

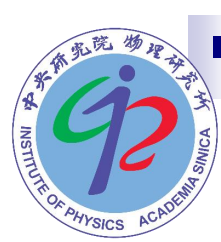


Chiral transport phenomena in core-collapse supernovae

Di-Lun Yang

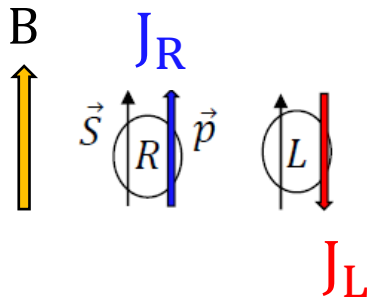
Institute of Physics, Academia Sinica

(AAC workshop, Aug. 2, 2023)



Anomalous (chiral) transport in chiral matter

- Normal transport : Ohmic current : $\mathbf{J}_V = \sigma_e \mathbf{E}$
- Weyl (massless) fermions : \mathcal{P} -even \mathcal{T} -odd
 chirality=helicity ($\vec{S} \cdot \hat{p}$) vector-charge current : $\mathbf{J}_V = \mathbf{J}_R + \mathbf{J}_L$
 axial-charge current : $\mathbf{J}_5 = \mathbf{J}_R - \mathbf{J}_L$
- Anomalous transport (in chiral matter) : $\mathbf{J}_V = \frac{\mu_5}{2\pi^2} \mathbf{B}$
 Chiral magnetic effect (CME) \mathcal{P} -odd \mathcal{T} -even

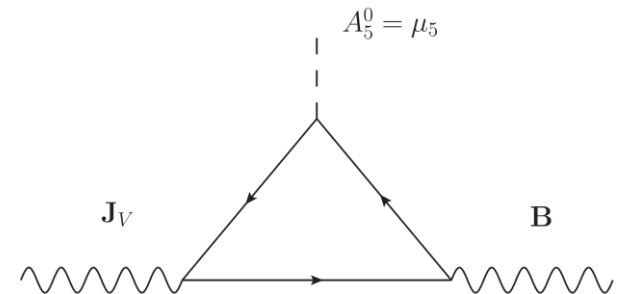


$J_V = J_R + J_L \neq 0$
 when $n_R - n_L = n_5 \neq 0$

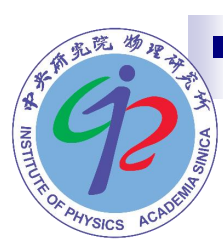
A. Vilenkin, PRD 22, 3080 (1980)
 K. Fukushima, D. Kharzeev, H. Warringa, PRD78, 074033 (2008)

- Chiral anomaly :

$$\partial_\mu J_{R/L}^\mu = \pm \frac{\mathbf{E} \cdot \mathbf{B}}{4\pi^2} \quad \Rightarrow \quad \partial_\mu J_5^\mu = \frac{\mathbf{E} \cdot \mathbf{B}}{2\pi^2}$$



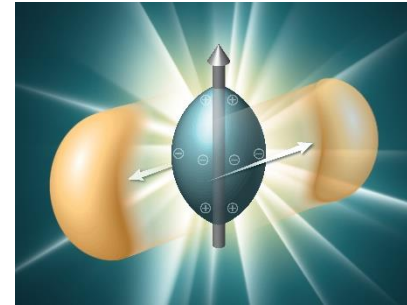
S. Adler, J. Bell, R. Jackiw, 69
 K. Fujikawa, 79



Where to find chiral transport in the real world?

- Nuclear physics : relativistic heavy ion collisions

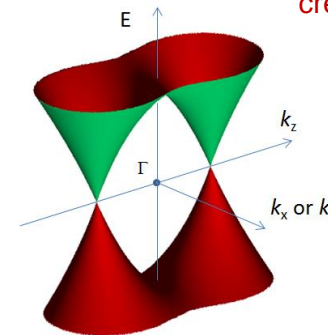
e.g. D. Kharzeev, et.al, Prog. Part. Nucl. Phys. 88, 1 (2016)



credit : BNL

- Condensed matter : Weyl semimetals

e.g. Q. Li, et.al., Nature Phys. 12 (2016) 550-554



Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

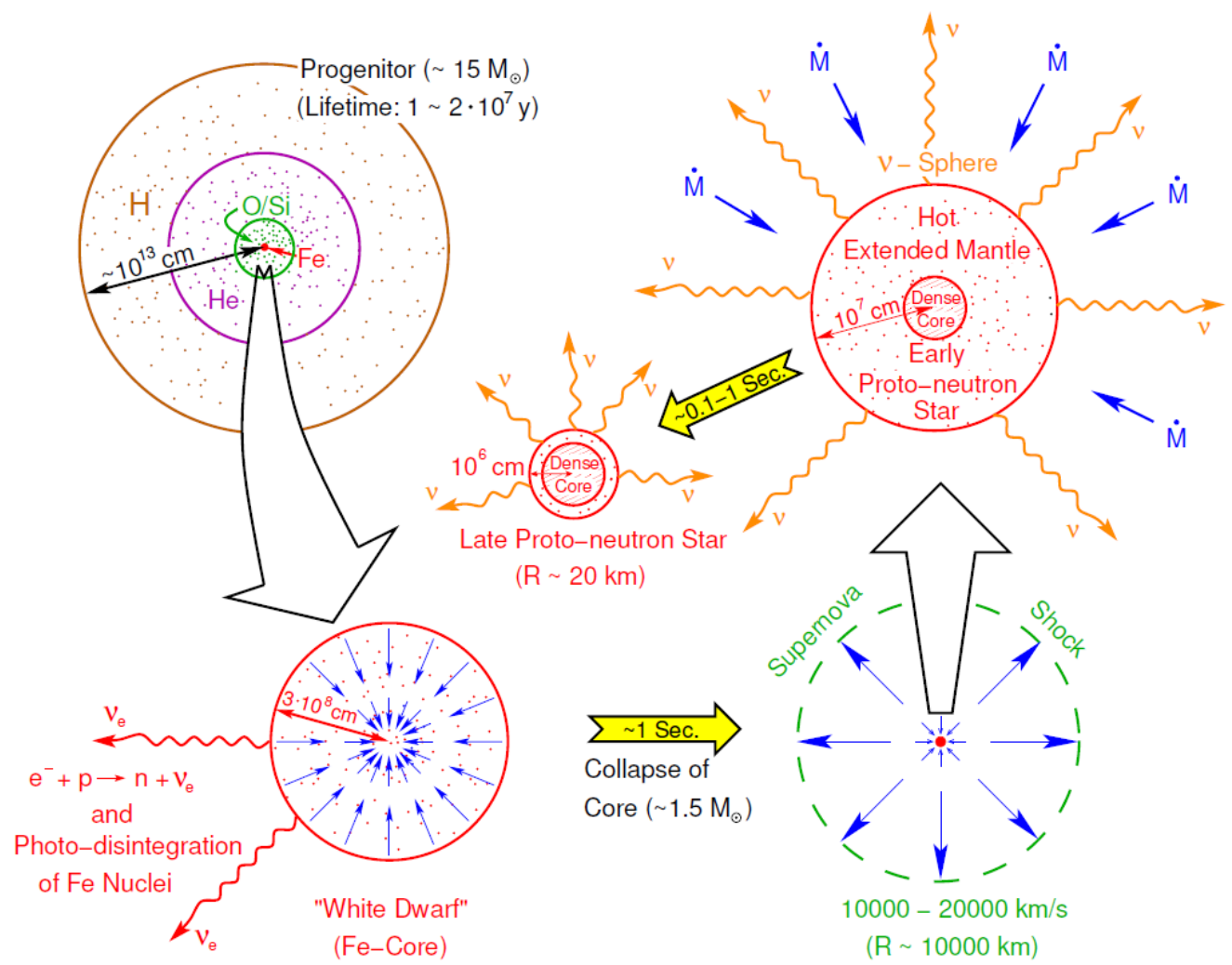
- Astrophysics : core-collapse supernovae

e.g. K. Kamada, N. Yamamoto, DY, Prog. Part. Nucl. Phys. 129 (2023) 104016

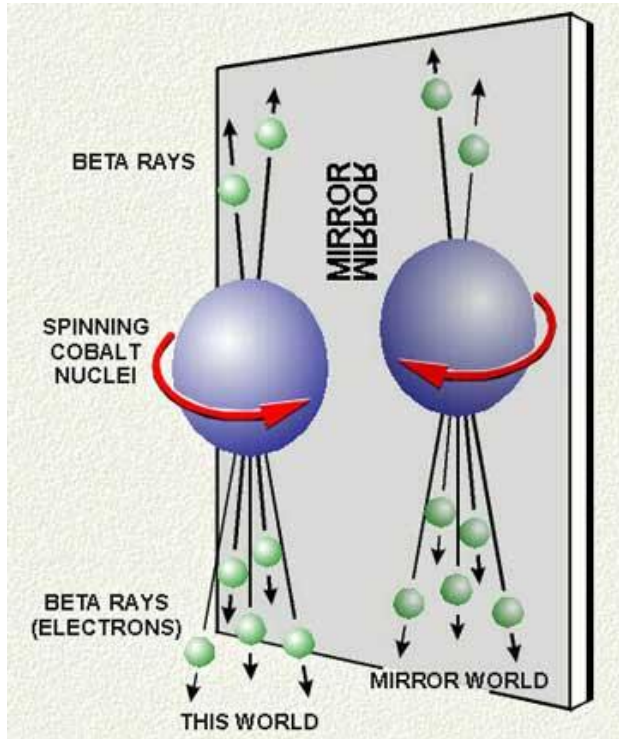


credit : RIKEN 3

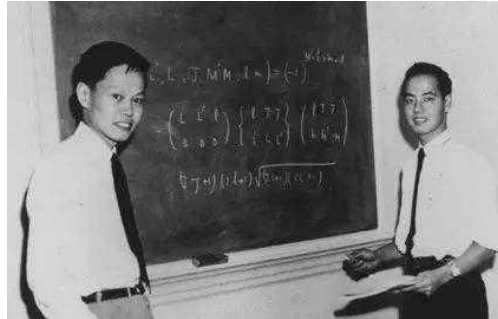
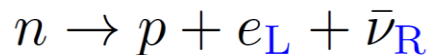
Evolution of core-collapse supernovae (CCSN)



Parity violation & weak interaction



<http://physics.nist.gov/GenInt/Parity/cover.html>



Lee & Yang (th) 1956



Wu et al., (exp) 1957

- Global parity violation in weak interaction
- Weak-interaction processes between leptons and nucleons are ubiquitous in CCSN.
- What will be the transport properties for (massless) fermions under parity (chirality) violation?

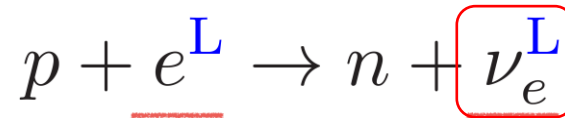
massless neutrinos & electrons considered ;
other quantum effects neglected : neutrino flavor oscillations

see the talk by M.- R. Wu

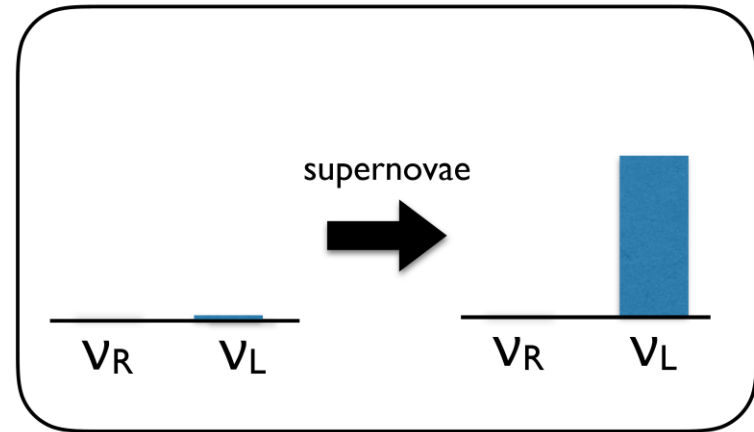
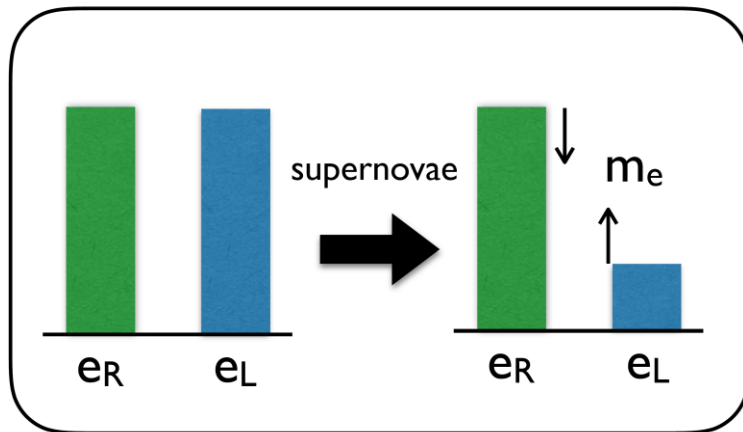
Generation of chirality imbalance

- Electron capture process in supernovae :

A. Ohnishi, N. Yamamoto, 2014, arXiv:1402.4760



an innate lefthander

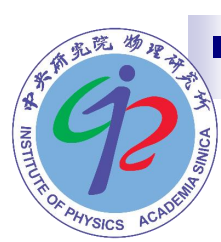


D. Grabowska, D.B. Kaplan, S. Reddy, PRD 91 (8) (2015) 085035

- Back-reaction from non-equilibrium neutrinos.

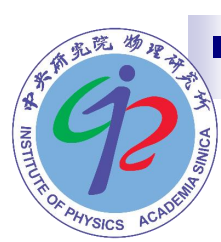
$$\nu_L^e(q) + n(k) \rightleftharpoons e_L(q') + p(k')$$

(relativistic electrons/neutrinos will be treated as massless pts.)



Chiral effects on dense stars?

- Three long-standing puzzles :
 - Pulsar kicks : the origin of momentum asymmetry for neutron stars?
 - Magnetars : the origin of strong & stable magnetic fields?
 - Explosions of CCSN with the observed energy?
- ❖ Chiral effects may provide possible explanations (qualitatively) in a consistent framework.
- Radiation hydrodynamics : e.g. S. W. Bruenn, *Astrophys. J. Suppl.* 58 (1985) 771.
magneto-hydrodynamics + relativistic kinetic theory (Boltzmann eq.)
matter (e, n, p) in equilibrium radiation (ν) out of equilibrium
- Constructing chiral radiation hydrodynamics :
chiral magneto-hydrodynamics + chiral kinetic theory
review : K. Kamada, N. Yamamoto, DY, *PPNP* 129 (2023) 104016



Chiral radiation transport equation

■ CKT for neutrinos : $\square_i f_q^{(\nu)} = \frac{1}{E_i} \left[(1 - f_q^{(\nu)}) \Gamma_q^{<} - f_q^{(\nu)} \Gamma_q^{>} \right]$

Boltzmann eq. in the inertial frame

collision term with quantum corrections

$$\square_i \equiv q \cdot D / E_i$$

N. Yamamoto & DY, APJ 895 (2020), 1

■ Neutrino absorption : $\bar{\Gamma}_q^{(ab)\lessgtr} \approx \bar{\Gamma}_q^{(0)\lessgtr} + \boxed{\hbar \bar{\Gamma}_q^{(\omega)\lessgtr}(q \cdot \omega) + \hbar \bar{\Gamma}_q^{(B)\lessgtr}(q \cdot B)}$

$$\nu_L^e(q) + n(k) \rightleftharpoons e_L(q') + p(k')$$

Fermi's EFT for weak int.

isoenergetic approx.:

NR approx., $M_n \approx M_p \approx M$

small-energy transf.

vorticity & magnetic field corrections :
breaking spherical symmetry & axisymmetry

analytic expressions : $\bar{\Gamma}_q^{(0)>} \approx \frac{1}{\pi \hbar^4 c^4} (g_V^2 + 3g_A^2) G_F^2 (q \cdot u)^3 (1 - f_{0,q}^{(e)}) \left(1 - \frac{3q \cdot u}{Mc^2} \right) \frac{n_n - n_p}{1 - e^{\beta(\mu_p - \mu_n)}}$

$$\bar{\Gamma}_q^{(B)>} \approx \frac{1}{2\pi \hbar^4 c^4 M} (g_V^2 + 3g_A^2) G_F^2 (q \cdot u) (1 - f_{0,q}^{(e)}) \left(1 - \frac{8q \cdot u}{3Mc^2} \right) \frac{n_n - n_p}{1 - e^{\beta(\mu_p - \mu_n)}}$$

$$\bar{\Gamma}_q^{(\omega)>} \approx \frac{1}{2\pi \hbar^4 c^4} (g_V^2 + 3g_A^2) G_F^2 (q \cdot u)^2 (1 - f_{0,q}^{(e)}) \left(\frac{2}{E_i} + \beta f_{0,q}^{(e)} \right) \frac{n_n - n_p}{1 - e^{\beta(\mu_p - \mu_n)}}$$

$\bar{\Gamma}_q^{(0)>}$: S. Reddy, M. Prakash, J. M. Lattimer, PRD58:013009,1998

Neutrino flux driven by magnetic fields

- Energy-momentum tensor & neutrino current : N. Yamamoto & DY, PRD 104, 123019 (2021)

$$T_{(\nu)}^{\mu\nu} = \int_q 4\pi\delta(q^2) \left(q^\mu q^\nu f_q^{(\nu)} - \hbar c q^{\{\mu} S_q^{\nu\}\rho} \mathcal{D}_\rho f_q^{(\nu)} \right),$$

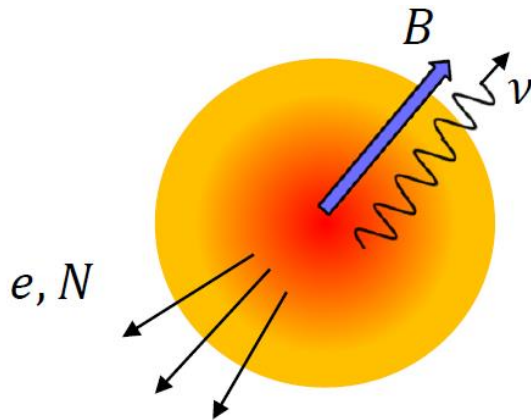
$$J_{(\nu)}^\mu = \int_q 4\pi\delta(q^2) \left(q^\mu f_q^{(\nu)} - \hbar c S_q^{\mu\rho} \mathcal{D}_\rho f_q^{(\nu)} \right), \quad \mathcal{D}_\mu f_q^{(\nu)} \equiv D_\mu f_q^{(\nu)} - \mathcal{C}_\mu [f_q^{(\nu)}]$$

- The momentum kick from neutrinos **near equilibrium** :

momentum con. : $\Delta T_e^{i0} = -\Delta T_\nu^{i0}, \quad \Delta T_e^{i0} = \mu_e \Delta j_e^i$

$\Rightarrow \Delta J_e^i = \xi_B B^i, \quad (\text{effective CME})$

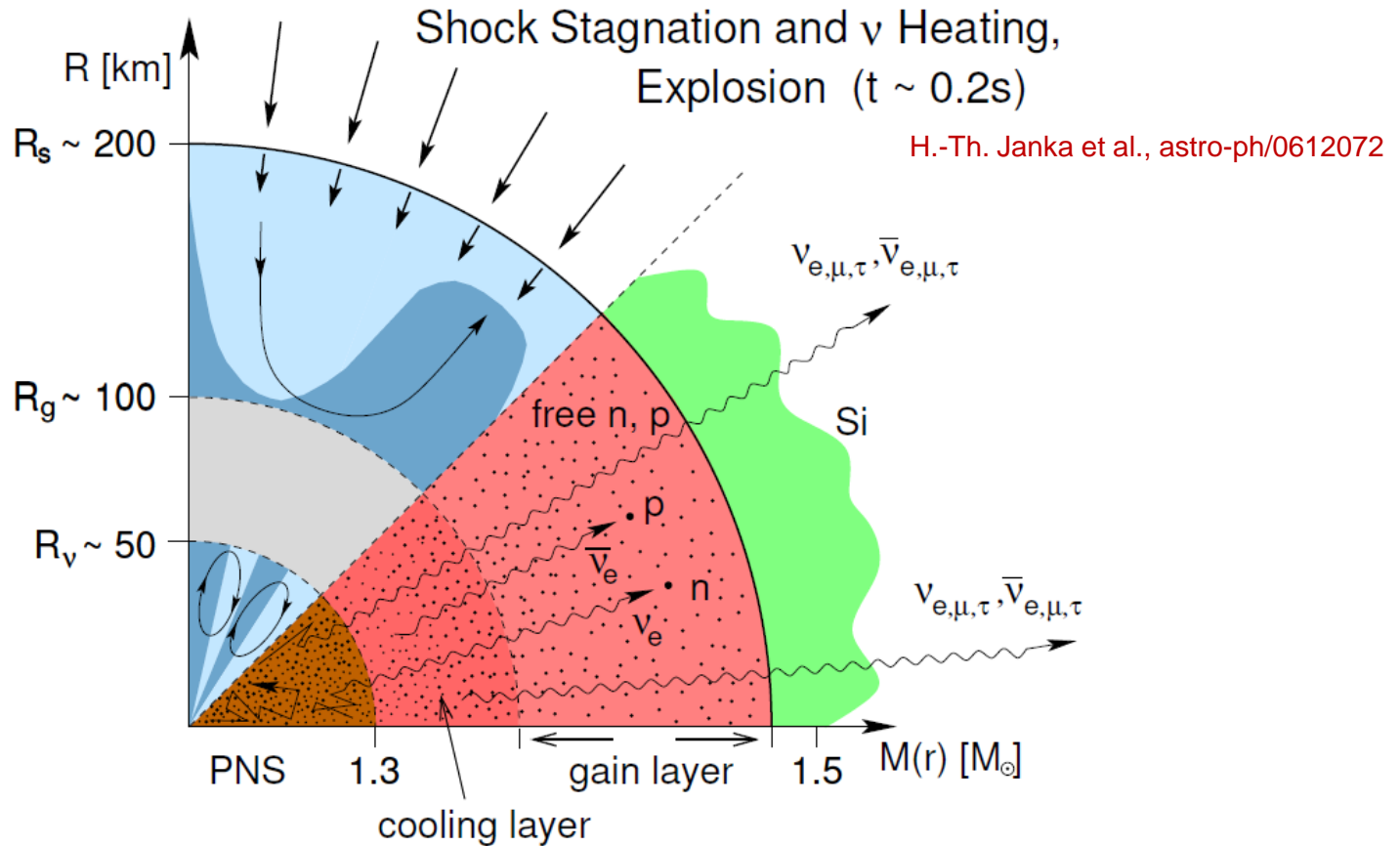
$$\xi_B = -\kappa (\nabla \cdot \mathbf{v}) \frac{\mu_\nu}{\mu_e}.$$



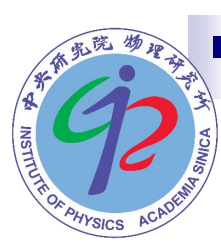
$$\kappa = \frac{1}{72\pi M G_F^2 (g_V^2 + 3g_A^2)} \frac{e^{2\beta(\mu_n - \mu_p)}}{n_n - n_p}$$

Shock revival by neutrino heating

- Heating by non-equilibrium neutrinos :



- ❖ How to have chiral effects from non-equilibrium neutrinos in the gain region?



Effective CME from neutrinos out of equilibrium

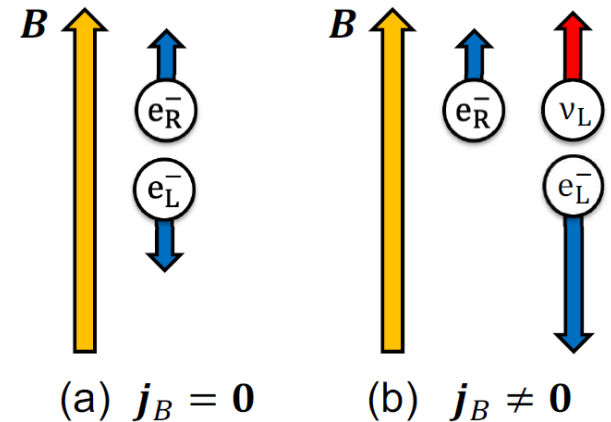
- Effective CME from neutrino radiation :

N. Yamamoto, DY, PRL 131, 012701 (2023)

$$j_B^\mu \approx \hbar e^2 \int \frac{d^4 q}{(2\pi)^3} \frac{\delta(q^2)}{q_0} (B^\mu q \cdot \partial_q - q \cdot B \partial_q^\mu) \delta f_W^{(e)} \equiv \xi_B B^\mu$$

$$\delta f_W^{(e)}(q, x) = -\frac{1}{q_0} \int_0^{x_0} dx'_0 F_W(q, x')|_c,$$

$$F_W \approx \frac{(q \cdot u)^3}{\pi} (g_V^2 + 3g_A^2) G_F^2 (n_p - n_n) \left[\frac{\bar{f}_q^{(e)}(1 - f_q^{(\nu)})}{1 - e^{\beta(\mu_n - \mu_p)}} + \frac{(1 - \bar{f}_q^{(e)})f_q^{(\nu)}}{1 - e^{\beta(\mu_p - \mu_n)}} \right]$$



- Approximate upper bounds (in the gain region) :

$$\xi_B^{\text{tot}} \approx -0.5 \text{ MeV} \quad \text{for } x_0 = 0.1 \text{ s (neutrino emission time)}$$

$$\square \text{ Kick velocity : } v_{\text{kick}} \sim \frac{|T_{B,\text{tot}}^{i0}|}{\rho_{\text{core}}} \approx \left(\frac{eB}{10^{13-14} \text{ G}} \right) \text{ km/s.} \quad \left. \begin{array}{l} B \sim 10^{15-16} \text{ G} \\ \text{for} \\ v_{\text{kick}} \sim 10^2 \text{ km/s} \end{array} \right\}$$

see also A. Vilenkin, Astrophys. J. 451, 700 (1995).

Chiral plasma instability

- Chiral plasma instability (CPI) :

M. Joyce, M. E. Shaposhnikov, PRL 79, 1193 (1997)

Y. Akamatsu, N. Yamamoto, PRL 111, 052002 (2013)

- Anomalous Maxwell's eq. :

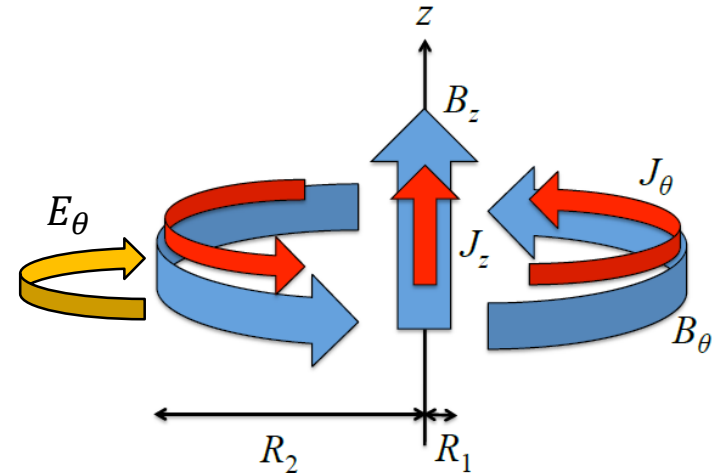
$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E}, \quad \nabla \times \mathbf{B} = \eta^{-1} \mathbf{E} + \boxed{\xi_B \mathbf{B}}$$

CME

$$\Rightarrow \frac{\partial \mathbf{B}}{\partial t} = \boxed{\eta \nabla^2 \mathbf{B}} + \boxed{\eta \nabla \times (\xi_B \mathbf{B})}$$

diffusion

CME (instability)



Y. Akamatsu, N. Yamamoto, PRD 90, 125031 (2014)

- Unstable modes at long wavelength : $\delta \mathbf{B} \propto e^{\sigma t + i \mathbf{k} \cdot \mathbf{x}}$

$$\sigma = \eta k (\xi_B - k) \quad (\text{for small } k, \text{ long wavelength})$$

- Helicity conservation :

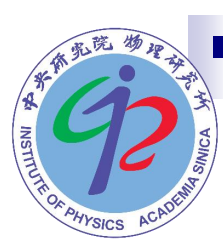
$$\frac{dH_{\text{tot}}}{dt} = 0,$$

$$H_{\text{tot}} \equiv N_5 + \frac{H_{\text{mag}}}{4\pi^2},$$

$$N_5 \equiv \int d^3x n_5, \quad H_{\text{mag}} \equiv \int d^3x \mathbf{A} \cdot \mathbf{B}.$$

exchange btw the (effective) axial charge & magnetic helicity

Y. Hirono, D. Kharzeev, Y. Yin, PRD 92, 125031 (2015)



Chiral magnetohydrodynamics

- Chiral magnetohydrodynamics (MHD) equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (17)$$

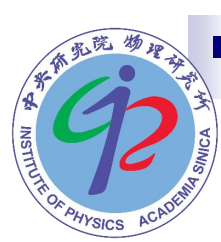
$$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla \cdot \left[\rho \mathbf{v} \mathbf{v} - \mathbf{B} \mathbf{B} + \left(P + \frac{B^2}{2} \right) \mathbf{I} \right] = \mathbf{S}, \quad (18)$$

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{1}{2} \rho v^2 + \frac{1}{\Gamma - 1} P + \frac{B^2}{2} \right) + \nabla \cdot \left[\left(\frac{1}{2} \rho v^2 + \frac{\Gamma}{\Gamma - 1} P \right) \mathbf{v} \right. \\ \left. + \mathbf{E} \times \mathbf{B} \right] = -\mathbf{S} \cdot \mathbf{v} - \Delta \mathbf{J} \cdot \mathbf{E}, \quad (19) \end{aligned}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} + \eta \nabla \times (\xi_B \mathbf{B}), \quad (20)$$

$$\frac{\partial n_{5,\text{eff}}}{\partial t} = \frac{1}{2\pi^2} \mathbf{E} \cdot \mathbf{B}, \quad (21)$$

$$\mathbf{S} = \rho \nu \nabla^2 \mathbf{v} + \frac{1}{3} \rho \nu \nabla (\nabla \cdot \mathbf{v})$$



Time evolution of the magnetic field

■ Numerical simulations :

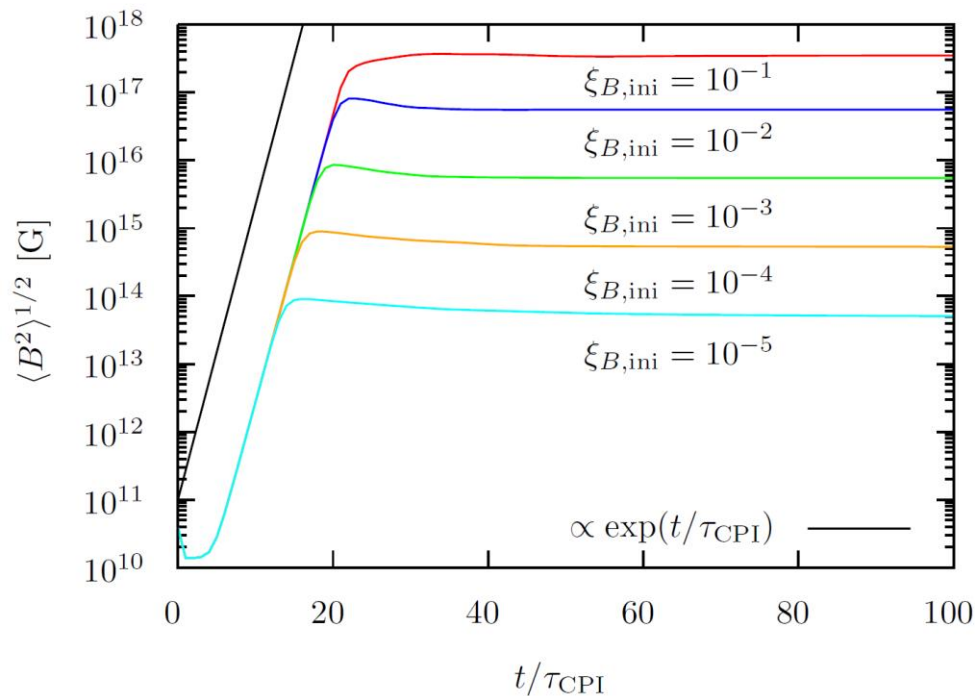
resistivity : $\eta = 1$ viscosity : $\nu = 0.01$

in the units of $100 \text{ MeV} = 1$

J. Matsumoto, N. Yamamoto, DY, PRD 105 (2022) 12, 123029

TABLE I. Summary of the simulation runs.

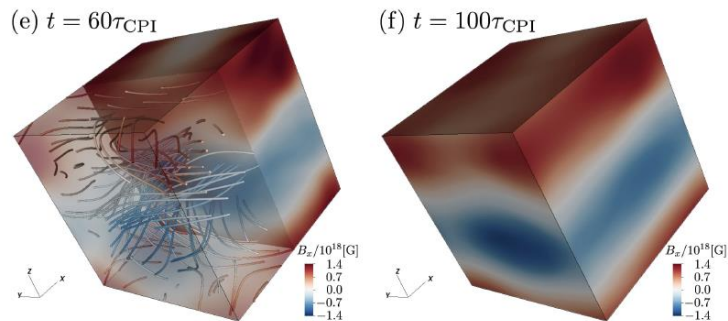
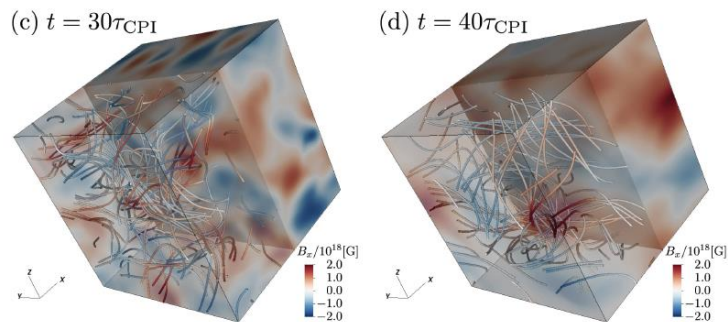
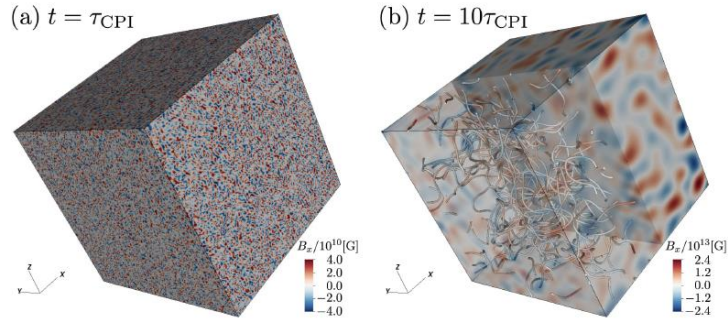
Name	L	$\xi_{B,\text{ini}}$	τ_{CPI}
Model 1	8×10^2	10^{-1}	4×10^2
Model 2	8×10^3	10^{-2}	4×10^4
Model 3	8×10^4	10^{-3}	4×10^6
Model 4	8×10^5	10^{-4}	4×10^8
Model 5	8×10^6	10^{-5}	4×10^{10}



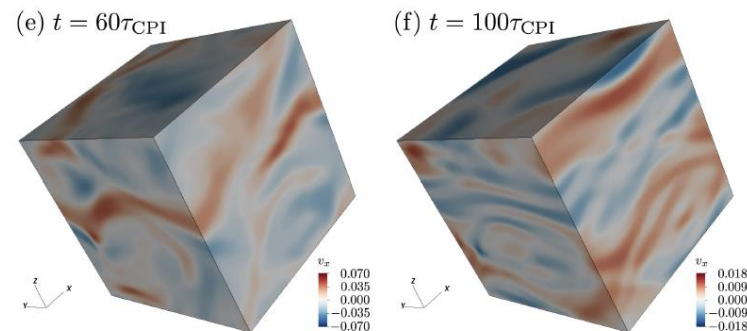
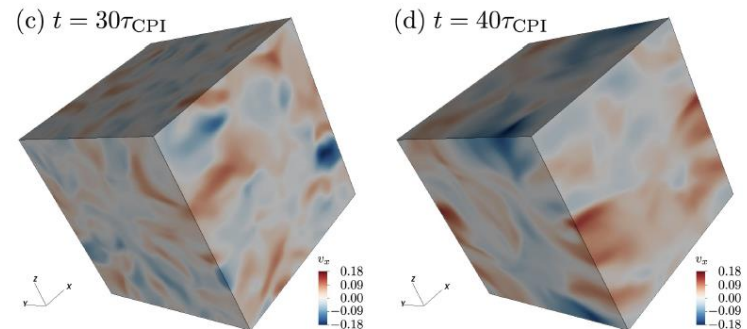
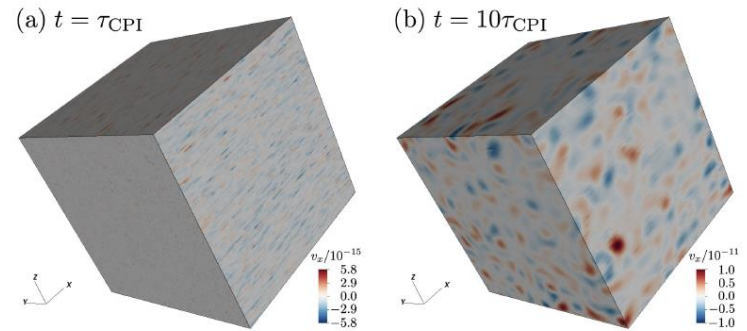
$$B_{\text{CPI}} \sim \mu_{5,\text{eff}}^2$$

Inverse cascade in local ChMHD simulations

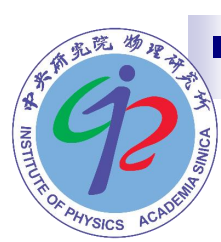
- Inverse cascade : [J. Matsumoto, N. Yamamoto, DY, PRD 105 \(2022\) 12, 123029](#)



magnetic field



fluid velocity



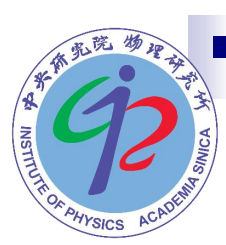
Summary & outlook

❖ Summary

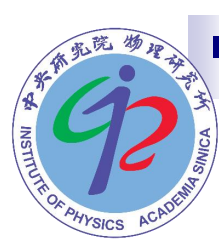
- ✓ The chiral effect for leptons due to “parity violation” from weak int. could qualitatively affect the supernova evolution.
- ✓ Neutrino radiation could induce an “effective CME”, which may result in pulsar kicks and dynamically generate strong magnetic fields in magnetars from “CPI”.
- ✓ Local ChMHD simulations show the “inverse cascade” through chiral effects, which may favor explosions of CCSN.

❖ Outlook

- ❑ Ultimate goal : comprehensive simulations for chiral radiation hydro.
- ❑ Chiral effects from vorticity, temperature/chemical-potential gradients, etc.
- ❑ Chiral effects + flavor oscillation for neutrinos



Thank you!



Chiral kinetic theory from QFT

- Wigner functions : $\dot{S}_L^<(q, x) \equiv \int_y e^{-\frac{iq \cdot y}{\hbar}} \langle \psi_L^\dagger(x, y/2) \psi_L(x, -y/2) \rangle \equiv \sigma^\mu \mathcal{L}_\mu^<(q, x)$

see e.g. Y. Hidaka, S. Pu, DY, PRD 95, 091901 (2017)

review : Y. Hidaka S. Pu, Q, Wang, DY, PPNP 127 (2022) 103989

- Perturbative solution up to $\mathcal{O}(\hbar)$: ($\sim \partial/q$: gradient expansion)

$$\mathcal{L}^{<\mu} = 2\pi \left[\delta(q^2) (q^\mu - \hbar S_{(n)}^{\mu\nu} \mathcal{D}_\nu) - \hbar \tilde{F}^{\mu\nu} q_\nu \delta'(q^2) \right] f_L,$$

$$\mathcal{D}_\mu \mathcal{L}_\nu^< \equiv (\nabla_\mu - \Gamma_{\mu\rho}^\lambda q^\rho \partial_{q^\lambda} + F_{\rho\mu} \partial_q^\rho) \mathcal{L}_\nu^< - \Sigma_\mu^< \mathcal{L}_\nu^> + \Sigma_\mu^> \mathcal{L}_\nu^<, \quad S_{(n)}^{\mu\nu} = \frac{\epsilon^{\mu\nu\alpha\beta} q_\alpha n_\beta}{2q \cdot n}.$$

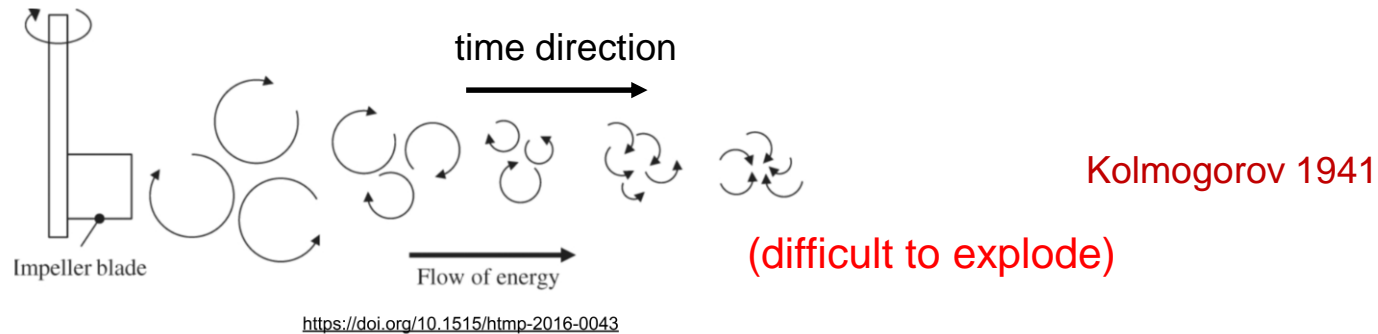
- Chiral kinetic equation :

$$0 = \delta(q^2 - \hbar F_{\alpha\beta} S_{(n)}^{\alpha\beta}) \left\{ \left[q \cdot \mathcal{D} - \hbar \left(\frac{S_{(n)}^{\mu\nu} F_{\mu\rho} n^\rho}{q \cdot n} + (D_\mu S_{(n)}^{\mu\nu}) \right) \mathcal{D}_\nu - \hbar S_{(n)}^{\mu\nu} (\nabla_\mu F_\nu^\lambda - q^\rho R_{\rho\mu\nu}^\lambda) \partial_{q^\lambda} \right] f_L \right. \\ \left. - \hbar \frac{\epsilon^{\mu\nu\alpha\beta} n_\nu}{2q \cdot n} ((1 - f_L) \Delta_\alpha^> \Sigma_\beta^< - f_L \Delta_\alpha^< \Sigma_\beta^>) \right\}.$$

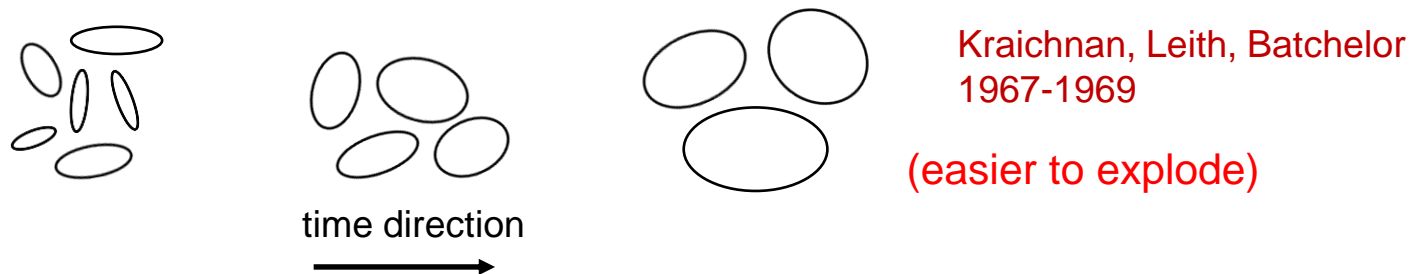
- Current and EM tensor : $J^\mu = 2 \int_q \mathcal{L}^{<\mu}, \quad T^{\mu\nu} = \int_q (\mathcal{L}^{<\mu} q^\nu + \mathcal{L}^{<\nu} q^\mu).$

Direct & inverse energy cascades

- 3D supernova simulations are more difficult to achieve explosion than 2D
- Turbulence in 3D : direct energy cascade



- Turbulence in 2D : inverse energy cascade



- Chiral effects in 3D : inverse cascade