Overview on spin transport theory in heavy ion collisions

Qun Wang

Department of Modern Physics
Univ of Science & Technology of China (USTC)

School of Mechanics and Physics
Auhui University of Science & Technology (AUST)

ExHIC-p workshop on polarization phenomena in nuclear collisions
March 14-17, 2024, IOP, Academia Sinica, Taipei

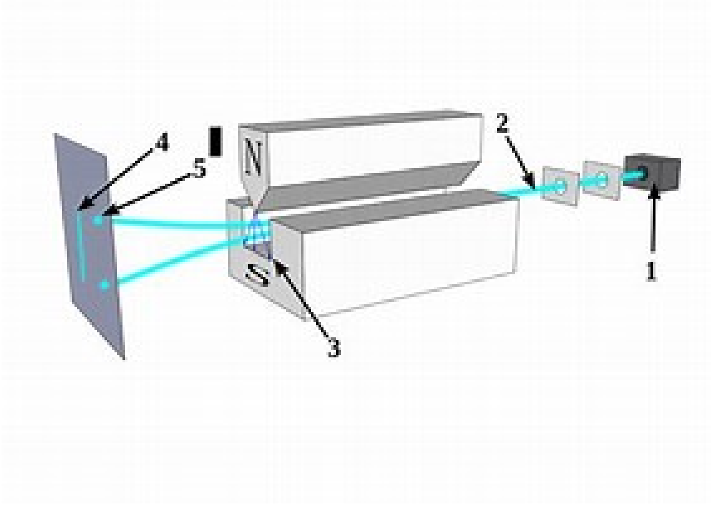
Outline

- **The 100-th anniversary of discovery of spin: early history**
- **A microscopic model for spin-vorticity coupling that emerges from spin-orbit coupling in parton-parton scatterings**
- **Spin Boltzmann (Kinetic) Equations for massive fermions and vector mesons**
- **Spin alignment of vector mesons with effective ϕ fields**
- **Summary**

100 years of spin

Discovery of electron spin

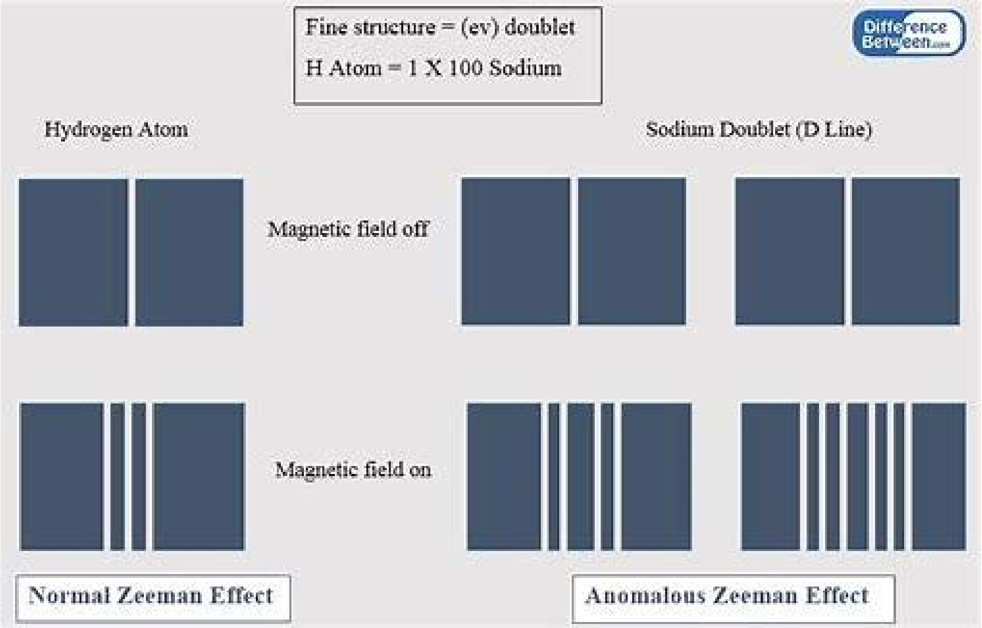
- **The classical picture of a particle's spin fails due to the problem of exceeding speed of light.**
- **Stern-Gerlach experiment (1922): first observation of two discrete quantum states of sliver atom with μ_B in non-homogeneous B field**



Otto Stern, Nobel prize in Physics 1943

Discovery of electron spin

- **Zeeman effect (1896), Anomalous Zeeman effect (1920s): quantization of orbital angular momentum and spin.**



Pieter Zeeman, Nobel prize in Physics 1903

Discovery of electron spin

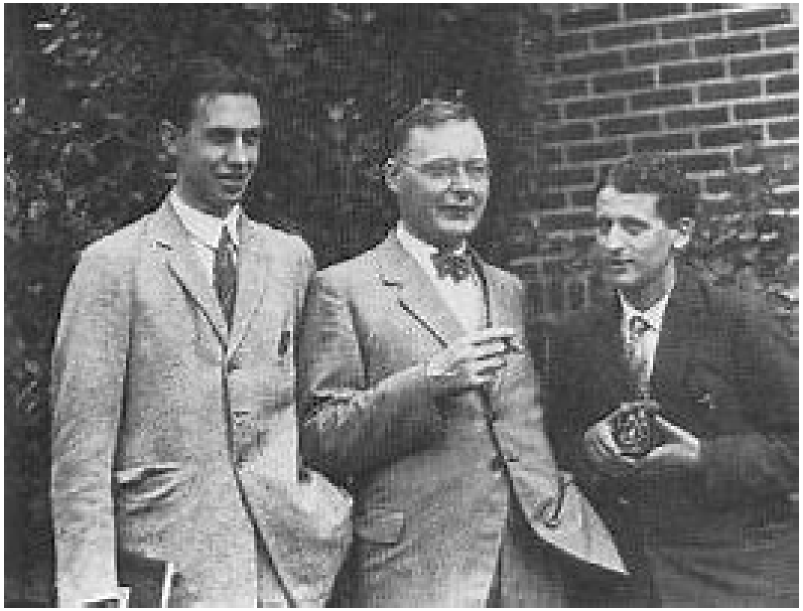
- **Fourth quantum number by Wolfgang Pauli (1924): to explain anomalous Zeeman effect, which takes only two values.**
- **Concept of electron spin by Ralph Kronig (1925): can explain even splitting of alkali spectra (over-estimated by factor 2), but opposed by Pauli and Bohr, not published**

Wolfgang Pauli,
Nobel prize in Physics 1945

Ralph Kronig

Discovery of electron spin

- **Electron spin by Uhlenbeck and Goudsmit (1925):**



George Uhlenbeck, Samuel Goudsmit

G.E. Uhlenbeck and S. Goudsmit, Naturwissenschaften 13 (1925) 953.

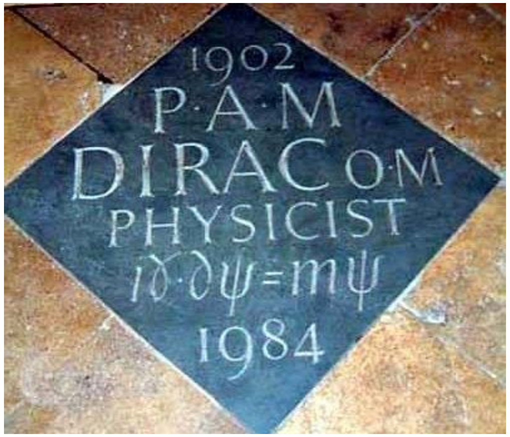
A subsequent publication by the same authors, Nature 117 (1926) 264.

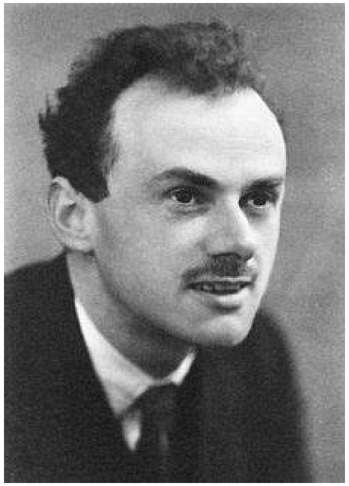
Discovery of electron spin

- **Pauli's non-relativistic theory for electron spin (1927): Schroedinger equation for particle with spin-1/2, Pauli spinor, Pauli matrices (dimension 2)**

$$\hat{H}|\psi\rangle = \left[\frac{1}{2m}\left[(\mathbf{p} - q\mathbf{A})^2 - q\hbar\boldsymbol{\sigma}\cdot\mathbf{B}\right] + q\phi\right]|\psi\rangle = i\hbar\frac{\partial}{\partial t}|\psi\rangle$$

- **Dirac equation (1928): relativistic extension of Pauli's theory, Dirac spinor, Dirac matrices (dimension 4)**





Paul Dirac Nobel Prize in physics 1933

Spin-orbit coupling from Dirac Equation

Dirac equation for spin-1/2 particle in a central potential $V(r) \sim -C/r$

$$\left(\boldsymbol{\alpha}\cdot\hat{\boldsymbol{P}}+\beta m+V\right)\begin{pmatrix}\chi\\ \psi\end{pmatrix}=E\begin{pmatrix}\chi\\ \psi\end{pmatrix}$$

$$\hat{\boldsymbol{P}}=-i\vec{\nabla}$$

$$\begin{pmatrix}E-V-m & -\boldsymbol{\sigma}\cdot\hat{\boldsymbol{P}}\\ -\boldsymbol{\sigma}\cdot\hat{\boldsymbol{P}} & E-V+m\end{pmatrix}\begin{pmatrix}\chi\\ \psi\end{pmatrix}=0$$

χ and ψ are Pauli spinors

Expressing ψ (small component) in terms of χ (large component) in non-relativistic approximation, we obtain spin-orbit coupling term $H_{S.O.}$

$$E_S\chi=\left[V+(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{P}})\frac{1}{E-V+m}(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{P}})\right]\chi$$

$$\frac{1}{E-V+m}\approx\frac{1}{2m}\left(1-\frac{E_S-V}{2m}\right)=\frac{1}{2m}-\frac{E_S-V}{4m^2}$$

$$\approx\left[\frac{\hat{\boldsymbol{P}}^2}{2m}+V-(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{P}})\frac{E_S-V}{4m^2}(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{P}})\right]\chi$$

$$E_S\equiv E-m$$

Effective Hamiltonian

$$=\left[\frac{\hat{\boldsymbol{P}}^2}{2m}+V-\frac{\hat{\boldsymbol{P}}^4}{8m^3}-i\frac{(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{P}})\times[\hat{\boldsymbol{P}},V]}{4m^2}-\frac{\hat{\boldsymbol{P}}\cdot[\hat{\boldsymbol{P}},V]}{4m^2}\right]\chi$$

spin-orbit coupling is a relativistic effect and can be derived from Dirac equation !

$$H_{S.O.}=-i\frac{(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{P}})\times[\hat{\boldsymbol{P}},V]}{4m^2}=C\frac{\boldsymbol{\sigma}\cdot(\boldsymbol{r}\times\hat{\boldsymbol{P}})}{4m^2r^3}$$

$$[\hat{\boldsymbol{P}},V]=[-i\nabla,V]=-iC\frac{\boldsymbol{r}}{r^3}$$

Rotation and Spin in HIC

Global OAM and Magnetic field in HIC

- **Huge global orbital angular momenta are produced**

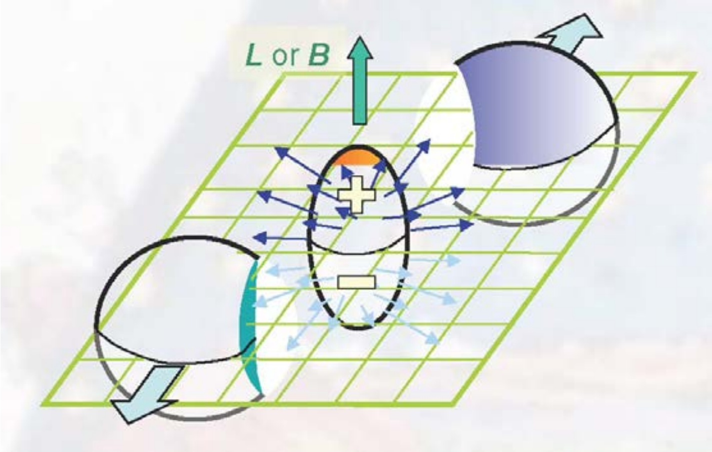
$$\mathrm{L} \sim 10^5 \hbar$$

- **Very strong magnetic fields are produced**

$$\mathrm{B} \sim m_\pi^2 \sim 10^{18}\ \mathrm{Gauss}$$

- **How do orbital angular momenta be transferred to the matter in HIC?**

- **How is spin coupled to local vorticity in the fluid?**



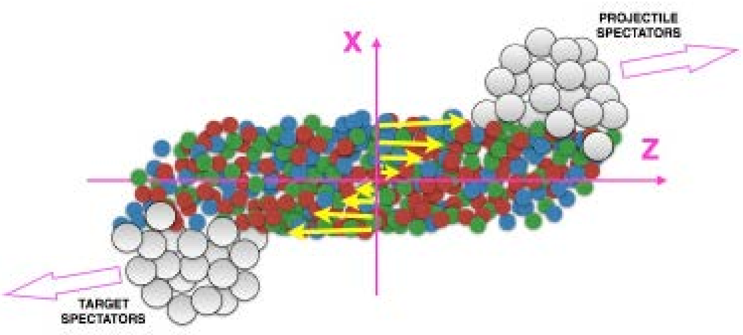
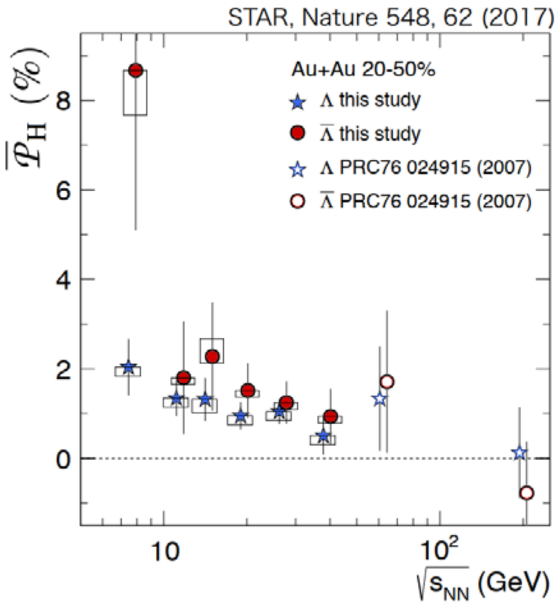


Figure taken from
Becattini et al, 1610.02506

STAR: global polarization of Λ hyperon



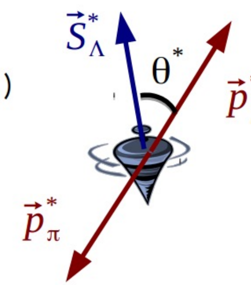
Experiments: AiHong Tang's talk

parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P_\Lambda} \cdot \mathbf{p_p^*})$$

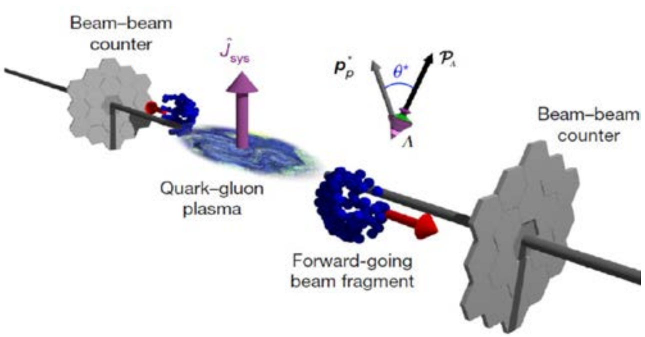
α: Λ decay parameter (=0.642±0.013)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame

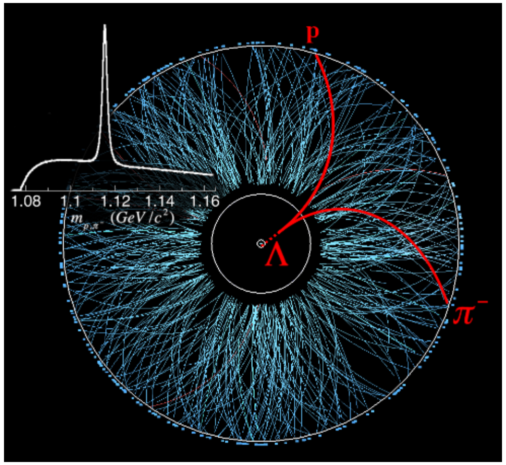


$$\Lambda \to p + \pi^+$$

(BR: 63.9%, cτ~7.9 cm)

Updated by BES III, PRL129, 131801 (2022)





ω = (9 ± 1)x10^{21}/s, the largest angular velocity that has ever been observed in any system

Some theoretical works on spin polarization of massive fermions

Microscopic models based on spin-orbit couplings
[Liang and Wang (2005); Gao, Chen, Deng, Liang, QW, Wang (2008); Zhang, Fang, QW, Wang (2019).]

Quantum statistical theory
[Zubarev (1979); Weert (1982); Becattini et al. (2012-2020); Hayat, et al. (2015); Floerchinger (2016).]

Spin hydrodynamical model
[Florkowski, Friman, Jaiswal, Ryblewski,Speranza (2017-2018); Montenegro, Tinti, Torrieri (2017-2019). Hattori, Hongo, Huang, Matsuo, Taya (2019)]

Kinetic theory in Wigner function formalism
[Early works: Heinz (1983); Elze, Gyulassy, Vasak (1986); Vasak, Gyulassy and Elze (1987); Zhuang, Heinz (1996).
Recent developments: Fang, Pang, QW, Wang (2016); Weickgenannt, Sheng, Speranza, QW, Rischke (2019); Gao, Liang (2019); Wang, Guo, Shi, Zhuang (2019); Hattori, Hidaka, Yang (2019); Lin (2022).]

Spin transport theory with local and non-local collisions in Wigner function formalism
[Yang, Hattori, Hidaka (2020); Weickgenannt, Speranza, Sheng, Wang, Rischke (2020); Wagner, Weickgenannt, Rischke (2022)]

Local polarization and shear induced polarization
[Fu, Liu, Pang, Song, Yin (2021); Becattini, Buzzegoli, Inghirami, Karpenko (2021); Yi, Pu, Yang (2021)]

Spin transport model in other methods
[Li and Yee (2019); Kapusta, Rrapaj, Rudaz (2020)]

There are huge number of papers in this field on the market !

Some review articles on polarization in HIC

1. Global and local spin polarization in heavy ion collisions: a brief overview, [phenomenology]
   QW, Nucl. Phys. A 967, 225 (2017).

2. Relativistic hydrodynamics for spin-polarized fluids, [theory]
   Florkowski, Kumar, R. Ryblewski, Prog. Part. Nucl. Phys. 108, 103709 (2019).

3. Polarization and Vorticity in the Quark–Gluon Plasma, [phenomenology]
   Becattini, Lisa, Ann. Rev. Nucl. Part. Sci. 70, 395 (2020).

4. Vorticity and Spin Polarization in Heavy Ion Collisions: Transport Models, [phenomenology]
   Huang, Liao, QW, Xia, Lect. Notes Phys. 987, 281 (2021).

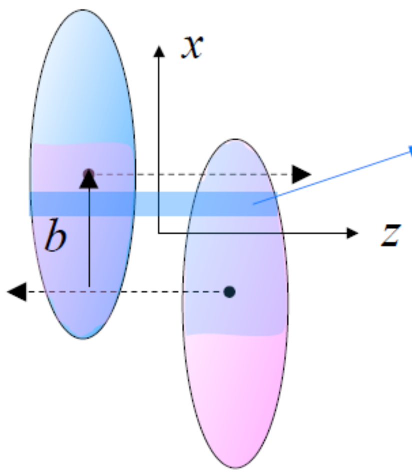
5. Global polarization effect and spin-orbit coupling in strong interaction, [phenomenology]
   Gao, Liang, QW, Wang, Lect. Notes Phys. 987, 195 (2021).

6. Spin and polarization: a new direction in relativistic heavy ion physics, [theory+phenom.]
   Becattini, Rept. Prog. Phys. 85, No.12, 122301 (2022)

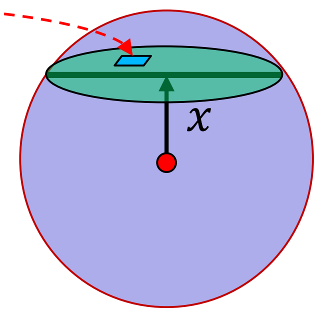
7. Foundations and applications of quantum kinetic theory, [theory]
   Hidaka, Pu, QW, Yang, Prog. Part. Nucl. Phys. 127, 103989 (2022).

8. Spin polarization in relativistic heavy-ion collisions, [theory+experimet]
   Becattini, Buzzegoli, Niida, Pu, Tang, QW, 2402.04540.

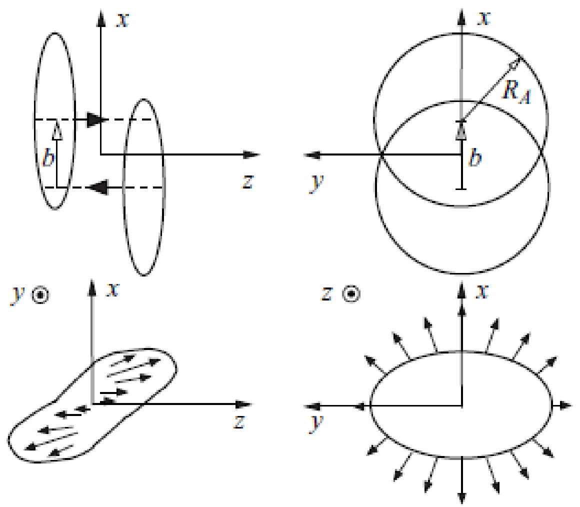
Global OAM in HIC



$$\frac{dN_{part}^{P,T}}{dx} = \int dy\, dz\, \rho_A^{P,T}(x, y, z, b)$$



Number of participant nucleons per unit x in projectile or target nuclei

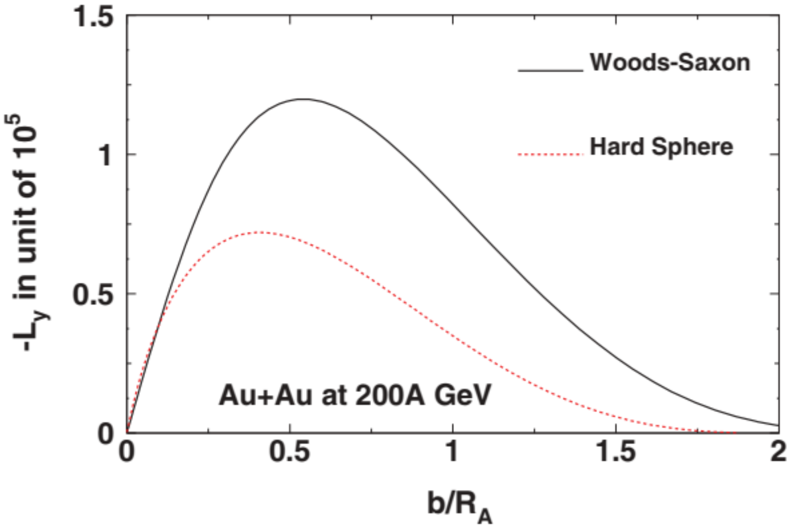


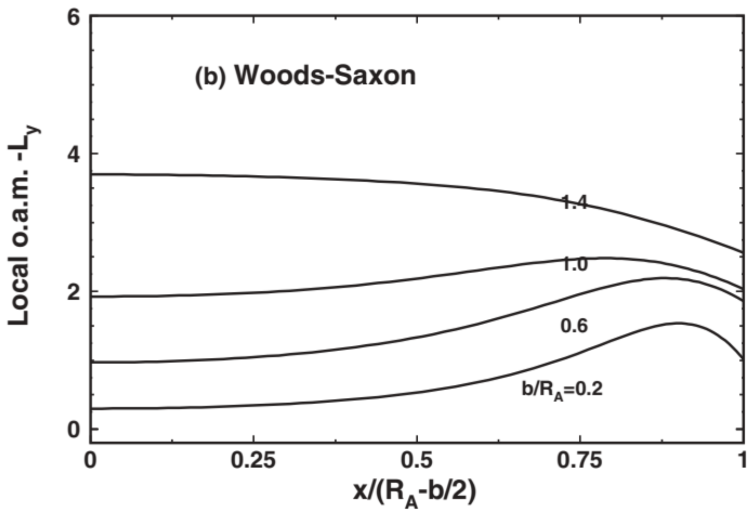
Collective longitudinal momentum per produced parton

$$p_z(x,b) = \frac{\sqrt{s}}{2c(s)} \frac{\frac{dN_{\rm part}^P}{dx} - \frac{dN_{\rm part}^T}{dx}}{\frac{dN_{\rm part}^P}{dx} + \frac{dN_{\rm part}^T}{dx}}$$

Liang, Wang PRL (2005); Gao, Chen, Deng, et al. PRC (2008)

Global and local OAM





Global OAM in y-direction

$$L_y = -p_{\rm in} \int x dx \left(\frac{dN_{\rm part}^P}{dx} - \frac{dN_{\rm part}^T}{dx} \right)$$

Local OAM in y-direction

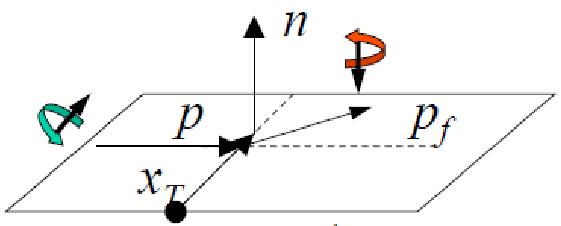
$$\begin{aligned} L_y &= -\Delta x \Delta p_z \\ &= -(\Delta x)^2 \frac{dp_z}{dx} \sim \beta \omega_y \end{aligned}$$

Liang, Wang PRL 94, 102301 (2005); Gao, Chen, Deng, et al. PRC 77, 044902 (2008)

Emergence of spin-vorticity coupling from spin-orbit coupling

Quark polarization in potential scatterings

- Quark scatterings at small angle in static potential at impact parameter x_T
- Unpolarized and polarized cross sections



$$A^0(q_T) = \frac{1}{q_T^2 + \mu^2}$$

screening mass

$$\mu \sim T\sqrt{\alpha_S}$$

$$\frac{d\sigma}{d^2\vec{x}_T} = \frac{d\sigma_+}{d^2\vec{x}_T} + \frac{d\sigma_-}{d^2\vec{x}_T} = 4C_T\alpha_s^2 K_0(\mu x_T)$$

$$\frac{d\Delta\sigma}{d^2\vec{x}_T} = \frac{d\sigma_+}{d^2\vec{x}_T} - \frac{d\sigma_-}{d^2\vec{x}_T} \propto \vec{n}\cdot(\vec{x}_T \times \vec{p})$$

Spin quantization direction   OAM   Spin-orbit coupling

- Polarization for small angle scattering and $m_q \gg p, \mu$

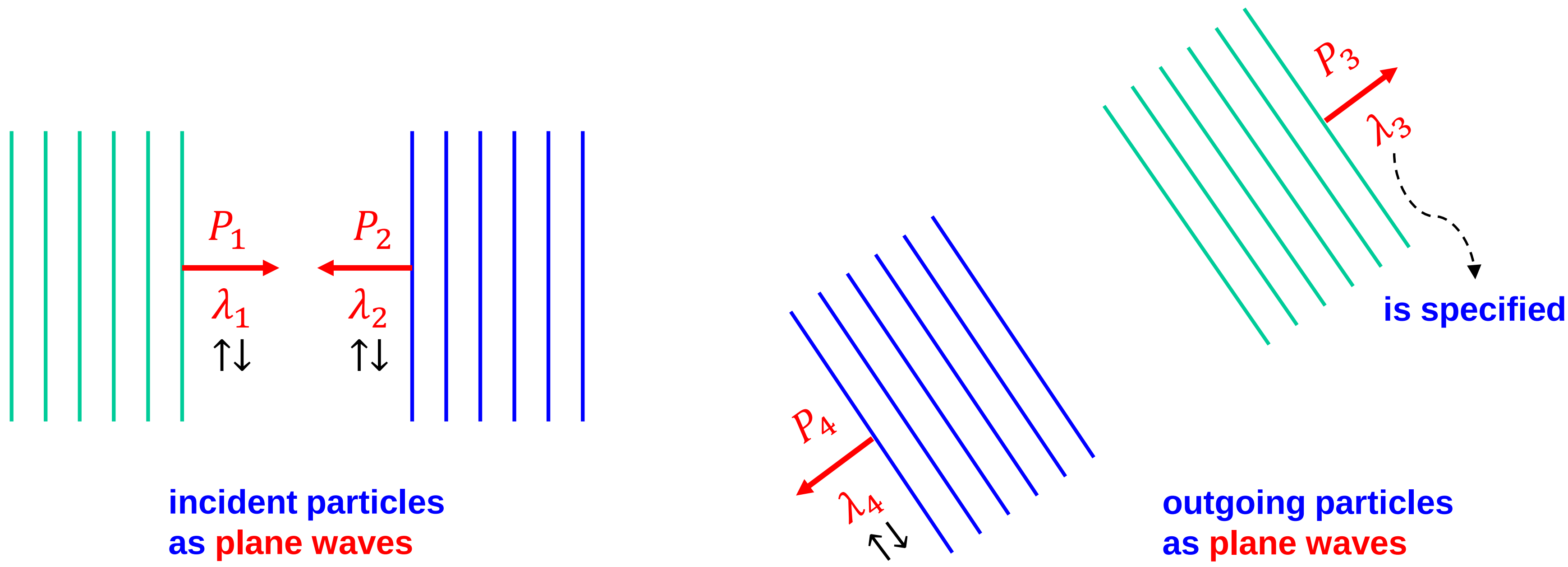
$$P_q \approx -\pi\frac{\mu p}{4m_q^2} \sim -\frac{\Delta E_{LS}}{E_0}$$

Liang, Wang, PRL 94, 102301(2005)

- With initial polarization P_i, the final polarization P_f after one scattering is $P_f = P_i - \frac{(1-P_i^2)\pi\mu p}{2E(E+m) - P_i\pi\mu p}.$

Huang, Huovinen, Wang, PRC84, 054910(2011)

Collisions of particles as plane waves



Particle collisions as plane waves:
since there is no favored position for particles, so the OAM vanishing

$$\langle \hat{x} \times \hat{p} \rangle = 0 \quad \Longrightarrow \quad \left(\frac{d\sigma}{d\Omega}\right)_{\lambda_3=\uparrow} = \left(\frac{d\sigma}{d\Omega}\right)_{\lambda_3=\downarrow}$$

Quark-quark scattering at fixed impact parameter

For the quark-quark scattering of spin-momentum states

$$q_1(P_1,\lambda_1)+q_2(P_2,\lambda_2)\rightarrow q_1(P_3,\lambda_3)+q_2(P_4,\lambda_4)$$

where $P_i=(E_i,\vec{p}_i)$ and λ_i denote spin states, the difference cross section (λ_3 is specified)

$c_{qq}=2/9$ (color factor)

$$d\sigma_{\lambda_3}=\frac{c_{qq}}{4F}\sum_{\lambda_1\lambda_2\lambda_4}\mathcal{M}(Q)\mathcal{M}^*(Q)(2\pi)^4\delta^{(4)}(P_1+P_2-P_3-P_4)\frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}\frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}$$

fixed

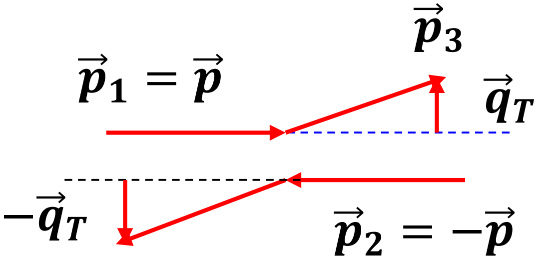
sum over ↑↓

$Q=P_3-P_1=P_2-P_4$ (momentum transfer)

$F=4\sqrt{(P_1\cdot P_2)^2-m_1^2m_2^2}$ (flux factor)

Integrate $\vec{p}_4$ and $p_{3z}^{\pm}=\pm\sqrt{p^2-q_T^2}$

to remove $\delta^{(4)}(P_1+P_2-P_3-P_4)$



Quark-quark scattering at fixed impact parameter

We obtain $d\sigma_{\lambda_3}$ for scattered quark with spin state λ_3

$$d\sigma_{\lambda_3} = \frac{c_{qq}}{16F} \sum_{\lambda_1\lambda_2\lambda_4} \sum_{i=+,-} \frac{1}{(E_1+E_2)|p_{3z}^i|} \mathcal{M}(Q_i)\mathcal{M}^*(Q_i) \frac{d^2\vec{q}_T}{(2\pi)^2}$$

for small angle scattering, only $i = +$ is relevant

Jacobian

momentum transfer in small angle scattering

Then we can introduce impact parameter $\vec{x}_T = (x_T, \phi)$

$$d\sigma_{\lambda_3} = \frac{c_{qq}}{16F} \sum_{\lambda_1\lambda_2\lambda_4} \int d^2\vec{x}_T \int \frac{d^2\vec{q}_T}{(2\pi)^2} \frac{d^2\vec{k}_T}{(2\pi)^2} \frac{1}{(E_1+E_2)|p_{3z}^i|} e^{i(\vec{k}_T - \vec{q}_T)\cdot\vec{x}_T} \frac{\mathcal{M}(\vec{q}_T)\mathcal{M}^*(\vec{k}_T)}{\Lambda(\vec{q}_T)\Lambda^*(\vec{k}_T)}$$

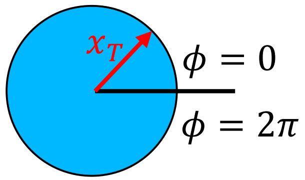
$\Rightarrow d^2\sigma_{\lambda_3}/d^2\vec{x}_T$

1

$\Lambda = \sqrt{(E_1+E_2)|p_{3z}^+(q_T)|}$

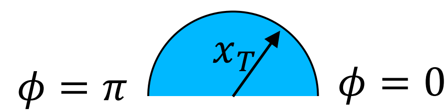
If we integrate over $\vec{x}_T$ in whole space we obtain

$$\sigma_{\lambda_3} = \int_0^\infty dx_T\, x_T \int_0^{2\pi} d\phi \frac{d^2\sigma_{\lambda_3}}{d^2\vec{x}_T} \Longrightarrow \sigma_\uparrow = \sigma_\downarrow$$



Quark-quark scattering at fixed impact parameter

If we integrate over $\vec{x}_T$ in half-space we obtain



$$\sigma_{\lambda_3} = \int_0^\infty dx_T\, x_T \int_0^\pi d\phi \frac{d^2\sigma_{\lambda_3}}{d^2\vec{x}_T} \Longrightarrow \sigma_\uparrow \neq \sigma_\downarrow$$

The differential cross section for spin-independent and spin-dependent part

$$\frac{d^2\sigma_{\lambda_3}}{d^2\vec{x}_T} = \frac{d^2\sigma}{d^2\vec{x}_T} + \lambda_3 \frac{d^2\Delta\sigma}{d^2\vec{x}_T}$$

$$\frac{d^2\sigma}{d^2\vec{x}_T} = \frac{1}{2}\left(\frac{d^2\sigma_\uparrow}{d^2\vec{x}_T} + \frac{d^2\sigma_\downarrow}{d^2\vec{x}_T}\right) = F(x_T)$$

$$\frac{d^2\Delta\sigma}{d^2\vec{x}_T} = \frac{1}{2}\left(\frac{d^2\sigma_\uparrow}{d^2\vec{x}_T} - \frac{d^2\sigma_\downarrow}{d^2\vec{x}_T}\right) = \underline{\vec{n}\cdot(\vec{x}_T \times \vec{p})}\Delta F(x_T)$$

spin-orbit coupling

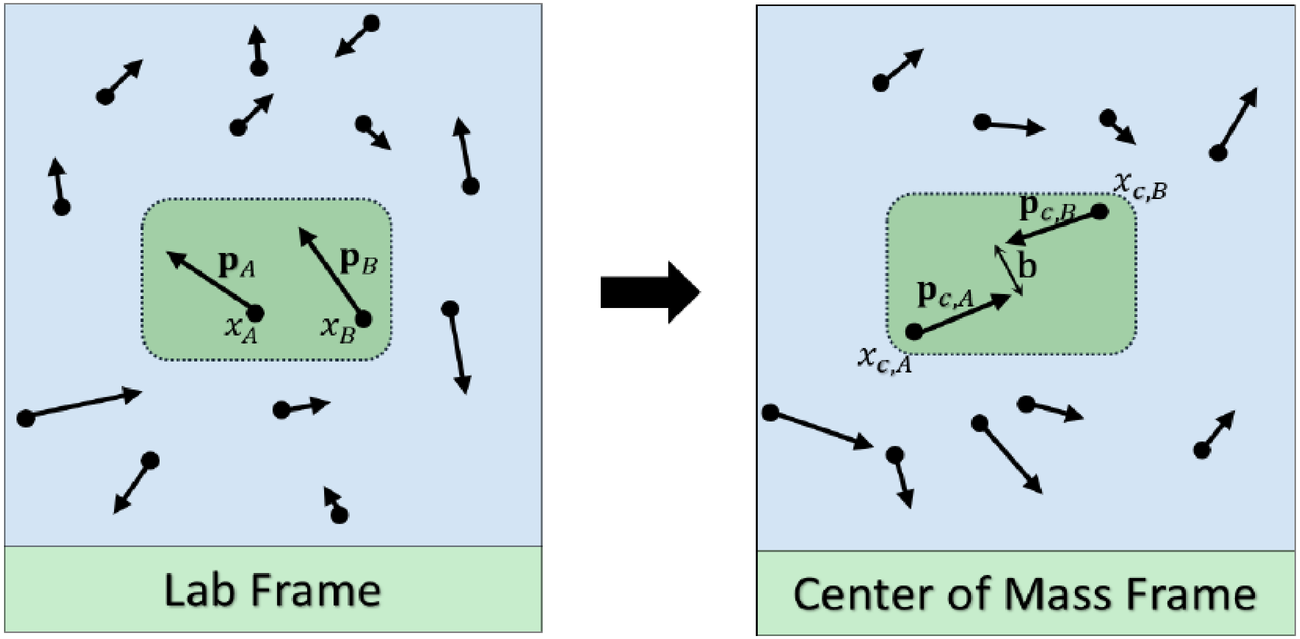
$$\Delta\sigma = \int_0^\infty dx_T x_T \int_0^\pi d\phi \frac{d^2\Delta\sigma}{d^2\vec{x}_T}$$

$$\sigma = \int_0^\infty dx_T x_T \int_0^{2\pi} d\phi \frac{d^2\Delta\sigma}{d^2\vec{x}_T}$$

$$P_q = \frac{\Delta\sigma}{\sigma}$$

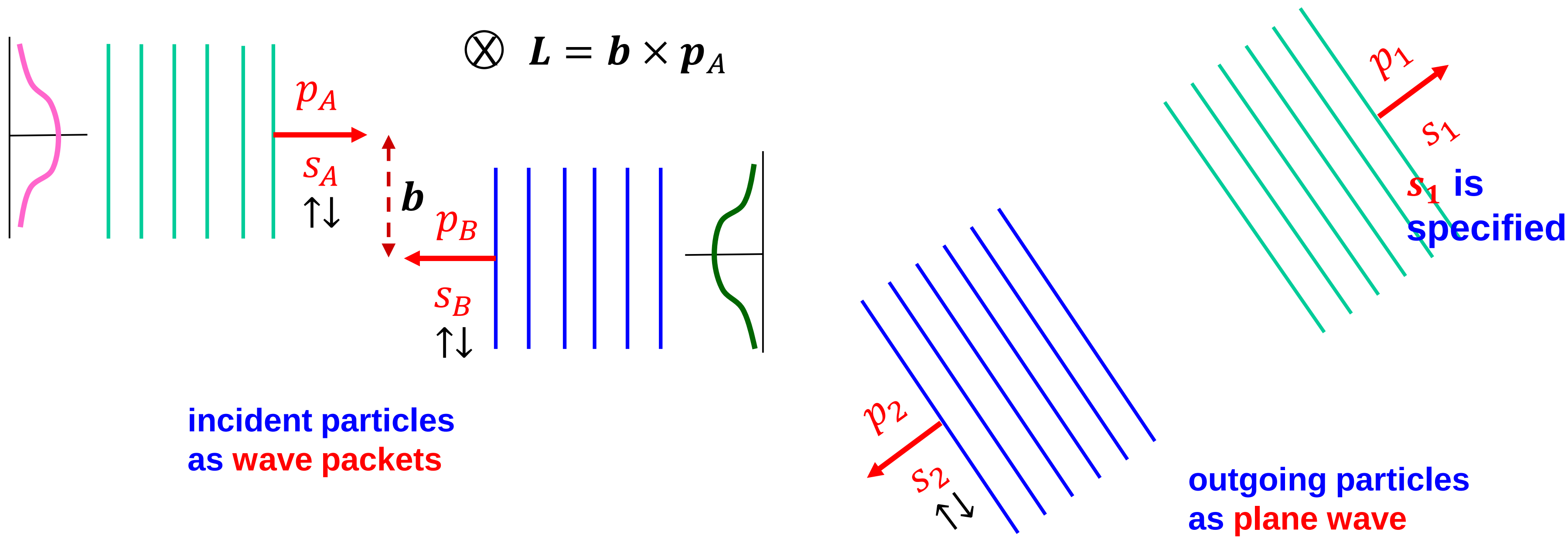
Gao, Chen, Deng, et al., PRC 77, 044902 (2008)

Ensemble average in thermal QGP for global polarization through spin-orbit couplings in parton scatterings



Zhang, Fang, QW, Wang, PRC 100, 064904 (2019)

Collisions of particles as wave packets



Particle collisions as wave packets: there is a transverse distance between two wave packets (impact parameter) giving non-vanishing OAM and then the polarization of one final particle

$$\boldsymbol{L} = \boldsymbol{b} \times \boldsymbol{p}_A \Longrightarrow \left(\frac{d\sigma}{d\Omega}\right)_{s_1=\uparrow} \neq \left(\frac{d\sigma}{d\Omega}\right)_{s_1=\downarrow}$$

Collisions of particles at different space-time points

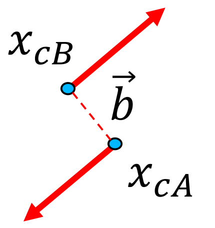
- **Two incident particles at $x_A = (t_A, \mathrm{x}_A)$ and $x_B = (t_B, \mathrm{x}_B)$ in the lab frame**

$$\boxed{\mathrm{x}_A \neq \mathrm{x}_B}$$

$$t_A = t_B \Longrightarrow t_{c,A} \neq t_{c,B}$$

$$t_A \neq t_B \Longrightarrow t_{c,A} = t_{c,B}$$

$$\boxed{\mathrm{x}_{c,A} \neq \mathrm{x}_{c,B}}$$



CM frame

- **We impose the causality condition in CM frame for scattering of particles at two different space-time points (the time interval and longitudinal distance of two space-time points should be small enough for scattering to take place)**

$$\Delta t_c = t_{c,A} - t_{c,B} = 0$$

$$\Delta x_{c,L} = \hat{\mathrm{p}}_{c,A} \cdot (\mathrm{x}_{c,A} - \mathrm{x}_{c,B}) = 0$$

CM frame: collisions take place at the same time and longitudinal position but displaced by impact parameter

From spin-orbit coupling to spin-vorticity coupling: ensemble average

- **Quark polarization rate per unit volume: 10D + 6D integration**

$$\frac{d^4\mathbf{P}_{AB\to 12}(X)}{dX^4} = \frac{\pi}{(2\pi)^4}\frac{\partial(\beta u_\rho)}{\partial X^\nu}\int \frac{d^3p_A}{(2\pi)^3 2E_A}\frac{d^3p_B}{(2\pi)^3 2E_B}$$

$$\times|v_{c,A}-v_{c,B}|[\Lambda^{-1}]^\nu_{\ j}\mathbf{e}_{c,i}\epsilon_{ikh}\hat{\mathbf{p}}^h_{c,A}$$

$$\times f_A(X,p_A)\, f_B(X,p_B)\,(p^\rho_A - p^\rho_B)\Theta_{jk}(\mathbf{p}_{c,A})$$

$$\equiv \frac{\partial(\beta u_\rho)}{\partial X^\nu}\mathbf{W}^{\rho\nu}$$

6D integral

Lorentz boost

10D integral

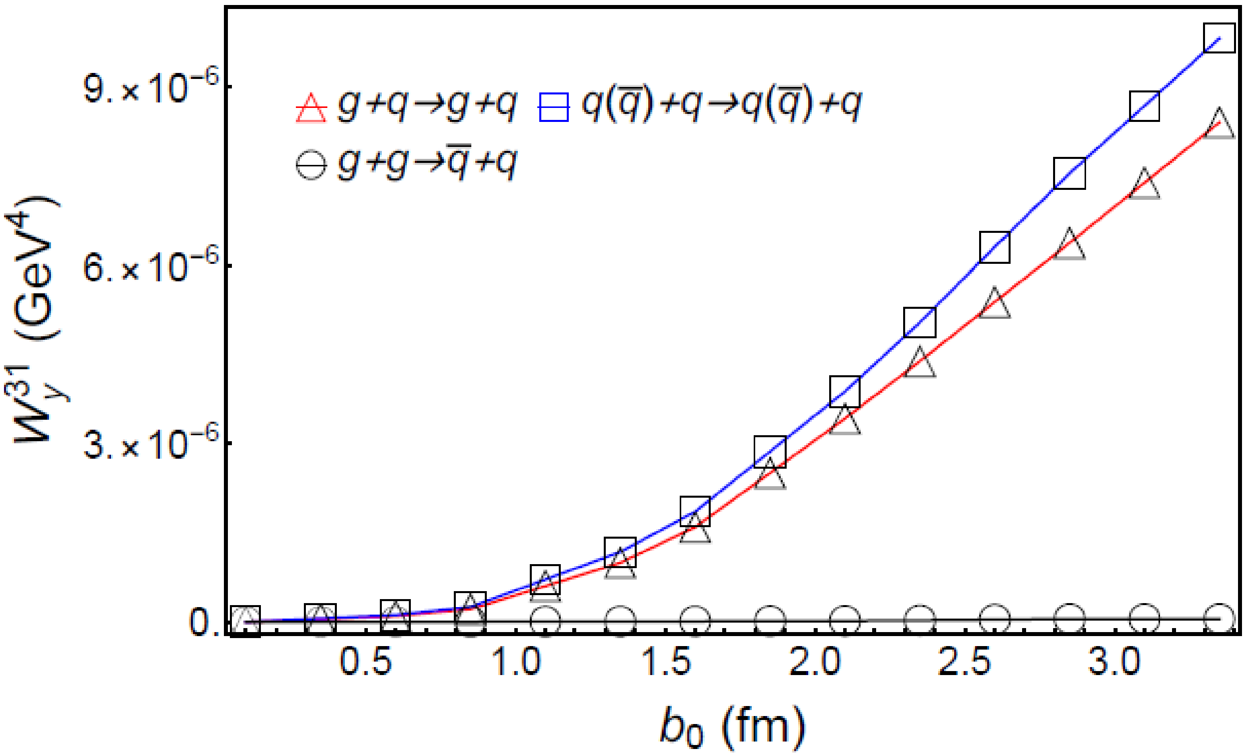
16D integral !!

- **Numerical challenge !!!** **We have developed ZMCintegral-3.0, a Monte Carlo integration package that runs on multi-GPUs** **[Wu, Zhang, Pang, QW, Comp. Phys. Comm. (2020) (1902.07916)]**

- **Another challenge:** **there are more than 5000 terms in polarized amplitude squared for 2-to-2 parton scatterings**

$$I_M^{q_a q_b\to q_a q_b}(s_2) = \sum_{s_A,s_B,s_1}\sum_{i,j,k,l}\mathcal{M}\left(\{s_A,k_A;s_B,k_B\}\to\{s_1,p_1;s_2,p_2\}\right)\mathcal{M}^*\left(\{s_A,k'_A;s_B,k'_B\}\to\{s_1,p_1;s_2,p_2\}\right)$$

Numerical results for quark polarization



The cutoff b_0 is of the order of hydro length scale $1/\partial u(x)$ and larger than interaction scale $1/m_D$: $b_0 \sim \frac{1}{\partial u(x)} > \frac{1}{m_D}$

$$\frac{d^4 \mathrm{P}_{AB\to 12}(X)}{dX^4} = 2W\nabla_X \times (\beta \mathbf{u})$$

Zhang, Fang, QW, et al., PRC 100, 064904 (2019)

Numerical results and comparison with data

AMPT transport model

-- Li, Pang, QW, Xia, PRC96, 054908(2017)

-- Wei, Deng, Huang, PRC99, 014905(2019)

UrQMD + vHLLE hydro

-- Karpenko, Becattini, EPJC 77, 213(2017)

PICR hydro

-- Xie, Wang, Csernai, PRC 95,031901(2017)

Chiral Kinetic Equation + Collisions

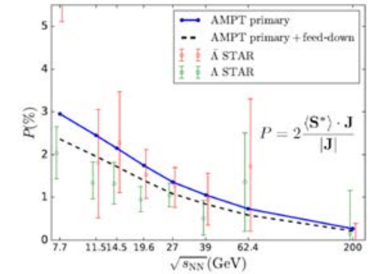
-- Sun, Ko, PRC96, 024906(2017)

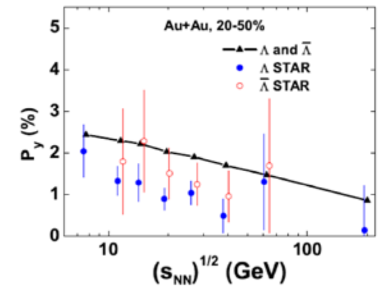
-- Liu, Sun, Ko, PRL125, 062301(2020)

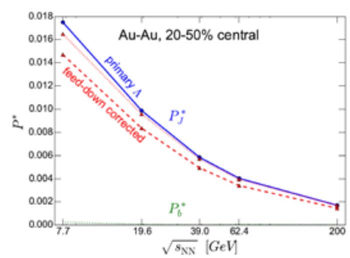
AVE+3FD

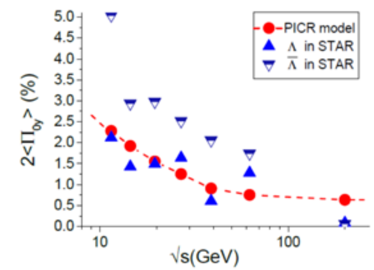
-- Ivanov, 2006.14328

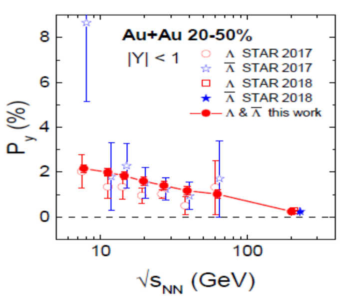
Other works …

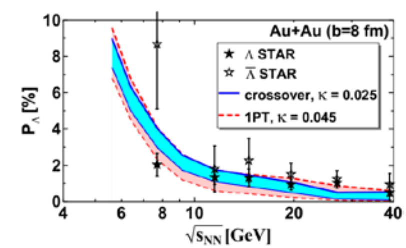












Quantum kinetic equations with spin or Spin Boltzmann (kinetic) equations with Wigner functions

QKT for massive fermions in Wigner functions

- **Wigner function (4x4 matrix) for spin 1/2 massive fermions**

$$W_{\alpha\beta}(x,p) = \int d^4y \exp\left(\frac{i}{\hbar}p\cdot y\right)\left\langle \overline{\psi}_\beta\left(x-\frac{y}{2}\right)\psi_\alpha\left(x+\frac{y}{2}\right)\right\rangle$$

Heinz (1983); Vasak-Gyulassy-Elze (1987); Zhuang-Heinz (1996); Iancu-Blaizot (2001); QW-Redlich-Stoecker-Greiner (2002)

- **Wigner function decomposition in 16 generators of Clifford algebra**

spin 4-vector

$$W = \frac{1}{4}\left[\mathscr{F} + i\gamma^5\mathscr{P} + \gamma^\mu\mathscr{V}_\mu + \gamma^5\gamma^\mu\mathscr{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu}\right]$$

scalar p-scalar vector axial-vector tensor

$$j^\mu = \int d^4p\mathscr{V}^\mu, \qquad j_5^\mu = \int d^4p\mathscr{A}^\mu, \qquad T^{\mu\nu} = \int d^4p p^\mu\mathscr{V}^\nu$$

Recent reviews:
Hidaka-Pu-QW-Yang, PPNP (2022)
Gao-Liang-QW, IJMPA (2021)

Vasak-Gyulassy-Elze, Ann. Phys. 173, 462 (1987);
Elze-Gyulassy-Vasak, Nucl. Phys. B 276, 706 (1986);

Polarization from different sources in QKT with Wigner functions (without collisions)

Axial vector component of WF (spin vector) has many contributions

$$j_5^\mu = j^\mu_{5,\mathrm{thermal}} + j^\mu_{5,\mathrm{shear}} + j^\mu_{5,\mathrm{accel}} + j^\mu_{5,\mathrm{chemical}} + j^\mu_{5,\mathrm{EM}}$$

Thermal vorticity

$$j^\mu_{5,\mathrm{thermal}} = a\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}p_\nu\partial_\alpha\frac{u_\beta}{T}$$

$$\partial_{(\sigma}u_{\nu)} \equiv \frac{1}{2}\left(\partial_\mu u_\nu + \partial_\nu u_\mu\right)$$

Shear viscous tensor

$$j^\mu_{5,\mathrm{shear}} = -a\frac{1}{T(u\cdot p)}\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta p^\sigma\partial_{(\sigma}u_{\nu)}$$

Fluid acceleration

$$j^\mu_{5,\mathrm{accel}} = a\frac{1}{2T}\epsilon^{\mu\nu\alpha\beta}p_\nu u_\alpha\left(Du_\beta - \frac{1}{T}\partial_\beta T\right)$$

Gradient of chemical potential

$$j^\mu_{5,\mathrm{chemical}} = a\frac{1}{(u\cdot p)}\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta\partial_\nu\frac{\mu}{T}$$

Electromagnetic fields

$$j^\mu_{5,\mathrm{EM}} = a\frac{1}{T(u\cdot p)}\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta E_\nu + a\frac{B^\mu}{T}$$

Hidaka, Pu, Yang (2018); Yi, Pu, Yang (2021)

Becattini, et al, (2021)
Fu, Liu, et al., (2021)

Spin DOF: Matrix Valued Spin Distributions (MVSD)

Relativistic MVSD for fermion in QFT

$$p^\mu \equiv \frac{1}{2}(p_1^\mu + p_2^\mu) \qquad q^\mu \equiv p_1^\mu - p_2^\mu$$

$$f_{rs}(x,p) \equiv \int \frac{d^4q}{2(2\pi)^3} \exp\left(-\frac{i}{\hbar} q\cdot x\right) \delta(p\cdot q) \langle a^\dagger(s,\mathbf{p}_2) a(r,\mathbf{p}_1)\rangle$$

on-shell condition

$r, s = \uparrow, \downarrow$ or $1, 2$

Relativistic MVSD can be parameterized in un-polarized and polarized parts

$j = 1, 2, 3$

$$f_{rs}^{(+)}(x,\mathbf{p}) = \frac{1}{2} f_q(x,\mathbf{p}) \left[\delta_{rs} - P_\mu^q(x,\mathbf{p}) n_j^{(+)\mu}(\mathbf{p}) \tau_{rs}^j\right],$$

$$f_{rs}^{(-)}(x,-\mathbf{p}) = \frac{1}{2} f_{\overline{q}}(x,-\mathbf{p}) \left[\delta_{rs} - P_\mu^{\overline{q}}(x,-\mathbf{p}) n_j^{(-)\mu}(\mathbf{p}) \tau_{rs}^j\right],$$

Pauli matrices in spin space (rs-space)

un-polarized dist.

spin polarization dist.

Four-vectors of three basis directions in rest frame of q and $\overline{q}$ (one is the spin quantization direction)

MVSD:
Becattini et al. (2013)
Sheng, Weickgenannt, et al. (2021)
Sheng, QW, Rischke (2022)

Spin Boltzmann equation for massive fermions

- **At leading order spin Boltzmann equation (SBE) with local collision terms**

$$\frac{1}{E_p} p \cdot \partial_x \mathrm{tr}\left[f^{(0)}(x,p)\right] = \mathscr{C}_{\mathrm{scalar}}\left[f^{(0)}\right]$$

$$\frac{1}{E_p} p \cdot \partial_x \mathrm{tr}\left[n_j^{(+)\mu}\tau_j f^{(0)}(x,p)\right] = \mathscr{C}^{\mu}_{\mathrm{pol}}\left[f^{(0)}\right]$$

$\Longrightarrow$ $f_{rs}^{(0)}(x,p)$

- **At next-to-leading order, SBE describes how $f^{(1)}(x,p)$ evolves for given $f^{(0)}(x,p)$ and $\partial_x f^{(0)}(x,p)$ [non-local terms]**

$$\frac{1}{E_p} p \cdot \partial_x \mathrm{tr}\left[f^{(1)}(x,p)\right] = \mathscr{C}_{\mathrm{scalar}}\left[f^{(0)}, \partial_x f^{(0)}, f^{(1)}\right]$$

$$\frac{1}{E_p} p \cdot \partial_x \mathrm{tr}\left[n_j^{(+)\mu}\tau_j f^{(1)}(x,p)\right] = \mathscr{C}^{\mu}_{\mathrm{pol}}\left[f^{(0)}, \partial_x f^{(0)}, f^{(1)}\right]$$

determined by leading order SBE

$\partial_\mu u_\nu,\ \partial_\mu T,\ \partial_\mu \mu_B$

Convenient for simulation !

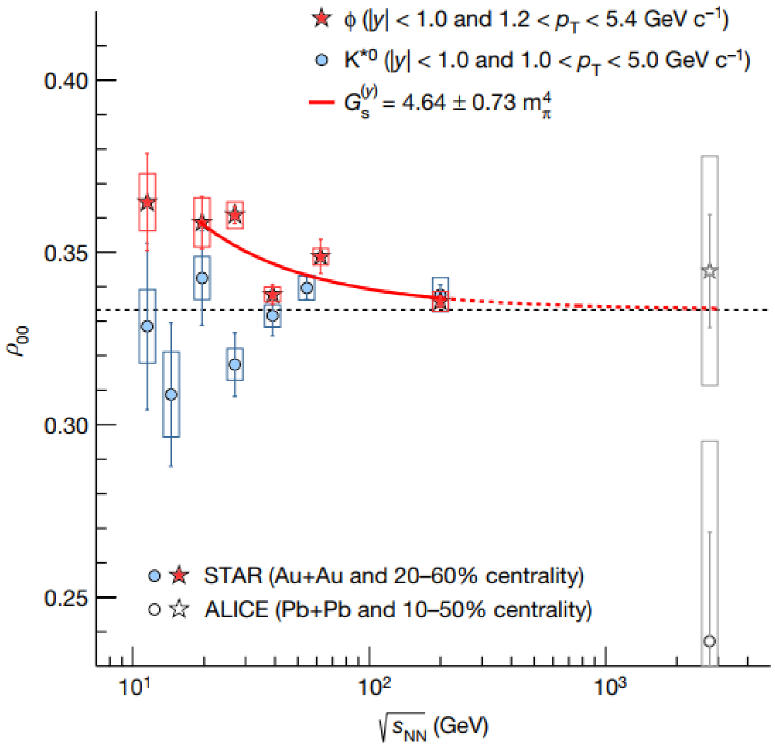
Sheng, Speranza, Rischke, QW, Weickgenannt (2021)

spin transport for massive fermions from WF or KB equation was also studied in: Yang, Hattori, Hidaka (2020); Gao, Liang (2021); Wang, Zhuang (2021)

Spin alignment for vector mesons

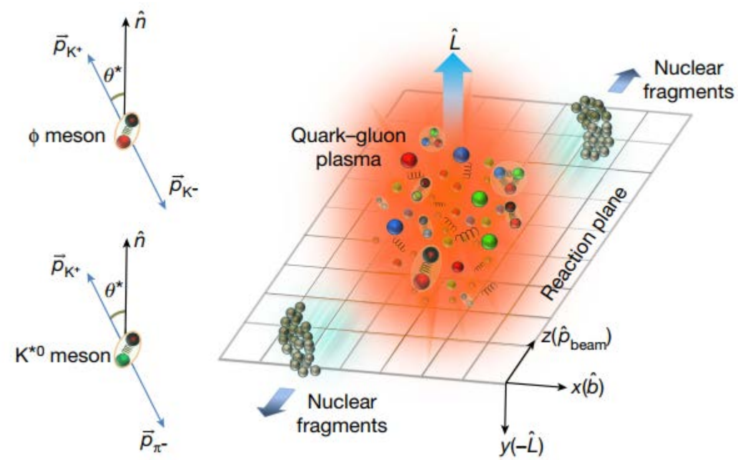
STAR: global spin alignments of vector mesons

STAR Collab., Nature 614, 244 (2023)



NR-QCM with local correlation or fluctuation of strong force fields

Experiments: AiHong Tang's talk



$$\frac{dN}{d\cos\theta} = \frac{3}{4}\left[1 - \rho_{00} + (3\rho_{00} - 1)\cos^2\theta\right]$$

Theory predictions:
Sheng, Oliva, QW (2020);
Sheng, Oliva, et al., (2022)

Some of early theoretical works on spin alignment of vector mesons in HIC

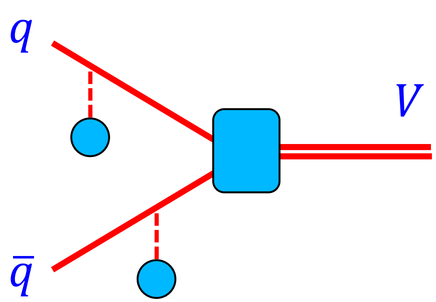
1. Spin alignment of vector mesons in non-central A+A collisions,
[Z.T. Liang, X.-N. Wang, PLB(2005); nucl-th/0411101]
2. Quark coalescence model for polarized vector mesons and baryons,
[Y.-G. Yang, Q. Wang, et al., PRC(2018); 1711.06008.]
3. What can we learn from the global spin alignment of ϕ mesons in heavy-ion collisions?
[X.-L. Sheng, L. Oliva, Q. Wang, PRD (2020); 1910.13684.]
4. Improved quark coalescence model for spin alignment and polarization of hadrons,
[X.-L. Sheng, Q.Wang, X.-N. Wang, PRD(2020); 2007.05106.]
5. Local spin alignment of vector mesons in relativistic heavy-ion collisions,
[X.-L. Xia, H. Li, X.-G. Huang, et al., PLB(2021); 2010.01474.]
6. Spin alignment of vector mesons as a probe of spin hydrodynamics and freeze-out,
[K. J. Goncalves, G.Torrieri, PRC (2022); 2104.12941.]
7. Helicity polarization in relativistic heavy ion collisions,
[J.-H. Gao, PRD(2021); 2105.08293]
8. Spin Alignment of Vector Mesons in Heavy-Ion Collisions,
[X.-L. Sheng, L. Oliva, et al., PRL(2023); 2205.15689.]
9. Relativistic spin dynamics for vector mesons,
[X.-L. Sheng, L. Oliva, et al., PRD(2024); 2206.05868.]
10. Tensor Polarization and Spectral Properties of Vector Meson in QCD Medium,
[F. Li, S.Y.F. Liu, 2206.11890.]
11. Generating tensor polarization from shear stress,
[D. Wagner, N. Weickgenannt, Enrico Speranza, PRR (2023); 2207.01111.]
12. Spin alignment of vector mesons by glasma fields,
[A. Kumar, B. Mueller, D.-L. Yang, PRD(2023); 2304.04181.]
13. Spin polarization and spin alignment from quantum kinetic theory with self-energy corrections,
[S. Fang, S. Pu, D.-L. Yang, PRD (2024); 2311.15197.]
14. Linear response theory for spin alignment of vector mesons in thermal media,
[W.-B. Dong, Y.-L. Yin, et al., to appear in PRD (2024); 2311.18400.]
15. Spin alignment of vector mesons by second-order hydrodynamic gradients,
[A. Kumar, P. Gubler, D.-L. Yang, 2312.16900.]

Spin evolution with Spin Boltzmann (Kinetic) Equation for vector mesons in quark coalescence model

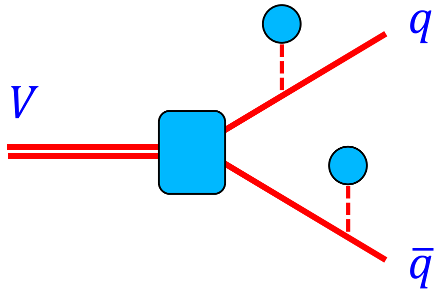
Sheng, Oliva, et al., 2206.05868, 2205.15689

For details: Xin-Li Sheng's talk

Review on QKE and SKE based on Wigner functions:
Hidaka, Pu, QW, Yang, Prog. Part. Nucl. Phys. 127 (2022) 103989



Quark coalescence to V-meson



V-meson dissociation to quarks

Quark coalescence model:
Greco, Ko, Levai (2003);
Fries, Mueller et al (2003);
Yang, Hwa (2003).

Wigner functions for vector mesons

The Wigner function can be defined from $G^{<}_{\mu\nu}(x_1, x_2)$ [or equivalently $G^{>}_{\mu\nu}(x_1, x_2)$] by taking a Fourier transform with respect to the relative position $y = x_1 - x_2$

$$G^{<}_{\mu\nu}(x,p) \equiv \int d^4y\, e^{ip\cdot y} G^{<}_{\mu\nu}(x_1,x_2) = \int d^4y\, e^{ip\cdot y} \left\langle A^{\dagger}_{\nu}(x_2) A_{\mu}(x_1) \right\rangle$$

Inserting the quantized field, we obtain the WF at the leading order $O(\hbar)$

$$\begin{aligned} G^{(0)<}_{\mu\nu}(x,p) &= 2\pi \sum_{\lambda_1,\lambda_2} \delta\left(p^2 - m_V^2\right) \Big\{ \theta(p^0) \epsilon_{\mu}(\lambda_1, \mathbf{p})\, \epsilon^{*}_{\nu}(\lambda_2, \mathbf{p})\, f^{(0)}_{\lambda_1\lambda_2}(x, \mathbf{p}) \\ &+ \theta(-p^0) \epsilon^{*}_{\mu}(\lambda_1, -\mathbf{p})\, \epsilon_{\nu}(\lambda_2, -\mathbf{p}) \left[\delta_{\lambda_2\lambda_1} + f^{(0)}_{\lambda_2\lambda_1}(x, -\mathbf{p}) \right] \Big\} \end{aligned}$$

$$\begin{aligned} \epsilon_0 &= \mathbf{n}_y \\ \epsilon_{+1} &= -\frac{1}{\sqrt{2}}(\mathbf{n}_z + i\mathbf{n}_x) \\ \epsilon_{-1} &= \frac{1}{\sqrt{2}}(\mathbf{n}_z - i\mathbf{n}_x) \end{aligned}$$

$$\epsilon^{\mu}(\lambda, \mathbf{k}) = \left(\frac{\mathbf{k}\cdot\epsilon_{\lambda}}{m_V}, \epsilon_{\lambda} + \frac{\mathbf{k}\cdot\epsilon_{\lambda}}{m_V(E^V_{\mathbf{k}} + m_V)}\mathbf{k} \right)$$

where the MVSD for vector meson is defined as

$$f^{(0)}_{\lambda_1\lambda_2}(x, \mathbf{p}) \equiv \int \frac{d^4u}{2(2\pi)^3} \delta(p\cdot u) e^{-iu\cdot x} \left\langle a^{\dagger}_{V}\left(\lambda_2, \mathbf{p} - \frac{\mathbf{u}}{2}\right) a_V\left(\lambda_1, \mathbf{p} + \frac{\mathbf{u}}{2}\right) \right\rangle$$

$\rho_{\lambda_1\lambda_2}$ **spin density matrix $\lambda_1, \lambda_2 = 1, 0, -1$**

Wigner functions for vector mesons

The decomposition of MVSD (spin density matrix)

$$f^{(0)}_{\lambda_1\lambda_2} = \mathrm{Tr}(f^{(0)}) \left(\frac{1}{3} + \frac{1}{2} \underline{P_i \Sigma_i} + \underline{T_{ij} \Sigma_{ij}} \right)_{\lambda_1\lambda_2}$$

$i, j = 1, 2, 3$ and $\lambda_1, \lambda_2 = 1, 0, -1$

Polarization part (cannot be measured in strong decay)

tensor part (can be measured in strong decay)

where Σ_i and Σ_{ij} are 3×3 traceless matrices and defined as

$$\Sigma_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \Sigma_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\Sigma_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad \Sigma_{ij} = \frac{1}{2}(\Sigma_i \Sigma_j + \Sigma_j \Sigma_i) - \frac{2}{3}\delta_{ij}$$

Wigner functions for vector mesons

From Kadanoff-Baym equation for Wigner functions, we obtain the spin Boltzmann (kinetic) equation in quasi-particle approximation

$$\frac{p}{E_p^V}\cdot\partial_x f_{\lambda_1\lambda_2}(x,\mathbf{p})\approx R_{\lambda_1\lambda_2}^{\mathrm{coal}}(\mathbf{p})-R^{\mathrm{diss}}(\mathbf{p})f_{\lambda_1\lambda_2}(x,\mathbf{p})$$

where $R_{\lambda_1\lambda_2}^{coal}$ and R^{diss} denote the coalescence and dissociation rates for the vector meson, i.e. the rates of $q\bar{q}\rightarrow M$ and $M\rightarrow q\bar{q}$ respectively. Schematically the formal solution reads

$$\begin{aligned} f_{\lambda_1\lambda_2}(x,\mathbf{p}) & \sim \frac{R_{\lambda_1\lambda_2}^{\mathrm{coal}}(\mathbf{p})}{R^{\mathrm{diss}}(\mathbf{p})}\left[1-\exp\left(-R^{\mathrm{diss}}(\mathbf{p})\Delta t\right)\right] \\ & \sim \begin{cases} R_{\lambda_1\lambda_2}^{\mathrm{coal}}(\mathbf{p})\Delta t, & \text{for } \Delta t\ll 1/R^{\mathrm{diss}}(\mathbf{p}) \\ \frac{R_{\lambda_1\lambda_2}^{\mathrm{coal}}(\mathbf{p})}{R^{\mathrm{diss}}(\mathbf{p})}, & \text{for } \Delta t\gg 1/R^{\mathrm{diss}}(\mathbf{p}) \end{cases} \end{aligned}$$

$\rho_{\lambda_1\lambda_2}$ **spin density matrix $\lambda_1,\lambda_2=1,0,-1$**

Wigner functions for vector mesons

The spin density matrix element can be put into a compact form with an explicit dependence on the polarization vector of the quark and antiquark

$$\begin{aligned}\rho^{V}_{\lambda_1\lambda_2}(x,\mathbf{p}) &= \frac{\Delta t}{32}\int\frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}\frac{1}{E^{\overline{q}}_{p'}E^{q}_{\mathbf{p}-\mathbf{p}'}E^{V}_{p}}f_{\overline{q}}(x,\mathbf{p}')f_q(x,\mathbf{p}-\mathbf{p}')\\ &\quad\times 2\pi\hbar\delta\left(E^V_p - E^{\overline{q}}_{p'} - E^{q}_{\mathbf{p}-\mathbf{p}'}\right)\epsilon^{*}_{\alpha}(\lambda_1,\mathbf{p})\epsilon_{\beta}(\lambda_2,\mathbf{p})\\ &\quad\times\mathrm{Tr}\left\{\Gamma^{\beta}\left(p'\cdot\gamma - m_{\overline{q}}\right)\left[1+\gamma_5\gamma\cdot\underline{P^{\overline{q}}}(x,\mathbf{p}')\right]\Gamma^{\alpha}\right.\\ &\quad\left.\times\left[(p-p')\cdot\gamma + m_q\right]\left[1+\gamma_5\gamma\cdot\underline{P^{q}}(x,\mathbf{p}-\mathbf{p}')\right]\right\}\end{aligned}$$

where the polarization for s and $\overline{s}$ are given by

Details: Xin-Li Sheng's talk

$$\begin{aligned}P^{\mu}_{s}(x,\mathbf{p}) &= \frac{g_\phi}{4m_s E^s_p T_{\mathrm{eff}}}\epsilon^{\mu\nu\rho\sigma}\underline{F^{\phi}_{\rho\sigma}}p_\nu\left[1-f_s(x,\mathbf{p})\right]\\ P^{\mu}_{\overline{s}}(x,\mathbf{p}) &= -\frac{g_\phi}{4m_s E^{\overline{s}}_p T_{\mathrm{eff}}}\epsilon^{\mu\nu\rho\sigma}\underline{F^{\phi}_{\rho\sigma}}p_\nu\left[1-f_{\overline{s}}(x,\mathbf{p})\right]\end{aligned}$$

Effective ϕ field strength tensor

Spin density matrix element for vector mesons

The fusion (coalescence) collision kernel can be evaluated in the rest frame of ϕ meson, which gives ρ_{00}^{ϕ}

$$\rho_{00}(x,\mathbf{0}) \approx \frac{1}{3} + C_1\left[\frac{1}{3}\boldsymbol{\omega}'\cdot\boldsymbol{\omega}' - (\boldsymbol{\epsilon}_0\cdot\boldsymbol{\omega}')^2\right] + C_2\left[\frac{1}{3}\boldsymbol{\varepsilon}'\cdot\boldsymbol{\varepsilon}' - (\boldsymbol{\epsilon}_0\cdot\boldsymbol{\varepsilon}')^2\right] - \frac{4g_\phi^2}{m_\phi^2 T_{\rm eff}^2}C_1\left[\frac{1}{3}\mathbf{B}'_\phi\cdot\mathbf{B}'_\phi - (\boldsymbol{\epsilon}_0\cdot\mathbf{B}'_\phi)^2\right] - \frac{4g_\phi^2}{m_\phi^2 T_{\rm eff}^2}C_2\left[\frac{1}{3}\mathbf{E}'_\phi\cdot\mathbf{E}'_\phi - (\boldsymbol{\epsilon}_0\cdot\mathbf{E}'_\phi)^2\right],$$

$$C_1 = \frac{8m_s^4 + 16m_s^2 m_\phi^2 + 3m_\phi^4}{120m_s^2(m_\phi^2 + 2m_s^2)},$$

$$C_2 = \frac{8m_s^4 - 14m_s^2 m_\phi^2 + 3m_\phi^4}{120m_s^2(m_\phi^2 + 2m_s^2)}.$$

rest frame of ϕ meson

spin quantization direction

All fields with prime are defined in the rest frame of ϕ meson

Features:

(1) Perfect factorization of x and p dependence;

(2) Perfect cancellation for mixing terms (protected by symmetry): all fields appear in squares, i.e. ρ_{00}^{ϕ} measures fluctuations of fields.

Surprising results!

Lorentz transformation for ϕ fields

We can express ρ_{00}^{ϕ} in terms of ϕ fields in the lab frame and obtain the dependence on momenta of ϕ mesons through Lorentz transformation

$$\mathbf{B}'_{\phi} = \gamma \mathbf{B}_{\phi} - \gamma \mathbf{v} \times \mathbf{E}_{\phi} + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{B}_{\phi}}{v^2}\mathbf{v},$$

$$\mathbf{E}'_{\phi} = \gamma \mathbf{E}_{\phi} + \gamma \mathbf{v} \times \mathbf{B}_{\phi} + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{E}_{\phi}}{v^2}\mathbf{v},$$

where $\gamma = E_{\mathbf{k}}^{\phi}/m_{\phi}$ **and** $\mathbf{v} = \mathbf{k}/E_{\mathbf{k}}^{\phi}$

Then we obtain factorization form of $\left\langle \rho_{00}^{\phi} \right\rangle$ in terms of lab-frame fields

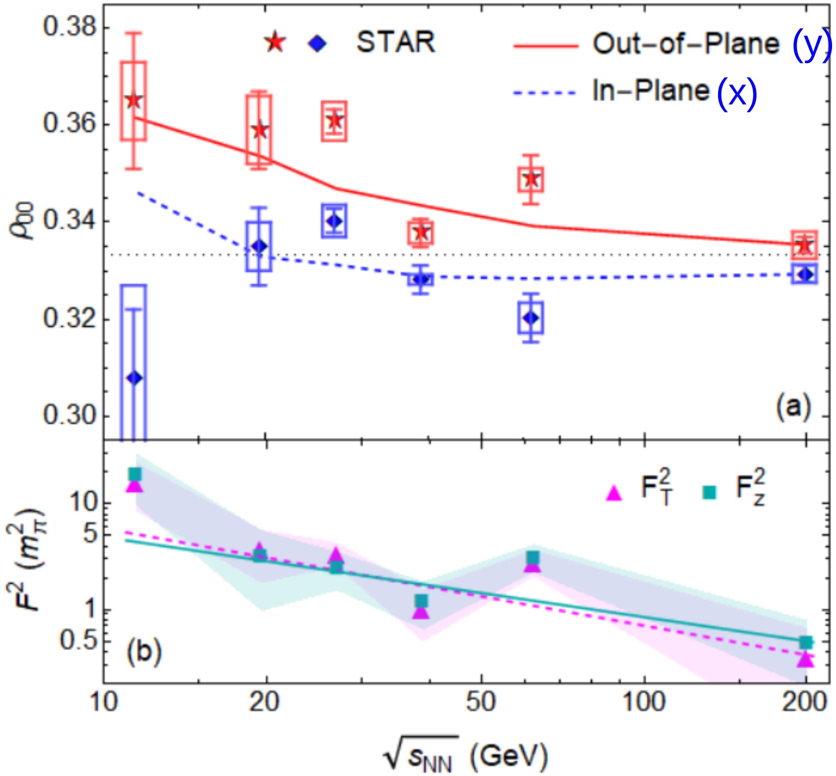
$$\left\langle \overline{\rho}_{00}^{\phi}(x,\mathbf{p}) \right\rangle_{x,\mathbf{p}} \approx \frac{1}{3} + \frac{1}{3}\sum_{i=1,2,3} \langle I_{B,i}(\mathbf{p})\rangle \frac{1}{m_{\phi}^2}\left[\langle \boldsymbol{\omega}_i^2\rangle - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\text{eff}}^2}\left\langle (\mathbf{B}_i^{\phi})^2 \right\rangle\right] + \frac{1}{3}\sum_{i=1,2,3} \langle I_{E,i}(\mathbf{p})\rangle \frac{1}{m_{\phi}^2}\left[\langle \boldsymbol{\varepsilon}_i^2\rangle - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\text{eff}}^2}\left\langle (\mathbf{E}_i^{\phi})^2 \right\rangle\right]$$

space-time average

momentum average

three basis directions in lab frame

STAR data on ρ_{00}^{y} and ρ_{00}^{x}



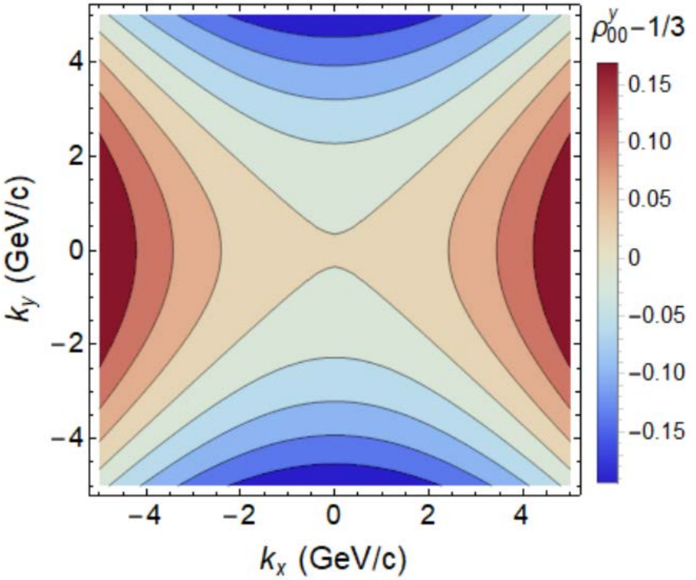
$$F_T^2 \equiv \langle E_{x,y}^2 \rangle = \langle B_{x,y}^2 \rangle, \quad F_z^2 \equiv \langle E_z^2 \rangle = \langle B_z^2 \rangle$$

(a) The STAR's data on phi meson's ρ_{00}^{y} (out-of-plane, red stars) and ρ_{00}^{x} (in-plane, blue diamonds) in 0-80% Au+Au collisions as functions of collision energies. The red-solid line and blue-dashed line are calculated with values of F_T^2 and F_z^2 from fitted curves in (b).

(b) Values of F_T^2 (magenta triangles) and F_z^2 (cyan squares) with shaded error bands extracted from the STAR's data on the phi meson's ρ_{00}^{y} and ρ_{00}^{x} in (c). The magenta-dashed line (cyan-solid line) is a fit to the extracted F_T^2 (F_z^2) as a function of $\sqrt{s_{NN}}$ (see the text).

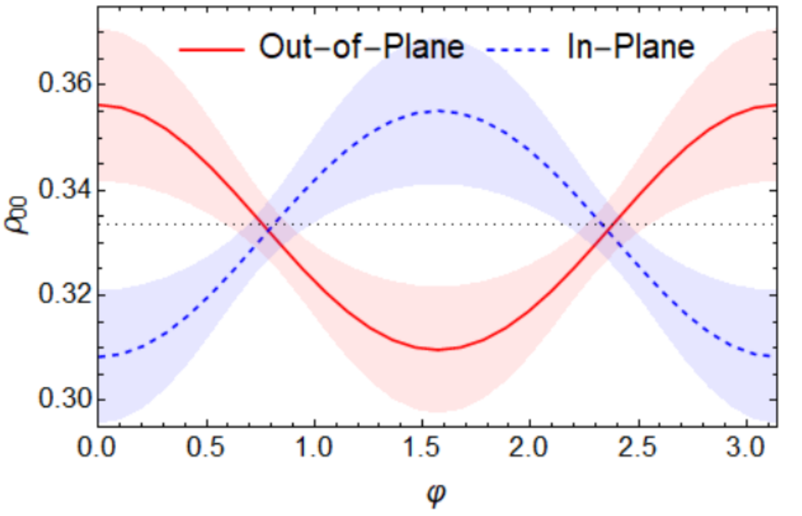
Details: Xin-Li Sheng's talk

Prediction on ρ_{00}^{y} and ρ_{00}^{x}



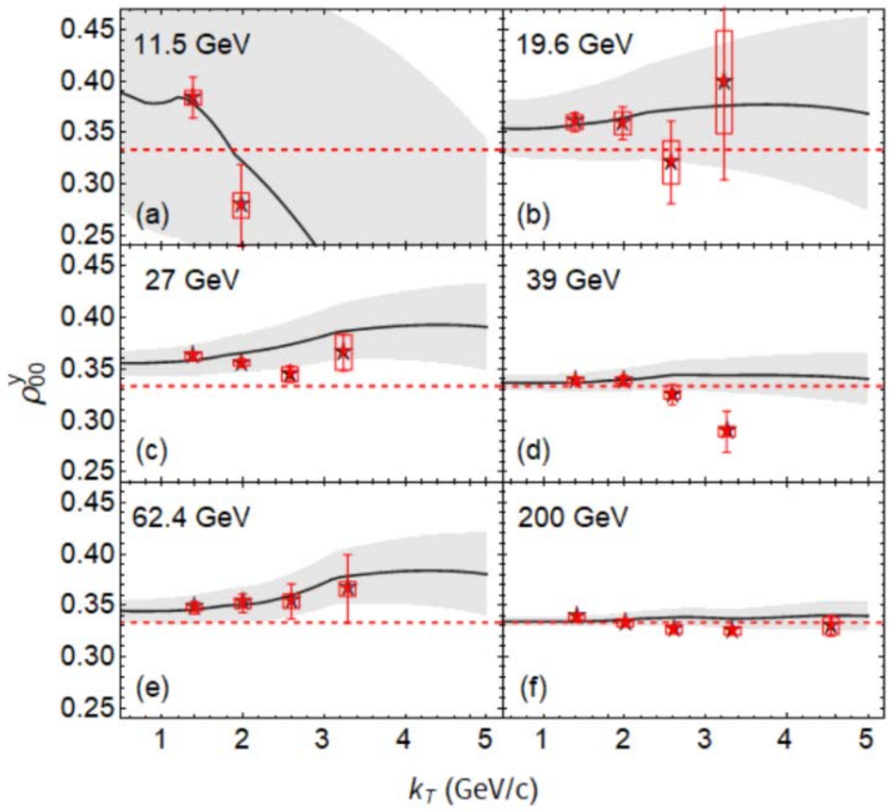
Contour plot of $\rho_{00}^{y} - 1/3$ for ϕ mesons as a function of k_x and k_y in 0-80% Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

Details: Xin-Li Sheng's talk



Calculated ρ_{00}^{y} (out-of-plane) and ρ_{00}^{x} (in plane) of ϕ mesons as functions of the azimuthal angle φ in 0-80% Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Shaded error bands are from the extracted parameters F_T^2 and F_z^2.

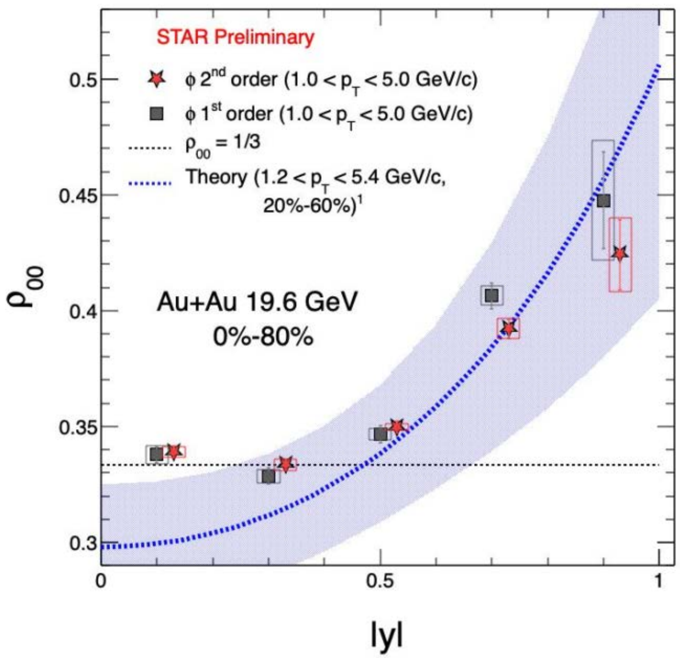
Transverse momentum spectra of ρ_{00}^{y}

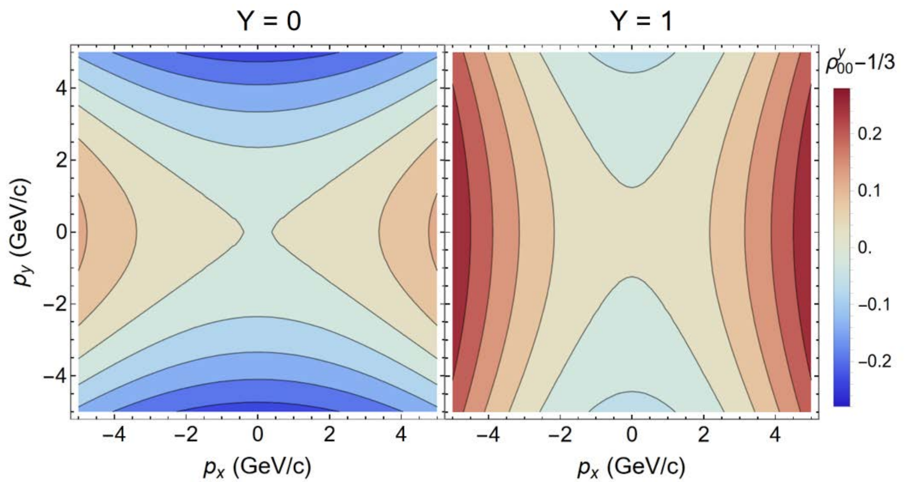


Calculated ρ_{00}^{y} (solid line) of ϕ mesons as functions of transverse momenta in 0-80% Au+Au collisions at different colliding energies in comparison with STAR data. Shaded error bands are from the extracted parameters F_T^2 and F_z^2 .

Details: Xin-Li Sheng's talk

STAR's new measurements and our prediction on rapidity dependence of ρ_{00}^{y}





Details: **Xin-Li Sheng's talk**

If B^2 and E^2 is isotropic in all directions in lab frame, we have simple formula with clear physics

$$\langle \delta\rho_{00}^{y}\rangle(\mathbf{p}) = \frac{8}{3m_\phi^4}(C_1+C_2)F^2\left(\frac{p_x^2+p_z^2}{2}-p_y^2\right)$$
$$\propto \frac{1}{2}p_T^2\left[3\cos(2\varphi)-1\right]+\left(m_\phi^2+p_T^2\right)\sinh^2 Y$$

Sheng, Pu, QW, PRC(2023); 2308.14038

Vector fields in Chiral quark model

Nuclear Physics B234 (1984) 189–212

Citations: 2272 (till September 22, 2023)

Fernandez, Valcarce, Straub, Faessler (1993)
Zhang, et al, (1997); Li, Ye, Lu (1997); Zhao, Li, Bennhold (1998)

CHIRAL QUARKS AND THE NON-RELATIVISTIC QUARK MODEL*

Aneesh MANOHAR and Howard GEORGI

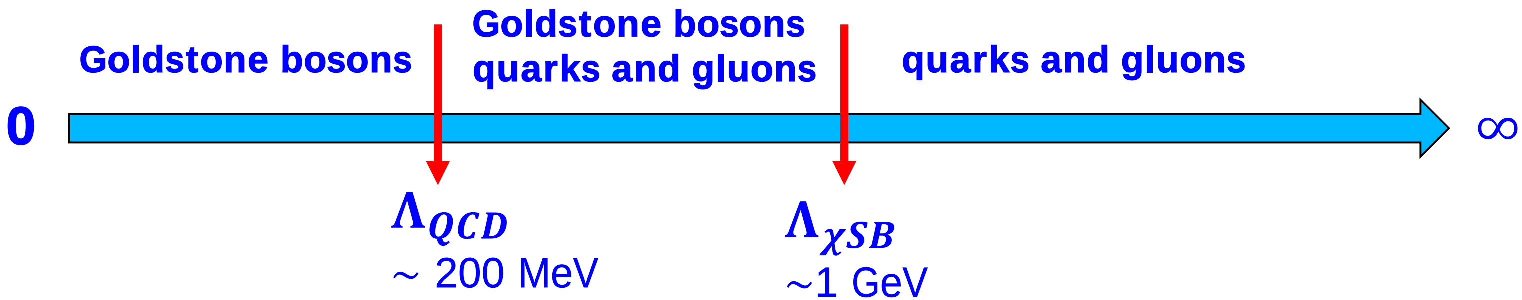
Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA

Received 18 July 1983

We study some of the consequences of an effective lagrangian for quarks, gluons and goldstone bosons in the region between the chiral symmetry breaking and confinement scales. This provides an understanding of many of the successes of the non-relativistic quark model. It also suggests a resolution to the puzzle of the hyperon non-leptonic decays.

Vector fields in Chiral quark model

- **Scale for strong interaction in dynamical process**



- **SU(3) Goldstone bosons by 3×3 matrix Σ and ξ,**

$$\Sigma = \exp\left(i\frac{2\chi}{f}\right) = \exp\left(i\frac{\chi}{f}\right)\exp\left(i\frac{\chi}{f}\right) = \xi\xi$$

$f = 93$ **MeV**

$$\chi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

Vector field in Chiral quark model

- **Σ and ξ transform under $SU_L(3)\times SU_R(3)$ as**

$$\Sigma \to L\Sigma R^\dagger, \qquad \xi \to L\xi U^\dagger = U\xi R^\dagger$$

- **A set of color and flavor triplet quarks** $\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \qquad \psi = U\psi$

- **Lagrangian is invariant under $SU_L(3)\times SU_R(3)$ transformation**

$$\begin{aligned}\mathcal{L} &= \overline{\psi}\left[i\gamma_\mu\left(\partial^\mu + igG^\mu\right) + g_V\gamma_\mu V^\mu\right]\psi + g_A\overline{\psi}\gamma_5\gamma_\mu A^\mu\psi \\ &+\frac{1}{4}f^2\mathrm{Tr}\left(\partial^\mu\Sigma^\dagger\partial_\mu\Sigma\right) - \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}\end{aligned}$$

3x3 matrix

$$\begin{aligned}V^\mu &= \frac{1}{2}\left(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger\right) \\ A^\mu &= \frac{1}{2}i\left(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger\right)\end{aligned}$$

Effective vector fields induced by currents Goldstone boson fields

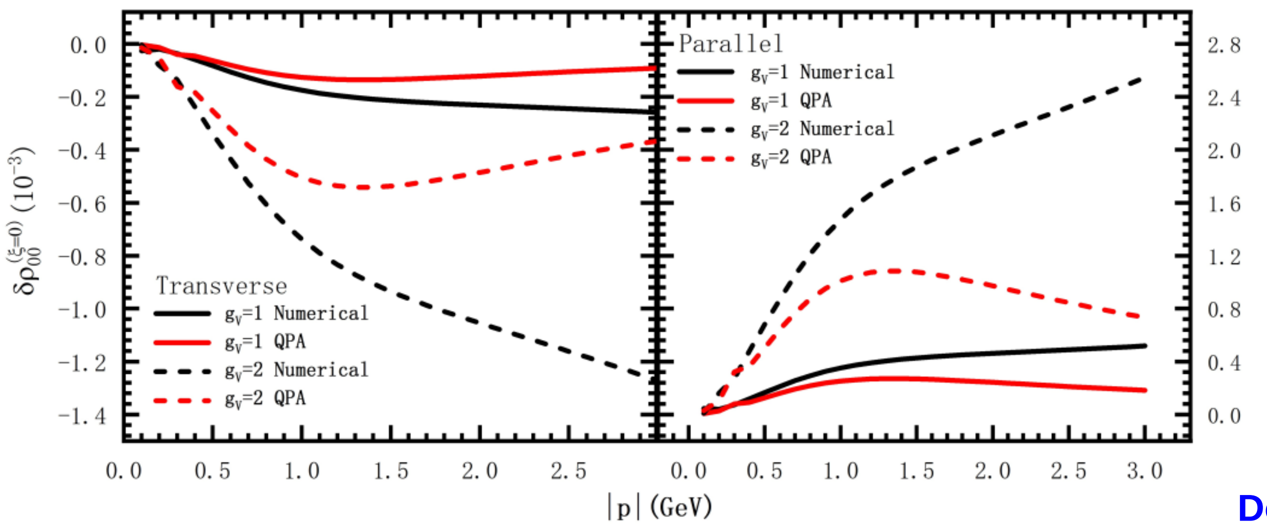
Spin alignment from self-energy and shear stress tensor in linear response theory

The correction to ρ_{00} from self-energy and shear stress tensor $\xi_{\mu\nu}$

$$\delta\rho_{00}(\mathbf{p}) = \delta\rho_{00}^{(\xi=0)}(\mathbf{p}) + \xi_{\mu\nu}C^{\mu\nu}(\mathbf{p})$$

$$\delta\rho_{00}^{(\xi=0)} \sim (\Delta E_T - \Delta E_L)$$

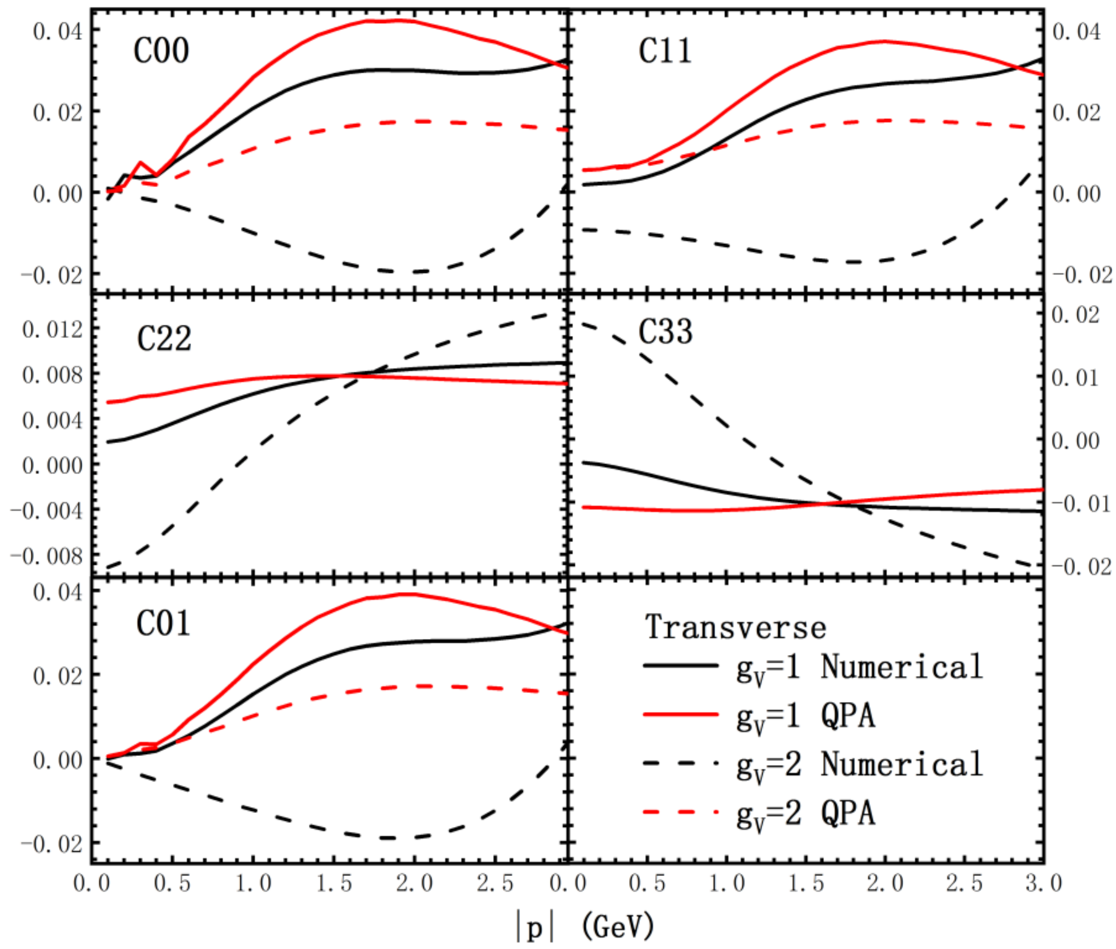
$$\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu)$$



The numerical results for $\delta\rho_{00}^{(\xi=0)}$ for the transverse (left) and parallel (right) configurations in which the momentum is transverse and parallel to the spin quantization direction z respectively. The results under the quasi-particle approximation (QPA) are shown for comparison.

Dong, Yin, Sheng, et al., 2311.18400, to appear in PRD

Spin alignment from self-energy and shear stress tensor in linear response theory



The numerical results for $C^{\mu\nu}$ for the transverse configuration in which the momentum is perpendicular to the spin quantization direction z. The results under the quasi-particle approximation (QPA) are shown for comparison.

Dong, Yin, Sheng, et al., 2311.18400, to appear in PRD

Take-home messages

- P_Λ **measures the fields (net mean field),** ρ_{00}^{ϕ} **measures field squared (field correlation or fluctuation).**
- **The vector strong force field is induced by the current of pseudo-Goldstone boson during the hadronization**

Questions for answers in the future

Global polarization:

- **We really need a comprehensive simulation solving the spin Boltzmann equation or spin hydro which includes non-equilibrium effects**

Spin alignment of vector mesons:

- **Any connection with QCD sum rules and QCD vacuum properties? Any connection with quark or gluon condensates (trace anomaly)?**
- **Implication for J/Psi polarization (gluon fields)?**
- **Any connection with effects from glasma fields? (Kuma, Mueller, Yang, 2023)**
- **Any connection with spin correlation of $\Lambda\bar{\Lambda}$? (Lv, Yu, et al., 2402.13721)**

Summary

Spin Boltzmann equation with local and non-local collisions

Spin hydrodynamics
Local and global equilibrium of spin

Particle scatterings in quantum field theory