Consistency Check of ANSYS and COMSOL simulations in the bulk deformation of ETM induced by Photon Calibrator

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Special thanks to
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LIGO Scientific Collaboration

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Outline

- Purpose of this study
- Crosscheck of internal resonance modes
- Crosscheck of displacement ratio of deformation
- Future studies

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Introduction

- Observation-4 of gravitational wave search has started at May 24th.
- Current sensitivity in LIGO is 130 Mpc at Hanford, 150 Mpc at Livingston.

(https://ldas-jobs.ligo-wa.caltech.edu/~detchar/summary/day/20230519/)

(https://ldas-jobs.ligo-la.caltech.edu/~detchar/summary/day/20230519/)

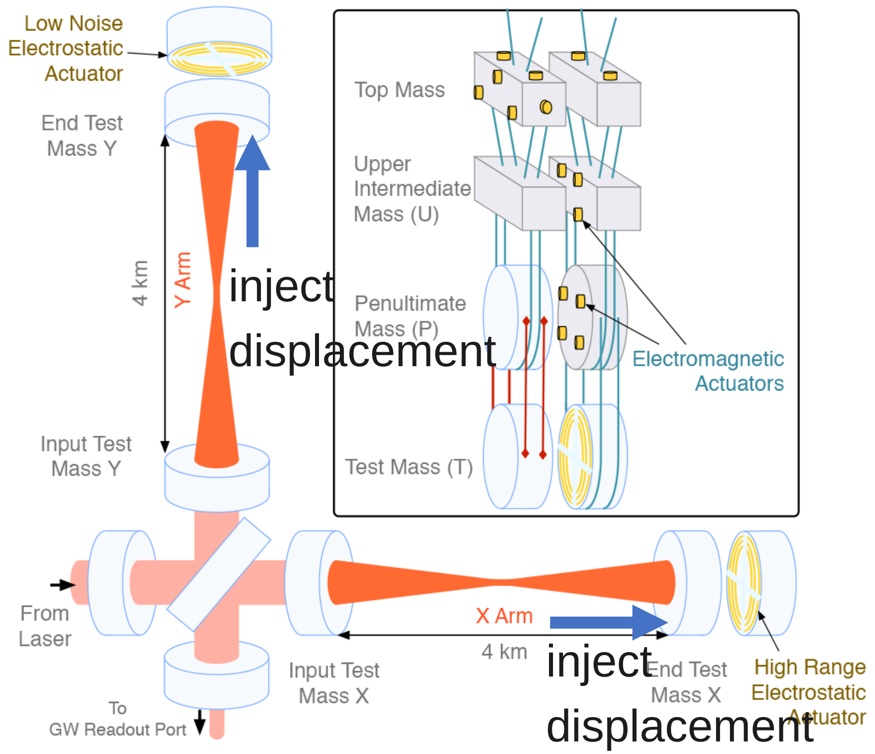
- We are in calibration group of LIGO scientific collaboration.
- When we detect a gravitational event, we have to provide its systematic error.
- We have calibration instruments. We have to mitigate its uncertainty.
- Bulk deformation of an end test mass (ETM) caused by a calibration instrument induces error in high frequency signal.

Calibration of interferometer

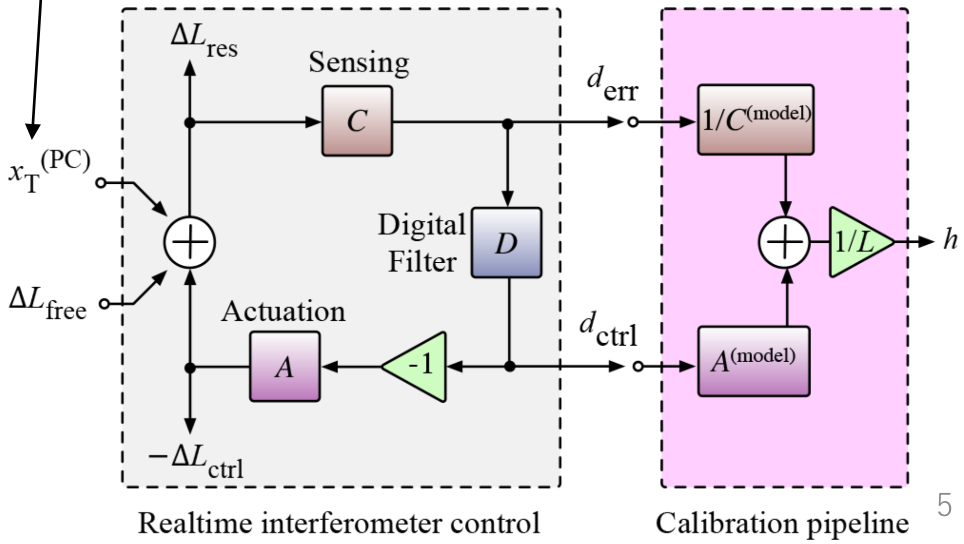
We have to calibrate response of the interferometer to reconstruct spacetime fluctuation from photosensor voltage.

Sensor signal → Mirror displacement

Inject known actuation force



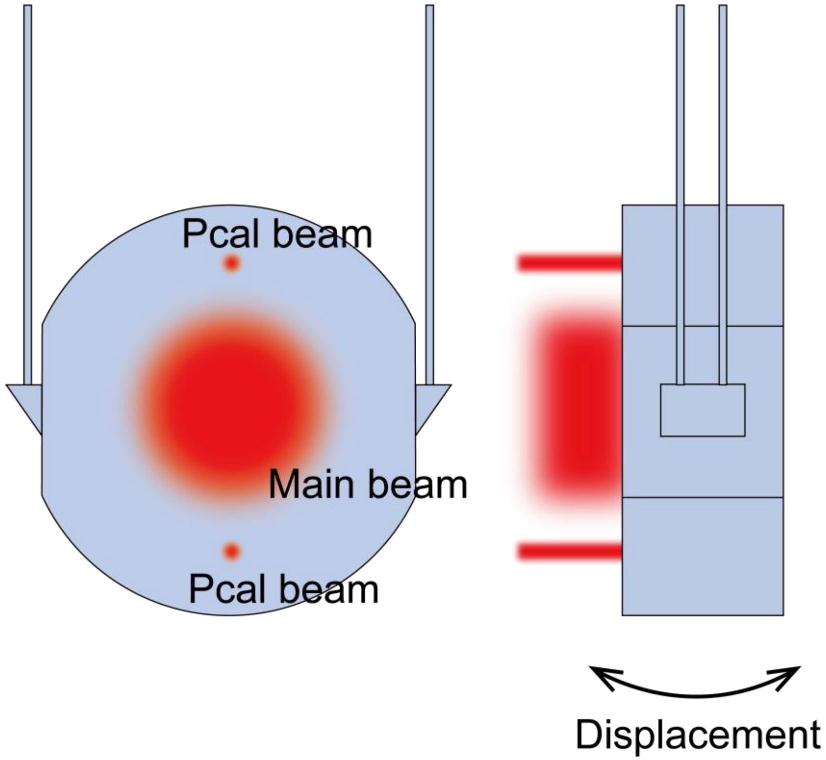
Then we can know sensing (C) and actuation (A) function.



Photon calibrator

One of our calibration instruments is Photon calibrator (Pcal).

We actuate an ETM by tiny photon pressure of laser.



Uncertainty of Pcal is 0.28% in LIGO.
(https://dcc.ligo.org/LIGO-G2300462)

Error from **deformation** is no longer negligible.

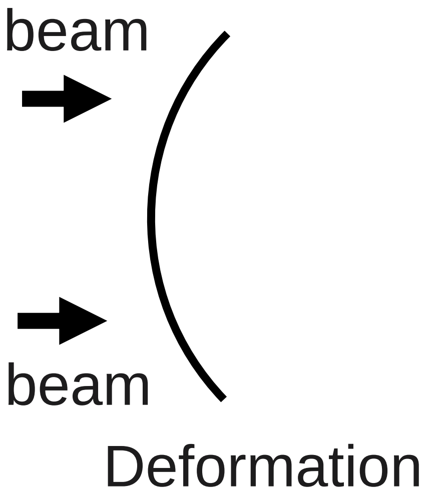
$$\underbrace{\vec{x}}_{\text{Mirror displacement}} = -\underbrace{\frac{2P\cos\theta}{c}}_{\text{Power-to-force const.}}\underbrace{\frac{1}{M\omega^2}}_{\text{Transfer function of suspension}}\left\{1+\underbrace{\frac{M}{I}\left(\vec{a}\cdot\vec{b}\right)}_{\text{Rotation}}\right\}\underbrace{\mathcal{G}(f)}_{\textbf{Deformation}}$$

Photon Calibrator and bulk deformation

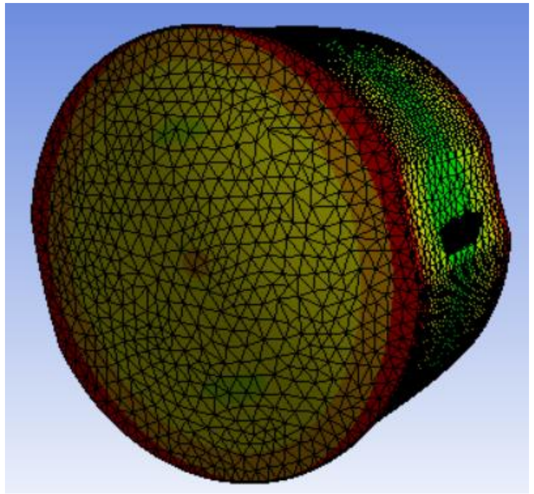
Beams of Pcal cause bulk deformation of the ETM.

It induces error of sub-%-order in calibration.

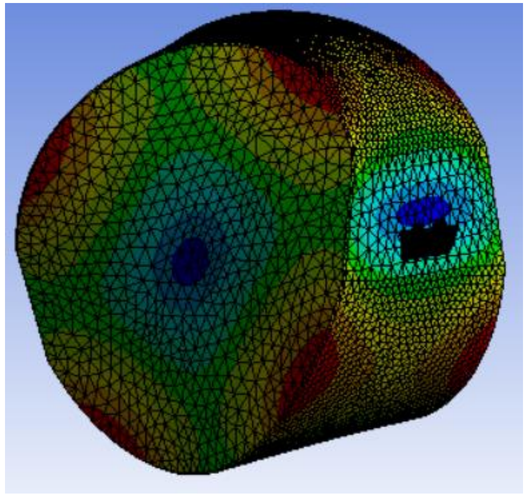
It rises in the kHz region, which is important for burst and neutron star EoS.



Drumhead mode



Butterfly mode



Estimation of bulk deformation

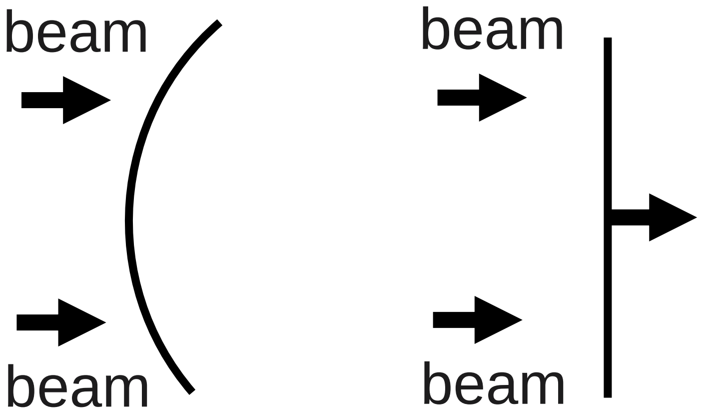
An index of bulk deformation is **displacement ratio** from the rigid mass motion $(1/M\omega^2)$.

We should estimate it by simulation.

$$\mathcal{G}_{(a_i,b)}(f) = \left[1 + \frac{\sum_m x_m(f)}{x_{\mathrm{rigid}}(f)}\right] = \left[\frac{\mathrm{x}_{\mathrm{total}}}{\mathrm{x}_{\mathrm{rigid}}}\right]_{\mathrm{FEA}}$$

$$\mathrm{x}_{\mathrm{total}}(f) = \underbrace{k_I}_{\text{Normalization by main beam power}} \int_\Omega \underbrace{\mathrm{w}(x,y;f)}_{\text{Surface displacement}} \cdot \underbrace{I(x,y,z)}_{\text{Main beam profile}} dxdy$$

Previous studies used ANSYS and COMSOL.



Deformation

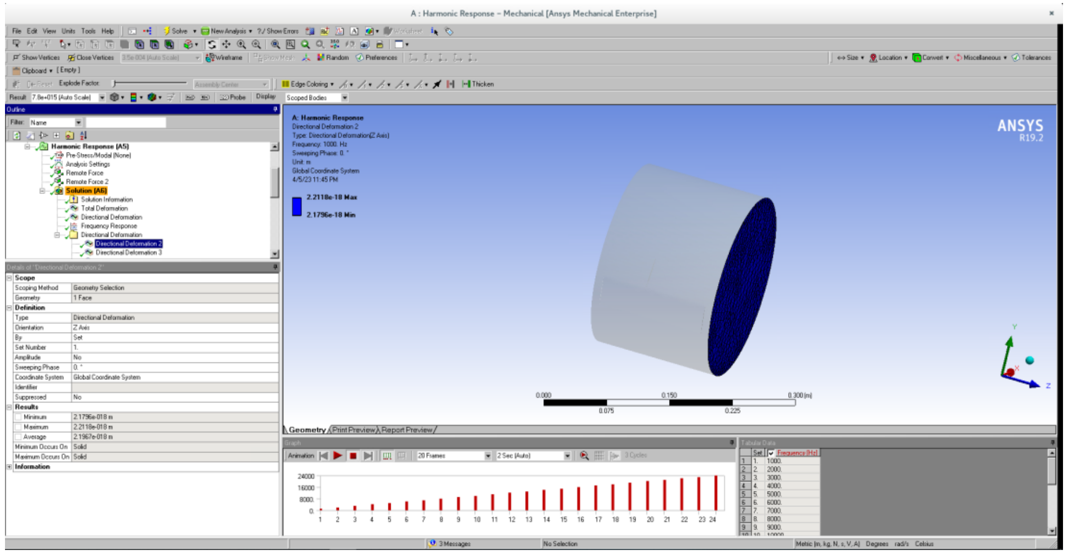
Rigid mass motion

ANSYS and COMSOL

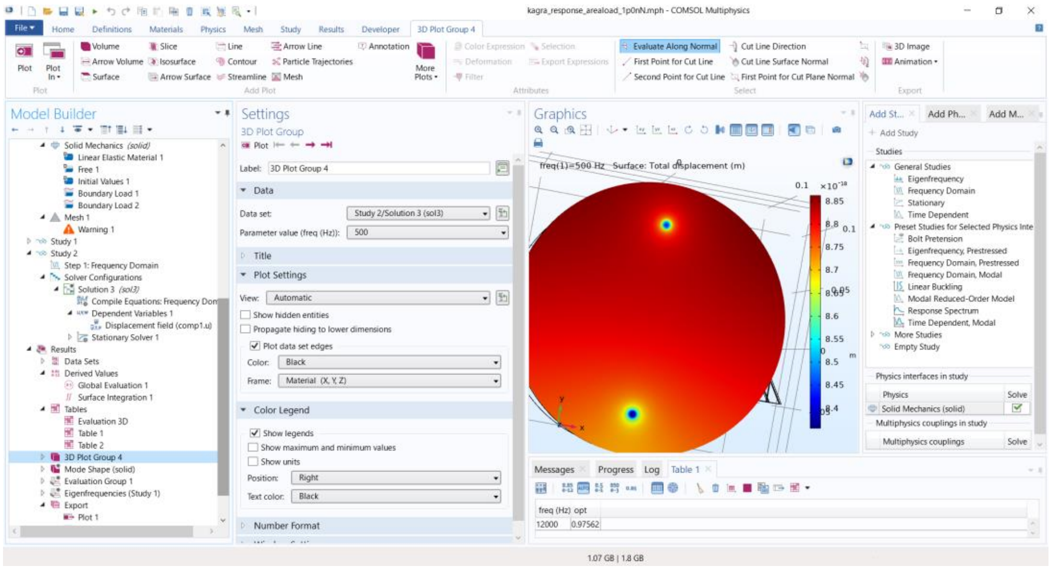
ANSYS and COMSOL are widely used finite-element analysis software.

We cross-checked results from these two software.

ANSYS GUI



COMSOL GUI



Research flow

Modal analysis

- Check deformation resonant modes
- Seek optimal Pcal beam positions

↓

Harmonic response analysis

- Determine optimal beam points

↓

Study of beam misalignment

(Previous studies)

LIGO, COMSOL
https://scholarsbank.uoregon.edu/xmlui/handle/1794/24553

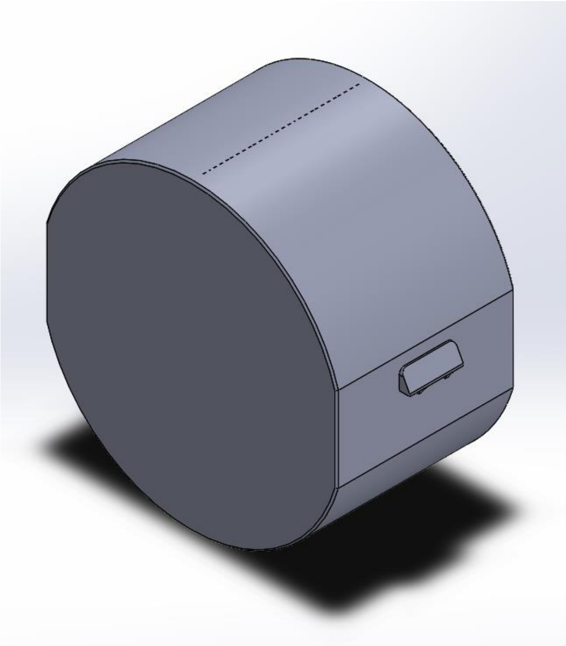
https://dcc.ligo.org/LIGO-T1700213

KAGRA, ANSYS & COMSOL https://arxiv.org/abs/2302.12180

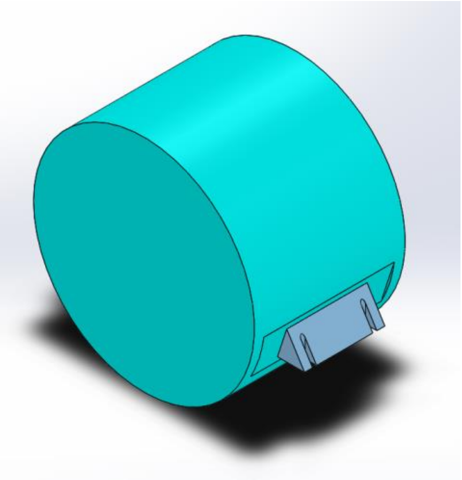
ETM parameters

	aLIGO	KAGRA
Diameter (mm)	340	220
Thickness (mm)	200	150
Weight (kg)	39.618	22.994
Materials	Silica (body)	Sapphire
Density (kg/m^3)	2203	4000
Poisson's ratio	0.1631	0.3
Young's modulus(GPa)	72.6	400
Main beam radius(mm)	62	35.3

aLIGO ETM



KAGRA ETM



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Modal analysis

Modal analysis is a method for detecting resonant eigenfrequencies.

We checked two fundamental deformation modes: Butterfly and Drumhead.

(Essential settings in ANSYS)

- Used “Modal” solver.
- Mesh size was Default and Resolution was 6. Mesh defeature was No.

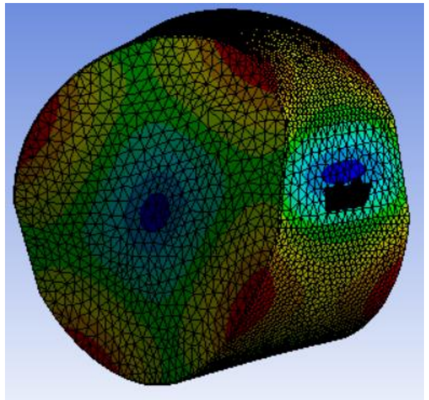
(Essential settings in COMSOL)

- Used “Eigenfrequency” study.
- Mesh size was Fine.

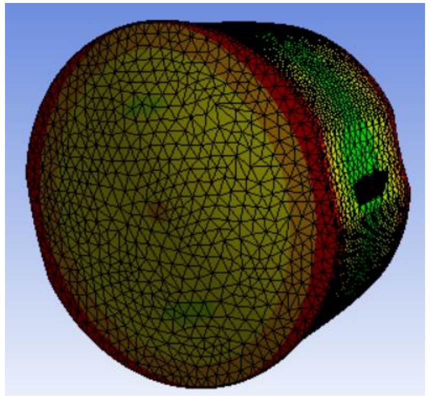
Modal resonant frequencies

Two software showed the consistent modal results within 1 Hz.

aLIGO
Butterfly (5946.2 Hz)



Drumhead (8152.9 Hz)



aLIGO

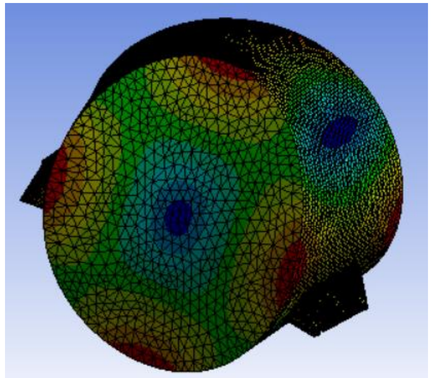
ANSYS modes (Hz)	COMSOL modes (Hz)
5946.2 (butterfly)	5946.0 (butterfly)
6051 (butterfly)	6051.0 (butterfly)
8152.9 (drumhead)	8152.9 (drumhead)

KAGRA

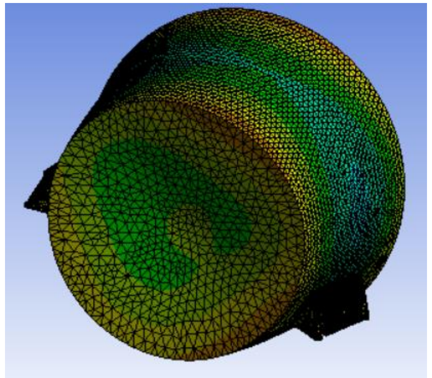
ANSYS modes (Hz)	COMSOL modes (Hz)
15913 (butterfly)	15913.7 (butterfly)
15978 (butterfly)	15978.6 (butterfly)
…	…
23658 (drumhead)	23658.7 (drumhead)

KAGRA has higher resonant frequency because of higher stiffness.

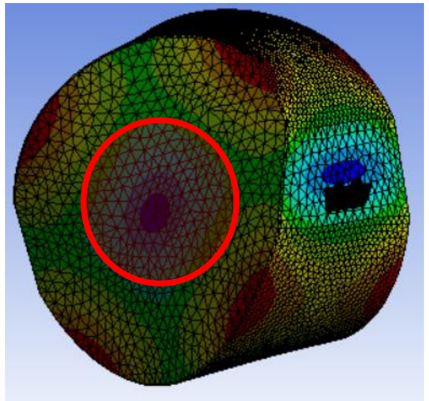
KAGRA
Butterfly (15913 Hz)



Drumhead (23658 Hz)

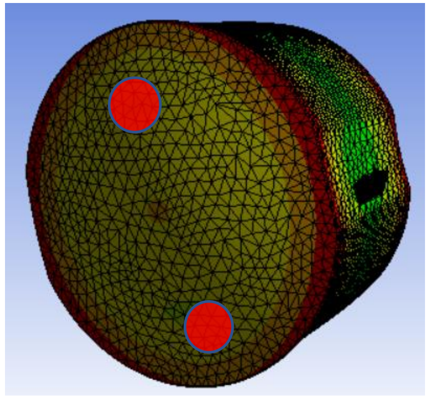


Mitigate deformation



Butterfly mode is neglected by integrating deformation over cross section of the main beam.

-> Need to estimate residual deformation.



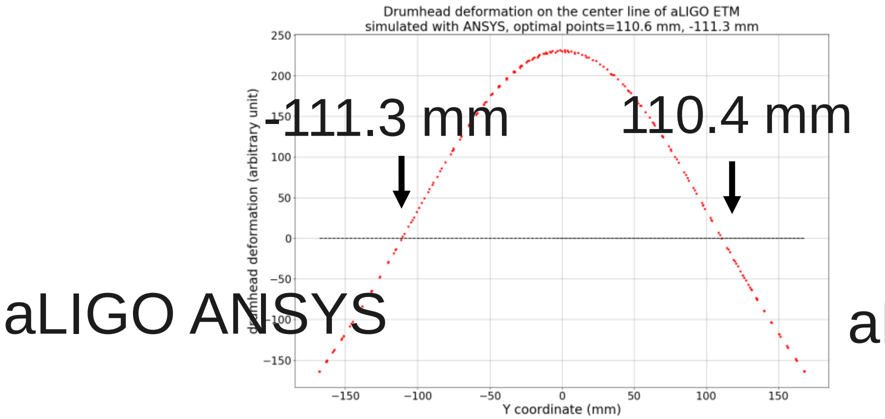
Drumhead mode is suppressed by injecting two Pcal beams at symmetric positions.

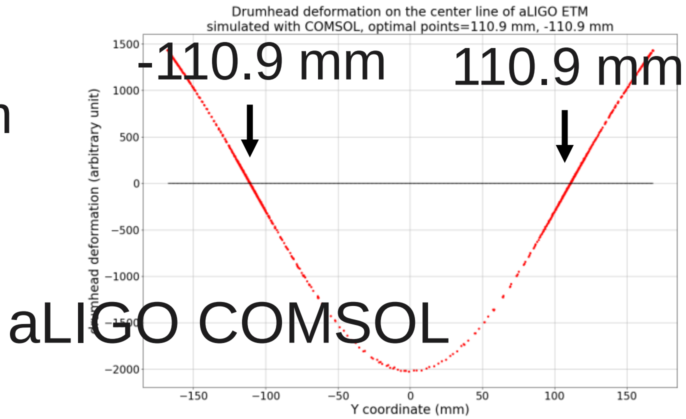
-> Need to optimize beam positions.

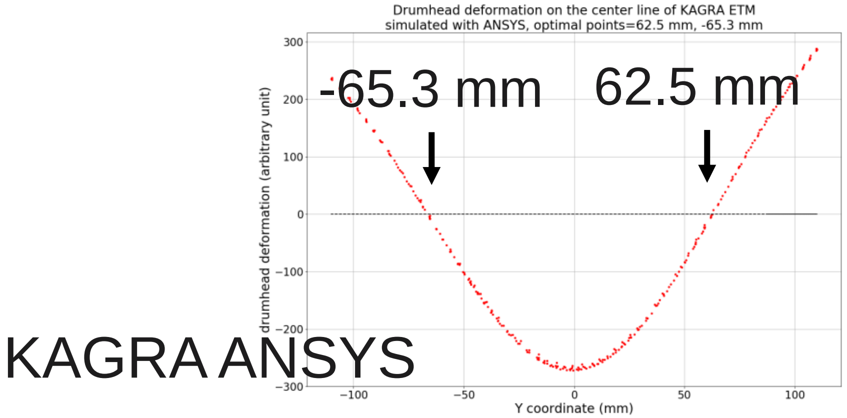
Drumhead node to inject Pcal laser

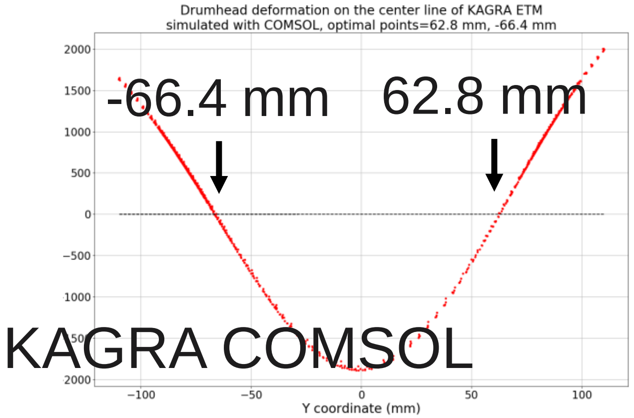
Pcal beam positions should be at nodes of the drumhead mode.

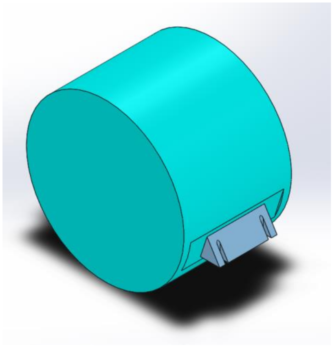
ANSYS and COMSOL gave consistent node positions within 1.1 mm discrepancy.











KAGRA's drumhead nodes are asymmetric because of the ears.

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Harmonic response

Harmonic response analysis is a method to apply oscillating load and see deformation.

We determined Pcal beam positions which minimize displacement ratio.

Our ANSYS settings were based on discussion with Prof. Sadakazu Haino.

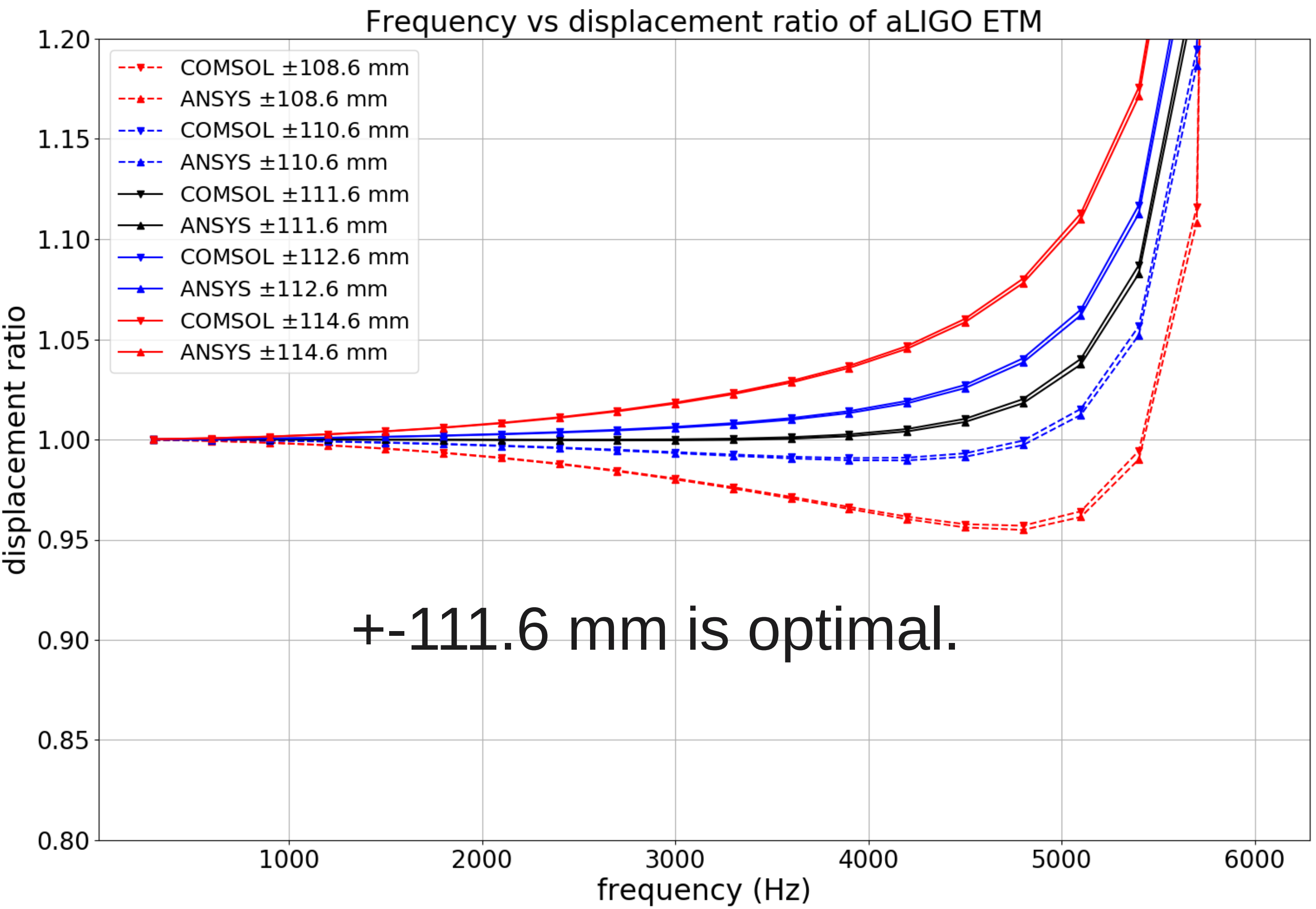
- Used “Harmonic response” solver.
- Applied Remote forces on the mirror surface and set Pinball region to the Pcal beam radius.
- Mesh size was 5 mm on the mirror surface. Other parts were Default and Resolution was 6. Mesh defeature was No.
- Displacement ratio was calculated offline.

Our COMSOL settings followed Dr. Nicola De Lillo’s document written for LIGO.
(https://dcc.ligo.org/LIGO-T1700213)

- Used “Frequency Domain” study.
- Applied Harmonic loads on Point loads on the mirror surface.
- Mesh size was Fine.
- Displacement ratio was calculated within COMSOL.

aLIGO result with COMSOL&ANSYS

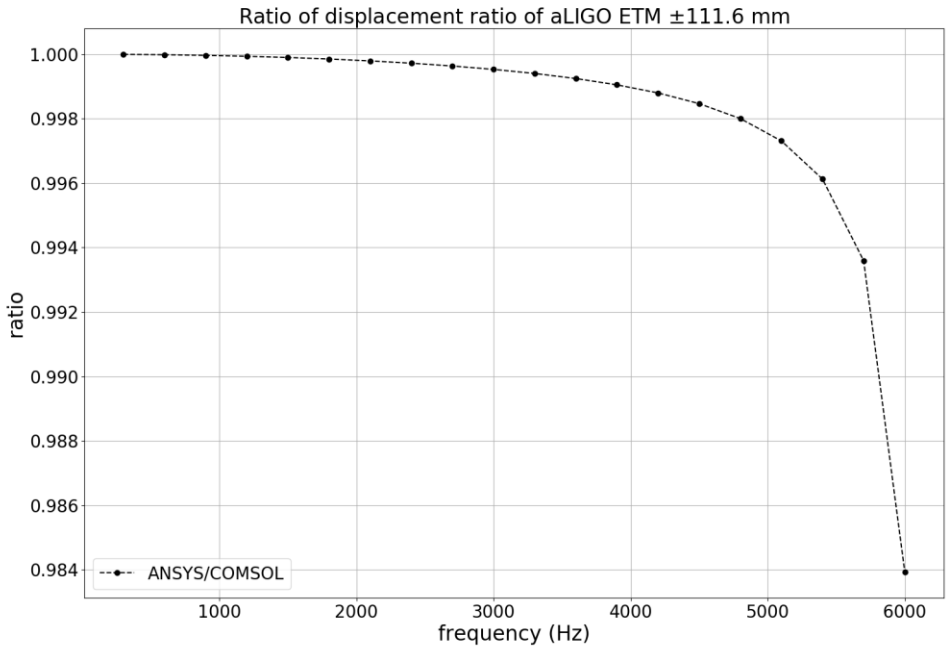
Displacement ratios were consistent in the two software.



+-111.6 mm is optimal.

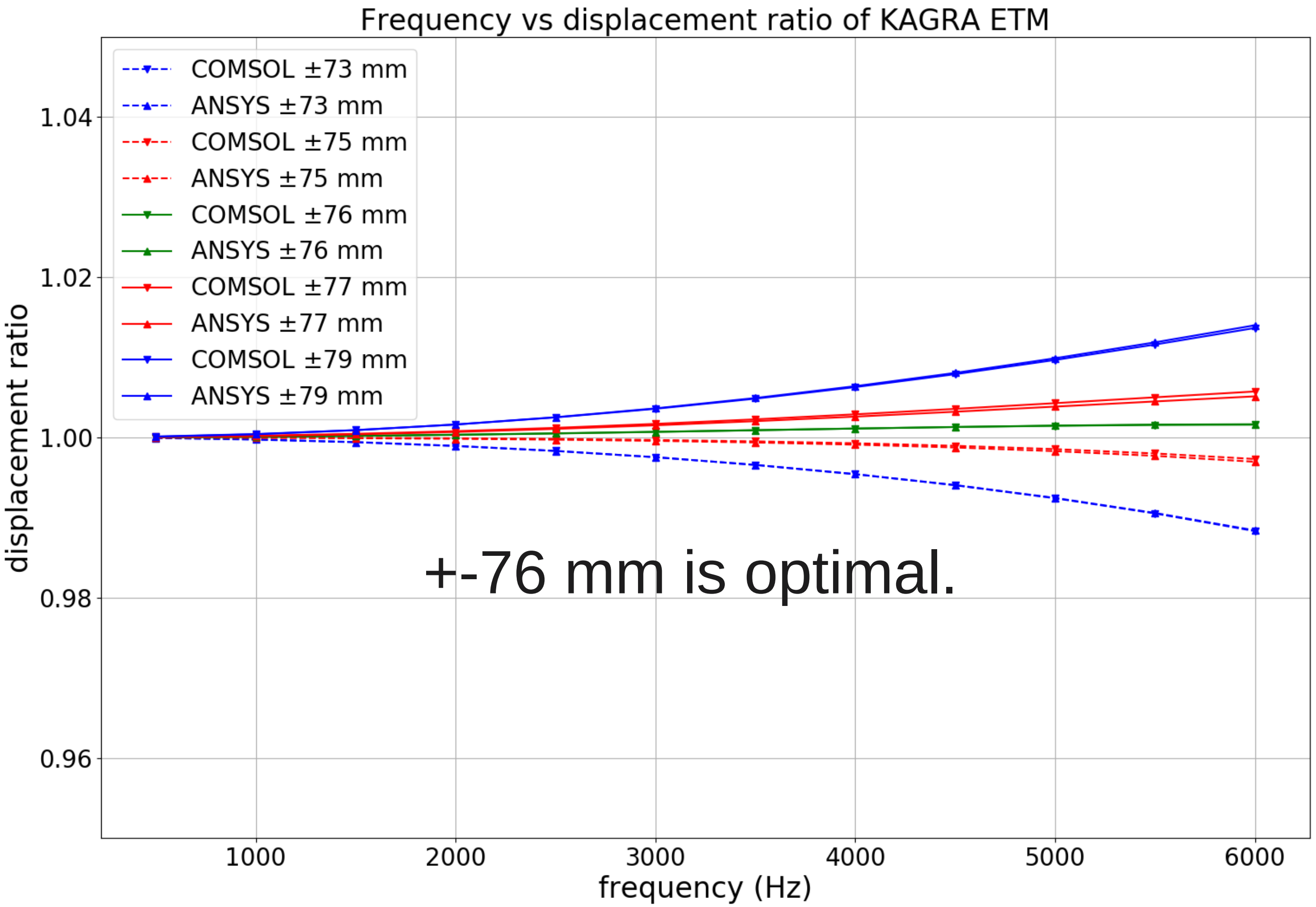
Discrepancy of two software result is below 0.3% in <5000 Hz region.

It rises near butterfly mode frequency.

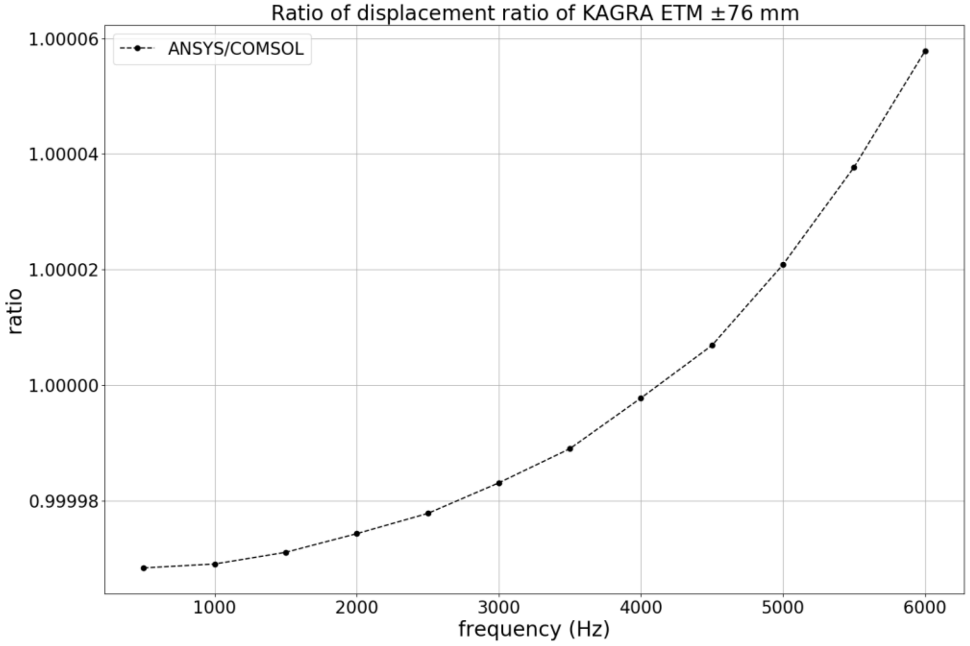


KAGRA result with COMSOL&ANSYS

Displacement ratios were consistent in the two software.



Discrepancy of two software result is below 0.006% in <6000 Hz region.



Result

We performed modal and harmonic response analysis with two software, ANSYS and COMSOL.

We got consistent results with these two software.

aLIGO ANSYS	aLIGO COMSOL	Value	KAGRA ANSYS	KAGRA COMSOL
5946.2	5946.0	**Butterfly mode freq. (Hz)**	15913	15913.7
8152.9	8152.9	**Drumhead mode freq. (Hz)**	23658	23658.7
-111.3/110.4	-110.9/110.9	**Drumhead node (mm)**	-65.3/62.5	-66.4/62.8
±111.6	±111.6	**Optimal Pcal beam positions (mm)**	±76	±76

Conclusion & Discussion

- ANSYS and COMSOL give consistent results of resonant frequencies and displacement ratio.

aLIGO ANSYS	aLIGO COMSOL	Value	KAGRA ANSYS	KAGRA COMSOL
-111.3/110.4	-110.9/110.9	**Drumhead node (mm)**	-65.3/62.5	-66.4/62.8
±111.6	±111.6	**Optimal Pcal beam positions (mm)**	±76	±76

- On the other hand, drumhead node varies among conditions.
  - **ANSYS / COMSOL** -> ANSYS has more degrees of freedom. Mesh size in simulation and node selection in analysis.
  - **+Y / -Y asymmetry** -> Possibly due to the ears. KAGRA has larger asymmetry because it has flat surfaces at the lower part for ears.
  - **Discrepancy from harmonic response analysis** -> Also due to ears of KAGRA. Asymmetry allows butterfly mode to remain after integration over the main beam.

- Therefore optimizing Pcal beam position by displacement ratio is important.

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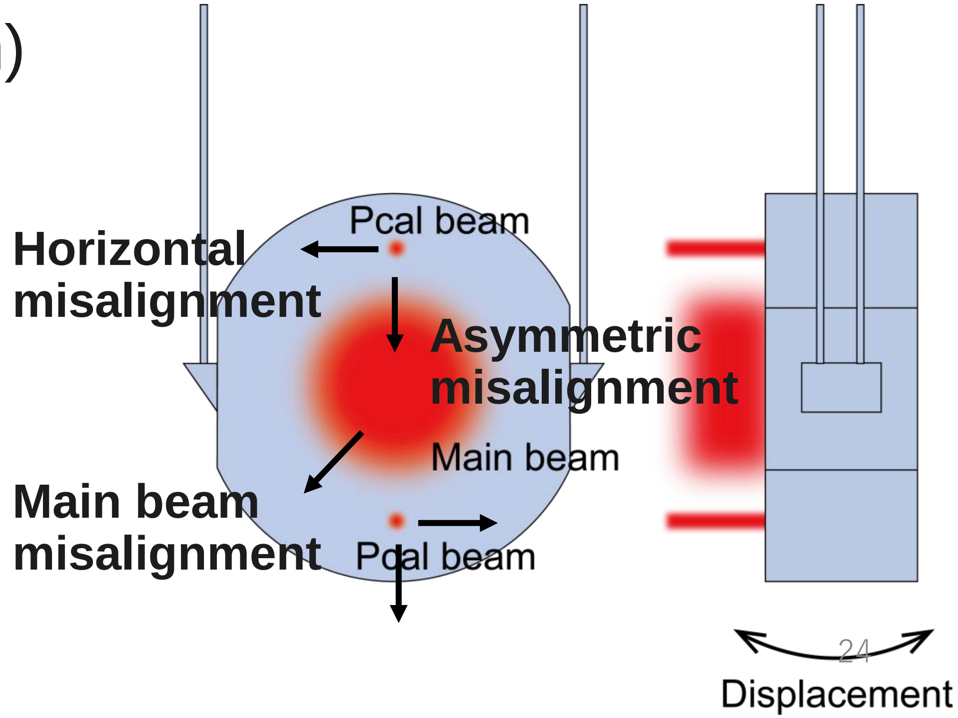
Future studies

We should make a detailed document of procedure for each software.

Using these simulation settings and analysis flow, we can perform consistent study in multiple environment.

We can start further studies of deformation by misaligned beams.

- Suspended mirror (more realistic assumption)
- Asymmetric misalignment
- Horizontal misalignment
- Main beam misalignment



Summary

- We studied bulk deformation of aLIGO and KAGRA mirrors.
- We performed modal analysis to seek the optimal Pcal beam positions.
- We performed harmonic response analysis to determine the optimal points more accurately.
- All results were consistent among ANSYS and COMSOL software.
- Next step is evaluating bulk deformation caused by various cases of beam misalignment.

Appendix

Analysis process: displacement ratio

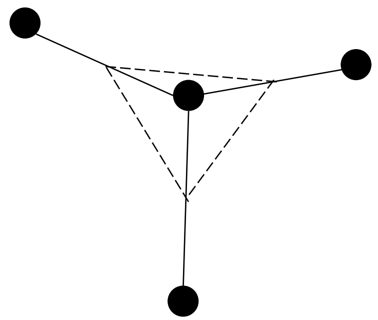
- Calculate displacement ratio G by the following formula.

$$\mathcal{G}_{(a_i,b)}(f) = \left[1 + \frac{\sum_m x_m(f)}{x_{\mathrm{rigid}}(f)}\right] = \left[\frac{\mathrm{x}_{\mathrm{total}}}{\mathrm{x}_{\mathrm{rigid}}}\right]_{\mathrm{FEA}}$$

$$\mathrm{x}_{\mathrm{total}}(f) = \underbrace{k_I}_{\text{normalization}} \int_\Omega \underbrace{\mathrm{w}(x, y; f)}_{\text{Simulated surface deformation}} \cdot \underbrace{I(x, y, z)}_{\text{Main beam profile}} dxdy$$

$$I(x, y, z) = \exp\left(-2\frac{(x - x_0)^2 + (y - y_0)^2}{\underbrace{\omega(z)^2}_{\text{Main beam radius}}}\right)$$

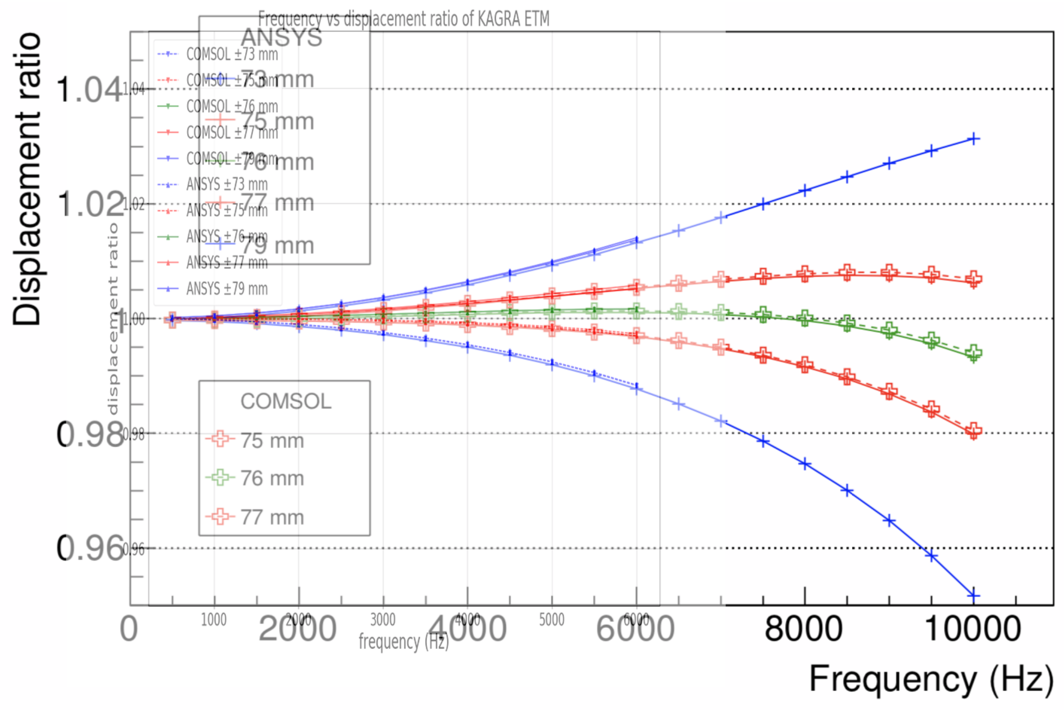
We used area around each node point as dxdy.



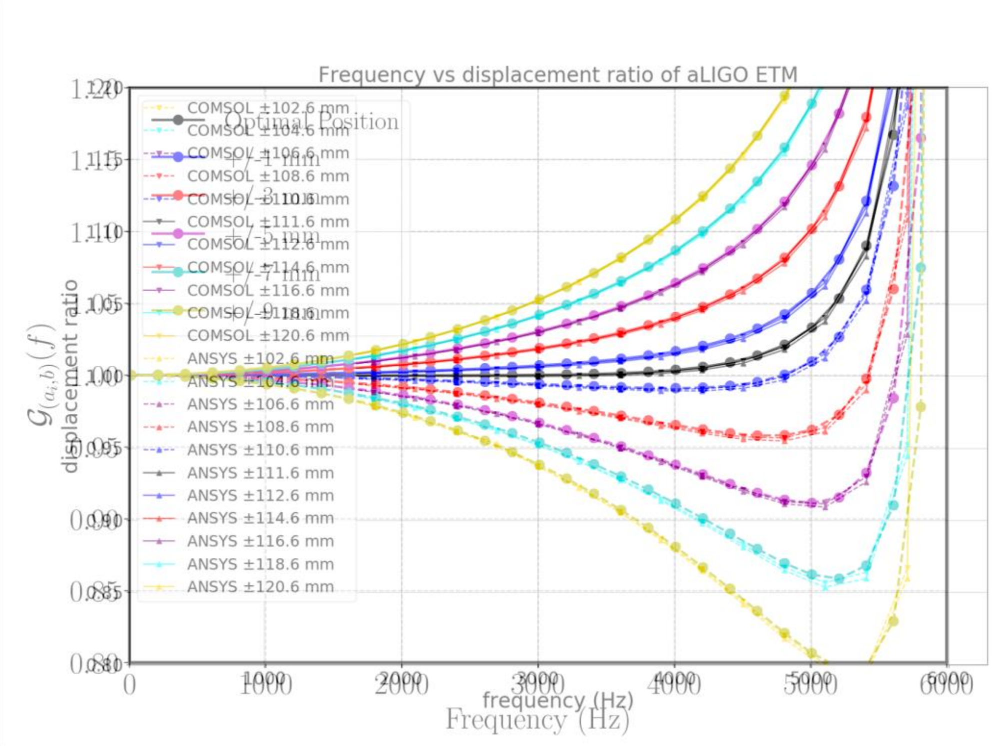
Consistency with previous studies

Our displacement ratios were also consistent with previous studies done by Sadakazu, Sudarshan, and Nicola.

Overplot with KAGRA's Pcal paper



Overplot with Sudarshan's thesis



Nicola’s setting for COMSOL

- Mesh: Physical control, Fine
- Pcal beam point: 111.6 mm as optimal. Add two *point* in *Component -> Geometry*. Set position and force in z axis. Then add them in *Solid Mechanics -> Point load*. Right click each *Point load* and check *Harmonic Perturbation*. Also don’t forget to set *Linear perturbation* in *Study -> Solver Configurations -> Stationary Solver -> General -> Linearity* .
- Force: 3.3333e-9 N per beam.
- Frequency: Write *range(300,300,6000)* in *Study -> Frequency Domain -> Study Settings -> Frequencies* .
- Mass: Write mass1.mass in *Results -> Derived Values -> Global Evaluation -> Expression.* When it is evaluated into some table, it should be 39.618 kg.
- Normalization with the main beam: Define *Analytic function* as *an1* in *Global Definitions*. Write *exp(-2*(x^2+y^2)/(0.062^2))* in *Definition -> Representation*. Arguments are *x,y*. Set *Plot Parameters* as *[-0.15,0.15]* for both x and y. Then write *510.894*w*an1(x,y)/(3.3333e-9*2)*mass1.mass*(2*pi*freq)^2* in *Results -> Derived Values -> Surface Integration -> Expression*. This 510.894 was calculated by python in advance with the same x,y range. I set steps of x,y as 0.001.
- Main beam radius: 0.062 m. This is used in Gaussian profile defined as *an1*.
- Fitting: No. Calculate raw data in COMSOL.

(ETM parameters)

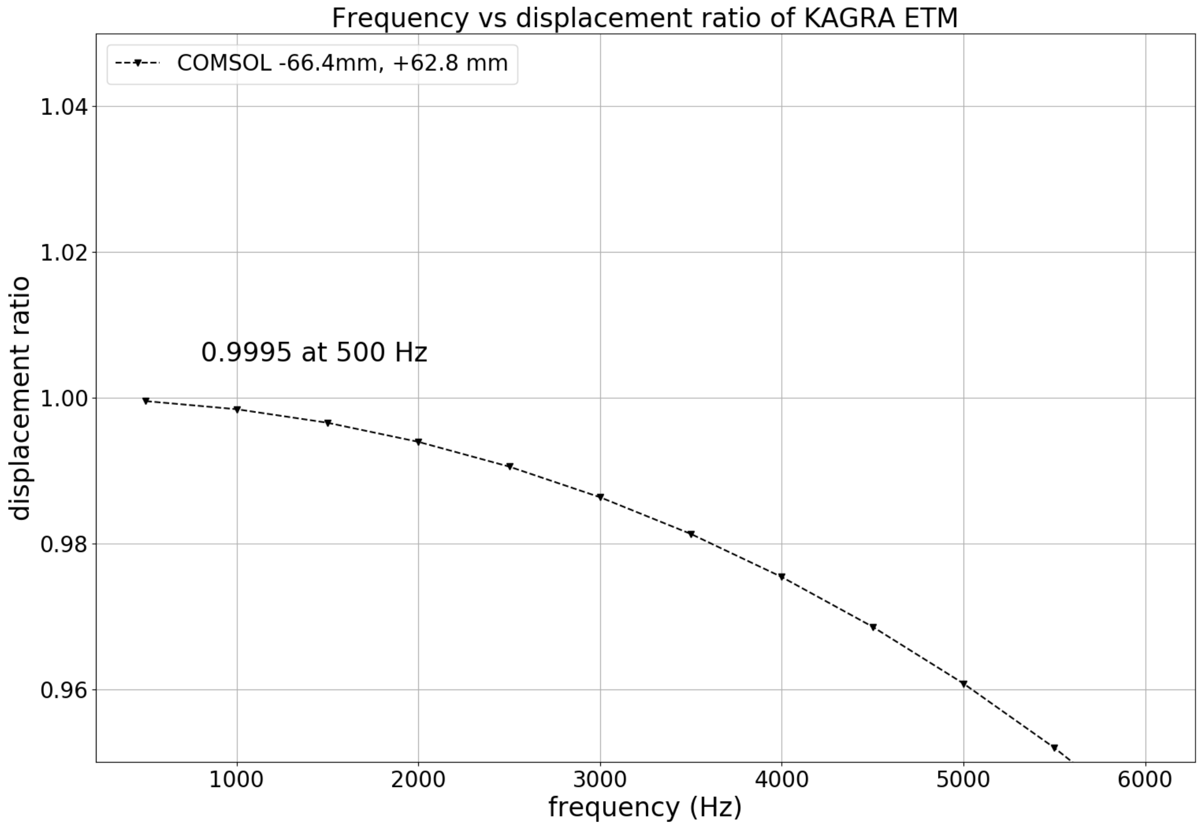
<Main body> Silica. Density: 2203 kg/m^3, Poisson’s ratio: 0.1631, Young’s modulus: 72.6 GPa.

<Ears> Suprasil 312. Density: 2200 kg/m^3, Poisson’s ratio: 0.17, Young’s modulus: 70.0 GPa.

If inject Pcal to KAGRA's drumhead node

KAGRA's drumhead nodes were -66.4 mm and +62.8 mm.

If we inject Pcal here, the displacement ratio is 0.9995 at 500 Hz. (0.05% error)



If we inject to optimal points (+-76 mm), displacement ratio is 1.00005 at 500 Hz.

(Difference of ten times)