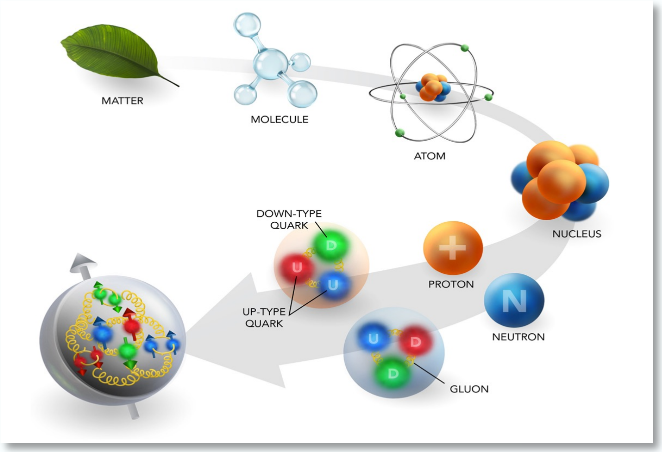
Lecture 2: TMD theory

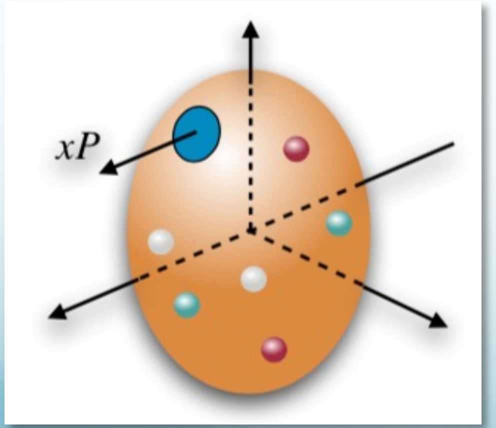
- TMD basic, parton model, symmetry

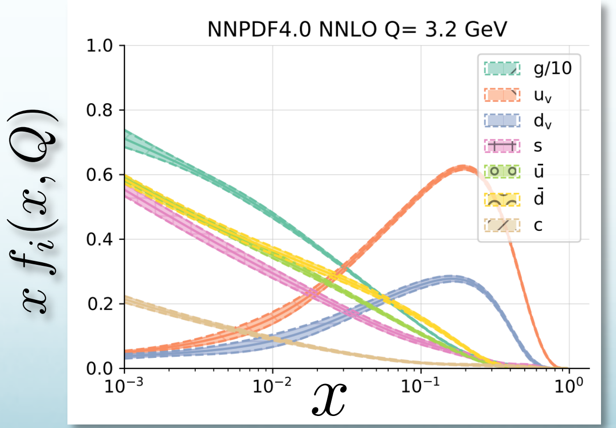
EIC era: Imaging of proton and nucleus

- Unraveling the mysteries of relativistic hadronic bound states



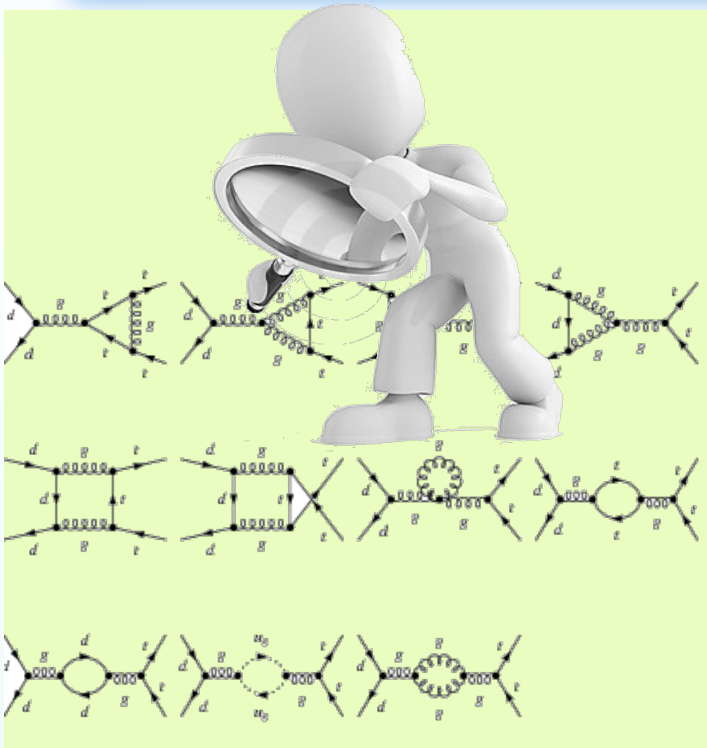
- Beyond 1D: collinear PDFs provide 1D structure – longitudinal motion

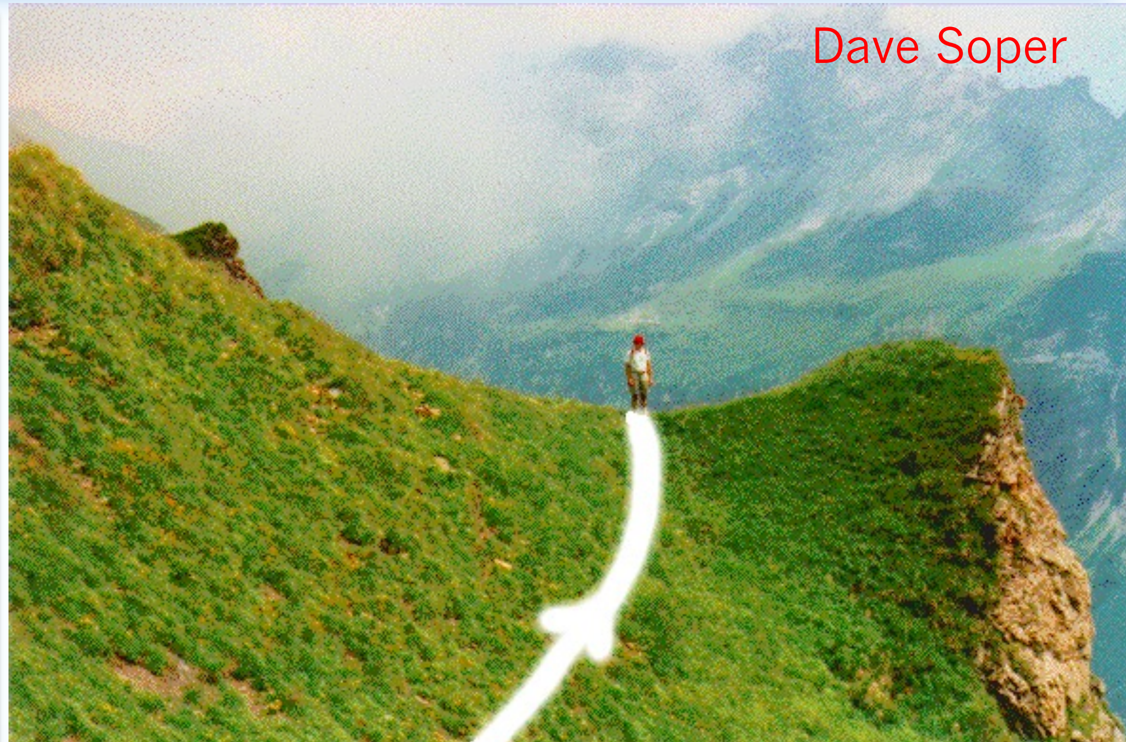




[Proton Movie]

Direction 1: deep in purturbative direction





Jets, Jet substructure, W/Z/H+Jet, …
NLO, NNLO, Resummation, multi-loops/legs, …

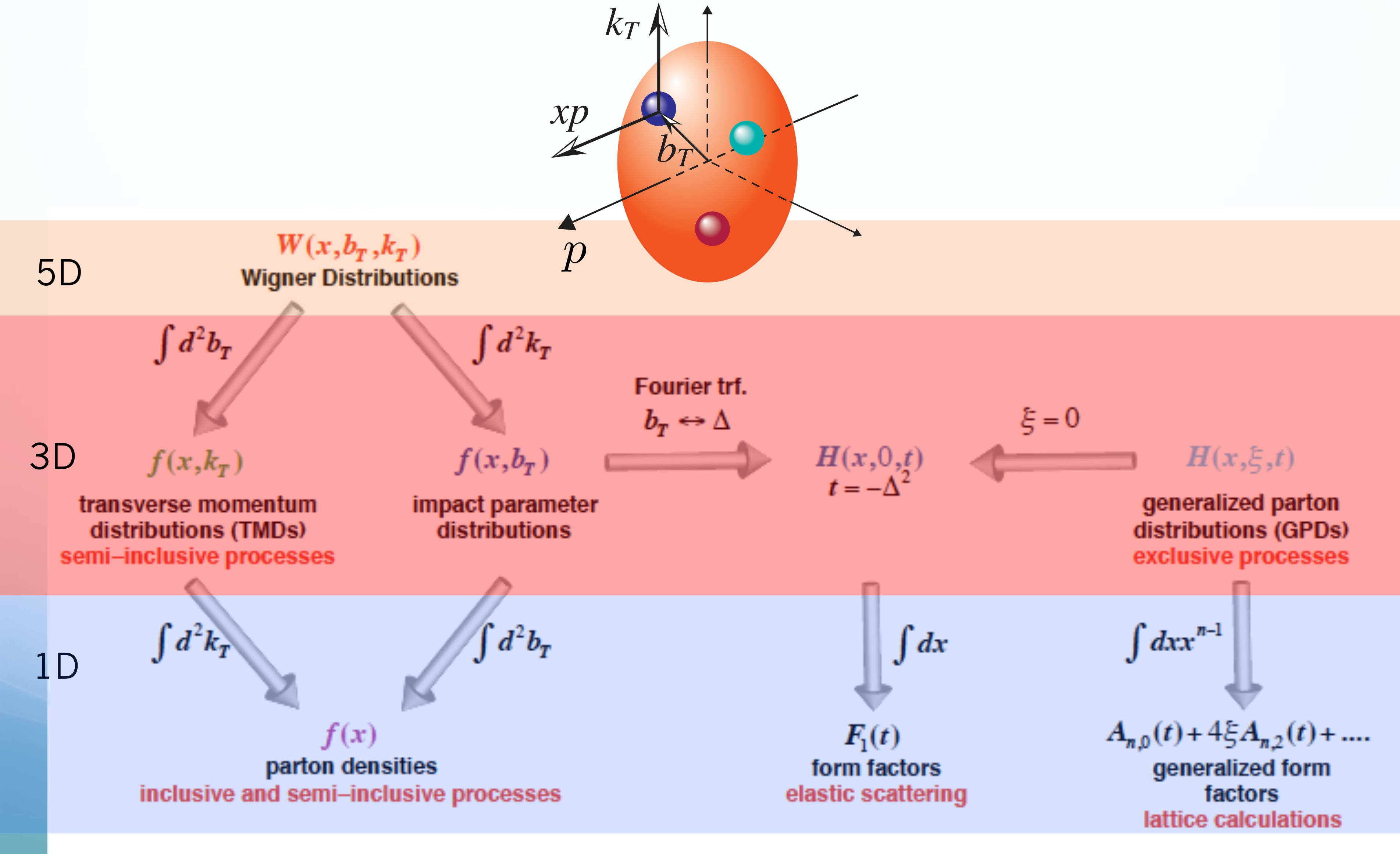
$$\sigma(Q) = f_{q,g}(x) \otimes \hat{\sigma}_{q,g}(Q)$$

Direction 2: quantum tomography of nucleons

$$\sigma(Q) = f_{\mathrm{q,g}}(x) \otimes \hat{\sigma}_{q,g}(Q)$$

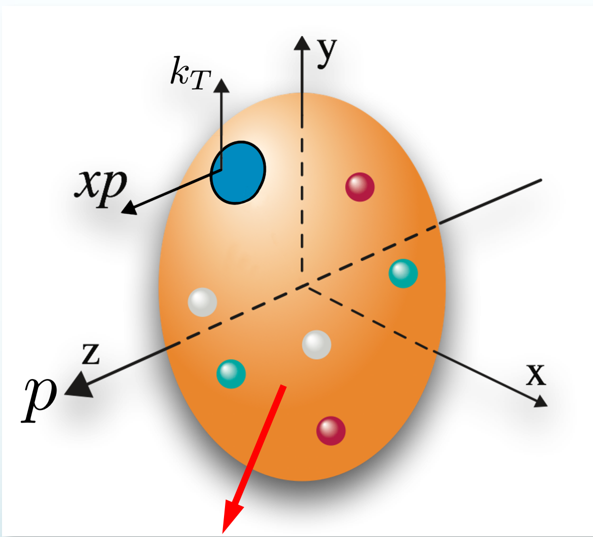
Unified view: internal landscape

- Wigner distributions: a quantum version of phase-space distribution

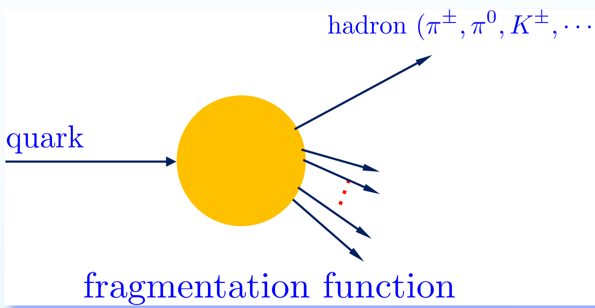


Transverse Momentum Dependent distributions (TMDs)

- 3D imaging in momentum space
  - Both longitudinal and transverse motion
  - What are the quantum correlations between the motion of the quarks/gluons, their spin and the spin of the proton? (TMD PDFs)
  - Similarly precision information on hadronization (TMD FFs)

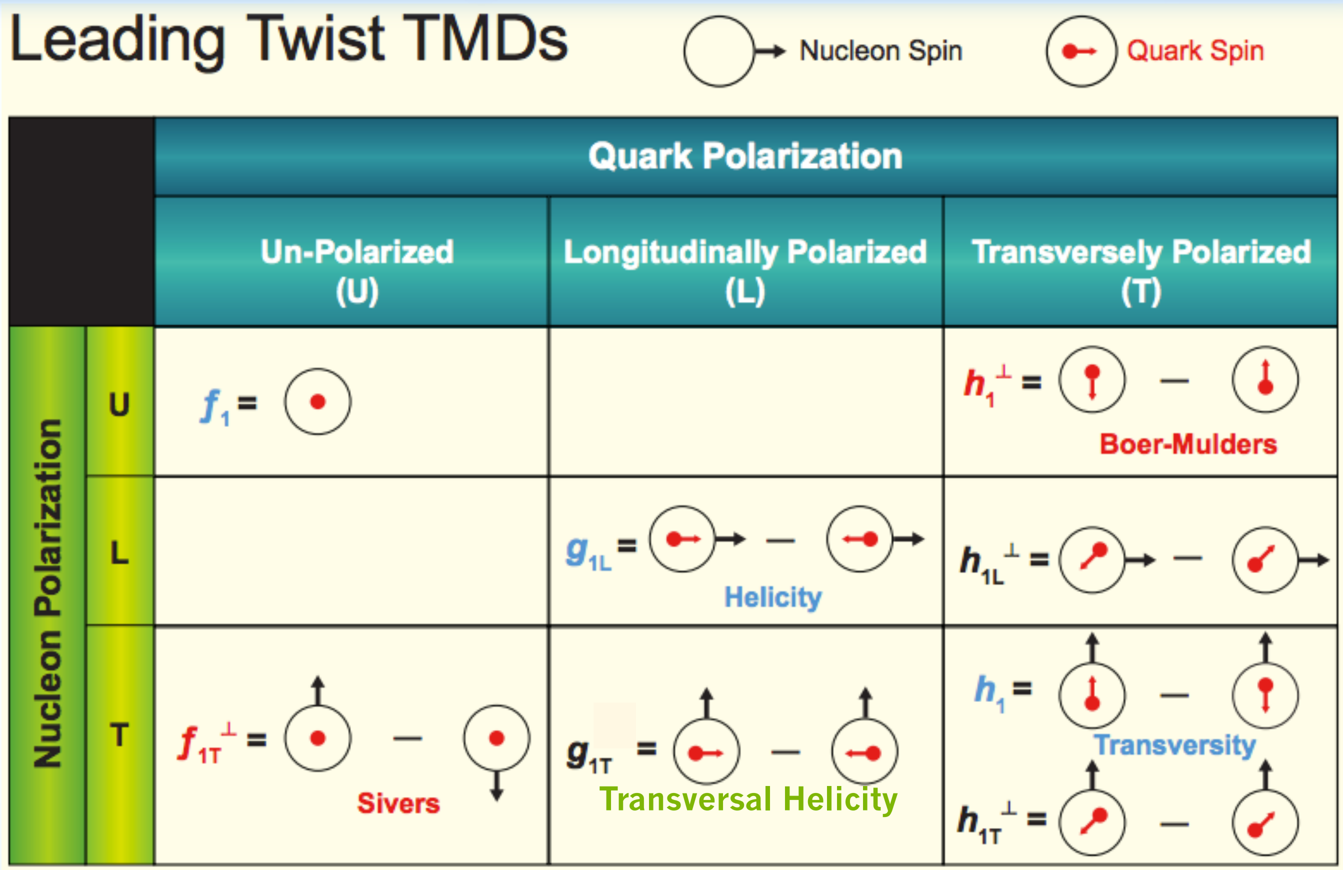


Proton spin



TMDs with polarization

TMD PDFs

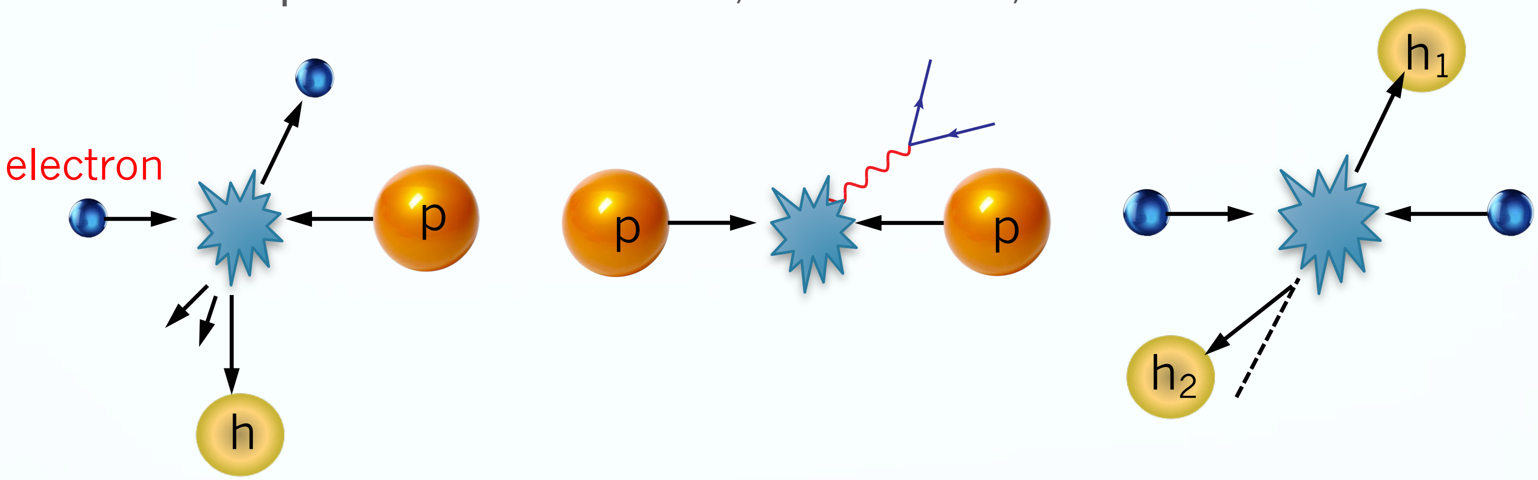


TMD FFs

	Quark Polarization U	Quark Polarization L	Quark Polarization T
Pion	D_1		$H_1^\perp$ **Collins**

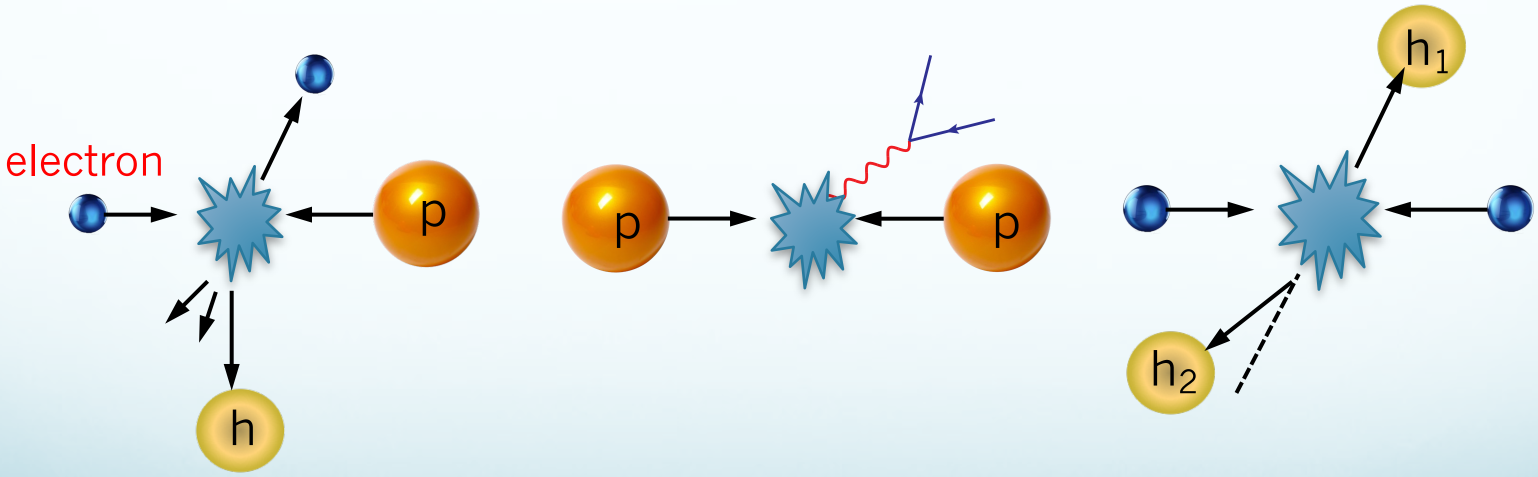
Processes to extract TMDs

- Standard processes: SIDIS, Drell-Yan, dihadron in e^+e^-



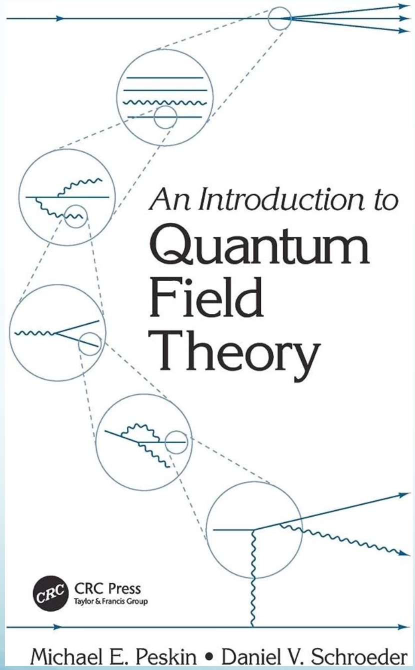
TMD parton model

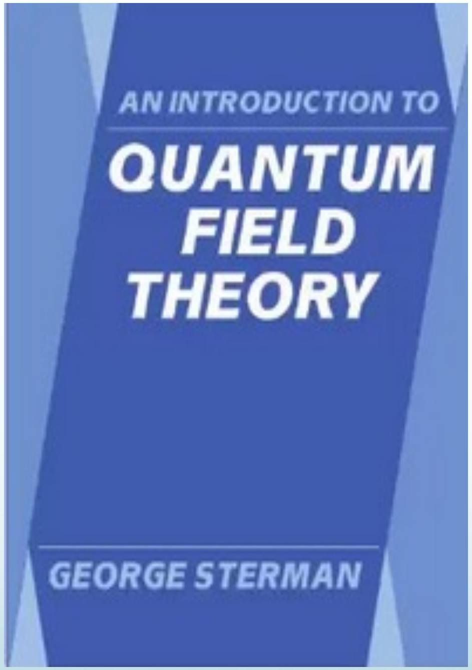
- Two major questions we have to answer
  - How many TMD distributions are needed in order to fully characterize the proton structure?
  - How are the cross section (observables) connected to the TMDs?

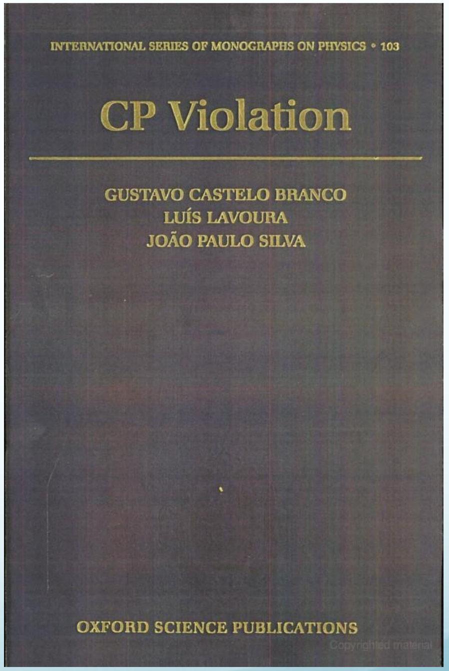


Good textbooks

- Understand C, P, T discrete symmetry properties of the correlation function
  - Most textbooks on quantum field theory will give discussion on this topic, such as Peskin, Sterman: appendix of Sterman's book
  - If you want extensive discussion, see this book





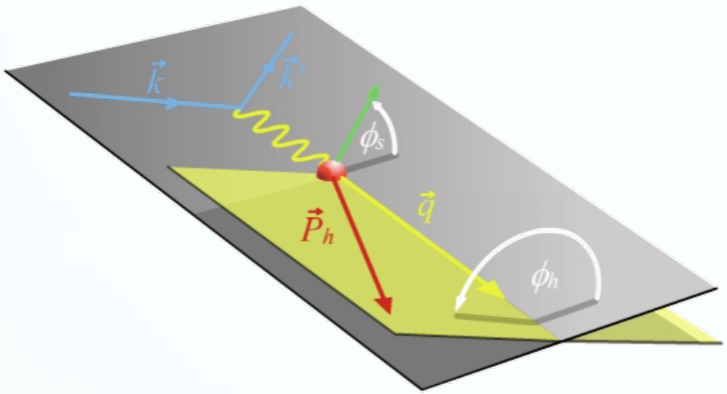


Operator analysis

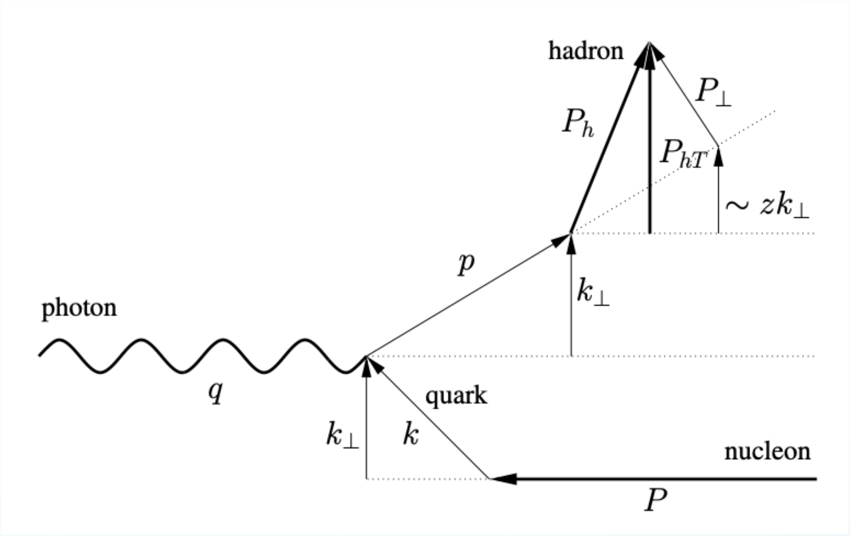
- Operator analysis to figure out how many distributions are needed to characterize the nucleon structure
- For details, see
  - Mulders, Tangerman hep-ph/9403227
  - Mulders, Tangerman hep-ph/9510301
  - Mulders, http://www.nat.vu.nl/~mulders/correlations-0new.pdf
- From now on, we will look at this hand-writing note

TMD parton model

- Here I provide a detailed note for deriving SIDIS cross section



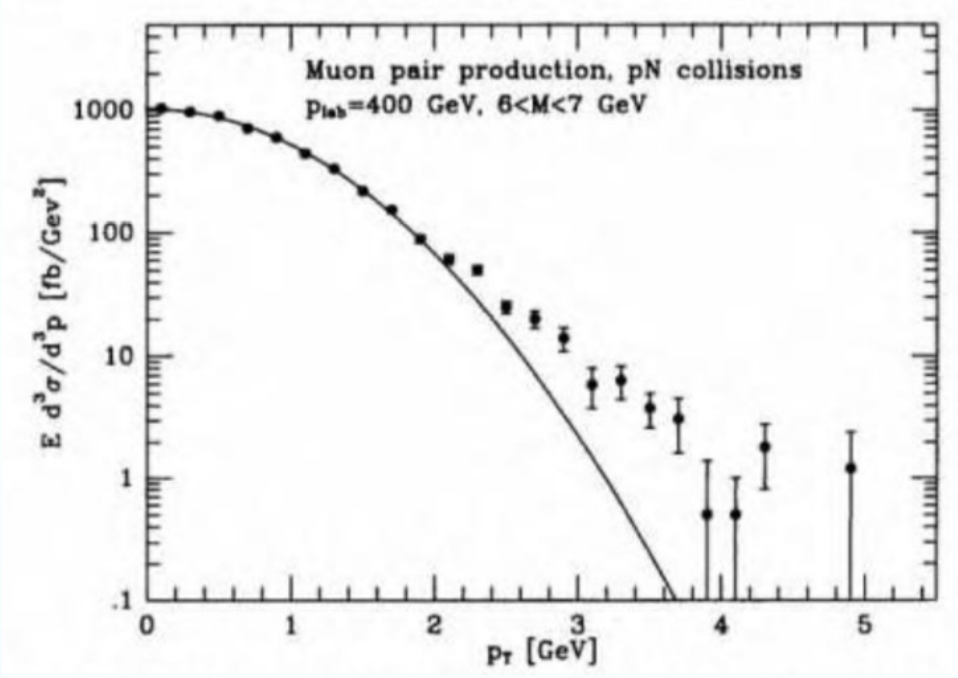
$$\ell + p^{\uparrow} \to \ell' + \pi(p_T) + X$$



$$\frac{d\sigma}{dx_B dy dz_h d^2 P_{hT}} = \sigma_0 \sum_q e_q^2 \int d^2 k_T d^2 p_T f_{q/p}(x_B, k_T^2) D_{h/q}(z_h, p_T^2) \delta^2\left(z_h \vec{k}_T + \vec{p}_T - \vec{P}_{hT}\right)$$

Parton model for TMDs

- At low pT, the distribution is described by a Gaussian form



$$f_q(x, k_T^2) = f_q(x) g(k_T)$$

$$g(k_T) = \frac{1}{\pi \langle k_T^2 \rangle} e^{-k_T^2/\langle k_T^2 \rangle}$$

since $$\int d^2 k_T f_q(x, k_T^2) = f_q(x)$$

$$\rightarrow \int d^2 k_T g(k_T) = 1$$

TMD parton model phenomenology

- In the early days of TMD studies, one typically uses a Gaussian model to describe transverse momentum dependence

$$f_{q/p}(x,k_\perp) = f_{q/p}(x)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}$$

$$D_{h/q}(z,p_\perp) = D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

$$\begin{aligned}\frac{d\sigma^{\ell+p\to\ell' hX}}{dx_B\,dQ^2\,dz_h\,dP_T^2} &= \frac{2\,\pi^2\alpha^2}{(x_B\,s)^2}\,\frac{[1+(1-y)^2]}{y^2}\\ &\times \sum_q e_q^2 \int d^2\boldsymbol{k}_\perp\,d^2\boldsymbol{p}_\perp\,\delta^{(2)}\Big(\boldsymbol{P}_T - z_h\boldsymbol{k}_\perp - \boldsymbol{p}_\perp\Big)\,f_{q/p}(x,k_\perp)\,D_{h/q}(z,p_\perp)\\ &\equiv \frac{2\,\pi^2\alpha^2}{(x_B\,s)^2}\,\frac{[1+(1-y)^2]}{y^2}\,F_{UU}\,.\end{aligned}$$

$$F_{UU} = \sum_q e_q^2\,f_{q/p}(x_B)\,D_{h/q}(z_h)\,\frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle}$$

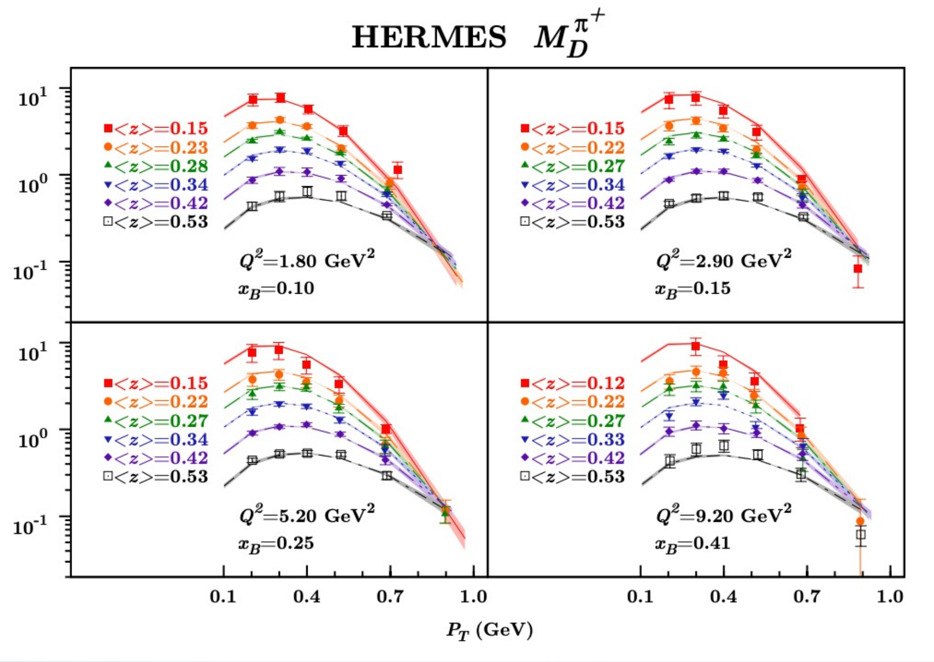
$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2\,\langle k_\perp^2\rangle\,.$$

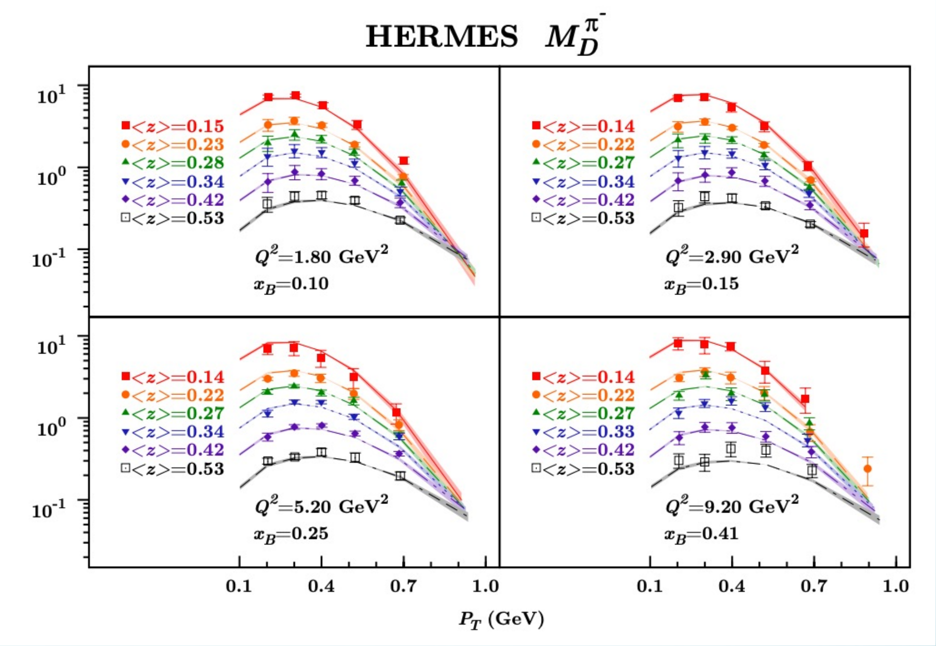
TMD parton model works well

- When the energy (Q) range spans relatively small, TMD parton model does a decent job [fitted parameters]

$$\langle k_\perp^2 \rangle = 0.57 \pm 0.08 \text{ GeV}^2$$
$$\langle p_\perp^2 \rangle = 0.12 \pm 0.01 \text{ GeV}^2$$

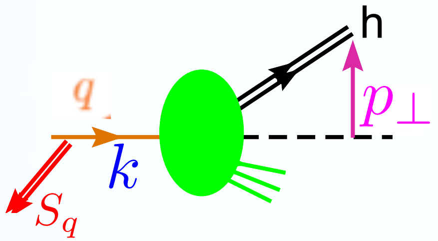
Anselmino, et.al. 1312.6261





Polarized TMDs: Collins function

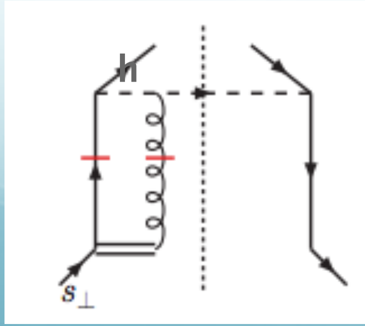
- Collins function: unpolarized hadron from a transversely polarized quark



$$D_{h/q}(z,p_\perp) = D_1^q(z,p_\perp^2) + \frac{1}{zM_h} H_1^{\perp q}(z,p_\perp^2)\vec{S}_q \cdot \left(\hat{k}\times p_\perp\right)$$

Spin-independent Spin-dependent

- ✓ 2002: A. Metz studied the universality property of Collins function in a model-dependent way – very subtle – finally found it is universal between SIDIS and e+e-
- ✓ 2004: Collins and Metz have general arguments
- ✓ 2008: Yuan generalizes to pp
- ✓ 2010: Boer, Kang, Vogelsang, and Yuan perform perturbative tail calculation, demonstrate the gauge link does not contribute

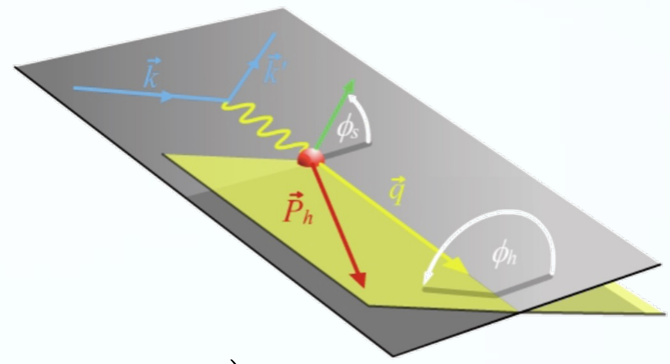


$$H_1^{\perp \mathrm{SIDIS}}(z,p_\perp^2) = H_1^{\perp e^+e^-}(z,p_\perp^2) = H_1^{\perp \mathrm{pp}}(z,p_\perp^2)$$

Metz 02, Collins, Metz 04, Yuan 08,
Boer, Kang, Vogelsang, Yuan, PRL 10, …

Polarized TMDs: measurements

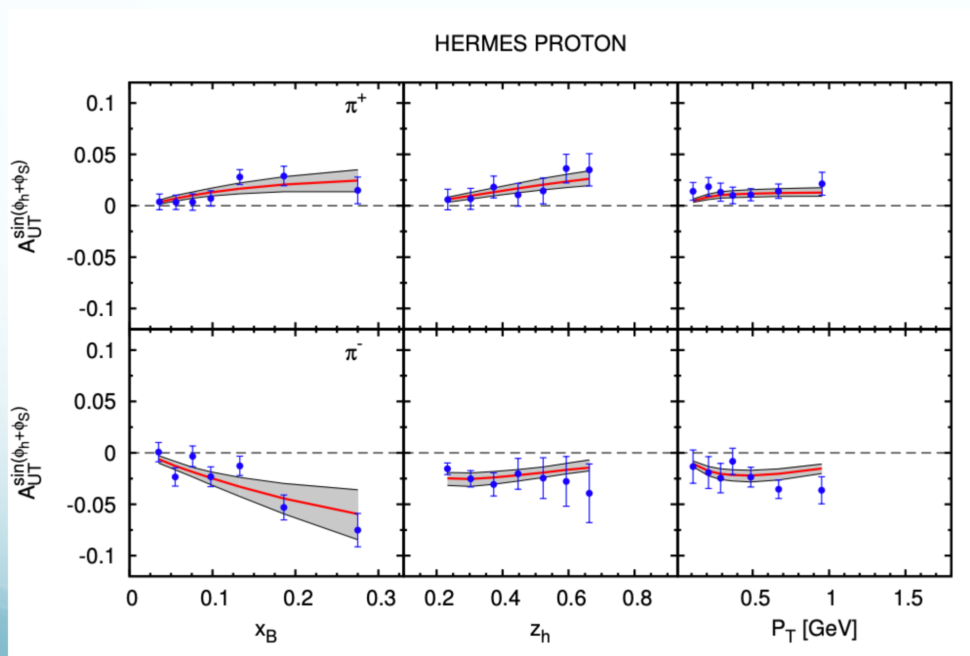
- Collins asymmetry in SIDIS



$$\frac{d^5\sigma(S_\perp)}{dx_B dy dz_h d^2 P_{h\perp}} = \sigma_0(x_B, y, Q^2)\left[F_{UU} + \sin(\phi_h+\phi_s)\,\frac{2(1-y)}{1+(1-y)^2}\,F_{UT}^{\sin(\phi_h+\phi_s)} + \ldots\right]$$

$$A_{UT}^{\sin(\phi_h+\phi_s)} \equiv 2\langle\sin(\phi_h+\phi_s)\rangle = \frac{\sigma_0(x_B,y,Q^2)}{\sigma_0(x_B,y,Q^2)}\,\frac{2(1-y)}{1+(1-y)^2}\,\frac{F_{UT}^{\sin(\phi_h+\phi_s)}}{F_{UU}}.$$

$$F_{UT}^{\sin(\phi_h+\phi_s)} \propto \int d^2k_T d^2p_T\, h_1(x_B,k_T) H_1^\perp(z,p_T)\,\delta^2\left(z_h\vec{k}_T + \vec{p}_T - \vec{P}_{hT}\right)$$



Anselmino, et.al. 1510.05389

Summary

- Operator analysis allows us to determine the independent TMD correlators
- To probe a small transverse momentum, one has to measure a small transverse momentum for the final state particles
- TMD parton model works well when the Q range spans relatively small