Scalar Gravitational Waves Can Be Generated Even Without Direct Coupling Between Dark Energy and Ordinary Matter

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Outline

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Introduction

- Waves propagating in the universe is affected by the geometry of spacetime.
- It was found that the universe is accelerating expanding, known as Dark energy dominated era.
- Could gravitational waves reveal the what the Dark energy is?

Setup

- For our purpose, we consider the linearised theory with spatially flat Friedmann-Robertson-Walker (FRW) background
- The background dynamics is driven by a minimally coupled scalar field $\overline{\varphi}$ subject to potential $V(\overline{\varphi})$.

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv \mathcal{H}^2 = \frac{8\pi G_\mathrm{N}}{3}\left(\frac{1}{2}\dot{\overline{\varphi}}^2 + a^2 V[\overline{\varphi}]\right) \tag{1}$$

$$\ddot{\overline{\varphi}} + 2\mathcal{H}\dot{\overline{\varphi}} + a^2 V'(\overline{\varphi}) = 0 \tag{2}$$

- The metric tensor and scalar field are written as

$$g_{\mu\nu} = a^2(\eta)(\eta_{\mu\nu} + \chi_{\mu\nu}(\eta, \vec{x})), \qquad \varphi = \overline{\varphi}(\eta) + \psi(\eta, \vec{x}). \tag{3}$$

Setup

It is better to express the final results in terms of observables rather than $\overline{\varphi}$ and $V(\overline{\varphi})$. The equation of state parameter is defined as

$$w = \frac{\frac{1}{2}\overline{\varphi}^2 - a^2 V(\overline{\varphi})}{\frac{1}{2}\overline{\varphi}^2 + a^2 V(\overline{\varphi})} \tag{4}$$

There is evidence that the equation of state w is close to -1. Hence,

$$\delta w \equiv w + 1. \tag{5}$$

Setup

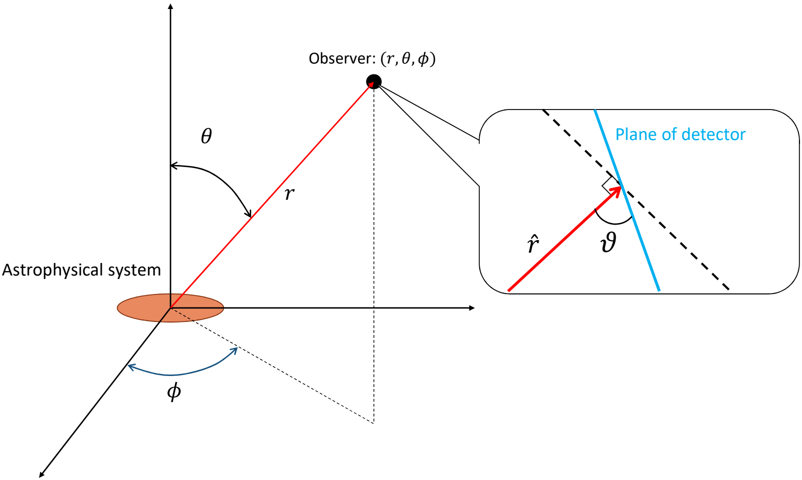

Figure 1: Setup

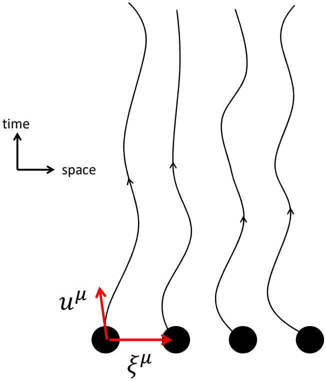

Figure 2: Detector

Separation vector ξ^μ is described by

$$\frac{D^2\xi^\mu}{d\tau^2} = -R^\mu{}_{\alpha\nu\beta}u^\alpha u^\beta \xi^\nu \tag{6}$$

For free-falling and co-moving observers,

$$\begin{aligned} \ddot{\xi}^i &= -a^{-2}R^i{}_{0j0}\xi^j \\ &\approx -a^2\delta_1 C^i{}_{0j0}\xi^j + (\text{trace part}) \end{aligned} \tag{7}$$

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First order Gauge-invariant perturbation theory

Gauge-invariant perturbation theory: Use the perturbations $\chi_{\mu\nu}$ and ψ to construct gauge-invariant quantities.

Motivation:

- Gauge ambiguity of interpretation in gauge-fixing approach.
- It is not always possible to find an appropriate choice of gauge which can simplify the calculation.
- It works!!

First order Gauge-invariant perturbation theory

There are two gauge-invariant metric perturbations propagating in the spacetime.

- Bardeen scalar Φ
- Transverse-traceless tensor $D_{ij} \equiv \chi_{ij}^{\mathrm{TT}}$.

Their equations can be written in the form

$$(\partial^2 + U(\eta))W(\eta, \vec{x}) = \mathcal{J}(\eta, \vec{x}) \tag{8}$$

where ∂^2 is the wave operator in Minkowski. It was shown[1] that the retarded Green's function takes the form

$$\mathcal{G}_U^+ = \frac{\delta(T-R)}{4\pi R} + \frac{\Theta(T-R)}{4\pi} \frac{\partial}{\partial \bar{\sigma}} \mathcal{V}(\eta, \eta', \bar{\sigma}) \tag{9}$$

[1] Chu, 2015.

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Scalar tidal forces

The light cone contribution of scalar contribution of Weyl tensor in far-zone and non-relativistic limit is

$$\delta_1 C^{(\Phi|\gamma)i}{}_{0j0} \approx -\frac{G_{\rm N}}{2a(\eta)r}(\delta_{ij} - 3\widehat{r}_i\widehat{r}_j)\mathcal{H}(\eta)\mathcal{H}(\eta_r)\sqrt{\delta w(\eta)\delta w(\eta_r)}\left(M(\eta_r) + \frac{\ddot{Q}_{\ell\ell}(\eta_r)}{2a^2(\eta_r)}\right). \tag{10}$$

where $\eta_r = \eta - r$.

Scalar tidal forces: Polarisation pattern

For co-moving observers, the relative acceleration of a pair of masses is

$$\ddot{\xi}^{\mu}\widehat{e}_{\mu} \approx -\frac{G_{\mathrm{N}}}{2a(\eta)r} \times \left(3\sin^2\vartheta - 2\right)\mathcal{H}(\eta)\mathcal{H}(\eta_r)\sqrt{\delta w(\eta)\delta w(\eta_r)}\left(M(\eta_r) + \frac{\ddot{Q}_{\ell\ell}(\eta_r)}{2a^2(\eta_r)}\right). \quad (11)$$

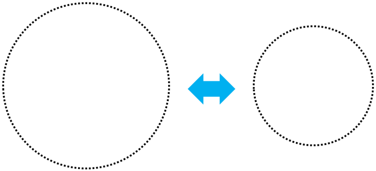

Figure 3: Polarisation pattern

Outline

Binary system

- Total mass: m, reduced mass: μ, Eccentricity: e, Polar angle: ψ
- Separation: $d = \frac{r_0(1-e^2)}{1+e\cos\psi}$, Angular velocity: $\omega_a^2 = \frac{G_{\rm N}m}{r_0^3}$

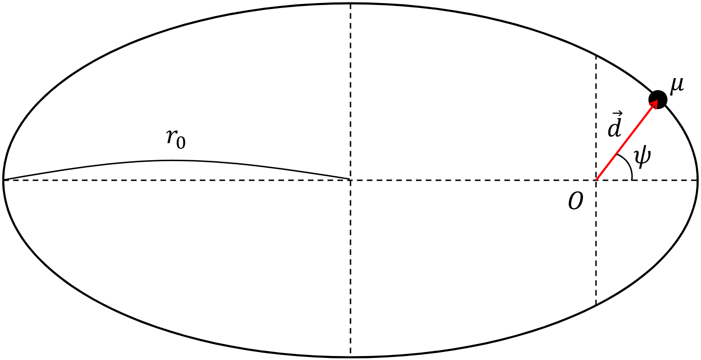

Figure 4: Setup of binary system

Binary system in Newtonian limit

We apply our light cone result of $\delta_1 C^{i}{}_{0j0}$ to the binary system in Newtonian limit.

$$\delta_1 C^{(\Phi|\gamma)i}{}_{0j0} \approx -\frac{G_{\rm N}^{5/3}\omega_a^{2/3}m^{2/3}\mu}{2a(\eta)r}\frac{e}{1-e^2} \times (\delta_{ij}-3\widehat{r}_i\widehat{r}_j)\mathcal{H}(\eta)\mathcal{H}(\eta_r)\sqrt{\delta w(\eta)\delta w(\eta_r)}\cos\psi. \tag{12}$$

$$\delta_1 C^{(g|\gamma)i}{}_{0j0} \approx -\frac{8G_{\rm N}^{5/3}\omega_a^{8/3}m^{2/3}\mu}{a(\eta)r} \times \left(\frac{1+\cos^2\theta}{2}\cos[2(\psi-\phi)]\epsilon^{+}_{ij} + \cos\theta\sin[2(\psi-\phi)]\epsilon^{\times}_{ij}\right), \tag{13}$$

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Conclusions

- Scalar gravitational waves can be excited by isolated astrophysical system even though the scalar field is minimally coupled to gravity.
- Tidal forces induced by scalar gravitational waves are sensitive to equation of state parameter δw.
- For a Newtonian binary system,
  1. The scalar gravitational waves are sensitive to eccentricity e.
  2. The frequency of scalar gravitational waves is half of usual tensor mode.

Reference

Chu, Y.-Z. (2015). Transverse traceless gravitational waves in a spatially flat FLRW universe: Causal structure from dimensional reduction. *Phys. Rev. D*, *92*(12), 124038.

Thank you for listening

Q&A