QCD at small Bjorken x:
the Color Glass Condensate (CGC)

Jamal Jalilian-Marian

Baruch College, City University of New York

New York NY

TIDC Autumn School On Electron-Ion Collider (EIC)
Physics Department, National Taiwan University

color averaging

McLerran-Venugopalan (93)

$$< \mathbf{O}(\rho) > \equiv \int \mathbf{D}[\rho]\, \mathbf{O}(\rho)\, \mathbf{W}[\rho]$$

$$\mathbf{W}[\rho] \simeq \mathbf{e}^{-\int \mathbf{d^2 x_t} \frac{\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{a}}(\mathbf{x_t})}{\mathbf{2}\,\mu^{\mathbf{2}}}} \qquad \mu^{\mathbf{2}} \equiv \frac{\mathbf{g^2\, A}}{\mathbf{S}_\perp}$$

$$\mathbf{T(r_t)} \equiv \frac{\mathbf{1}}{\mathbf{N_c}} < \mathbf{Tr}\,[\mathbf{1} - \mathbf{V(r_t)^\dagger\, V(0)}] > \sim \mathbf{1} - \mathbf{e}^{-[\mathbf{r_t^2\, Q_s^2}]\, \mathbf{log}(\frac{\mathbf{1}}{\mathbf{r_t}\,\Lambda_{\mathbf{QCD}}})}$$

$$\mathbf{r_t} \equiv \mathbf{x_t} - \mathbf{y_t}$$

$r_t \ll \frac{1}{Q_s}$ $\quad T(r_t) \longrightarrow r_t^2\, Q_s^2\, log(\frac{1}{r_t\, \Lambda_{QCD}})$ *color transparency*

$r_t \gg \frac{1}{Q_s}$ $\quad T(r_t) \longrightarrow 1$ *perturbative unitarization*

Solution of BFKL evolution equation

$$\frac{d}{dy}T(x_t,y_t)=\frac{N_c\alpha_s}{2\pi^2}\int d^2z_t\,\frac{(x_t-y_t)^2}{(x_t-z_t)^2(y_t-z_t)^2}\left[T(x_t,z_t)+T(z_t,y_t)-T(x_t,y_t)\right]$$

$$\mathbf{T(x,k_\perp)\sim(\frac{1}{x})^{\alpha_P-1}\,exp\left\{-\frac{ln^2(\frac{k_\perp}{k_{0\perp}})}{\bar{\alpha}\;ln(1/x)}\right\}}$$

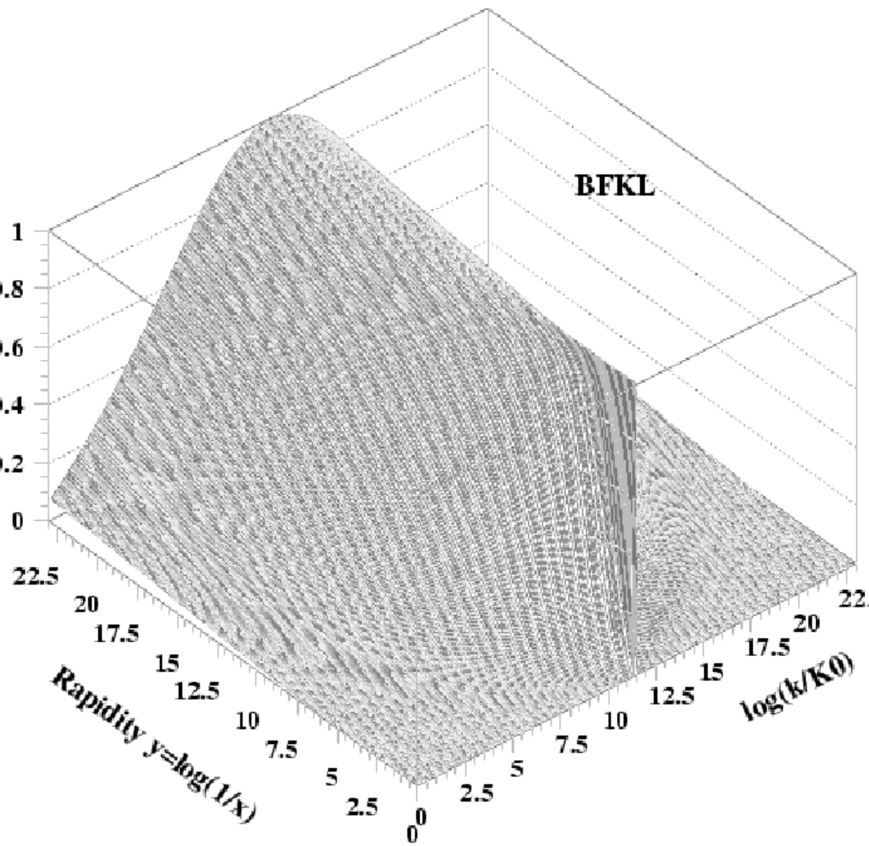
with

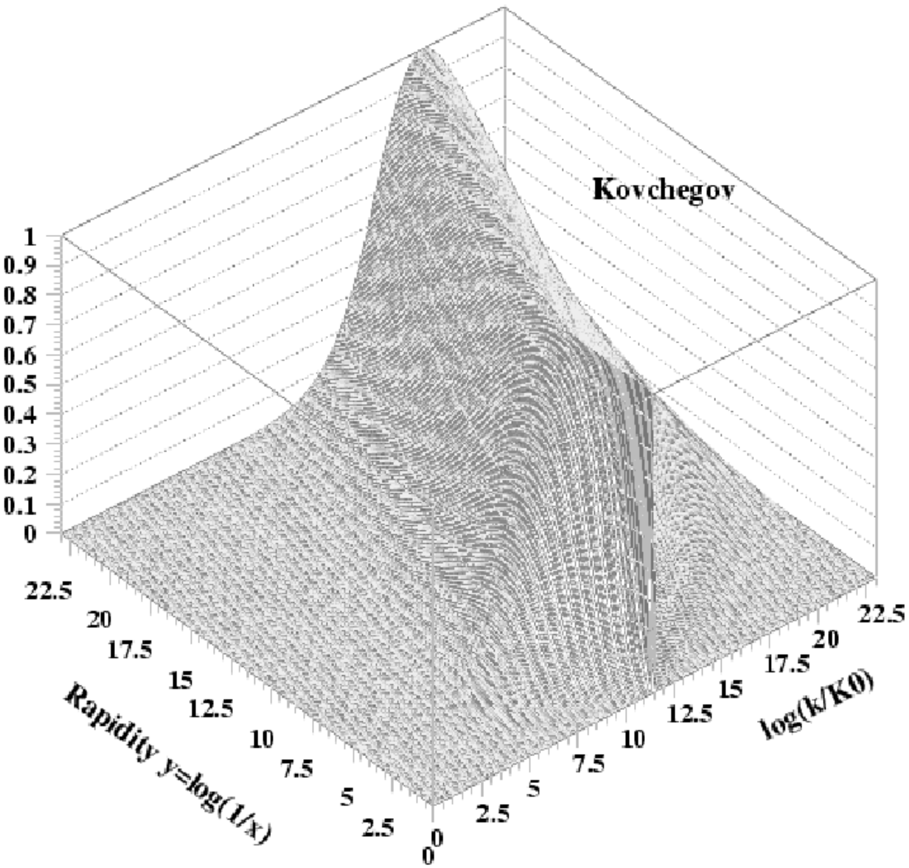
$$\alpha_{\mathbf{P}}-\mathbf{1}=\frac{\mathbf{4\,N_c}\,\alpha_{\mathbf{s}}}{\pi}\;\ln\mathbf{2}$$

BFKL predicts a fast rise of gluon distribution: unitarity?

diffusion of momenta into infrared?

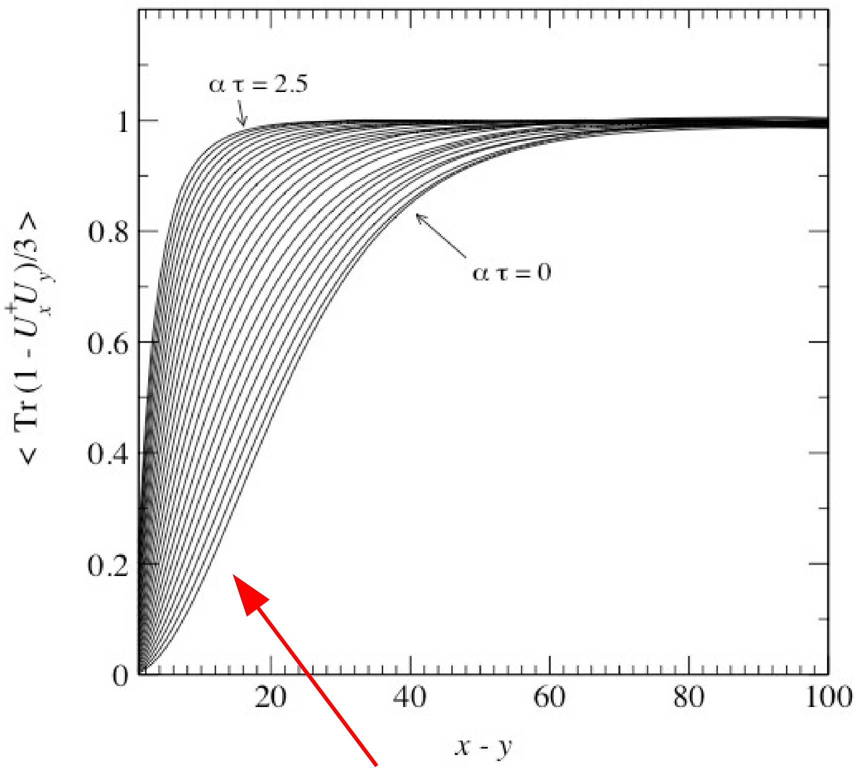
BFKL vs BK evolution





A. Stasto et al.

Solution of BK evolution equation



$\sim \mathbf{r_t^2\, xG(x, 1/r_t^2)}$

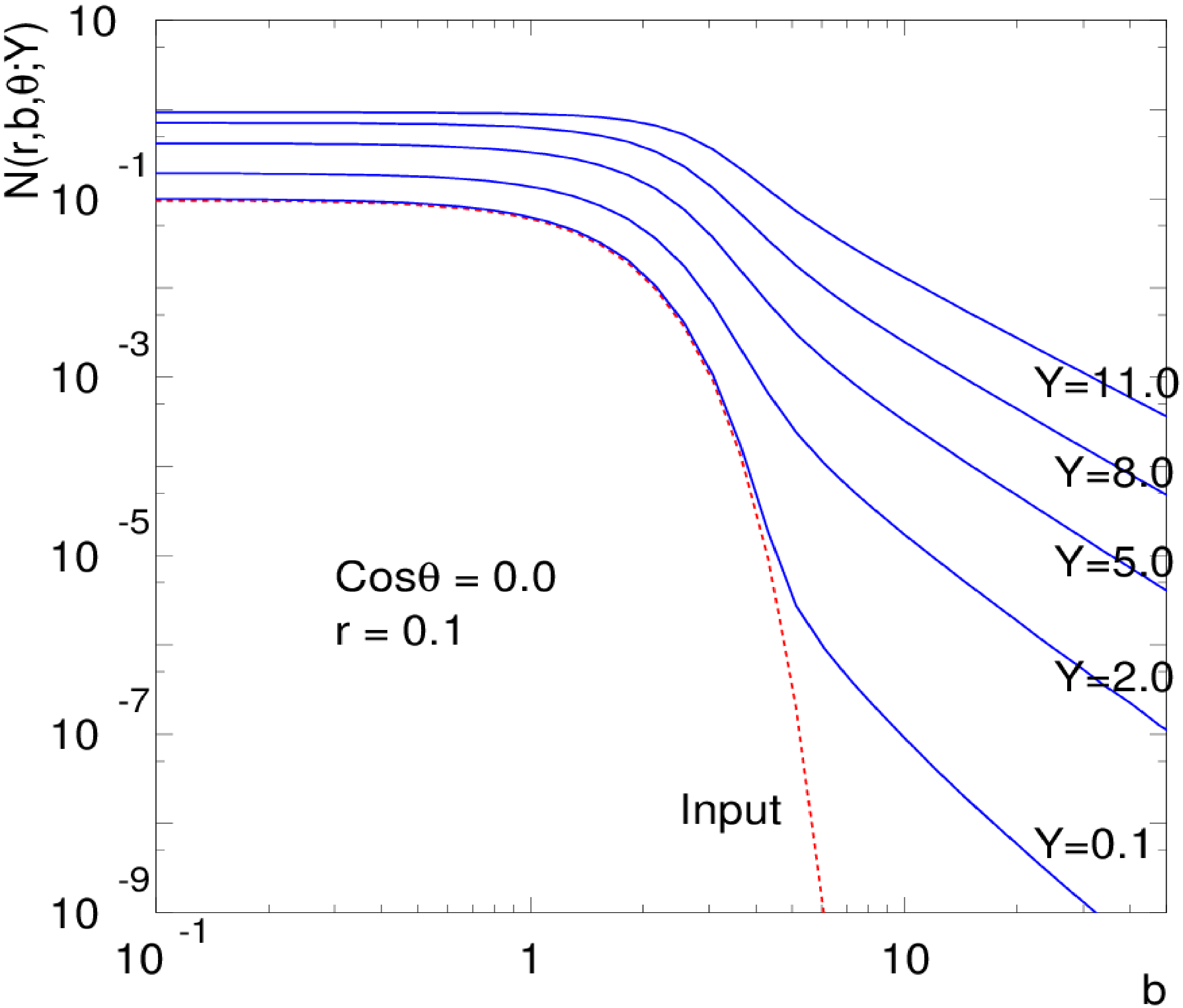
$$\tilde{T}(p_t) \sim \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right] \qquad Q_s^2 \ll p_t^2$$

$$\tilde{T}(p_t) \sim \log\left[\frac{Q_s^2}{p_t^2}\right] \qquad Q_s^2 \gg p_t^2$$

$$\tilde{T}(p_t) \sim \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]^{\gamma} \qquad Q_s^2 < p_t^2$$

Rummukainen-Weigert, NPA739 (2004) 183

b_t dependence of solution of BK



Golec-Biernat, Stasto 2003

Quadrupole

$$< Q(r,\bar{r},\bar{s},s) > \equiv \frac{1}{N_c} < Tr\, V(r)\, V^{\dagger}(\bar{r})\, V(\bar{s})\, V^{\dagger}(s) >$$

line config.: $r=\bar{s},\ \bar{r}=s,\ z\equiv r-\bar{r}$

square config.: $r-\bar{s}=\bar{r}-s=r-\bar{r}=\cdots\equiv z$

"naive" Gaussian: $Q=S^2$

Gaussian $Q_{|}(z)\approx\frac{N_c+1}{2}[S(z)]^{2\frac{N_c+2}{N_c+1}}-\frac{N_c-1}{2}[S(z)]^{2\frac{N_c-2}{N_c-1}}$

$$Q_{sq}(z)=[S(z)]^2\left[\frac{N_c+1}{2}\left(\frac{S(z)}{S(\sqrt{2}z)}\right)^{\frac{2}{N_c+1}}-\frac{N_c-1}{2}\left(\frac{S(\sqrt{2}z)}{S(z)}\right)^{\frac{2}{N_c-1}}\right]$$

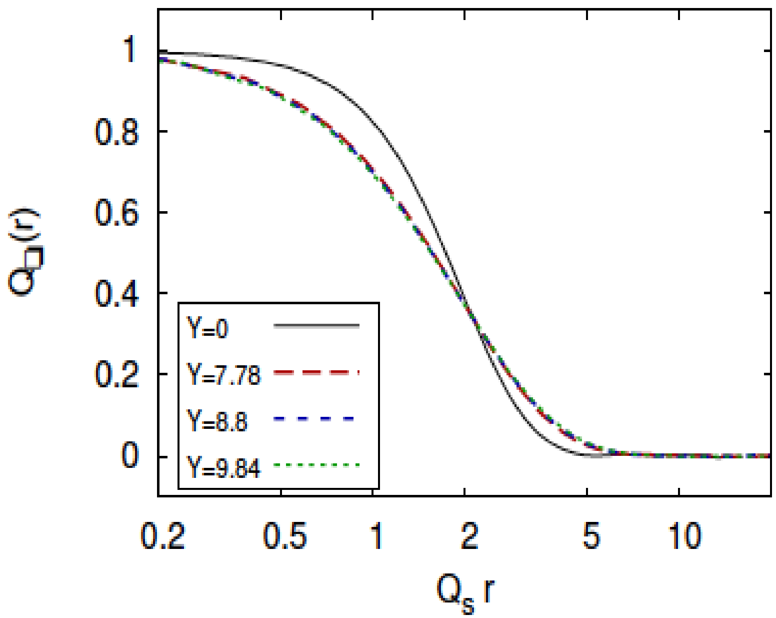
Gaussian + large Nc $Q_{|}(z)\rightarrow S^2(z)[1+2\,log[S(z)]]$

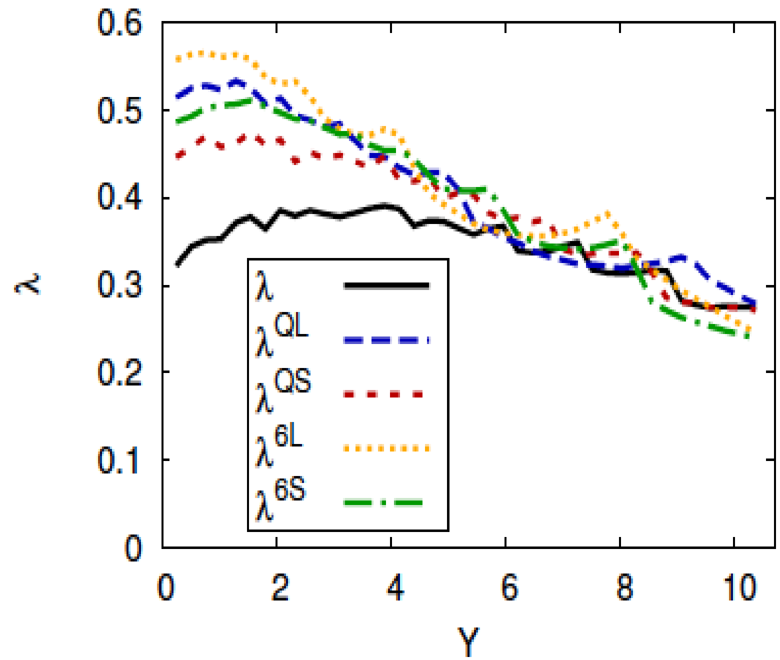
$$Q_{sq}(z)=S^2(z)\left[1+2\ln\left(\frac{S(z)}{S(\sqrt{2}z)}\right)\right]$$

Quadrupole: JIMWLK evolution

Dumitru-JJM-
Lappi-Schenke-Venugopalan:
PLB706 (2011) 219

$$< Q(r, \bar{r}, \bar{s}, s) > \equiv \frac{1}{N_c} \; < Tr\, V(r)\, V^{\dagger}(\bar{r})\, V(\bar{s})\, V^{\dagger}(s) >$$





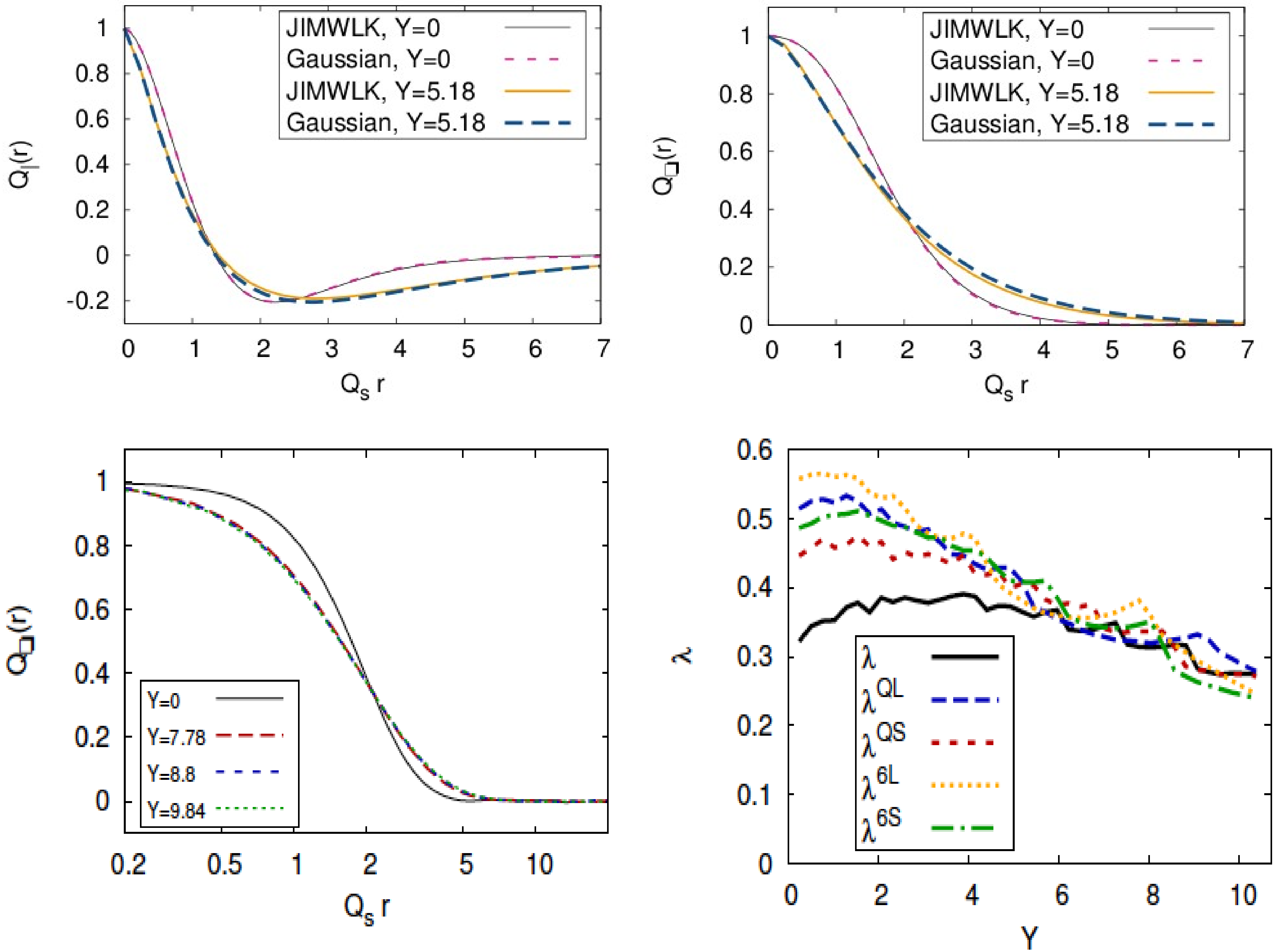
scaling

energy dependence

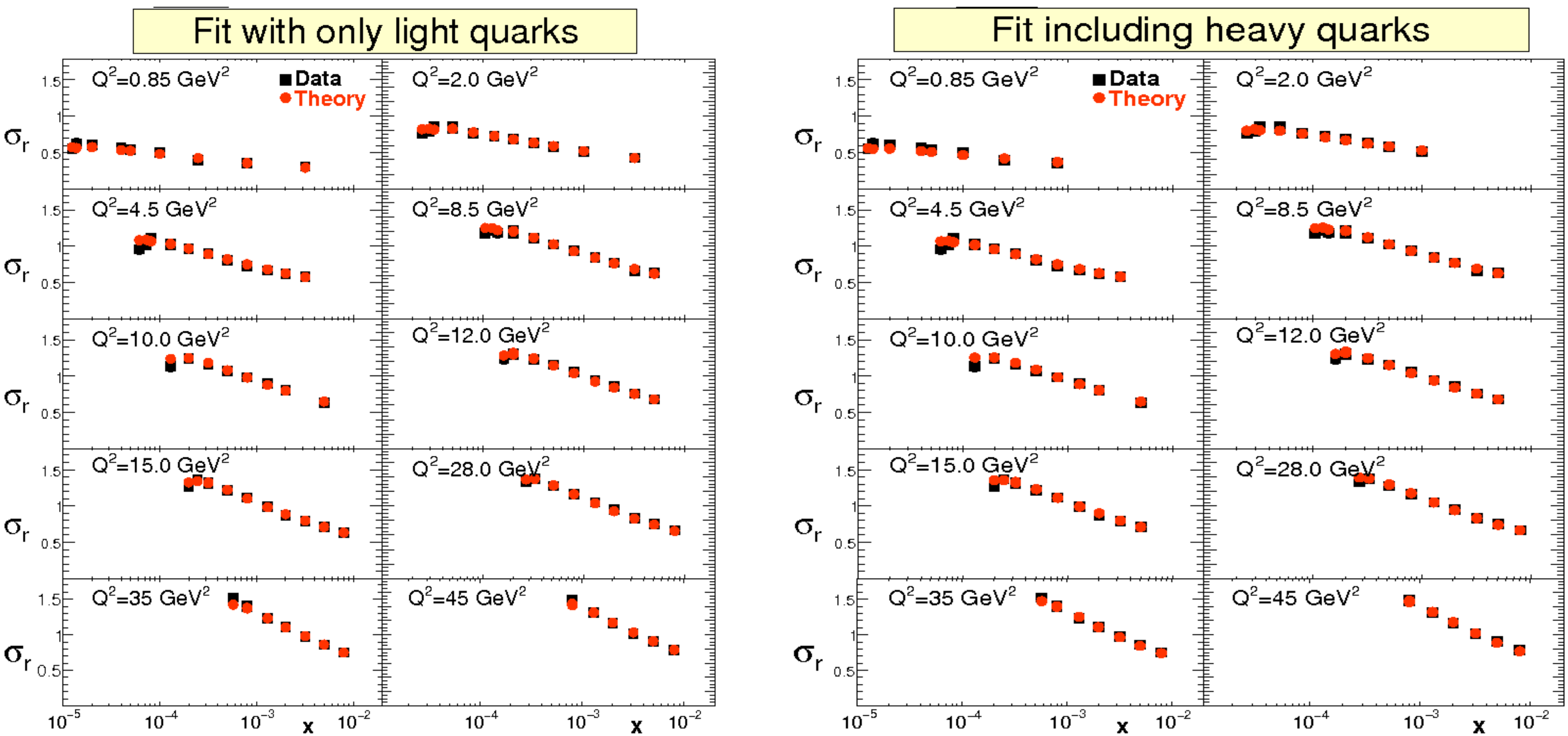
connection to/understanding from statistical physics: S. Munier,....

Fisher-Kolgomorov-Petrovsky-Piscounov (FKPP) equation

Quadrupole: $< Q(r, \bar{r}, \bar{s}, s) > \equiv \frac{1}{N_c} < Tr\, V(r)\, V^{\dagger}(\bar{r})\, V(\bar{s})\, V^{\dagger}(s) >$



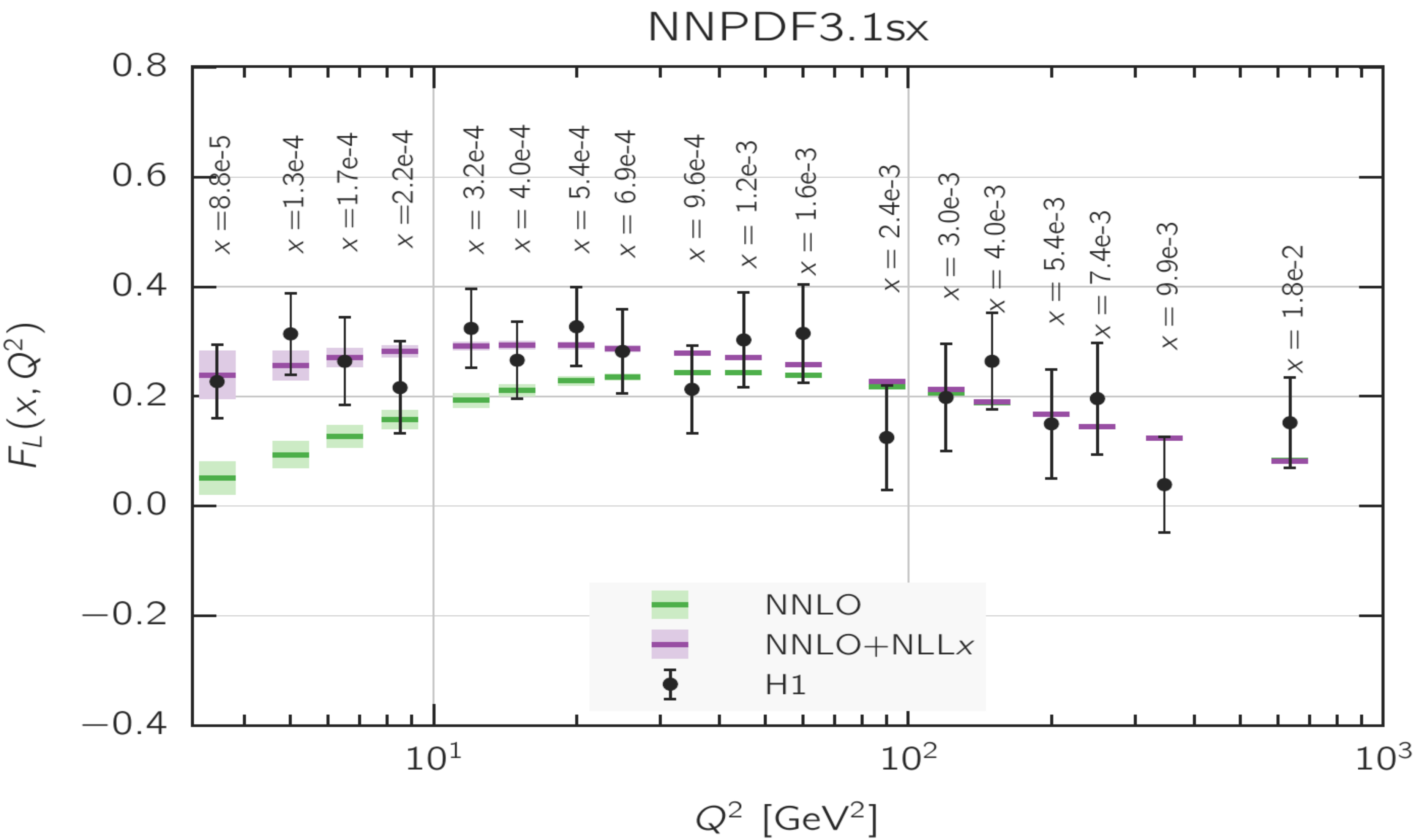
Structure functions at HERA



AAMQS(2010)

PQCD: DGLAP-based approaches also “work” : need more discriminatory observables

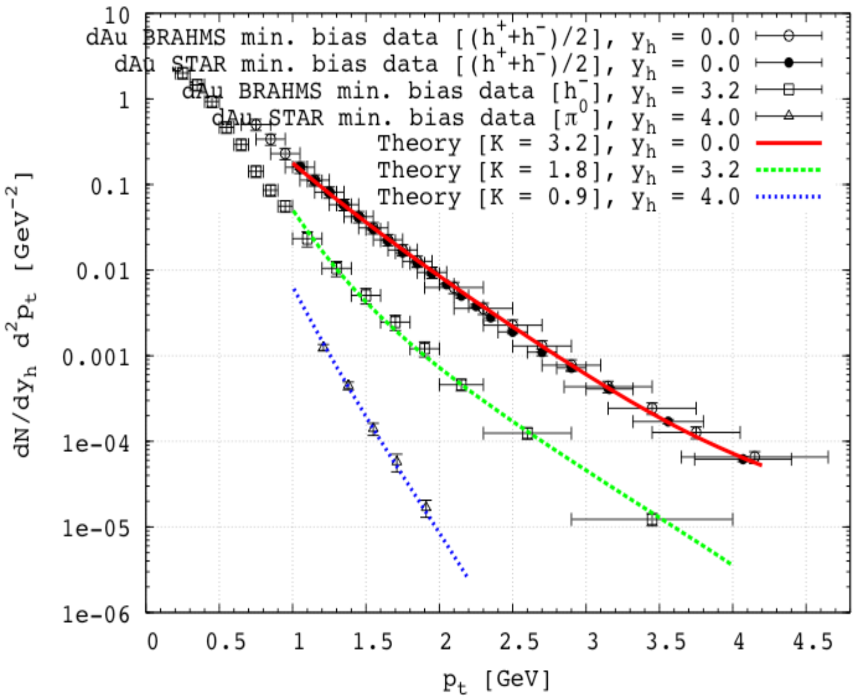
F_L at HERA

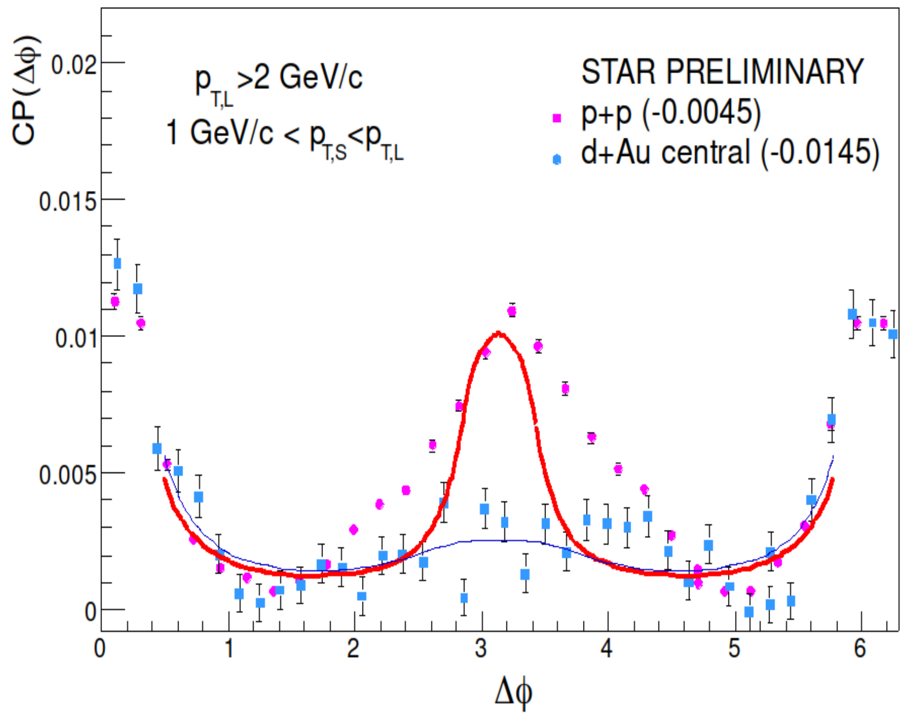


arXiv:1710.05935

CGC at RHIC

Single and double inclusive hadron production in dA collisions



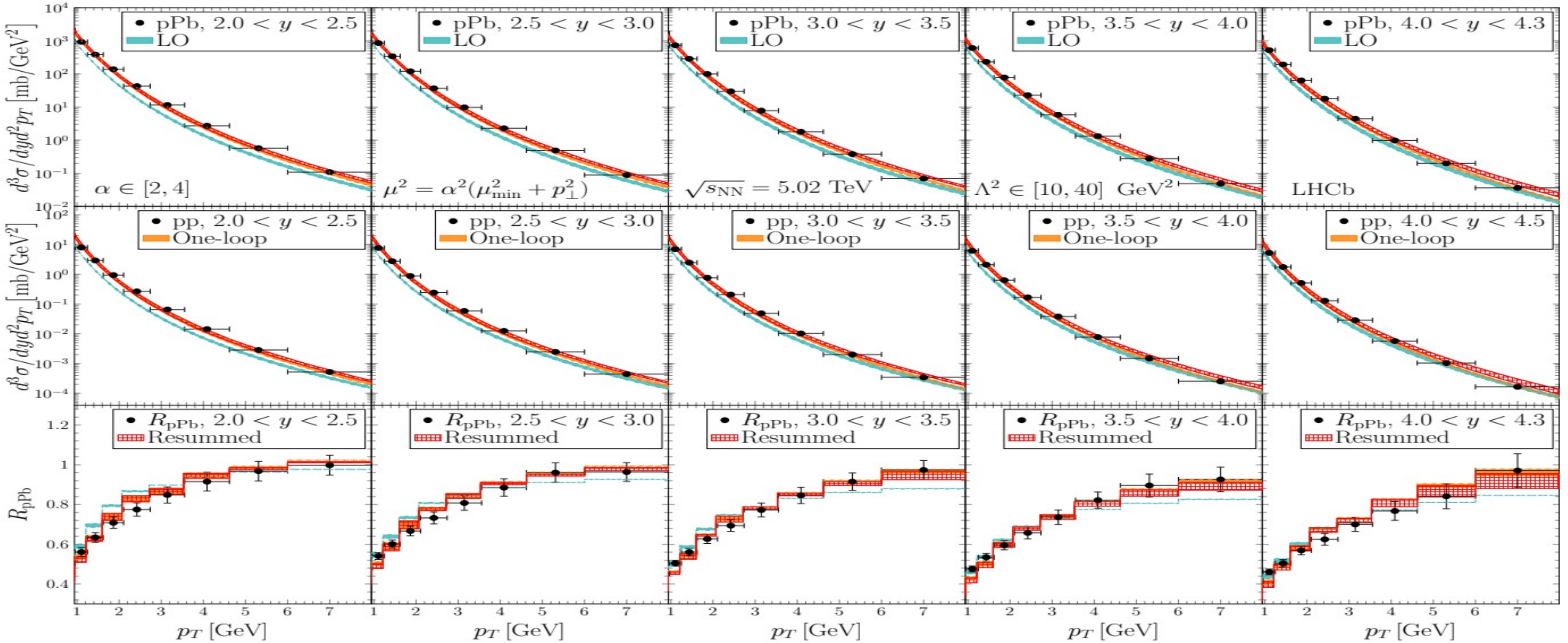


Dumitru,Hayashigaki, JJM, NPA770 (2006) 57

Albacete, Marquet, PRL105 (2010) 162301

CGC at NLO

Single inclusive hadron production in pA collisions: LHCb



Shi, Wang, Wei, Xiao, arXiv:2112.06975

Toward precision CGC at small x: inclusive DIS

NLO corrections to DIS structure functions:

Beuf (2017)
Beuf, Lappi, Paatelainen (2022)
....

NLO corrections to single inclusive hadron production in DIS:

Bergabo, JJM (2023)

NLO corrections to inclusive two-particle production in DIS:

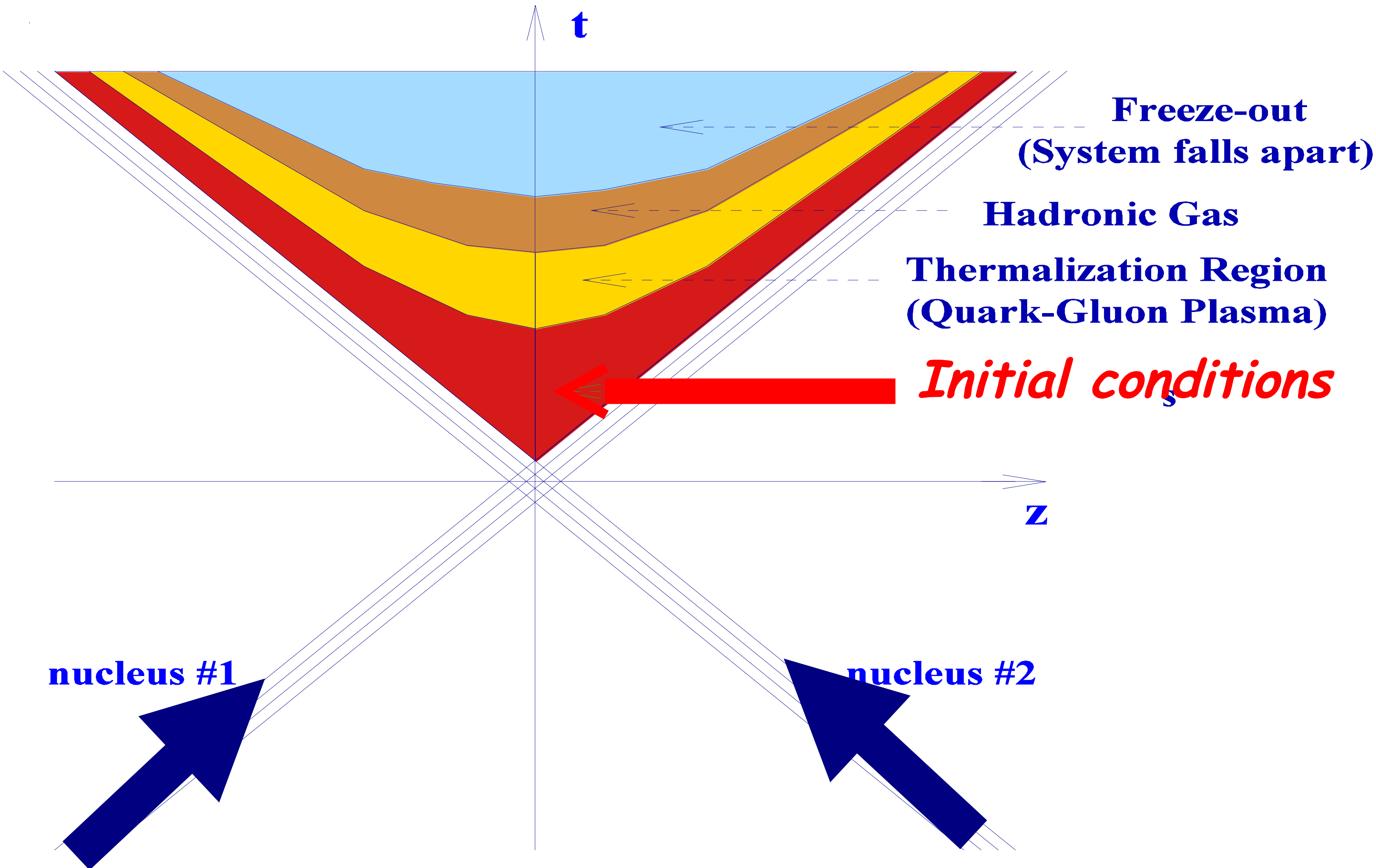
Bergabo, JJM (2022, 2023)
Taels, Altinoluk, Beuf, Marquet (2022)
Caucal, Salazar, Schenke, Venugopalan (2022)
Caucal, Salazar, Venugopalan (2021)
....

DIS: sub-eikonal corrections at small x

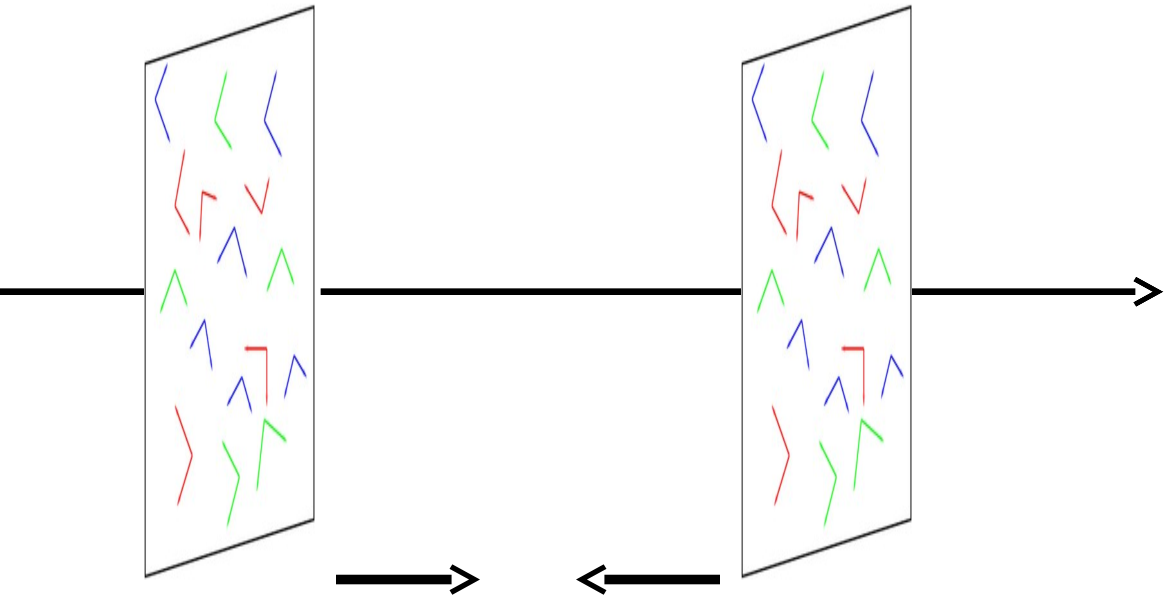
Altinoluk, Armesto, Beuf (2023)
Altinoluk, Beuf, Czajka, Tymowska (2021, 2022)
....

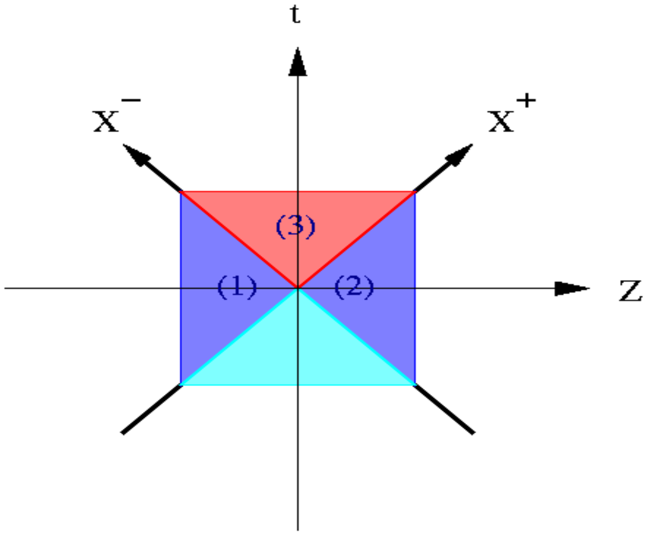
Significant work on exclusive production, diffraction, spin,TMDs

Space-Time History of a Heavy Ion Collision



Heavy Ion Collisions at High Energy:
Colliding Sheets of Color Glass





before the collision:

$$\mathbf{A}^+ = \mathbf{A}^- = \mathbf{0}$$

$$\mathbf{A}^\mathbf{i} = \mathbf{A}^\mathbf{i}_\mathbf{1} + \mathbf{A}^\mathbf{i}_\mathbf{2}$$

$$\mathbf{A}^\mathbf{i}_\mathbf{1} = \theta(\mathbf{x}^-)\theta(-\mathbf{x}^+)\alpha^\mathbf{i}_\mathbf{1}$$

$$\mathbf{A}^\mathbf{i}_\mathbf{2} = \theta(-\mathbf{x}^-)\theta(\mathbf{x}^+)\alpha^\mathbf{i}_\mathbf{2}$$

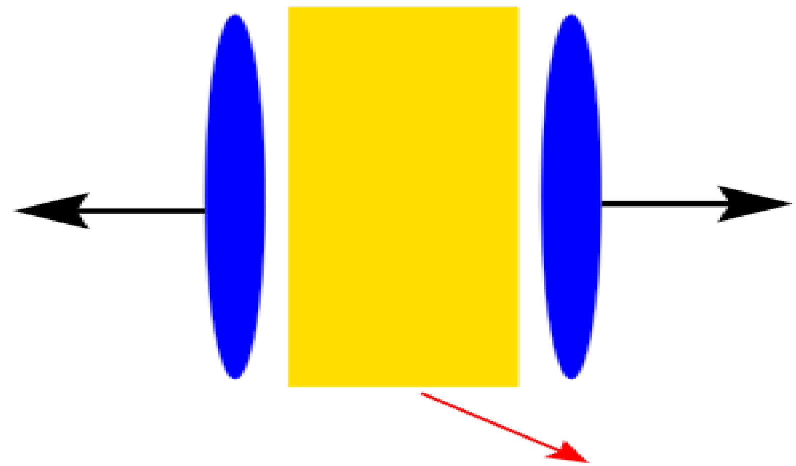
after the collision:

solve for $\mathbf{A}_\mu$

in the forward LC

Colliding Sheets of Color Glass at High Energies

solve the classical eqs. of motion in the forward light cone: subject to initial conditions given by one nucleus solution



GLASMA:strong color fields with occupation number ~ $\frac{1}{\alpha_s}$

initial energy and multiplicity of produced gluons depend on Q_s

$$\frac{1}{A_\perp}\frac{dE_\perp}{d\eta} = \frac{0.25}{g^2}Q_s^3 \qquad \frac{1}{A_\perp}\frac{dN}{d\eta} = \frac{0.3}{g^2}Q_s^2$$