TMD Theory and Phenomenology

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QCD in 1973

- Asymptotic freedom: papers (Gross-Wilczek, Politzer) published in PRL side by side on June 25, 1973

Ultraviolet Behavior of Non-Abelian Gauge Theories*

David J. Gross† and Frank Wilczek
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(Received 27 April 1973)

It is shown that a wide class of non-Abelian gauge theories have, up to calculable logarithmic corrections, free-field–theory asymptotic behavior. It is suggested that Bjorken scaling may be obtained from strong-interaction dynamics based on non-Abelian gauge symmetry.

Non-Abelian gauge theories have received much attention recently as a means of constructing unified and renormalizable theories of the weak and electromagnetic interactions.[1] In this note we report on an investigation of the ultraviolet (UV) asymptotic behavior of such theories. We have found that they possess the remarkable feature, perhaps unique among renormalizable theories, of asymptotically approaching free-field theory. Such asymptotically free theories will exhibit, for matrix elements of currents between on-mass-shell states, Bjorken scaling. We therefore suggest that one should look to a non-Abelian gauge theory of the strong interactions to provide the explanation for Bjorken scaling, which has so far eluded field-theoretic understanding.

The UV behavior of renormalizable field theories can be discussed using the renormalization-group equations,[2,3] which for a theory involving one field (say $g\varphi^4$) are

$$[m\partial/\partial m+\beta(g)\partial/\partial g-n\gamma(g)]\Gamma_{\rm asy}^{(n)}(g;P_1,\ldots,P_n)=0. \quad (1)$$

$\Gamma_{\rm asy}^{(n)}$ is the asymptotic part of the one-particle–irreducible renormalized n-particle Green's function, $\beta(g)$ and $\gamma(g)$ are finite functions of the renormalized coupling constant g, and m is either the renormalized mass or, in the case of massless particles, the Euclidean momentum at which the theory is renormalized.[4] If we set $P_i=\lambda q_i^0$, where q_i^0 are (nonexceptional) Euclidean momenta, then (1) determines the λ dependence of $\Gamma^{(n)}$:

$$\Gamma^{(n)}(g;P_i)=\lambda^D\Gamma^{(n)}(\bar g(g,t);q_i)\exp[-n\int_0^t\gamma(\bar g(g,t'))dt'], \quad (2)$$

where $t=\ln\lambda$, D is the dimension (in mass units) of $\Gamma^{(n)}$, and $\bar g$, the invariant coupling constant, is the solution of

$$d\bar g/dt=\beta(\bar g), \quad \bar g(g,0)=g. \quad (3)$$

The UV behavior of $\Gamma^{(n)}$ $(\lambda\to+\infty)$ is determined by the large-t behavior of $\bar g$ which in turn is controlled by the zeros of β: $\beta(g_f)=0$. These fixed points of the renormalization-group equations are said to be UV stable [infrared (IR) stable] if $\bar g\to g_f$ as $t\to+\infty$ $(-\infty)$ for $\bar g(0)$ near g_f. If the physical coupling constant is in the domain of attraction of a UV-stable fixed point, then

$$\Gamma^{(n)}(g;P_i)\underset{\lambda\to\infty}{\approx}\lambda^{D-n\gamma(g_f)}\Gamma^{(n)}(g_f;q_i)\exp\{-n\int_0^\infty[\gamma(\bar g(g,t))-\gamma(g_f)]dt\}, \quad (4)$$

so that $\gamma(g_f)$ is the anomalous dimension of the field. As Wilson has stressed, the UV behavior is determined by the theory at the fixed point $(g=g_f)$.[5]

In general, the dimensions of operators at a fixed point are not canonical, i.e., $\gamma(g_f)\neq 0$. If we wish to explain Bjorken scaling, we must assume the existence of a tower of operators with canonical dimensions. Recently, it has been argued for all but gauge theories, that this can only occur if the fixed point is at the origin, $g_f=0$, so that the theory is asymptotically free.[6,7] In that case the anomalous dimensions of all operators vanish, one obtains naive scaling up to finite and calculable powers of $\ln\lambda$, and the structure of operator products at short distances is that of free-field theory.[7] Therefore, the existence of such a fixed point, for a theory of the strong interactions, might explain Bjorken scaling and the success of naive light-cone or parton-model relations. Unfortunately, it appears that the fixed point at the origin, which is common to all theories, is not UV stable.[8,9] The only exception would seem to be non-Abelian gauge theories, which hitherto have not been explored in this re-

[14]Y. Nambu and G. Jona-Lasino, Phys. Rev. 122, 345 (1961); S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).
[15]K. Symanzik (to be published) has recently suggested that one consider a $\lambda\varphi^4$ theory with a negative λ to achieve UV stability at $\lambda=0$. However, one can show, using the renormalization-group equations, that in such theory the ground-state energy is unbounded from below (S. Coleman, private communication).
[16]W. A. Bardeen, H. Fritzsch, and M. Gell-Mann, CERN Report No. CERN-TH-1538, 1972 (to be published).
[17]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 28, 1494 (1972); S. Weinberg, Phys. Rev. D 5, 1962 (1972).
[18]For a review of this program, see S. L. Adler, in Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Illinois, 1972 (to be published).

Reliable Perturbative Results for Strong Interactions?*

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(Received 3 May 1973)

An explicit calculation shows perturbation theory to be arbitrarily good for the deep Euclidean Green's functions of any Yang-Mills theory and of many Yang-Mills theories with fermions. Under the hypothesis that spontaneous symmetry breakdown is of dynamical origin, these symmetric Green's functions are the asymptotic forms of the physically significant spontaneously broken solution, whose coupling could be strong.

Renormalization-group techniques hold great promise for studying short-distance and strong-coupling problems in field theory.[1,2] Symanzik[2] has emphasized the role that perturbation theory might play in approximating the otherwise unknown functions that occur in these discussions. But specific models in four dimensions that had been investigated yielded (in this context) disappointing results.[3] This note reports an intriguing contrary finding for any generalized Yang-Mills theory and theories including a wide class of fermion representations. For these one–coupling-constant theories (or generalizations involving product groups) the coefficient function in the Callan-Symanzik equations commonly called $\beta(g)$ is negative near $g=0$.

The constrast with quantum electrodynamics (QED) might be illuminating. Renormalization of QED must be carried out at off-mass-shell points because of infrared divergences. For small e^2, we expect perturbation theory to be good in some neighborhood of the normalization point. But what about the inevitable logarithms of momenta that grow as we approach the mass shell or as some momenta go to infinity? In QED, the mass-shell divergences do not occur in observable predictions, when we take due account of the experimental situation. The renormalization-group technique[4] provides a somewhat opaque analysis of this situation. Loosely speaking,[5] the effective coupling of soft photons goes to zero, compensating for the fact that there are more and more of them. But the large-p^2 divergence represents a real breakdown of perturbation theory. It is commonly said that for momenta such that $e^2\ln(p^2/m^2)\sim 1$, higher orders become comparable, and hence a calculation to any finite order is meaningless in this domain. The renormalization group technique shows that the effective coupling grows with momenta.

The behavior in the two momentum regimes is reversed in a Yang-Mills theory. The effective coupling goes to zero for large momenta, but as p^2's approach zero, higher-order corrections become comparable. Thus perturbation theory tells *nothing* about the mass-shell structure of the symmetric theory. Even for arbitrarily small g^2, there is no sense in which the interacting theory is a small perturbation on a free multiplet of massless vector mesons. The truly catastrophic infrared problem makes a symmetric particle interpretation impossible. Thus, though one can well approximate asymptotic Green's functions, to what particle states do they refer?

Consider theories defined by the Lagrangian

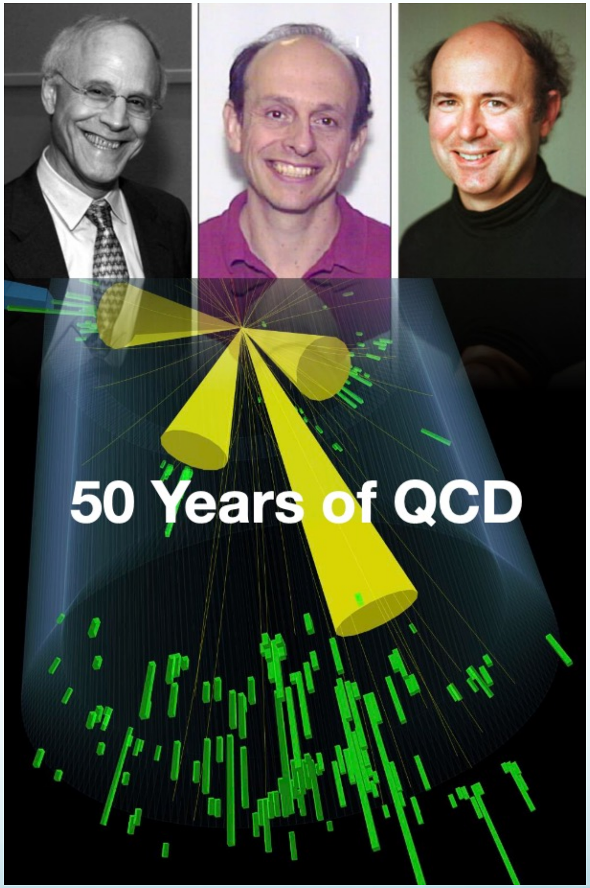
$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}^aF^{a\mu\nu}+i\bar\psi_i\gamma\cdot D_{ij}\psi_j, \quad (1)$$

where

$$F_{\mu\nu}^a=\partial_\mu A_\nu^a-\partial_\nu A_\mu^a+gf^{abc}A_\mu^bA_\nu^c,$$

50 Years of QCD

- A dedicated conference will be held at UCLA during Sept. 11 – 15, 2023

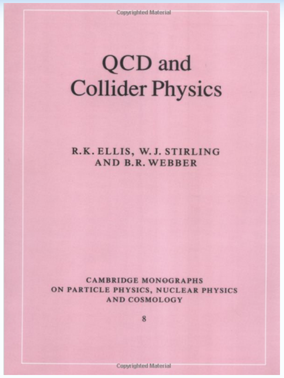


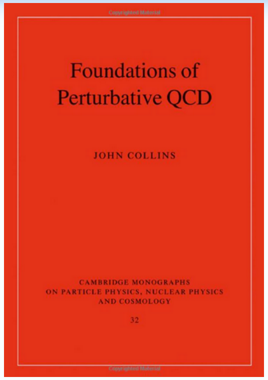
Outline

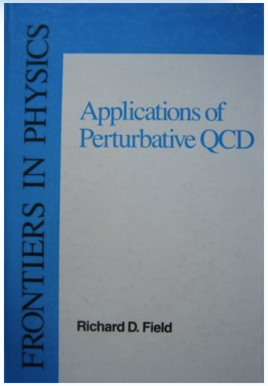
- Lecture 1: QCD factorization
  - Collinear factorization and collinear PDFs/FFs
- Lecture 2: TMD theory in standard processes
  - TMD parton model, operator analysis
- Lecture 3: TMD factorization and evolution
  - TMD evolution and global fitting
- Lecture 4: TMDs via jets and jet substructure
  - Jet TMD and jet fragmentation functions

Selected references on QCD

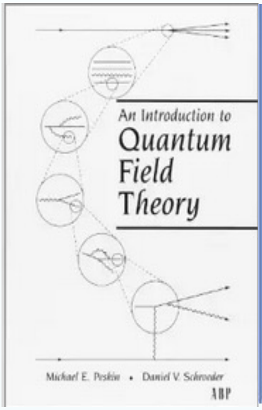
- Textbooks on QCD
  - QCD and Collider Physics – Ellis, Stirling, Webber
  - Foundations of Perturbative QCD: J. Collins
  - Applications of Perturbative QCD: R. Field

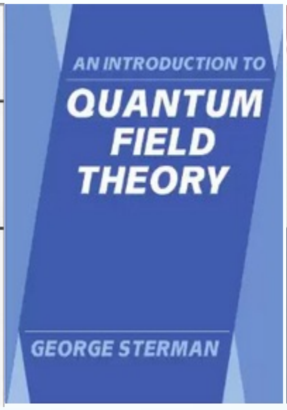


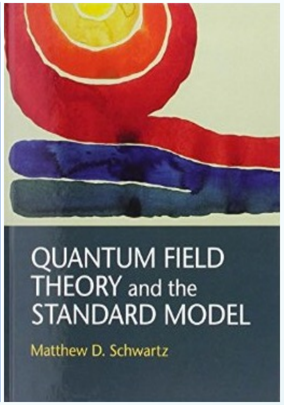




- Textbooks on Quantum Field Theory
  - An Introduction to Quantum Field Theory: Peksin & Schroeder, Sterman
  - Quantum Field Theory and the Standard Model: M. Schwartz

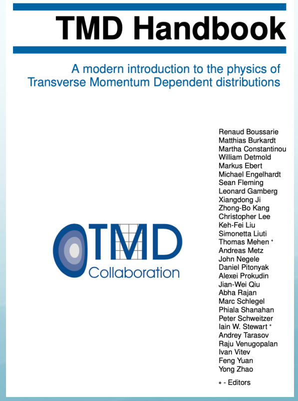






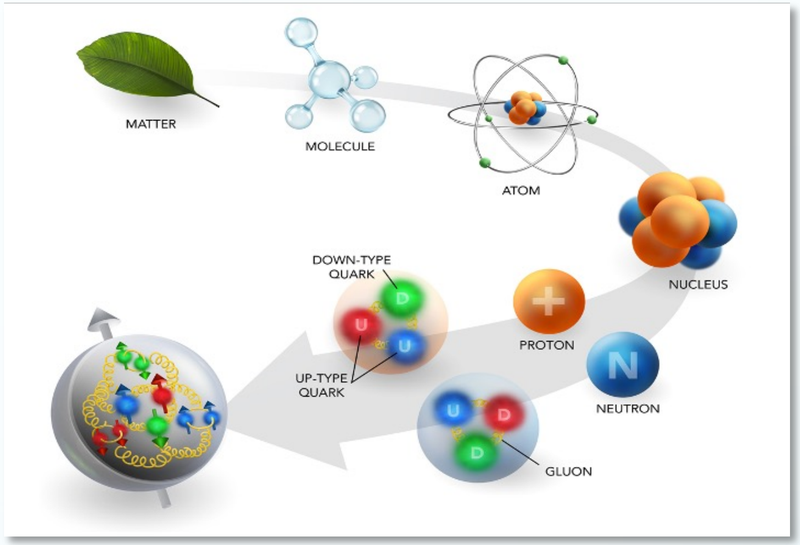
- CTEQ collaboration
  http://cteq.gitlab.io/

- TMD Handbook: 2304.03302

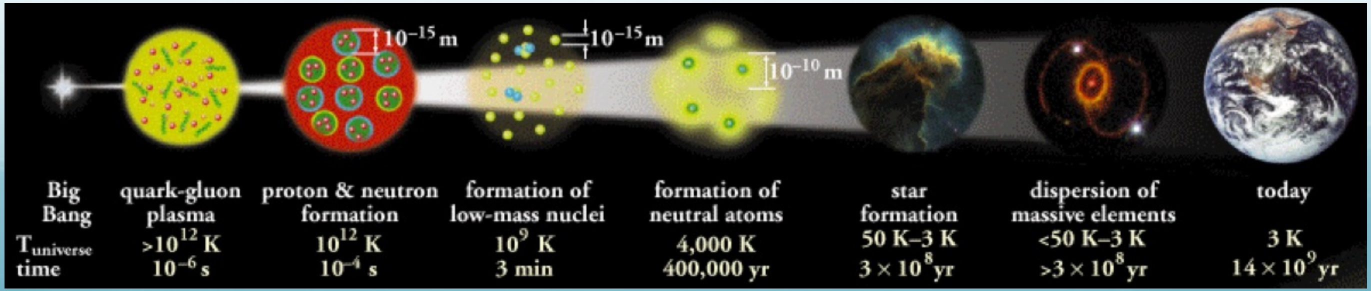


The structure of matter

- The exploration on the structure of matter has a really long history
  - Dalton 1803 (atom)
  - Rutherford 1911 (nucleus)
  - Chadwick 1932 (neutron)
  - Gell-Mann and Zweig 1964 (quark model)
  - Feynman 1969 (parton model), …



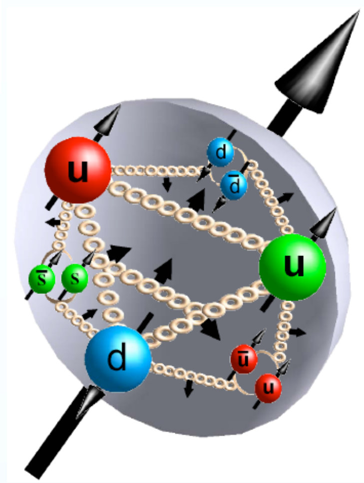
- Central goal of nuclear science
  - **To discover, explore, and understand all forms of nuclear matter and the associated dynamics**



Exploring the nucleon: fundamental importance in science

Know what we are made of:
Most abundant particles around us
Building blocks of all elements

Fundamental properties:
Proton mass, spin, magnetic moment, understand them in terms of the internal degrees of freedom

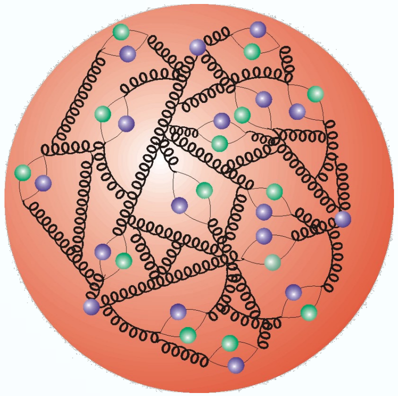


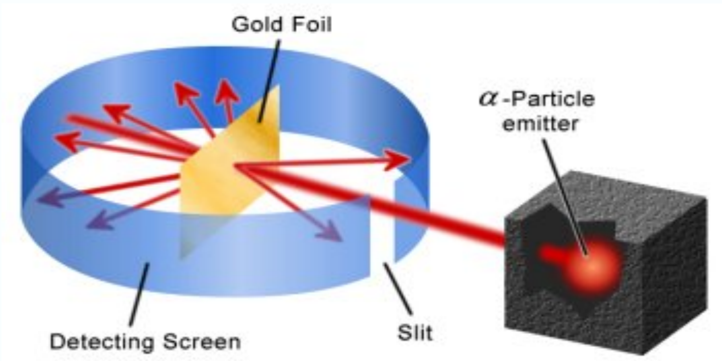
Tool for discovery:
Colliding high energy nucleons
New Physics beyond SM
LHC, Tevatron,
RHIC, HERA, ...

Exploring QCD and strong interaction:
Confinement,
Lattice QCD,
Asymptotic freedom,
perturbative QCD, ...

Hadron structure

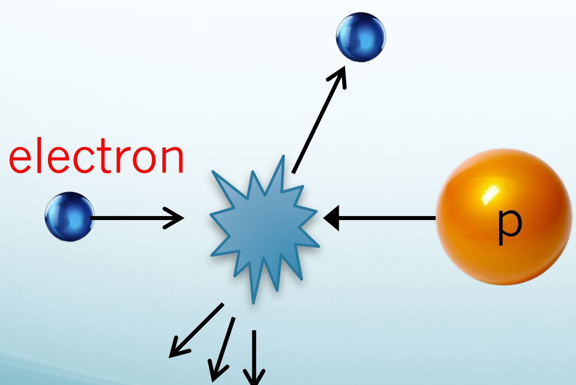
- Nucleon: quantum many-body system of quarks and gluons



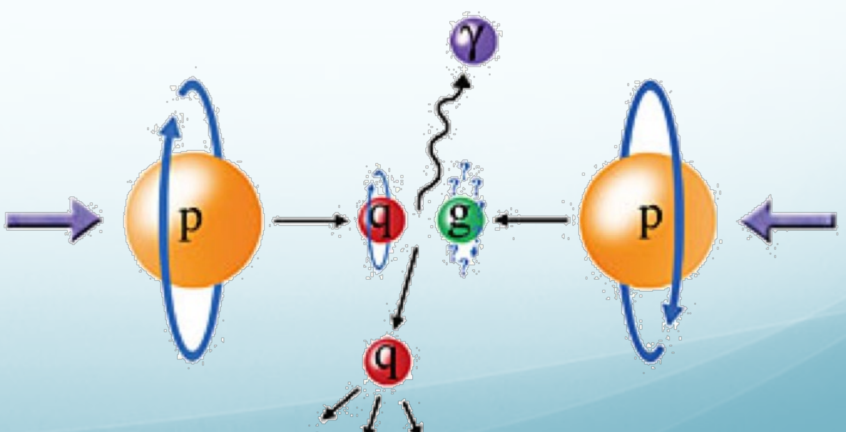


Rutherfold's experiment

- High energy scattering: to extract information on the nucleon structure, we send in a probe and measure the outcome of the collisions



Deep Inelastic Scattering (DIS)



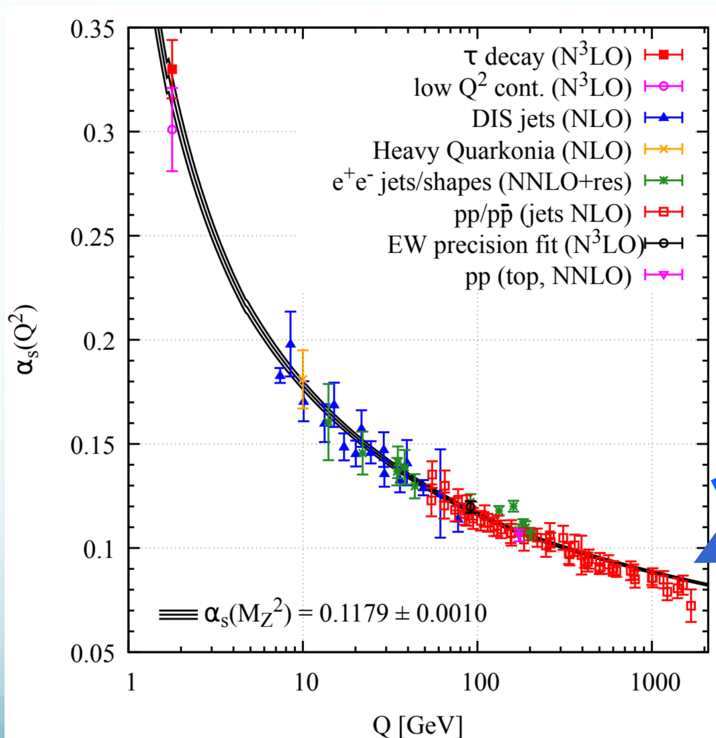
Proton-Proton collisions

QCD: Two faces

Asymptotic freedom

Confinement

No one has ever seen a free quark



High energy scale

Weak coupling

Low energy scale, strong coupling

Low energy scale: at the size of the proton

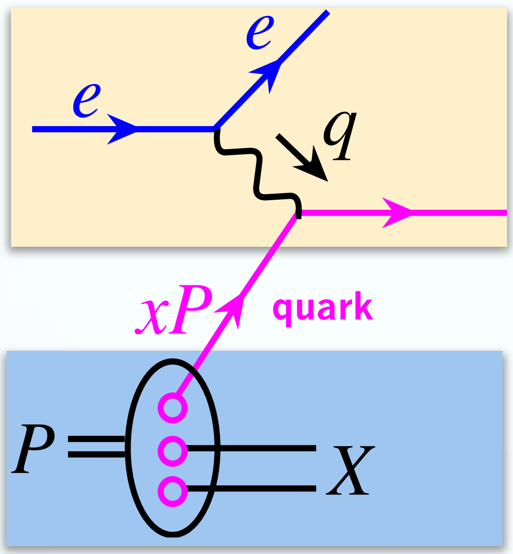
Millennium Prize Problem

Lecture 1: QCD factorization

- Collinear factorization, DGLAP, PDFs, FFs

QCD factorization

- Take deep inelastic scattering as an example



$\hat{\sigma}_{q,g}(Q)$

$Q = \sqrt{|q^2|}$

$f_{\mathrm{q,g/proton}}(x)$

Parton Distribution Functions (PDFs):
Probability for finding a parton in a proton with momentum fraction x

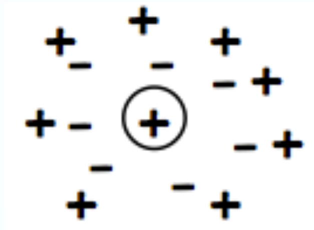
$$\sigma_{\mathrm{proton}}(Q) = f_{\mathrm{q,g/proton}}(x) \otimes \hat{\sigma}_{q,g}(Q)$$

measured **extracted** **calculable**

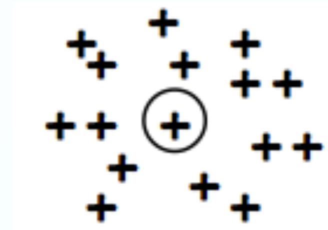
- Proton structure: encoded in PDFs
- QCD dynamics at high-energy scale *Q*

Understanding QCD: running coupling (asymptotic freedom)

- Gluon carries color charges
  - Strong coupling α_s depends on the distance (i.e., energy)



Screening: $\alpha_{em}(r) \uparrow$ as $r \downarrow$



Anti-screening: $\alpha_s(r) \downarrow$ as $r \downarrow$

Asymptotic Freedom ⇔ antiscreening

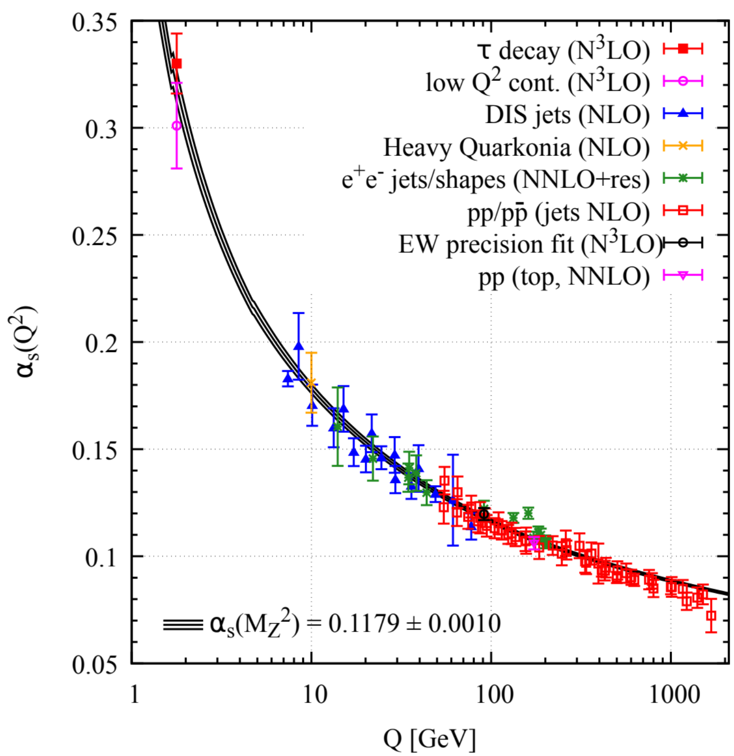
$$\text{QCD: } \frac{\partial \alpha_s(Q^2)}{\partial \ln Q^2} = \beta(\alpha_s) < 0$$

Compare

$$\text{QED: } \frac{\partial \alpha_{EM}(Q^2)}{\partial \ln Q^2} = \beta(\alpha_{EM}) > 0$$

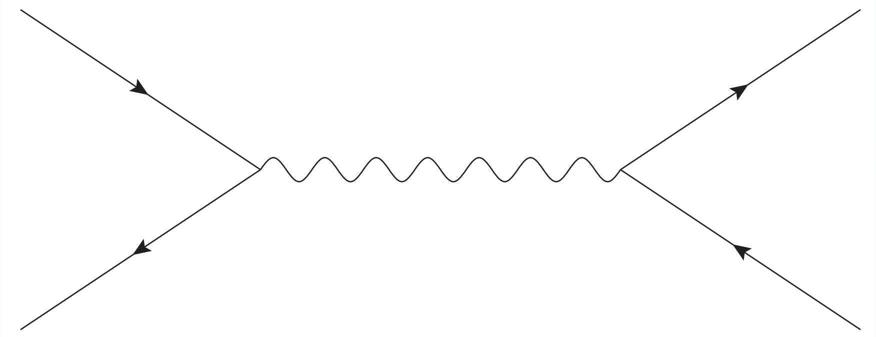
D.Gross, F.Willczek, Phys.Rev.Lett 30,(1973)
H.Politzer, Phys.Rev.Lett 30, (1973)

2004 Nobel Prize in Physics

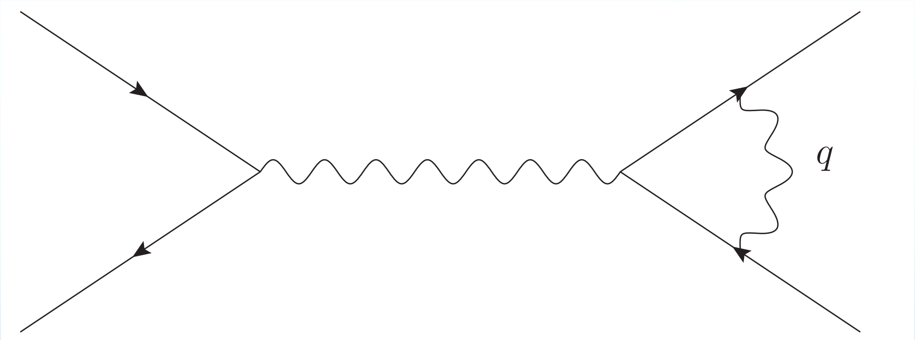


Why does the coupling constant run?

- Leading order calculation is simple: tree diagrams – always finite



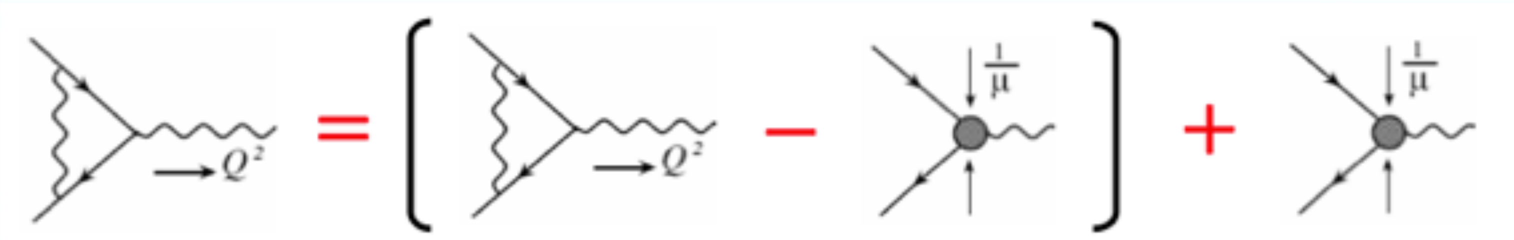
- Study a higher order Feynman diagram: one-loop, the diagram is divergent as q → ∞



- Make sense of the result: redefine the coupling constant to be physical

Renormalization (Redefine the coupling constant)

- Renormalization
  - UV divergence due to "high momentum" states
  - Experiments cannot resolve the details of these states

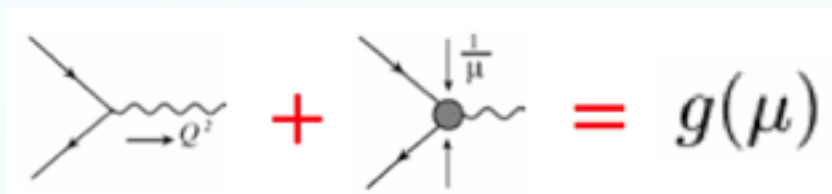


Low momentum state

High momentum state

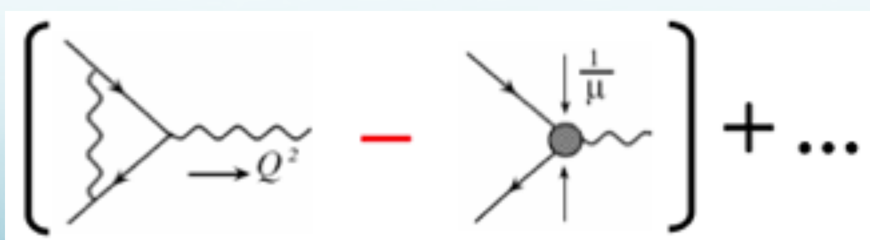
- Combine the "high momentum" states with leading order

LO:



Renormalized coupling

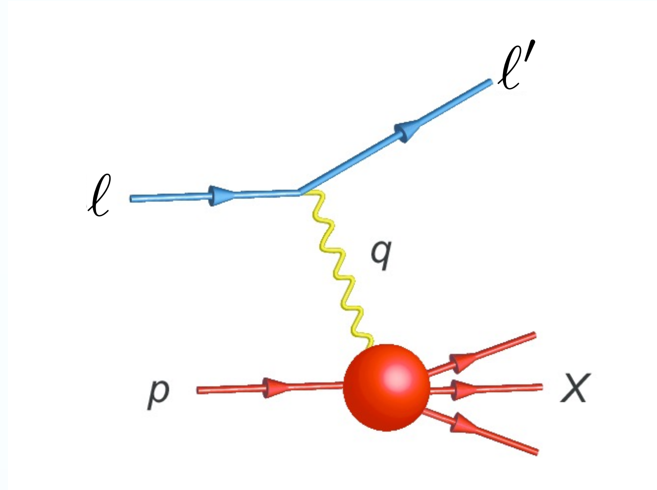
NLO:



No UV divergence!

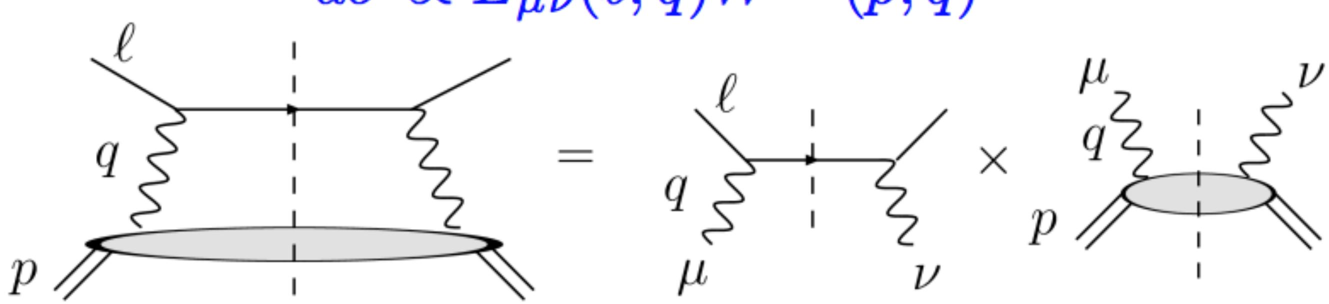
Simple study of Deep Inelastic Scattering (DIS): parton model

- DIS has been used a lot in extracting hadron structure



- Leptonic and hadronic tensor

$$d\sigma \propto L_{\mu\nu}(\ell, q) W^{\mu\nu}(p, q)$$



– Electron is elementary: $L_{\mu\nu}$ can be calculated perturbatively

Structure functions

- Hadronic tensor: Lorentz decomposition + parity invariance (for photon case) + time-reversal invariance + gauge invariance

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x_B, Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2\left(x_B, Q^2\right)$$

- All the information about hadron structure is contained in the structure functions

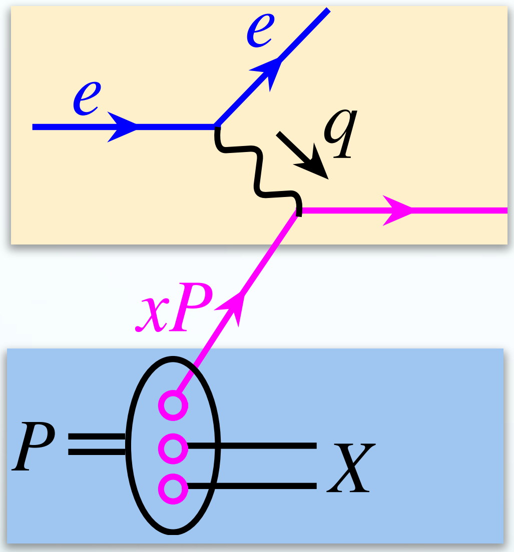
$$L_{\mu\nu} = 2\left(\ell_\mu \ell'_\nu + \ell'_\mu \ell_\nu - \ell\cdot\ell' g_{\mu\nu}\right)$$

$$\frac{d\sigma}{dx_B dQ^2} = \frac{4\pi\alpha^2_{\rm em}}{x_B Q^4}\left[(1-y)\, F_2(x_B, Q^2) + y^2 x_B F_1(x_B, Q^2)\right]$$

DIS at leading order: parton model

- Hadronic tensor at leading order

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1(x_B, Q^2) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2(x_B, Q^2)$$



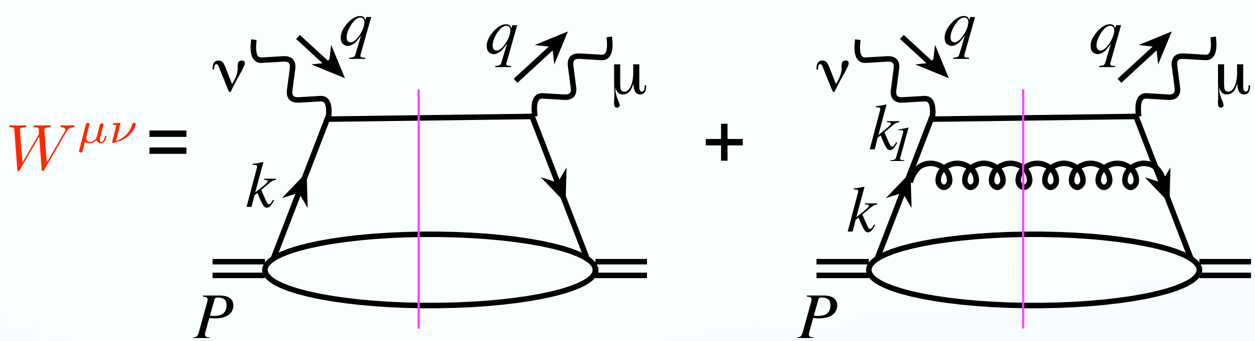
$\hat{\sigma}_{q,g}(Q)$

$f_{\mathrm{q,g/proton}}(x)$

$$2x_B F_1(x_B, Q^2) = F_2(x_B, Q^2) = x_B \sum_q e_q^2\, f_{q/p}(x_B, Q^2)$$

What about higher order?

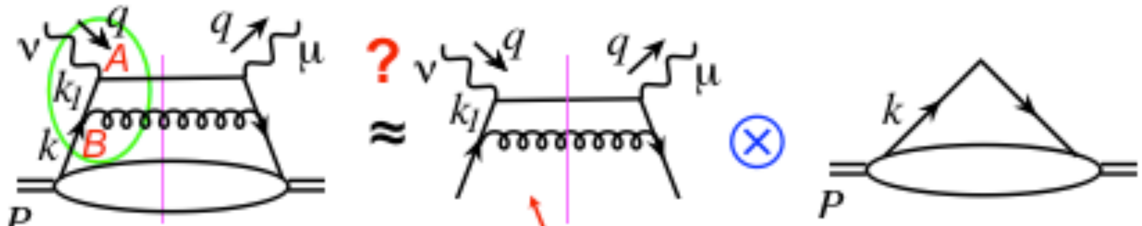
- Going beyond the leading order of the DIS, we face another divergence



- pQCD calculations: understand and make sense of all kinds of divergences
  - Ultraviolet (UV) divergence $k \to \infty$: renormalization (redefine coupling constant)
  - Collinear divergence $k//P$: redefine the PDFs and FFs
  - Soft divergence $k \to 0$: usually cancel between real and virtual diagrams for collinear PDFs/FFs; do not cancel for TMDs, leads to new evolution equations

QCD dynamics beyond tree level

- Going beyond leading order calculation



Collinear divergence!!! (from $k_1^2 \sim 0$)

$$\Rightarrow \int d^4k_1 \frac{i}{k_1^2+i\epsilon}\frac{-i}{k_1^2-i\epsilon} \Rightarrow \infty$$

$$k_1^2 = (k+k_g)^2 = 2EE_g(1-\cos\theta)$$

- $k_1^2 \sim 0$ intermediate quark is on-shell

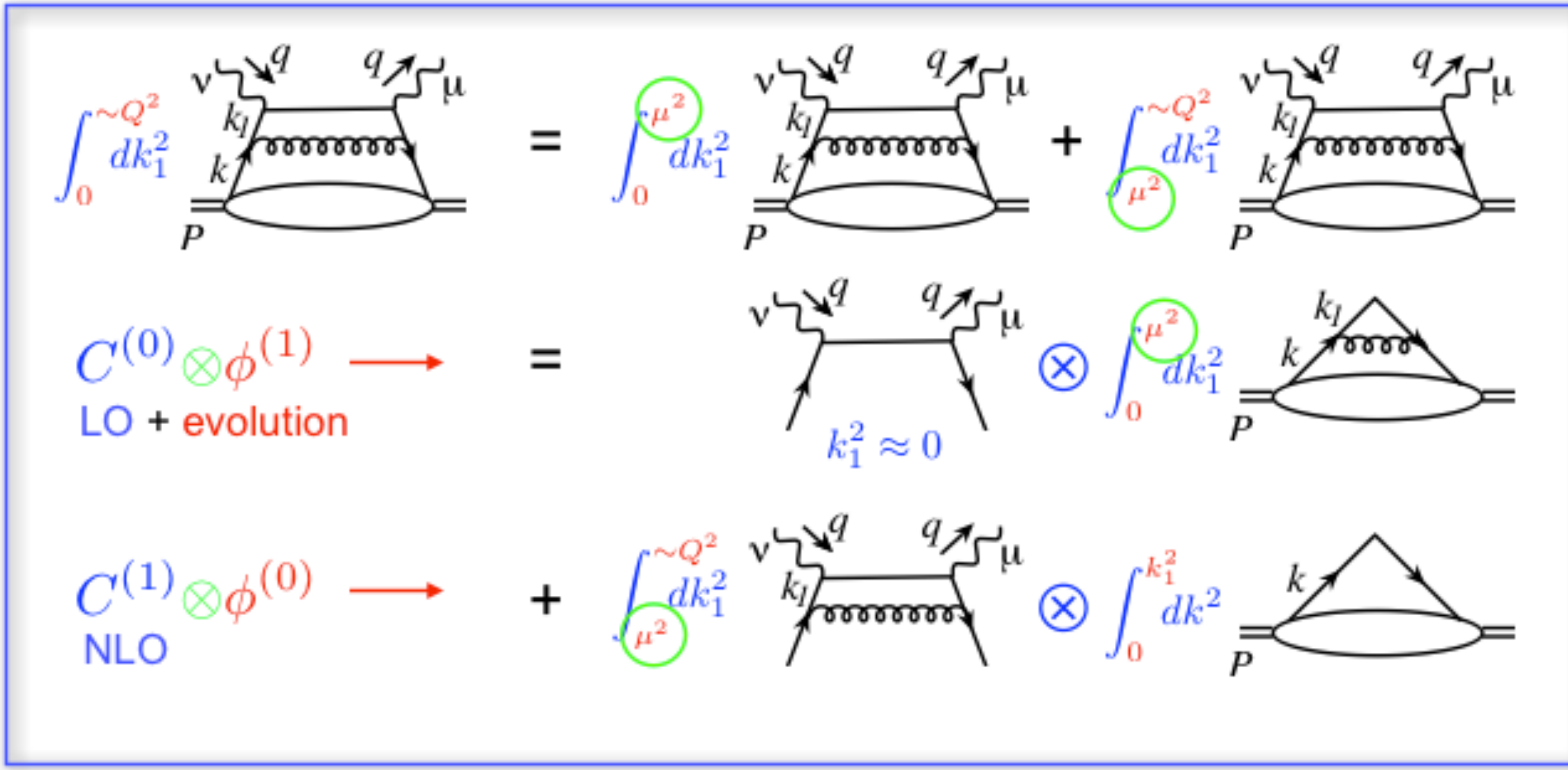
  $t_{AB} \rightarrow \infty$

- gluon radiation takes place long before the photon-quark interaction
  $\Longrightarrow$ a part of PDF

Partonic diagram has both long- and short-distance physics

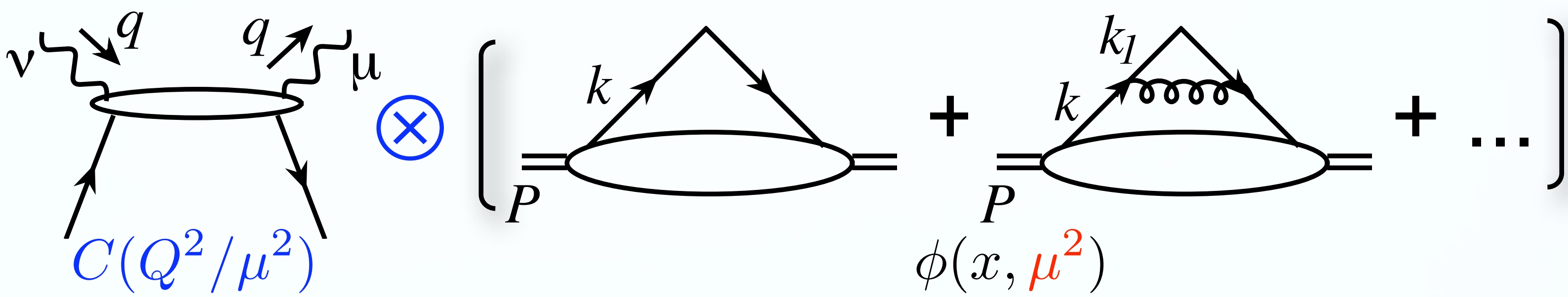
QCD factorization: beyond parton model

- Systematic remove all the long-distance physics into PDFs

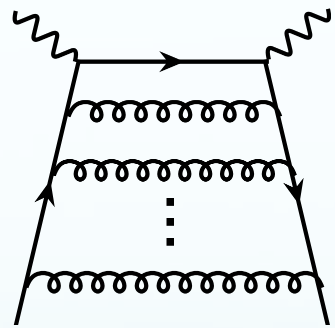


Scale-dependence of PDFs

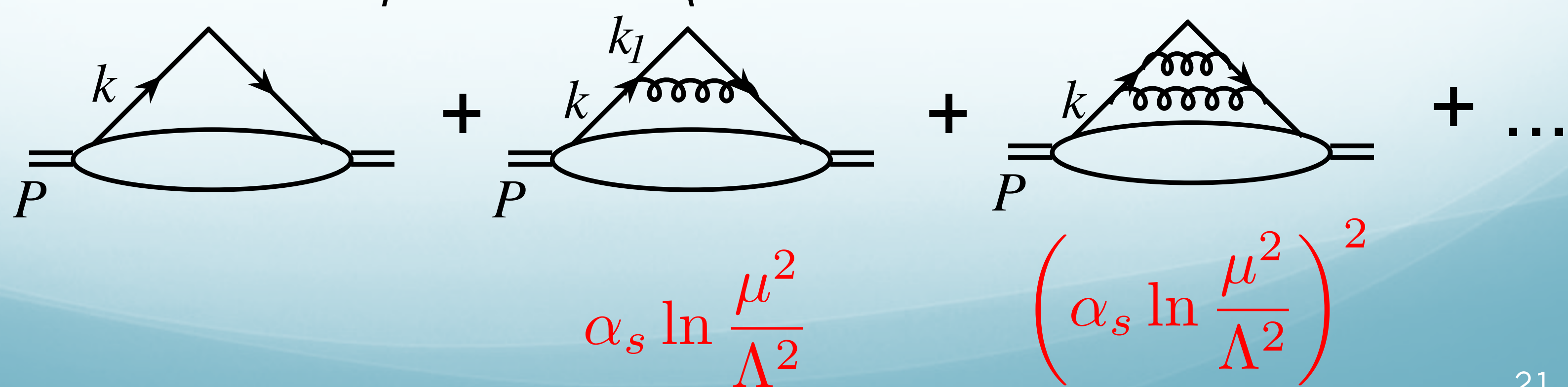
- Logarithmic contributions into parton distributions



- Going to even higher orders: QCD resummation of single logs



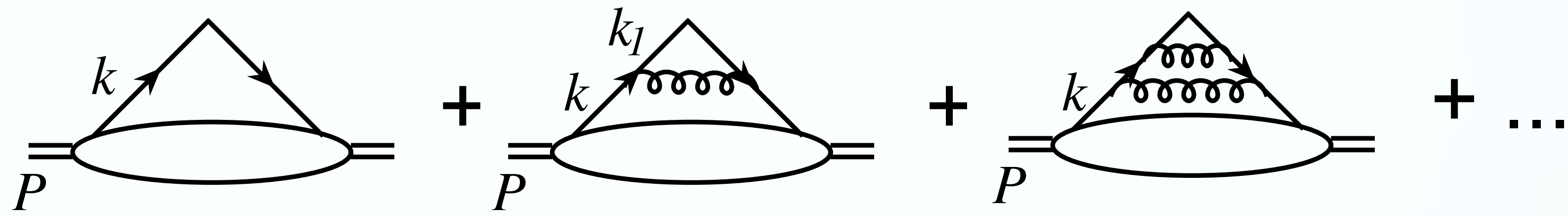
$\phi(x, \mu^2) =$



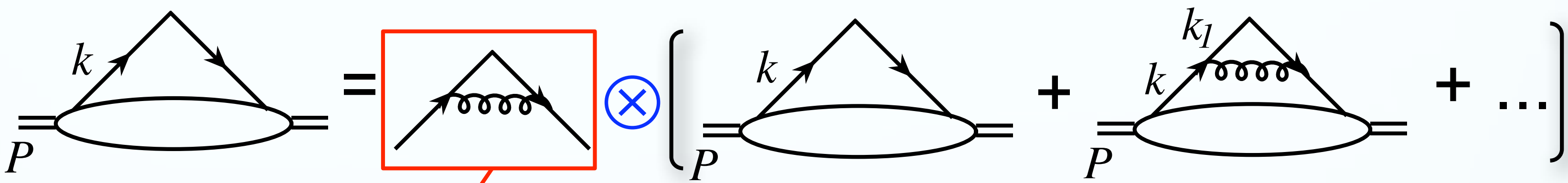
DGLAP evolution = resummation of single logs

- Evolution = Resum all the gluon radiation

$\phi(x, \mu^2) =$



$\phi(x, \mu^2)$ -



Evolution kernel
splitting function

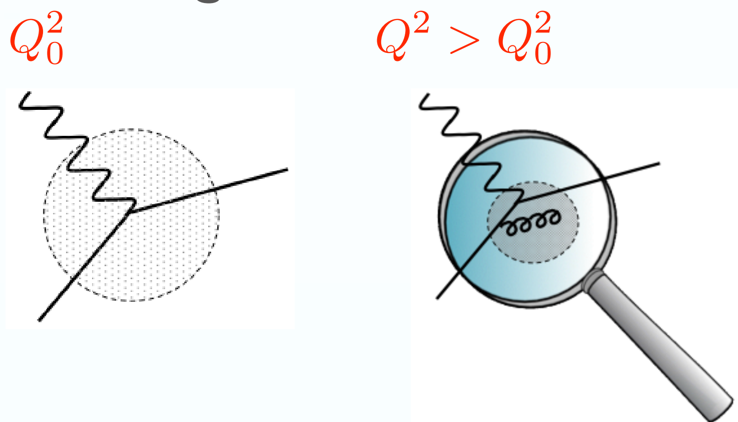
DGLAP Equation

$$\frac{\partial}{\partial \ln \mu^2} \phi_i(x, \mu^2) = \sum_j P_{ij}\left(\frac{x}{x'}\right) \otimes \phi_j(x', \mu^2)$$

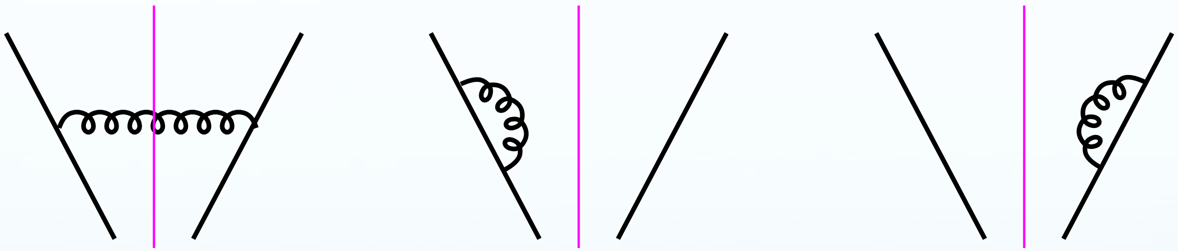
- By solving the evolution equation, one resums all the single logarithms of type $\left(\alpha_s \ln \frac{\mu^2}{\Lambda^2}\right)^n$

Evolutions of PDFs

- Perturbative change:



- Feynman diagrams for unpolarized PDFs



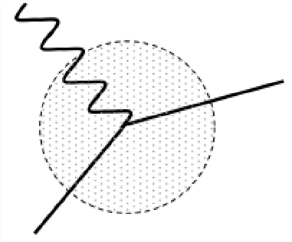
$$\frac{q(x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[P_{qq}(z)\, q(\xi,\mu_F)\right]$$

$$P_{qq}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

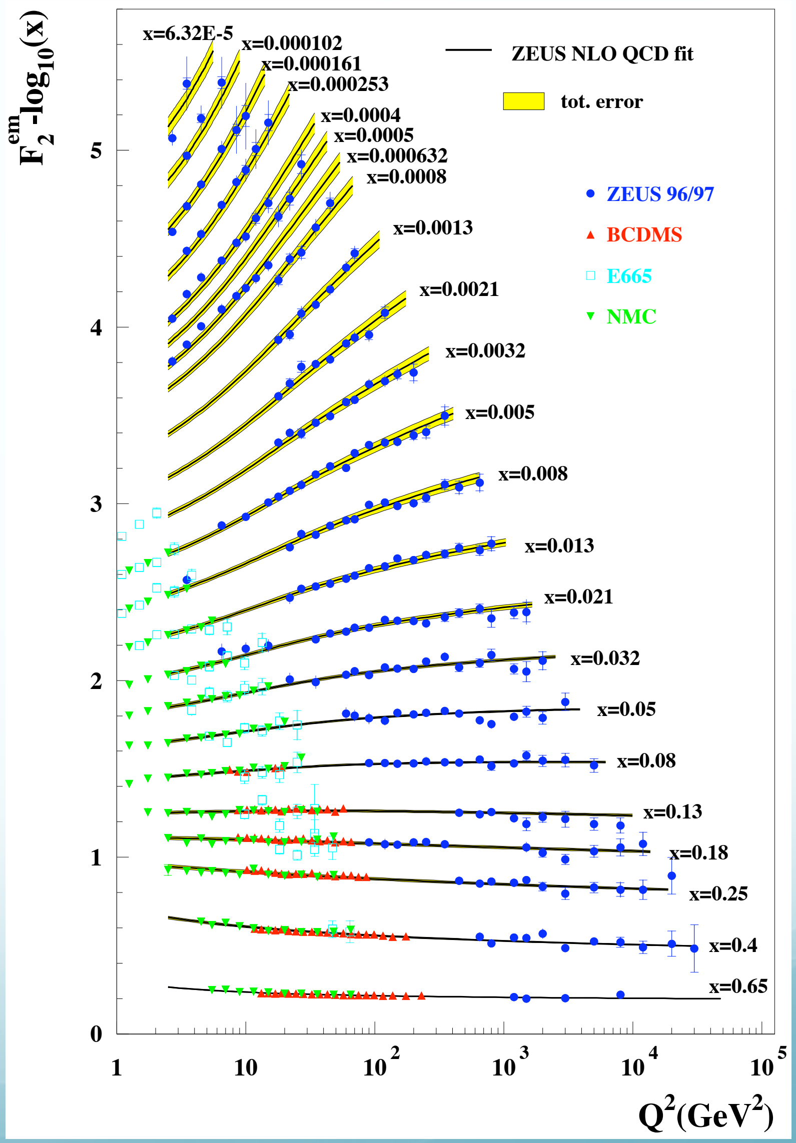
PDFs also depends on the scale of the probe

- Increase the energy scale, one sees parton picture differently

Q_0^2



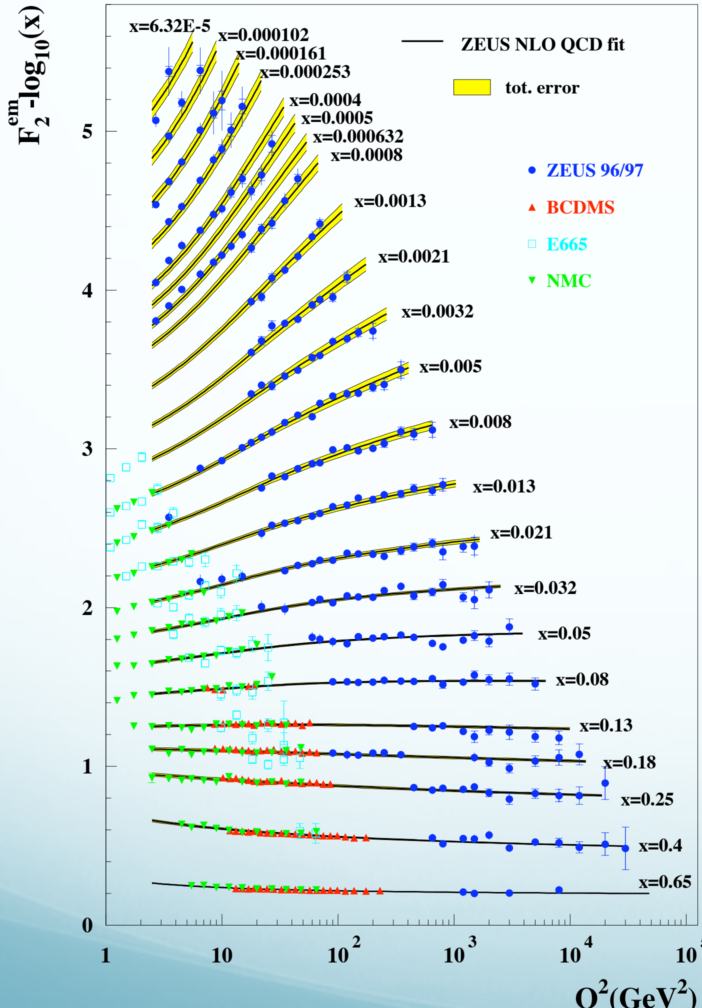
$Q^2 > Q_0^2$

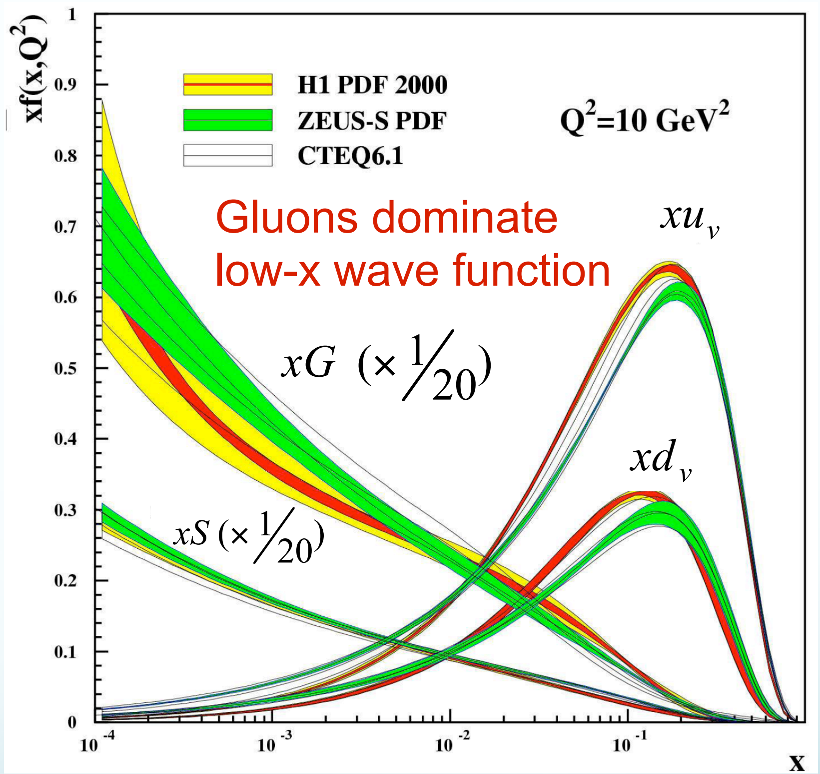


Success of QCD collinear factorization

- PDFs are universal and evolve via DGLAP equations

DIS: e+p





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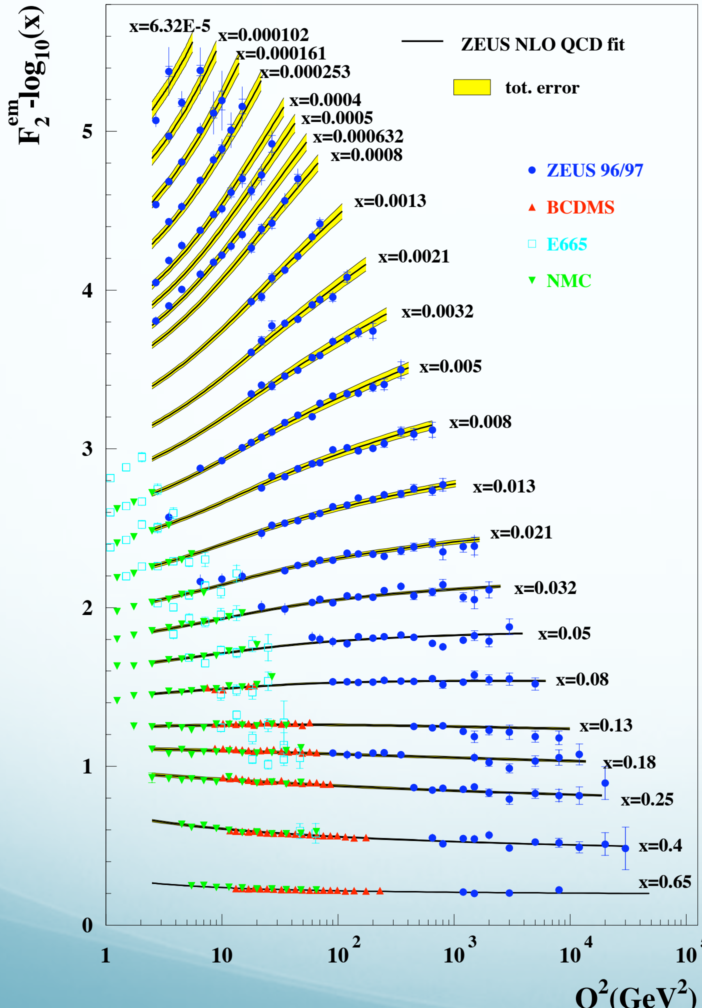
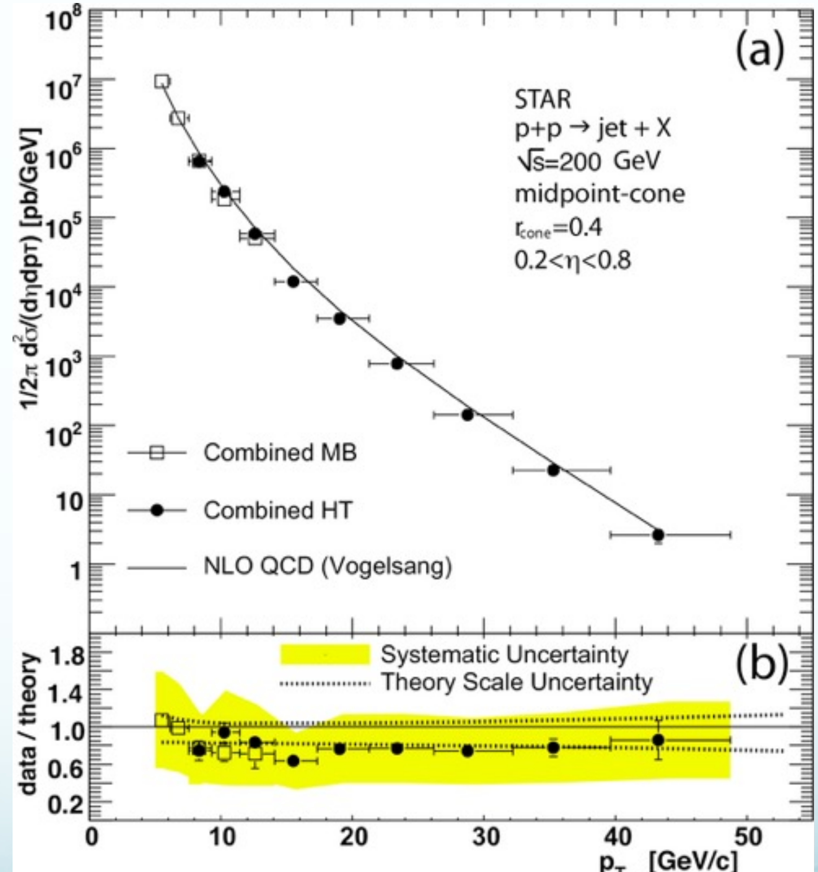


RHIC: p+p@200 GeV



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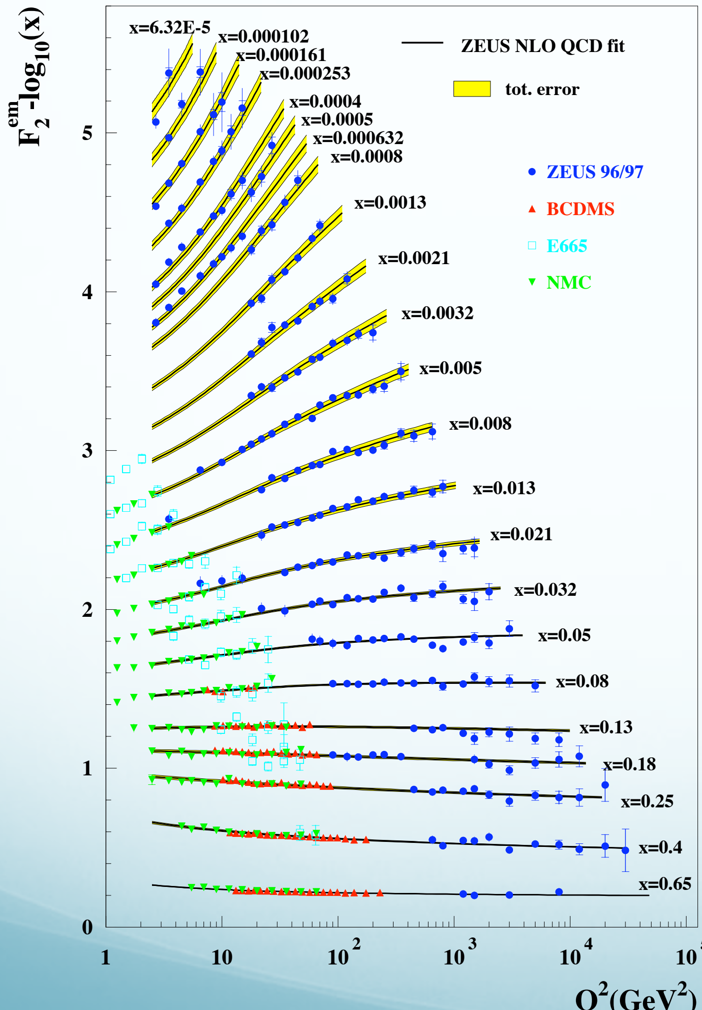
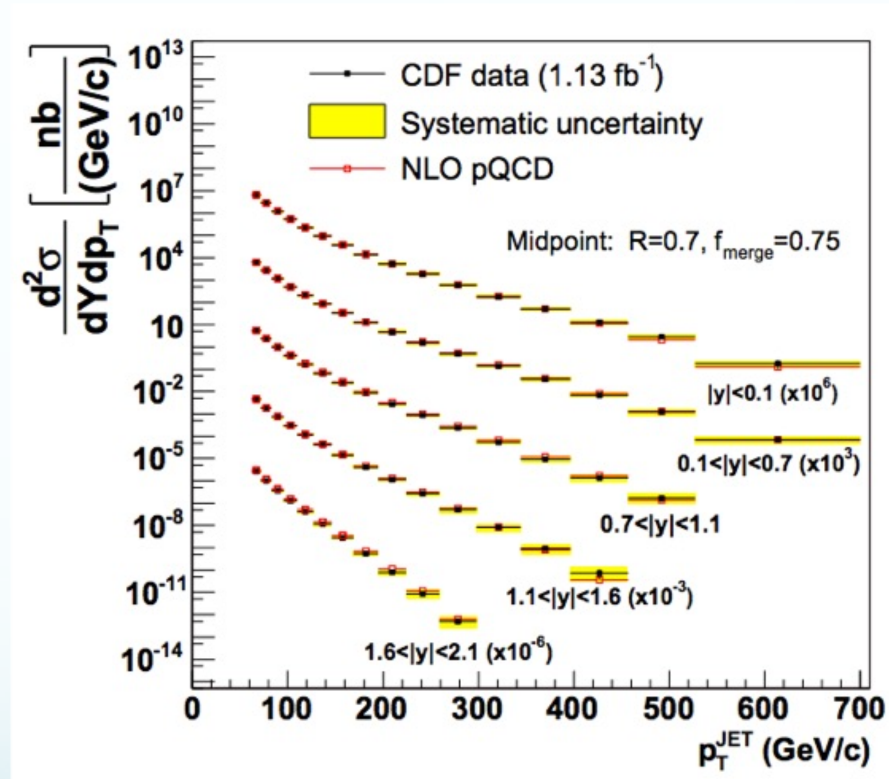


Tevatron: p+pbar@1.96 TeV



Success of QCD collinear factorization

- PDFs are universal and evolve via DGLAP equations

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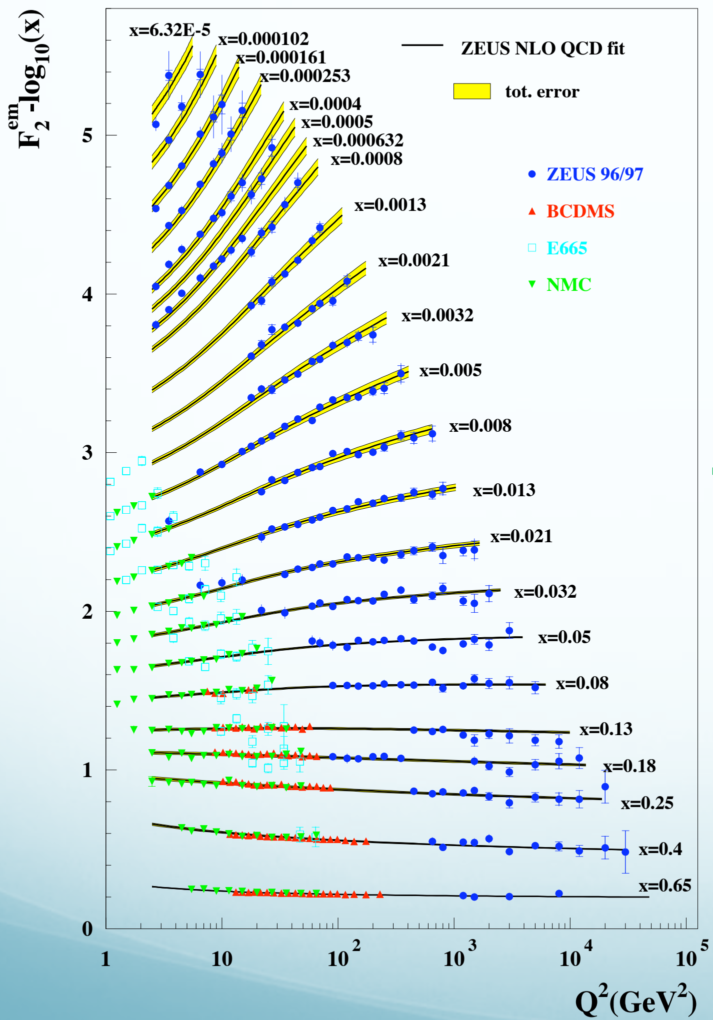
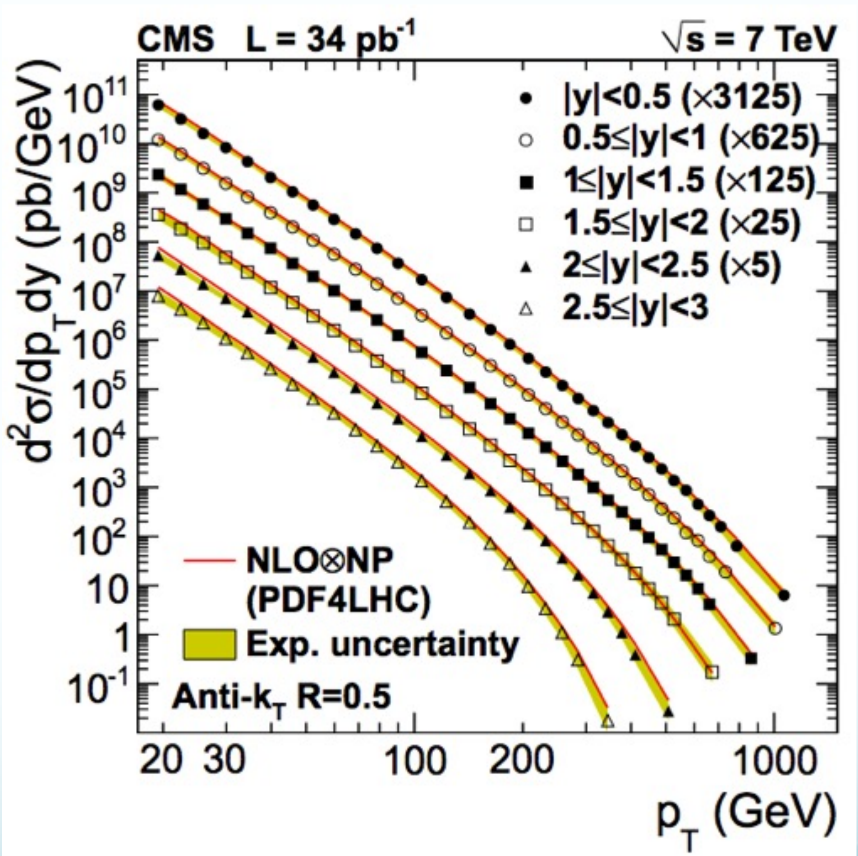
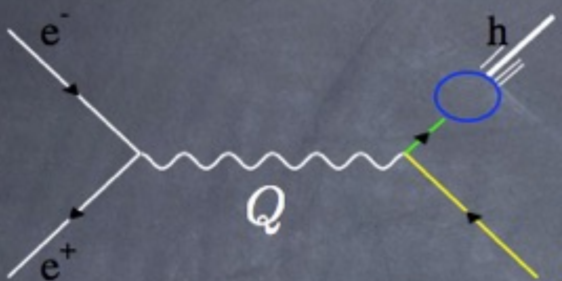


LHC: p+p@7 TeV



Same idea for hadron production

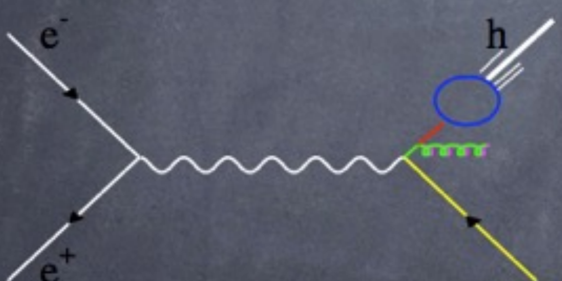
- Fragmentation function: another probability
  - Going to NLO, needs also to absorb the collinear divergence, and thus the scale dependence of the fragmentation function



$$z \equiv \frac{E_h}{E_q} = \frac{2E_h}{Q}$$

$$\frac{d\sigma}{dz}(e^+e^- \to h\,X) = \sum_q \sigma(e^+e^- \to q\bar{q})\,\left[D_q^h(z) + D_{\bar{q}}^h(z)\right]$$

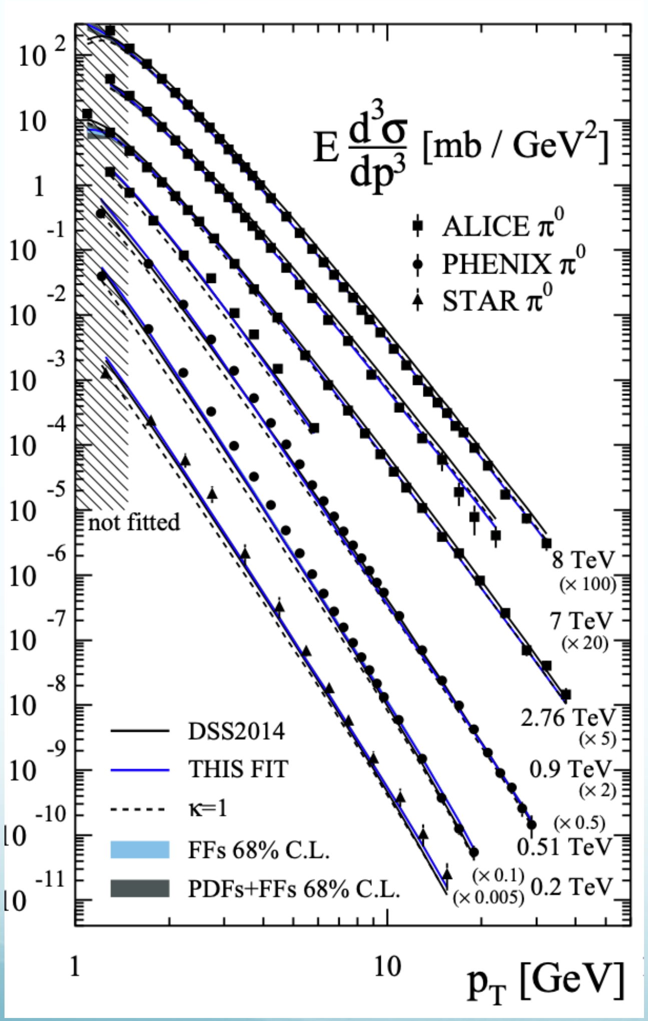
$$\sum_h \int_0^1 z\,D_q^h(z)\,dz = 1 \qquad \sum_q \int_0^1 \left[D_q^h(z) + D_{\bar{q}}^h(z)\right]\,dz = n_h$$

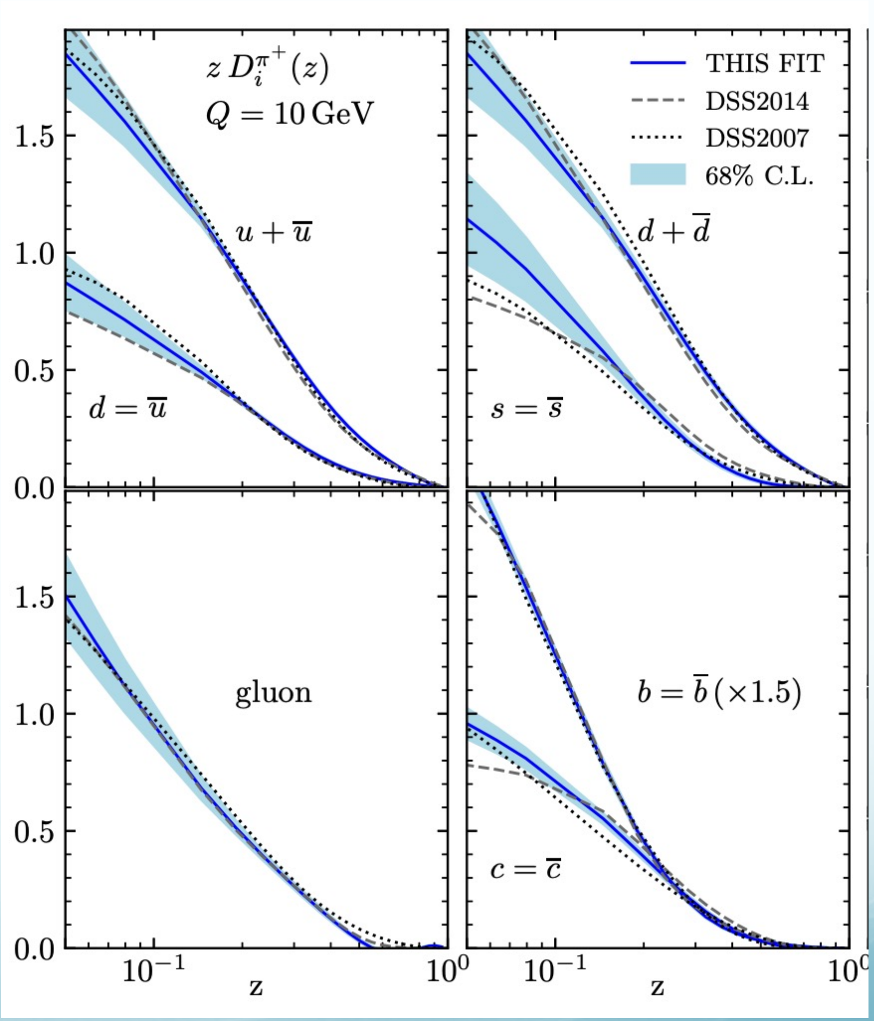


$$D_q^h(z) \longrightarrow D_q^h(z, Q^2)$$

It also works well

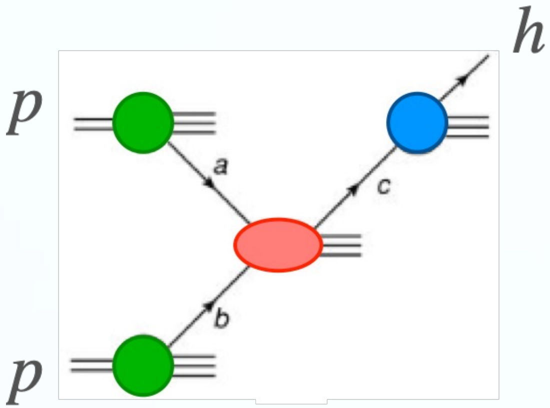
- DSS parameterizations for the fragmentation function





Single hadron production in p+p

- Single hadron production



$$p + p \to h + X$$

$$\frac{d\sigma^{p+p\to h+X}}{dp_T d\eta} = \sum_{a,b,c} f_a(x_a,\mu^2) \otimes f_b(x_b,\mu^2) \otimes \hat{\sigma}_{ab\to c} \otimes D_{h/c}(z,\mu^2)$$

Google Colab: collinear factorization example

- Please check this link to view, copy the code to your Google Drive to start playing with it
  - If possible, please create a google account before the lecture
  - Please download this experimental data file and upload to your own google drive

Summary

- Asymptotic freedom: allow one to calculate partonic cross sections
- Parton distribution functions and fragmentation functions
- DGLAP evolution