Parton Distributions on a Lattice

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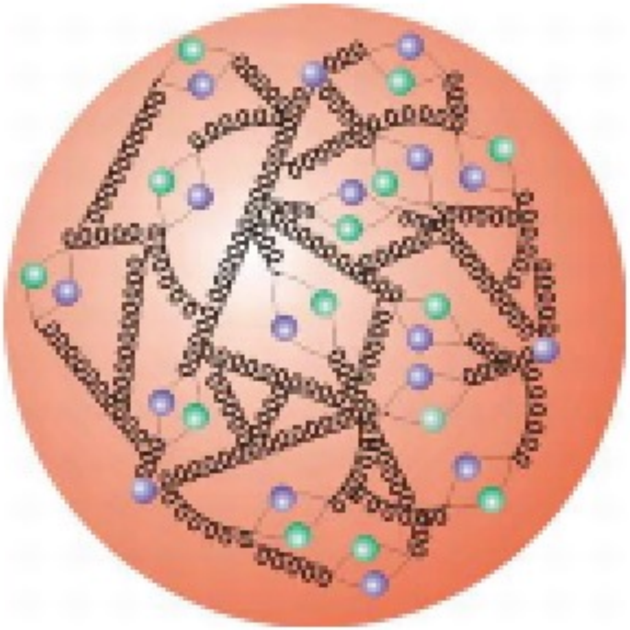
The complicated world inside a proton



Parton structures: 1d mom+spin PDF to 3d GPD & TMD to Wigner (and beyond?) [BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, …] to applications (Higgs, new physics…)

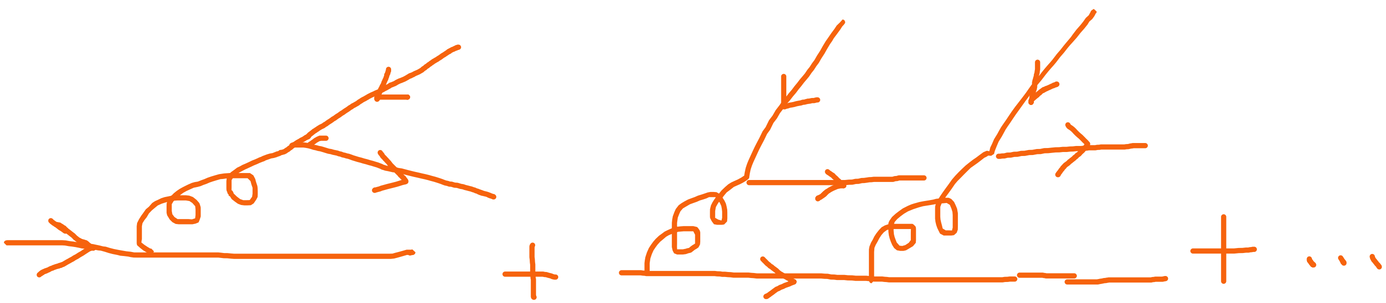
Can we determine these distributions theoretically?

PDFs from QCD---a light cone problem!

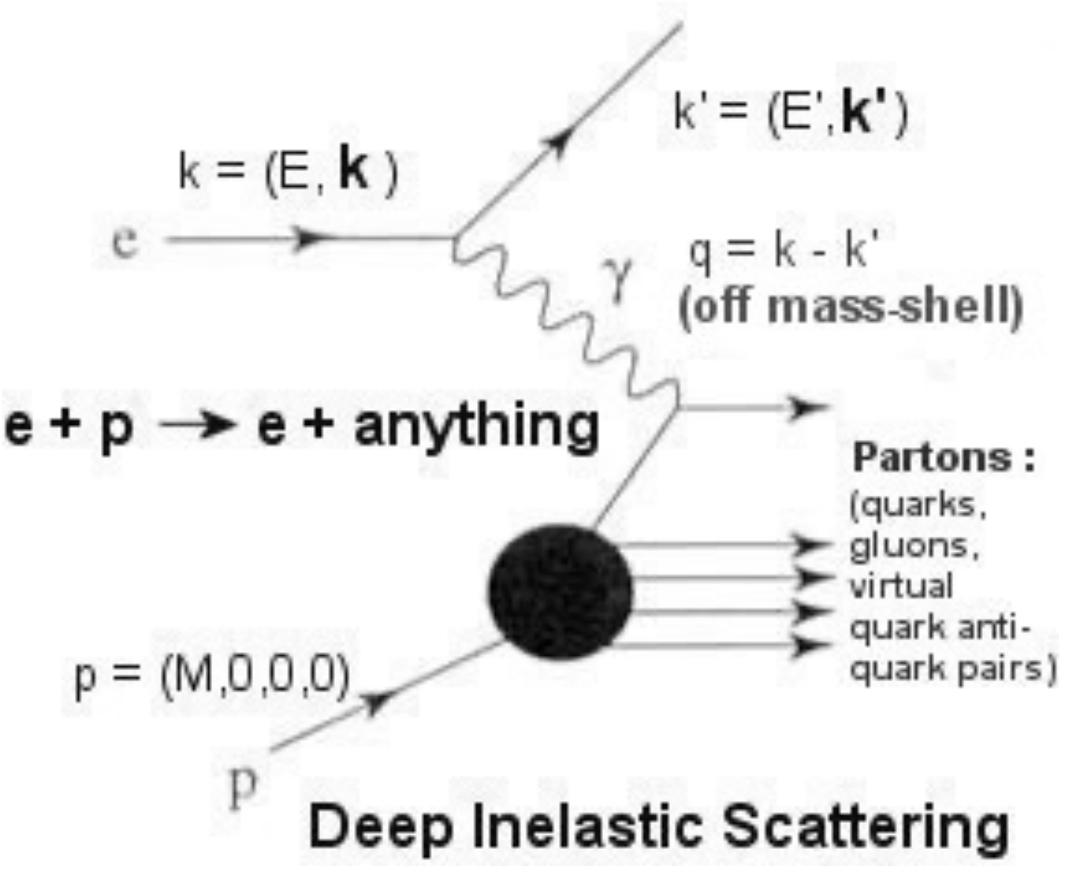
- The number of quark anti-quark pairs diverges (manifestation of non-perturbative nature of the problem): an infinite body problem!
- Lattice QCD
- Euclidean lattice: light cone operators cannot be distinguished from local operators

$$t^2 - \mathbf{r}^2 = 0$$

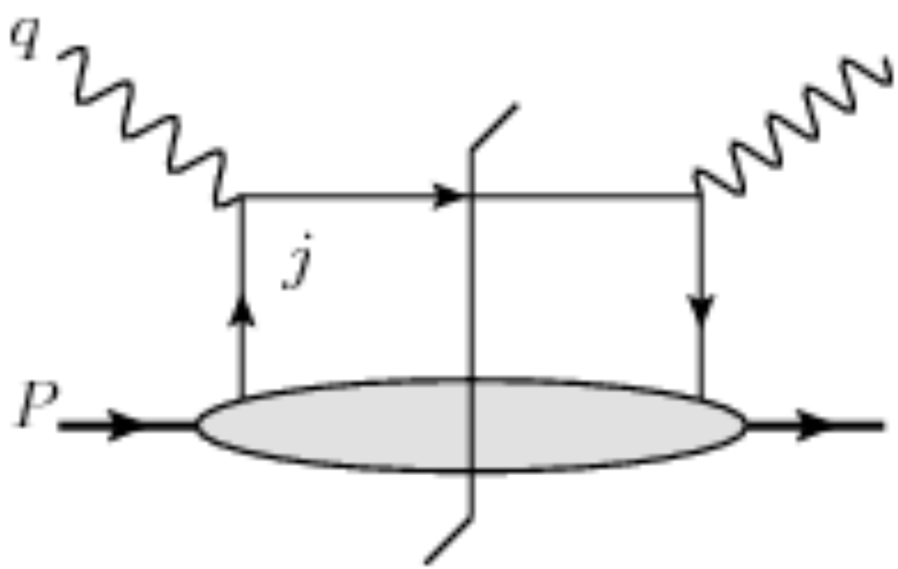
$$-t_E^2 - \mathbf{r}^2 = 0$$



Measuring Parton Distributions Using DIS experiments



Parton Distribution Function (PDF) in QCD



The struck parton moves on a light cone at the leading order in the twist-expansion.

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \langle P \,|\, \overline{\psi}(0)\lambda \cdot \gamma \Gamma \psi(\xi^- \lambda) \,|\, P \rangle$$

PDFs from QCD---a light cone problem!

- Euclidean lattice: light cone operators cannot be distinguished from local operators
- Moments of PDF given by local twist-2 operators (twist = dim - spin); limited to first few moments but carried out successfully

$$\langle x^n \rangle$$

Beyond the first few moments

- Smeared sources: Davoudi & Savage
- Gradient flow: Monahan & Orginos
- Current-current correlators: K.-F. Liu & S.-J. Dong; Braun & Müller; Detmold & Lin; QCDSF; Qiu & Ma
- Xiangdong Ji (Phys. Rev. Lett. 110 (2013) 262002): quasi-PDF: computing the x-dependence directly. (variation: pseudo-PDF, Radyushkin; w/ Karpie, Orginos, Zafeiropoulos)

Ji's idea

- Quark PDF in a proton: $(\lambda^2 = 0)$

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \langle P | \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda) | P \rangle$$

- Boost invariant in the z-direction, rest frame OK
- Quark bilinear op. always on the light cone
- What if the quark bilinear is slightly away from the light cone (space-like) in the proton rest frame?

- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.

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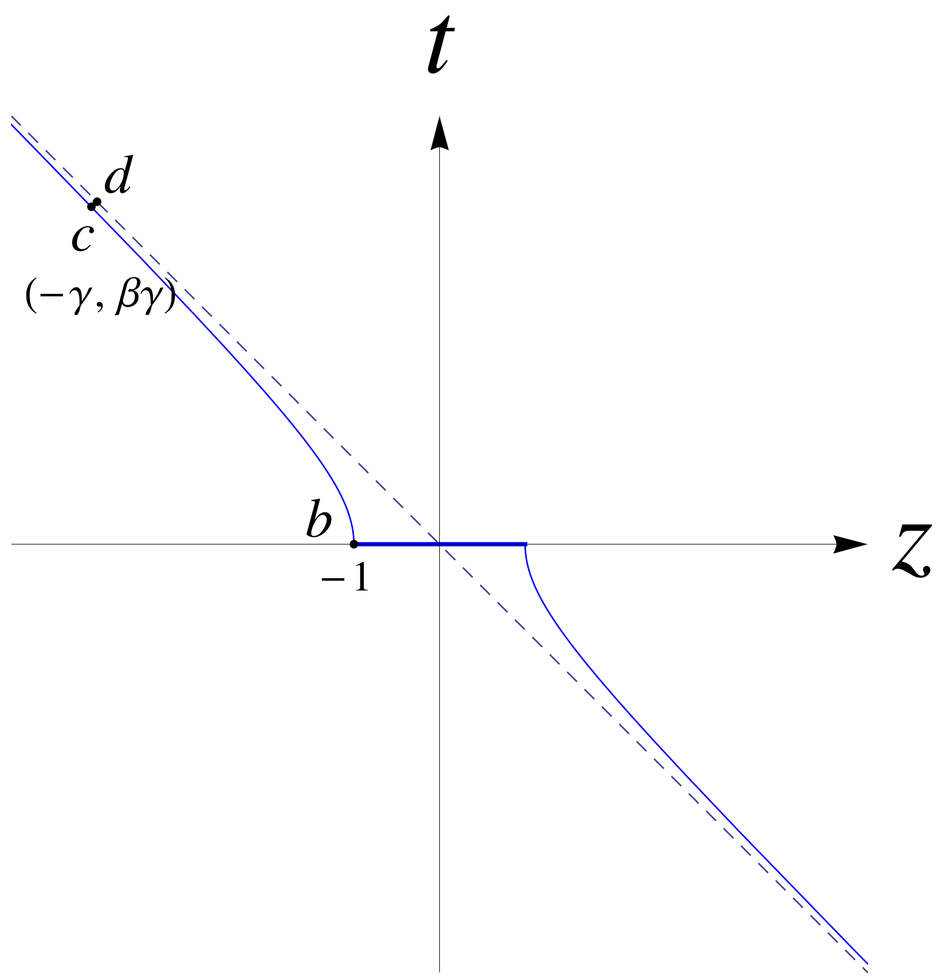

- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.
- Analogous to HQET: need power corrections & matching---LaMET (Large Momentum Effective Theory)

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- Analogous to HQET: need power corrections & matching---LaMET (Large Momentum Effective Theory)

$$\tilde{q}(x,\Lambda,P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\Lambda}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2},\frac{M^2}{P_z^2}\right) + \ldots.$$

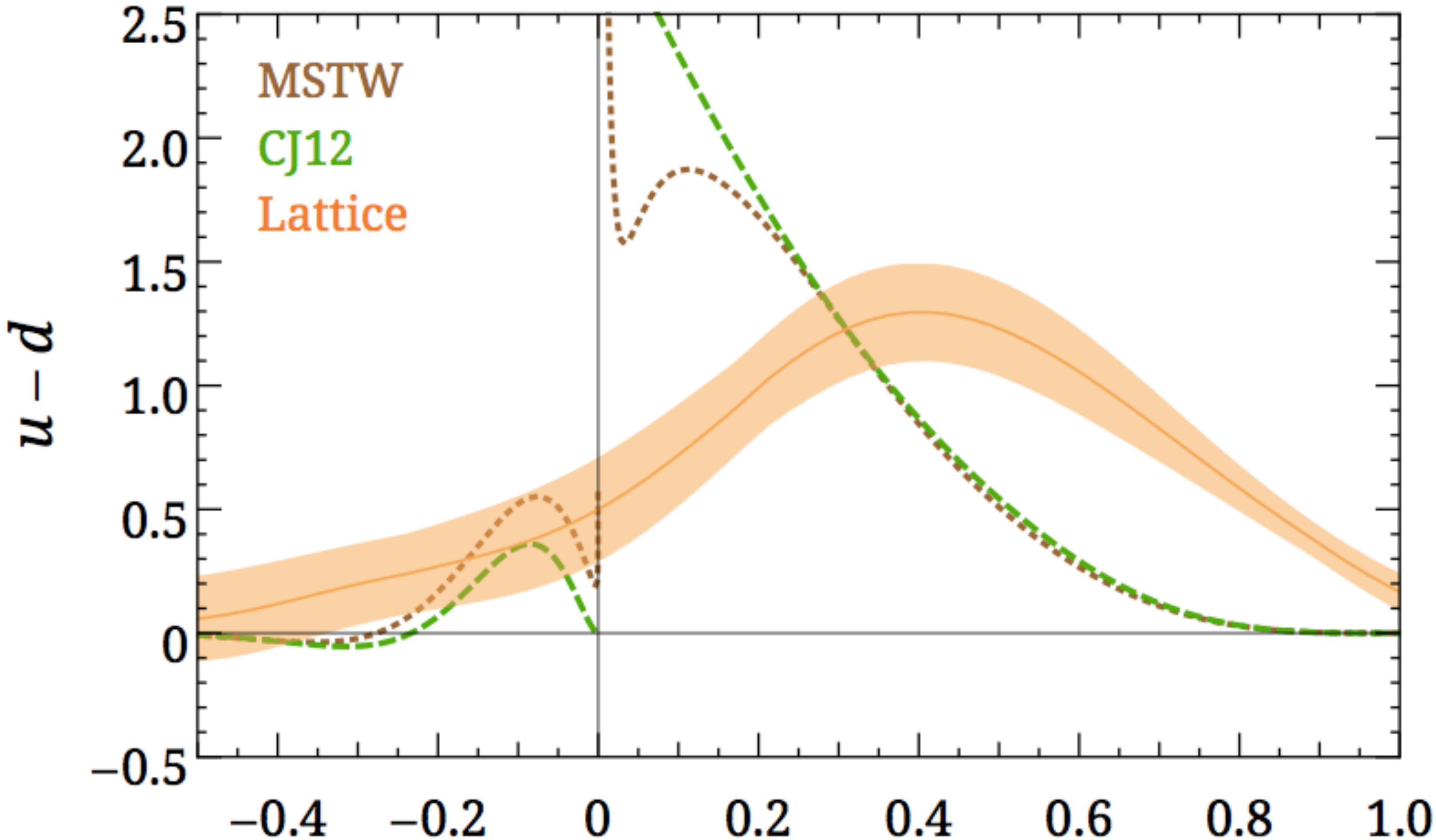
Matching

$$\tilde{q}(x,\Lambda,P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\Lambda}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2},\frac{M^2}{P_z^2}\right) + \ldots$$

Xiong, Ji, Zhang, Zhao (GPD: Ji, Schafer, Xiong, Zhang; Xiong, Zhang) Factorization (Ma, Qiu; Li; OPE: Izubuchi, Ji, Jin, Stewart, Zhao), Linear divergence, LPT (Ishikawa, Ma, Qiu, Yoshida; JWC, Ji, Zhang; Xiong, Luu, Meissner; Rossi, Testa; Constantinou et al.) Multiplictive Renormalizability (Ji, Zhang, Zhao; Ishikawa, Ma, Qiu, Yoshida; Green, Jansen, Steffens; Zhang, Ji, Schäfer, Wang, Zhao; Li, Ma, Qiu), RI (Monahan & Orginos; Yong & Stewart; Constantinou et al.; LP3), NPR(Constantinou et al.; LP3), E vs. M spaces (Carlson et al.; Briceno et al.), Renormalon (Braun, Vladimirov, Zhang; Liu, Chen), Modeling (Xing et al.,…),...

LaMET 1.0

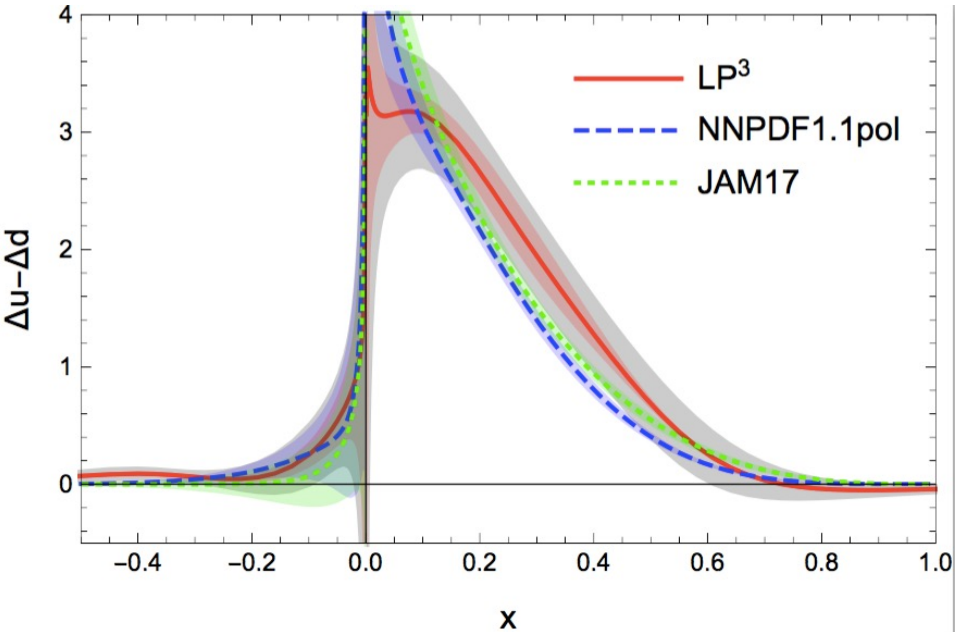
LP3 (1402.1462)

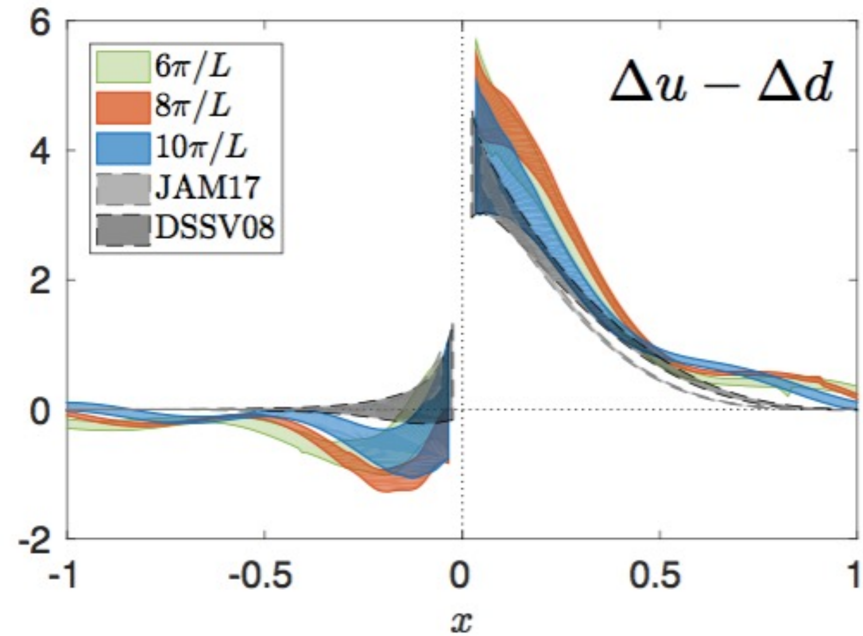


$q(-x) = -\bar{q}(x)$

LaMET 2.0

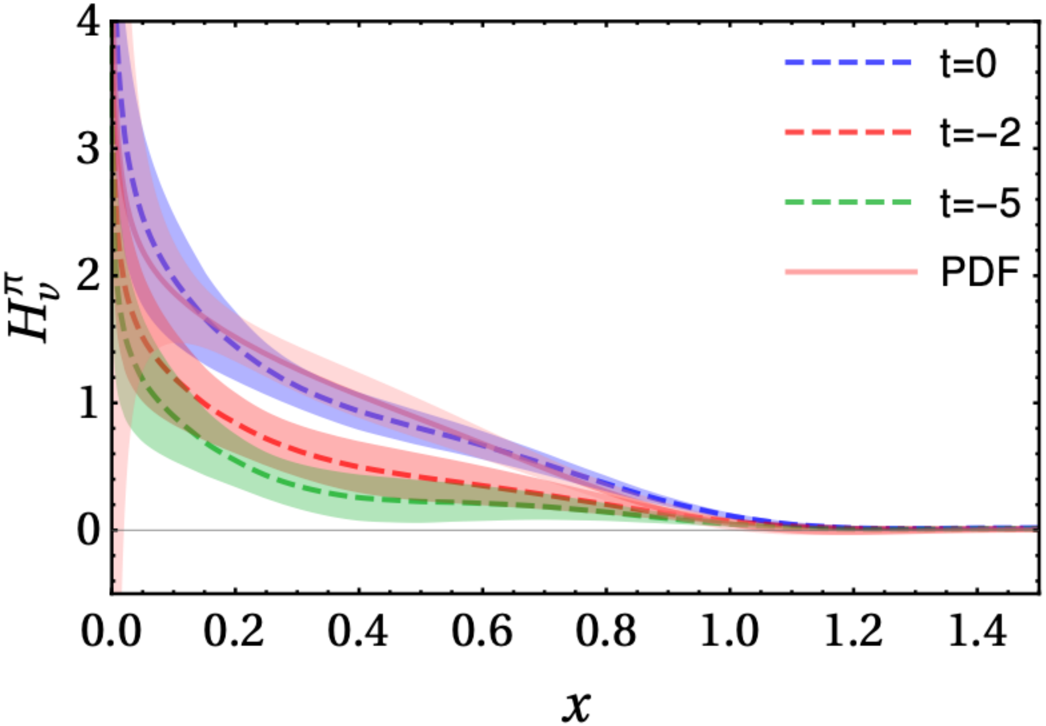
Compared with ETMC





LP3(1807.07431,PRL) ETMC(1803.02685,PRL)

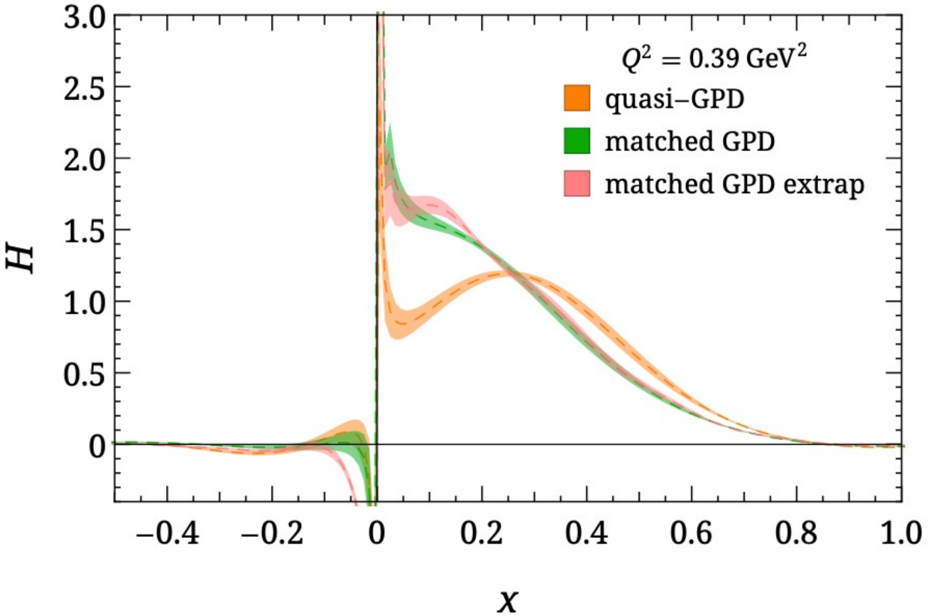
Pion Skewless GPD

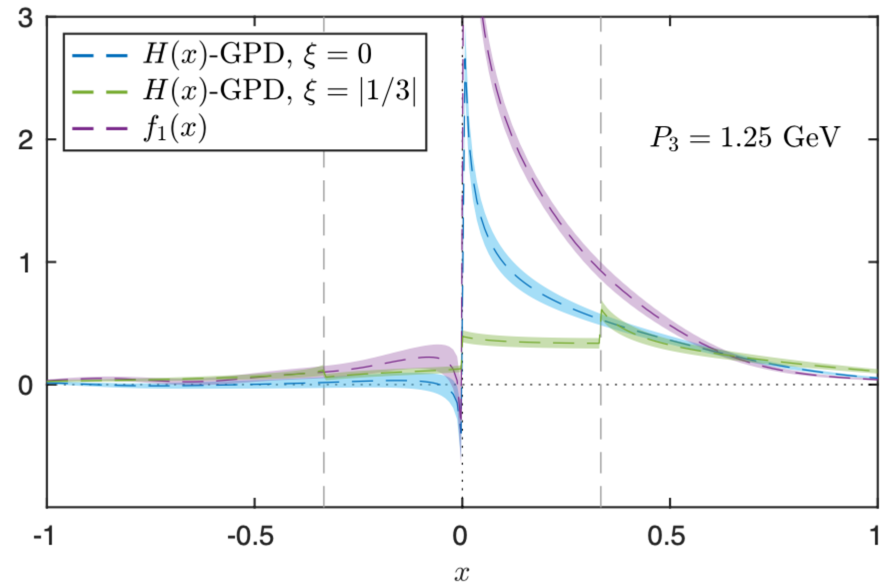


JWC, HW Lin, JH Zhang (1904.12376)

Nucleon GPD at Physical Pion Mass

HW Lin (2008.12474, PRL)





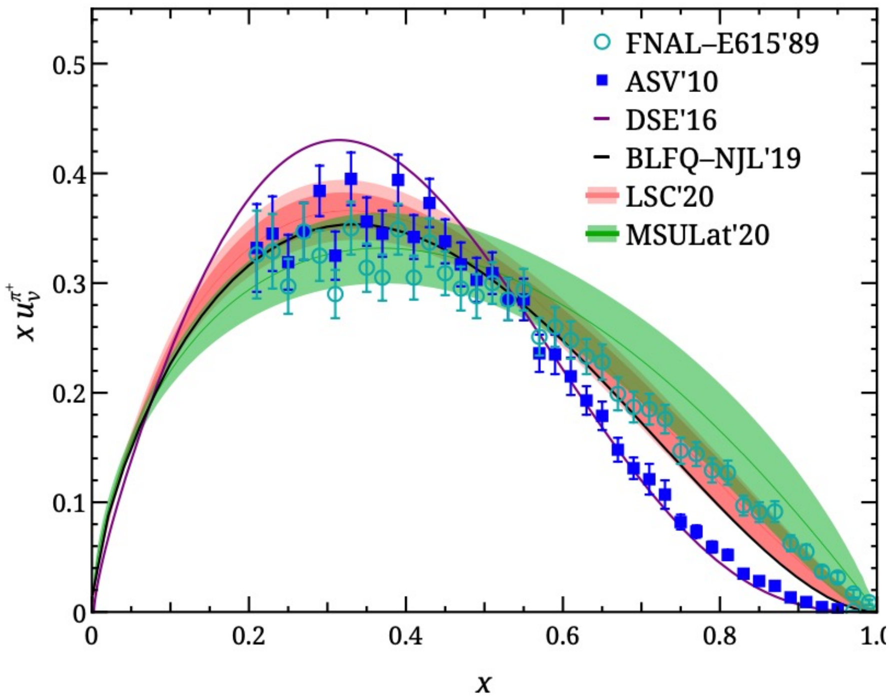
HW Lin (2008.12474, PRL)

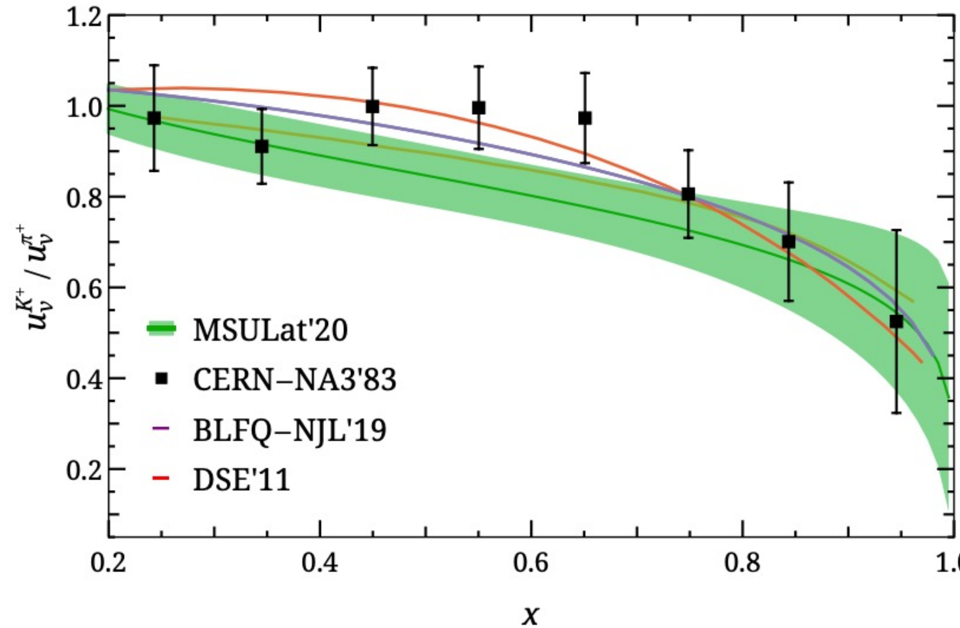
M_pi =140 MeV

ETMC (2008.10573, PRL)

M_pi = 260 MeV

Meson Valence Quark Distributions





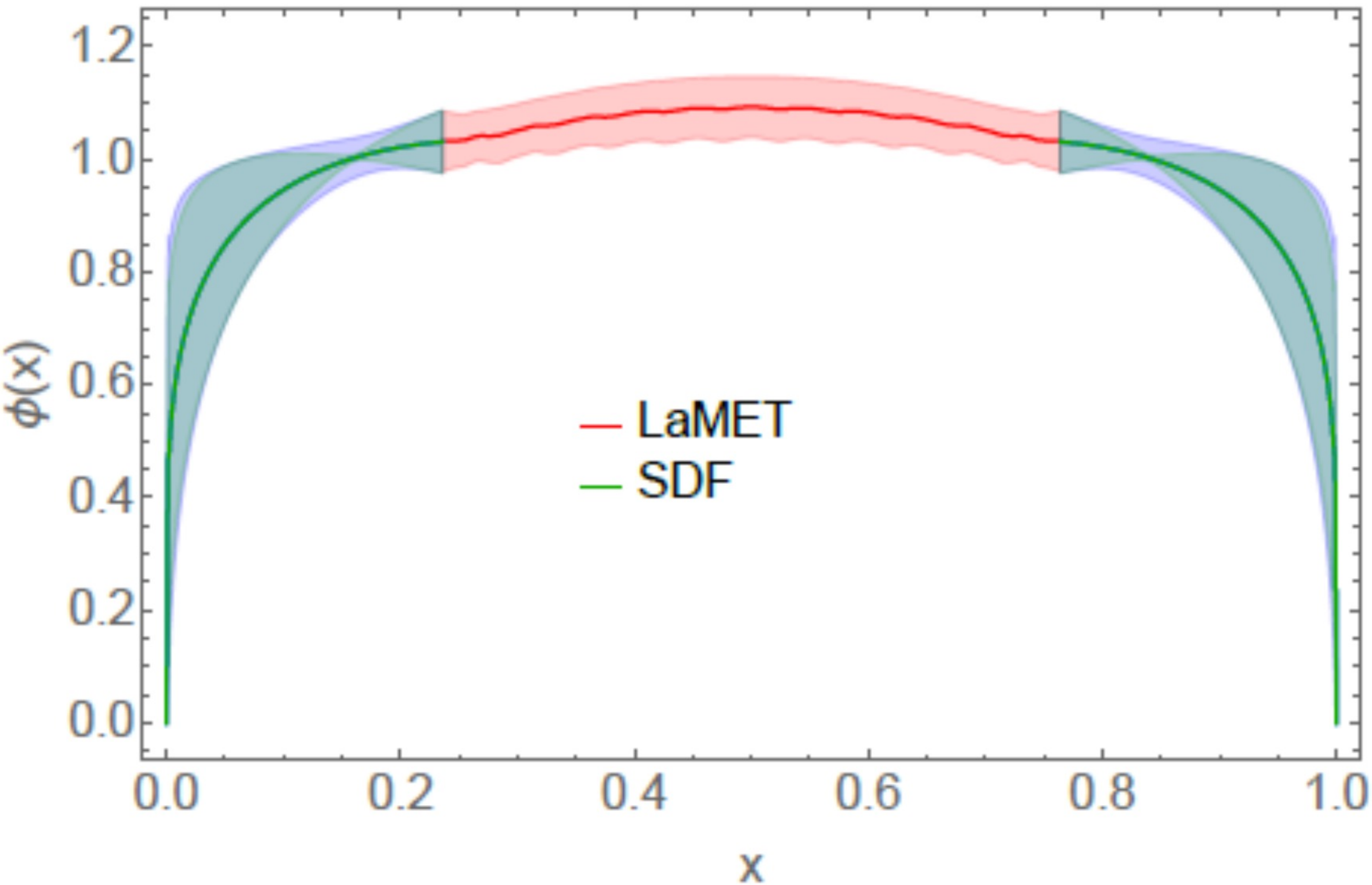
HW Lin, JWC, Z Fan, JH Zhang, R Zhang (2003.14128)

LaMET 3.0

Precision Control

- Hybrid Renormalization
- Resummation near x = 0 & 1 (Y. Su et al, 2209.01236)
- Leading UV renormalon (R. Zhang et al)
- Complementarity (X. Ji, 2209.09332): LaMET for intermediate x, then phenomenological forms near x = 0 & 1 w/ parameters determined by moments.

Pion Distribution Amplitude



R. Zhang et al

Outlook

- Rapid progress made since 2013
- Further error study (non-singlet)

  Know whether it works within 3 years (~20%)?
- Singlet PDF's: s, c, b and gluons

  Additional 3-5 yrs?
- If it works, complimentary to exp.: PDF (sea asymmetry, small and large x's, non-valence partons), DA, GPD, TMD, Wigner distributions …

Backup slides

First (isovector) LPDF Computation

- Lattice: $24^3 \times 64$

  $a \approx 0.12 \text{ fm} \quad L \approx 3 \text{ fm}$
- Fermions: MILC highly improved staggered quarks (HISQ) Clover (valence)

  $N_f = 2+1+1 \qquad M_\pi \approx 310 \text{ MeV}$
- Gauge fields/links: hypercubic (HYP) smearing, 461 config.

- $P^z = \frac{2\pi}{L} n = n \times 0.43 \; GeV$ $\qquad$ n = 1,2,3…

Lattice Setup (isovector proton PDF)

- Lattice: $64^3 \times 96$

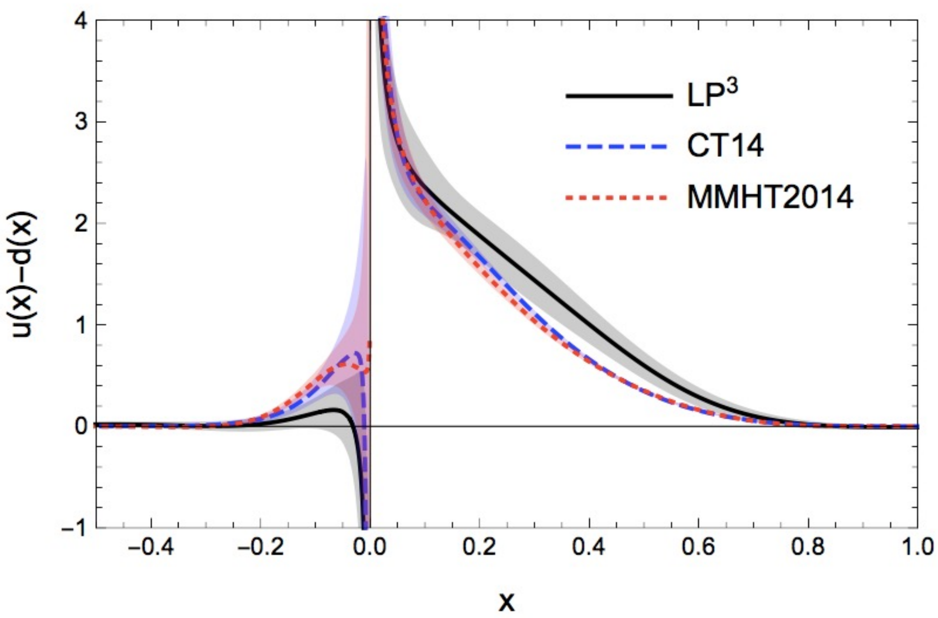
  $a = 0.09 \text{ fm} \qquad L \approx 5.8 \text{ fm}$
- Fermions: MILC highly improved staggered quarks (HISQ) Clover (valence)

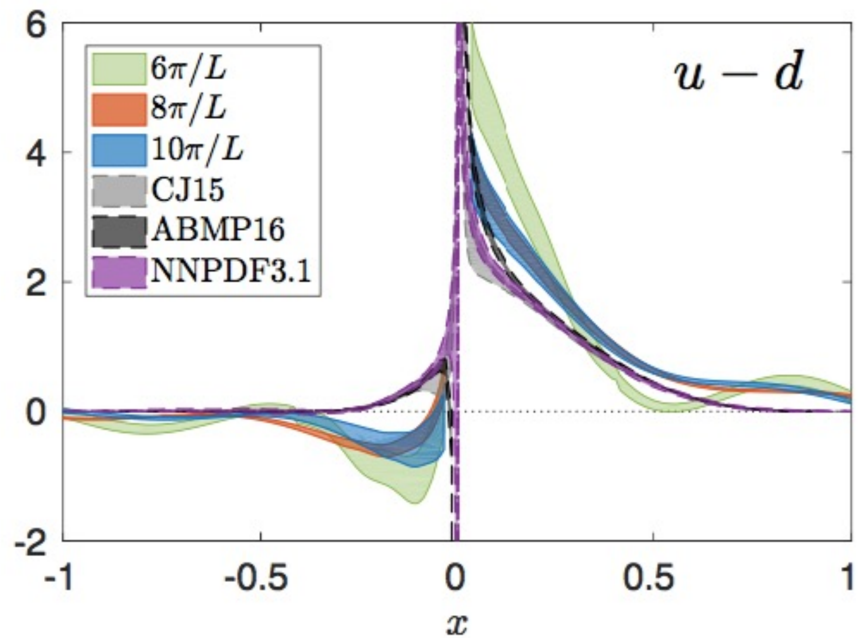
  $N_f = 2 + 1 + 1 \qquad M_\pi \approx 135 \text{ MeV}$
- Gauge fields/links: hypercubic (HYP) smearing (one step), 884 config.

- $P^z = n\frac{2\pi}{L}$ = 2.2, 2.4, 3.0 GeV (n = 10,12,14)

(high momentum smearing: Bali, Lang, Musch, Schafer)

Compared with ETMC

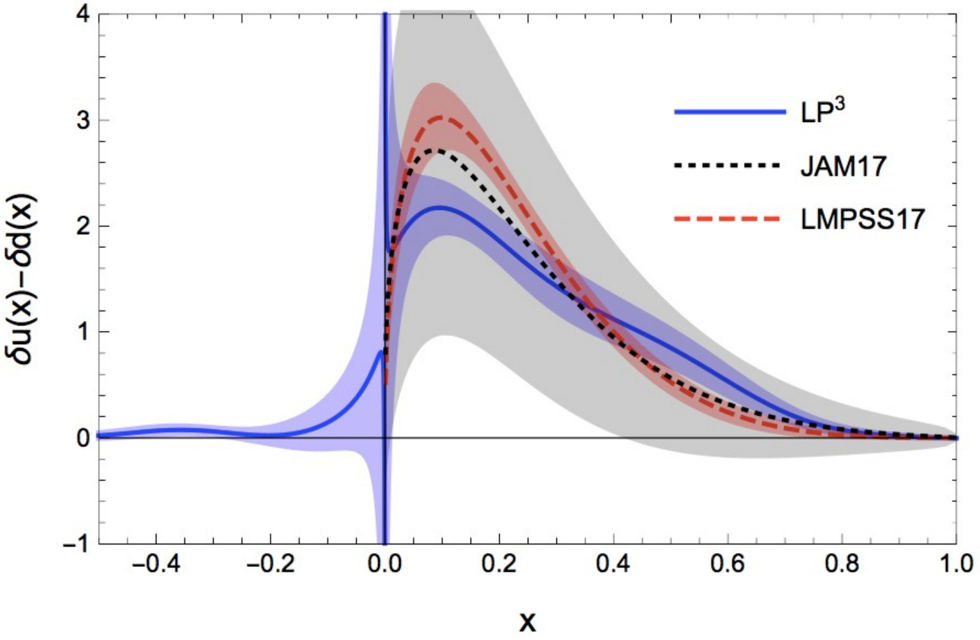


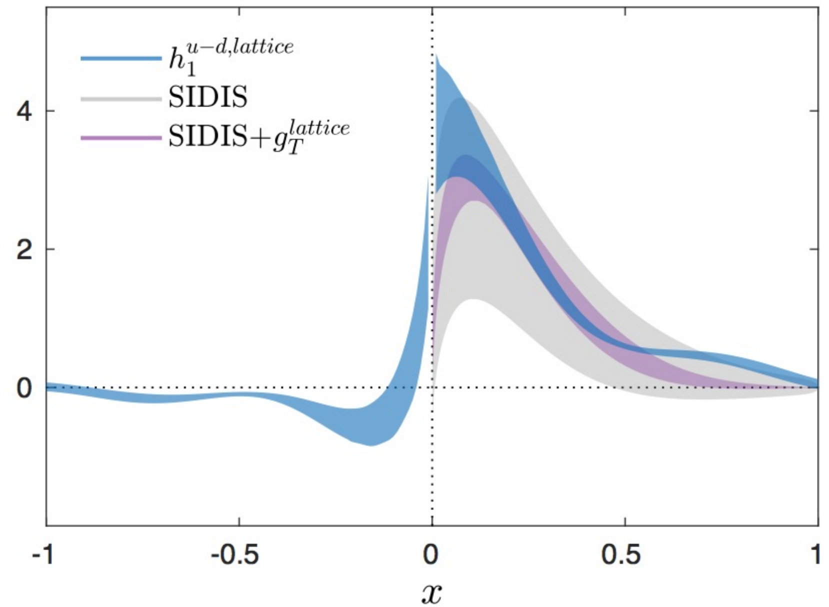


LP3(1803.04393)

ETMC(1803.02685)

Compared with ETMC





LP3 (1810.05043) ETMC(1803.02685)

Review: Ji's LPDF (LaMET)

$$\widetilde{q}(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{-ixzP^z} \langle P | \overline{\psi}(0) \lambda \cdot \gamma \Gamma \psi(z\lambda) | P \rangle$$
$$\equiv \int \frac{dz}{2\pi} e^{-ixzP^z} h(zP^z) P^z$$

$$\lambda^\mu = (0,0,0,1)$$

- Taylor expansion yields

$$\overline{\psi} \lambda \cdot \gamma \Gamma (\lambda \cdot D)^n \psi = \lambda_{\mu_1} \lambda_{\mu_2} \cdots \lambda_{\mu_n} O^{\mu_1 \cdots \mu_n}$$

op. symmetric but not traceless

$$(\lambda_{\mu_1} \lambda_{\mu_2} - g_{\mu_1 \mu_2} \lambda^2 / 4)$$

Review: Ji's LPDF (LaMET)

$$\left\langle P \left| O^{(\mu_1 \cdots \mu_n)} \right| P \right\rangle = 2a_n P^{(\mu_1} \cdots P^{\mu_n)}$$

- LHS: trace, twist-4 $\mathcal{O}(\Lambda_{QCD}^2/(P^z)^2)$ corrections, parametrized in this work
- RHS: trace $\mathcal{O}(M^2/(P^z)^2)$
- One loop matching $\alpha_s \ln P^z$, OPE

$$\tilde{q}(x, \Lambda, P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}, \frac{\Lambda}{P_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{P_z^2}, \frac{M^2}{P_z^2}\right) + \ldots .$$