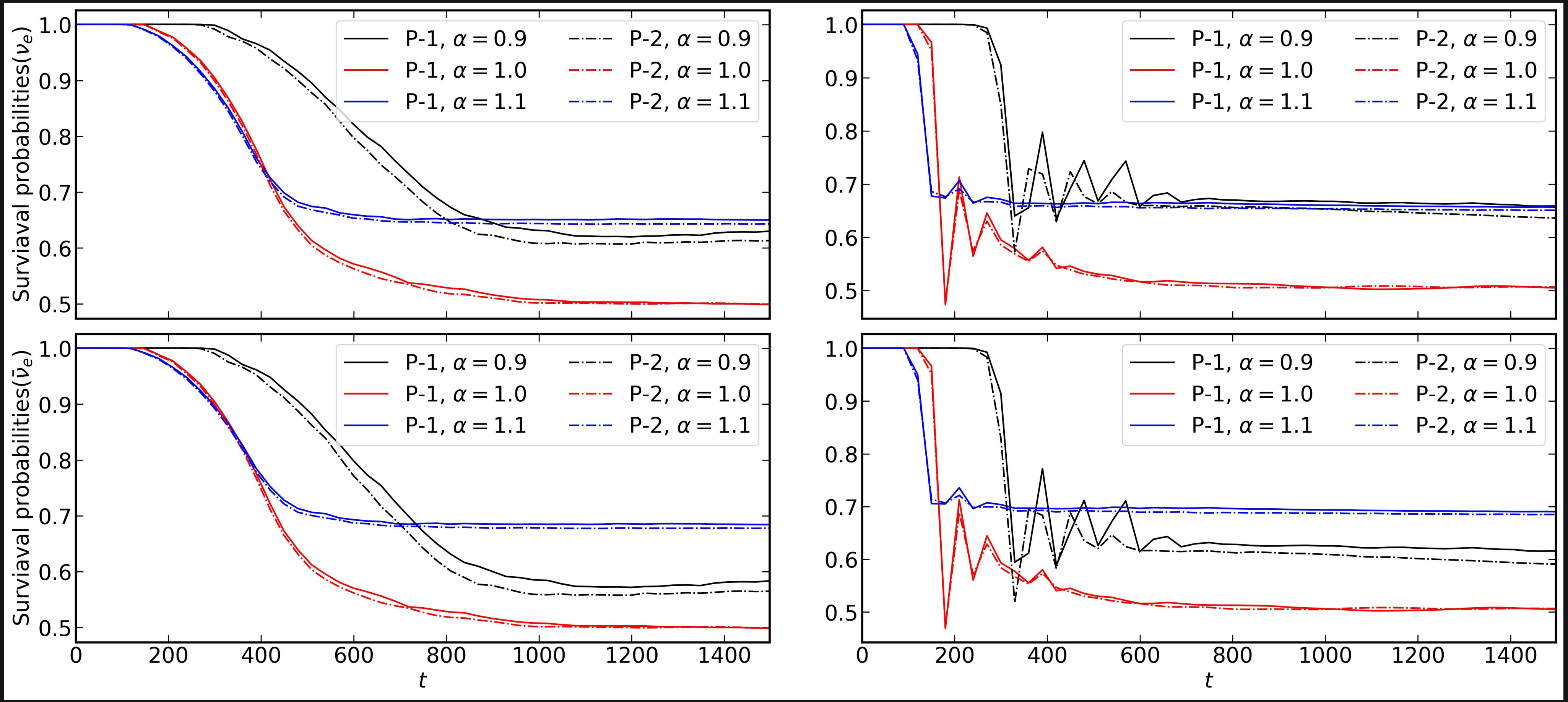
2D Simulation: Results

- Final angular distribution



2D Simulation: Results

- Evolution of the survival probabilities



1+2+2 D simulation: Takeaways

1. Although there are some subtle differences, the the outcome of the long term evolution in 1+2+2 dimensions qualitatively agrees with the findings from the simulation in 1+1+1 dimensions. .

2. Different evolution history and final state of different ‘ϕ -modes' for a given v_z even when the ELN spectrum has azimuthal symmetry indicate azimuthal symmetry breaking.

A few questions:

? What happens to the final state in multi-dimension.

? Effect of relaxation of reflection symmetry in ELN.

? Impact of collisions.

Conclusions

- The system evolve towards a quasi-steady state.
- Neutrinos at least in some regions of the angular spectrum reach nearly complete depolarization respecting νELN conservation.
- Flavor wave interactions can result in the speed up of flavor depolarization and production of small scale structures.
- Inferences made in the 1+1+1D are qualitatively valid in 1+2+2D as well.
- Presence of the symmetry breaking modes can alter the flavor evolution history of different \`ϕ-modes', resulting in ϕ-depependent final ELN distribution.

THANK YOU