Spin alignment of vector mesons by the color fields in the glasma

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Collaborators: **Prof. Di-Lun Yang, and Prof. Berndt Müller**

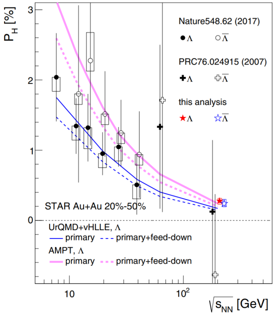
Based on Papers

AK, Di-Lun Yang, Berndt Müller, Phys. Rev. D 107, 076025 (2023); arXiv:2212.13354 [nucl-th]
AK, Di-Lun Yang, Berndt Müller, Phys. Rev. D 108, 016020 (2023); arXiv:2304.04181 [nucl-th]

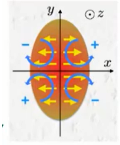
ExHIC-p Workshop on Polarization Phenomena in Nuclear Collisions, Institute of Physics Academia Sinica (ASIoP), Taipei, Taiwan, March 14-17, 2024

1. Motivation
2. QKT and spin polarization/alignments of particles due to color fields
3. Results
4. Summary and Outlook

Global and Local Spin polarization of Λ hyperons in RHIC experiment

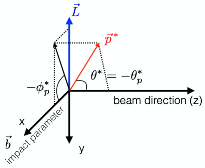


[J. Adam *et al.* (STAR), Phys. Rev. C 98, 014910(2018)]
UrQMD+vHLLE: I. Karpenko, F. Becattini, EPJC 77, 213 (2017)
AMPT: H. Li, L. Pang, Q. Wang, and X. Xia, PRC 96, 054908 (2017)



[S.Voloshin, EPJ Web Conf. 171, 07002 (2018)]

[F. Becattini, I. Karpenko, PRL 120 (2018) no.1, 012302, [1707.07984]]



$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_H \mathbf{P_H} \cdot \mathbf{p}_p^*)$$

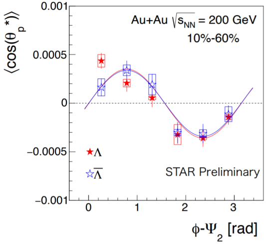
$$\langle \cos\theta_p^* \rangle = \int \frac{dN}{d\Omega^*} \cos\theta_p^* d\Omega^*$$

$$= \alpha_H P_z \langle (\cos\theta_p^*)^2 \rangle$$

$$\therefore\ P_z = \frac{\langle \cos\theta_p^* \rangle}{\alpha_H \langle (\cos\theta_p^*)^2 \rangle}$$

$$= \frac{3\langle \cos\theta_p^* \rangle}{\alpha_H} \quad \text{(if perfect detector)}$$

α_H: hyperon decay parameter
θ_p^*: θ of daughter proton in Λ rest frame



T. Niida, NPA 982 (2019) 511514 [1808.10482];

[J. Adam *et al.* (STAR), Phys. Rev. Lett. 123, 132301 (2019)]

Theoretical Status

Modified Cooper Frye formula, $\mathcal{P}^{\mu}(p) = \frac{\int d\Sigma \cdot p \mathcal{J}_5^{\mu}(p,X)}{2m \int d\Sigma_\mu \mathcal{N}^{\mu}(p,X)}$.

In the global equilibrium: $\mathcal{P}^{\mu}(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int d\Sigma_\lambda p^\lambda f_p(1-f_p)\varpi_{\rho\sigma}}{\int d\Sigma_\lambda p^\lambda n_F}$;

$$f_p = (1 + \exp[\beta \cdot p - \mu Q/T])^{-1}, \boxed{\varpi^{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)}$$

[F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338, 32 (2013), R. Fang, L. Pang, Q. Wang, X. Wang Phys. Rev. C 94, 024904 (2016)]

Describes the global polarization Λ-polarization.
Problem with explaining the local polarization $\longrightarrow$ spin sign problem

In Local equilibrium, thermal shear corrections defined in term of tensor $\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu)$ can play some role.

[F. Becattini el al., Phys. Lett. B 820 (2021) 136519, also Phys.Rev.Lett. 127, 272302 (2021)]

[Shuai Y. F. Liu, Yi Yin, JHEP 07 (2021) 188, arXiv:2103.09200 [hep-ph]]

This potentially resolves the spin sign problem.

[F. Becattini, M. Buzzegoli, A. Palermo, G. Inghirami, and I. Karpenko, Phys. Rev. Lett. 127, 272302 (2021), arXiv:2103.14621]

[B. Fu, S. Y. F. Liu, L. Pang, H. Song, Y. Yin, Phys. Rev. Lett. 127, 142301 (2021)], arXiv: 2103:10403 [hep-ph]]

[Wojciech Florkowski, AK, Aleksas Mazeliauskas, Radoslaw Ryblewski, Phys. Rev. C 105, 064901 (2022); arXiv:2112.02799 [hep-ph]]

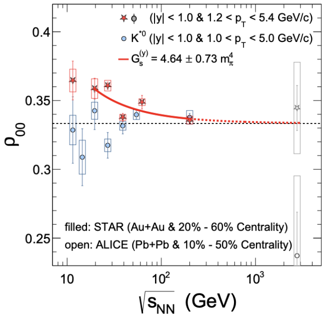
[Wojciech Florkowski, AK, Aleksas Mazeliauskas, Radoslaw Ryblewski, Phys. Rev. C 105, 064901 (2022); arXiv:2112.02799 [hep-ph]]

[Cong Yi, Shi Pu, Di-Lun Yang, Phys.Rev.C 104 (2021) 6, 064901; arXiv: 2106.00238 [hep-ph]]

Results are sensitive to EoS [S. K. Singh, J. Alam, arXiv: 2110.15604 [hep-ph]], freeze-out temperature, out-off equilibrium corrections should also be considered.

Global spin alignments of vector mesons in HICs

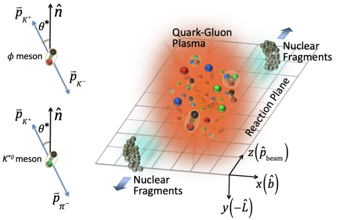
Deviation of ρ_{00} from 1/3 indicates net spin alignment



[M. S. Abdallah *et al.* (STAR), Nature 614 (2023) 7947, 244-248, arXiv:2204.02302 [hep-ph]]

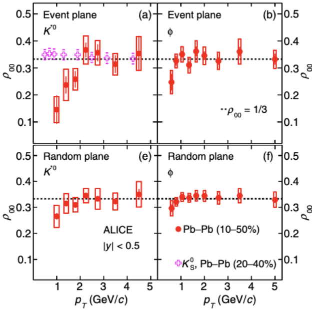
Spin alignment of vector mesons are measured by the ρ_{00}-component of spin density matrix ρ_{mn} with unit trace.

$$\boxed{\frac{\mathrm{d}N}{\mathrm{d}\cos\theta^*} \propto \left[1 - \rho_{00} + \cos^2\theta^*(3\rho_{00} - 1)\right]}$$

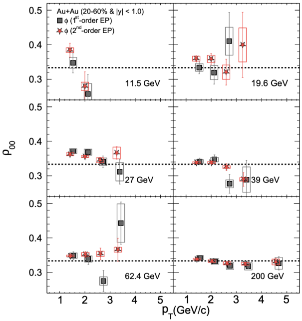


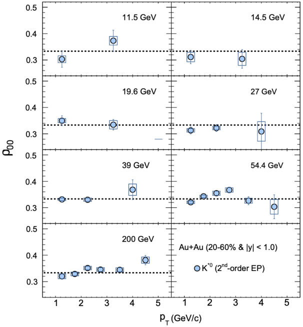
[M. S. Abdallah *et al.* (STAR), Nature 614 (2023) 7947, 244-248, arXiv:2204.02302 [hep-ph]]

LHC ($\sqrt{s_{NN}} = 2.76$ TeV) and RHIC data for ρ_{00} of K^{*0} and ϕ meson spin alignments



[S. Acharya et al. (ALICE), Phys. Rev. Lett. 125, 012301 (2020)]





[M.S. Abdallah et al. (STAR), Nature 614 (2023) 7947, 244-248,]

	ϕ	K^{*0}
ALICE	$\rho_{00} < 1/3 \quad (p_T \lesssim 1$ GeV)	$\rho_{00} < 1/3$
STAR	$\rho_{00} > 1/3$	$\rho_{00} \approx 1/3$

Theoretical Status

From the theoretical predictions of Global spin alignments are first made based on the quark coalescence model [Z.-T. Liang and X.-N. Wang, PLB 629, 20 (2005)]

$$\boxed{\rho_{00} \approx \frac{1}{3} + C_{VM} + C_{M} + C_{VE} + C_{E}}$$

$C_{VM} + C_{M} + C_{VE} + C_{E}$: are the contributions from vorticity & eletromagnetic fields in the quark coalescence scenario [X. L. Sheng et., al., Phys. Rev. D 101, 096005 (2020)]

$$C_{VM} + C_{M} + C_{VE} + C_{E}$$
$$= -\frac{1}{9}\langle \omega_y^2 \rangle + \frac{e^2}{81 m_s^2 T_{\mathrm{eff}}^2}\langle B_y^2 \rangle - \frac{\langle \mathbf{p}_\phi^2 \rangle}{27 m_s^2}\langle \epsilon_z^2 + \epsilon_x^2 \rangle + \frac{e^2 \langle \mathbf{p}_\phi^2 \rangle}{243 m_s^4 T_{\mathrm{eff}}^2}\langle E_z^2 + E_x^2 \rangle \quad \approx 10^{-5}$$

where, $\langle \mathbf{p}_\phi^2 \rangle \approx 9.18 m_\pi^2$. Thus, $\rho_{00} \approx 1/3$

Fragmentation contribution [Liang et., al., Phys. Lett. B 629, 20 (2005)]

$$\rho_{00}^{\mathrm{frag}} = \frac{1}{3} + C_F; \quad C_F = \frac{2}{9}\langle P_q^2 \rangle \approx 10^{-5}; \quad \text{Giving} \quad \rho_{00}^{\mathrm{frag}} \approx 1/3$$

Theoretical Status

Local spin alignments contribution [Xiao-Liang Xia et. al., Phys. Lett. B 817 (2021) 136325]

$$\rho_{00}^{\mathrm{L}} < 1/3$$

Like electric charges in motion can generate an EM field, quarks in motion can generate an effective $\phi-$ meson field. Such ϕ-meson field can polarize the quarks with a large magnitude due to strong interaction.

A model with vector meson strong force field should accommodate the large positive signal of $\phi-$ meson ρ_{00}
X. L. Sheng et., al., Phys. Rev. D 101, 096005 (2020);
X. L. Sheng et., al., Phys. Rev. D 102, 056013 (2020);

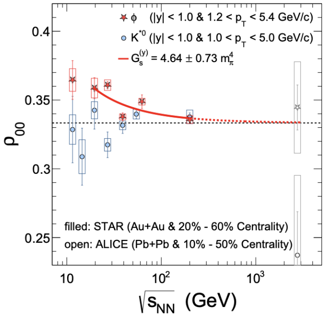
$$\boxed{\rho_{00} = \frac{1}{3} + C_{VM} + C_M + C_{VE} + C_E + C_\phi}$$

where, C_Λ is defined above and C_ϕ is the contribution from vector meson strong force field

$$C_\phi = \frac{G_s^{(y)}}{27 m_s^2 T_{\mathrm{eff}}^2}; \quad G_s^{(y)} = g_\phi^2 \left[3\langle B_{\phi,y}^2 \rangle + \frac{\langle \mathbf{p}_\phi^2 \rangle}{m_s^2} \langle E_{\phi,y}^2 \rangle - \frac{3}{2} \langle B_{\phi,z}^2 + B_{\phi,x}^2 \rangle - \frac{\langle \mathbf{p}_\phi^2 \rangle}{m_s^2} \langle E_{\phi,z}^2 + E_{\phi,x}^2 \rangle \right]$$

Theoretical Status

This works well for global spin alignment data for ϕ

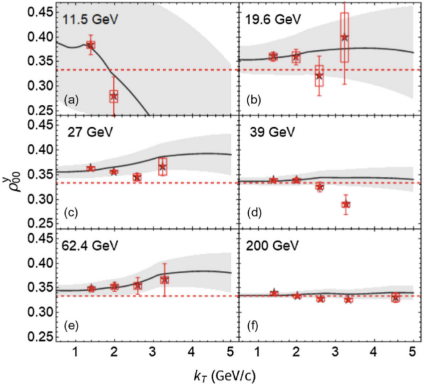


[M. S. Abdallah *et al.* (STAR), Nature 614 (2023) 7947, 244-248, arXiv:2204.02302 [hep-ph]]

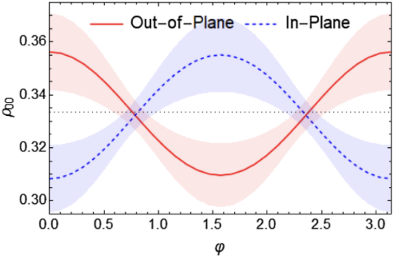
What about local spin alignment data??

Theoretical Status

- A better formula for the spin density matrix element ρ_{00}, that include the local fluctuations of the ϕ-meson fields during hadronization, was derived in the framework of relativistic quantum transport theory with an effective quark-meson model for strong interaction in quark coalescence scenario.

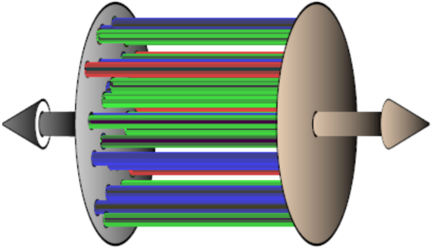


[X. L. Sheng et., al., Phys. Rev. Lett. 131, 042304 (2023)]

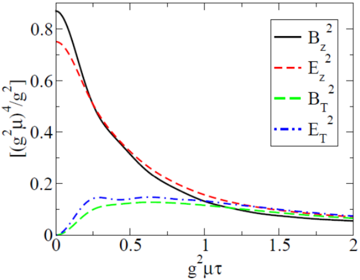


[X. L. Sheng et., al., Phys. Rev. Lett. 131, 042304 (2023)]

Other possible source of spin alignment??



[F. Gelis, E. Iancu, J. Jalilian-Marian, R. Venugopalan, Ann.Rev.Nucl.Part.Sci.60:463-489,2010]



[T. Lappi, Phys.Lett.B 643 (2006) 11-16]

Color flux tubes in the glasma phase : longitudinal chromo-EM fields in early times.

Instabilities can further enhance the fields.

Possibilty: spins of quarks gets dynamically polarized from the color fields in the glasma phase while traversing QGP.

Quantum Kinetic theory

QKT is based on [H.T. Elze, M. Gyulassy and D. Vasak, Transport Equations for the QCD Quark Wigner Operator, Nucl. Phys. B 276 (1986) 706]

$$\grave{S}^{<}(p,X)=\int\frac{d^4Y}{(2\pi)^4}e^{-\frac{ip\cdot Y}{\hbar}}\bar{\psi}\left(X+\frac{Y}{2}\right)U\left(X+\frac{Y}{2},X\right)\otimes U\left(X,X-\frac{Y}{2}\right)\psi\left(X-\frac{Y}{2}\right)$$

The dynamical evolution of $\grave{S}^{<}(p,X)$ is govern by the Kadanoff-Baym equation [H. T. Elze, M. Gyulassy, and D. Vasak, Phys. Lett., B177, 402–408, 1986]

$$\boxed{\left[\gamma^{\mu}\left(\hat{\Pi}_{\mu}+i\frac{\hbar}{2}\hat{\nabla}_{\mu}\right)-m\right]\grave{S}^{<}(p,X)=0.}$$

$$\begin{aligned}\hat{\Pi}_{\mu}\grave{S}^{<} &= p_{\mu}\grave{S}^{<}+\frac{i\hbar}{8}[F_{\nu\mu},\partial_p^{\nu}\grave{S}^{<}]_c+O(\hbar^2)\\ \hat{\nabla}_{\mu}\grave{S}^{<} &= D_{\mu}\grave{S}^{<}+\frac{1}{2}\{F_{\nu\mu},\partial_p^{\nu}\grave{S}^{<}\}_c-\frac{i\hbar}{24}[(\partial_p\cdot DF_{\nu\mu}),\partial_p^{\nu}\grave{S}^{<}]_c+O(\hbar^2)\end{aligned}$$

$D_{\mu}O=\partial_{\mu}O+i[A_{\mu},O]_c$, $A_{\mu}=t^aA^a_{\mu}$, $F_{\nu\mu}=t^aF^a_{\nu\mu}$, and $t^a=\frac{\lambda^a}{2}$ with λ^a being the Gell-Mann matrices. Gauge coupling g has been absorbed in gauge field A^a_{μ}. Moreover, $\{,\}_c$ and $[,]_c$ denote the anti-commutation and commutation relations in color space.

Quantum Kinetic Theory

- Clifford algebra decomposition[H. T. Elze, M. Gyulassy, and D. Vasak, Phys. Lett., B177, 402–408, 1986]

$$\grave{S}^{<}(p,X) = \left(\mathcal{F}(p,X) + i\gamma^5\mathcal{P}(p,X) + \gamma^\mu\mathcal{V}_\mu(p,X) + \gamma^5\gamma^\mu\mathcal{A}_\mu(p,X) + \frac{1}{2}\Sigma^{\mu\nu}\mathcal{S}_{\mu\nu}(p,X)\right)$$

- 10 different equations can be derived for the different coefficient functions.
- Adopting the power counting scheme: $\mathcal{V}^\mu \approx O(\hbar^0)$, $\mathcal{A}^\mu \approx O(\hbar)$ and perturbatively solving the equations up to order $\hbar$ we can find
- Only two relevant components: $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$
- Dynamical variable in $\mathcal{V}_\mu/\mathcal{A}_\mu$: $\underbrace{f_V(p,X)}_{\text{Vector charge distribution}}$ and $\underbrace{\tilde{a}^\mu(p,X)}_{\text{Spin four vector}}$

In $m \to 0$, $\tilde{a}^\mu(p,X) = p^\mu f_A(p,X)$

$$\boxed{\mathcal{V}^\mu = 2\pi\delta\left(p^2 - m^2\right)p^\mu f_V}$$

$$\boxed{\mathcal{A}^\mu = 2\pi\left[\delta(p^2 - m^2)\tilde{a}^\mu + \frac{\hbar}{2}p_\nu\delta'(p^2 - m^2)\{\tilde{F}^{\mu\nu}, f_V\}_c\right]}$$

- Finally one can derive dynamical equations for $f_V(p,X)$ and $\tilde{a}^\mu(p,X)$ (SKE and AKE)

Quantum Kinetic Theory

- Both the SKE and AKE are matrices in color space.
- Color decomposition: $O = O^s I + O^a t^a$,
- Color decomposition of SKE and AKE leads to coupled dynamical equations for the color singlet and color octet components of vector and axial charge distributions: f_V^s, f_V^a, $\tilde{a}^{s\mu}$, $\tilde{a}^{a\mu}$

$$p^\rho\left(\partial_\rho f_V^{\mathrm{s}} + \bar{C}_2 g F^a_{\nu\rho}\partial_p^\nu f_V^a\right) = 0,$$
$$p^\rho\left(\partial_\rho f_V^a + g F^a_{\nu\rho}\partial_p^\nu f^{\mathrm{s}}\right) = 0,$$

$$p^\rho\left(\partial_\rho \tilde{a}^{\mathrm{s}\mu} + \bar{C}_2 g F^a_{\nu\rho}\partial_p^\nu \tilde{a}^{a\mu}\right) - \frac{\hbar\bar{C}_2}{2}\epsilon^{\mu\nu\rho\sigma}p_\rho(\partial_\sigma g F^a_{\beta\nu})\partial_p^\beta f_V^a = 0,$$
$$p^\rho\left(\partial_\rho \tilde{a}^{a\mu} + g F^a_{\nu\rho}\partial_p^\nu \tilde{a}^{\mathrm{s}\mu}\right) - \frac{\hbar}{2}\epsilon^{\mu\nu\rho\sigma}p_\rho(\partial_\sigma g F^a_{\beta\nu})\partial_p^\beta f_V^{\mathrm{s}} = 0.$$

- Where $\bar{C}_2 = 1/(2N_c)$ with N_c the number of colors, and we have dropped the higher order terms in g responsible for the gauge links between color fields for brevity.

Quantum Kinetic Theory

- After solving one can obtain $\tilde{a}^{s\mu}$ and $\tilde{a}^{a\mu}$ in terms of f_V^s

$$\boxed{\tilde{a}^{s\mu}(p,X) = \frac{\hbar g^2 \bar{C}_2}{2}\epsilon^{\mu\nu\rho\sigma}\int_{k,X'}^{p,X}\partial_p^{\kappa}\int_{k',X''}^{p,X'} p^{\lambda}p_{\rho}\left(\partial_{X''\sigma}\left(F^a_{\kappa\lambda}(X')F^a_{\alpha\nu}(X'')\right) + \partial_{X'\sigma}\left(F^a_{\kappa\nu}(X')F^a_{\alpha\lambda}(X'')\right)\right)\partial_p^{\alpha} f_V^{\mathrm{s}}(p,X'')}$$

$$\boxed{\tilde{a}^{a\mu}(p,X) = g\int_{k,X'}^{p,X}\left[\frac{\hbar}{2}\epsilon^{\mu\nu\rho\sigma}p_{\rho}(\partial_{\sigma}F^a_{\beta\nu}(X'))\partial_p^{\beta} f_V^{\mathrm{s}}(p,X')\right]}.$$

where, $\int_{k,X'}^{p,X} \equiv \int d^4k \int \frac{d^4X'}{(2\pi)^4}\frac{e^{ik\cdot(X'-X)}}{p\cdot k + i\epsilon}$

Note: In priciple we need to introduce the collision terms in the dynamical equations characterized by $\mathcal{C}_{\mathrm{s}}$, $\mathcal{C}_{\mathrm{o}}^{a}$, $\mathcal{C}_{\mathrm{s}}^{\mu}$, and $\mathcal{C}_{\mathrm{o}}^{a\mu}$, which depend on details of scattering processes.

For AKEs, we may now postulate the relaxation-time forms,

$$\mathcal{C}_{\mathrm{s}}^{\mu} \approx -\frac{p_0(\tilde{a}^{\mathrm{s}\mu} - \tilde{a}^{\mathrm{s}\mu}_{\mathrm{eq}})}{\tau_{\mathrm{R}}^{\mathrm{s}}}, \quad \mathcal{C}_{\mathrm{o}}^{a\mu} \approx -\frac{p_0\tilde{a}^{a\mu}}{\tau_{\mathrm{R}}^{\mathrm{o}}},$$

In this case the solution for $\tilde{a}^{a\mu}(p,X)$ modifies by $\int_{k,X'}^{p,X} \equiv \int d^4k \int \frac{d^4X'}{(2\pi)^4}\frac{e^{ik\cdot(X'-X)}}{p\cdot k + ip0\tau_{\mathrm{R}}^{-1} + i\epsilon}$

Spin polarization of quarks

The axial current for quarks

$$J_5^\mu(X) \equiv \int \frac{d^4p}{(2\pi)^4} \mathcal{J}_5^\mu(p,X) = 4\int \frac{d^4p}{(2\pi)^4} \mathcal{A}^\mu(p,X)$$

$$\boxed{\mathcal{P}_q^i(\mathbf{p}) = \frac{\int d\Sigma \cdot p \mathrm{Tr_c}(\mathcal{J}_5^i(p,X))}{2m\int d\Sigma_\nu \mathrm{Tr_c}(\mathcal{N}^\nu(p,X)}) = \frac{\int d\Sigma \cdot p\mathcal{J}_5^{si}(\boldsymbol{p},X)}{2m\int d\Sigma_\mu \mathcal{N}^{s\mu}(\boldsymbol{p},X)}}$$

$\mathcal{N}^{s\mu}(\boldsymbol{p},X) = N_c\left(p^\mu f_V^s\right)_{p_0=\epsilon_{\boldsymbol{p}}}$

$\mathcal{J}_5^{s\mu}(\boldsymbol{p},X) = N_c\mathcal{A}^{s\mu}(p,X) = N_c\left(\boxed{\tilde{a}^{s\mu}} + \hbar\bar{C}_2\boxed{\mathcal{A}_Q^\mu}\right)_{p_0=\epsilon_{\boldsymbol{p}}}$, $\epsilon_{\boldsymbol{p}} = \sqrt{|\boldsymbol{p}|^2+m^2}$ is the onshell energy.

Non-dynamical source term: $\mathcal{A}_Q^\mu|_{p_0=\epsilon_{\boldsymbol{p}}} = \mathcal{A}_{Q1}^\mu + \mathcal{A}_{Q2}^\mu$

$$\mathcal{A}_{Q1}^\mu = \left[\frac{\partial_{p\kappa}}{2}\int_{k,X'}^{p,X} p^\beta\left(\tilde{F}^{a\mu\kappa}(X)F^a_{\alpha\beta}(X')\right)\partial_p^\alpha f_V^{\mathrm{s}}(p,X')\right]_{p_0=\epsilon_{\boldsymbol{p}}}$$

$$\mathcal{A}_{Q2}^\mu = \frac{1}{2\epsilon_{\boldsymbol{p}}^2}(p_{\perp\kappa} - \epsilon_{\boldsymbol{p}}^2\partial_{p_{\perp}\kappa})\left[\int_{k,X'}^{p,X} p^\beta\left(\tilde{F}^{a\mu\kappa}(X)F^a_{\alpha\beta}(X')\right)\partial_p^\alpha f_V^{\mathrm{s}}(p,X')\right]_{p_0=\epsilon_{\boldsymbol{p}}},$$

$V_{\perp\mu}$ represents the spatial component of an arbitrary four vector V_μ.

Spin polarization of quarks

- Non-dynamical source term depends on the color-field correlator only at the late time when spin freezes out. Dynamical contribution comes by integrating the color-field correlator over a whole period before the spin freeze-out.
- Non-dynamical source term can be dropped as compared to dynamical source term.
- Note that $\tilde{a}^{s\mu}(p, X)$ is an operator here and the ensemble average need to be taken in the end. Thus the spin polarization of quarks can be evaluated via

  $\langle \mathcal{P}^i_q \rangle \sim \langle \tilde{a}^{si}(p, X) \rangle$
- This implies that one has to evaluate the correlations of QCD field strength tensors

For practical applications, one has to convert the field strengths into chromo-electric and -magnetic fields, which are explicitly given by

$$F^a_{\mu\nu} = -\epsilon_{\mu\nu\alpha\beta} B^{a\alpha} \bar{n}^\beta + E^a_{[\mu} \bar{n}_{\nu]}, \quad \tilde{F}^{a\mu\nu} = B^{a[\mu} \bar{n}^{\nu]} + \epsilon^{\mu\nu\alpha\beta} E^a_\alpha \bar{n}_\beta,$$

where $\bar{n}^\mu = (1, \mathbf{0})$ denotes the temporal direction.

- Finally one has to evaluate the correlations of color-electromagnetic fields.

Color-field correlations in the glasma

• In the CGC picture, the dynamics of the soft modes (small-x) of gluons with large occupation numbers follows the classical Yang-Mills

$$[D_\nu, F^{\mu\nu}] = J^\nu$$

• Color indices are omitted. D_ν is the covariant derivative augmented by the non- Abelian field and $F^{\mu\nu}$ is the gluonic field strength tensor. J^ν is the source coming from valence quarks. .

In the linearized (abelianized) approximation the solution of Classical Yang-Mills equations with the CGC picture yields, [P. Guerrero-Rodriguez and T. Lappi, Phys. Rev. D 104, 014011 (2021)].

$$E^{c\eta}(\tau, x_\perp) = gf^{abc}\delta^{ij}\int \frac{d^2q_\perp}{(2\pi)^2}\int d^2u_\perp\, \alpha_1^{i,a}(u_\perp)\alpha_2^{j,b}(u_\perp) \times J_0(q\tau)e^{iq_\perp(x-u)_\perp},$$

$$B^{c\eta}(\tau, x_\perp) = gf^{abc}\epsilon^{ij}\int \frac{d^2q_\perp}{(2\pi)^2}\int d^2u_\perp\, \alpha_1^{i,a}(u_\perp)\alpha_2^{j,b}(u_\perp) \times J_0(q\tau)e^{iq_\perp(x-u)_\perp},$$

$$E_T^{ci}(\tau, x_\perp) = -igf^{abc}\epsilon^{ij}\epsilon^{kl}\int \frac{d^2q_\perp}{(2\pi)^2}\frac{q^j}{q}\int d^2u_\perp\, \alpha_1^{k,a}(u_\perp)\alpha_2^{l,b}(u_\perp) \times J_1(q\tau)e^{iq_\perp(x-u)_\perp},$$

$$B_T^{ci}(\tau, x_\perp) = -igf^{abc}\epsilon^{ij}\delta^{kl}\int \frac{d^2q_\perp}{(2\pi)^2}\frac{q^j}{q}\int d^2u_\perp\, \alpha_1^{k,a}(u_\perp)\alpha_2^{l,b}(u_\perp) \times J_1(q\tau)e^{iq_\perp(x-u)_\perp}.$$

Considering

$$\langle \alpha_1^{i,a}(u_\perp)\alpha_1^{j,b}(v_\perp)\rangle = \frac{\delta^{ab}}{2}\left(\delta^{ij}G(u_\perp, v_\perp) + \left(\delta^{ij} - \frac{2r^ir^j}{r^2}\right)h(u_\perp, v_\perp)\right)$$

where, $r = |r_\perp| = |u_\perp - v_\perp|$.

Color-field correlations in the glasma

We shall focus on the central rapidity regime, $\eta \to 0$. In this case the field correlations can be obtained following Ref.[P. Guerrero-Rodriguez and T. Lappi, Phys. Rev. D 104, 014011 (2021)]

$$\begin{aligned}\langle E_T^{ai}(X)B_T^{aj}(X')\rangle &= 0, \quad \langle E^{az}(X)B^{az}(X')\rangle = 0,\\ \langle E_T^{ai}(X)E^{az}(X')\rangle &= 0, \quad \langle B_T^{ai}(X)B^{az}(X')\rangle = 0\end{aligned}$$

The non-vanishing correlators

$$\langle E_T^{ai}(X')E_T^{aj}(X'')\rangle = -\bar{N}_c\epsilon^{in}\epsilon^{jm}\int_{\perp;q,u}^{X'}\int_{\perp;l,v}^{X''}\Omega_-(u_\perp,v_\perp)\frac{q^n l^m}{ql}J_1(qX_0')J_1(lX_0'')\theta(X_0')\theta(X_0''),$$

$$\langle B_T^{ai}(X')B_T^{aj}(X'')\rangle = -\bar{N}_c\epsilon^{in}\epsilon^{jm}\int_{\perp;q,u}^{X'}\int_{\perp;l,v}^{X''}\Omega_+(u_\perp,v_\perp)\frac{q^n l^m}{ql}J_1(qX_0')J_1(lX_0'')\theta(X_0')\theta(X_0''),$$

$$\langle E_T^{ai}(X')B^{az}(X'')\rangle = -i\bar{N}_c\epsilon^{in}\int_{\perp;q,u}^{X'}\int_{\perp;l,v}^{X''}\Omega_-(u_\perp,v_\perp)\frac{q^n}{q}J_1(qX_0')J_0(lX_0'')\theta(X_0')\theta(X_0''),$$

$$\langle B_T^{ai}(X')E^{az}(X'')\rangle = -i\bar{N}_c\epsilon^{in}\int_{\perp;q,u}^{X'}\int_{\perp;l,v}^{X''}\Omega_+(u_\perp,v_\perp)\frac{q^n}{q}J_1(qX_0')J_0(lX_0'')\theta(X_0')\theta(X_0''),$$

$$\langle E^{az}(X')E^{az}(X'')\rangle = \bar{N}_c\int_{\perp;q,u}^{X'}\int_{\perp;l,v}^{X''}\Omega_+(u_\perp,v_\perp)J_0(qX_0')J_0(lX_0'')\theta(X_0')\theta(X_0''),$$

$$\langle B^{az}(X')B^{az}(X'')\rangle = \bar{N}_c\int_{\perp;q,u}^{X'}\int_{\perp;l,v}^{X''}\Omega_-(u_\perp,v_\perp)J_0(qX_0')J_0(lX_0'')\theta(X_0')\theta(X_0'').$$

- Where, $\bar{N}_c \equiv \frac{1}{2}g^2N_c(N_c^2-1)$, $\Omega_\mp(u_\perp,v_\perp) = [G_1(u_\perp,v_\perp)G_2(u_\perp,v_\perp) \mp h_1(u_\perp,v_\perp)h_2(u_\perp,v_\perp)]$ $\int_{\perp;q,u}^{X'} \equiv \int\frac{d^2q_\perp}{(2\pi)^2}\int d^2u_\perp\, e^{iq_\perp(X'-u)_\perp}$

• In the small-momentum limit $\hat{p}^\mu_\perp \equiv p^\mu_\perp/p_0 \ll 1$ for $p_0 = \epsilon_{\boldsymbol{p}} \equiv \sqrt{\boldsymbol{p}^2 + m^2}$ being onshell. It turn out

$$\langle \tilde{a}^{sy}(p,X)\rangle \approx -\left(\frac{g^2\bar{C}_2}{2} p_0\left(\partial_{p0} f_V(p_0)\right)\right)\int_{k,X'}^{p,X}\int_{k',X''}^{p,X'}\Big[\partial_{X''0}\left\langle E^a_{[1}(X')E^a_{3]}(X'')\right\rangle$$
$$+(X''_0 - X'_0)\left(\partial^2_{X''1}\left\langle E^a_1(X')E^a_3(X'')\right\rangle + \partial_{X''2}\partial_{X''1}\left\langle E^a_2(X')E^a_3(X'')\right\rangle\right)\Big]$$

From the above defined field correlator, concludes $\langle \tilde{a}^{sy}(p,X)\rangle = 0$. By symmetry argument it can also shown $\langle \tilde{a}^{sx}(p,X)\rangle = 0$

• Not clear for the case of $\langle \tilde{a}^{sz}(p,X)\rangle$.

$$\boxed{\langle \tilde{a}^{sz}(p,X)\rangle \approx -\left(\frac{g^2\bar{C}_2}{2} p_0\left(\partial_{p0} f_V(p_0)\right)\right)\left(\mathcal{A}_a + \mathcal{A}_b + \mathcal{A}_c\right)}$$

$$\mathcal{A}_a = \int_{k,X'}^{p,X}\int_{k',X''}^{p,X'} \partial_{X''0}\langle E^a_{[2}(X')E^a_{1]}(X'')\rangle$$

$$\mathcal{A}_b = -\int_{k,X'}^{p,X}\int_{k',X''}^{p,X'} \left(\partial^1_{X'}\langle B^a_{[3}(X')E^a_{1]}(X'')\rangle + \partial^2_{X'}\langle B^a_{[3}(X')E^a_{2]}(X'')\rangle\right)$$

$$\mathcal{A}_c = \int_{k,X'}^{p,X}\int_{k',X''}^{p,X'}(X''_0 - X'_0)\left(\partial^1_{X''}\partial_{X''[1}\langle E^a_1(X')E^a_{2]}(X'')\rangle + \partial^2_{X''}\partial_{X''[1}\langle E^a_2(X')E^a_{2]}(X'')\rangle\right)$$

Analyis with GBW kind of distribution

We adopt the GBW distribution where $h_{1,2} = 0$ [P. Guerrero-Rodriguez and T. Lappi, Phys. Rev. D 104, 014011 (2021)]

$$\Omega_{\pm}(u_\perp, v_\perp) = \Omega(u_\perp, v_\perp) = \frac{Q_s^4}{g^4 N_c^2}\left(\frac{1-e^{-Q_s^2|u_\perp - v_\perp|^2/4}}{Q_s^2|u_\perp - v_\perp|^2/4}\right)^2 = \frac{Q_s^4}{g^4 N_c^2}\left(\frac{1-e^{-Q_s^2|s_\perp - t_\perp - r_\perp|^2/4}}{Q_s^2|s_\perp - t_\perp - r_\perp|^2/4}\right)^2.$$

• $\mathrm{r}_\perp \equiv X_\perp - Y_\perp, \quad s_\perp = X_\perp - u_\perp, \quad t_\perp = Y_\perp - v_\perp.$

• Using the above kind of gluon distribution in the color field correlation one can carry out the multidimensional integration's involved in the expressions.

•It can be shown $\langle \tilde{a}^{sz}(p, X)\rangle = 0$

Spin alignment of vector mesons

Spin alignment of vector mesons is determined by [Z.-T. Liang and X.-N. Wang, Phys. Lett. B 629, 20 (2005), Y.-G. Yang, R.-H. Fang, Q. Wang, and X.-N. Wang, Phys. Rev. C 97, 034917 (2018)]

$$\rho_{00} = \frac{1 - \langle \mathcal{P}_q^i \mathcal{P}_{\bar{q}}^i \rangle}{3 + \langle \mathcal{P}_q^i \mathcal{P}_{\bar{q}}^i \rangle}$$

Here, $\langle \mathcal{P}_q^i \mathcal{P}_{\bar{q}}^i \rangle \neq \langle \mathcal{P}_q^i \rangle \langle \mathcal{P}_{\bar{q}}^i \rangle$.
The above formula was derived based on the assumption that the spin of a quark and of an antiquark is fully polarized along the quantization axis. In a more generic case, as shown in Ref.[X.-L. Sheng, L. Oliva, Z.-T. Liang, Q. Wang, and X.-N. Wang (2022), 2206.05868] the above formula gets modified

$$\rho_{00} = \frac{1 + \sum_{j=x,y,z} \langle \mathcal{P}_q^j \mathcal{P}_{\bar{q}}^j \rangle - 2 \langle \mathcal{P}_q^i \mathcal{P}_{\bar{q}}^i \rangle}{3 + \sum_{j=x,y,z} \langle \mathcal{P}_q^j \mathcal{P}_{\bar{q}}^j \rangle}$$

or

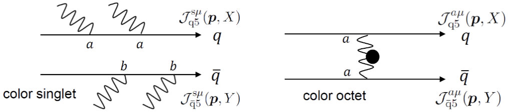
$$\rho_{00} \approx \frac{1}{3} + \frac{2}{9} \left(\langle \mathcal{P}_q^x \mathcal{P}_{\bar{q}}^x \rangle + \langle \mathcal{P}_q^z \mathcal{P}_{\bar{q}}^z \rangle - 2 \langle \mathcal{P}_q^y \mathcal{P}_{\bar{q}}^y \rangle \right).$$

Spin alignment of vector mesons

Based on a phenomenological construction in the quark model ρ_{00} one can be obtained by calculating [Avdhesh Kumar, Berndt Mueller, and Di-Lun Yang, Phys. Rev. D 107, 076025 (2023); arXiv:2212.13354v3 [nucl-th].

$$\langle \mathcal{P}^i_q(\boldsymbol{p})\mathcal{P}^i_{\bar{q}}(\boldsymbol{p})\rangle = \frac{\int d\Sigma_X \cdot p \int d\Sigma_Y \cdot p \langle \mathcal{J}^{si}_{q5}(\boldsymbol{p},X)\mathcal{J}^{si}_{\bar{q}5}(\boldsymbol{p},Y)\rangle}{4m^2\left(\int d\Sigma_X \cdot \mathcal{N}^s(\boldsymbol{p},X)\right)^2} = \frac{\int d\Sigma_X \cdot p \int d\Sigma_Y \cdot p \left(\langle \tilde{a}^{si}_q(\boldsymbol{p},X)\tilde{a}^{si}_{\bar{q}}(\boldsymbol{p},Y)\rangle\right)}{4m^2(\int d\Sigma_X \cdot p\, f^{\rm s}_{{\rm V}q}(\boldsymbol{p},X))^2},$$

- Here only the color-singlet components of $\mathcal{J}^i_{q5}$ and $\mathcal{J}^i_{\bar{q}5}$ contribute to both the spin polarization and correlation.
- By the symmetry of color-charge conjugation, we should have $\mathcal{J}^{si}_{\bar{q}5}(\boldsymbol{p},X) = \mathcal{J}^{si}_{q5}(\boldsymbol{p},X)$ and $\langle \mathcal{P}^i_q(\boldsymbol{p})\mathcal{P}^i_{\bar{q}}(\boldsymbol{p})\rangle$ is expected to be positive when having equal number of quarks and antiquarks.
- Our initial calculations were based on this phenomenological approximation.



[AK, Di-Lun Yang, Berndt Müller, Phys. Rev. D 108, 016020 (2023)]

Spin alignment of vector mesons

A new derivation of spin density matrix by Prof. Di-Lun Yang from the vector-meson kinetic equation in the quark-coalescence scenario gives
[AK, Di-Lun Yang, Berndt Müller, Phys.Rev.D 108, 016020 (2023); arXiv:2304.04181v1[nucl-th]].

$$\rho_{\lambda_1\lambda_1}(q) = \frac{\int d\Sigma_X \cdot q\ f^{\phi}_{\lambda_1}(q,X)}{\int d\ \Sigma_X \cdot q\ \sum_{\lambda=0,\pm 1}\left(f_\lambda(q,X)\right)} = \frac{\int d\Sigma_X \cdot q\ \epsilon^*_\mu(\lambda_1,q)\epsilon_\nu(\lambda_1,q)\mathcal{C}^{\mu\nu}_{\rm Coal}(q,X)}{\int d\ \Sigma_X \cdot q\ \sum_{\lambda=0,\pm 1}\epsilon^*_\mu(\lambda,q)\epsilon_\nu(\lambda,q)\mathcal{C}^{\mu\nu}_{\rm Coal}(q,X)},$$

where $\mathcal{C}^{\mu\nu}_{\rm Coal}(q,X)$ refers to the meson-quark interaction characterized by an effective Lagrangian. $\mathcal{L}_{\rm int} = g_\phi \Gamma \cdot V\bar{\psi}\psi$

It turns out that in meson rest frame and non-relativistic limit

$$\rho_{00}(q) = \frac{\int d\Sigma_X \cdot q f_0(q,X)}{\int d\Sigma_X \cdot q\left(f_0(q,X)+f_{+1}(q,X)+f_{-1}(q,X)\right)} = \frac{1-\mathrm{Tr}_c\langle\hat{\mathcal{P}}^y_q(\boldsymbol{q}/2)\hat{\mathcal{P}}^y_{\bar{q}}(\boldsymbol{q}/2)\rangle_{\boldsymbol{q}\approx 0}}{3-\sum_{i=x,y,z}\mathrm{Tr}_c\langle\hat{\mathcal{P}}^i_q(\boldsymbol{q}/2)\hat{\mathcal{P}}^i_{\bar{q}}(\boldsymbol{q}/2)\rangle_{\boldsymbol{q}\approx 0}},$$

which can be approximated as

$$\rho_{00} \approx \frac{1}{3}+\frac{1}{9}\mathrm{Tr}_c\left(\langle\hat{\mathcal{P}}^x_q\hat{\mathcal{P}}^x_{\bar{q}}\rangle+\langle\hat{\mathcal{P}}^z_q\hat{\mathcal{P}}^z_{\bar{q}}\rangle-2\langle\hat{\mathcal{P}}^y_q\hat{\mathcal{P}}^y_{\bar{q}}\rangle\right),$$

$$\mathrm{Tr_c}\langle\hat{\mathcal{P}}^i_q(\boldsymbol{q}/\mathbf{2})\hat{\mathcal{P}}^i_{\bar{q}}(\boldsymbol{q}/\mathbf{2})\rangle = \frac{\int d\Sigma_X \cdot q\mathrm{Tr_c}\left[\langle\mathcal{J}^i_{q5}(\boldsymbol{q}/\mathbf{2},X)\mathcal{J}^i_{\bar{q}5}(\boldsymbol{q}/\mathbf{2},X)\rangle\right]}{\int d\Sigma_X \cdot q\mathrm{Tr_c}\left[\mathcal{N}^0_q(\boldsymbol{q}/\mathbf{2},X)\mathcal{N}^0_{\bar{q}}(\boldsymbol{q}/\mathbf{2},X)\right]},$$

which will yield

$$\boxed{\mathrm{Tr_c}\langle\hat{\mathcal{P}}^i_q(\boldsymbol{q}/2)\hat{\mathcal{P}}^i_{\bar{q}}(\boldsymbol{q}/2)\rangle \approx \frac{\int d\Sigma_X \cdot q\big(\langle\tilde{a}^{si}_q(\boldsymbol{q}/2,X)\tilde{a}^{si}_{\bar{q}}(\boldsymbol{q}/2,X)\rangle + \langle\tilde{a}^{ai}_q(\boldsymbol{q}/2,X)\tilde{a}^{ai}_{\bar{q}}(\boldsymbol{q}/2,X)\rangle/(2N_c^2)\big)}{m_q m_{\bar{q}}\int d\Sigma_X \cdot q\big(f^{\mathrm{s}}_{\mathrm{V}q}(\boldsymbol{q}/2,X)f^{\mathrm{s}}_{\mathrm{V}\bar{q}}(\boldsymbol{q}/2,X)\big)}}$$

We can decompose

$$\mathrm{Tr_c}\langle\hat{\mathcal{P}}^i_q(\boldsymbol{q}/2)\hat{\mathcal{P}}^i_{\bar{q}}(\boldsymbol{q}/2)\rangle \approx \Pi^{ii}_{\mathrm{sin}} + \Pi^{ii}_{\mathrm{oct}},$$

$$\boxed{\Pi^{ii}_{\mathrm{sin}} \approx \frac{\int d\Sigma_X \cdot q\big(\langle\tilde{a}^{si}_q(\boldsymbol{q}/2,X)\tilde{a}^{si}_{\bar{q}}(\boldsymbol{q}/2,X)\rangle\big)}{m_q m_{\bar{q}}\int d\Sigma_X \cdot q\big(f^{\mathrm{s}}_{\mathrm{V}q}(\boldsymbol{q}/2,X)f^{\mathrm{s}}_{\mathrm{V}\bar{q}}(\boldsymbol{q}/2,X)\big)}},$$

$$\boxed{\Pi^{ii}_{\mathrm{oct}} \approx \frac{\int d\Sigma_X \cdot q\big(\langle\tilde{a}^{ai}_q(\boldsymbol{q}/2,X)\tilde{a}^{ai}_{\bar{q}}(\boldsymbol{q}/2,X)\rangle/(2N_c^2)\big)}{m_q m_{\bar{q}}\int d\Sigma_X \cdot q\big(f^{\mathrm{s}}_{\mathrm{V}q}(\boldsymbol{q}/2,X)f^{\mathrm{s}}_{\mathrm{V}\bar{q}}(\boldsymbol{q}/2,X)\big)}},$$

Results

- Color-singlet contribution led by local four-field correlations which can be calculated numerically.

- For $Q_s = 2$ GeV, $X_0^{\mathrm{th}} = 0.2$ fm, we find

$$\boxed{\Pi^{yy}_{\mathrm{sin}} = \Pi^{xx}_{\mathrm{sin}} \approx 2.4\Pi^{zz}_{\mathrm{sin}} \approx 3.5}$$

- Color octet contribution to the spin correlation can be expressed in term of two field correlators. We consider the quarks and antiquarks may emerge at the time later than the initial time with strongest color fields. In this scenario we will have

- For $Q_s = 2$ GeV, $X_0^{\mathrm{th}} = 0.2$ fm, we estimate

$$\boxed{\Pi^{zz}_{\mathrm{oct}} = -10.3}$$

Results

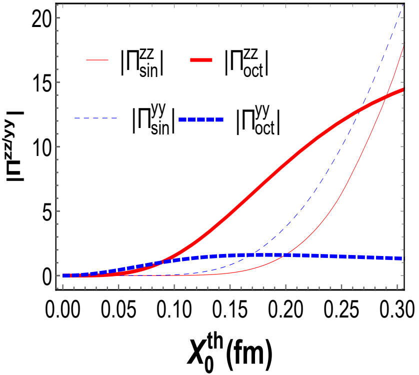


Figure: Magnitudes of spin correlations from the color-octet and color-singlet contributions for $Q_s = 2$ GeV.

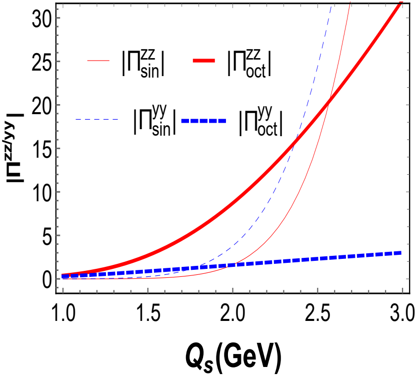


Figure: Magnitudes of spin correlations from the color-octet and color-singlet contributions for $X_0^{\rm th} = 0.2$ fm.

Results

The spin alignment of ϕ mesons from the glasma for $Q_s \approx 2$ GeV can be written in an approximate equation,

$$\rho_{00} \sim \frac{1}{3 + 10\, e^{-2X_0^{\mathrm{eq}}/\tau_{\mathrm{R}}^{\mathrm{o}}}}$$

where X_0^{eq} represents the freeze-out time at chemical equilibrium of the QGP and recall $\tau_{\mathrm{R}}^{\mathrm{o}}$ is an unknown parameter characterizing the effect of spin relaxation.

- In the heavy-quark limit $m \gg T$, we may adopt the following relaxation rate [M. Hongo et al., JHEP 08, 263 (2022)]

$$(\tau_{\mathrm{R}}^{\mathrm{o}})^{-1} \approx \frac{g^2 C_2(F) m_D^2 T}{6\pi m^2} \ln g,$$

- where $C_2(F) = (N_c^2 - 1)/(2N_c)$ and $m_D^2 = g^2 T^2 (2N_c + N_f)/6$. Taking $N_c = N_f = 3$, $\alpha_s = g^2/(4\pi) \approx 1/3$, and $T \sim 200$ MeV as the average temperature of QGP, one obtains $(\tau_{\mathrm{R}}^{\mathrm{o}})^{-1} \approx 0.04$ GeV. For $X_0^{\mathrm{eq}} \approx 0.5$ fm, it is found $e^{-2X_0^{\mathrm{eq}}/\tau_{\mathrm{R}}^{\mathrm{o}}} \approx 0.11$.

Thus, $\boxed{\rho_{00} \approx 0.24}$

Momentum-dependent analysis

To approximate the spin alignment result with finite vector meson momentum in lab frame we simply have to conduct the Lorentz boost of the color field correlator.

For the glasma fields:

$$\rho_{00} - \frac{1}{3} \approx \frac{\hbar^2 g^2 (v_x^2 - 2v_y^2 - 1) \int d\Sigma_X \cdot q \langle B^{az}(0, \boldsymbol{X}) B^{az}(0, \boldsymbol{X}) \rangle (\partial_{\epsilon_{\boldsymbol{q}/2}} \tilde{f}_V(\epsilon_{\boldsymbol{q}/2}, 0))^2}{72 N_c^2 m^2 \int d\Sigma_X \cdot q f^{\mathrm{th}}_{\mathrm{V}q}(\epsilon_{\boldsymbol{q}/2}) f^{\mathrm{th}}_{\mathrm{V}\bar{q}}(\epsilon_{\boldsymbol{q}/2})}$$

For the internal color fields characterizing an effective potential:

$$\rho_{00} - \frac{1}{3} \approx \frac{\hbar^2 g^2 (v_x^2 - 2v_y^2) \int d\Sigma_X \cdot q \langle B^{a}(0, \boldsymbol{X}) B^{a}(0, \boldsymbol{X}) \rangle (\partial_{\epsilon_{\boldsymbol{q}/2}} \tilde{f}_V(\epsilon_{\boldsymbol{q}/2}, 0))^2}{36 N_c^2 m^2 \int d\Sigma_X \cdot q f^{\mathrm{th}}_{\mathrm{V}q}(\epsilon_{\boldsymbol{q}/2}) f^{\mathrm{th}}_{\mathrm{V}\bar{q}}(\epsilon_{\boldsymbol{q}/2})}$$

	small-$\mathrm{P_T}$	large-$\mathrm{P_T}$	central	non-central
glasma	$\rho_{00}^{\phi,J/\psi} < 1/3$	$\rho_{00}^{\phi,J/\psi} \lesssim 1/3$	$\rho_{00}^{\phi,J/\psi} < 1/3$	$\rho_{00}^{\phi,J/\psi} \lesssim 1/3$
effective potential	$\lvert\rho_{00}^{\phi,J/\psi} - 1/3\rvert \gtrsim 0$	$\lvert\rho_{00}^{\phi,J/\psi} - 1/3\rvert > 0$	$\rho_{00}^{\phi,J/\psi} < 1/3$	$\rho_{00}^{\phi,J/\psi} > 1/3$

- Effective potential can play a more dominant role in low-energy collisions, where the contribution from external color fields vanishes.

Summary and Outlook

- We gave a brief overview of experimental and theoretical studies related to spin polarization and alignment of hadrons.
- Experimental results for the spin polarization and alignments suggest that non-local equilibrium effect (color fields in the glasma) may play important role.
- Using the quantum kinetic theory description for weakly coupled system of quarks and gluons with background color fields we derived the expressions for dynamical spin polarization arising from the correlation of color fields in the glasma.
- Our calculations based on color field correlations obtained by solving the Yang-Mills equations in the linearized (abelian) limit indicate that dynamical spin polarization of quarks is zero.
- We have estimated spin alignment of vector mesons arising from the color fields in the glasma using newly derived formula for the ρ_{00}-component of spin density matrix from the vector-meson kinetic equation in the quark-coalescence scenario. The new expression of ρ_{00}-component of spin density matrix, unlike the phenomenological one adopted in our previous work, involves contributions from spin correlations of both the color-singlet and color-octet components of the axial-charge current densities for quarks and antiquarks that are dynamically generated by the fluctuating color fields.

Summary and Outlook

- Our final result found to be in agreement with the experimental measurement for ρ_{00} of $\phi-$ mesons at small transverse momenta.
- Finally, we present the spin alignment results using the momentum dependent analysis.

Outlook:

- While deriving the spin alignment formula the explicit $O(\hbar^2)$ and higher order contribution in the $\hbar$ expansion of Wigner function were neglected. If we keep expansion of Wigner function up to the $O(\hbar^2)$ then second order hydrodynamic gradients will also come into play. Our estimates (at $\sqrt{s_{NN}} = 130$ GeV) using the simple thermal model indicate that such contribution are higher than the vorticity contribution.

  [Avdhesh Kumar, Di-Lun Yang, Philipp Gubler, Accepted in Phys. Rev. D; arXiv:2312.16900 [nucl-th].
- Hydrodynamic calculations are needed to estimate the contribution by second-order hydrodynamic gradients for the full scan of beam energies.

THANK YOU FOR YOUR ATTENTION