Prospects of lattice computations for TMD physics in Taiwan

C.-J. David Lin

National Yang Ming Chiao Tung University
國立陽明交通大學

EIC Asia workshop @ NCKU
30/01/2024

Outline

- ★ TMDPDFs and lattice QCD: what and how
- ★ Existing strategies and numerical results
- ★ Our approach
- ★ Outlook

What and how

The long-term goal

Leading-twist TMDPDFs

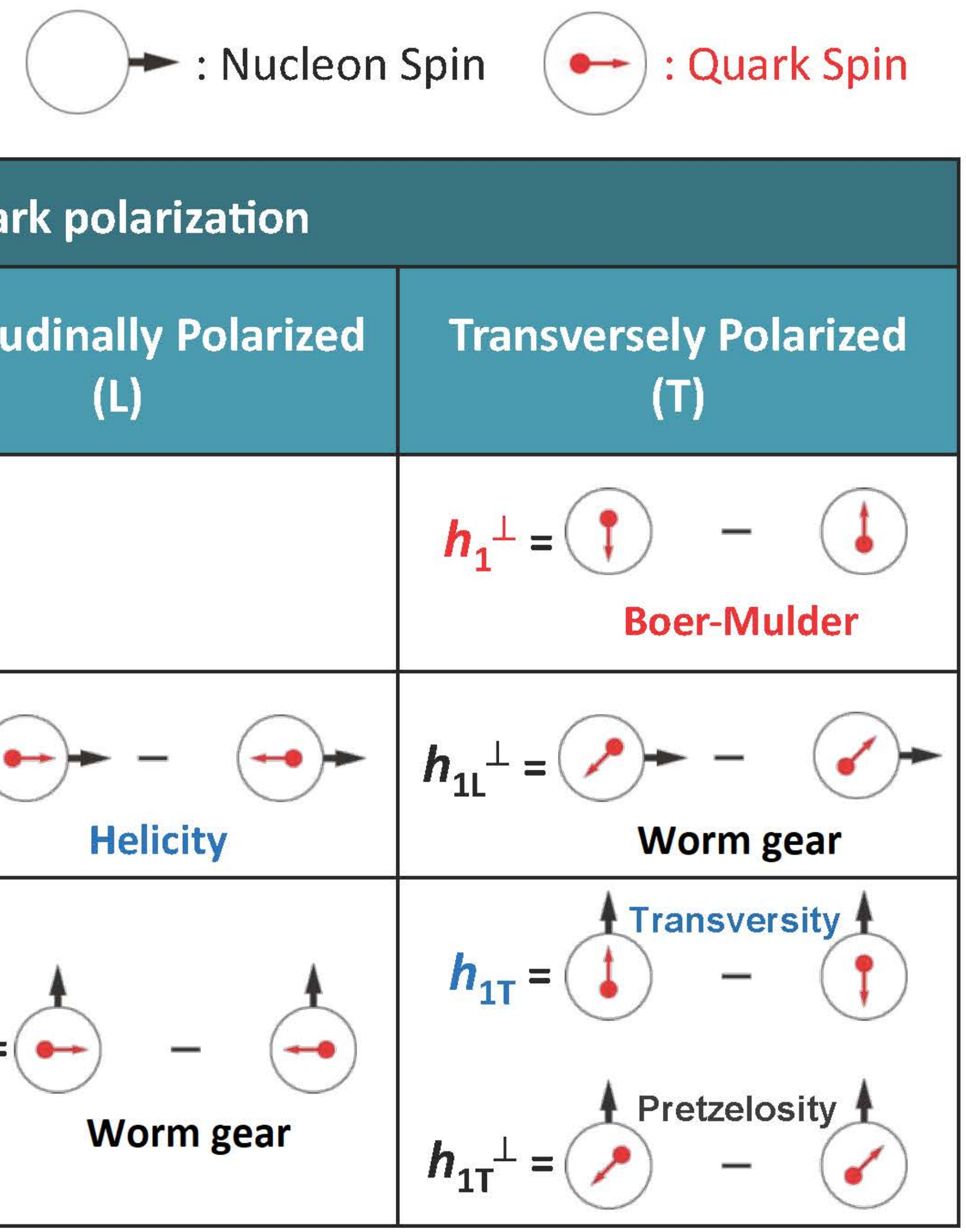
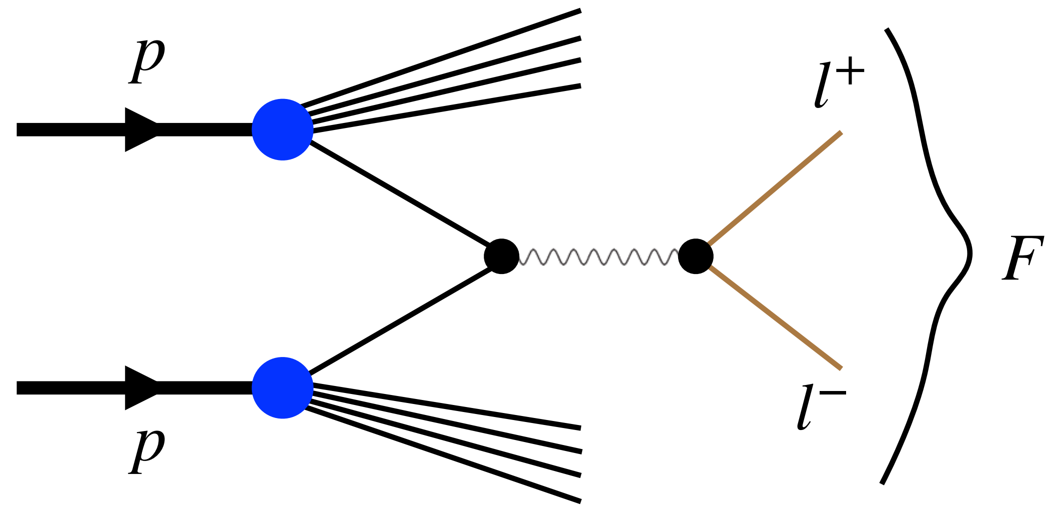


Figure from J. Arrington *et al*., arXiv:2022.13357

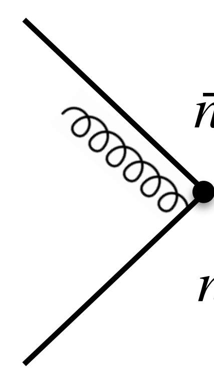
Drell-Yan factorisation and TMDPDF



$$\frac{d\sigma}{dQdYd^2q_T} = \sum_{ij} H_{ij}(Q,\mu) \int d^2b_T \; e^{i\vec{b}_T\cdot\vec{q}_T} \; f_i^{\rm TMD}(x_i,\vec{b}_T,\mu,\zeta_i) \; f_j^{\rm TMD}(x_j,\vec{b}_T,\mu,\zeta_j)$$

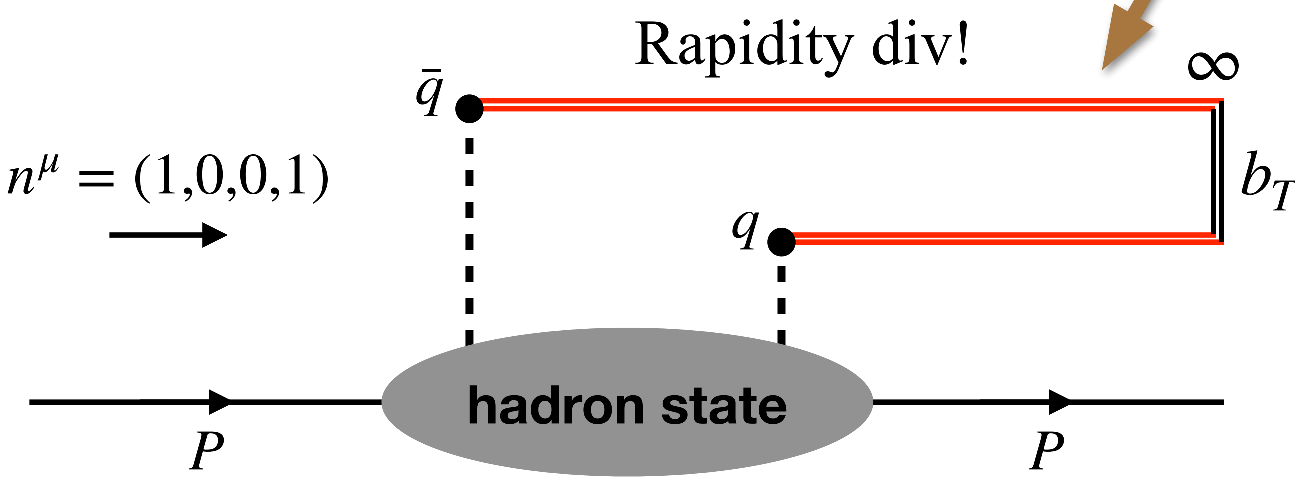
$$\times \left[1 + \mathcal{O}\left(\frac{q_T^2}{Q^2}, \frac{\Lambda_{\rm QCD}^2}{Q^2}\right)\right]$$

$\zeta_{i,j}$ from "rapidity divergence" and $\zeta_i\zeta_j = Q^4$

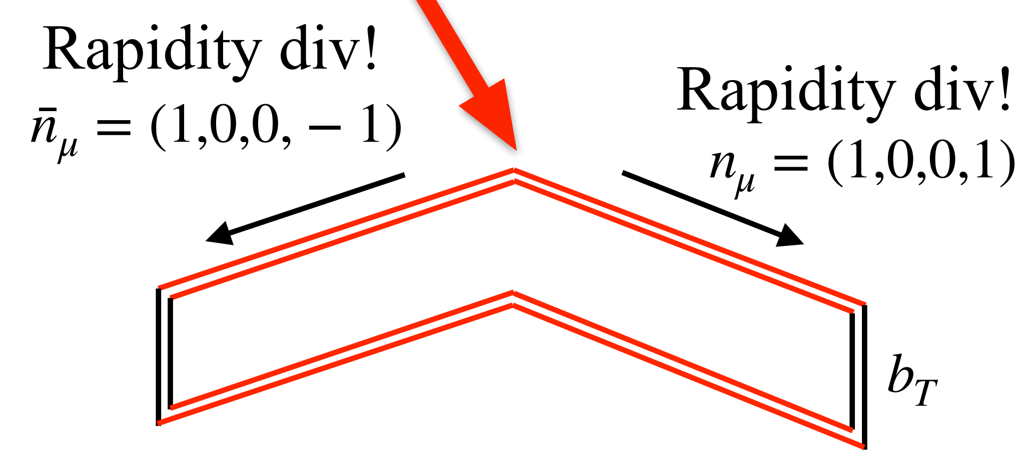


Drell-Yan factorisation and TMDPDF

$$f^{\text{TMD}}(x, \vec{b}_T, \mu, \zeta) = B(x, \vec{b}_T, \mu, \zeta/\nu^2)\,\sqrt{\mathcal{S}(b_T, \mu, \nu)}$$



“Beam function”

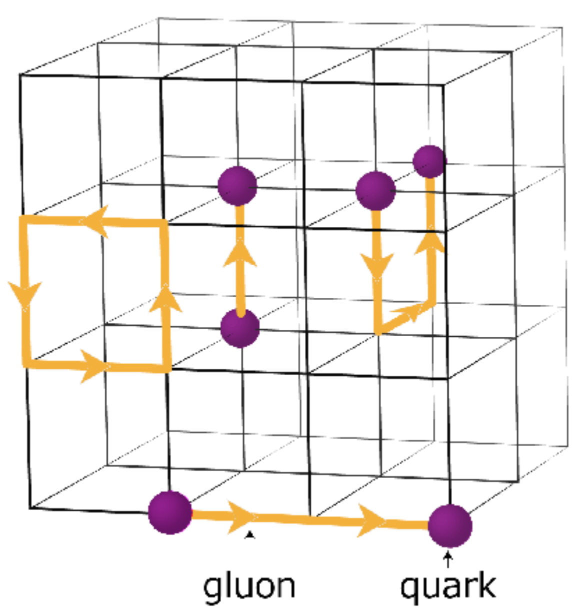


“Soft function”

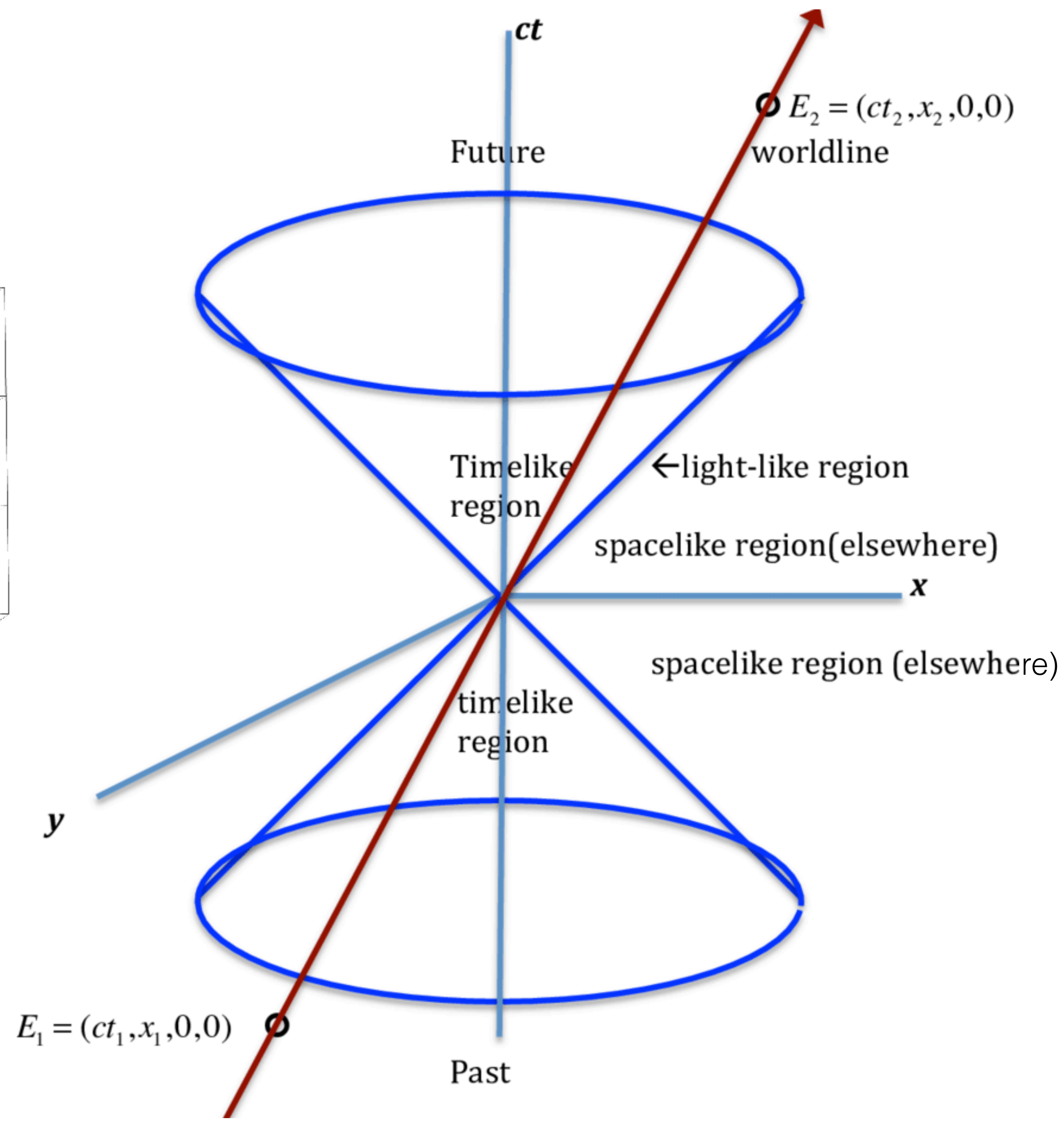
And the “Collins-Super (CS) kernel” for evolution in ν (ζ)

$\mathcal{S}(b_T, \mu, \nu) \Rightarrow \mathcal{S}_I(b_T, \mu), K(b_T, \mu) \Rightarrow$ both are *universal*

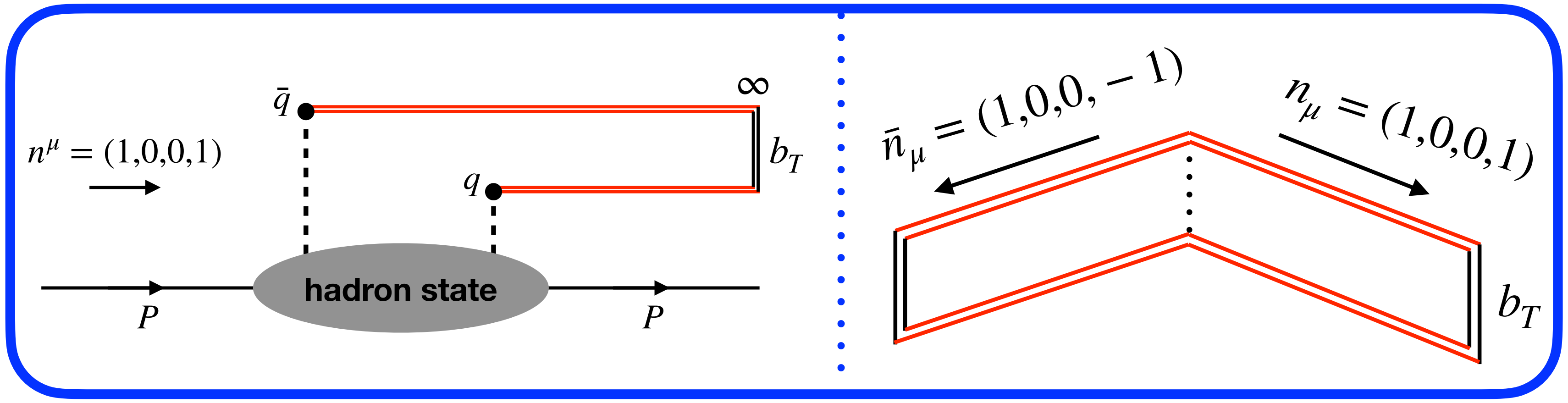
Challenges in parton physics from lattice QCD

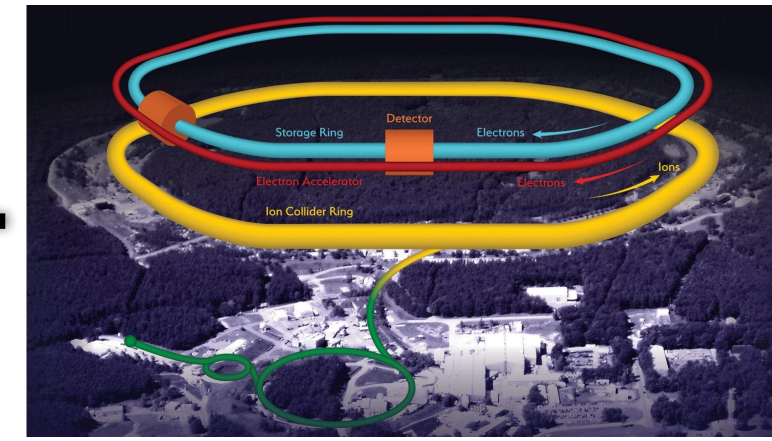


Euclidean space



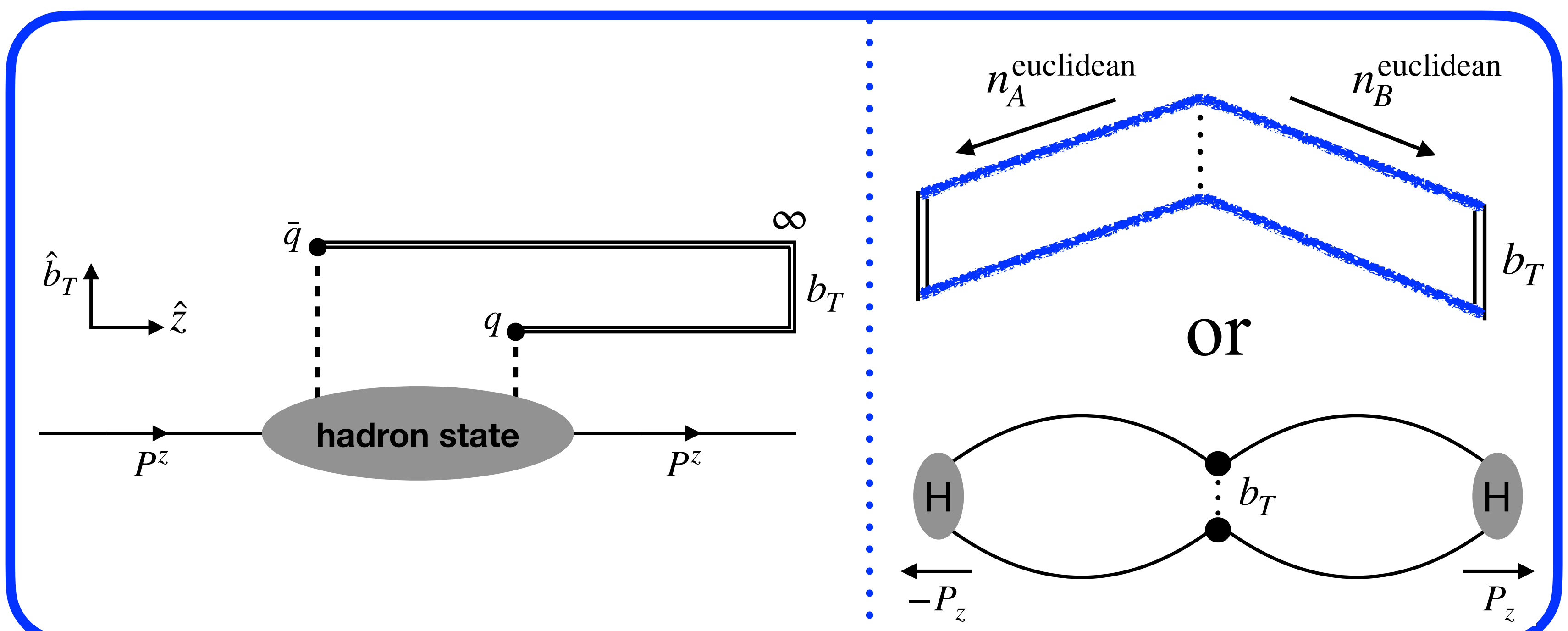
TMDPDF from LQCD

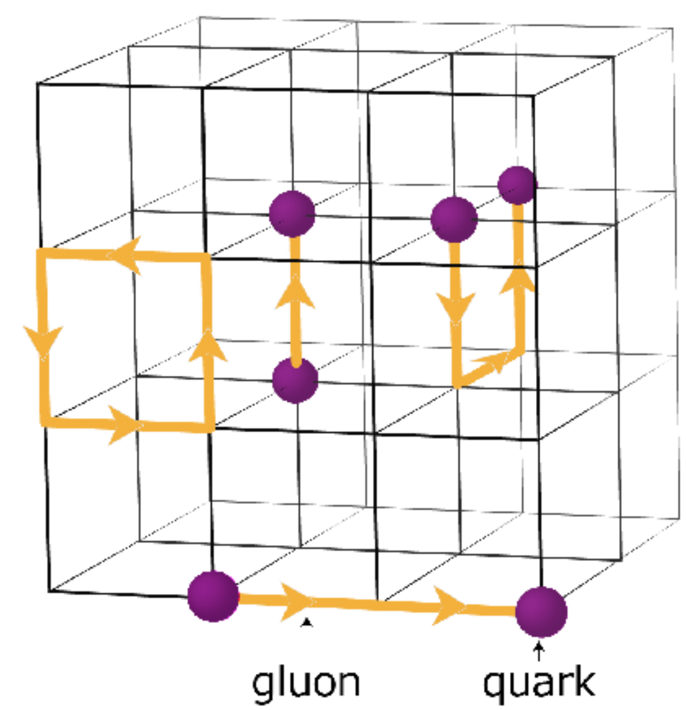




Minkowski, light-cone

Perturbation theory, $K(\mu, b_T)$, $S(\mu, b_T)$





Euclidean, space-like

Relating quasi-TMDPDF to TMDPDF

M.A. Ebert, S.T. Schindler, I.W. Stewart, Y. Zhao, JHEP 04 (2022) 178

$$\tilde{f}^{\rm TMD}(x, \vec{b}_T, \mu, P^z) = \underbrace{C^{\rm TMD}(\mu, xP^z)}_{\text{pertub. theo.}}\, g_S(b_T, \mu) \exp\left[\frac{1}{2} K(b_T, \mu) \log \frac{(2xP^z)^2}{\zeta}\right] \times f^{\rm TMD}(x, \vec{b}_T, \mu, \zeta) + \mathcal{O}\left(\frac{q_T^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}\right)$$

★ To obtain $f^{\rm TMD}$, one computes $\tilde{f}^{\rm TMD}$ with lattice QCD

★ Also need non-perturbative calculation of

The Collins-Soper kernel, $K(b_T, \mu)$

The soft function, $g_S(b_T, \mu) \sim \sqrt{S_I(b_T, \mu)}$

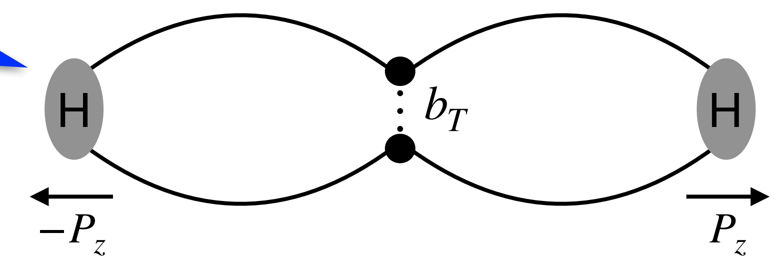
Existing lattice results

Soft function from the lattice

X. Ji, Y. Liu, Y.-S. Liu, Nucl. Phys. **B955** (2020) 115054, Phys. Lett. **B811** (2020) 135946

★ Compute the form factor

$$F(b_T, P^z) = \langle \pi(-p^z) \,|\, \bar{u}\Gamma u(b_T)\ \bar{d}\Gamma d(0) \,|\, \pi(P^z) \rangle$$

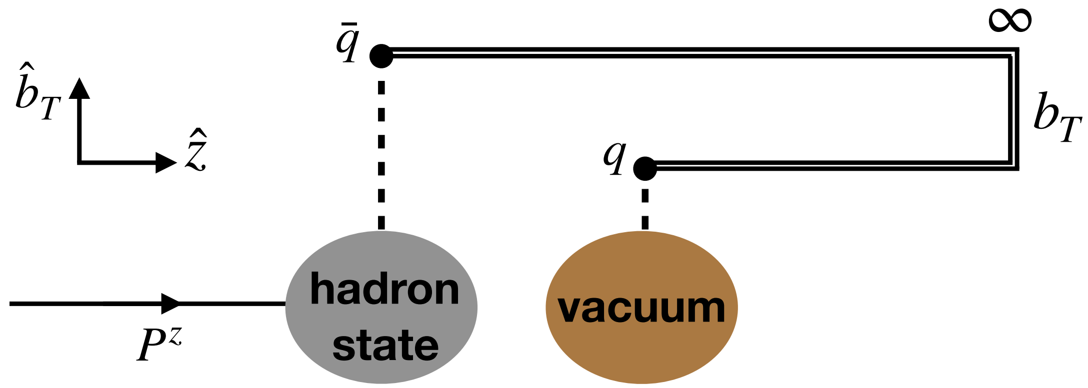


★ At large P^z , it factorises to

$$F(b_T, P^z) = S_I(b_T, \mu) \int_0^1 dx dx' \, H_\Gamma(x, x', P^z, \mu)$$

$$\times \, \Phi^\dagger(x', b_T, -P^z)\, \Phi(x, b_T, P^z) \; + \; \mathcal{O}\left(\frac{b_T}{P^z}, \frac{\Lambda_{\rm QCD}}{P^z} \right)$$

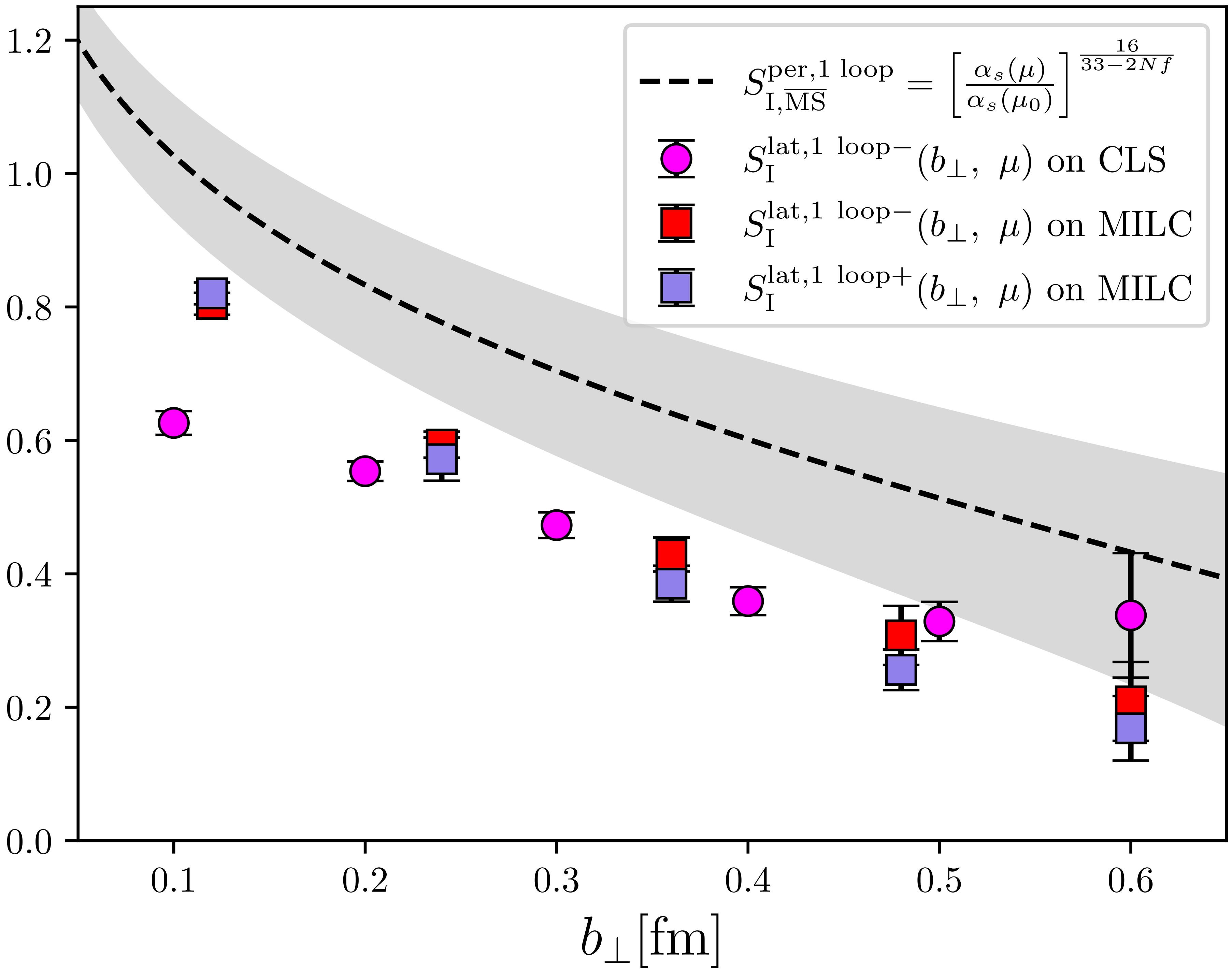
→ perturbative



quasi pion TMD wave function

Soft function from the lattice

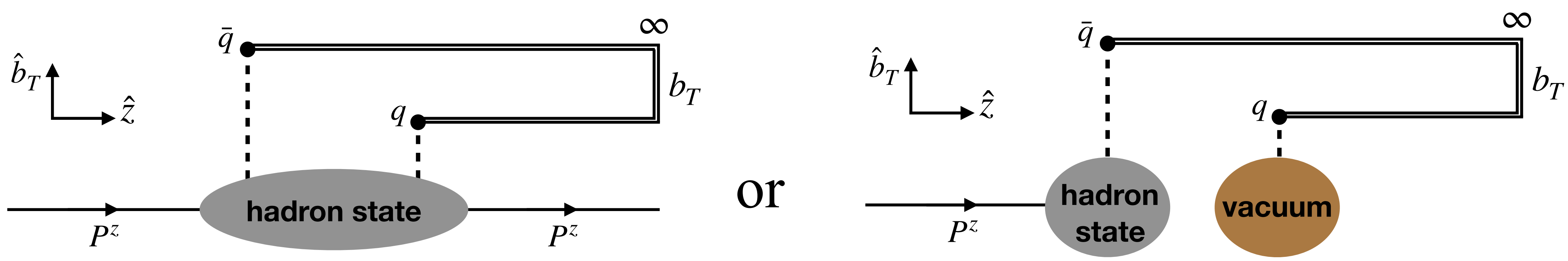
LPC Collaboration, JHEP **08** (2023) 172



CS kernel from the lattice

M. Ebert, I. Stewart, Y. Zhao, Phys. Rev., **D99** (2019) 034505

★ Compute qTMDPDF ($\tilde{f}^{\rm TMD}$) or qTMDWF ($\tilde{\Phi}^{\rm TMD}$)

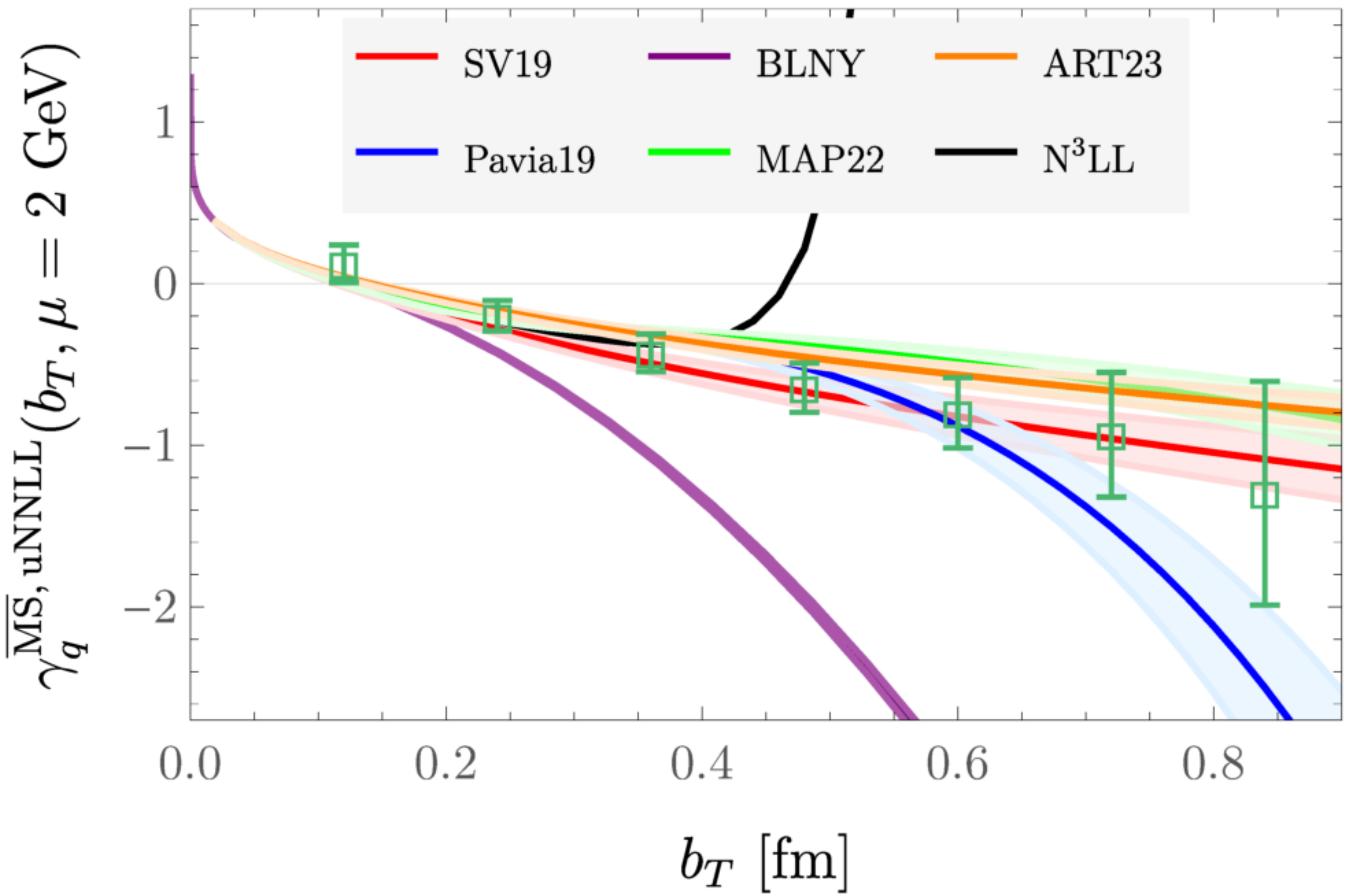


★ Determine the CS kernel from the ratio (at large P^z)

$$K(\mu, b_T) = \frac{1}{\log(P_1^z/P_2^z)} \log \frac{C^{\rm TMD}(\mu, xP_2^z)\ \tilde{\Phi}^{\rm TMD}(x, \vec{b}_T, \mu, P_1^z)}{\underbrace{C^{\rm TMD}(\mu, xP_1^z)}_{\text{perturbative}}\ \tilde{\Phi}^{\rm TMD}(x, \vec{b}_T, \mu, P_2^z)}$$

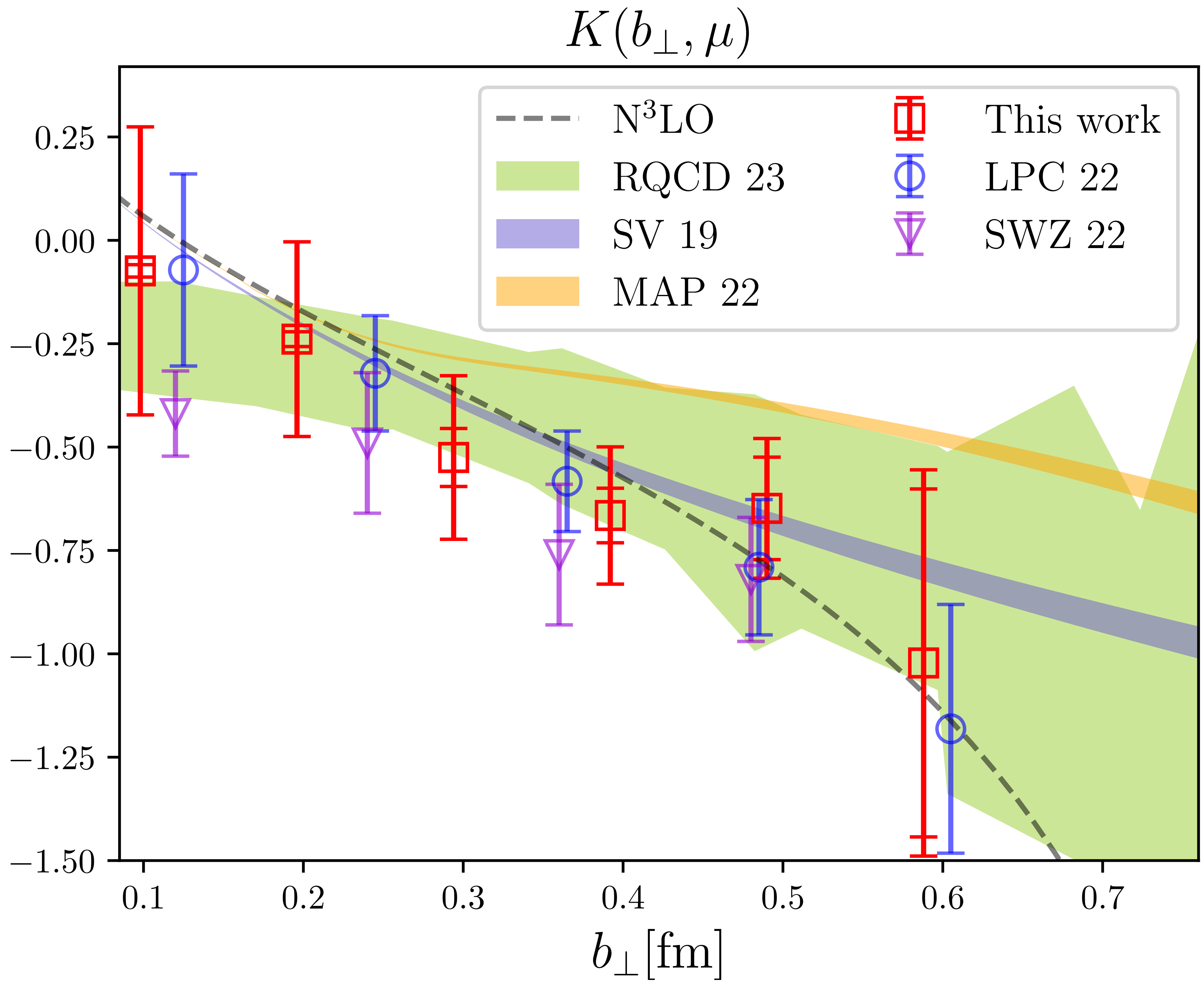
CS kernel from the lattice

A. Avhadiev, P. Shanahan, M. Wagman, Y. Zhao, Phys. Rev. **D198** (2023) 11, 114505

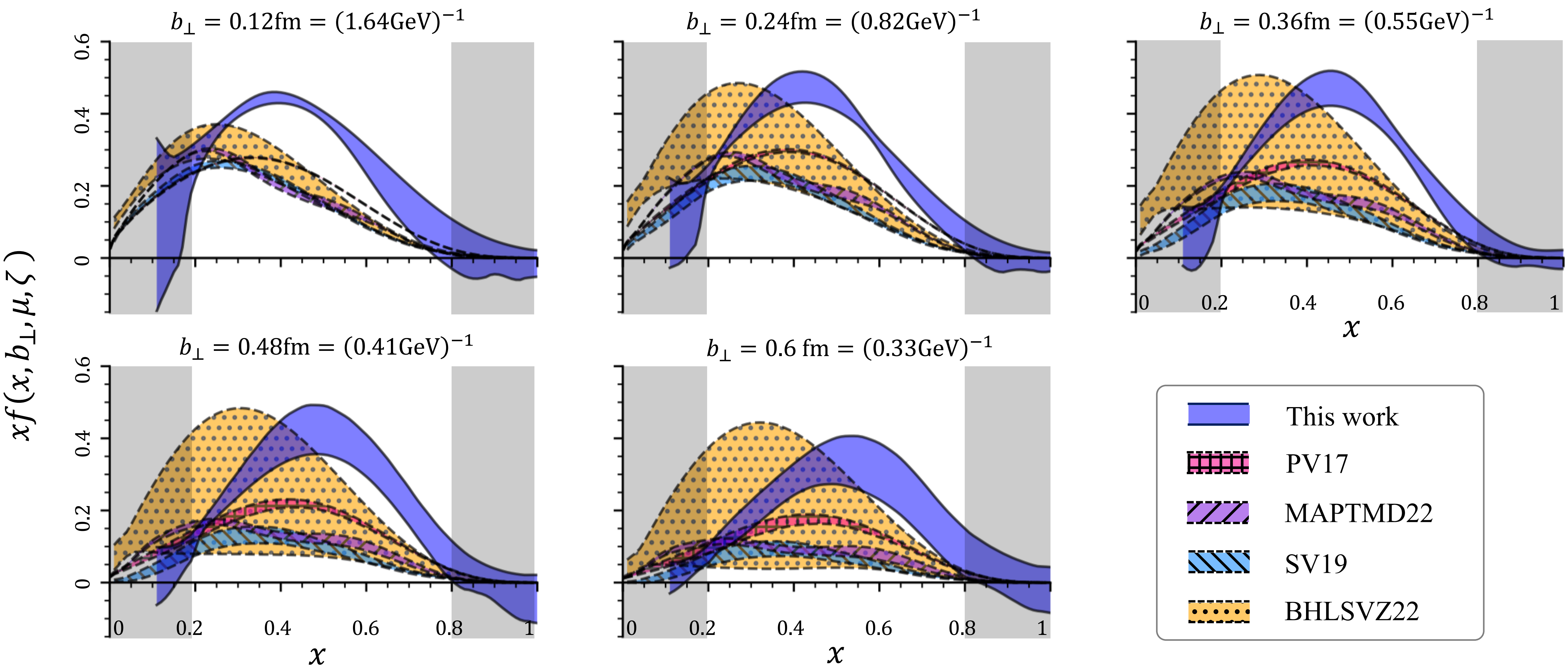


CS kernel from the lattice

LPC Collaboration, JHEP **08** (2023) 172



Unpolarised TMDPDF from the lattice



LPC Collaboration, J.-C. He *et al.*, arXiv: 2211.02340

Our approach

Taiwan lattice community and TMDPDF

★ Three numerical lattice PIs, all have projects on QCD

- → Ting-Wai Chiu @ Academia Sinica
- → Anthony Francis @ NYCU
- → C.-J. D. L. @ NYCU

★ A few phenomenologists working with lattice practitioners

- → Jiunn-Wei Chen, George W.-S. Hou @ NTU

★ The NYCU group is working on a TMD-physics project

- → New approach for soft function and CS kernel

A. Francis *et al.*, arXiv: 2312.04315

NYCU TMDPDF initiative

Anthony Francis

C.-J. David Lin

Wayne Morris

collaborators

William Detmold
(MIT)

Issaku Kanamori
(RIKEN)

Yong Zhao
(Argonne Nat'l Lab)

Need of new approaches for Soft function and CS kernel

★ Recent, previous lattice calculations involve pion states

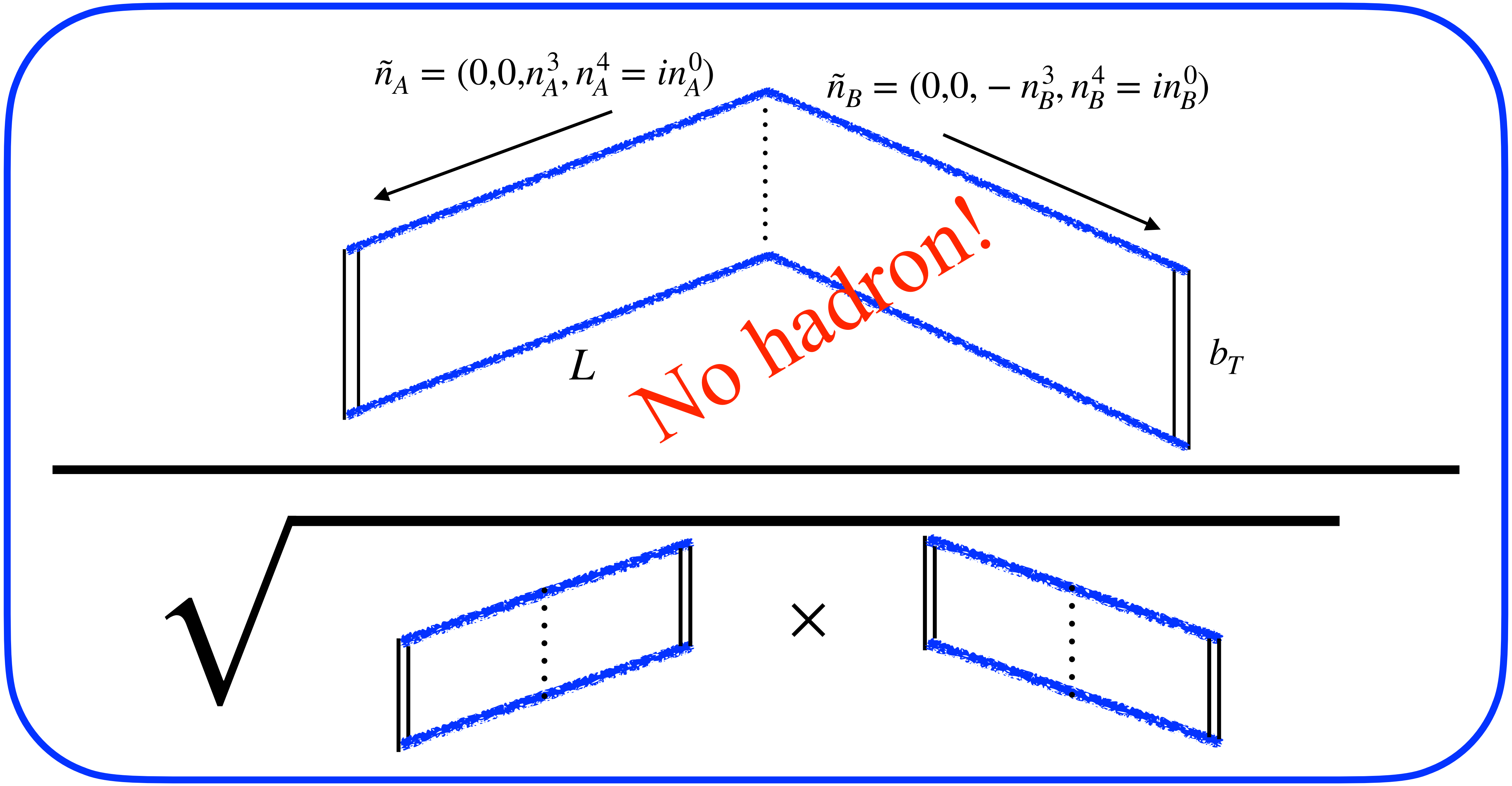
→ Universality?

★ Need calculations with other hadrons

★ Can one proceed without hadrons?

Our approach:

Soft function and CS kernel from Euclidean Wilson loops



Gives the Collins soft function in *Minkowski* space

Related to $S_I(b_T, \mu)$ and $K(b_T, \mu)$

Our approach:
Soft function and CS kernel from Euclidean Wilson loops

Off-light-cone regularisation in Collins' soft function, $S_C(b_T, \mu, y_A, y_B)$

★ One-loop results show:

➡ Collins soft function with space-like regularisation can be obtained

➡ Rapidities are related to the directional vectors of the Wilson lines

$$r_{a,b} \equiv \frac{n^3_{A,B}}{n^0_{A,B}} = \frac{1 + \mathrm{e}^{\pm y_{A,B}}}{1 - \mathrm{e}^{\pm y_{A,B}}}$$

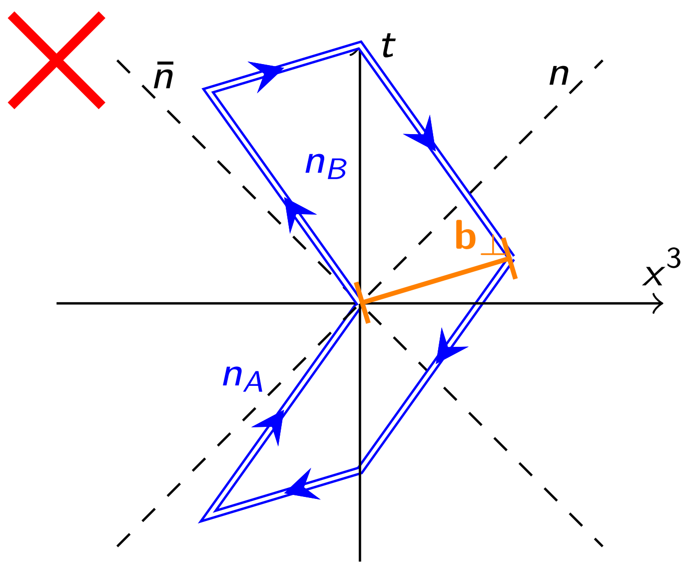
➡ Finite-length effects are of $O(b_T^4/L^4)$ or smaller

★ Determine $S_I(b_T, \mu)$ and $K(b_T, \mu)$ *via* varying $r_{a,b}$ and fitting to

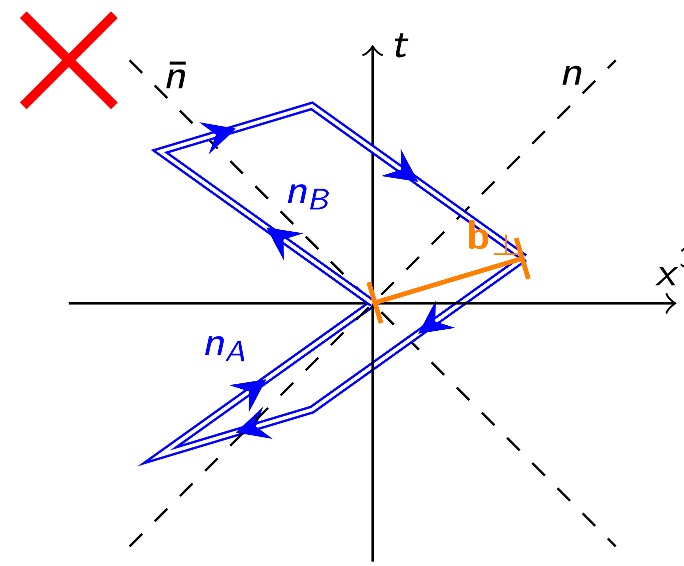
$$S_C(b_T, \mu, y_A, y_B) = S_I(b_T, \mu)\, \mathrm{e}^{2K(b_T,\mu)\times(y_A - y_B)}$$

Rapidity regularisation in our approach

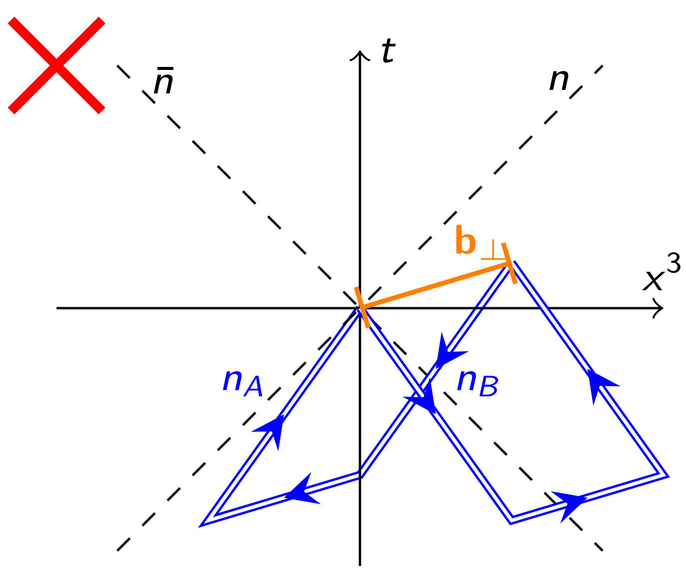
What can we reconstruct in Minkowski space?



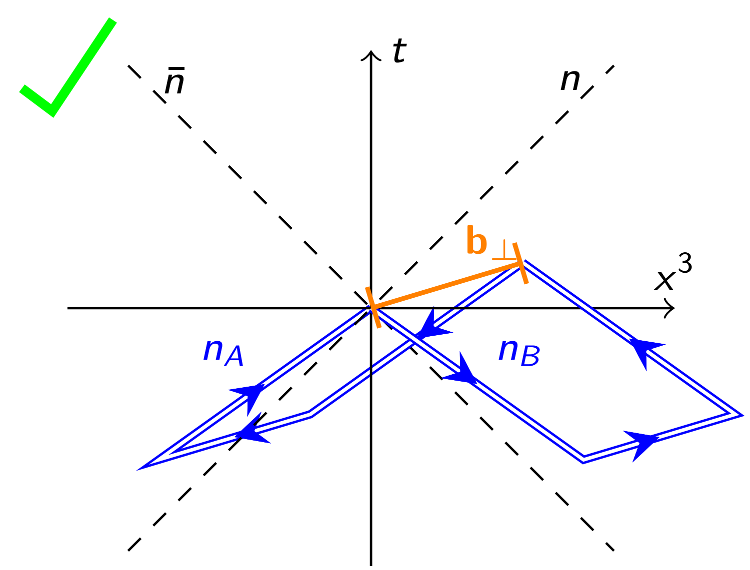
$|r_a| < 1, \quad |r_b| < 1, \quad n_A^0 n_B^0 (r_a r_b + 1) < 0$



$|r_a| > 1, \quad |r_b| > 1, \quad n_A^0 n_B^0 (r_a r_b + 1) < 0$



$|r_a| < 1, \quad |r_b| < 1, \quad n_A^0 n_B^0 (r_a r_b + 1) > 0$



$|r_a| > 1, \quad |r_b| > 1, \quad n_A^0 n_B^0 (r_a r_b + 1) > 0$

Our approach:

Soft function and CS kernel from Euclidean Wilson loops

★ Numerical implementation similar to moving HQET

X. Ji, Y. Liu, Y.-S. Liu, Nucl. Phys. **B955** (2020) 115054

→ non-static colour sources

J.E. Mandula, M.C. Ogilvie, Phys. Rev. **D45** (1992) 7, R2183

U. Aglietti *et al.*, Phys. Lett. **B294** (1992) 281

U. Aglietti, Nucl. Phys. **B421** (1994) 191

★ Our exploratory study shows promising statistical accuracy

→ Stay tuned for results of $K(b_T, \mu)$ and $S_I(b_T, \mu)$

Conclusion and outlook

★ Quasi-TMDPDF strategy available and tested

→ Exploratory numerical works available

→ Learning about the potential size of systematics

★ Need for alternative strategy

→ *e.g.*, CS kernel and soft function from Wilson loops

★ *And…*

This is not even the end of the beginning!

Numerical lattice-QCD results hitherto

★ The soft function

LPC Collaboration, Q.-A. Zhang *et. al.*, Phys. Rev., Lett. **125** (2020) 192001
Y. Li *et al.*, Phys. Rev., Lett. **128** (2022) 062002
LPC Collaboration, JHEP **08** (2023) 172

★ The Collins-Soper kernel

P. Shanahan, M. Wagman, Y. Zhao, Phys. Rev., **D102** (2020) 0141511
LPC Collaboration, Q.-A. Zhang *et. al.*, Phys. Rev., Lett. **125** (2020) 192001
P. Shanahan, M. Wagman, Y. Zhao, Phys. Rev., **D104** (2021) 114502
Y. Li *et al.*, Phys. Rev., Lett. **128** (2022) 062002
M. Schlemmer *et al.*, JHEP **08** (2021) 004
H.-M. Chu *et al.*, Phys. Rev. **D106** (2022) 3, 034509
LPC Collaboration, JHEP **08** (2023) 172
A. Avhadiev, P. Shanahan, M. Wagman, Y. Zhao, Phys. Rev. **D198** (2023) 11, 114505
H.-T. Shu *et al.*, Phys. Rev. **D108** (2022) 7, 074519

★ Unpolarised TMDPDF

LPC Collaboration, J.-C. He *et al.*, arXiv: 2211.02340