QCD at small Bjorken x:
the Color Glass Condensate (CGC)

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Outline of lectures

Lecture 1: introduction to DIS, QCD, small x, motivation for CGC
(electronic presentation)

Lecture 2: gluon saturation, eikonal scattering, Wilson lines, dipoles,...
(blackboard presentation)

Lecture 3: quantum corrections, BK evolution equation, solutions
(blackboard presentation)

Lecture 4: applications to high energy collisions, phenomenology,....
(blackboard presentation)

Literature

Textbook: Quantum Chromodynamics at high energy

by kovechegov and Levin

Reviews:

The Color Glass Condensate and high energy scattering in QCD:

Iancu and Venugopalan

The Color Glass Condensate: Gelis, Iancu, JJM and Venugopalan

Saturation physics and deuteron-gold collisions at RHIC: JJM and Kovchegov

....

Mining for gluon saturation at colliders: Morreale and Salazar

Many thanks to my friends and colleagues, A. Deshpande, M. Strattman and B. Surrow, whose slides I have freely used

Quantum ChromoDynamics (QCD)

Theory of strong interactions between quarks and gluons $\mathbf{SU(N_c)}$

$\mathbf{N_c = 3}$

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \sum^{\mathrm{f}} \bar{\Psi}^\alpha_i \left[i \not{D} - m_f \right]^{ij}_{\alpha\beta} \Psi^\beta_j$$

$$G^a_{\mu\nu}(x) \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

color index:
$$a, b, c = 1, \cdots, 8$$
$$\alpha, \beta = 1, 2, 3$$

f^{abc} *group structure constant*

Lorentz index: $\mu, \nu = 0, 1, 2, 3$

$\not{D} \equiv D_\mu \gamma^\mu$ *with* $\{ \gamma^\mu , \gamma^\nu \} = 2\, g^{\mu\nu}$

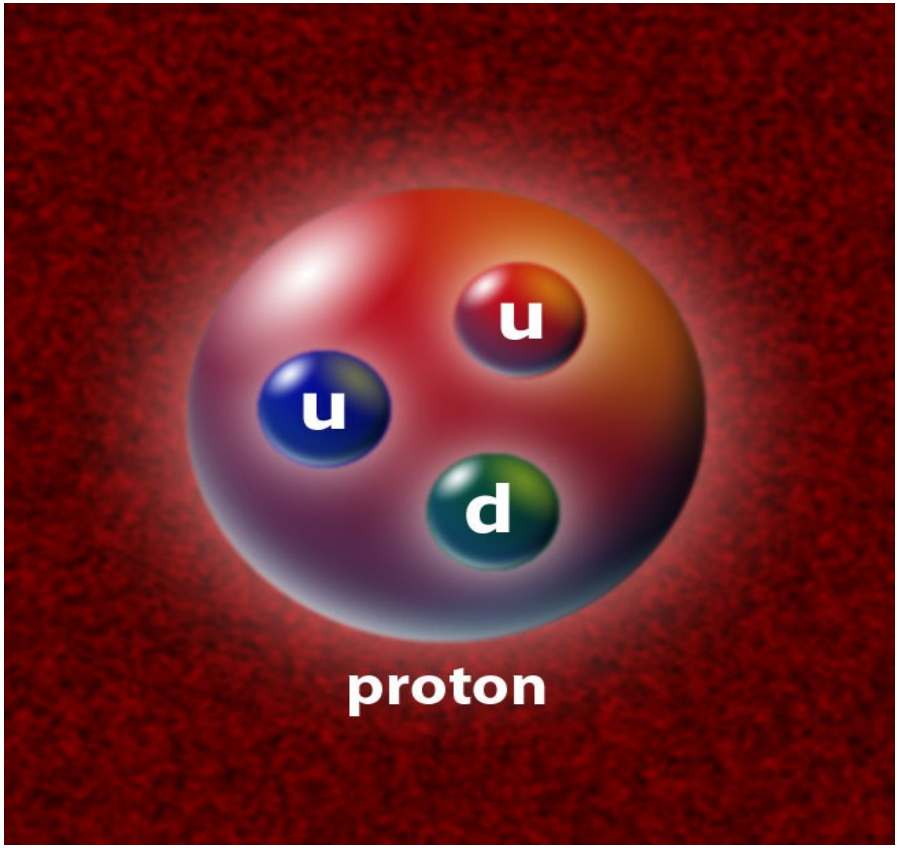
spinor index: $i, j = 1, 2, 3, 4$

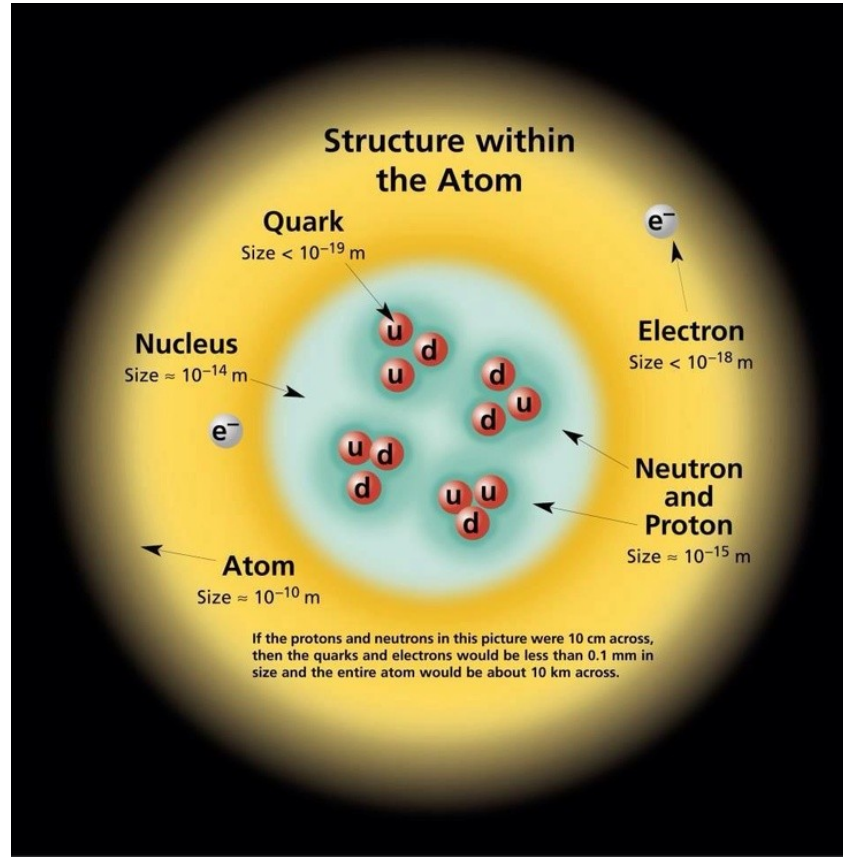
$D_\mu \equiv \partial_\mu + i g A_\mu$ *covariant derivative*

Quarks:
Fermions, spin 1/2
4x1 spinor, come in N_c *colors*
6 flavors (up, down, ..., top)
carry electric charge

Gluons:
Bosons, spin 1
come in $N_c^2 - 1$ *colors*
flavor blind
have no electric charge

Quantum ChromoDynamics (QCD)

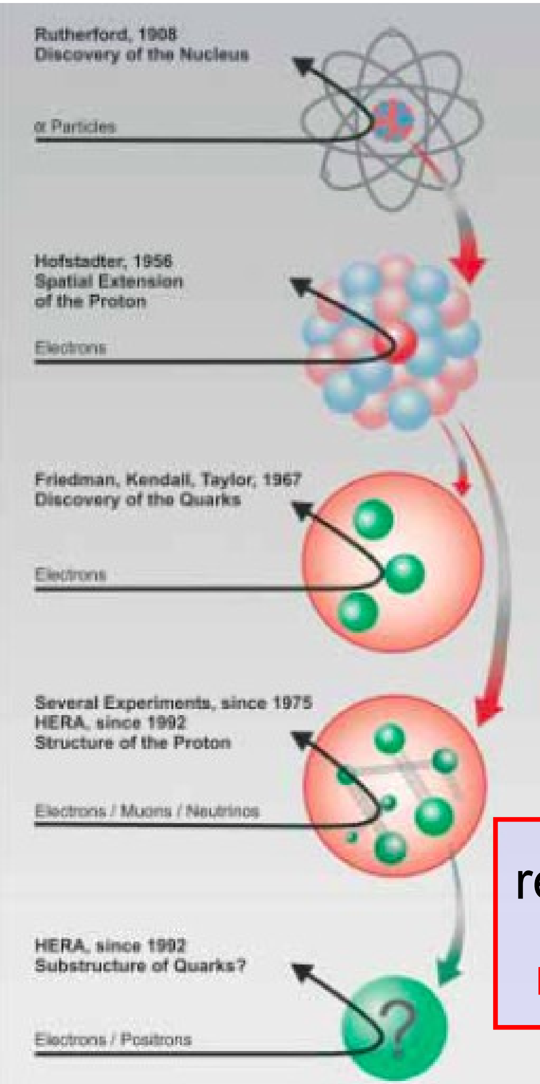




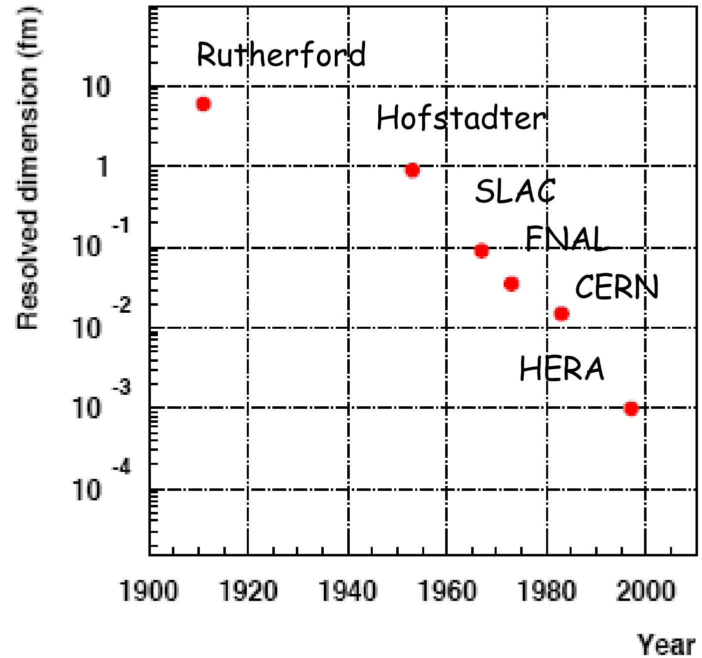
strong force confining quarks inside a proton

(and keeping protons inside a nucleus)

Deep Inelastic Scattering (DIS)

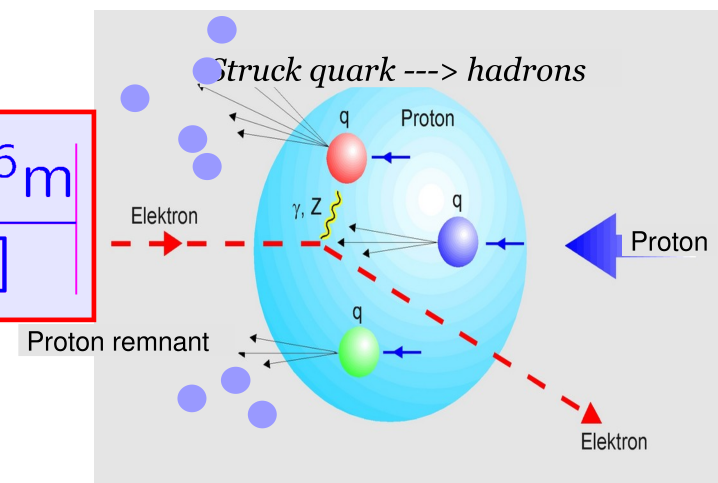


Probing
smaller distances
requires
larger momentum
transfer q
(small wavelength)

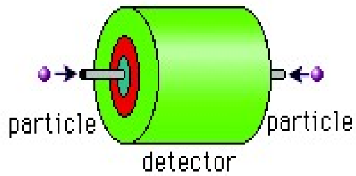


resolution: $\frac{\hbar}{Q} \approx \frac{2 \times 10^{-16}\text{m}}{Q[\text{GeV}]}$

r » 1/Q

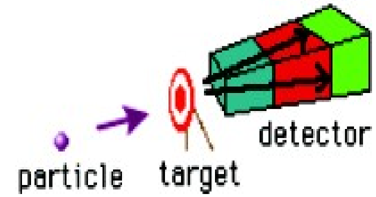


Collider experiment: Electron-Proton collisions at HERA (DESY, Hamburg, Germany)



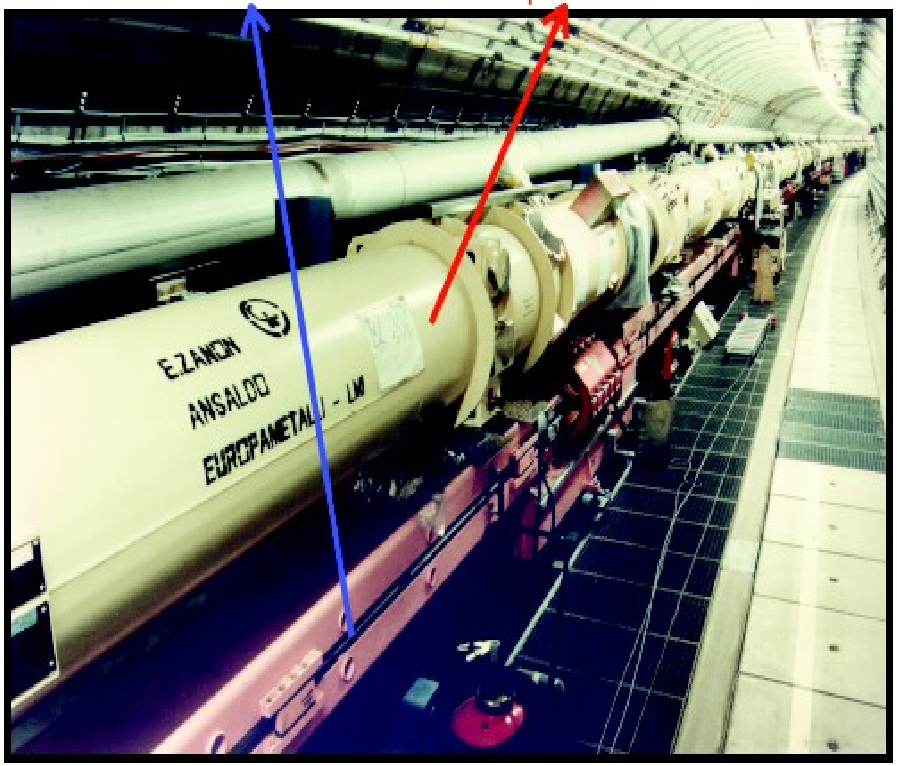
Equivalent to fixed target of

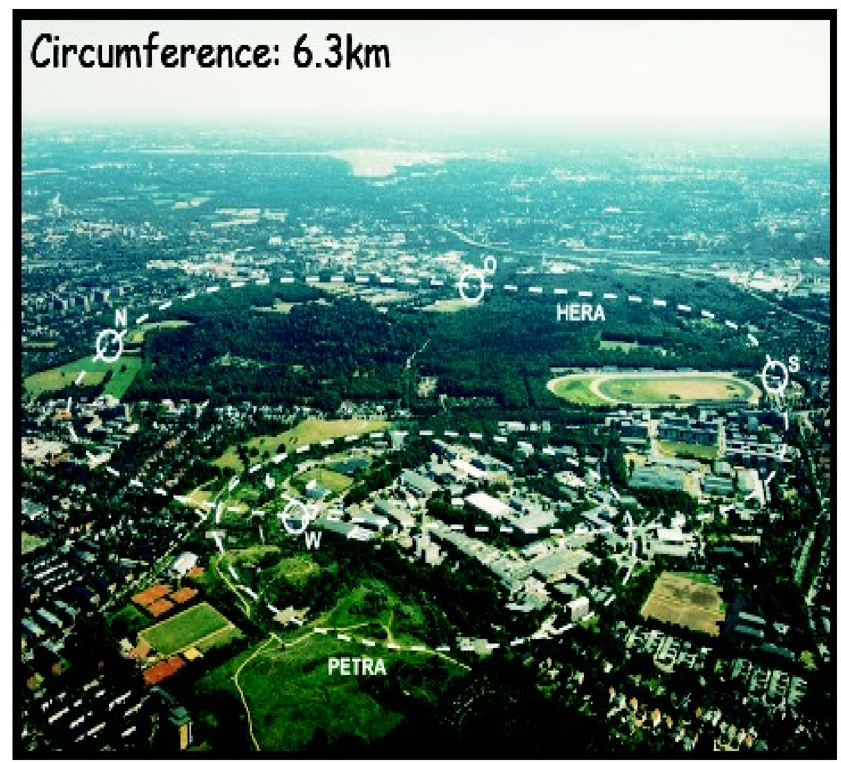
E_e = 50600 GeV:



E_e = 27.5 GeV

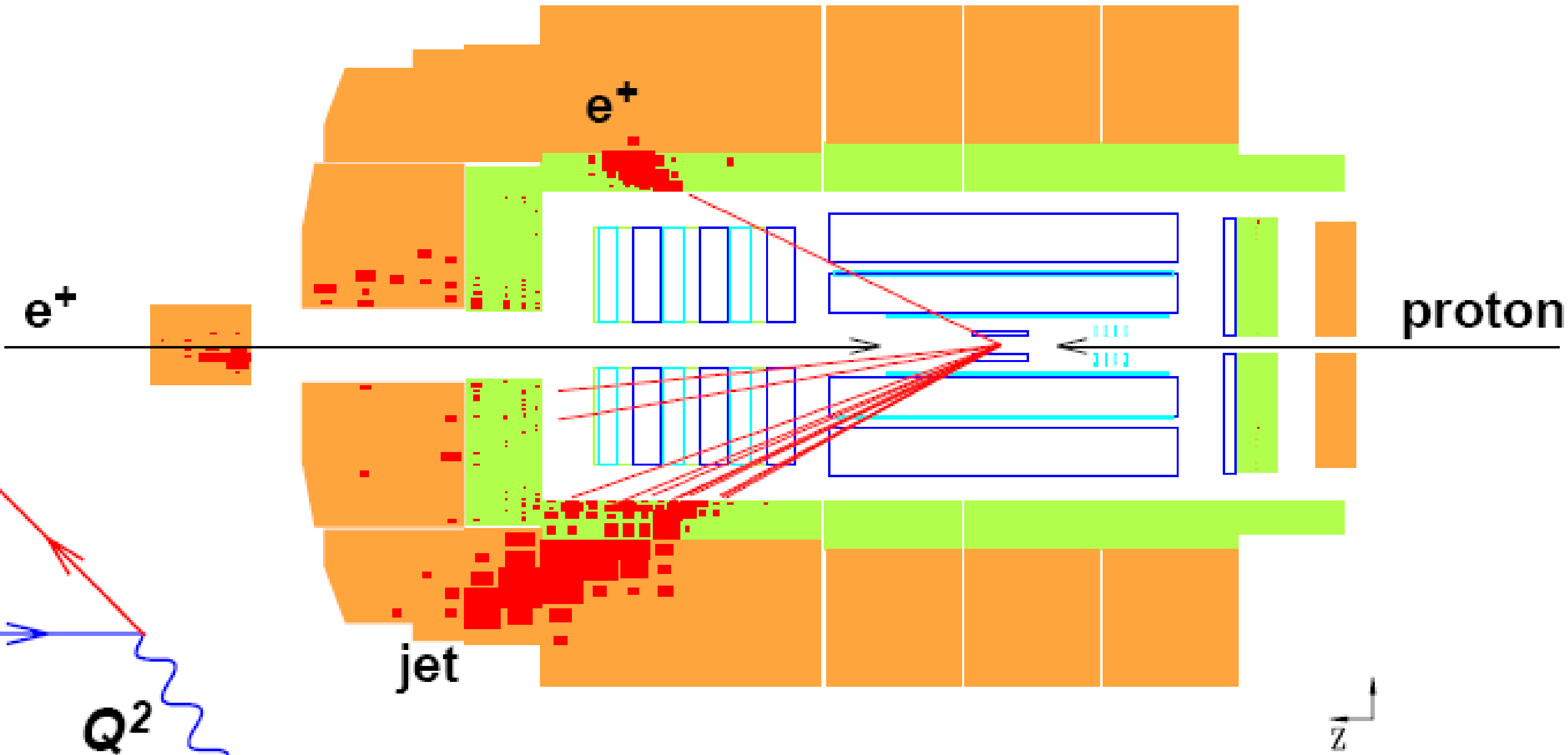
E_p = 920 GeV

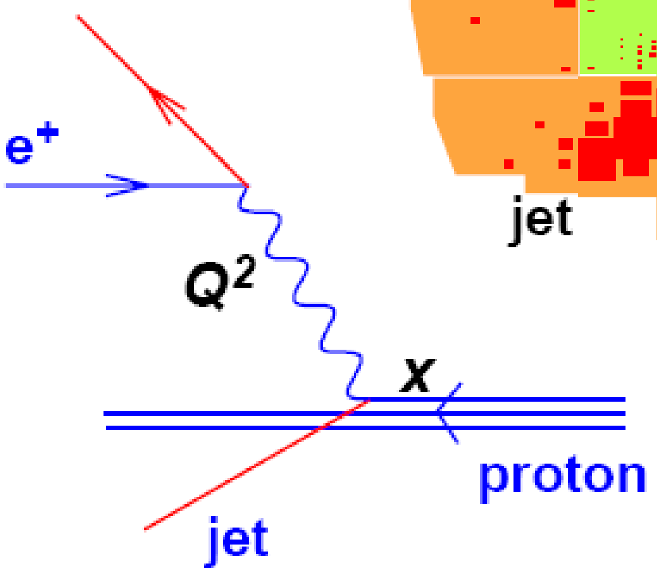




A DIS event

Q^2 = 25030 GeV^2; y = 0.56; **x=0.50**



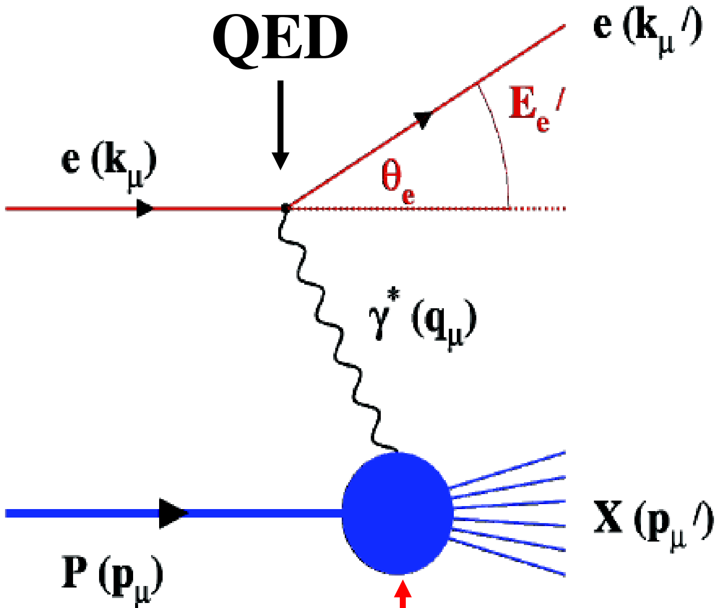


H1 Run 122145 Event 69506

Date 19/09/1995

Deep Inelastic Scattering (DIS)

probing hadron structure



QCD
(structure functions)

Kinematic Invariants

$$Q^2 = -q^2 = -(k_\mu - k'_\mu)^2$$

$$Q^2 = 4E_e E'_e \sin^2\left(\frac{\theta'_e}{2}\right)$$

Measure of resolution power

$$y = \frac{pq}{pk} = 1 - \frac{E'_e}{E_e}\cos^2\left(\frac{\theta'_e}{2}\right)$$

Measure of inelasticity

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

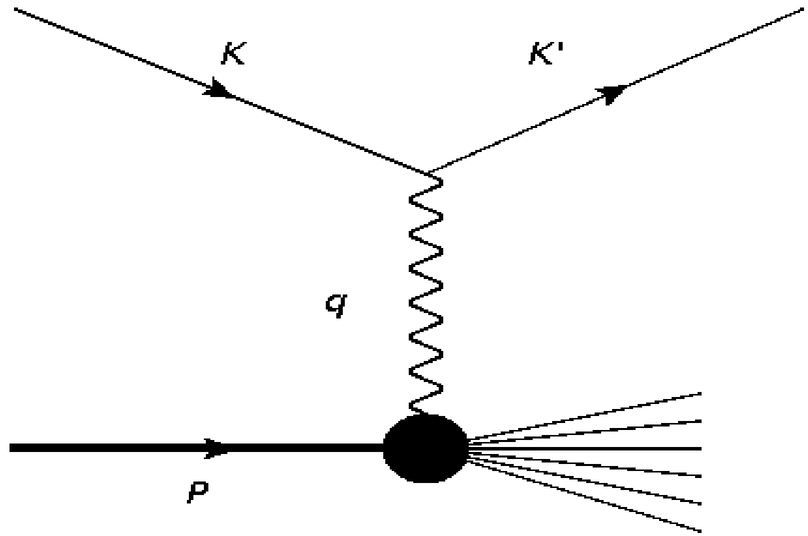
Measure of momentum fraction of struck quark

$$\mathbf{s \equiv (p + k)^2}$$

Deep Inelastic Scattering

A first analysis of DIS does not require any knowledge of QCD!

we know how a photon (electroweak vector bosons) couples:



$$\mathbf{d}\sigma = \frac{4\alpha^2}{s} \frac{d^3\mathbf{k}'}{2|\mathbf{k}'|} \quad \frac{1}{Q^4} \quad L^{\mu\nu}(k,q) \quad W_{\mu\nu}(p,q)$$

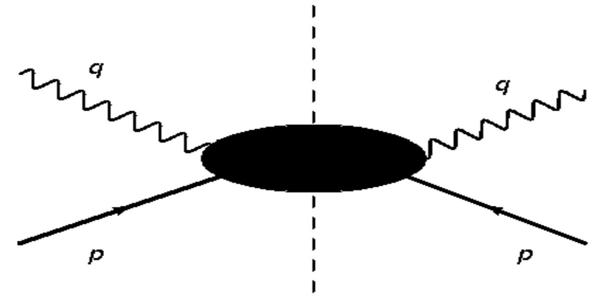
lepton phase space — **photon propagator²** — **leptonic tensor** — **hadronic tensor: contains all QCD dynamics**

with $\mathrm{L}^{\mu\nu}(k,q) \equiv 2\left(k^\mu\, k'^{\,\nu} + k^\nu\, k'^{\,\mu} - g^{\mu\nu}\, k\cdot k'\right)$

Deep Inelastic Scattering

strong interactions: contained in the hadronic tensor $W_{\mu\nu}(p,q)$

given by square of to all orders in QCD coupling constant $\gamma^\star(q)\,h(p) \longrightarrow X$

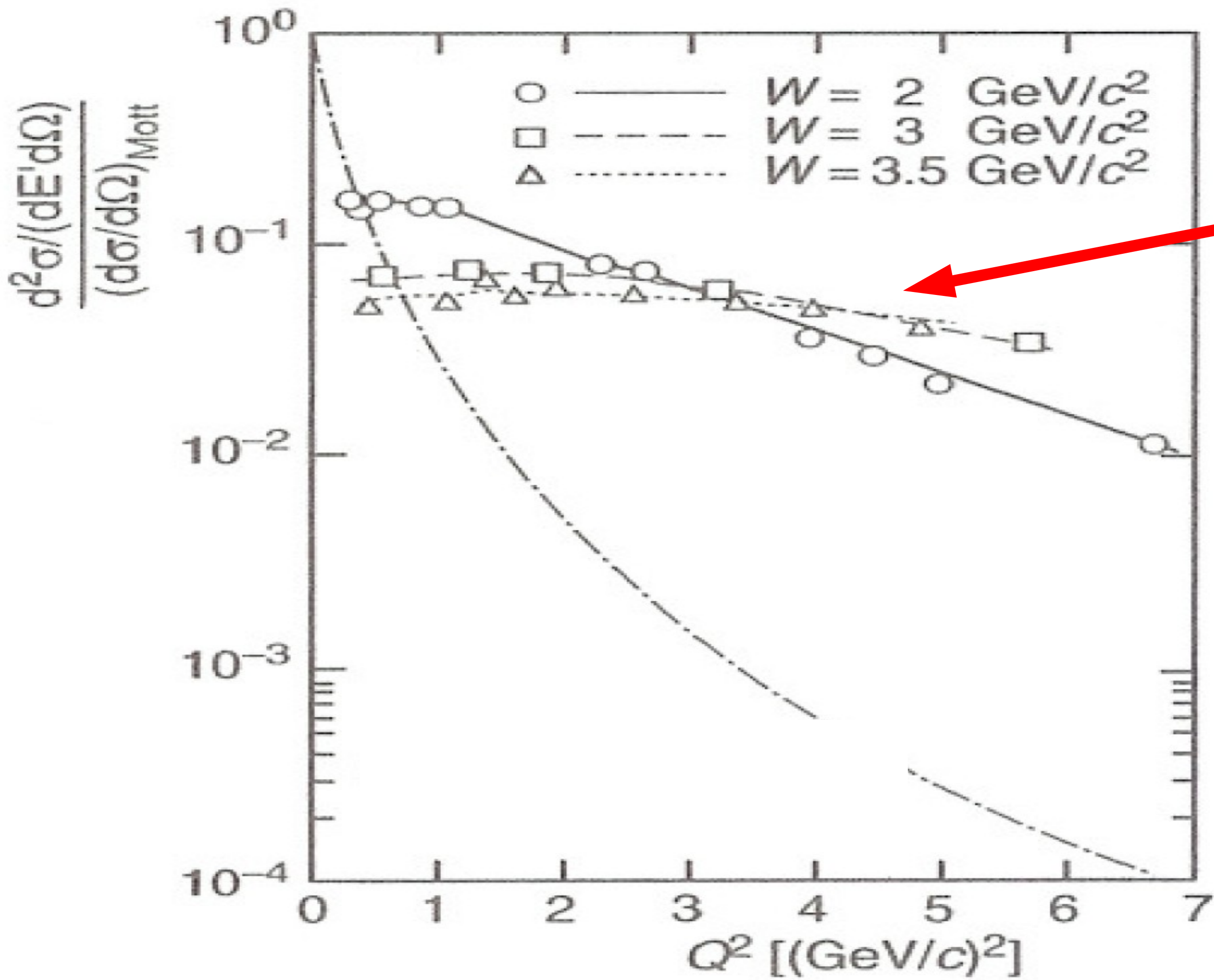


Lorentz + parity symmetries + current conservation:

$$W^{\mu\nu} = -\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) F_1(x,Q^2) + \left(p^\mu - \frac{p\cdot q}{q^2}\,q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}\,q^\nu\right)\frac{1}{p\cdot q}F_2(x,Q^2)$$

structure functions

Early DIS experiments: SLAC-MIT



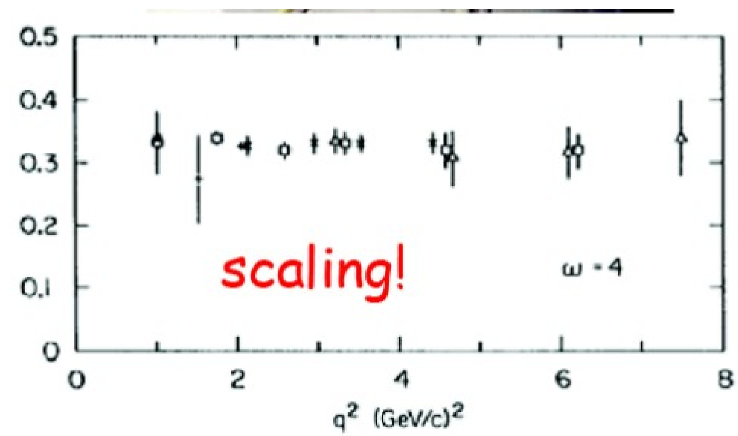
scale invariance:
scattering on
point-like objects

Nobel prize 1990: Friedman, Kendall, Taylor

Parton model of a hadron

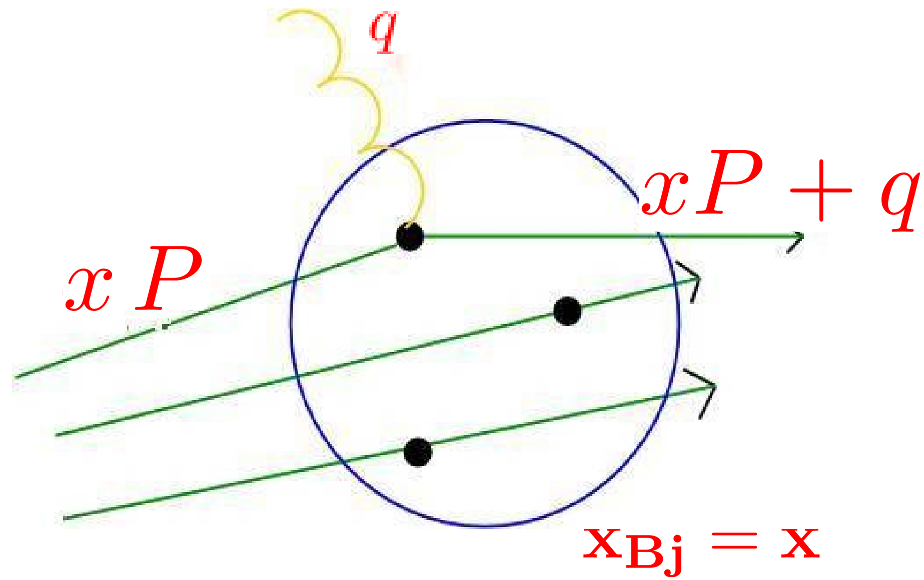
Bjorken: $Q^2, S \to \infty \qquad x_{Bj} = \frac{Q^2}{S}$

structure functions depend only on x_{Bj}



Feynman:

parton constituents of proton are "free" on time scale $1/Q << 1/\Lambda$ (interaction time scale between partons)



$$\mathbf{F_2(x) \equiv \sum^{f} e_f^2 \, x[q_f(x) + \bar{q}_f(x)]}$$

space-time picture of DIS

light cone variables: separation of large and small components of vectors under a boost

$$P^+ \equiv \frac{E + P_z}{\sqrt{2}} \qquad \mathrm{p}\cdot x = p^+ x^- + p^- x^+ - p_\perp \cdot x_\perp$$

$$P^- \equiv \frac{E - P_z}{\sqrt{2}}$$

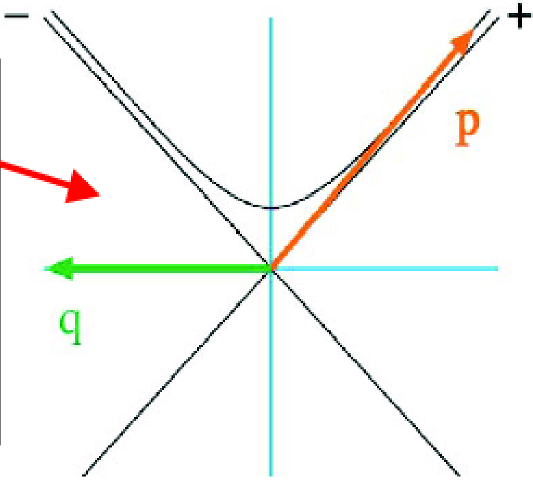
$$P_t = P_t$$

$$(\mathbf{V^+}, \mathbf{V^-}, \mathbf{V_t}) \to (\mathbf{e}^\omega \mathbf{V^+}, \mathbf{e}^{-\omega} \mathbf{V^-}, \mathbf{V_t})$$

$$\text{with} \quad \mathrm{e}^\omega = \frac{\mathbf{Q}}{\mathrm{x\, m_h}}$$

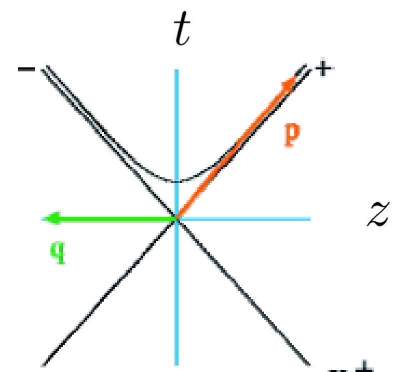
$$\begin{aligned} t &\longrightarrow \gamma(t - \beta\, z) \\ x &\longrightarrow x \\ y &\longrightarrow y \\ z &\longrightarrow \gamma(z - \beta t) \end{aligned}$$

4-vector	hadron rest frame	Breit frame
$(p^+, p^-, \vec{p}_T)$	$\frac{1}{\sqrt{2}}(m_h, m_h, \vec{0})$	$\frac{1}{\sqrt{2}}(\frac{Q}{x}, \frac{x m_h^2}{Q}, \vec{0})$
$(q^+, q^-, \vec{q}_T)$	$\frac{1}{\sqrt{2}}(-m_h x, \frac{Q^2}{m_h x}, \vec{0})$	$\frac{1}{\sqrt{2}}(-Q, Q, \vec{0})$



space-time picture of DIS

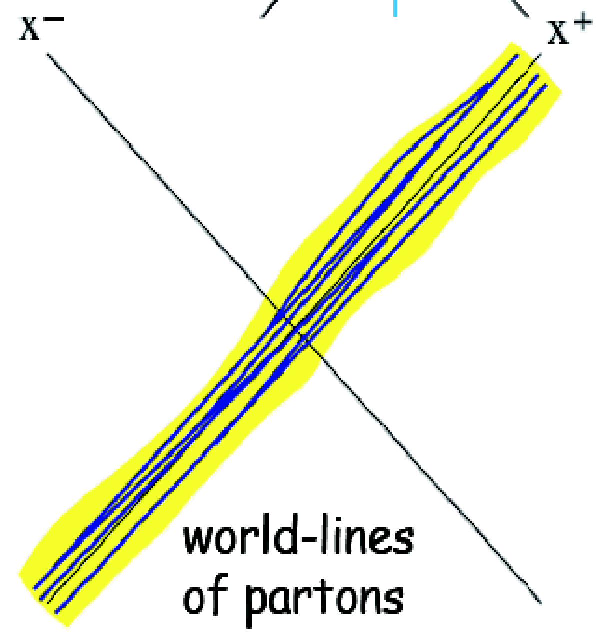
simple estimate for typical time-scale of interactions among the partons inside a fast-moving hadron:



rest frame: $\Delta x^{+} \sim \Delta x^{-} \sim \frac{1}{m}$

Breit frame: $\Delta x^{+} \sim \frac{1}{m}\frac{Q}{m} = \frac{Q}{m^2}$ large

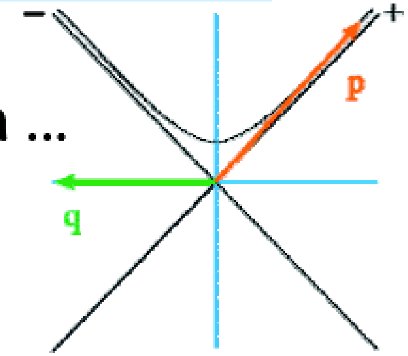
$\Delta x^{-} \sim \frac{1}{m}\frac{m}{Q} = \frac{1}{Q}$ small



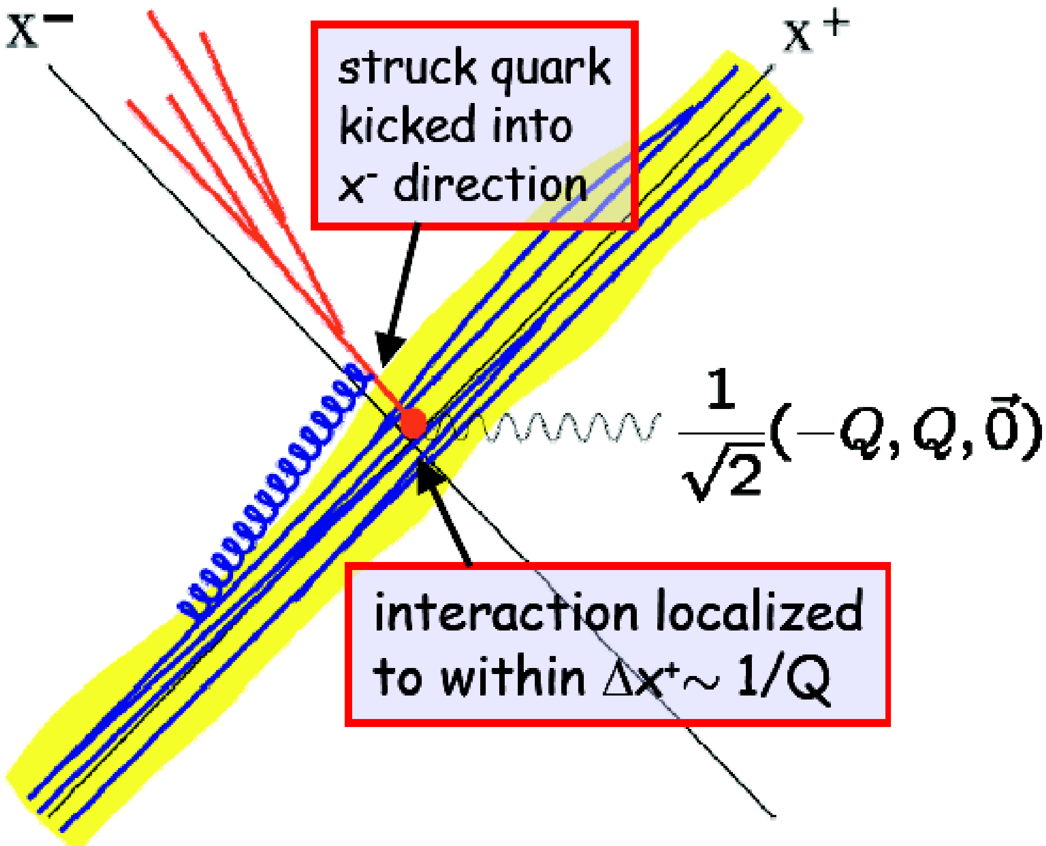
interactions between partons are spread out inside a fast moving hadron

How does this compare with the time-scale of the hard scattering?

space-time picture of DIS



now let the virtual photon meet our fast moving hadron ...

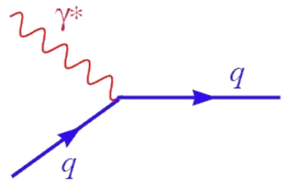


upshot:

- partons are free during the hard interaction
- hadron effectively consists of partons that have momenta $(p_i^+, p_i^-, \vec{p}_i)$
- convenient to introduce momentum fractions
  $0 < \xi_i \equiv p_i^+/p^+ < 1$

DIS in the QCD-improved parton model

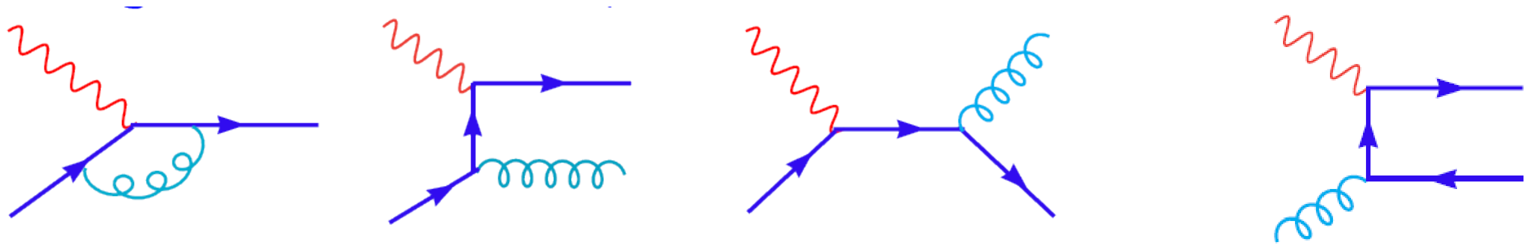
we got a long way (parton model) *without* invoking QCD



now we have to study **QCD dynamics in DIS**

– this leads to similar problems already encountered in e^+e^-

let's try to compute the **$O(\alpha_s)$ QCD corrections** to the naive picture



α_s corrections to the LO process photon-gluon fusion

caveat: ***expect divergencies***

related to soft/collinear emission or from loops

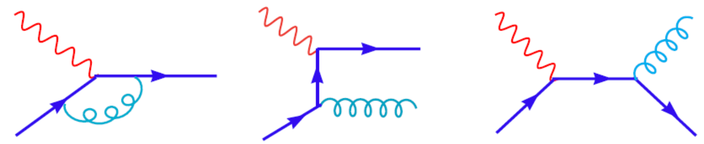
what to do with infinities?

introduce "**regulator**" in the intermediate stages, remove it at the end

general structure of the QCD corrections [O(α_s)]

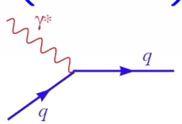
using small quark/gluon mass as a regulator:

$$\left.\frac{d^2\hat{\sigma}}{dx\,dQ^2}\right|_{F_2} \equiv \hat{F}_2^q$$



LO

$$= e_q^2 x \left[\delta(1-x) + \frac{\alpha_s(\mu_r)}{4\pi}\left[P_{qq}(x)\ln\frac{Q^2}{m_g^2} + C_2^q(x)\right]\right]$$

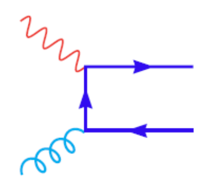


large logarithms (collinear emission)

finite coefficients

$$\left.\frac{d^2\hat{\sigma}}{dx\,dQ^2}\right|_{F_2} \equiv \hat{F}_2^g$$

$$= \sum_q e_q^2 x \left[0 + \frac{\alpha_s(\mu_r)}{4\pi}\left[P_{qg}(x)\ln\frac{Q^2}{m_q^2} + C_2^g(x)\right]\right]$$



divergences absorbed into pdf

$$\mathbf{F_2(x, Q^2) \equiv \sum_f e_f^2\, x[q_f(x, Q^2) + \bar{q}_f(x, Q^2)]}$$

properties of LO splitting functions

soft gluon divergence (z=1)
regulated by plus distribution

$$P_{qq}^{(0)} = P_{\bar{q}\bar{q}}^{(0)} = C_F\left[\left(\frac{1+z^2}{1-z}\right)_+ + \frac{3}{2}\delta(1-z)\right]$$

$$P_{qg}^{(0)} = P_{\bar{q}g}^{(0)} = T_R\left(z^2 + (1-z)\right)$$

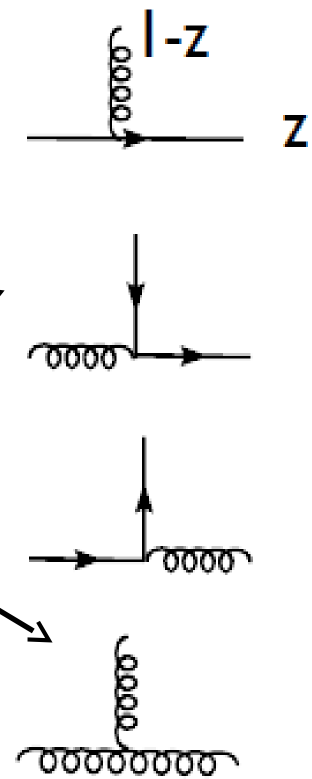
$$P_{gq}^{(0)} = P_{g\bar{q}}^{(0)} = C_F\frac{1+(1-z)^2}{z}$$

soft gluon divergence (z=0)
not reached, always z > 0

$$P_{gg}^{(0)} = 2C_A\left[z\left(\frac{1}{1-z}\right)_+ + \frac{1-z}{z} + z(1-z) + b_0\delta(1-z)\right]$$

soft gluon divergence (z=1)
regulated by plus distribution

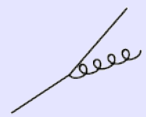
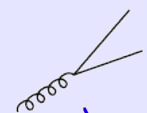
symmetric under
z -> (1-z)
except virtuals



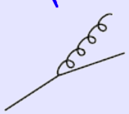
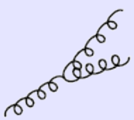
involves "**plus distribution**" $\int_0^1 dz\,[g(z)]_+\,f(z) \equiv \int_0^1 dz\,g(z)\,[f(z)-f(1)]$

condition: f(z) sufficiently smooth for z → 1

DGLAP “evolution” equation

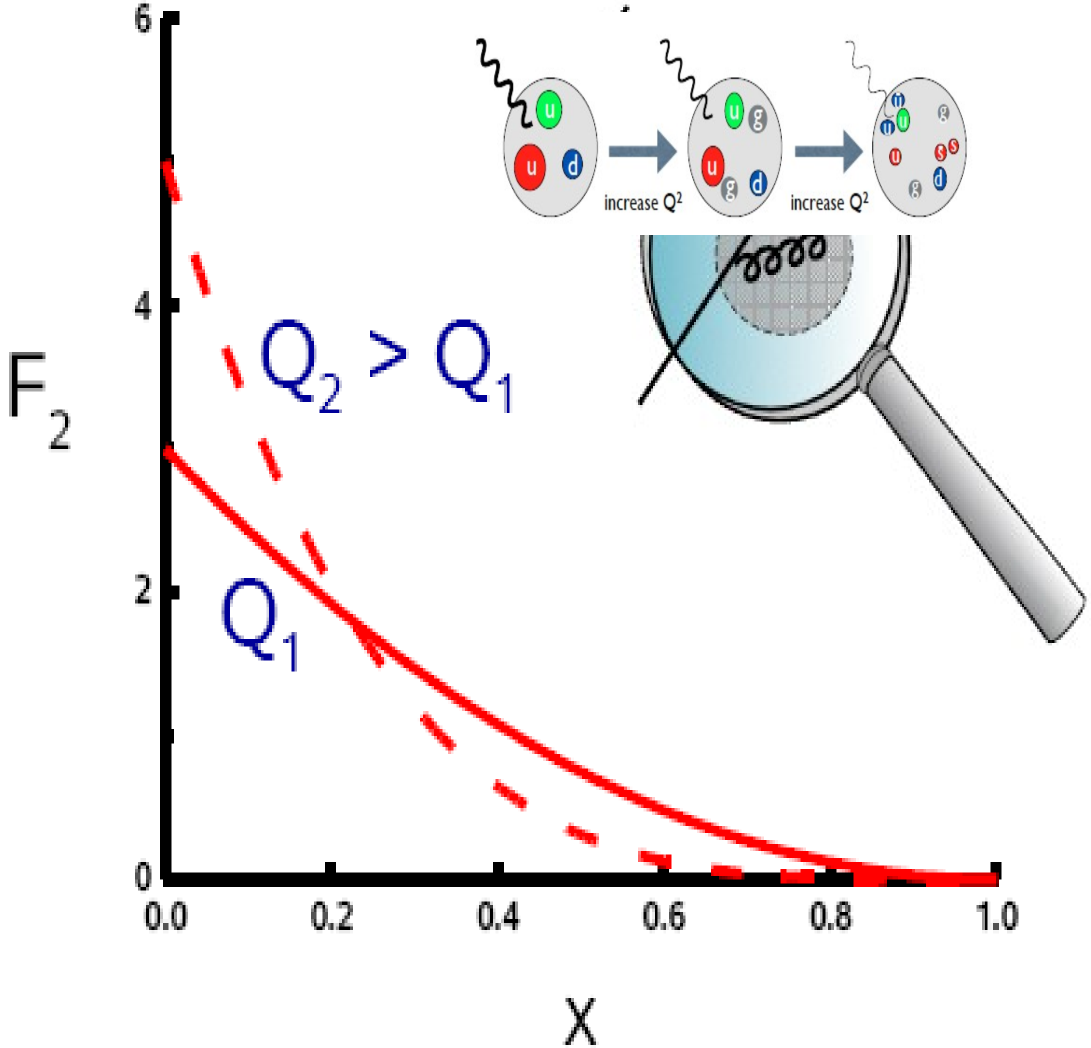
$$\frac{d}{d\ln\mu}\begin{pmatrix} q(x,\mu) \\ g(x,\mu) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \mathcal{P}_{qq} & \mathcal{P}_{qg} \\ \mathcal{P}_{gq} & \mathcal{P}_{gg} \end{pmatrix}_{(z,\alpha_s)} \cdot \begin{pmatrix} q(x/z,\mu) \\ g(x/z),\mu) \end{pmatrix}$$

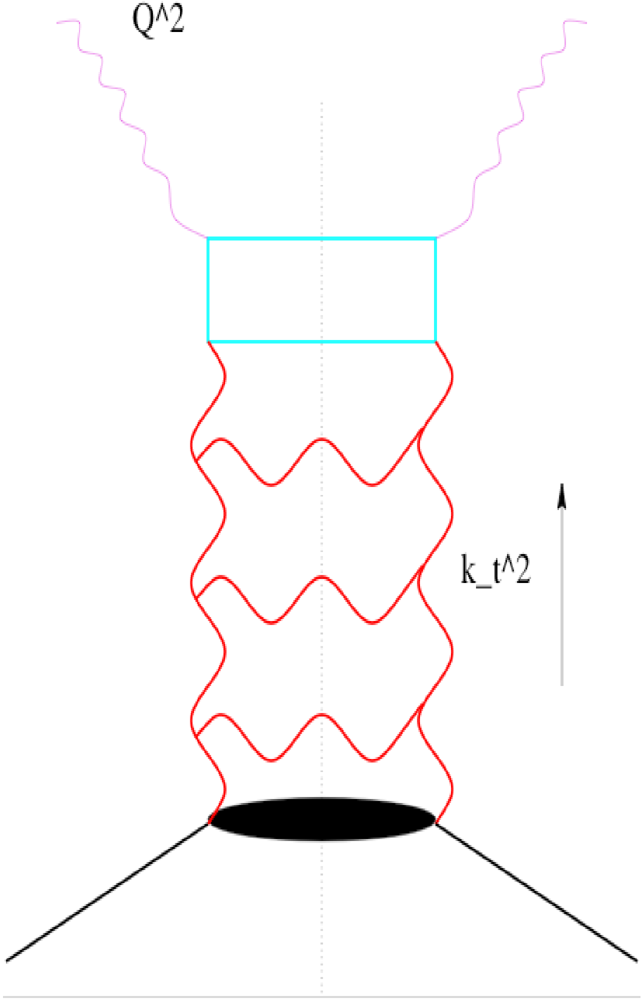



DGLAP "evolution" equation:

scale dependence of parton distribution functions

Dokshitzer-**G**ribov-**L**ipatov-**A**ltarelli-**P**arisi

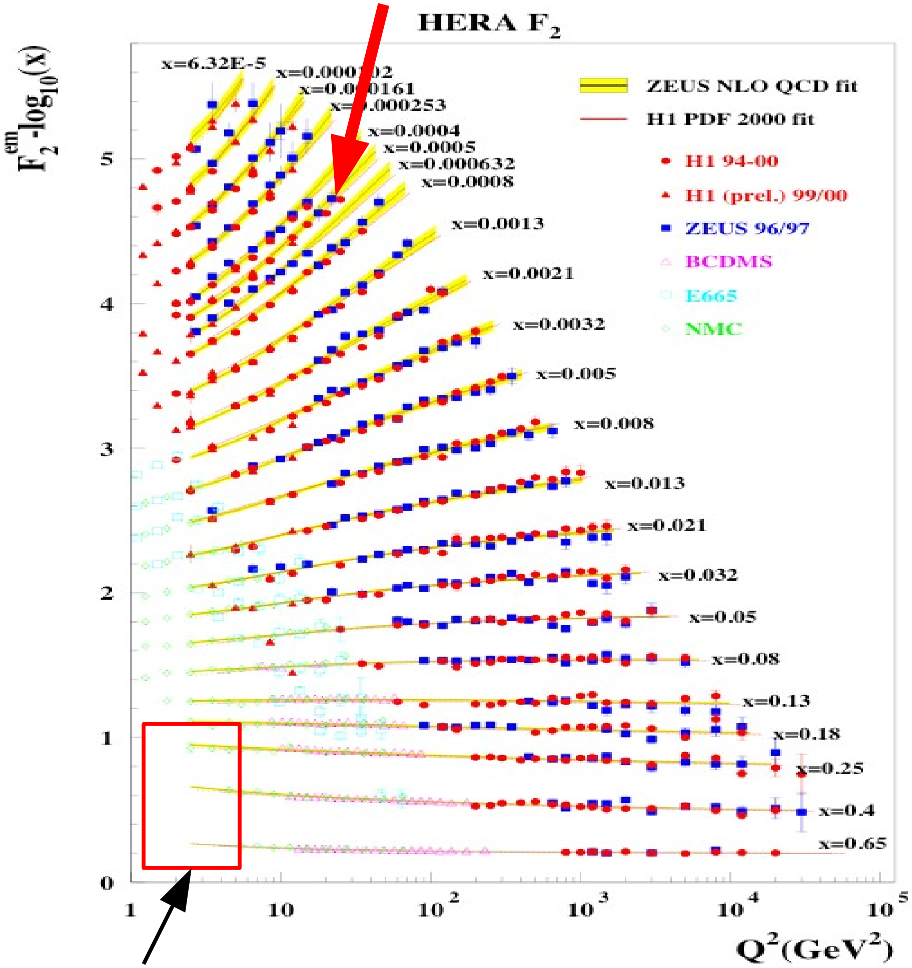




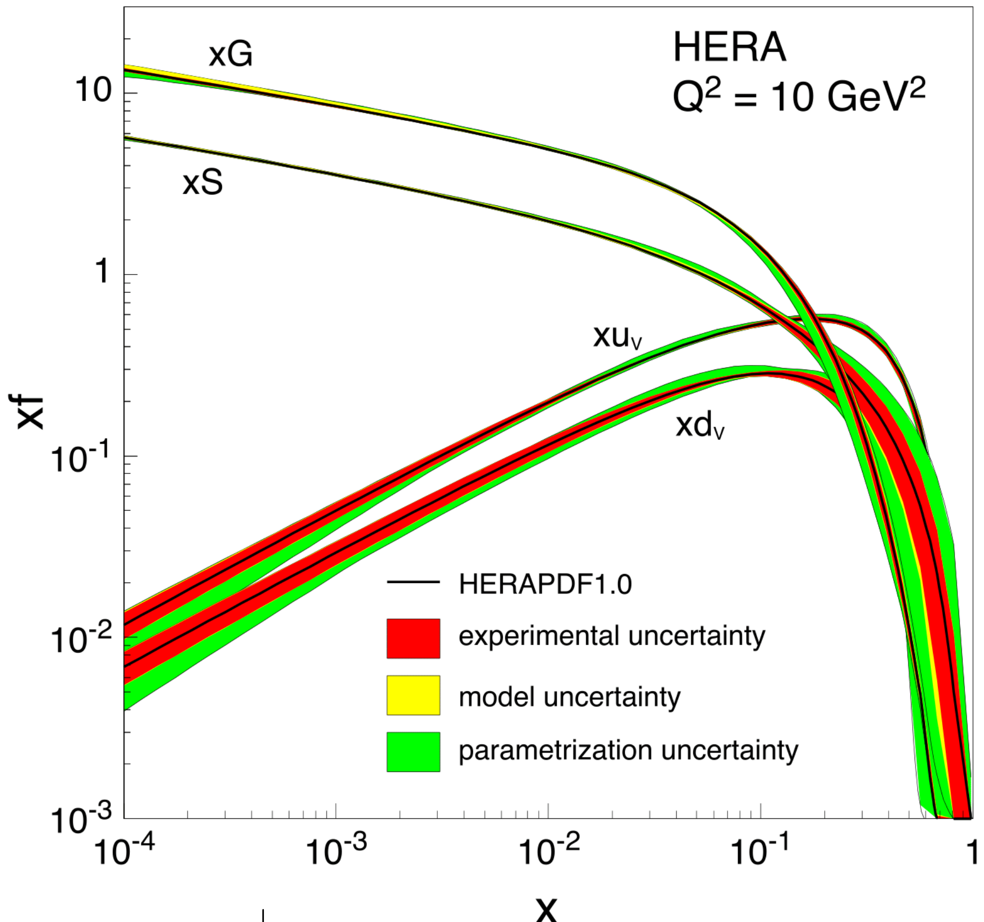
Deep Inelastic Scattering

$$F_2 \equiv \sum_{f=q,\bar{q}} e_f^2 \, xq(x,Q^2)$$

QCD: scaling violations



early experiments (SLAC,...):
scale invariance of hadron structure



$$x = \frac{p^+}{P^+}$$

***x** is the fraction of hadron energy carried by a parton*

What drives the growth of parton distributions?

Splitting functions at leading order $O(\alpha_S{}^0)$ $\quad (x \neq 1)$

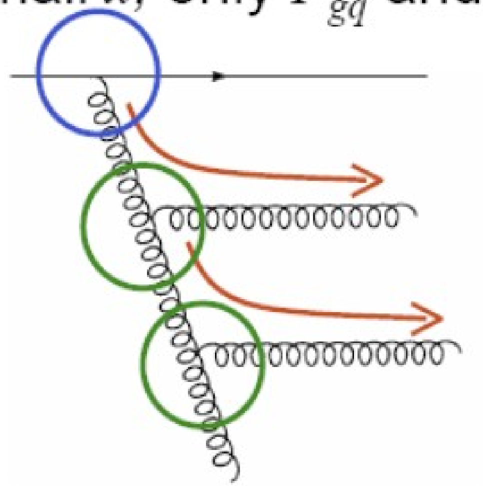
$$P_{qq}^{(0)}(x) = C_F \frac{1+x^2}{1-x}$$

$$P_{qg}^{(0)}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]$$

$$P_{gq}^{(0)}(x) = C_F \frac{1+(1-x)^2}{x}$$

$$P_{gg}^{(0)}(x) = 2C_A \left[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x)\right]$$

At small x, only P_{gq} and P_{gg} are relevant.



→ Gluon dominant at small *x*!

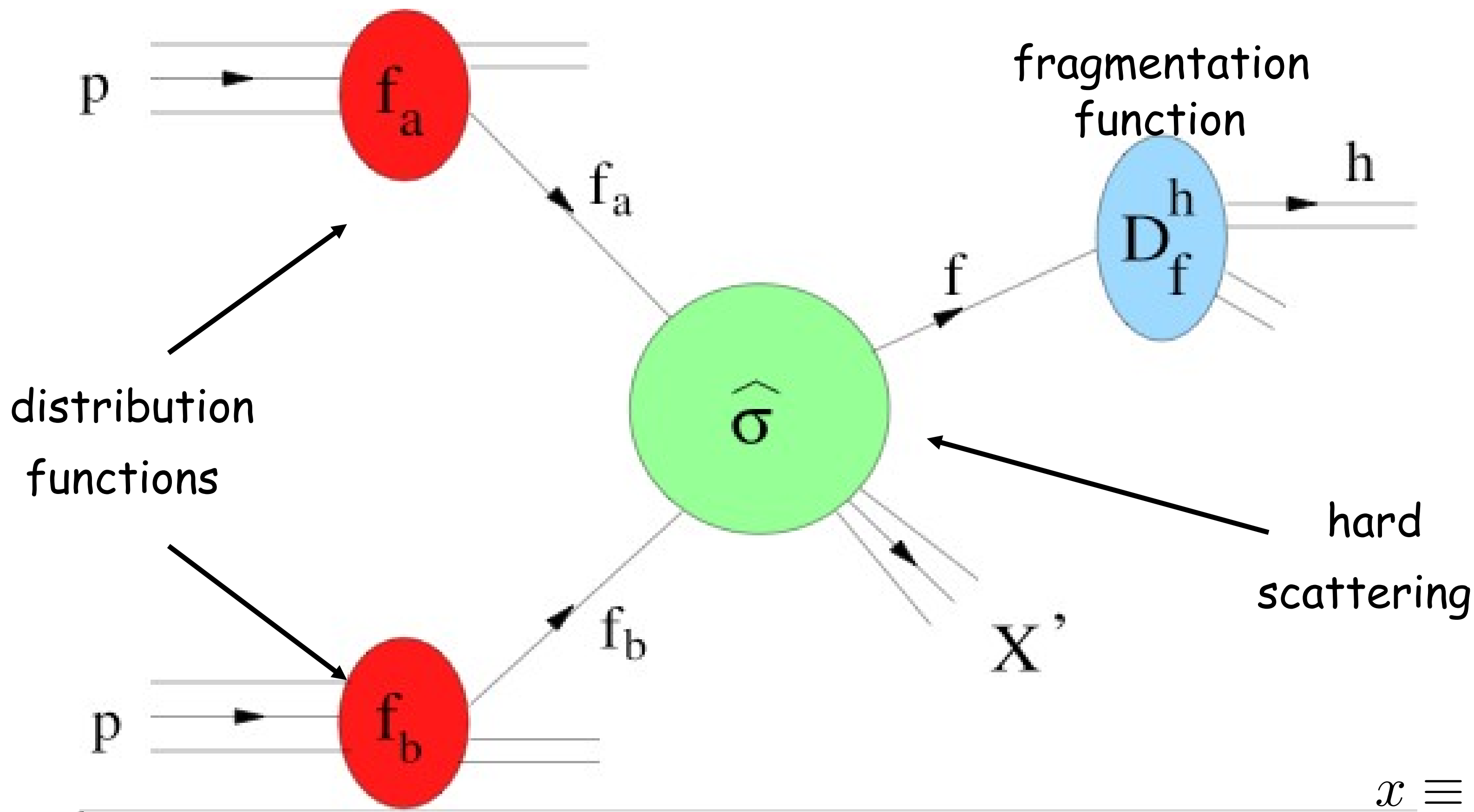
The double log approximation (DLA) of DGLAP is easily solved.

-- increase of gluon distribution at small x

$$\mathbf{xg(x, Q^2) \sim e^{\sqrt{\alpha_s \,(\log 1/x)\,(\log Q^2)}}}$$

QCD in proton-proton collisions

collinear factorization: separation of soft (long distance) and hard (short distance)



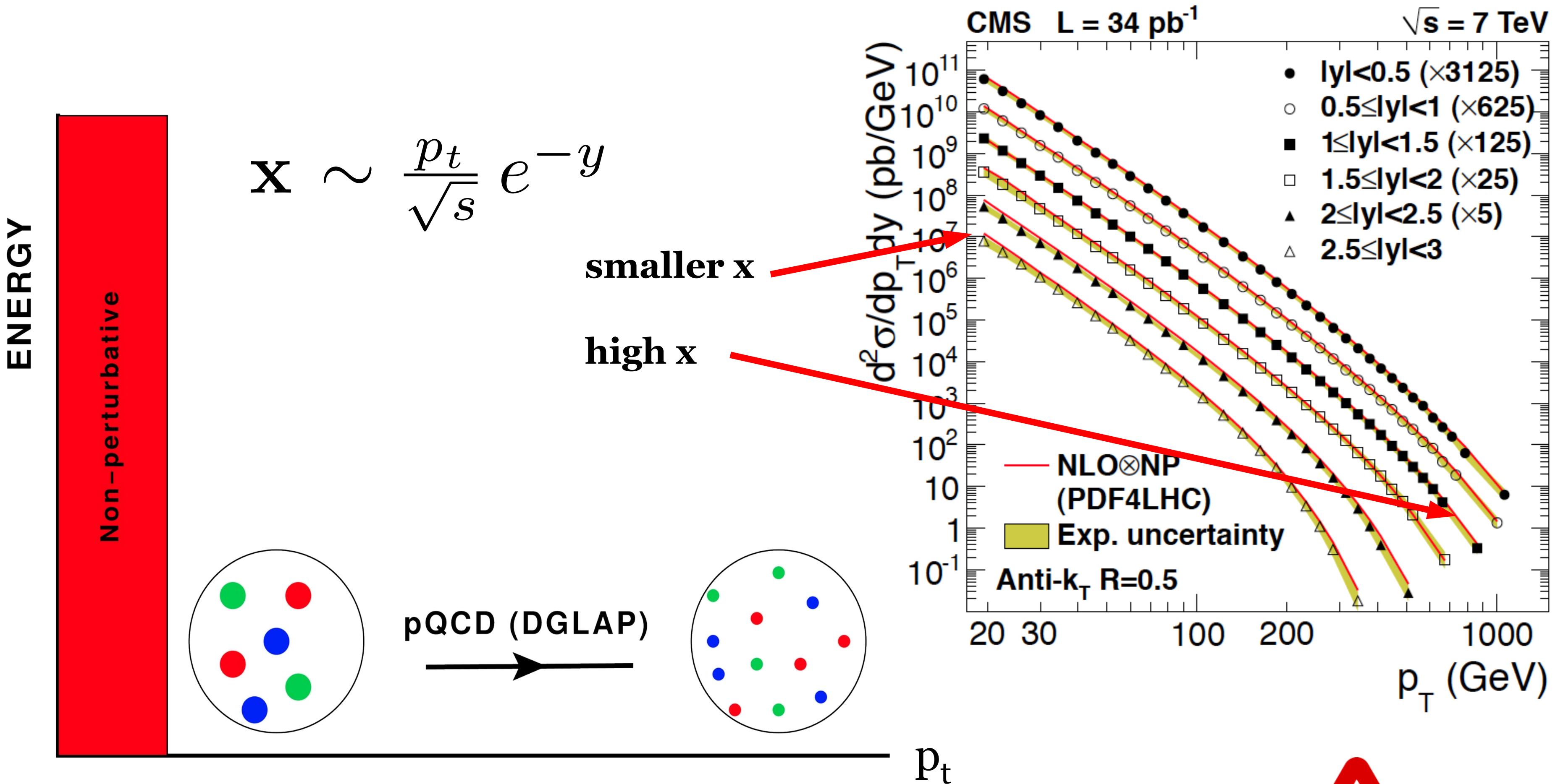
$$x \equiv \frac{p^+}{P^+}$$

$$\frac{d\sigma^{pp\to h\,X}}{d^2p_t\,dy} \sim f_a(x_1) \otimes f_b(x_2) \otimes \hat{\sigma} \otimes D_f^h(z) + \cdots\cdots$$

power corrections

pQCD: the standard paradigm

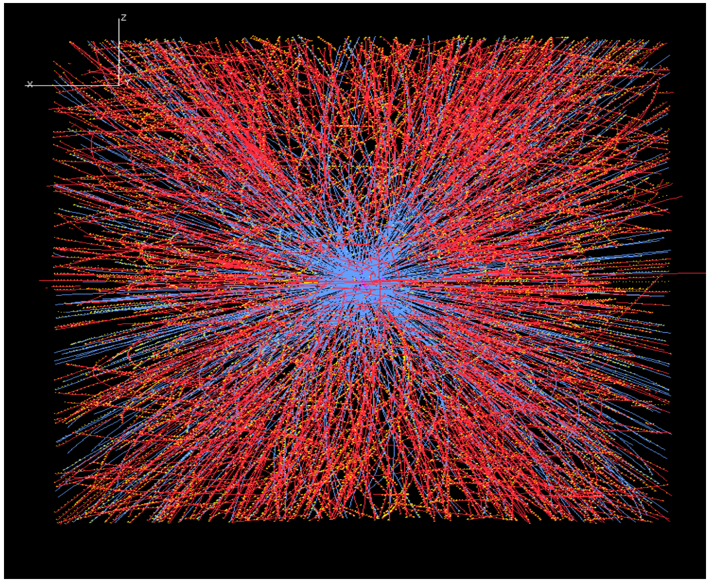
$$\mathbf{x} \sim \frac{p_t}{\sqrt{s}}\, e^{-y}$$

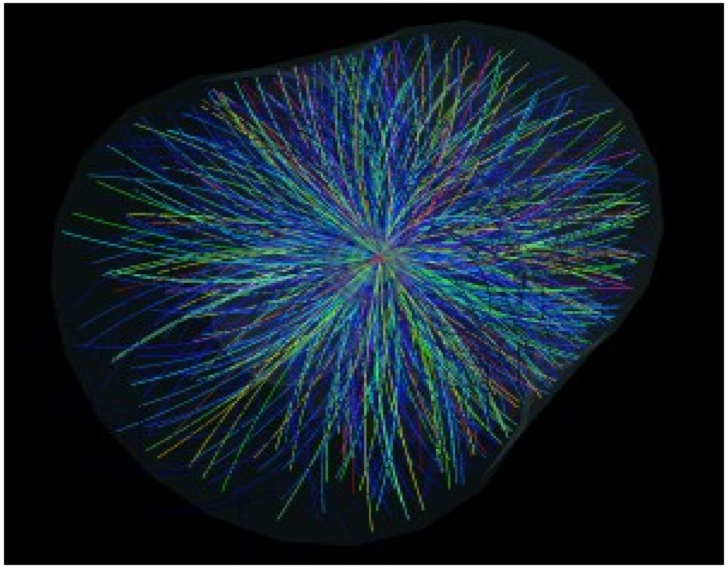


bulk of QCD phenomena happens at low p_t (<u>small x</u>)

Nucleus-Nucleus (AA) Collisions:

Quark-Gluon Plasma





$$\sqrt{S} \sim 200\,\mathrm{GeV}$$

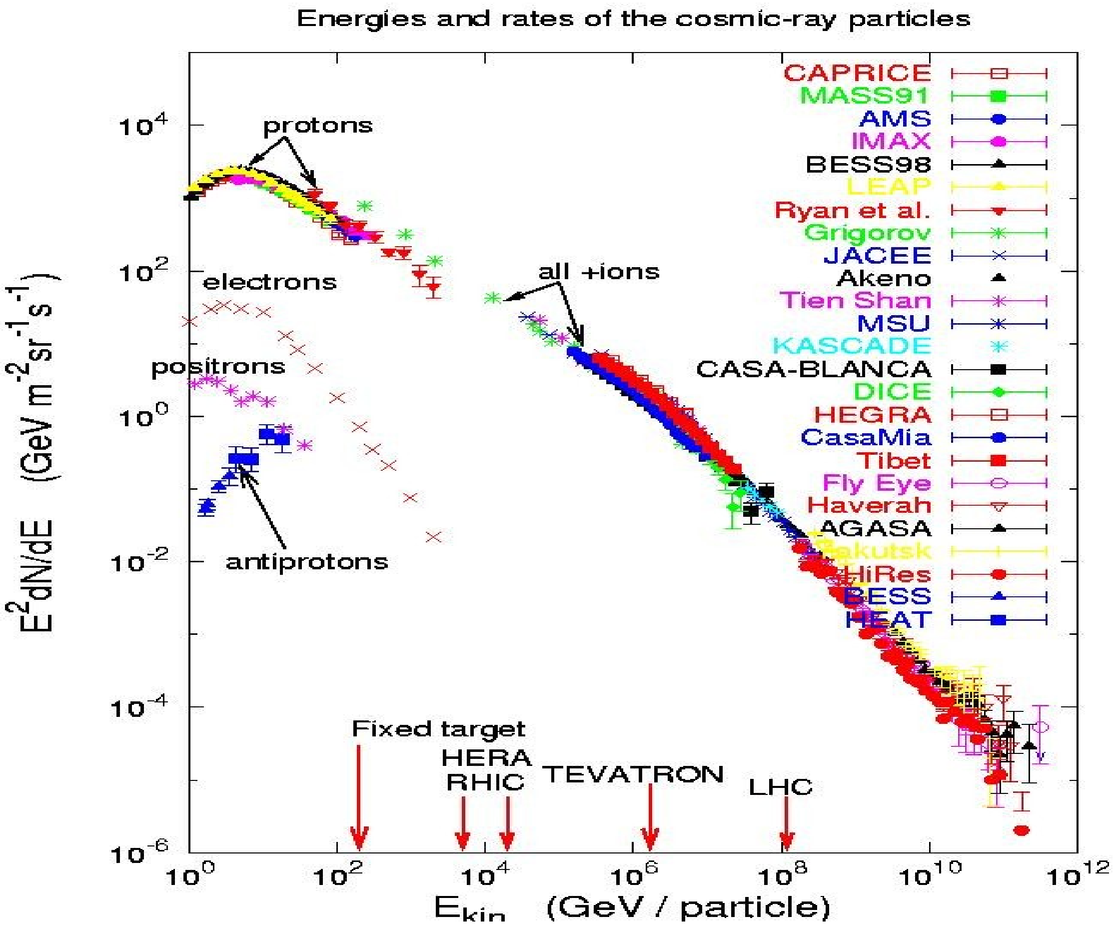
$$\sqrt{S} \sim 5\,\mathrm{TeV}$$

$$\mathrm{RHIC}: \ \frac{dN_{ch}}{d\eta} \sim 700$$

$$\mathrm{LHC}: \ \frac{dN_{ch}}{d\eta} \sim 1600$$

$$x \sim \frac{p_t}{\sqrt{S}}\, e^{-y} \to 0$$

High Energy Cosmic Rays



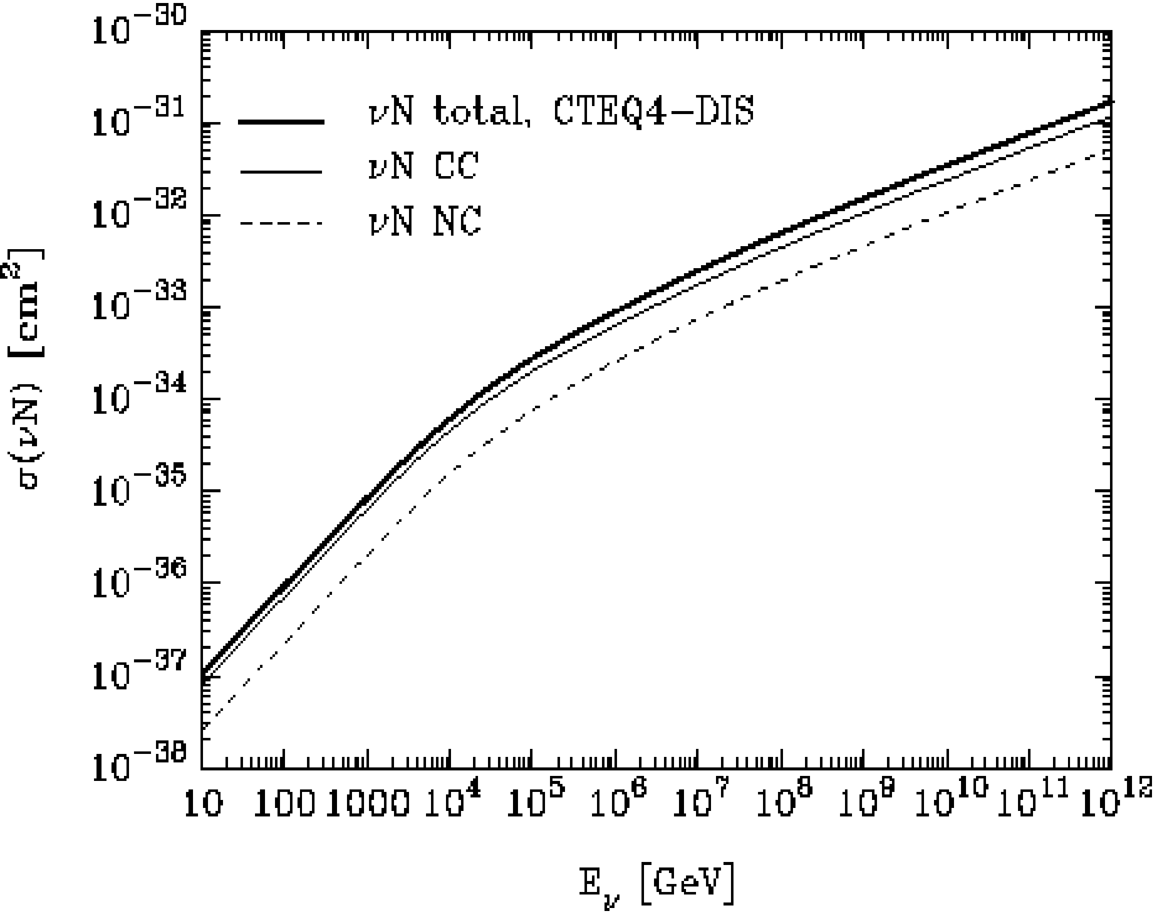
$$p\,A \to X$$

$$\sqrt{S} \sim 10^{2-3}\,\mathrm{TeV}$$

most particles/energy are in the forward rapidity region and have low p_t

$$x \sim \frac{p_t}{\sqrt{S}}\, e^{-y} \to 0$$

Ultra-High Energy Neutrinos



$$\nu\,\mathbf{N} \rightarrow \nu\,\mathbf{X}$$

$$\sqrt{\mathbf{S}} \sim \mathbf{10^{2-3}TeV}$$

$$\frac{\mathbf{M_z}}{\sqrt{\mathbf{S}}} \rightarrow \mathbf{0}$$

total cross section dominated by Q ~ M_z

need to understand structure of hadrons at *very small x*

QCD in the Regge-Gribov limit

recall $x_{Bj} \equiv \frac{Q^2}{S}$ $\quad S \rightarrow \infty\,,\; Q^2 \text{ fixed} : x_{Bj} \rightarrow 0$

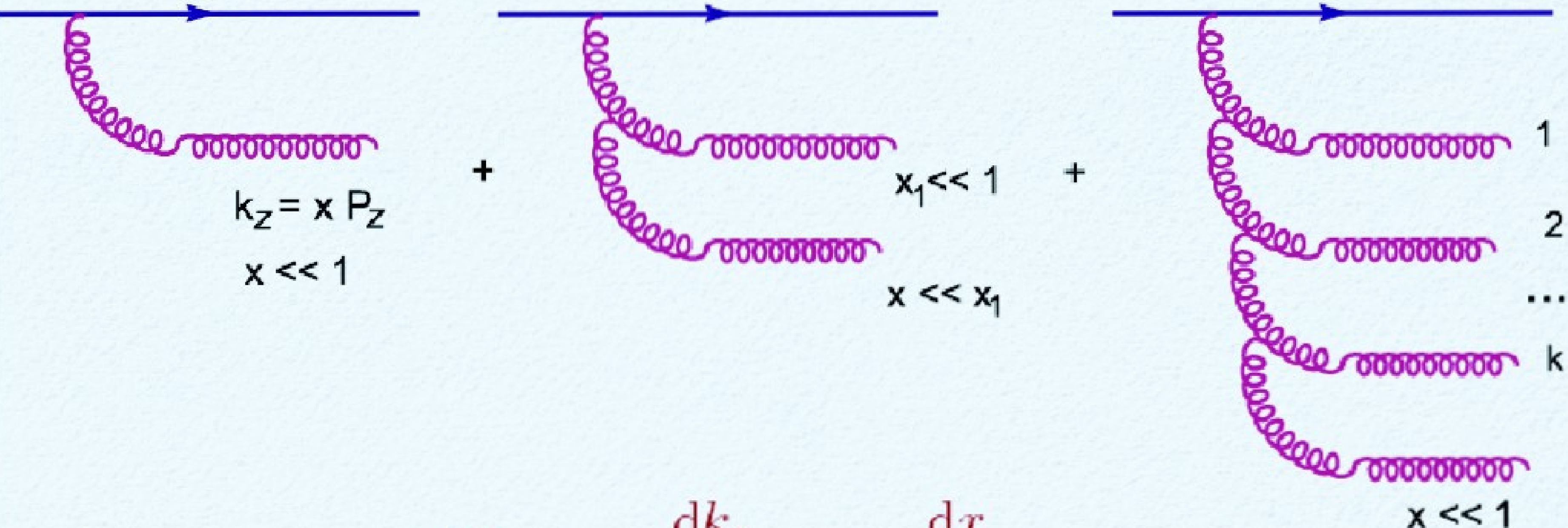
Regge

Gribov

gluon radiation at small x :pQCD

The infrared sensitivity of bremsstrahlung favors the emission of 'soft' (= small–x) gluons

$$\mathbf{P_{gg}(x) \sim \frac{1}{x} \text{ for } x \to 0}$$



$$d\mathcal{P} \propto \alpha_s \frac{dk_z}{k_z} = \alpha_s \frac{dx}{x}$$

The 'price' of an additional gluon:

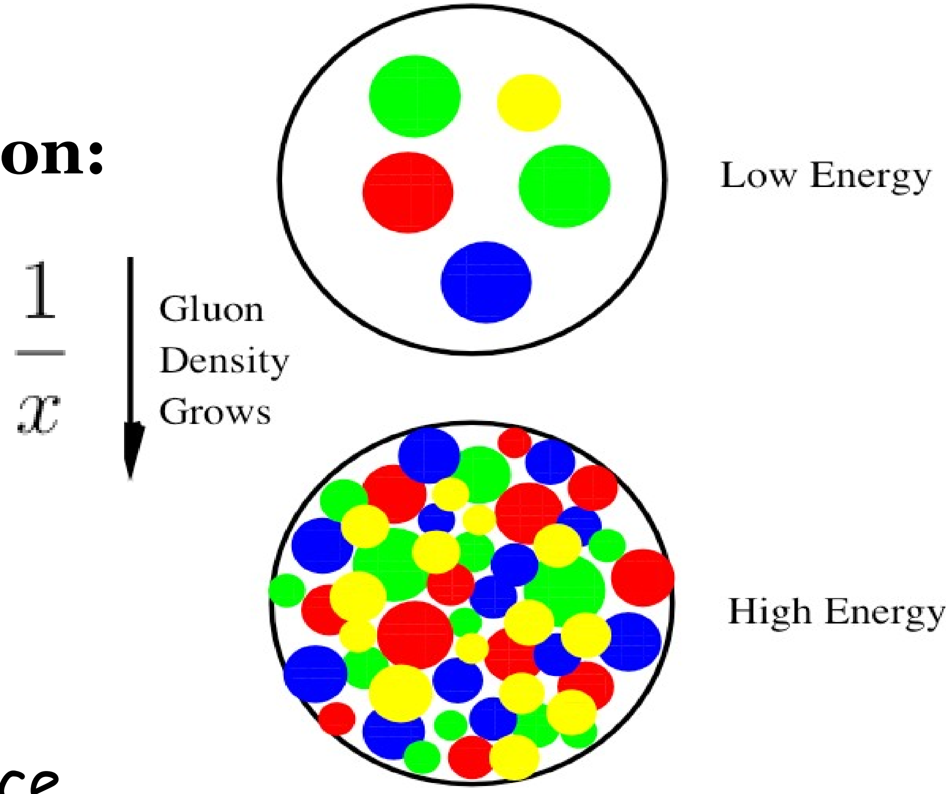
$$\mathcal{P}(1) \propto \alpha_s \int_x^1 \frac{dx_1}{x_1} = \alpha_s \ln \frac{1}{x}$$

number of gluons grows fast

$$n \sim e^{\alpha_s \ln 1/x}$$

Resolving the nucleus/hadron:

Regge-Gribov limit



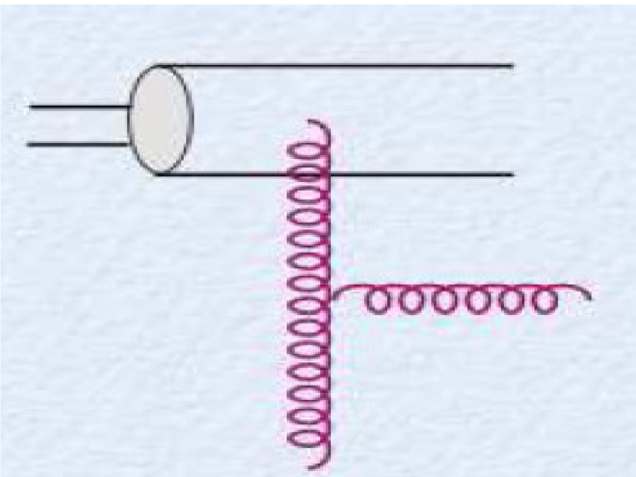
radiated gluons have the same size ($1/Q^2$) - the number of partons increase due to the increased longitudinal phase space

hadron/nucleus becomes a dense system of gluons:
concept of a quasi-free parton is not useful

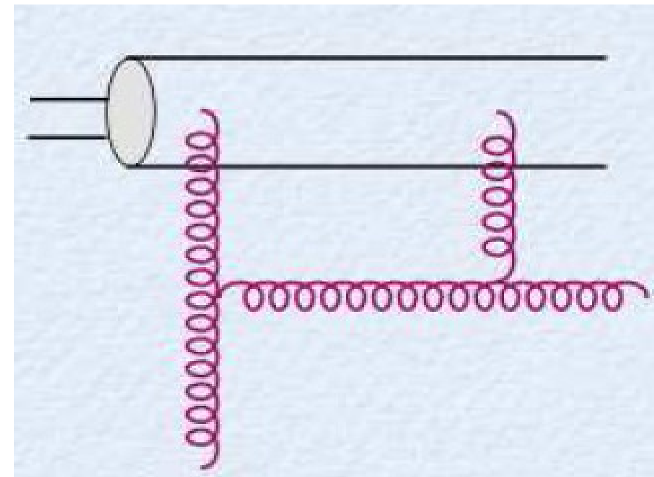
Physics of strong color fields in QCD, multi-particle production- possibly discover novel universal properties of theory in this limit

break down of pQCD at small x

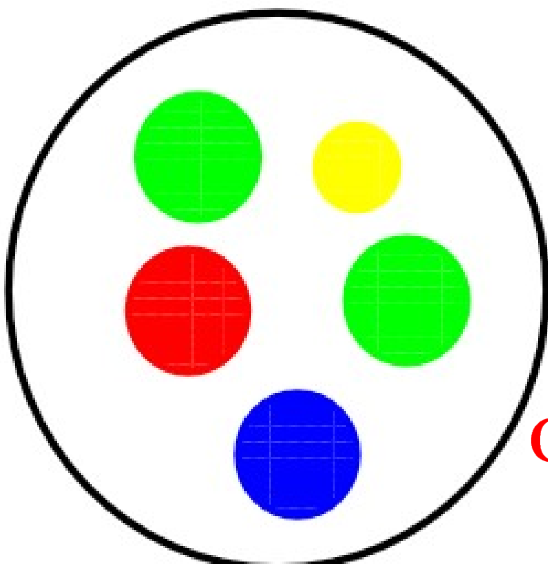
“attractive” bremsstrahlung *vs.* *“repulsive” recombination*



included in pQCD

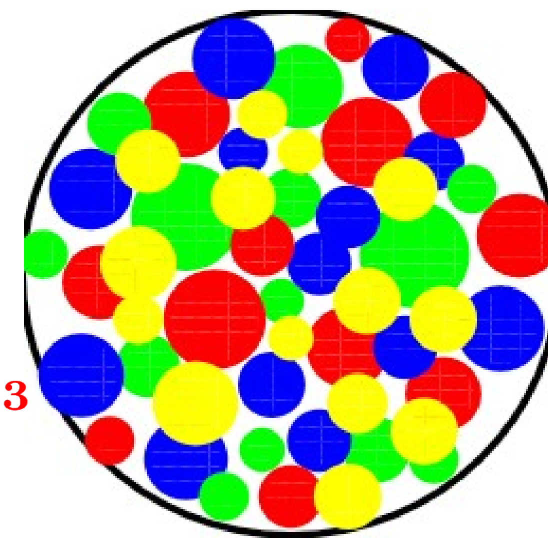


not included in pQCD
(collinear factorization)

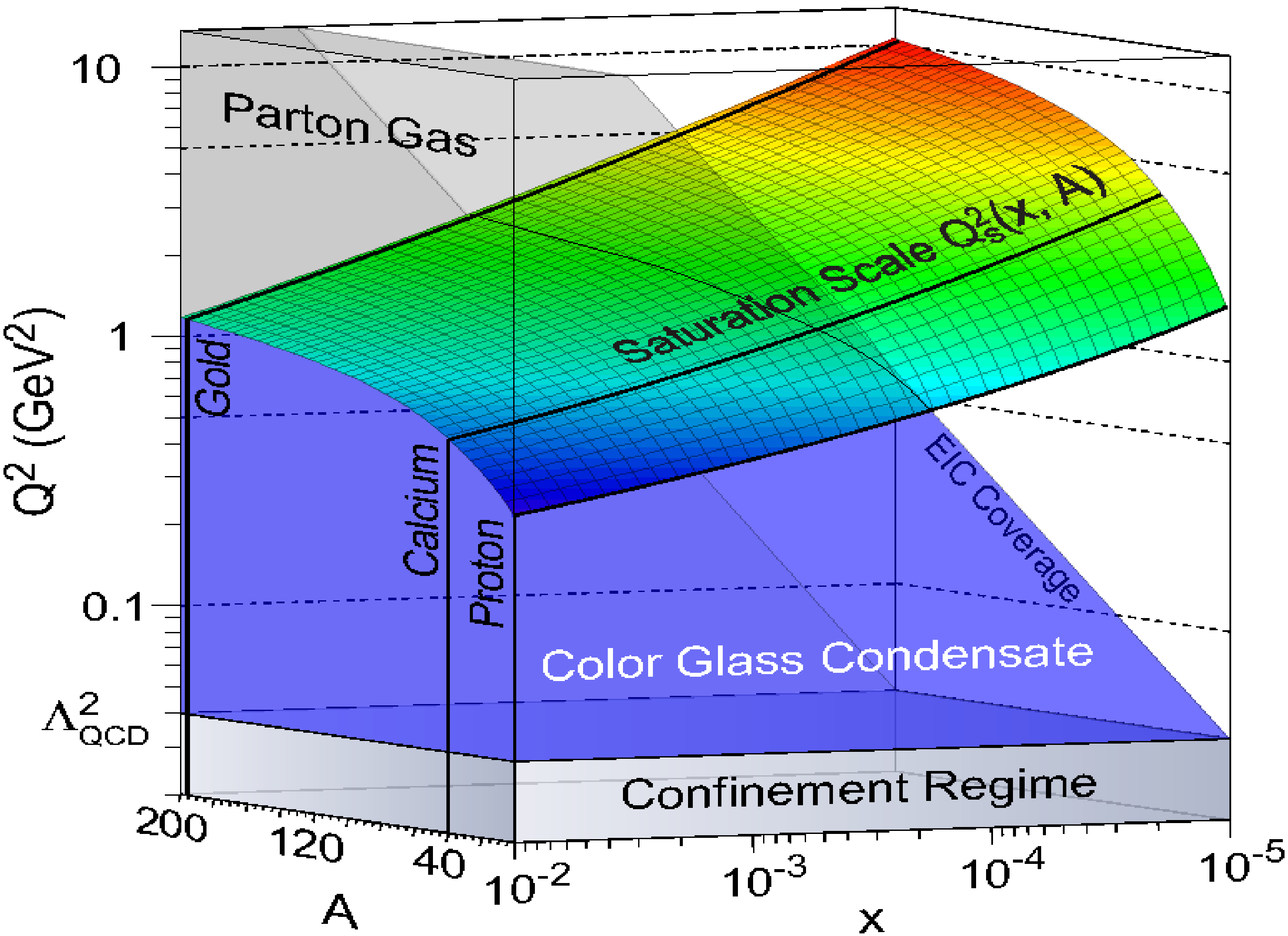


$$\frac{\alpha_s}{Q^2}\,\frac{xG(x,Q^2)}{\pi r^2}\sim 1$$

$$Q_s^2(x, b_t, A) \sim A^{1/3}\,(\frac{1}{x})^{0.3}$$

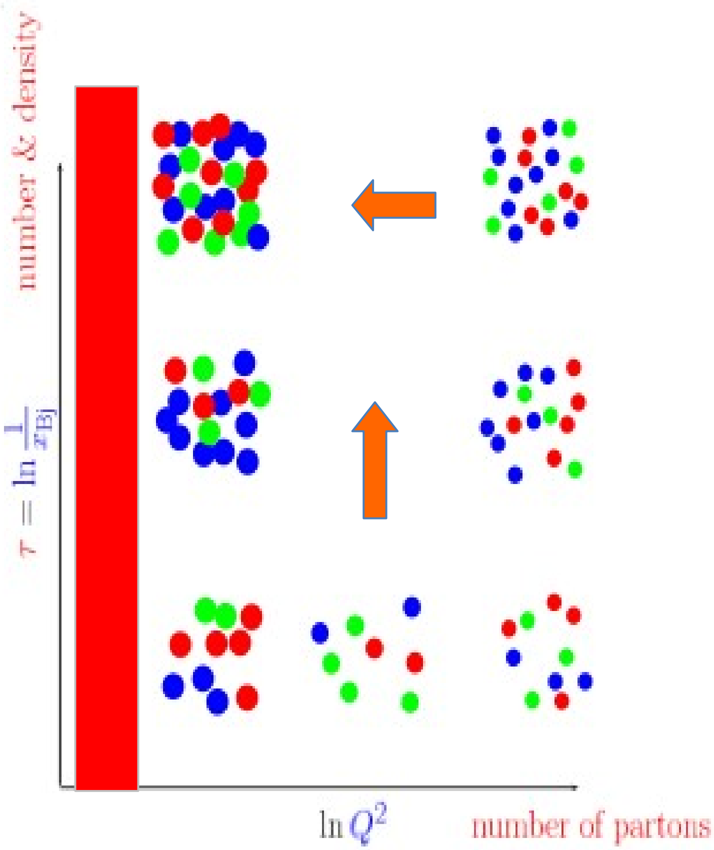


The Saturation Scale Q_s



X 9/4 for gluons

QCD at small x:
many-body dynamics of universal gluonic matter (CGC)



How does this happen ?

How do correlation functions evolve ?

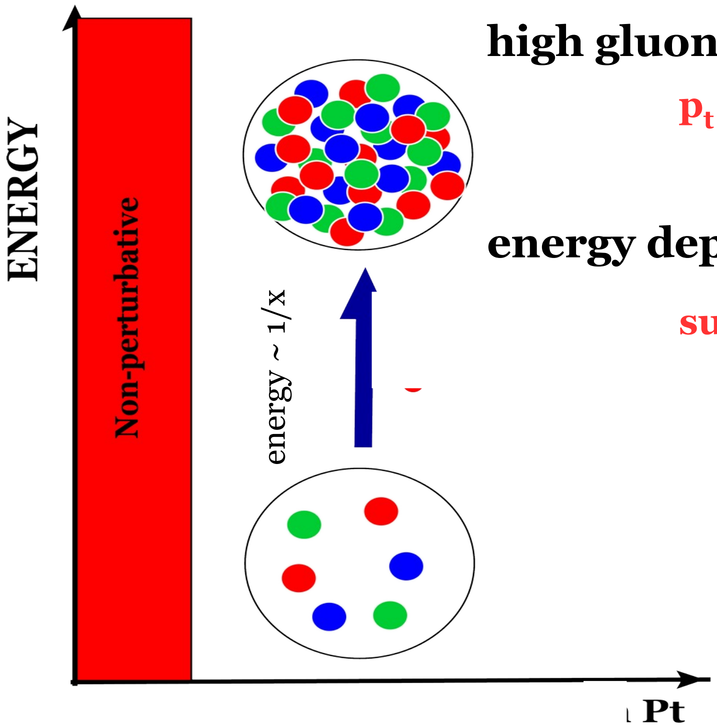
Are there scaling laws ?

Can CGC explain aspects of HIC ?

Initial conditions for hydro?
Thermalization ?
Long range rapidity correlations ?
Azimuthal angular correlations ?
Nuclear modification factor ?

QCD at small x:
a new approach is needed

QCD at high energy/small x: gluon saturation



high gluon density: **Eikonal multiple scattering**

p_t broadening (generic to multiple scattering)

energy dependence: **x-evolution via JIMWLK/BK**

suppression of spectra/away side peaks

$$\mathbf{Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}}$$

$$Q_s^2(x = 3 \times 10^{-4}) \sim 1\, GeV^2$$

for a proton target (quarks)

a framework for multi-particle production in QCD at small x/low p_t

$$\mathbf{x \leq 0.01} \qquad \alpha_s \ln\left(x_v/x\right) \sim 1$$