New topics in GPD

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Contents

- Chiral and trace anomalies in GPDs

Bhattacharya, YH, Vogelsang 2210.13419; 2305.09431

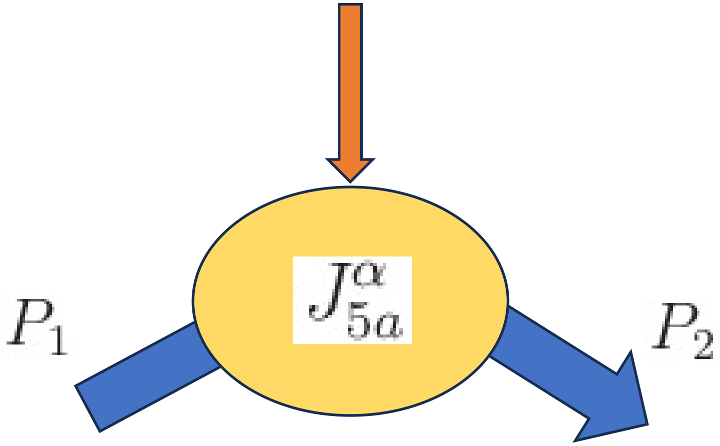
- Gravitational form factors

YH, Strikman 2102.12631
Martin-Caro, Huidobro, YH 2304.05994; 2312.12984
YH 2311.14470

Circa 1960: Isovector axial form factors

Noether current of SU(2) chiral symmetry $q \to e^{i\alpha^a \tau^a \gamma_5} q$

$$J^{\alpha}_{5a} = \sum_q \bar{q}\gamma^{\alpha}\gamma_5 \frac{\tau^a}{2} q$$



$$t = (P_2 - P_1)^2 \equiv l^2$$

Nucleon form factors

$$\langle P_2 | J^{\alpha}_{5a} | P_1 \rangle = \bar{u}(P_2)\left[\gamma^{\alpha}\gamma_5 F_A(t) + \frac{l^{\alpha}\gamma_5}{2M} F_P(t)\right]\frac{\tau^a}{2} u(P_1)$$

pseudovector pseudoscalar

Chiral symmetry breaking and pion pole

In massless QCD, the current is conserved $\partial_\alpha J^\alpha_{5a} = 0$

$$2MF_A(t) + \frac{tF_P(t)}{2M} = 0 \quad \Rightarrow \quad F_P(t) \approx \frac{-4M^2 g_A^{(3)}}{t}$$

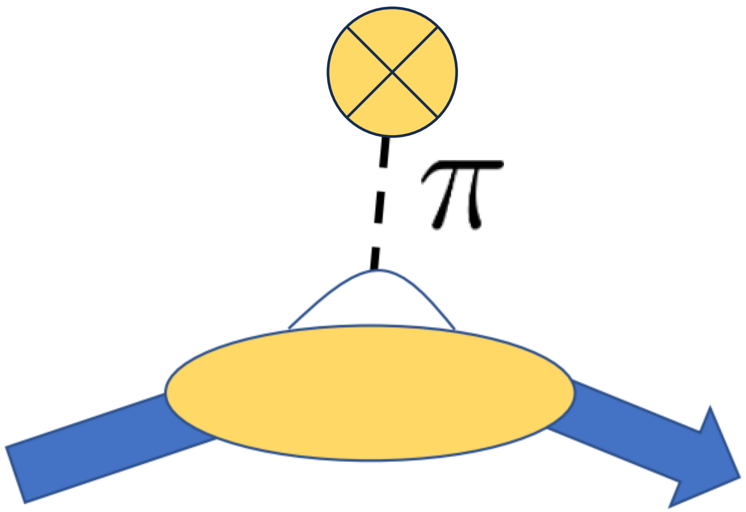
Pole at $t = 0$ from massless particle exchange

In real QCD with finite quark masses ,

$$\frac{1}{t} \to \frac{1}{t - m_\pi^2}$$

Pion nearly massless due to spontaneously broken chiral symmetry Nambu (1960)

Pion pole in GPD



$$F_P(t) = \int_{-1}^{1} dx \left(\tilde{E}_u(x,\xi,t) - \tilde{E}_d(x,\xi,t)\right) \approx \frac{-4M^2 g_A^{(3)}}{t}$$

Massless pole already in GPD in the ERBL region

$$\tilde{E}_u(x,\xi,t) - \tilde{E}_d(x,\xi,t) \sim \frac{\theta(\xi - |x|)}{t}$$

Penttinen, Polyakov, Goeke (1999)

??

First indication from lattice QCD? (Note that $\xi = 0$ in their paper.)
Bhattacharya et al. (2023)

Singlet axial form factors

Nucleon form factor of $J_5^\alpha = \sum_q \bar{q}\gamma^\alpha\gamma_5 q$

$$\langle P_2|J_5^\alpha|P_1\rangle = \bar{u}(P_2)\left[\gamma^\alpha\gamma_5 g_A(t) + \frac{l^\alpha\gamma_5}{2M}g_P(t)\right]u(P_1)$$

$g_A(0) = \Delta\Sigma$ quark spin contribution to the nucleon spin

In massless QCD, the current is conserved due to axial U(1) symmetry

$$2Mg_A(t) + \frac{tg_P(t)}{2M} = 0 \Rightarrow \frac{g_P(t)}{2M} \approx -\frac{2M\Delta\Sigma}{t}$$

Pole at $t = 0$ from massless η_0 meson exchange

Chiral anomaly

Quantum mechanically, the current is not conserved $\partial_\alpha J_5^\alpha = -\frac{n_f \alpha_s}{4\pi} F^{\mu\nu}\tilde{F}_{\mu\nu}$

$$\Rightarrow \quad \frac{g_P(t)}{2M} = \frac{1}{t}\left(i\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)} - 2Mg_A(t)\right)$$

anomaly pole $\qquad$ η_0 pole

In real QCD, there is no massless pole in $g_P(t)$ due to pole cancellation

Pole shifted to the physical η' meson mass via resummation of $1/N_c$ series Witten (1979), Veneziano (1979)

$$\frac{1}{t} + \frac{m_{\eta'}^2}{t^2} + \frac{m_{\eta'}^4}{t^3} + \cdots = \frac{1}{t - m_{\eta'}^2}$$

Any implications for the corresponding GPD? $g_P(t) = \sum_q \int_{-1}^{1} dx \tilde{E}_q(x,\xi,t)$

Gravitational form factors

QCD energy momentum tensor

$$\Theta^{\mu\nu} = \sum_f \bar{\psi}_f \gamma^{(\mu} i D^{\nu)} \psi_f - F^{\mu\rho} F^{\nu}{}_{\rho} + \frac{g^{\mu\nu}}{4} F^{\alpha\beta} F_{\alpha\beta}$$

Nucleon form factors

$$\langle P_2 | \Theta^{\alpha\beta} | P_1 \rangle = \bar{u}(P_2) \left[A(t) \frac{P^\alpha P^\beta}{M} + (A(t) + B(t)) \frac{P^{(\alpha} i \sigma^{\beta)\lambda} l_\lambda}{2M} + D(t) \frac{l^\alpha l^\beta - g^{\alpha\beta} t}{4M} \right] u(P_1)$$

In massless QCD, $\Theta^{\alpha\beta}$ is traceless due to conformal symmetry

$$A(t) + \frac{B(t)}{4M^2} t - \frac{3D(t)}{4M^2} t = 0 \qquad \frac{3}{4} D(t) \approx \frac{M^2}{t} A(t) \qquad (t \to 0)$$

Pole at $t = 0$ from massless glueball exchange

Trace anomaly

Quantum mechanically, the trace is nonzero $$(\Theta)^{\alpha}_{\alpha} = \frac{\beta(g)}{2g} F^{\mu\nu} F_{\mu\nu}$$

$$\Rightarrow \quad \frac{3}{4} D(t) \approx -\frac{M}{t} \left(\frac{\langle P_2 | \frac{\beta(g)}{2g} F^2 | P_1 \rangle}{\bar{u}(P_2) u(P_1)} - M A(t) \right)$$

anomaly pole glueball pole

In real QCD, there is no massless pole in $D(t)$ due to pole cancellation

Poles in $D(t)$ at physical glueball masses. Fujita, YH, Sugimoto, Ueda (2022)

Take-home message

Anomalies relate form factors

Chiral anomaly

$$2Mg_A(t) + \frac{tg_P(t)}{2M} = i\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}$$

Trace anomaly

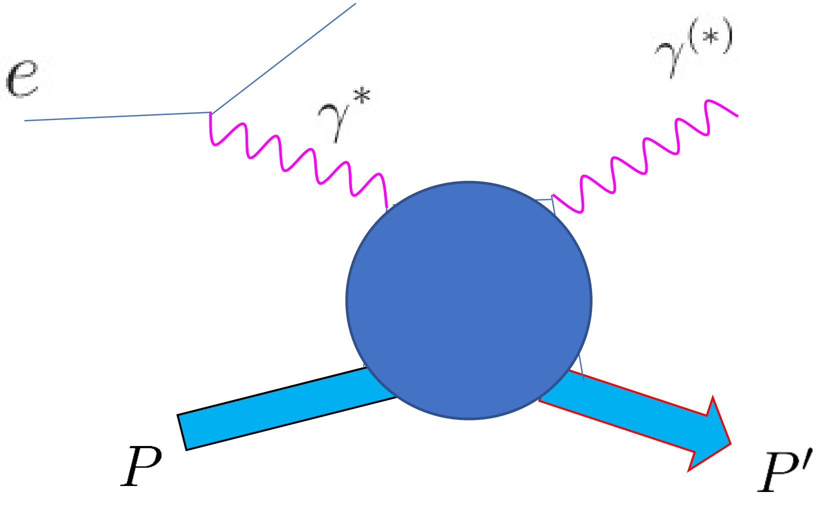
$$M\left(A(t) + \frac{B(t)}{4M^2}t - \frac{3D(t)}{4M^2}t\right)\bar{u}(P_2)u(P_1) = \langle P_2|\frac{\beta(g)}{2g}F^{\mu\nu}F_{\mu\nu}|P_1\rangle$$

Form factors are moments of GPDs

$$g_P(t) = \sum_q \int_{-1}^{1} dx\tilde{E}_q(x,\xi,t) \qquad A_q(t) + \xi^2 D_q(t) = \int_{-1}^{1} dx x H_q(x,\xi,t)$$

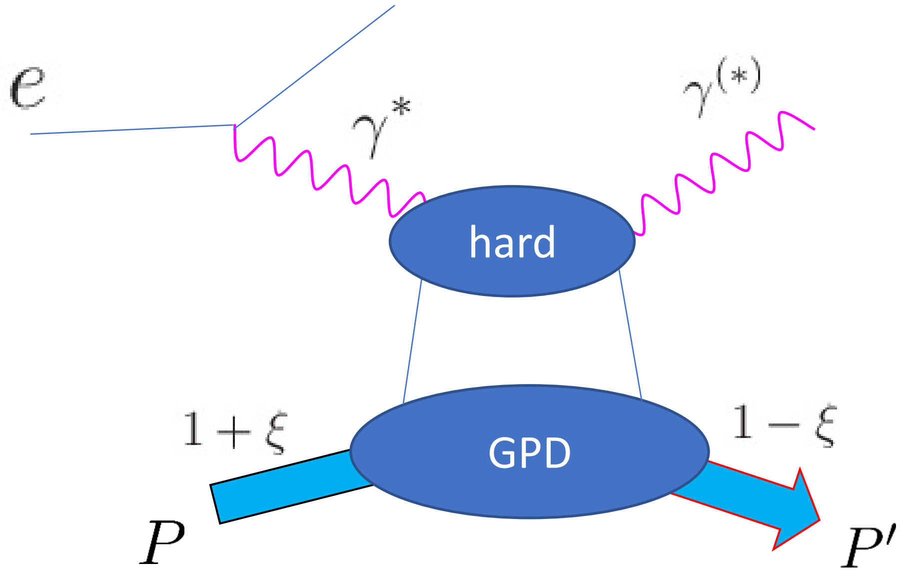
Anomalies relate/constrain GPDs!

Deeply Virtual Compton Scattering



Generalized
Bjorken limit

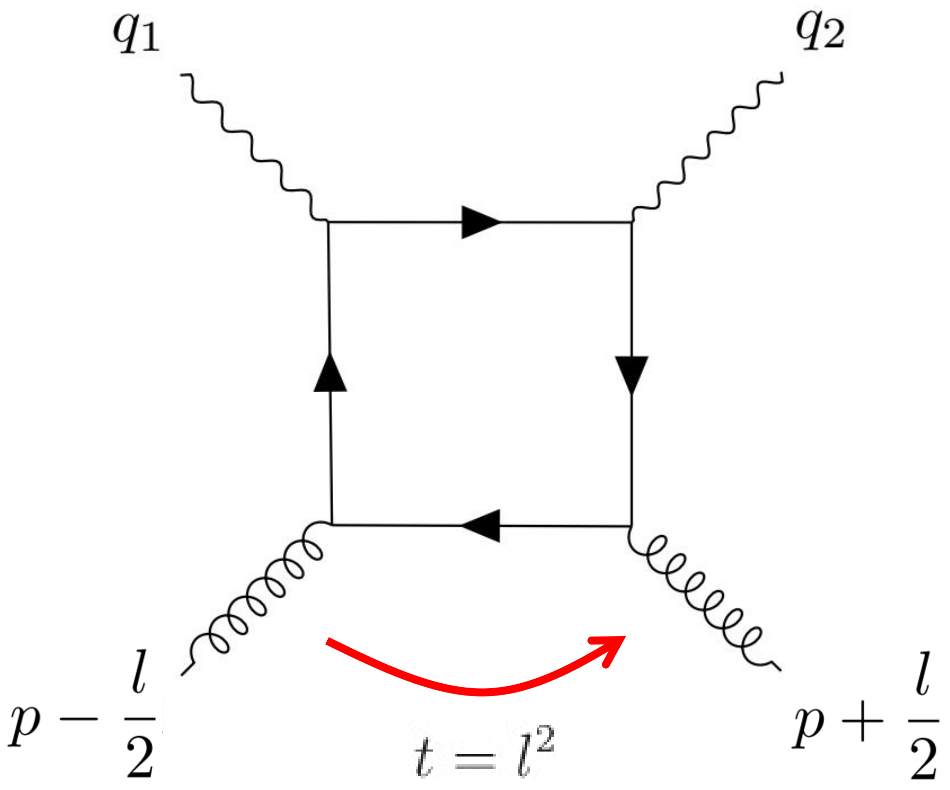
$Q^2 \gg |t|, \Lambda^2_{\mathrm{QCD}}$



Factorization proof Collins, Freund (1998); Ji, Osborne (1998)

$$T^{\mu\nu}(x_B,\xi,t) = \sum_{a=q,g} \int \frac{dx}{x} C_a^{\mu\nu}\left(\frac{x_B}{x},\frac{\xi}{x}\right) f_a(x,\xi,t) + \mathcal{O}(1/Q^2)$$

Box diagram (off-forward)



In all previous works on DVCS, the hard part was computed at $\xi \neq 0$ and $t = 0$

Naively, introducing $t \neq 0$ only produces higher twist corrections of order t/Q^2

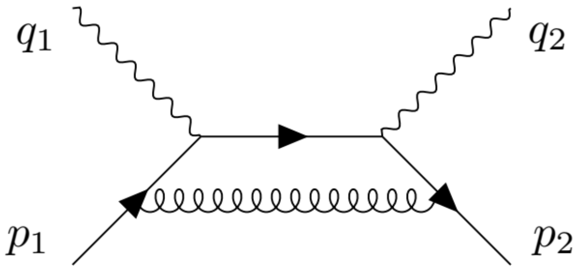
However, calculations with $t \neq 0$ can reveal anomaly poles. Tarasov, Venugopalan (2019,2021)

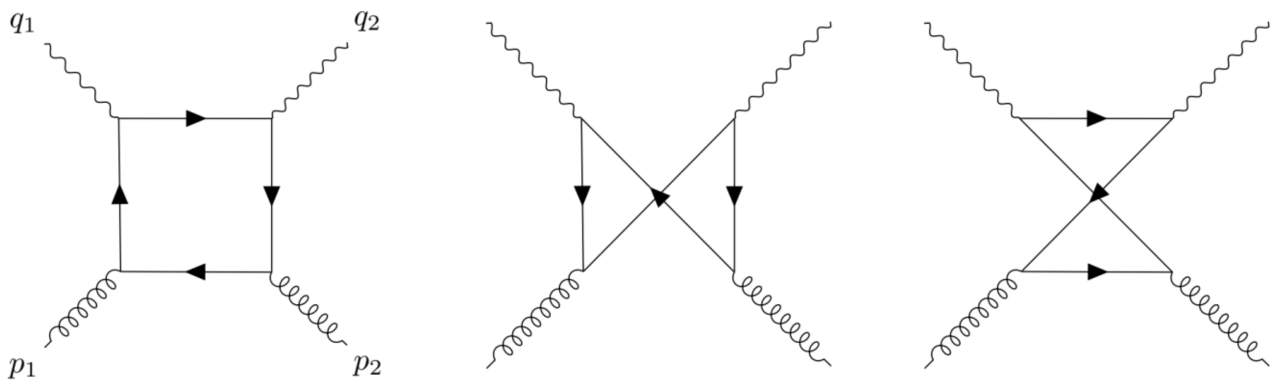
Work in the regime $\Lambda^2_{QCD} \ll |t| \ll Q^2$

$t \neq 0$ naturally regulates the collinear singularity $\frac{1}{\epsilon} \to \ln \frac{Q^2}{-t}$

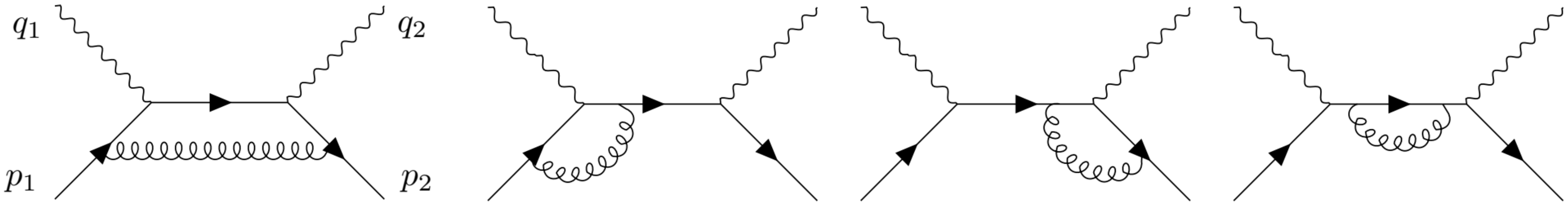
One-loop calculation with $t \neq 0$

$$T^{\mu\nu} = \frac{g_\perp^{\mu\nu}}{2P^+}\bar{u}(P_2)\left[\gamma^+\mathcal{H} + \frac{i\sigma^{+\nu}l_\nu}{2M}\mathcal{E}\right]u(P_2) - i\frac{\epsilon_\perp^{\mu\nu}}{2P^+}\bar{u}(P_2)\left[\gamma^+\gamma_5\tilde{\mathcal{H}} + \frac{\gamma^5 l^+}{2M}\tilde{\mathcal{E}}\right]u(P_2)$$





Sudakov poles in the quark channel



$$\delta C_1^q(\hat{x},\hat{\xi}) = \boxed{-\frac{\left(\frac{-t}{Q^2}\right)^{-\epsilon_{IR}}}{\epsilon_{\rm IR}^2(1-\hat{x})} - \frac{3\left(\frac{-t}{Q^2}\right)^{-\epsilon_{IR}}}{2\epsilon_{\rm IR}(1-\hat{x})}} + \frac{1-2\hat{x}-2\hat{x}^2+3\hat{\xi}^2}{2(1-\hat{x})(1-\hat{\xi}^2)}\ln\frac{\hat{x}-1}{\hat{x}} + \frac{(\hat{x}-\hat{\xi})(-1+\hat{x}^2+3\hat{x}\hat{\xi}+3\hat{\xi}^2)}{(1-\hat{x}^2)(1-\hat{\xi}^2)\hat{\xi}}\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}$$
$$+ \frac{1+\hat{x}^2-2\hat{\xi}^2}{2(1-\hat{x})(1-\hat{\xi}^2)}\ln^2\frac{\hat{x}-1}{\hat{x}} + \frac{\hat{x}}{2(1-\hat{\xi}^2)\hat{\xi}}\ln^2\frac{\hat{x}-\hat{\xi}}{\hat{x}} + \frac{-1-\hat{x}^2+2\hat{\xi}^2}{2(1-\hat{x}^2)(1-\hat{\xi}^2)}\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}\ln\frac{\hat{x}+\hat{\xi}}{\hat{x}} + \frac{\pi^2-54}{12(1-\hat{x})}$$
$$+ \frac{\hat{x}}{(1-\hat{\xi}^2)\hat{\xi}}\mathrm{Li}_2\frac{-2\hat{\xi}}{\hat{x}-\hat{\xi}} + \frac{1+\hat{x}^2-2\hat{\xi}^2}{(1-\hat{x})(1-\hat{\xi}^2)}\left(\mathrm{Li}_2\frac{1-\hat{\xi}}{1-\hat{x}} + \mathrm{Li}_2\frac{1+\hat{\xi}}{1-\hat{x}}\right) + (\hat{x}\to-\hat{x}),$$

Anomaly poles in the gluon channel

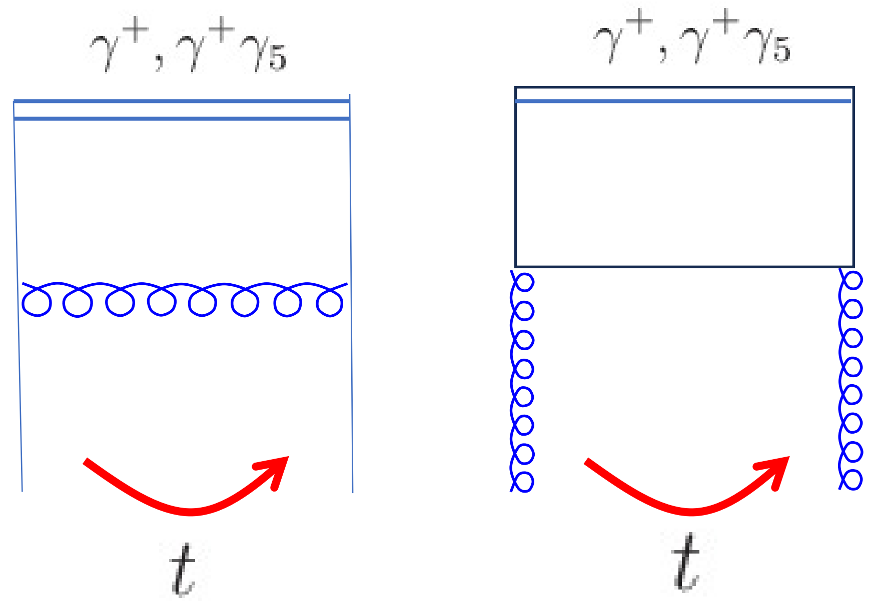
$$\tilde{\mathcal{E}} \sim \frac{\alpha_s}{t}\tilde{A}\otimes\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\frac{\langle P_2|F^{\mu\nu}(-z^-/2)W\tilde{F}_{\mu\nu}(z^-/2)|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}$$

$$\mathcal{H} \sim -\mathcal{E} \sim \frac{\alpha_s}{t}A\otimes\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\frac{\langle P_2|F^{\mu\nu}(-z^-/2)WF_{\mu\nu}(z^-/2)|P_1\rangle}{\bar{u}(P_2)u(P_1)}$$

twist-four GPDs featuring anomaly operators $F^{\mu\nu}F_{\mu\nu}$ $F^{\mu\nu}\tilde{F}_{\mu\nu}$ (nonlocal version)

not suppressed by $1/Q^2$

GPDs at one-loop



Compute quark GPD of a quark and gluon keeping $t \neq 0$

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p_2|\bar{q}(-z/2)W\gamma^+\gamma_5 q(z^-/2)|p_1\rangle$$

$$= \frac{\alpha_s T_R}{2\pi}\left[(1-\xi^2)\, i\epsilon^{+p\epsilon_2^*\epsilon_1}\left(\frac{2x-1-\xi^2}{(1-\xi^2)^2}\left(\frac{\left(\frac{\tilde{\mu}^2}{-l^2}\right)^{\epsilon}}{\epsilon_{\mathrm{UV}}} - \ln\frac{(1-x)^2}{1-\xi^2}\right) - 2\frac{1-x}{(1-\xi^2)^2}\right) + \frac{2il^+\epsilon^{\epsilon_1\epsilon_2^* lp}}{t}\frac{1-x}{1-\xi^2}\right]$$

Absorb the $1/t$ poles of Compton amplitude into twist-2 GPDs

Similarly, $\frac{1}{\epsilon_{\mathrm{IR}}}, \frac{1}{\epsilon_{\mathrm{IR}}^2}$ can also be absorbed.

→ Factorization restored

The fate of anomaly poles

After absorbed into twist-2 GPD, the anomaly pole becomes a part of the GPD

$$\sum_q (\tilde{E}_q(x,\xi,t) + \tilde{E}_q(-x,\xi,t)) = \frac{T_R n_f \alpha_s}{\pi} \frac{M^2}{t} \tilde{C}^{\rm anom} \otimes \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \frac{\langle P_2 | F^{\mu\nu}(-z^-/2) W \tilde{F}_{\mu\nu}(z^-/2) | P_1 \rangle}{\bar{u}(P_2)\gamma_5 u(P_1)} + \cdots$$

$$\int dx$$

$$\frac{g_P(t)}{2M} = \frac{1}{t}\left(i \frac{\langle P_2 | \frac{n_f \alpha_s}{4\pi} F\tilde{F} | P_1 \rangle}{\bar{u}(P_2)\gamma_5 u(P_1)} - 2M g_A(t) \right)$$

exactly reproduce the anomaly pole!

Twist-2 and twist-4 GPDs related by the chiral anomaly

Cancel the pole at $t=0$ with the nonperturbative η_0 meson pole.

Witten-Veneziano scenario at the GPD level.

D-term and gluon condensate

Trace anomaly pole induces the Polyakov-Weiss D-term of unpol GPDs

$$H_q^{\rm PW}(x,\xi,t) = -E_q^{\rm PW}(x,\xi,t) = \theta(\xi-|x|)D_q(x/\xi,t)$$

$$\sum_q D_q(z,t) \approx -\frac{T_R n_f \alpha_s}{\pi} z(1-|z|)\frac{M}{t}\left(\frac{\langle P_2|F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)} - \frac{\langle P_2|F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)}\bigg|_{t=0}\right) + \cdots$$

anomaly pole

$\int dx$

$$\sum_q D_q(t) \approx -\frac{M}{t}\left(\frac{\langle P_2|\frac{T_R n_f \alpha_s}{6\pi}F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)} - \frac{\langle P_2|\frac{T_R n_f \alpha_s}{6\pi}F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)}\bigg|_{t=0}\right) + \cdots$$

Compare with the full relation

$$\frac{3}{4}D(t) \approx -\frac{M}{t}\left(\frac{\langle P_2|\frac{\beta(g)}{2g}F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)} - MA(t)\right)$$

Conclusions (Part-1)

Bhattacharya, YH, Vogelsang,
2210.13419; 2305.09431

Anomalies relate form factors
Form factors are moments of GPDs

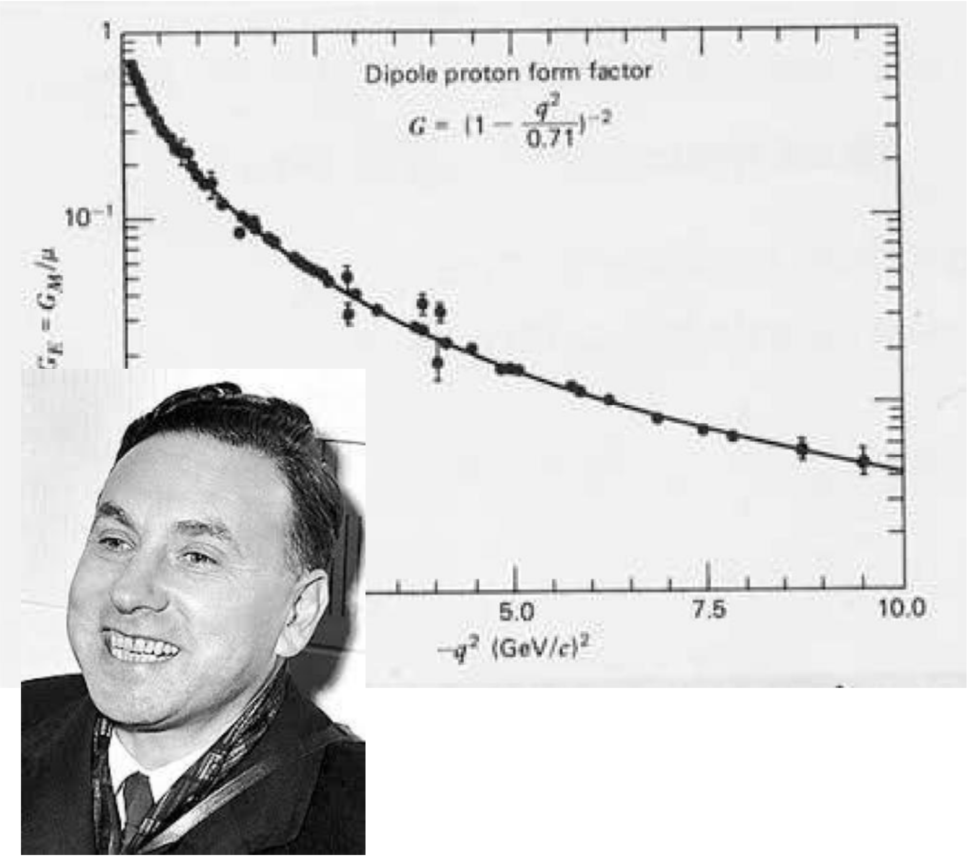
→ Anomalies relate GPDs

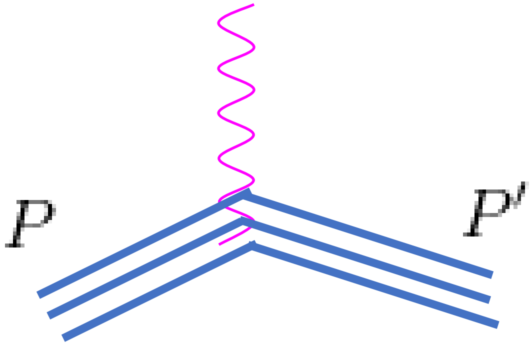
GPDs encode profound aspects of QCD such as chiral symmetry breaking and the origin of mass.

→ Connection to broader QCD community

Proton electromagnetic form factors

$$\langle p'|J^\mu(0)|p\rangle = \bar{u}(p')\left[\gamma^\mu F_1 + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}F_2\right]u(p)$$





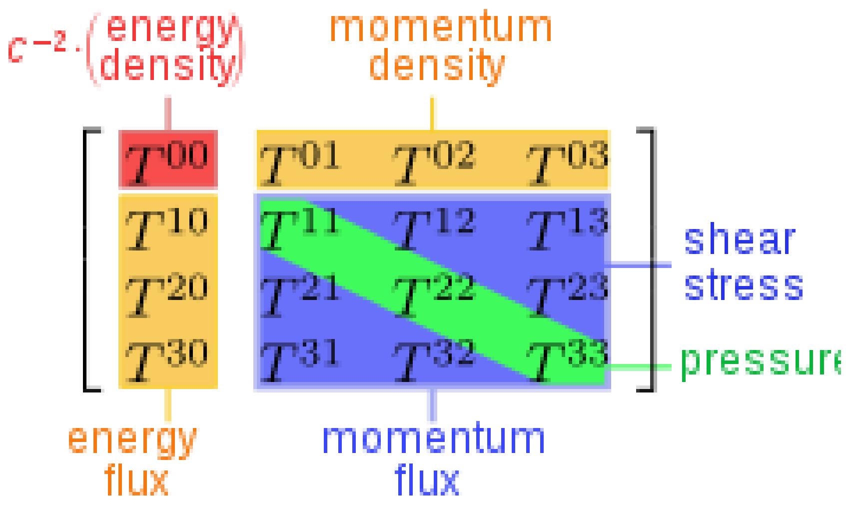
PRad (2019)

$$r_p = 0.831 \pm 0.007_{\text{stat}} \pm 0.012_{\text{syst}}$$

70 years of experimental study.

Charge radius measured to percent-level accuracy!

Radius zoo



Charge radius

$$\langle r^2\rangle_c = \frac{\int d\boldsymbol{x} x^2 \rho_c(\boldsymbol{x})}{\int d\boldsymbol{x} \rho_c(\boldsymbol{x})} = \frac{6}{G_E(0)}\frac{dG_E(t)}{dt}\Big|_{t=0}$$

Baryon number radius

$$\langle r^2\rangle_B = \frac{\int d\boldsymbol{x} x^2 \rho_B(\boldsymbol{x})}{\int d\boldsymbol{x} \rho_B(\boldsymbol{x})}$$

Mass radius

$$\langle r^2\rangle_m = \frac{\int d\boldsymbol{x}\, x^2 T^{00}(\boldsymbol{x})}{\int d\boldsymbol{x} T^{00}(\boldsymbol{x})} = 6\frac{dA(t)}{dt}\Big|_{t=0} - \frac{3D(0)}{2M^2}$$

Scalar radius

$$\langle r^2\rangle_s = \frac{\int d\boldsymbol{x}\, x^2 T^{\mu}_{\mu}(\boldsymbol{x})}{\int d\boldsymbol{x} T^{\mu}_{\mu}(\boldsymbol{x})} = 6\frac{dA(t)}{dt}\Big|_{t=0} - \frac{9D(0)}{2M^2}$$

Tensor radius

$$\langle r^2\rangle_t \equiv \frac{\int d\boldsymbol{x}\, x^2 \left(T^{00}(\boldsymbol{x}) + \frac{1}{2}T_{ii}(\boldsymbol{x})\right)}{\int d\boldsymbol{x}\left(T^{00} + \frac{1}{2}T_{ii}\right)} = 6\frac{dA(t)}{dt}\Big|_{t=0}$$

Mechanical radius

$$\langle r^2\rangle_{mech} = \frac{\int d\boldsymbol{x} x^2 \frac{x_i x_j}{x^2} T_{ij}(\boldsymbol{x})}{\int d\boldsymbol{x} \frac{x_i x_j}{x^2} T_{ij}(\boldsymbol{x})} = \frac{6D(0)}{\int_{-\infty}^{0} dt D(t)}$$

….

D-term: the last global unknown

$D(0)$ is a fundamental constant of the proton!

The value, even the sign, is unknown at the moment.

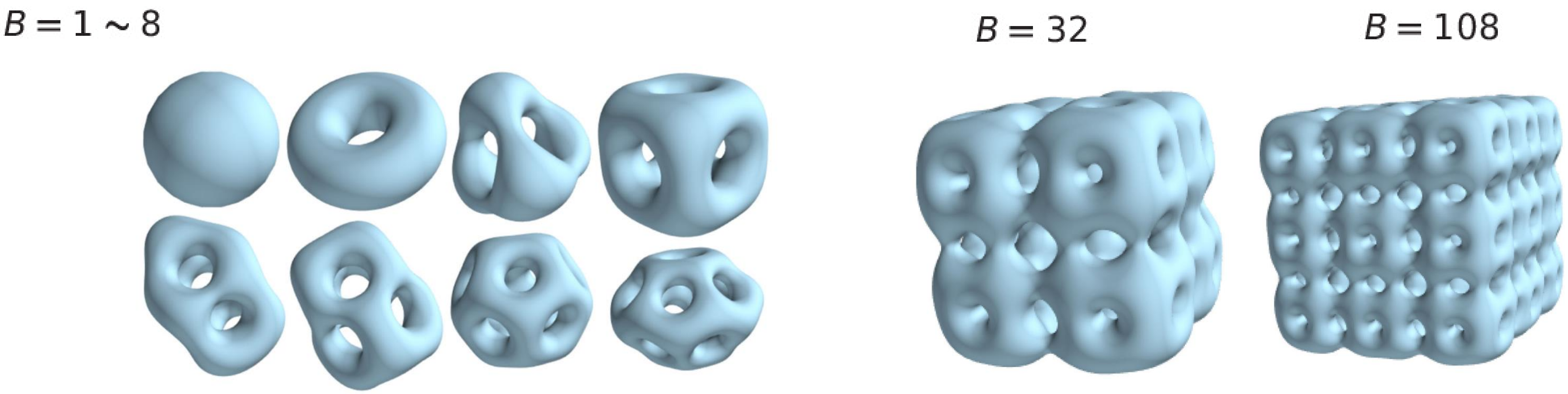
Spatial components of the energy momentum tensor
→ May be interpreted as radial force (`pressure') exerted by quarks and gluons Polyakov (2003)

$$T^{ij}(\boldsymbol{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)s(r) + \delta^{ij}\,p(r) \qquad D = M\int d^3r\, r^2 p(r)$$

Conjecture: Stable hadrons must have a negative D-term $D(t=0) < 0$

D-term of atomic nuclei in the Skyrme model

Martin-Caro, Huidobro, YH 2304.05994

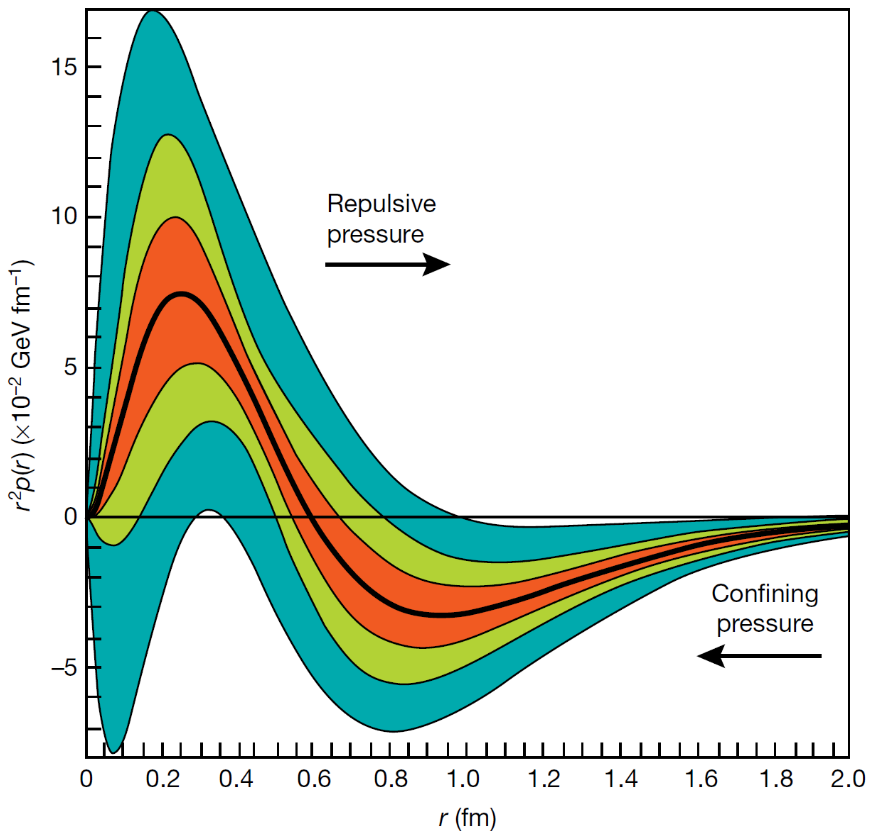


B	1	2	3	4	5	6	7	$8a$	$8b$	32	108
$D(0)$	-3.701	-13.126	-26.757	-43.304	-62.72	-85.95	-106.596	-128.368	-140.816	-1.874×10^3	-2.152×10^4

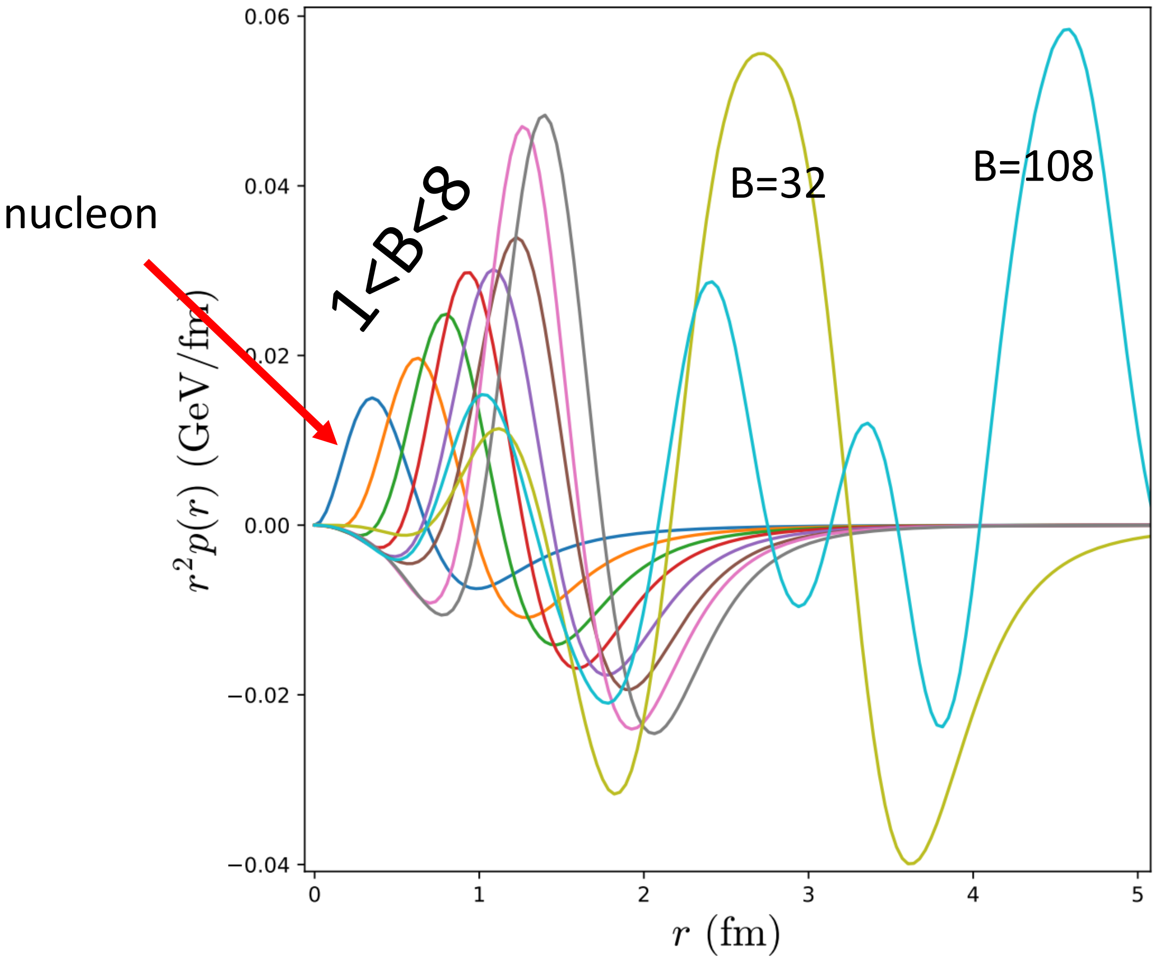
The value D(0) grows quickly with increasing B

cf. Polyakov (2003); Liuti, Taneja (2005); Guzey, Siddikov (2005)

`Pressure' inside nucleon and nuclei



Burkert, Elouadrhiri, Girod (2018)



Martin-Caro, Huidobro, YH, 2312.12984

Experimental study of GFFs?

- Introduced theoretically in the 60s.
- Received far less attention than EM form factors, not because they are less interesting/important.
- The obvious reason: We cannot measure them directly!

One-graviton exchange cross section $$\frac{d\sigma}{dt} \sim G_N^2 \frac{s^2}{t^2}$$

$$G_N \sim 1/M_P^2 \qquad M_P \sim 10^{19}\ \text{GeV}$$

- There are, however, indirect ways to probe them

1 graviton $\approx$ 2 photons or 2 gluons

Indirect measurement of GFFs

Decompose into the quark and gluon parts

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{u}(P')\Big[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M} + \bar{C}_{q,g}M\eta^{\mu\nu}\Big]u(P)$$

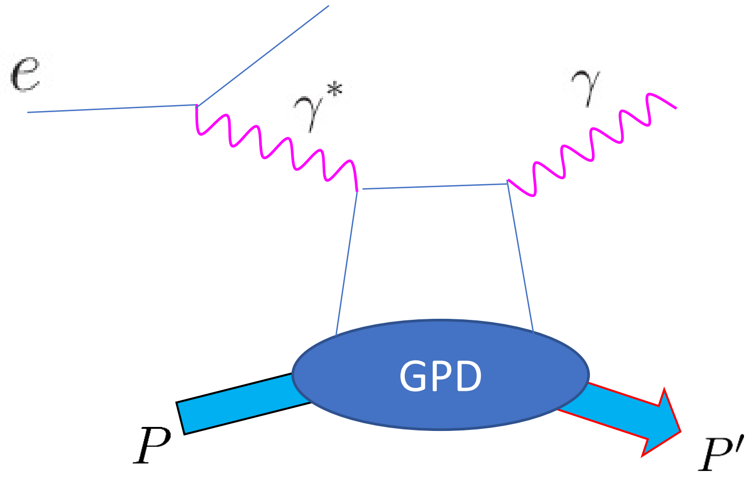
Use the connection to Generalized Parton Distributions (GPDs)

$$\int dx x H_{q.g}(x,\xi,t) = A_{q,g}(t) + D_{q,g}(t)\xi^2$$

Extract GPDs from exclusive processes via global analysis.

Quark D-term from Deeply Virtual Compton Scattering (DVCS)

$$D = D_u + D_d + D_s + D_g + \cdots$$



$D_{u,d}$ related to the subtraction constant in the dispersion relation for the Compton form factor Teryaev (2005)

$$\mathrm{Re}\mathcal{H}_q(\xi,t) = \frac{1}{\pi}\int_{-1}^{1} dx \mathrm{P}\frac{\mathrm{Im}\mathcal{H}_q(x,t)}{\xi - x} + 2\int_{-1}^{1} dz \frac{D_q(z,t)}{1-z}$$

$$\int_{-1}^{1} dz z D_q(z,t) = D_q(t)$$

1 graviton $\approx$ 2 photons

After all, 1 graviton $\neq$ 2 photons

$$\int_{-1}^{1} dz \frac{D_q(z,t)}{1-z}$$

what is measurable

$$\int_{-1}^{1} dz z D_q(z,t)$$

what we want

2-photon state couples to operators with arbitrary spin.
How can one isolate the spin-2 component?

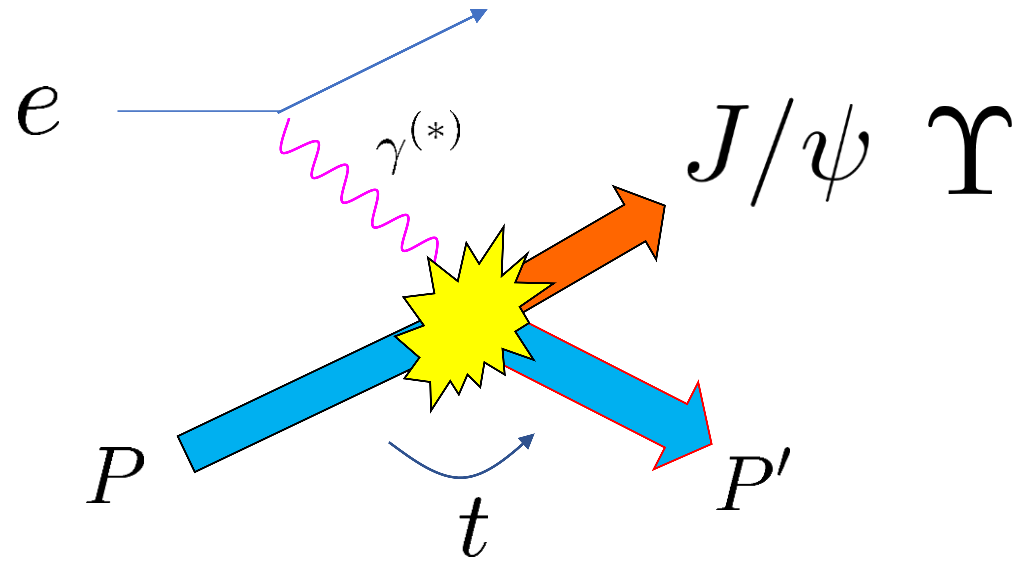
$$\frac{1}{1-z} = 1 + z + z^2 + z^3 + \cdots$$

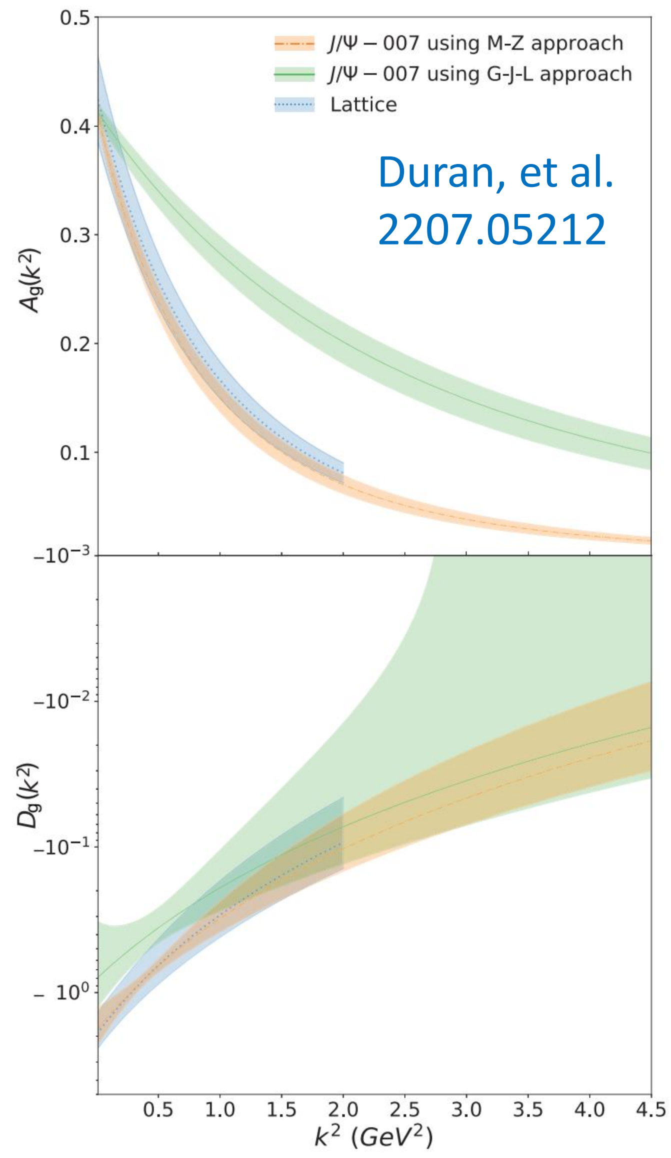
spin-2 (energy momentum tensor)

spin-4

Challenging. At least a large lever-arm in Q^2 is necessary. Only the EIC can do it.

Quarkonium photo-(electro-)production near threshold





Ongoing experiments at JLab, future measurement at EIC?

Originally proposed by Kharzeev, Satz, Syamtomov, Zinovev (1997) to probe the gluon condensate.

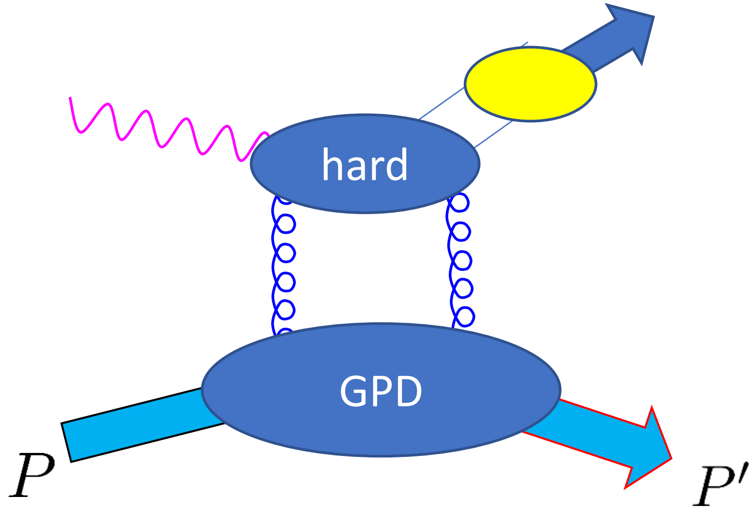
One can also study gluon GFFs in this process YH, Yang (2018)

GPD factorization

1 graviton $\approx$ 2 gluons

Light-cone dominance when $Q^2 \to \infty$ or $M_{QQ} \to \infty$

GPD factorization at high energy Collins, Frankfurt, Strikman (1996)
Ivanov, Schafer, Szymanowski, Krasnikov (2004)



Amplitude proportional to Compton form factor

$$\int_{-1}^{1} \frac{dx}{x} \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H_g(x, \xi, t)$$

Gluon GPD

Skewness $\xi = \frac{P^+ - P'^+}{P + +P'^+}$

1 graviton $\neq$ 2 gluons

what is measurable

$$\int_{-1}^{1} \frac{dx}{x} \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H_g(x, \xi, t)$$

what we want

$$\int_{-1}^{1} dx H_g(x, \xi, t)$$

in the previous subsection. Therefore, our conclusion of no direct connection between the near threshold photo-production of heavy quarkonium state and the gluonic gravitational form factors is consistent with the GPD formalism.

Sun, Tong, Yuan (2021)

Essentially the same problem as in the extraction of D-term from DVCS

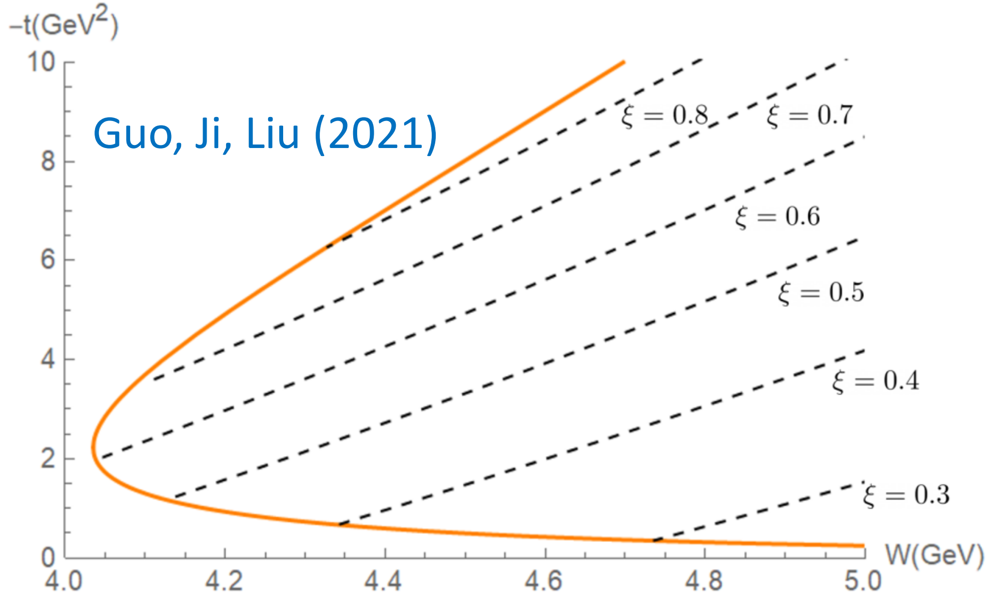
GPD factorization allows us to study this reaction from first principles.

But it also means that we are dealing with infinitely many operators with arbitrary spin.
How can one extract the spin-2 component?

Skewness!

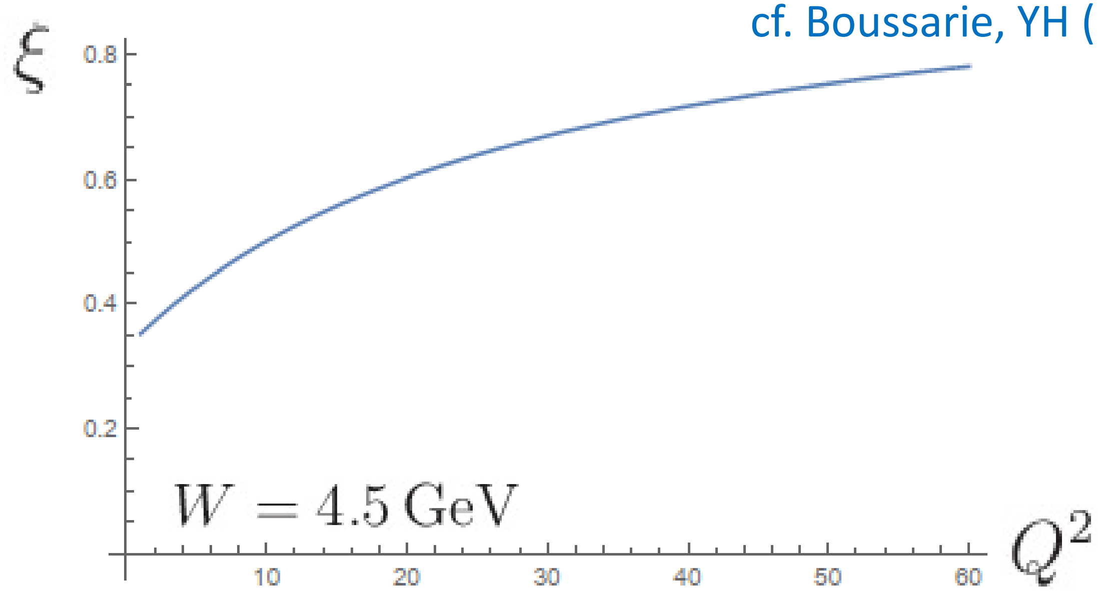
Threshold production characterized by large values of skewness YH, Strikman (2021)

J/ψ photo-production



J/ψ electro-production $|t| \ll Q^2$

cf. Boussarie, YH (2020)



$\xi \approx 1$ in the ideal limit $Q^2 \to \infty$ or $m_V \to \infty$

Energy momentum tensor strikes back

YH, Strikman 2102.12631
Guo, Ji, Liu 2103.11506

If $\xi \approx 1$, one can Taylor expand.

spin=2 (energy momentum tensor)

$$\int_{-1}^{1} \frac{dx}{x}\left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right) H_g(x,\xi,t) \approx 2\int dx(1+x^2+x^4+\cdots)H_g(x,\xi,t)$$

spin=4 spin=6

80% of the scattering amplitude comes from the energy momentum tensor, dominating over all the other twist-2 operators combined!

In practice, $\xi < 1$, and one deals with asymptotic series Guo, Ji, Yuan (2023)

Large-Q^2 and/or large quark mass (Υ production) needed. Only the EIC can do it.

Direct measurement of GFFs?

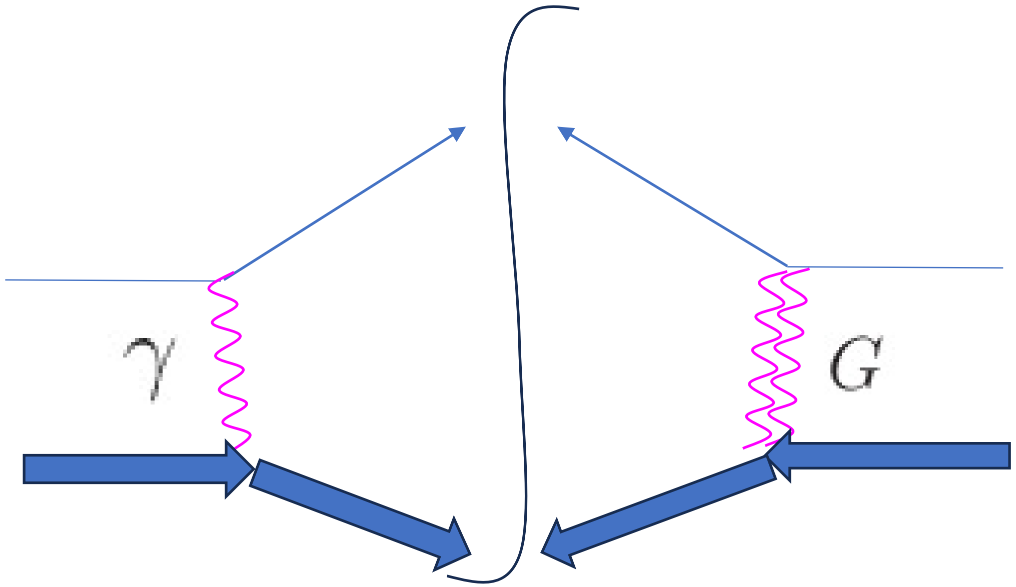
- Graviton exchange suppressed by the Planck energy $M_P \sim 10^{19}\,\mathrm{GeV}$
- But in some BSM scenarios, the effective Planck energy could be in the TeV region. E.g., extra dimension models.
- These models typically predict massive gravitons.
- Long history of searches for modification of Newton’s law/GR

Graviton-photon interference in elastic ep, eA

YH, 2311.14470

$$\delta\mathcal{L} = \kappa h_{\mu\nu} T^{\mu\nu}$$

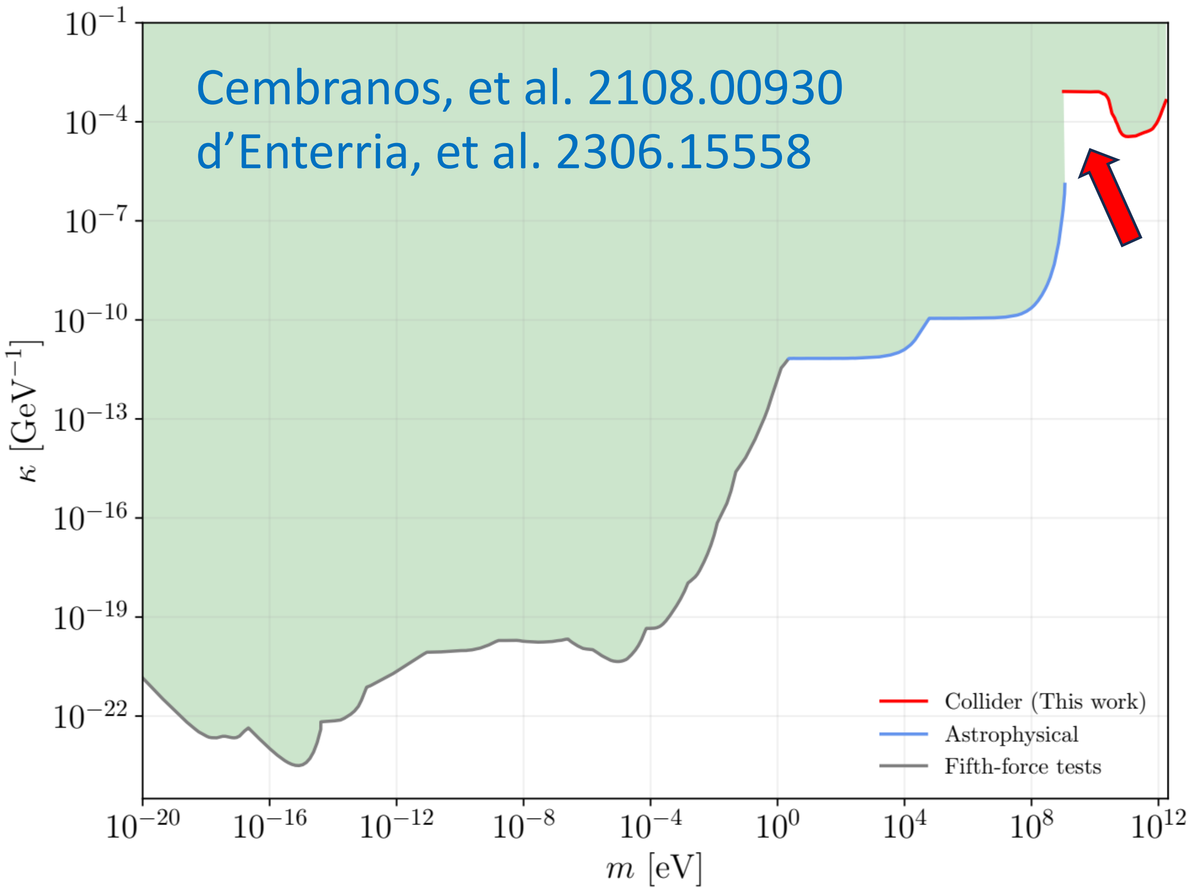
assume $\quad \kappa \sim 1\,\mathrm{TeV}^{-1}$

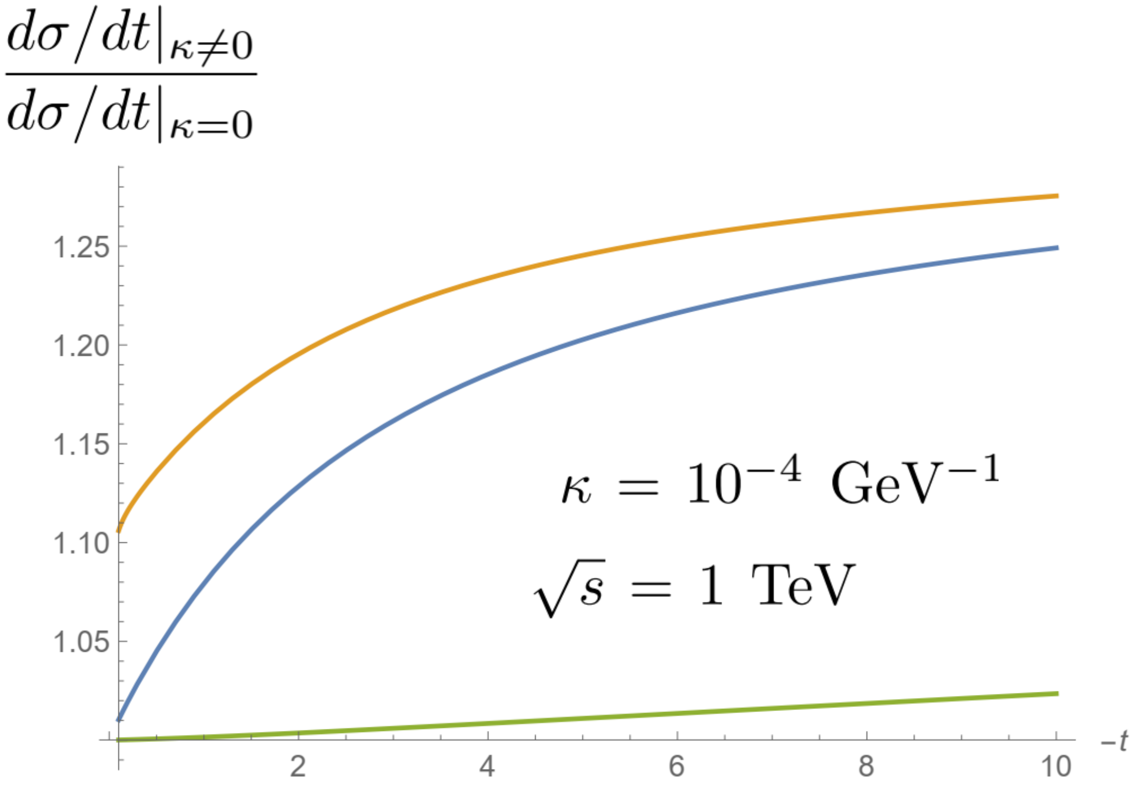


Rosenbluth

$$\frac{d\sigma}{dt} = \frac{4\pi\alpha_{em}^2}{t^2}\left\{\left(1+\frac{t-2M^2}{s}+\frac{M^4}{s^2}\right)\left(F_1^2(t)-\frac{tF_2^2(t)}{4M^2}\right)+\frac{t^2}{2s^2}(F_1(t)+F_2(t))^2\right\}$$
$$+\frac{\alpha_{em}\kappa^2 s}{t(t-m^2)}\left\{\left(1+\frac{3(t-2M^2)}{2s}\right)\left(A(t)F_1(t)-\frac{tB(t)F_2(t)}{4M^2}\right)+\mathcal{O}(s^{-2})\right\}+\mathcal{O}(\kappa^4)$$

Evading the LHC constraints





Where to look for? LHeC

`MuIC', Muon-ion collider (future upgrade of EIC) Acosta, Li 2107.02073

Perhaps also at HERA and EIC, if $\kappa \sim 10^{-3}\ \mathrm{GeV}^{-1}$

Conclusions (Part-2)

- GFFs: one of the frontiers of hadronic physics.
- EM form factors 70-year histroy, GFF just the beginning!
- Indirect measurements from DVCS, quarkonium threshold production. Challenging to extract the spin-2 component. Large skewness can help.
- Direct measurement via a massive graviton. New EIC-BSM connection