Heavy Quarkonium Production and Polarization in Small Systems

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ExHIC-p workshop on polarization phenomena in nuclear collisions
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Outline:

I. Background: Quarkonium production models

II. Hadronic quarkonium production of high p_T and low p_T

III. Forward production and nuclear effects

SEIKEI

Quarkonia as tools

p+p collisions (Small systems)

- Elementary quarkonium production mechanism

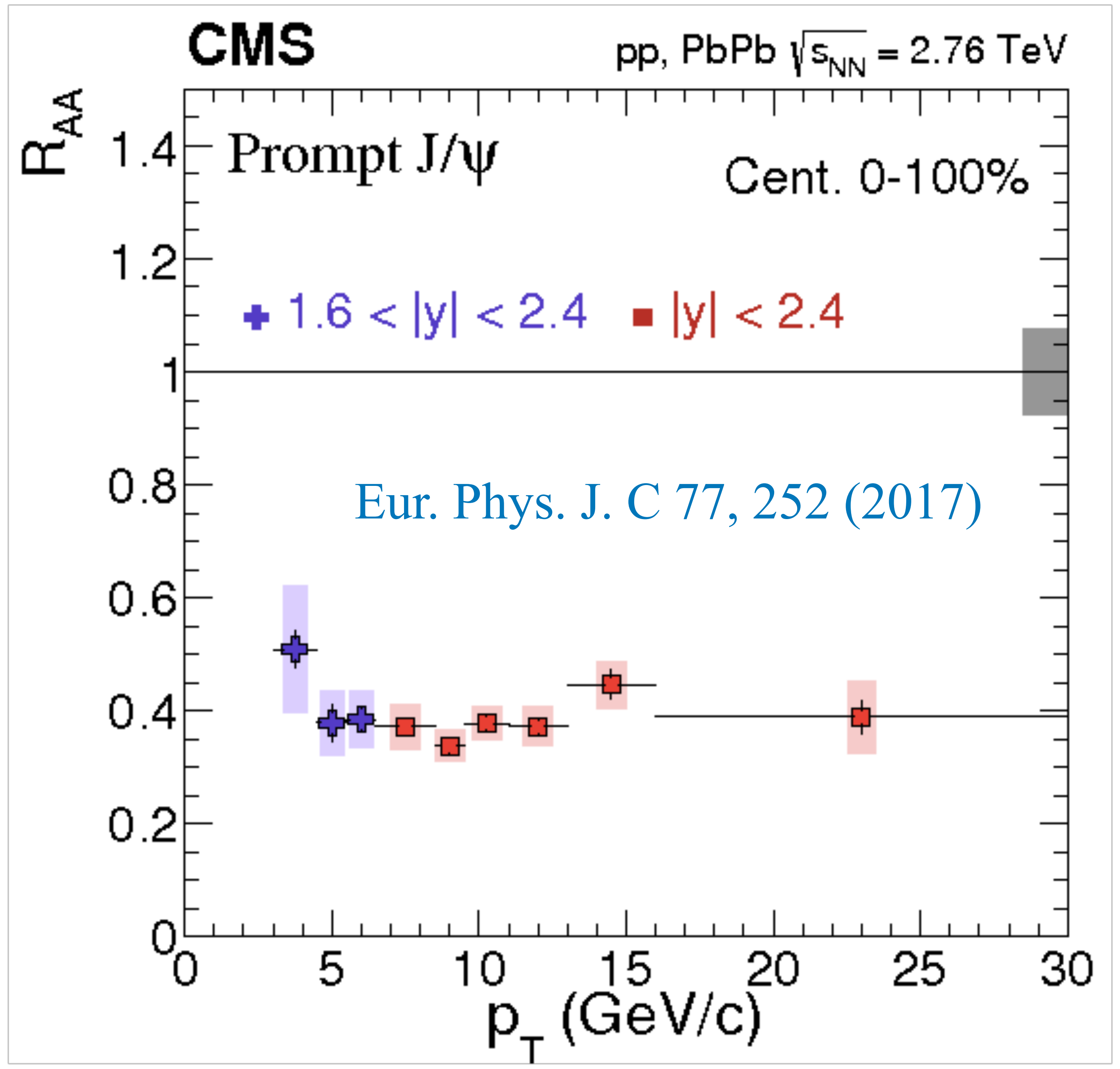
p/d/He+A collisions (Small systems)

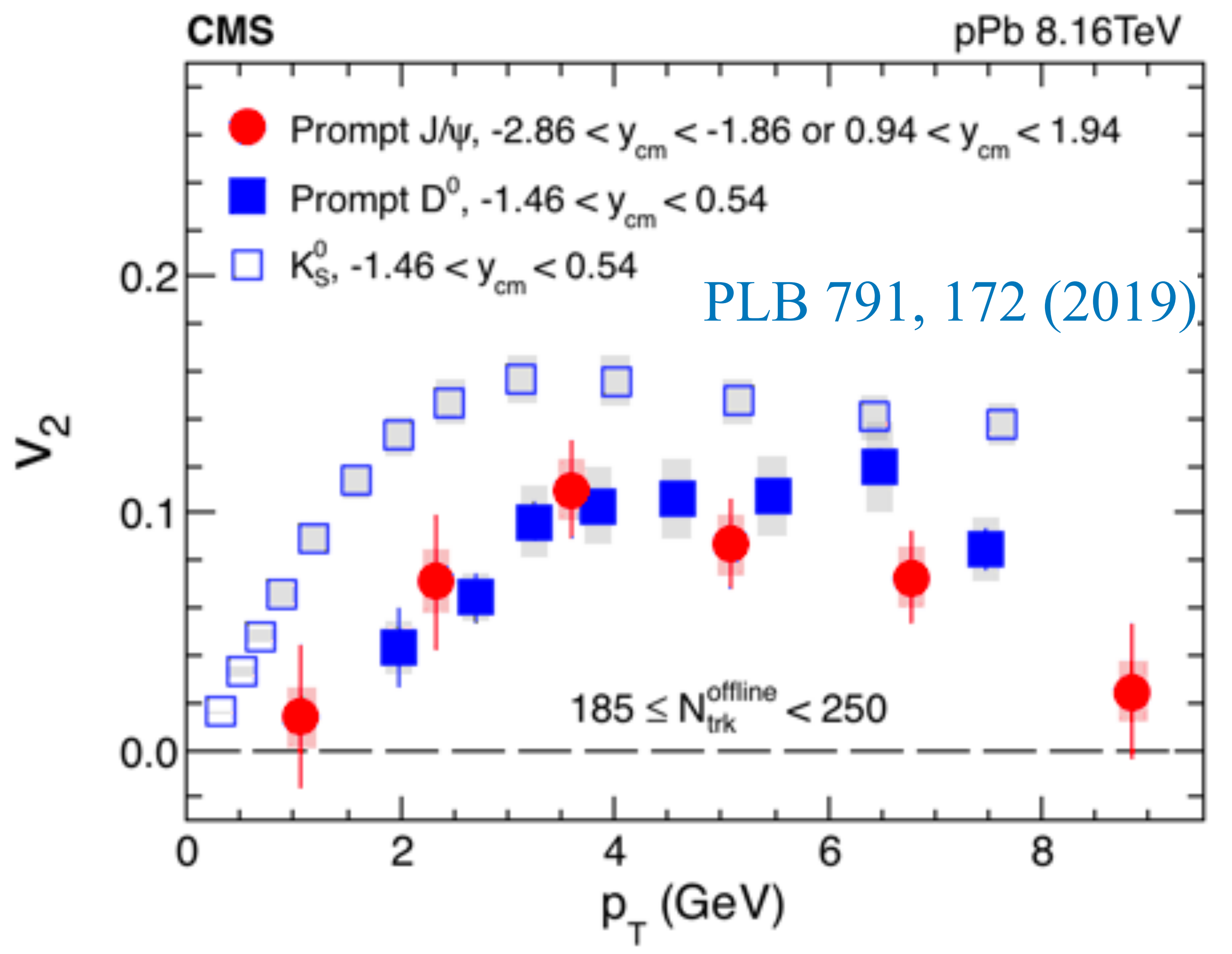
- Cold Nuclear Matter effects (nPDFs, energy loss, saturation, multiple-scattering, …)
- The nuclear medium, as a filter, diagnoses the quarkonium production mechanism.

A+A collisions (Large systems)

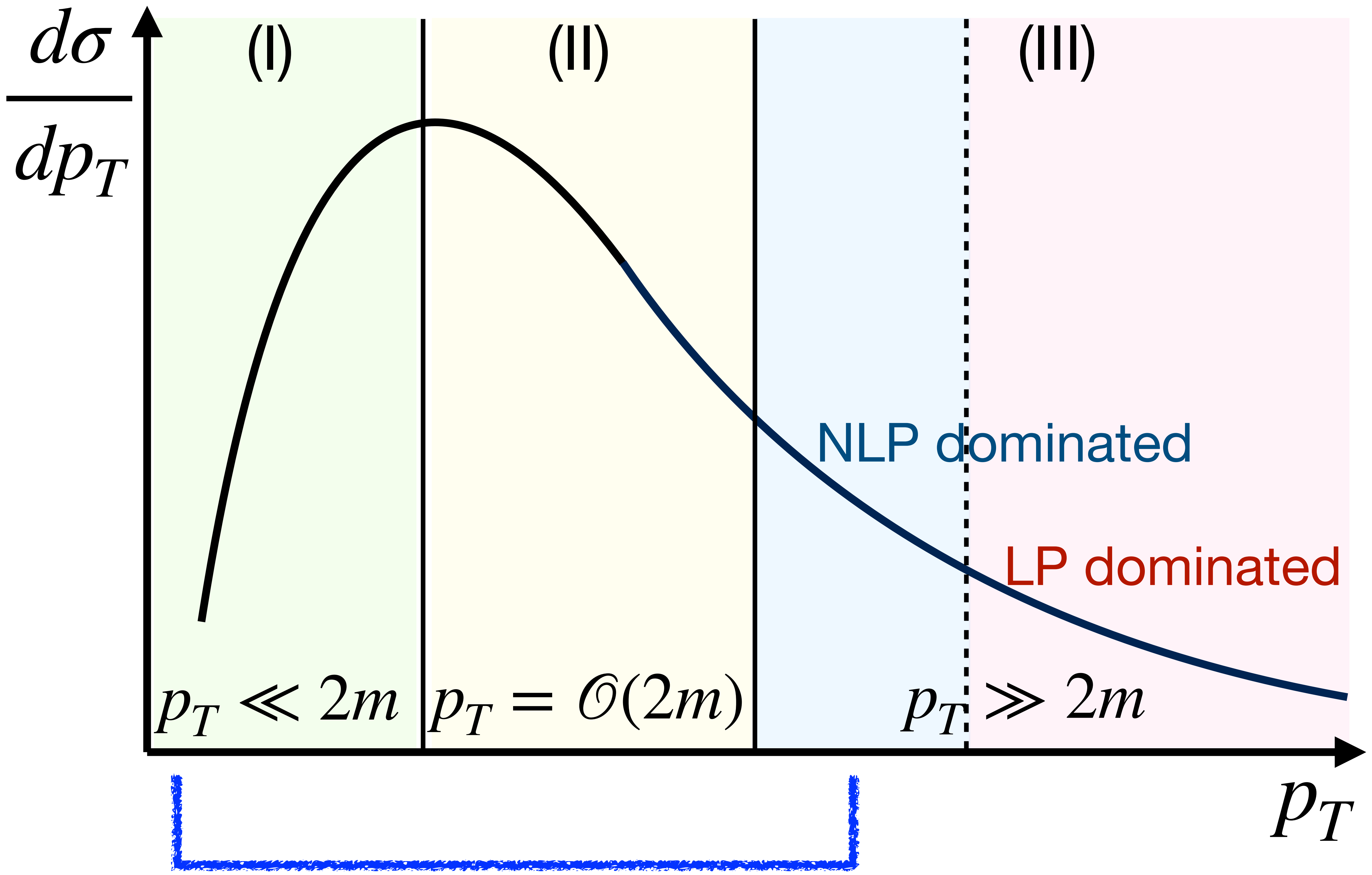
- Quarkonium dissociation, regeneration, collectivity.
- The space-time evolution of quarkonium formation involves various QCD and QED effects.

Quarkonium production in small systems (pp, pA) gives essential inputs for discussion of the QCD medium effect!





Hadronic quarkonium production: at a glance



The p_T spectrum can be modified by cold nuclear effects.

(I) TMD factorization + CEM or NRQCD

Berger, Qiu and Wang, PRD 71, 034007 (2005)
Sun, Yuan and Yuan, PRD88, 054008 (2013)
…

CGC framework + CEM or NRQCD (forward)

Ma, Venugopalan, PRL113, 19, 192301 (2014)
KW, Xiao, PRD92, 11, 111502 (2015)
…

(II) NRQCD factorization

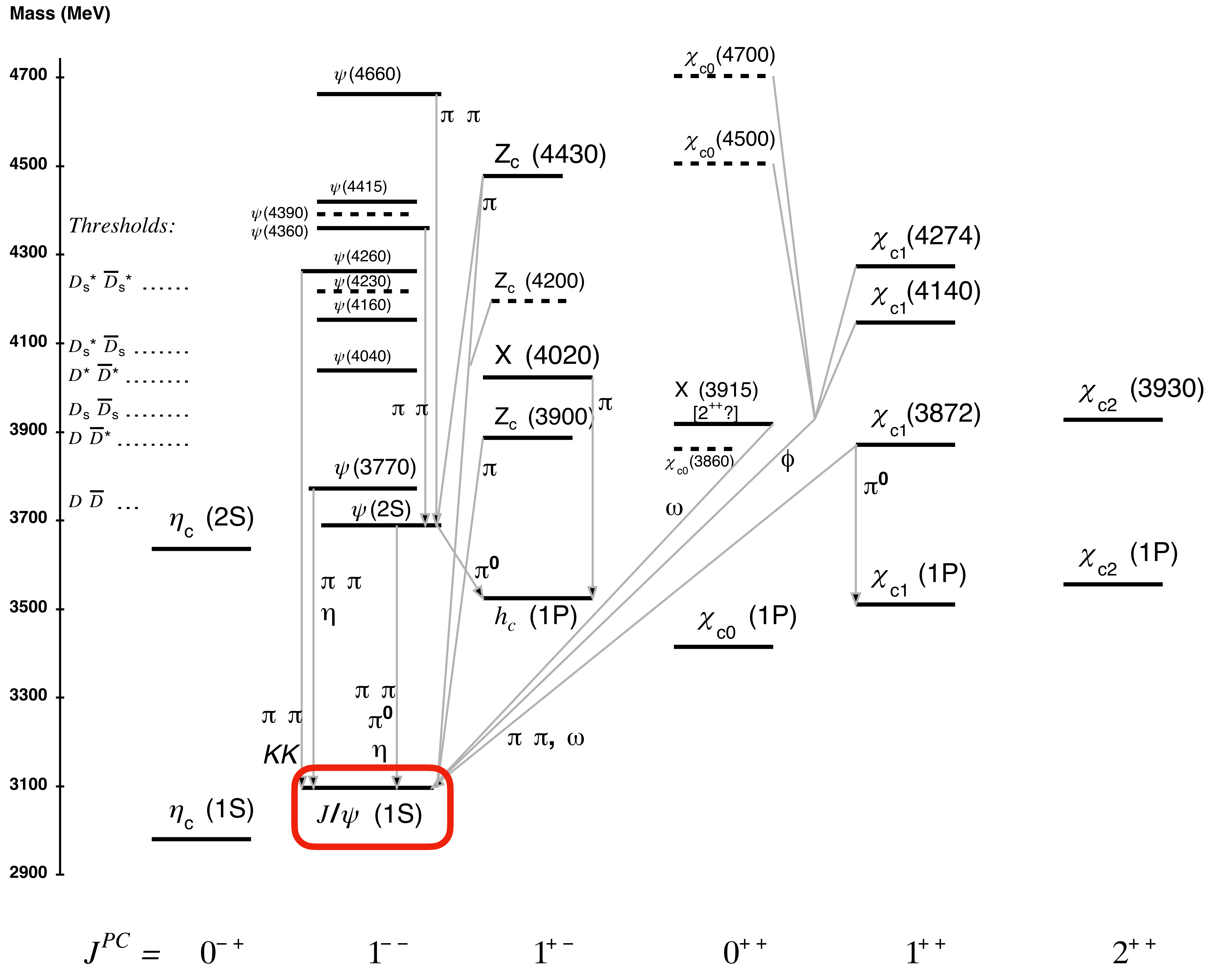
Butenschoen, Kniehl, PRD84, 051501 (2011)
Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 (2012)
Gong, Wan, Wang, Zhang, PRL110, 042002 (2013)
…

(III) QCD factorization w/ Fragmentation Functions

Kang, Qiu and Sterman, PRL108, 102002 (2012)
Bodwin, Chung, Kim, Lee, PRL113, 022001 (2014)
Ma, Qiu, Sterman, Zhang, PRL113, 14, 142002 (2014)
…

Need to choose a proper framework to explore QCD medium effects precisely!

I. Background: Quarkonium production models

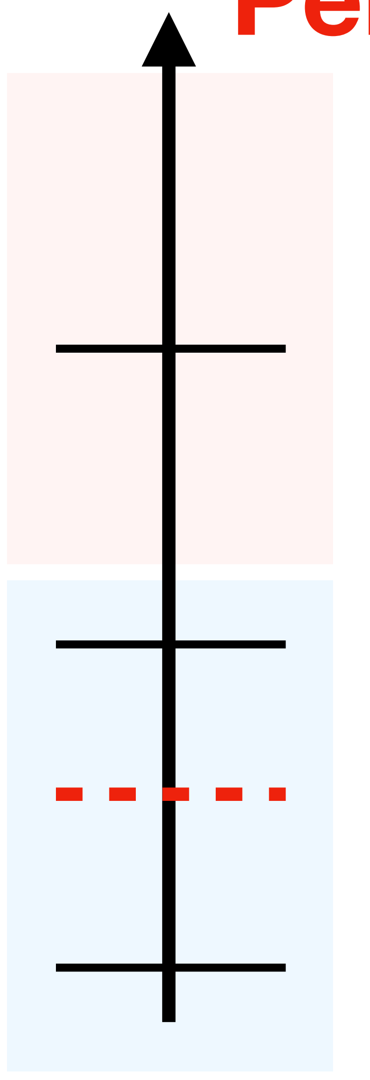


Inclusive direct production of S-wave quarkonium will be considered.

Scales in heavy quarkonium production (1/2)

Quarkonium, the bound state of a heavy quark and antiquark pair ($Q\bar{Q}$), involves multi-scale.

Perturbative



$\sim m$ Hard: production of $Q\bar{Q}$ [pQCD]

$\sim mv$ Soft: relative momentum [NRQCD]

$\sim \Lambda_{\mathrm{QCD}}$

$\sim mv^2$ Ultra-soft: binding energy [pNRQCD]

Non-Perturbative

data from PDG

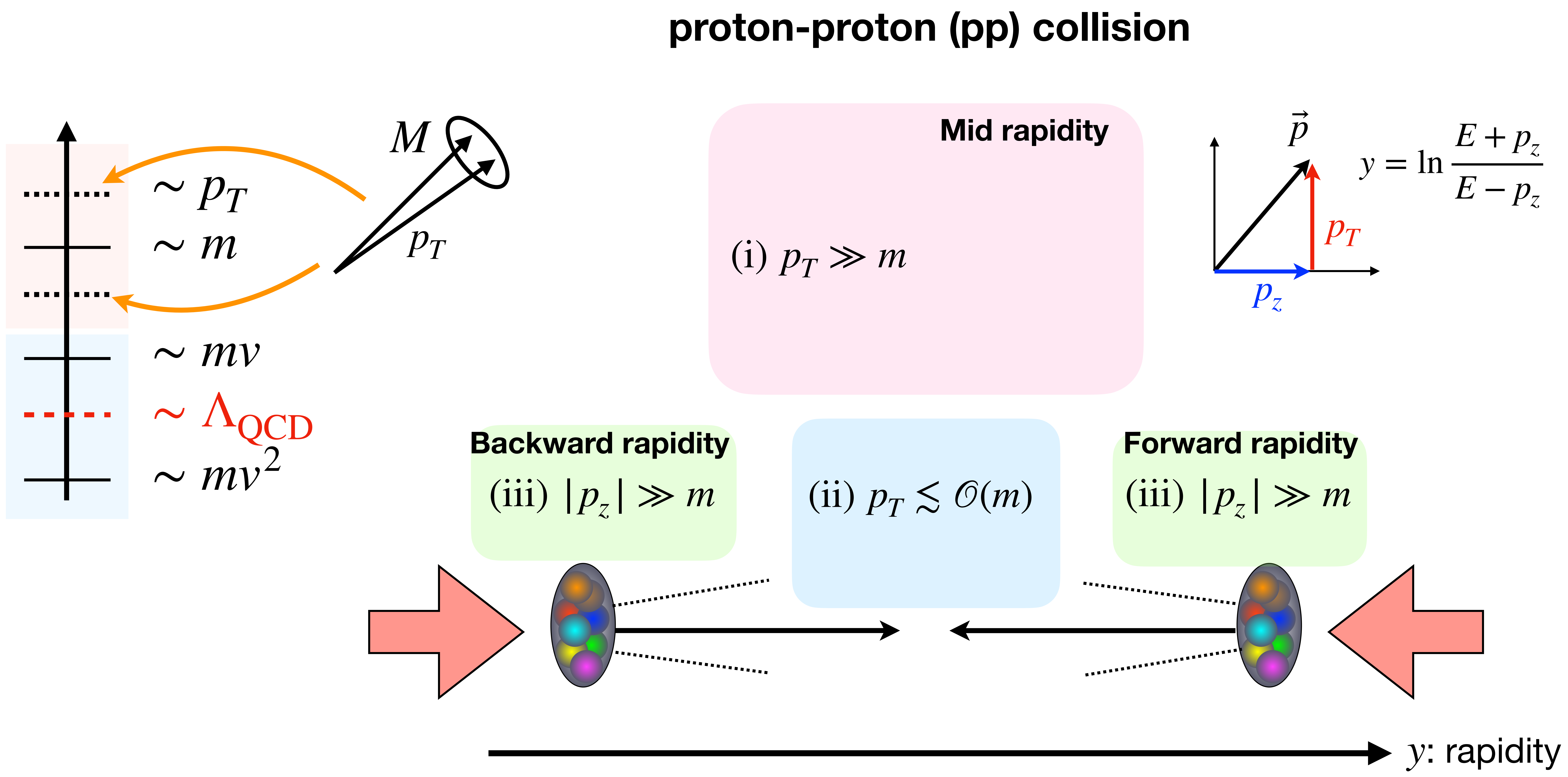
Flavor	Mass
u	~ 2 - 2.5 MeV
d	~ 4.5 - 5 MeV
s	~ 90 - 100 MeV
c	~ 1.25 - 1.3 GeV
b	~ 4.15 - 4.2 GeV
t	~ 173 GeV

$\sim \Lambda_{\mathrm{QCD}}$ (between s and c)

- ‘Heavy’ quark mass, $m \gg \Lambda_{\mathrm{QCD}}$, makes perturbation theory more reliable.
- Nonrelativistic system in the quarkonium rest frame with heavy quark velocity:

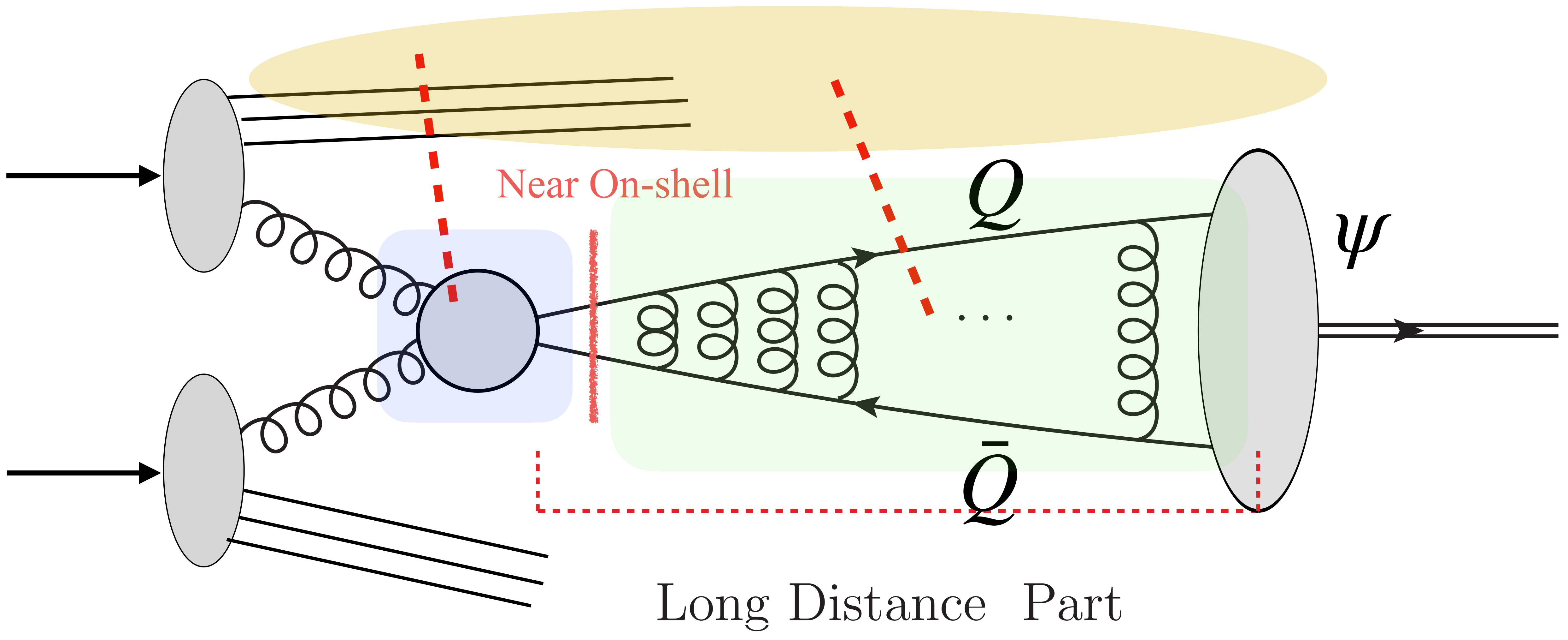
  $v^2 \sim 0.3\ (c\bar{c}),\ v^2 \sim 0.1\ (b\bar{b}).$
- Well separated momentum scales, $m \gg mv \gg mv^2$, result in effective theory.

Scales in heavy quarkonium production (2/2)



Quarkonium's momentum (p_T, p_z) is crucial to pin down the production mechanism!

Emergence of a heavy quarkonium and factorization



- Interactions between $Q\bar{Q}$ and soft partons can be suppressed by powers of $1/M$.
- $Q\bar{Q}$ could suffer from spectator interactions, suppressed by powers of $1/p_T$ or $1/p_z$.
- If the relative momentum $q^2 = (p_Q - p_{\bar{Q}})^2$ is large enough, radiative soft gluons could break factorization.

Factorization between $Q\bar{Q}$ production and nonperturbative bound state formation:

hard part, involving PDFs

$$d\sigma_{A+B\to\psi+X} = \sum_n \int dq^2\, d\hat{\sigma}_{A+B\to Q\bar{Q}[n]+X}(M^2, q^2)\, F_{Q\bar{Q}[n]\to\psi}(q^2) + \mathcal{O}\left(\frac{q^2}{M^2}\right)$$

n: quantum states of the pair

transition distribution

power corrections

Modern approaches for the bound state formation (1/2)

Qiu, Vary and Zhang, PRL88, 232301 (2002)
Bodwin, Braaten and Lee, PRD72, 014004 (2005)
Bodwin, Braaten and Lepage, PRD51, 1125 (1995)

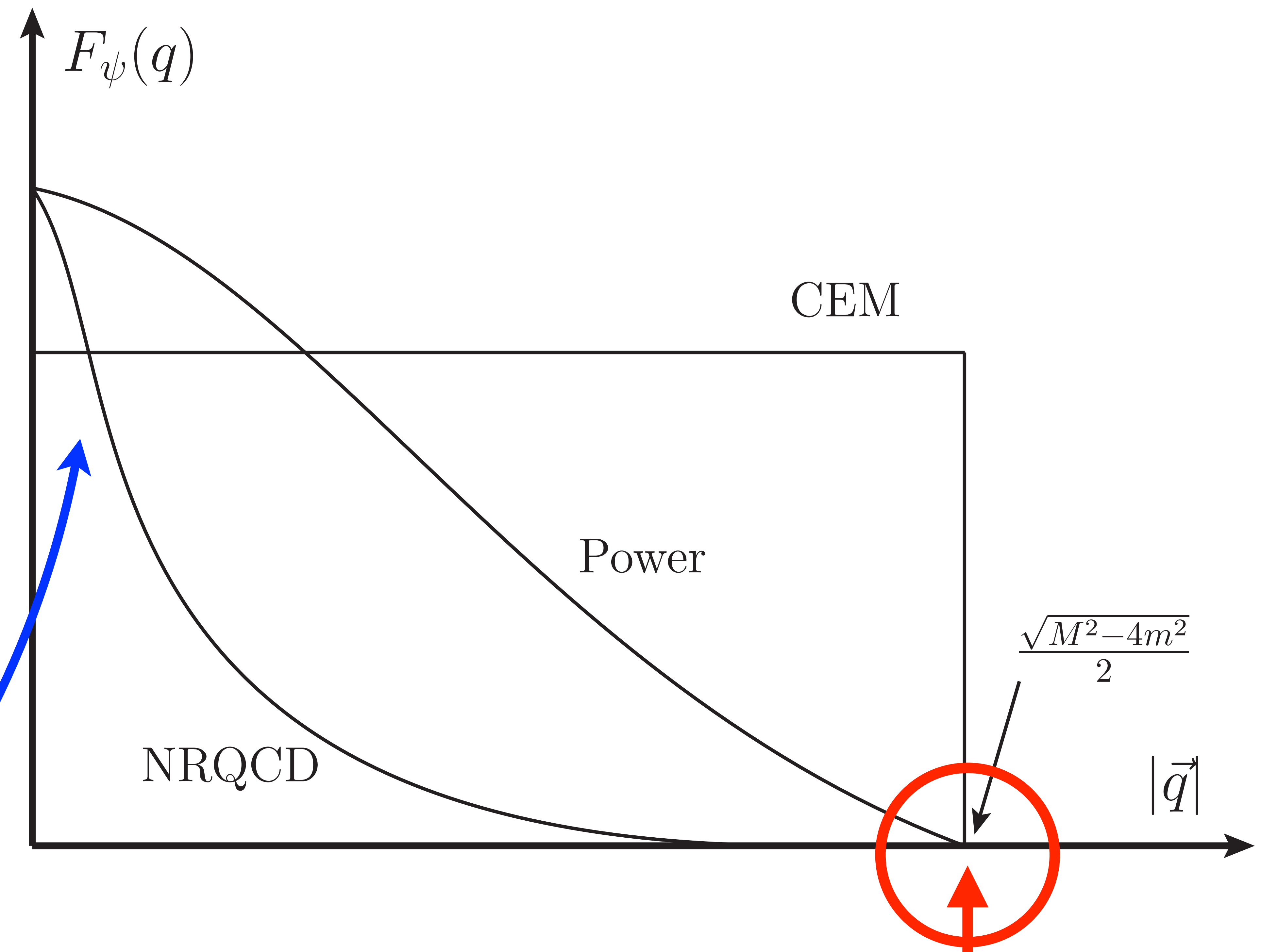
1. Color Evaporation Model: useful for phenomenology, but the long distance part is blinded.

$$d\sigma_\psi \approx F_\psi \int_0^{(2M_D)^2-(2m)^2} dq^2 d\hat{\sigma}_{Q\bar{Q}}(q^2)$$

2. Color Singlet Model (direct production at the early stage)

$$d\sigma_\psi \approx d\hat{\sigma}_{Q\bar{Q}}(q^2=0)\int dq^2 F_\psi(q^2) = |\Psi(0)|^2 d\hat{\sigma}_{Q\bar{Q}}$$

wave-function at origin



Open heavy flavor threshold

$F_\psi(q^2)$ should be peaked at $q^2=0$, and its moment is the square of the wave-function at origin.

Modern approaches for the bound state formation

3. NRQCD factorization approach: CS + CO contributions.

$$d\sigma_\psi \approx \sum_n \int dq^2\, F_{Q\bar{Q}[n]\to\psi}(q^2) \sum_m \frac{[q^2]^m}{m!} \frac{d^m d\hat{\sigma}_{A+B\to Q\bar{Q}[n]+X}(M^2,q^2)}{d^m q^2}\Big|_{q^2=0}$$

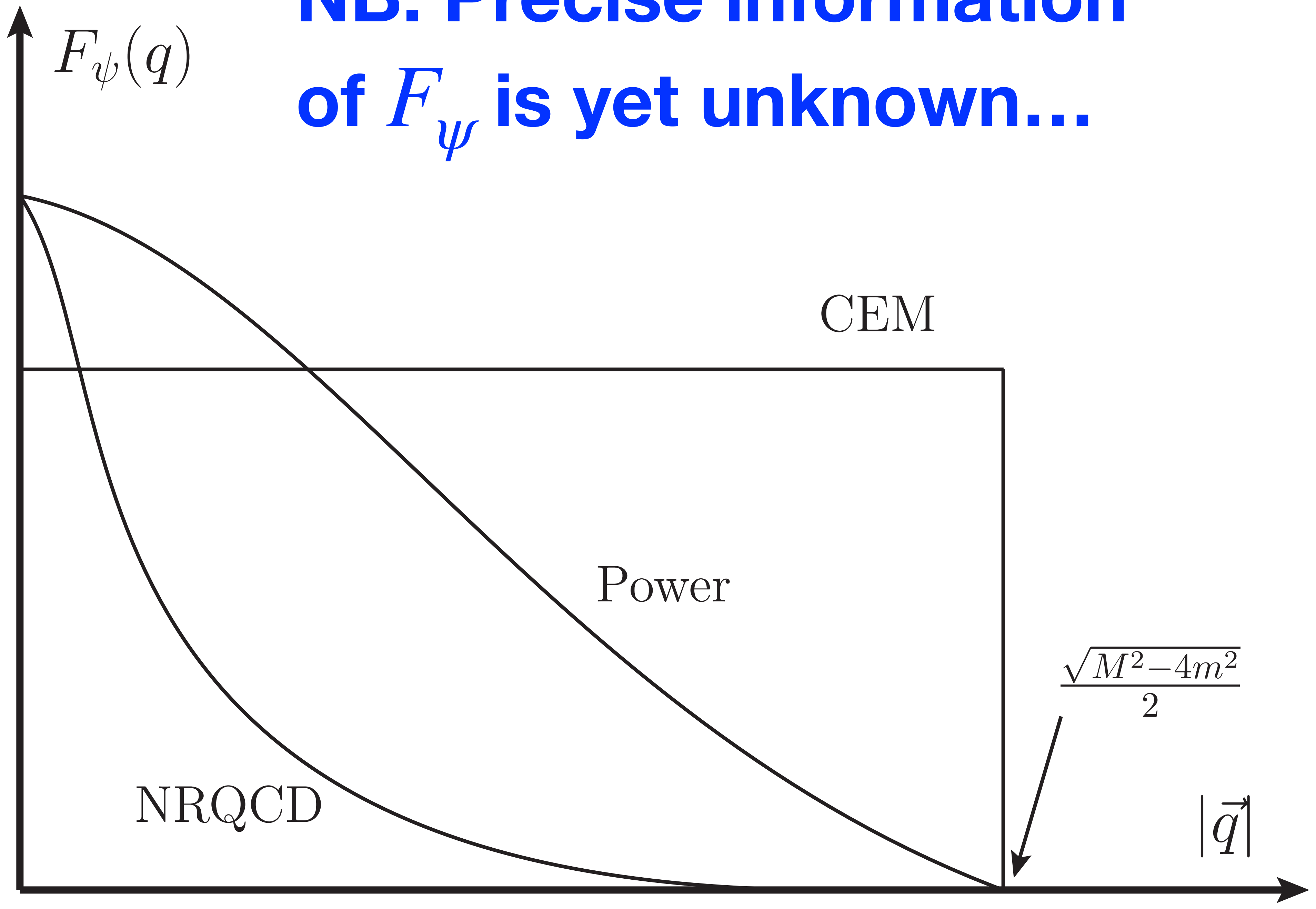
$$= \sum_{n,m} \frac{d^m d\hat{\sigma}_{A+B\to Q\bar{Q}[n]+X}(M^2,q^2)}{d^m q^2}\Big|_{q^2=0} \int dq^2 \frac{[q^2]^m}{m!} F_{Q\bar{Q}[n]\to\psi}(q^2)$$

$$\approx \sum_\kappa d\hat{\sigma}_{Q\bar{Q}[\kappa]}(q^2=0)\langle \mathcal{O}_{Q\bar{Q}[\kappa]\to\psi}\rangle$$

Long-Distance Matrix Elements (LDMEs), $\langle \mathcal{O}_{Q\bar{Q}[\kappa]}\rangle$, are **infinite-parameters** and organized by the power of the quark velocity $v^2 \sim q_T^2/m^2 < 1$ and α_s:

4-leading channels $\kappa = {}^3S_1^{[1]}, {}^1S_0^{[8]}, {}^3S_1^{[8]}, {}^3P_J^{[8]}$ with $J = 0, 1, 2$

NB: Precise information of F_ψ is yet unknown…



Braaten and Yuan, PRL71, 1673 (1993)
Braaten, Doncheski, Fleming and Mangano, PLB333, 548 (1994)
Cacciari and Greco, PRL73, 1586 (1994)
Braaten and Fleming, PRL74, 3327 (1995)

4. Fragmentation Function approach only at high p_T ($\gg m$).

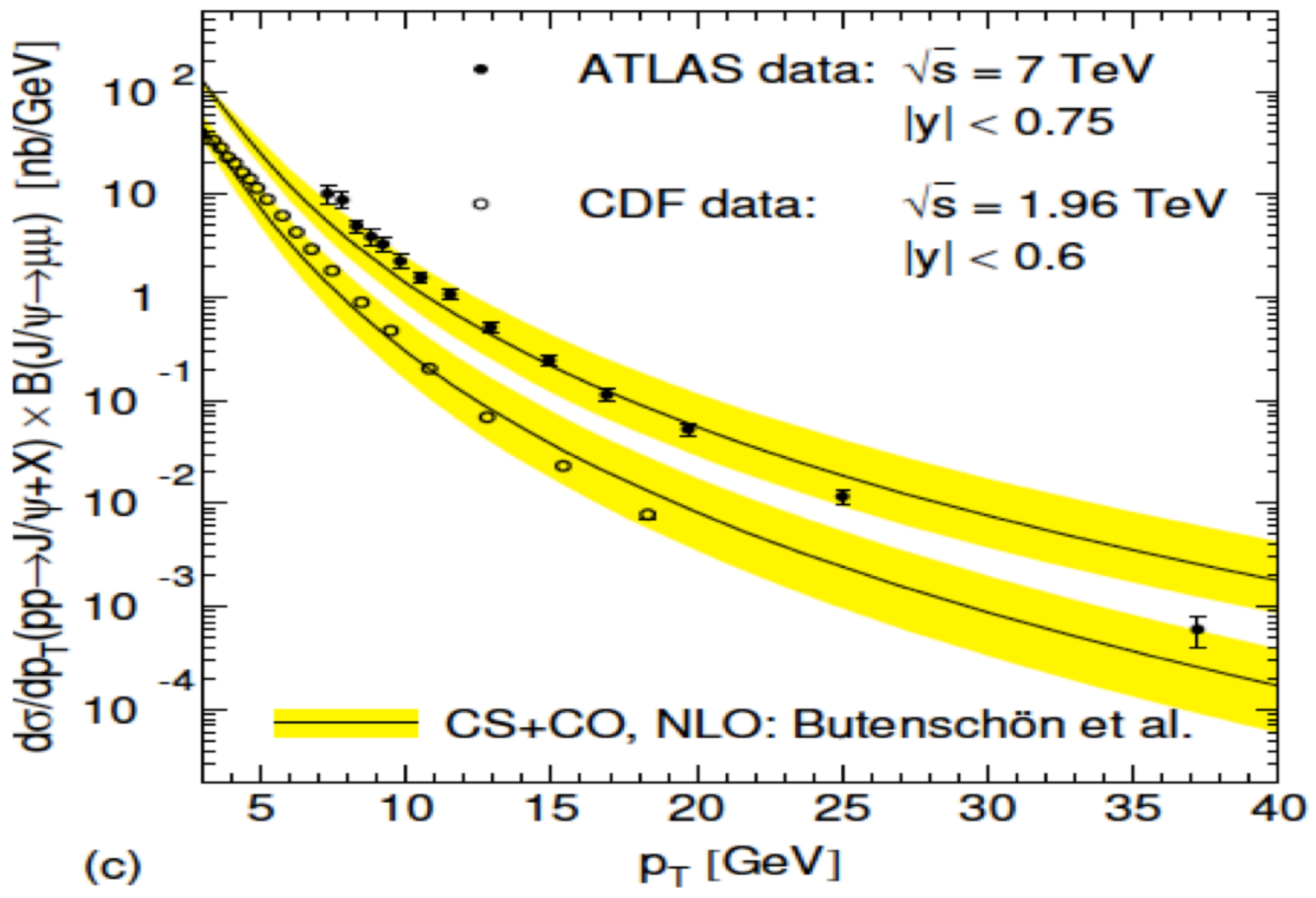
$$d\sigma_\psi \approx \sum_i \int dz D_{i\to\psi}(z)\, d\hat{\sigma}_i(z)$$

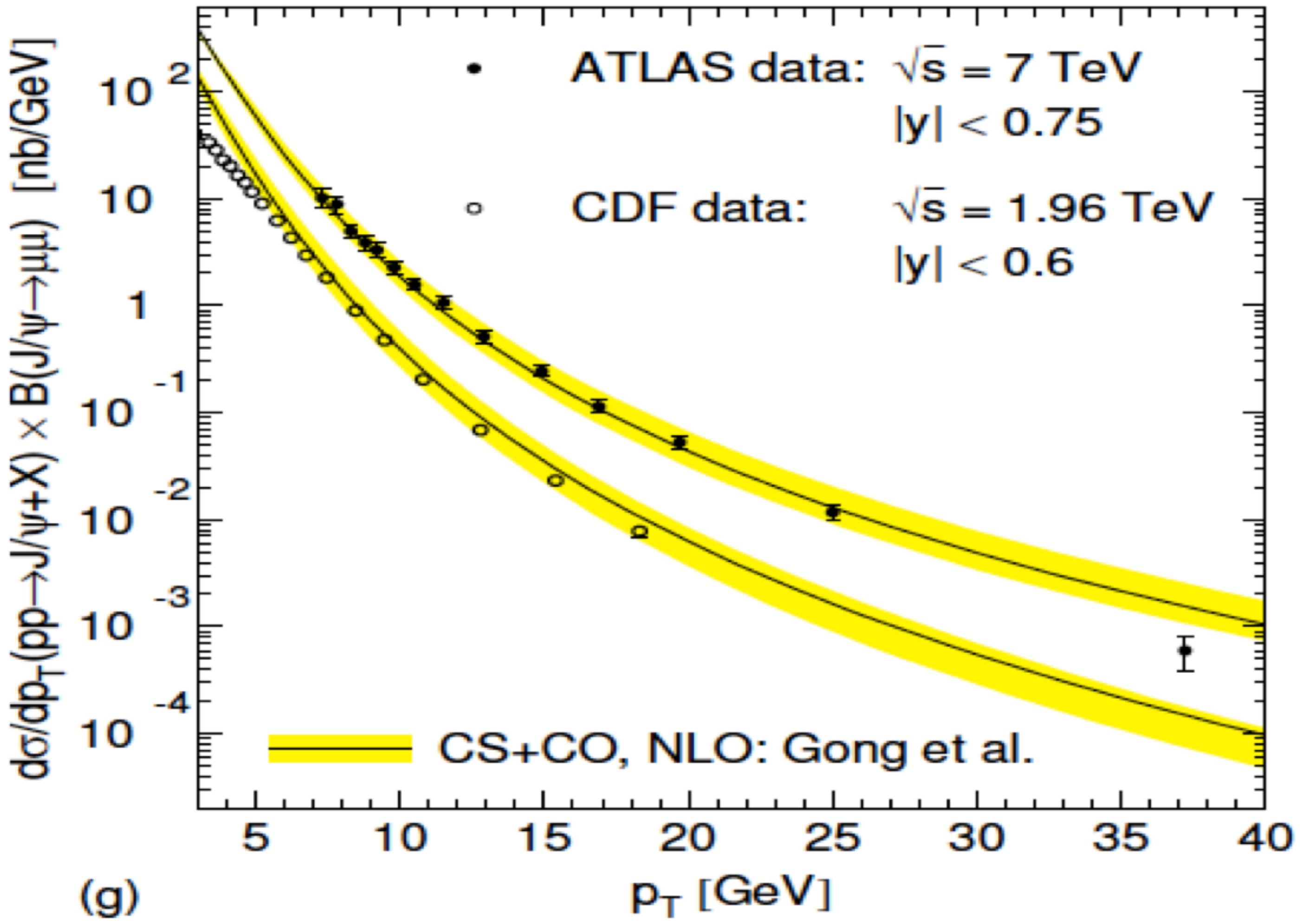
NRQCD vs. data: lack of universality of LDMEs

$$d\sigma_\psi \approx \sum_\kappa d\hat{\sigma}_{Q\bar{Q}[\kappa]}(q^2=0)\langle \mathcal{O}_{Q\bar{Q}[\kappa]\to\psi}\rangle$$

→ **Global data fitting**

	$\langle \mathcal{O}(^3S_1^{[1]})\rangle$ GeV^3	$\langle \mathcal{O}(^1S_0^{[8]})\rangle$ 10^{-2} GeV^3	$\langle \mathcal{O}(^3S_1^{[8]})\rangle$ 10^{-2} GeV^3	$\langle \mathcal{O}(^3P_0^{[8]})\rangle$ 10^{-2} GeV^5
Set I (Butenschoen *et al.*)	1.32	3.04	0.16	−0.91
Set II (Chao *et al.*)	1.16	8.9	0.30	1.26
Set III (Gong *et al.*)	1.16	9.7	−0.46	−2.14
Set IV (Bodwin *et al.*)	-	9.9	1.1	1.1

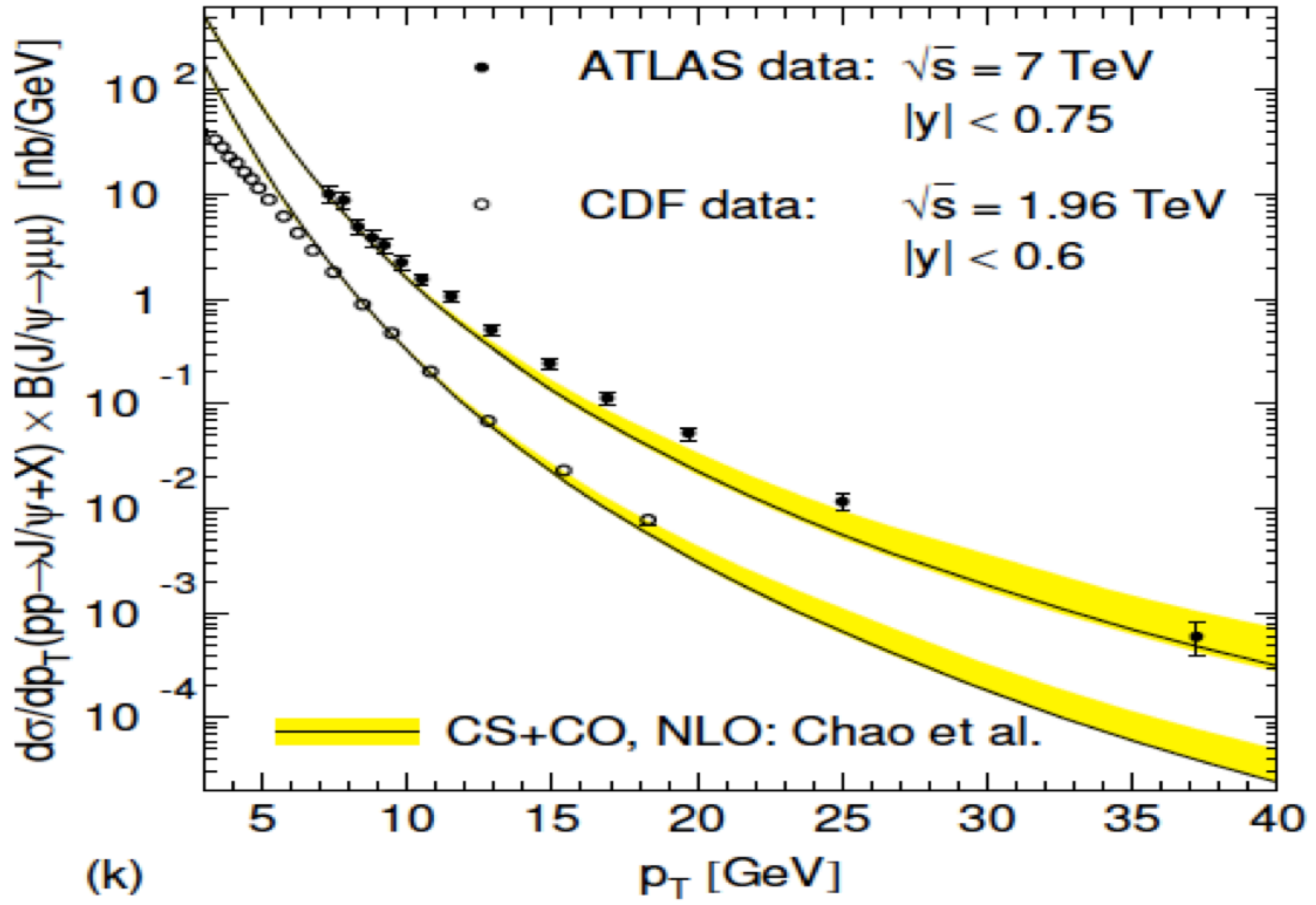


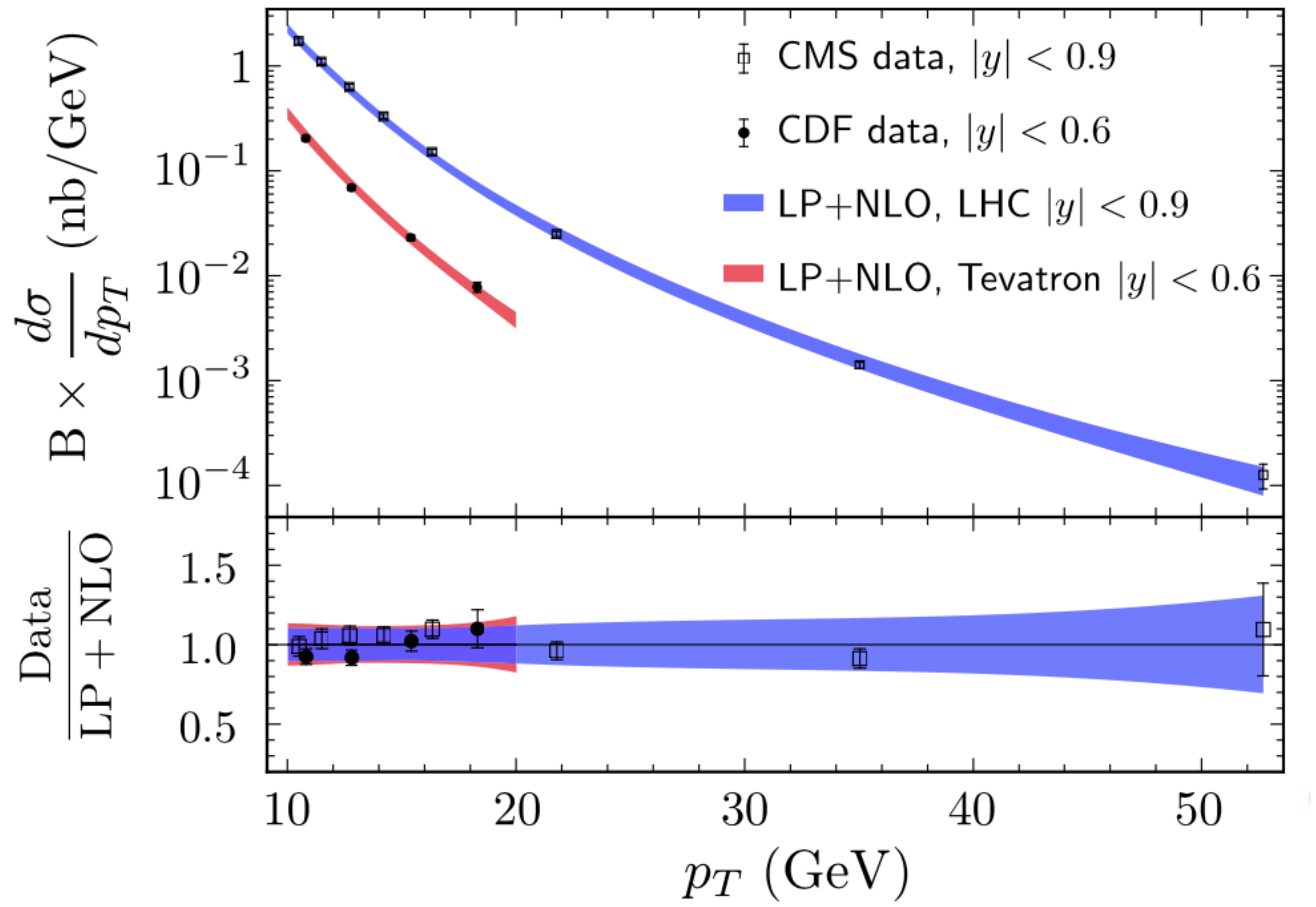


LDMEs should be universality, however:

- Numbers are not the same.
- Not even the sign.

Much more work is needed!





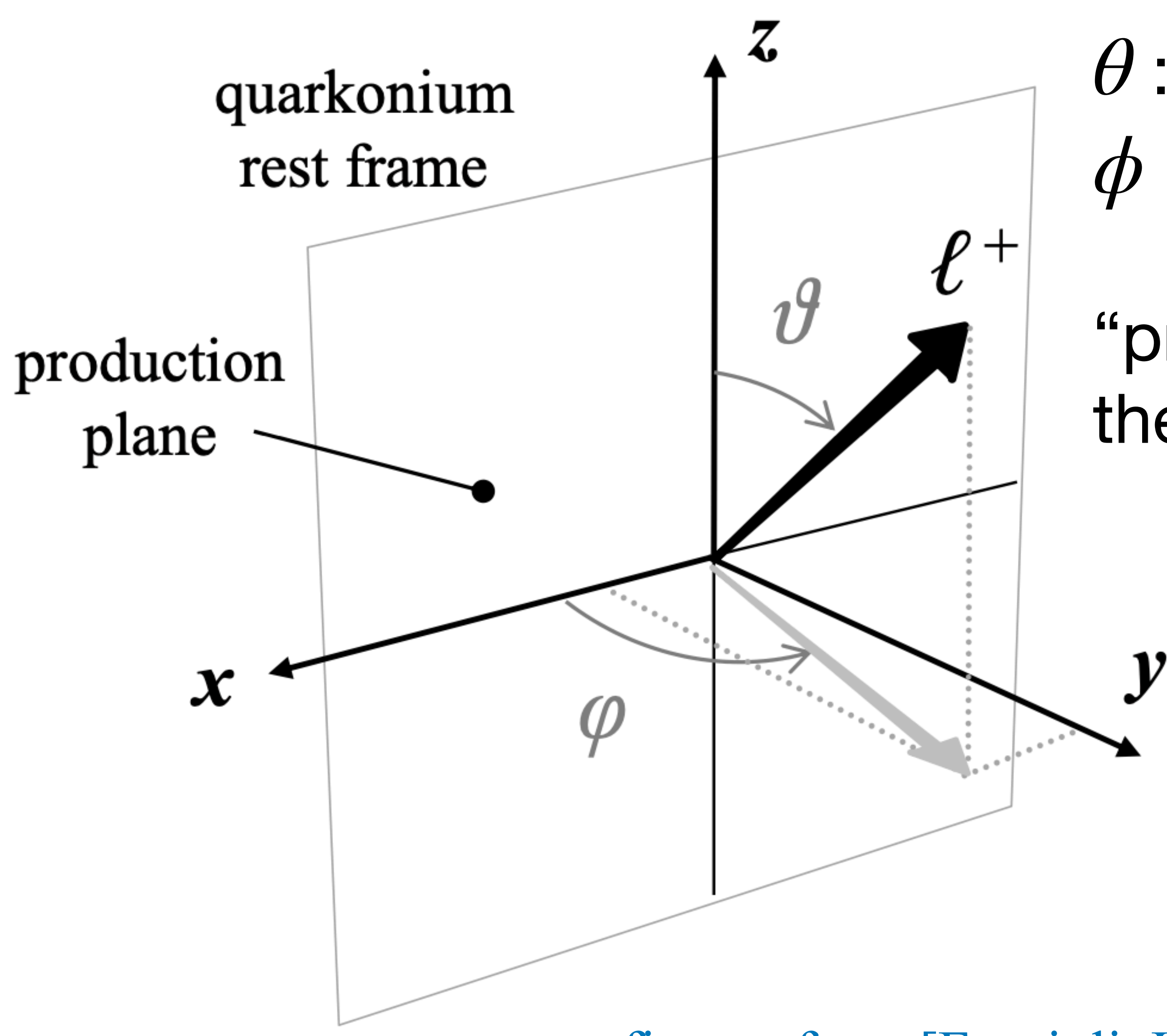
Fits in NRQCD

Butenschoen, Kniehl, PRD84, 051501 (2011).
Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 (2012).
Gong, Wan, Wang, Zhang, PRL110, 042002 (2013).
Bodwin, Chung, Kim, Lee, PRL113, 022001 (2014).
…

Fits in pNRQCD

Brambilla, Chung, Vairo, Wang, PRD105, no.11, L111503 (2022).

Polarization of quarkonium: a crucial observable

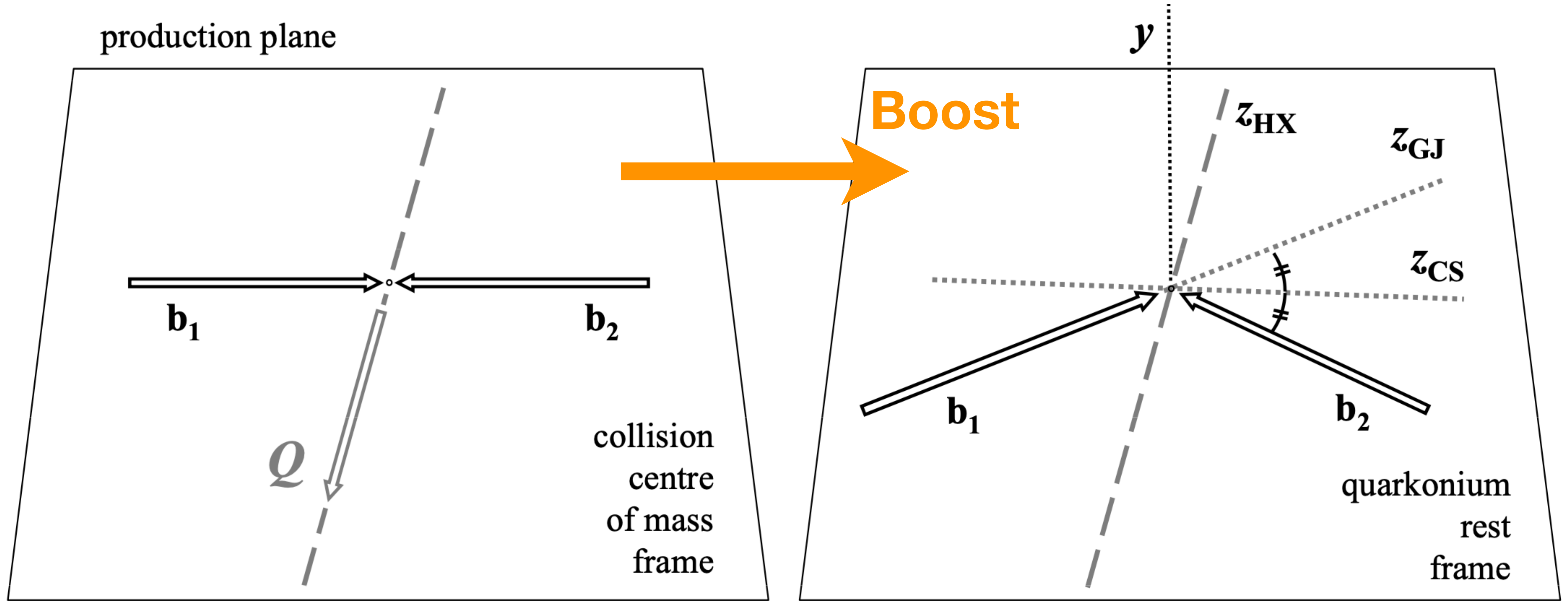


θ : polar angle

ϕ : azimuthal angle

"production plane" contains the momenta of the beams

figures from [Faccioli, Lourenco, Seixas and Wohri, Eur. Phys. J. C69, 657-673 (2010)]



The production rate is not very sensitive to the details of hadronization

→ Other observables or scales are needed.

$$\frac{d\sigma^{J/\psi(\to l^+l^-)}}{d\Omega} \propto 1 + \lambda_\theta \cos^2\theta + \lambda_\phi \sin^2\theta \cos 2\phi + \lambda_{\theta\phi} \sin 2\theta \cos\phi$$

Transverse pol.: $\lambda_\theta = +1$ (photon-like)

Longitudinal pol.: $\lambda_\theta = -1$

Unpolarized: $\lambda_\theta = 0$

The decay reference frame is not unique.

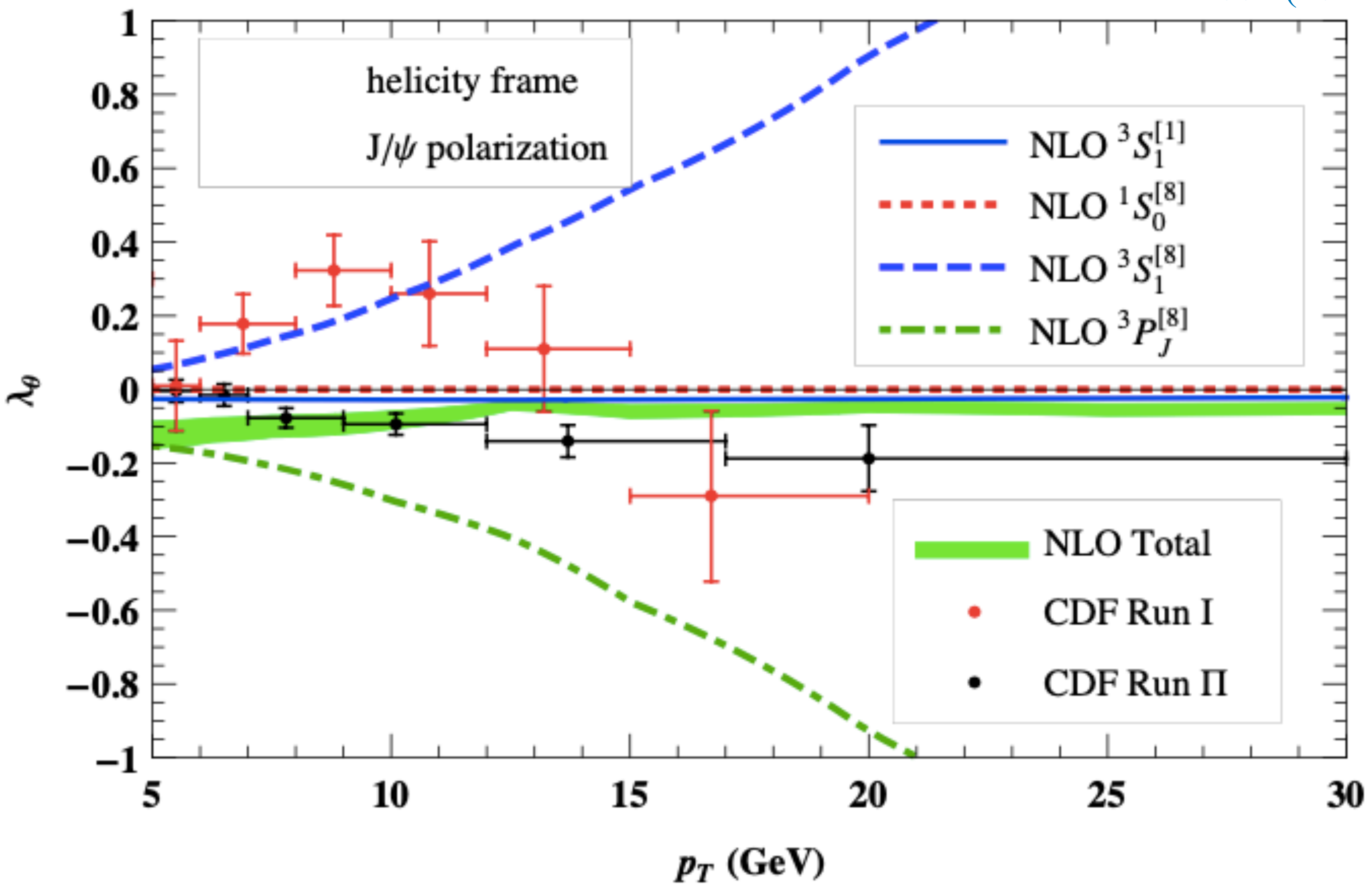
HX: Helicity frame (particle's direction)

GJ: Gottfried-Jackson frame (b1)

CS: Collins-Soper frame (bisector btw b1 and -b2)

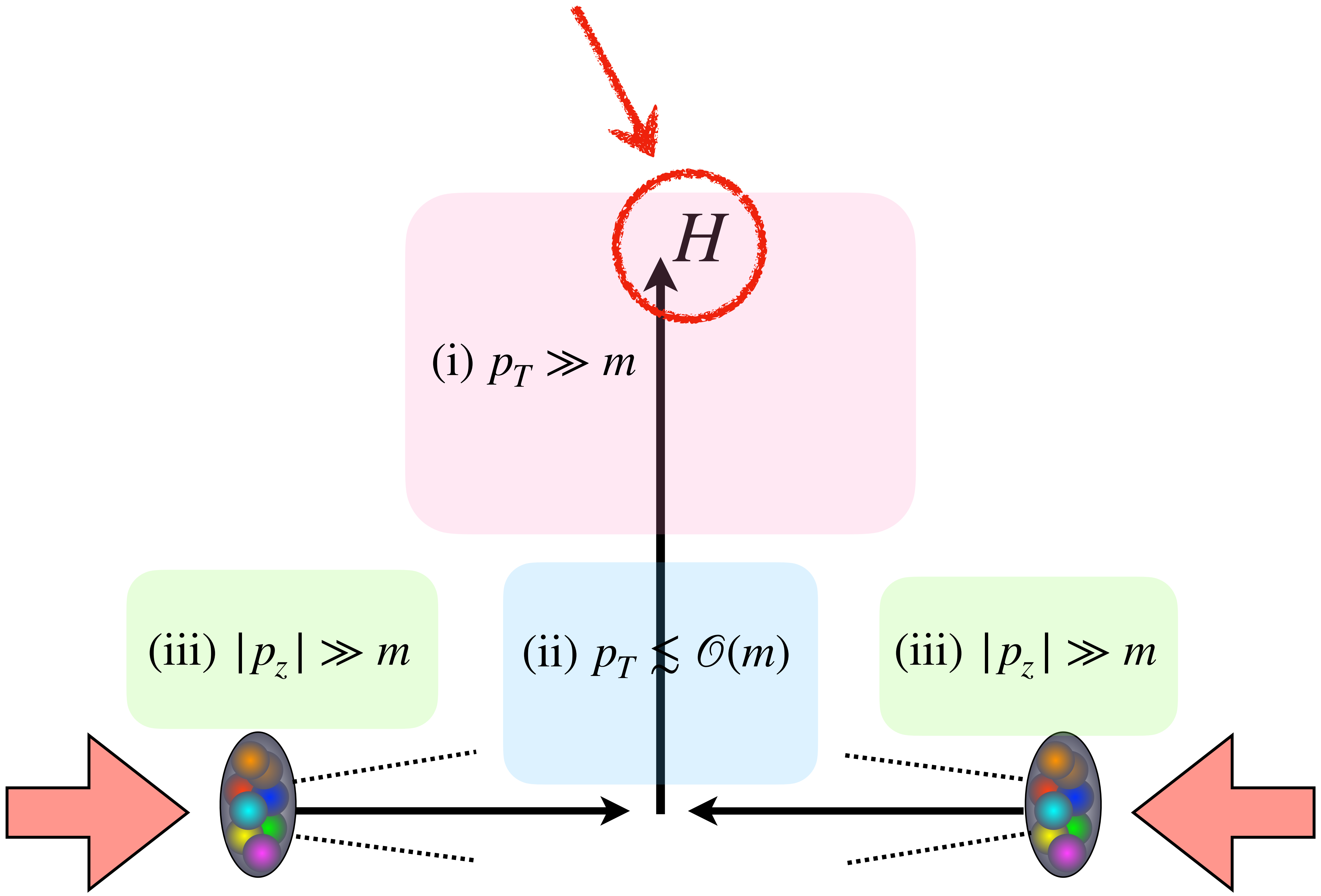
Polarization in NRQCD at NLO

Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 (2012)



- **NB**: transverse components of hard parts in the NRQCD for producing $Q\bar{Q}$ in P-state can be negative: cancellations between different intermediate states happen.
- The simultaneous description of both the p_T spectrum and polarization is an issue yet.

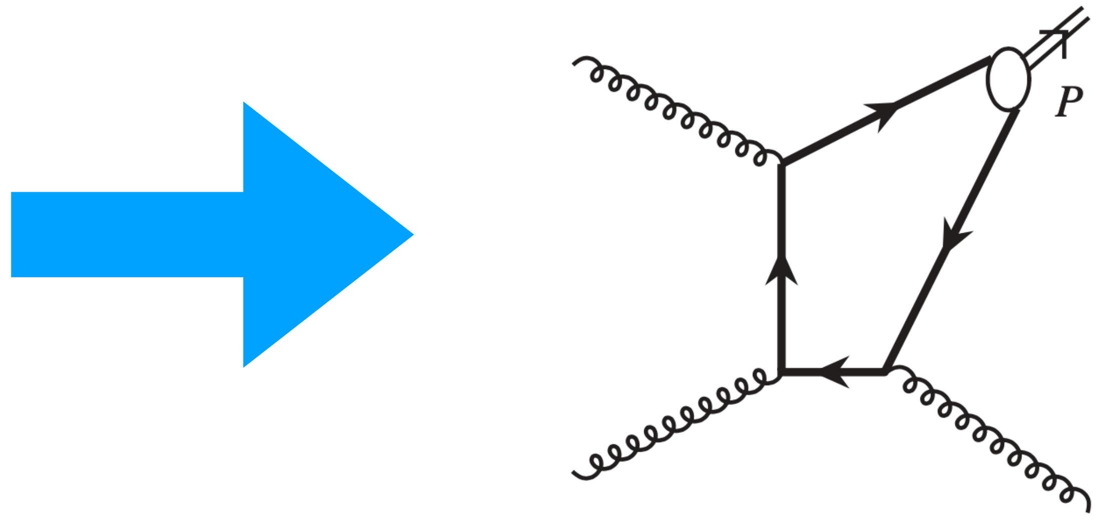
II-1. Hadronic quarkonium production of high p_T



Importance of higher order corrections at high p_T (1/2)

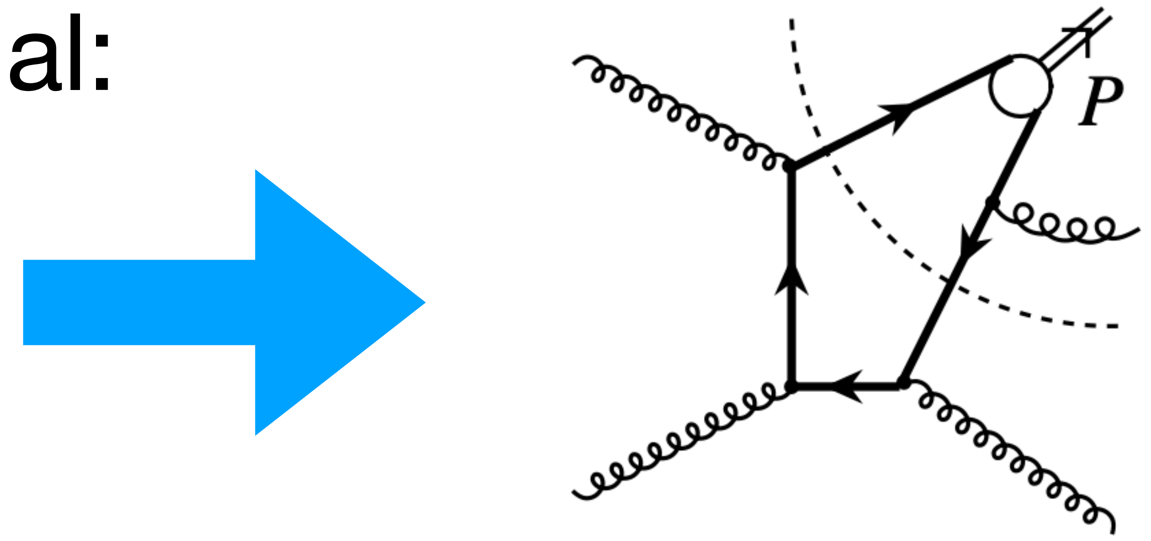
- At LO in CSM:

$$d\sigma(Q\bar{Q}[^3S_1^{[1]}]) \propto \frac{\alpha_s^3 m^4}{p_T^8}$$



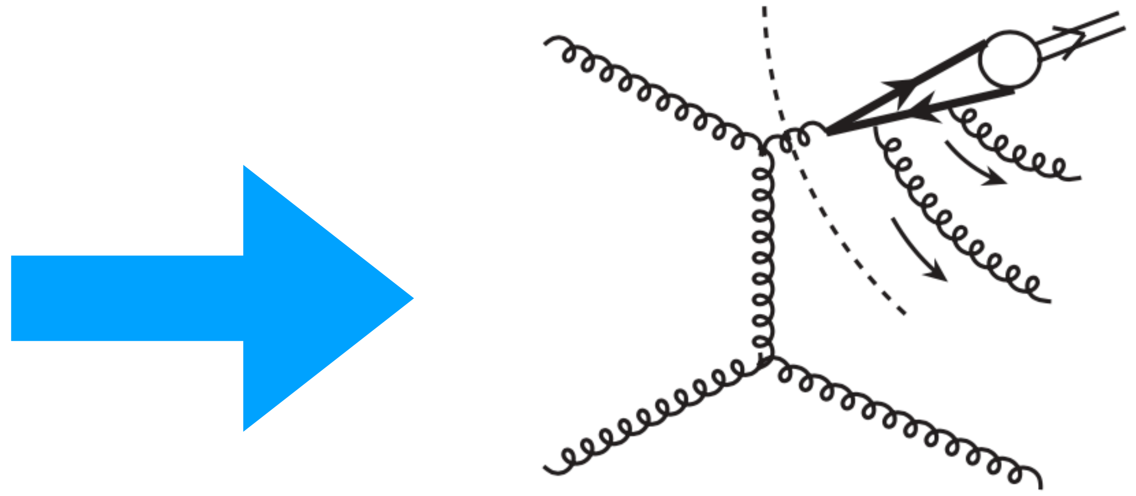
- At high p_T higher order corrections must be essential:

$$d\sigma(Q\bar{Q}[^3S_1^{[1]}]) \propto \frac{\alpha_s^3 m^4}{p_T^8} \times \frac{\alpha_s p_T^2}{m^2} = \frac{\alpha_s^4 m^2}{p_T^6}$$



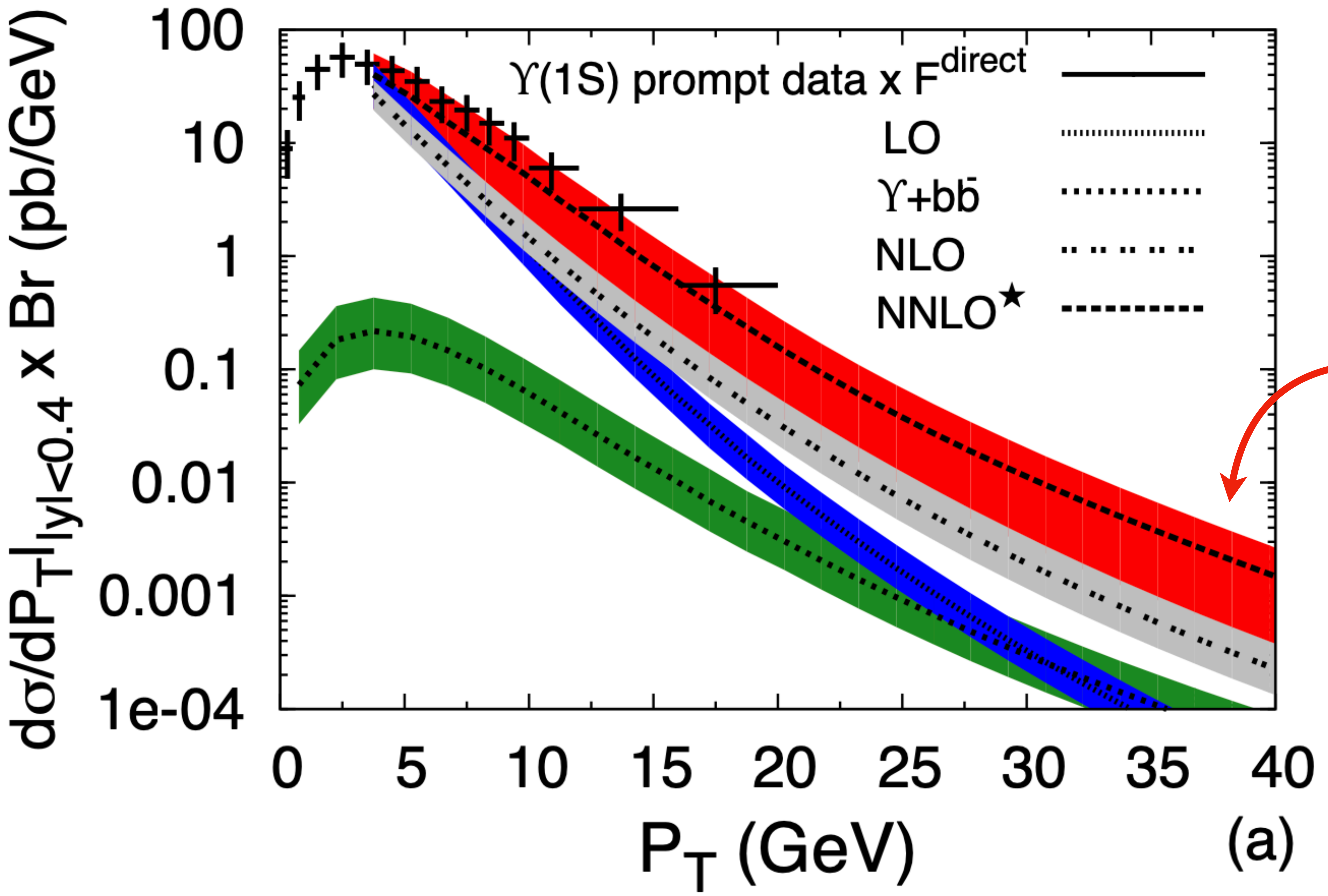
- The gluon jet fragmentation at high p_T:

$$d\sigma \propto \frac{\alpha_s^5}{p_T^4} \quad \& \quad d\sigma \propto \frac{\alpha_s^2}{p_\perp^4} \times \alpha_s^3 \ln\left(\frac{p_T^2}{m^2}\right)$$



The later is enhanced even if $\alpha_s \ll 1$; we may not obtain reliable predictions by considering only diagrams in the naive α_s expansion as well as v expansion.

Importance of higher order corrections at high p_T (2/2)



Artoisenet, Campbell, Lansberg, Maltoni, Tramontano, PRL101, 152001 (2008)

When $p_T \gg m$, the naive α_s expansion is not reliable!

- Consider the $1/p_T$ expansion first: leading power and subleading power contributions are factorizable.
- Then, expand each of the contributions in powers of α_s; those are calculable up to NLO.
- Leading order (LO) in $\alpha_s \neq$ Leading power (LP) in $1/p_T$

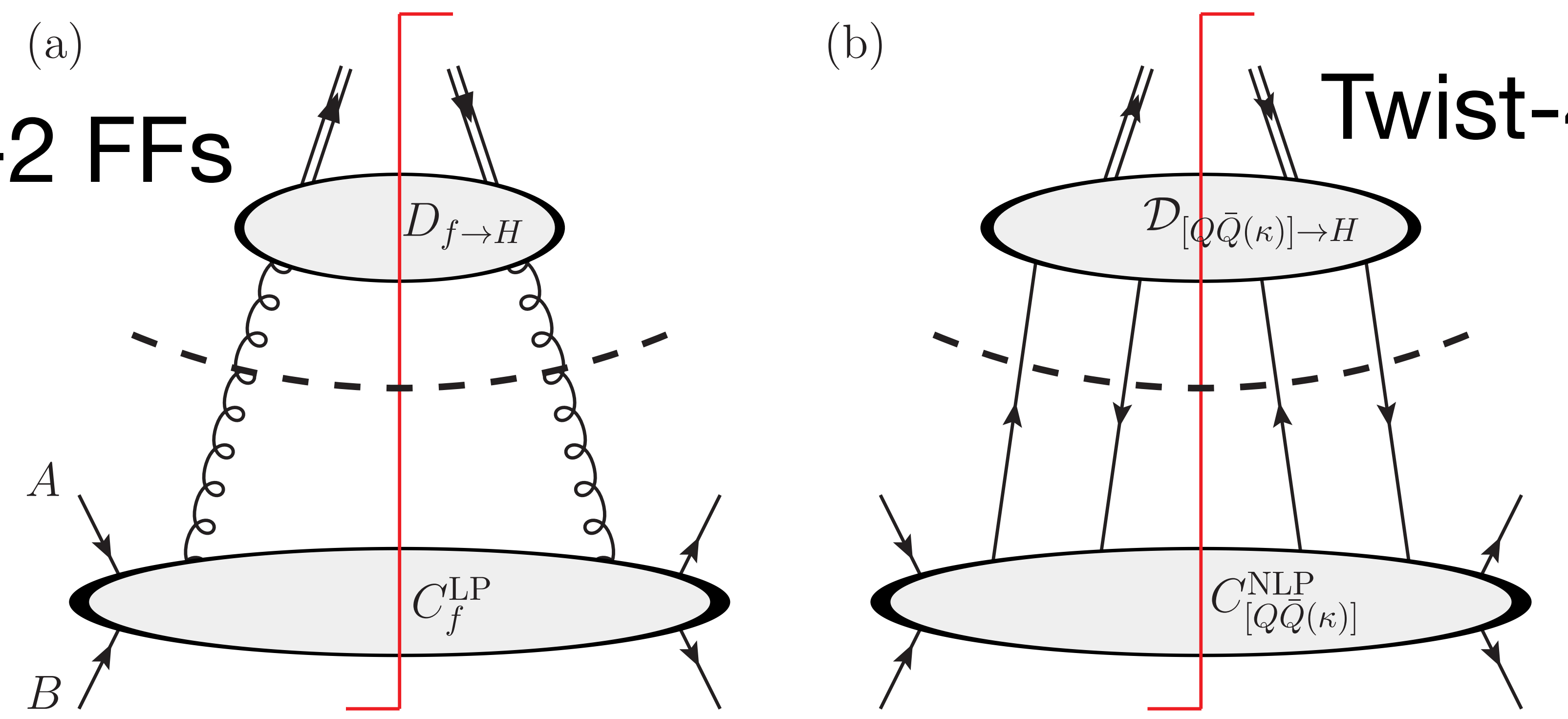
QCD factorization approach

Nayak, Qiu, Sterman, PRD72 (2005) 114012
Kang, Qiu, Sterman, PRL108 (2012) 102002
Kang, Ma, Qiu, Sterman, PRD90 (2014) 3, 034006, PRD91 (2015) 1, 014030

Leading power (LP) up to NLO

$$d\sigma^{\rm QCD-Res}_{A+B\to[f,Q\bar{Q}]\to H+X}(\mu) = \sum_{f=q,\bar{q},g} C^{\rm LP}_{A+B\to[f]+X}(\mu)\otimes D_{[f]\to H}(\mu)$$
$$+\frac{1}{p_\perp^2}\left[\sum_n C^{\rm NLP}_{A+B\to[Q\bar{Q}(n)]+X}(\mu)\otimes \mathscr{D}_{[Q\bar{Q}(n)]\to H}(\mu)\right]$$

Subleading power (NLP) at LO



pQCD projection operators

$$n = (v, a, t)^{[1,8]} = (\gamma^+, \gamma^+\gamma^5, \gamma^+\gamma_\perp^i)^{[1,8]}$$

v: vector
a: axial vector } **important at high p_T**
t: tensor — **suppressed at high p_T**

subtract double counting

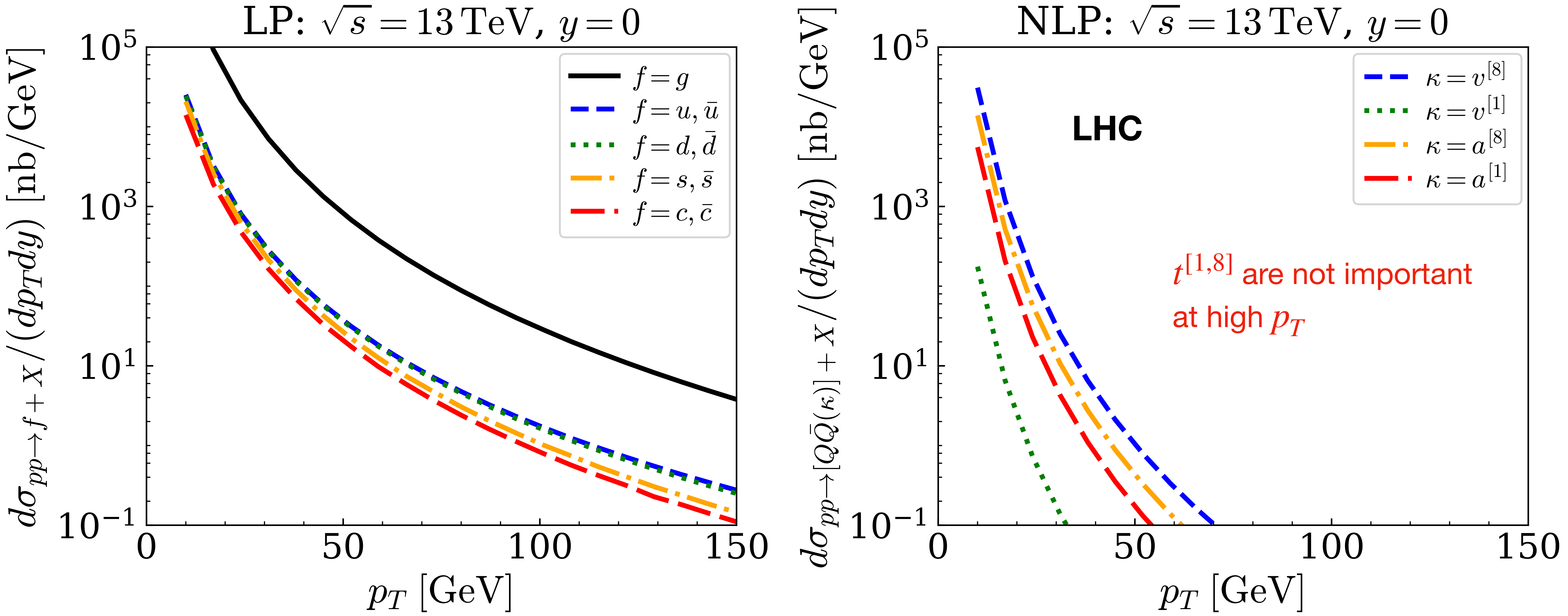
Matching condition

$$d\sigma_{A+B\to H+X}(m\neq 0) = d\sigma^{\rm QCD-Evol}_{A+B\to H+X}(m=0) + d\sigma^{\rm NRQCD-(n)}_{A+B\to H+X}(m\neq 0) - d\sigma^{\rm QCD-(n)}_{A+B\to H+X}(m=0)$$

$$\Rightarrow \begin{cases} d\sigma^{\rm QCD-Evol}_{A+B\to H+X} & \text{when} \quad p_\perp \gg m;\ d\sigma^{\rm NRQCD-(n)} \approx d\sigma^{\rm QCD-(n)} \\ d\sigma^{\rm NRQCD-(n)}_{A+B\to H+X} & \text{when} \quad p_\perp \to m;\ d\sigma^{\rm QCD-Evol} \approx d\sigma^{\rm QCD-(n)} \end{cases}$$

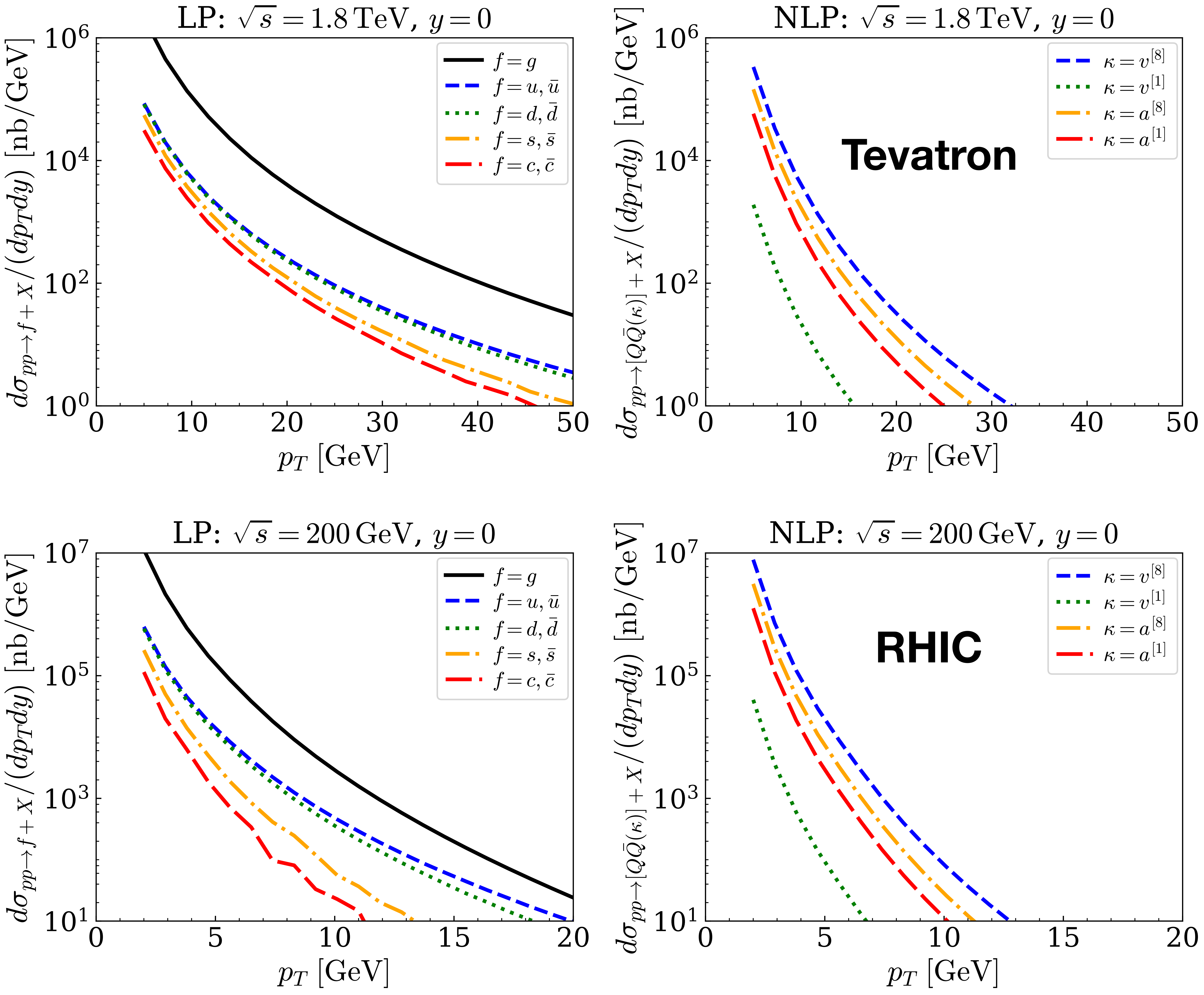
Partonic subleading power corrections (1/2)

Lee, Qiu, Sterman, **KW** (2021)



- Power corrections are suppressed by $1/p_T^2$ but sizable at moderate p_T ($p_T \gtrsim \mathcal{O}(2m)$).
- LP and NLP have different shapes in p_T.

Partonic subleading power corrections (2/2)



When you will study medium effects at RHIC, considering only the hadronization of $Q\bar{Q}$ is Okay.

Renormalization group improvement

Kang, Ma, Qiu, Sterman, PRD 90, 3, 034006 (2014)

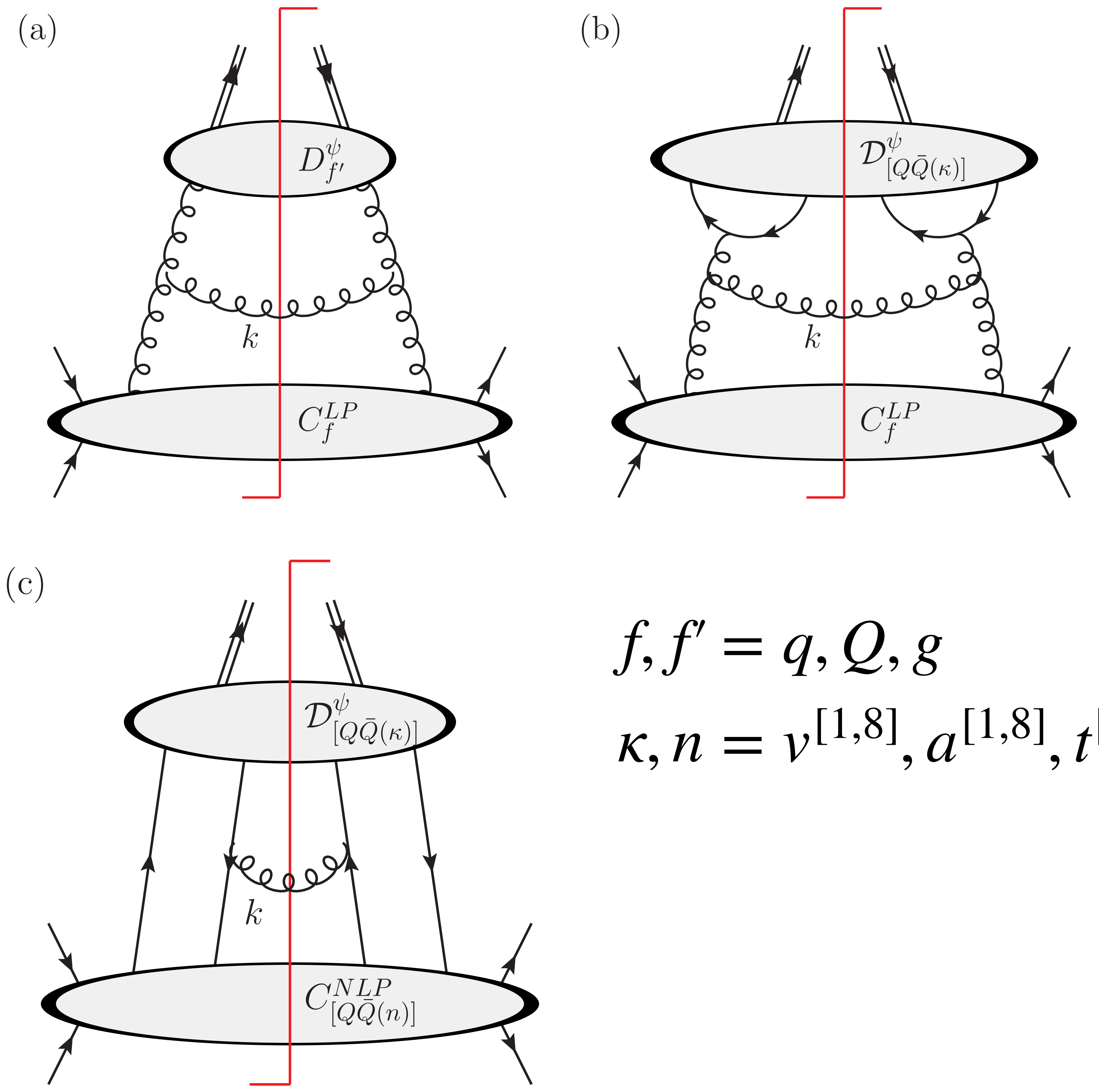
- **Twist-2 evolution equation: DGLAP + quark pair power corrections:**

$$\frac{\partial D_{[f]\to H}}{\partial \ln \mu^2} = \gamma_{[f]\to[f']} \otimes D_{[f']\to H} + \frac{1}{\mu^2}\gamma_{[f]\to[Q\bar{Q}(\kappa)]} \otimes \mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}$$

The inhomogeneous term is added to the **slope**, not to the FF itself.

- **Twist-4 "DGLAP like" evolution equation:**

$$\frac{\partial \mathcal{D}_{[Q\bar{Q}(n)]\to H}}{\partial \ln \mu^2} = \Gamma_{[Q\bar{Q}(n)]\to[Q\bar{Q}(\kappa)]} \otimes \mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}$$

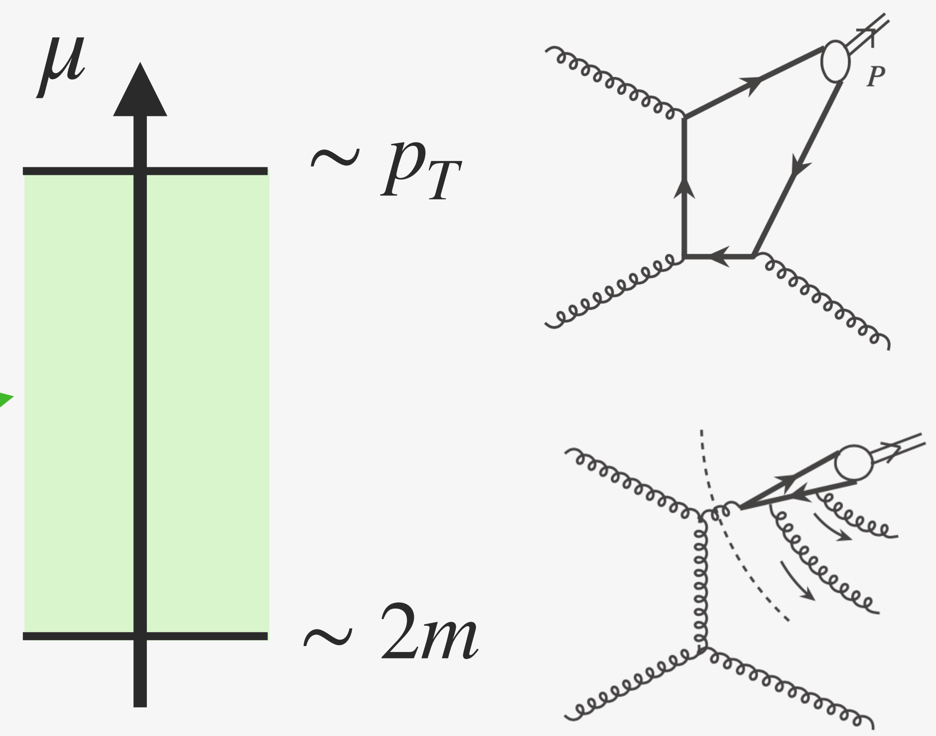


$f, f' = q, Q, g$

$\kappa, n = v^{[1,8]}, a^{[1,8]}, t^{[1,8]}$

The RG improved factorized cross section covers all events in which the heavy quark pair can be produced:

1. at the short-distance (p_T): early stage (**NLP**)
2. at the input scale ($2m$): later stage (**LP**)
3. in-between (**Quark pair power correction**)



Evolution of DP FFs in u, v-space

Lee, Qiu, Sterman, **KW**, arXiv:2211.12648 [hep-ph]

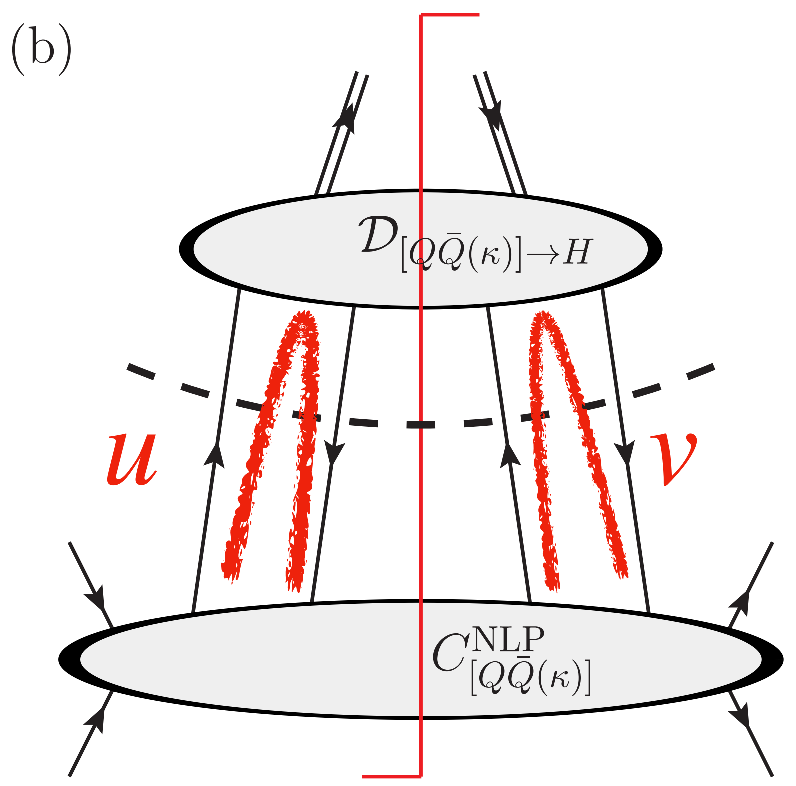
Consider the derivative of a test function:

$$D'_{\kappa\to n}(z,u,v) \equiv \frac{2\pi}{\alpha_s}\frac{dD_{\kappa\to n}(z,u,v)}{d\ln\mu^2},$$

$$D(z,u,v) \to D_z(z)D_u(u)D_v(v),$$

$$D_z(z,\alpha) = \frac{z^\alpha(1-z)^\beta}{B[1+\alpha,1+\beta]},$$

$$D_{u,v}(x,\gamma) = \frac{x^\gamma(1-x)^\gamma}{B[1+\gamma,1+\gamma]},$$

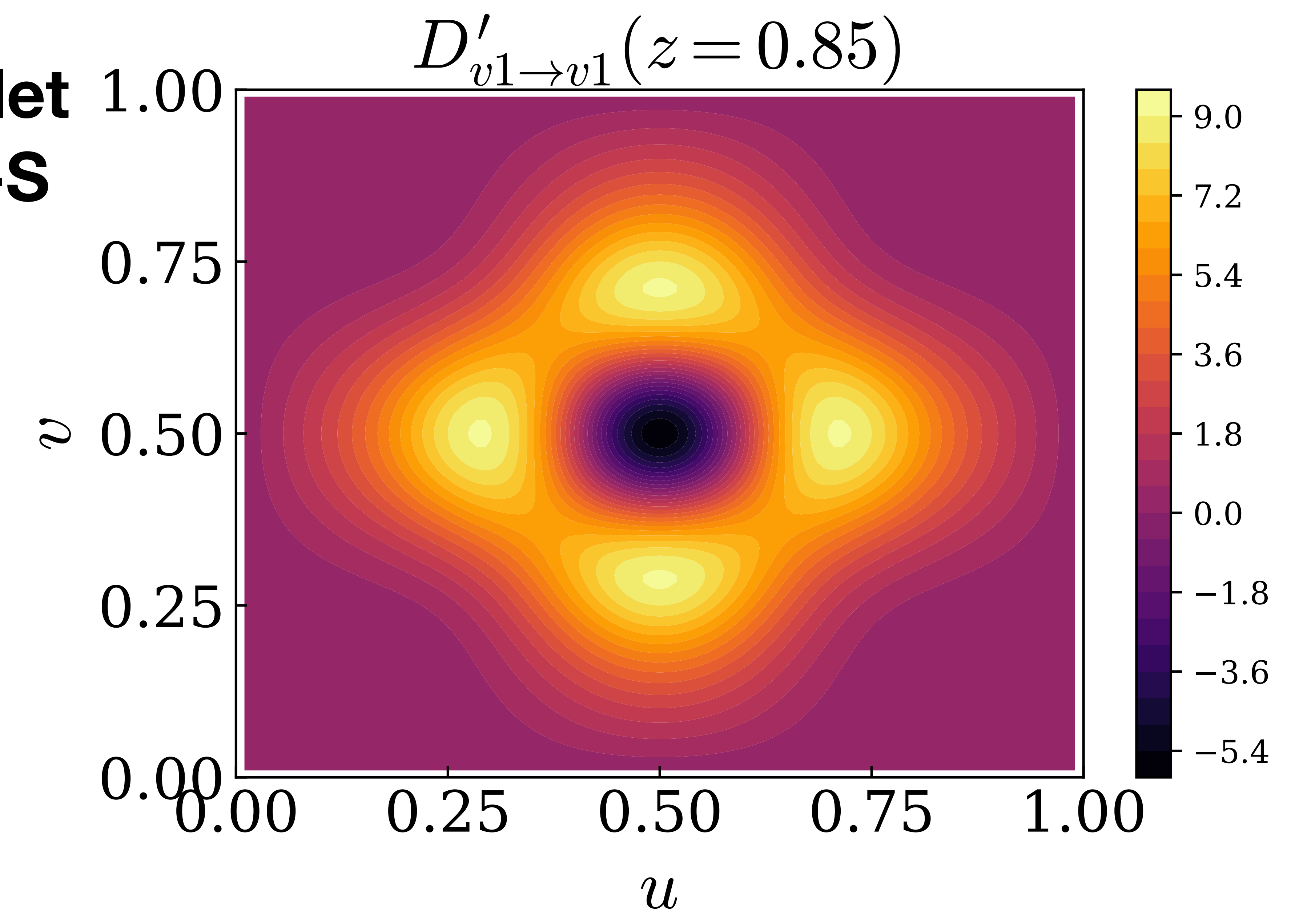


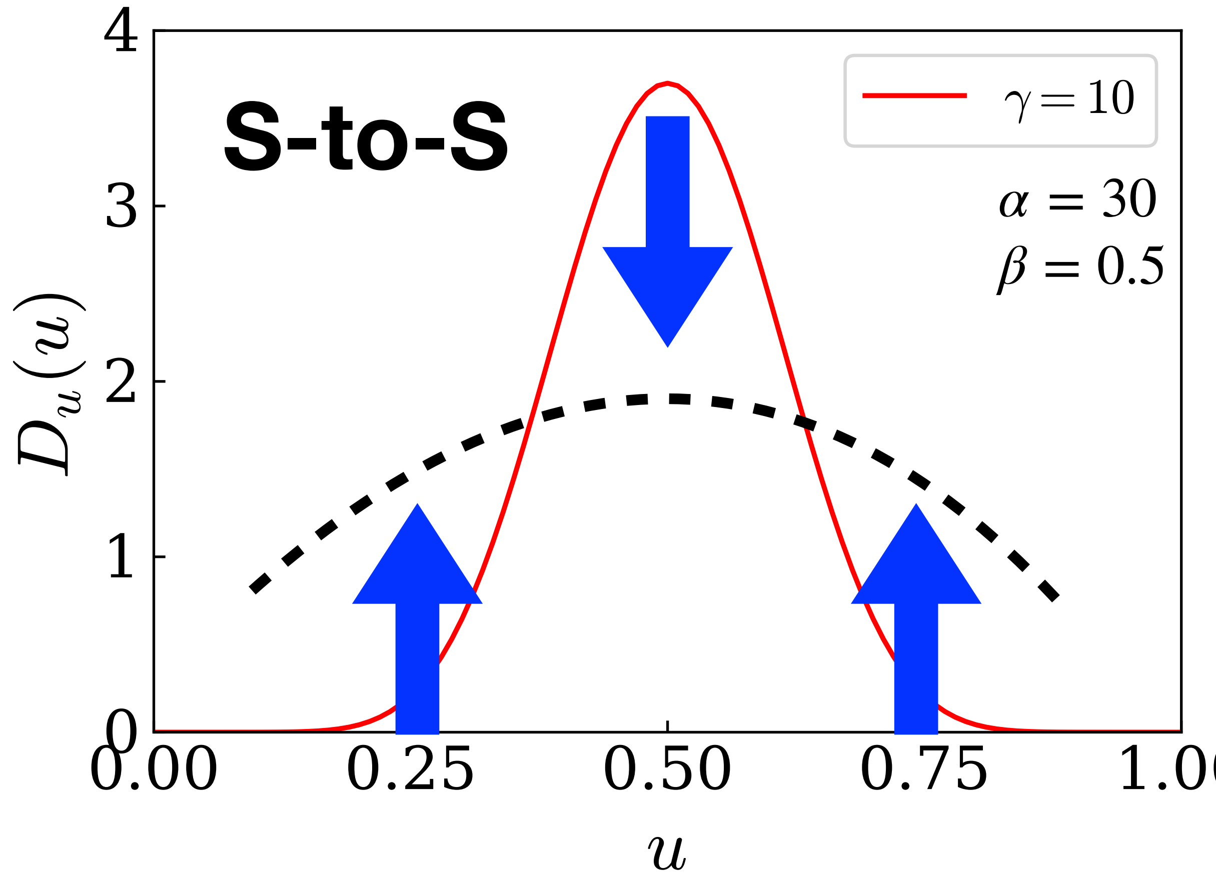
amplitude : $p_Q = up_c$

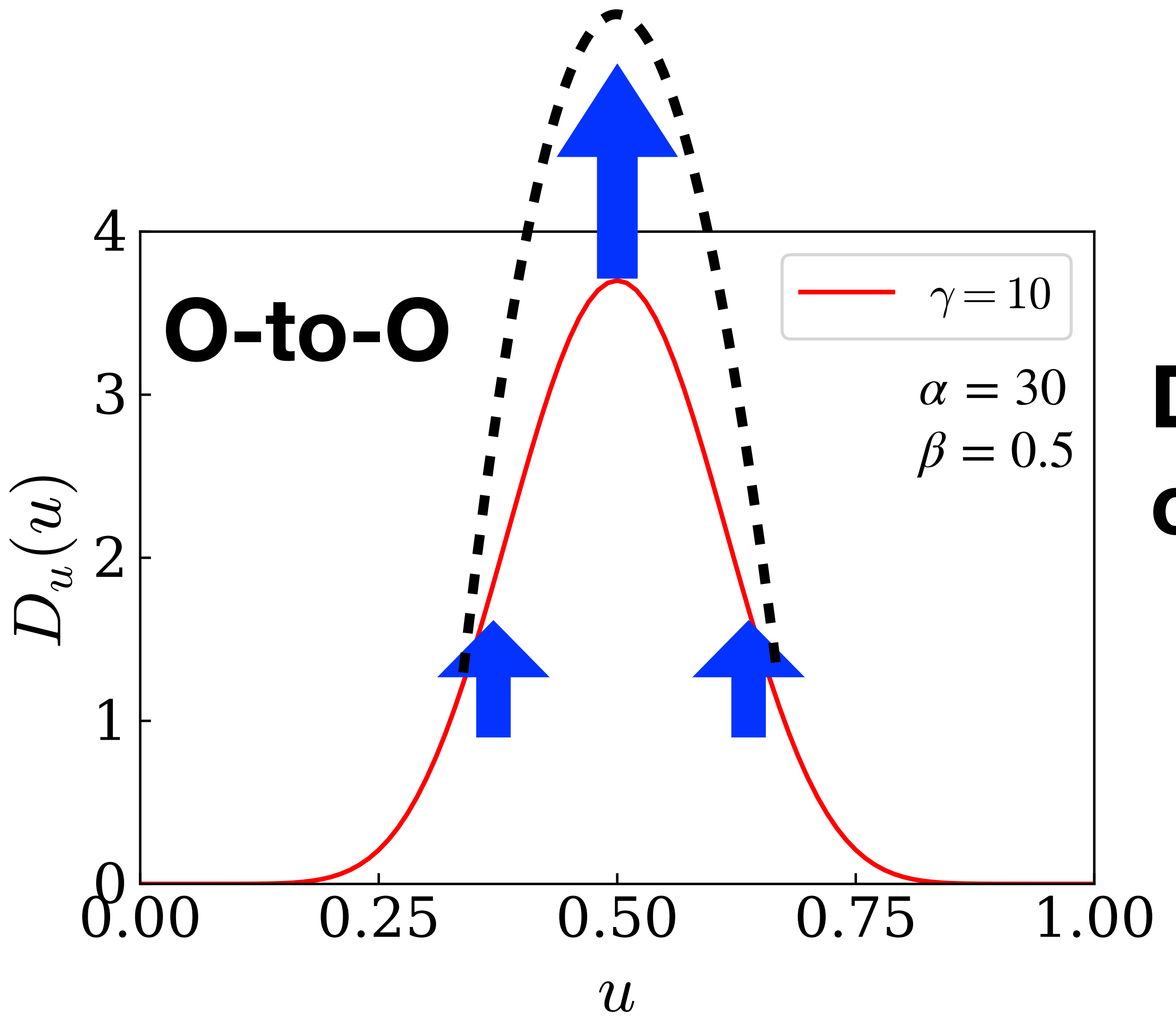
C.C. : $p_Q = vp_c$

with $zp_c^+ = p^+$

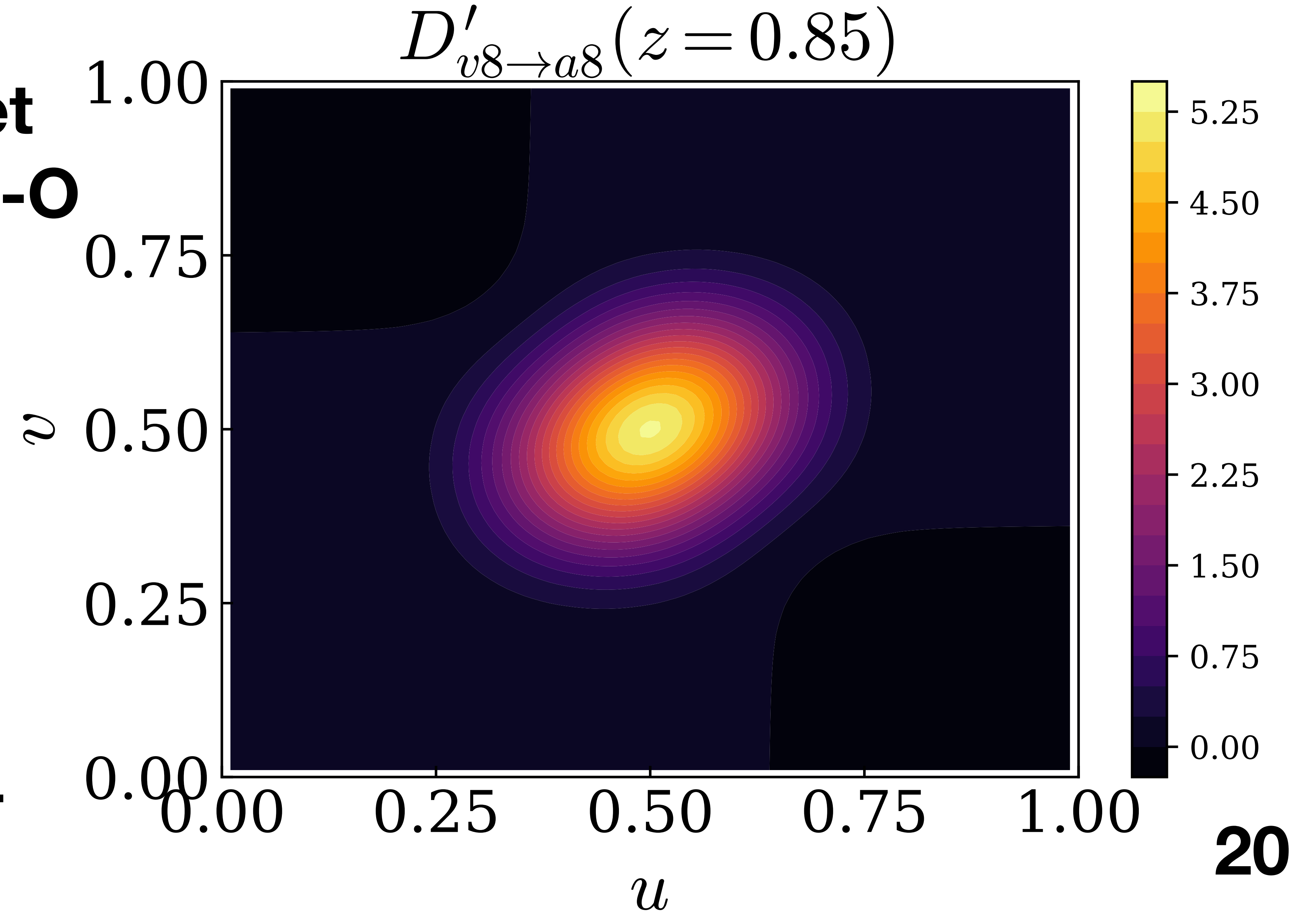
Diagonal singlet channel: S-to-S







Diagonal octet channel: O-to-O

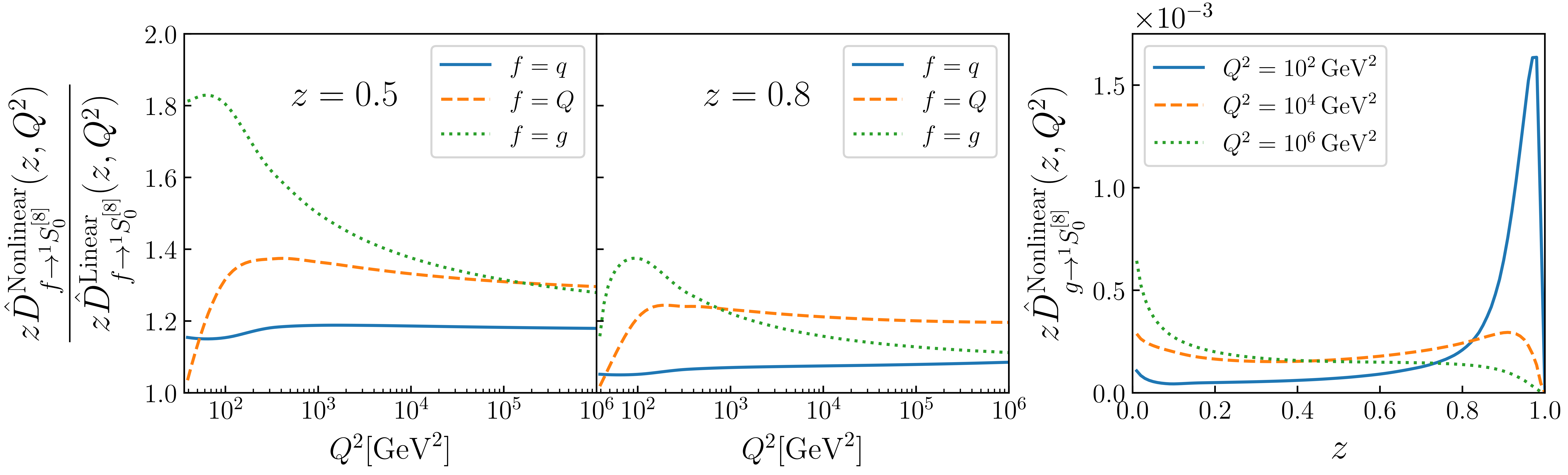


- S-to-S DP FFs get **broader** in u, v-space after evolution.
- O-to-O DP FFs become **narrower** with a large peak around $u = v = 1/2$.
- Off-diagonal channels: similar to O-to-O.

Quark pair corrections to SP FFs

Lee, Qiu, Sterman, **KW**, SciPost Phys. Proc.8, 143 (2022)

Lee, Qiu, Sterman, **KW**, in preparation.



$\alpha = 30, \beta = 0.5$

$\mu_0 = 4m_c = 6\,\text{GeV}$

$\mu_\Lambda = m_c = 1.5\,\text{GeV}$

The quark pair corrections to DGLAP evolution remain significant even at high $\mu^2 \sim p_T^2$.

$$\frac{\partial D_{f\to H}}{\partial \ln \mu^2} = \gamma_{f\to f'} \otimes D_{f'\to H} + \frac{1}{\mu^2}\gamma_{f\to[Q\bar{Q}(\kappa)]} \otimes \mathscr{D}_{[Q\bar{Q}(\kappa)]\to H}$$

$$\frac{\partial D^{\text{Inhomogeneous}}_{f\to H}}{\partial \ln \mu^2} \sim \frac{\partial D^{\text{Homogeneous}}_{f\to H}}{\partial \ln \mu^2}$$

$\mu^2 \to \infty$: the slope of $D_{f\to H}$ is the same as LP DGLAP.

Mueller and Qiu, NPB268, 427 (1986)

Qiu, NPB291, 746 (1987)

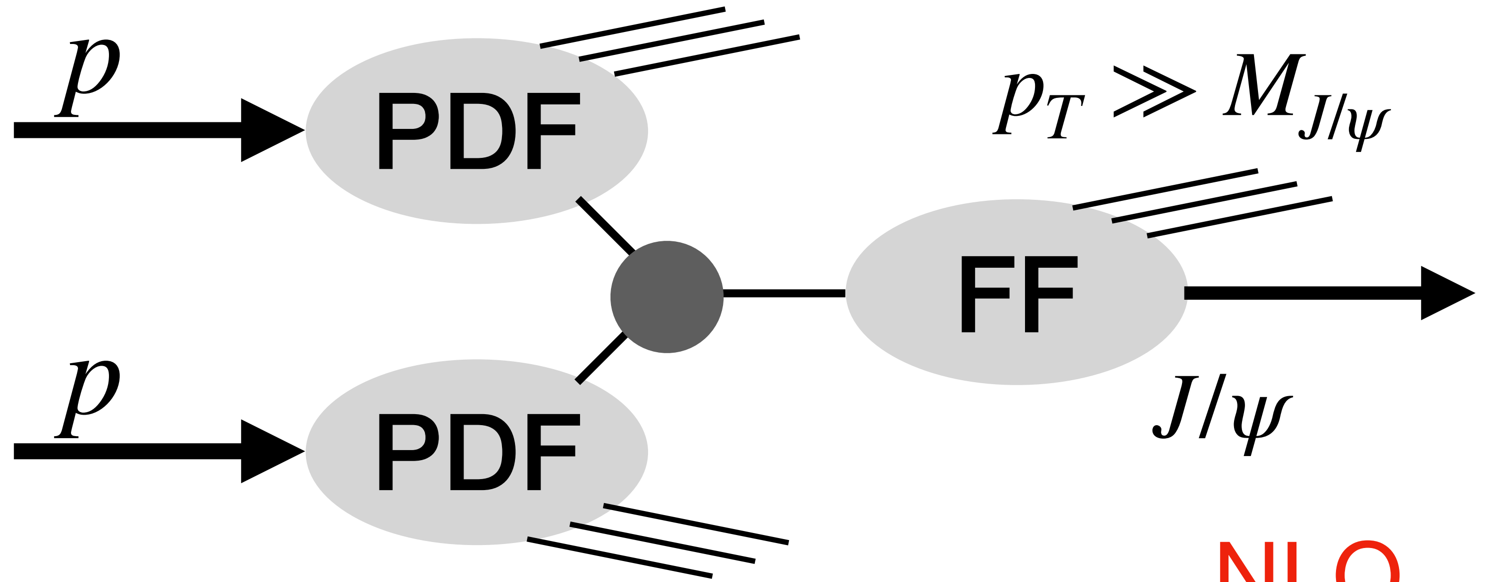
Eskola, Honkanen, Kolhinen, Qiu and Salgado, NPB660, 211 (2003)

The power corrections effect at low μ^2 does not go away fast: **analogous to nonlinear gluon recombination effects to gluon PDF at small-x and large μ^2.**

Phenomenology

$p + p \to J/\psi + X$

Lee, Qiu, Sterman, **KW**, arXiv:2211.12648 [hep-ph]



$$\frac{d\sigma_{p+p\to J/\psi+X}}{dp_T} \approx f_{i/p} \otimes f_{j/p} \otimes \left[D_k^{J/\psi} \otimes C_{ij\to k} + D_{c\bar{c}}^{J/\psi} \otimes C_{ij\to c\bar{c}} \right]$$

NLO ($C_{ij\to k}$) $\mathrm{LO} \times K_{\mathrm{NLP}}$ ($C_{ij\to c\bar{c}}$)

Input fragmentation functions:

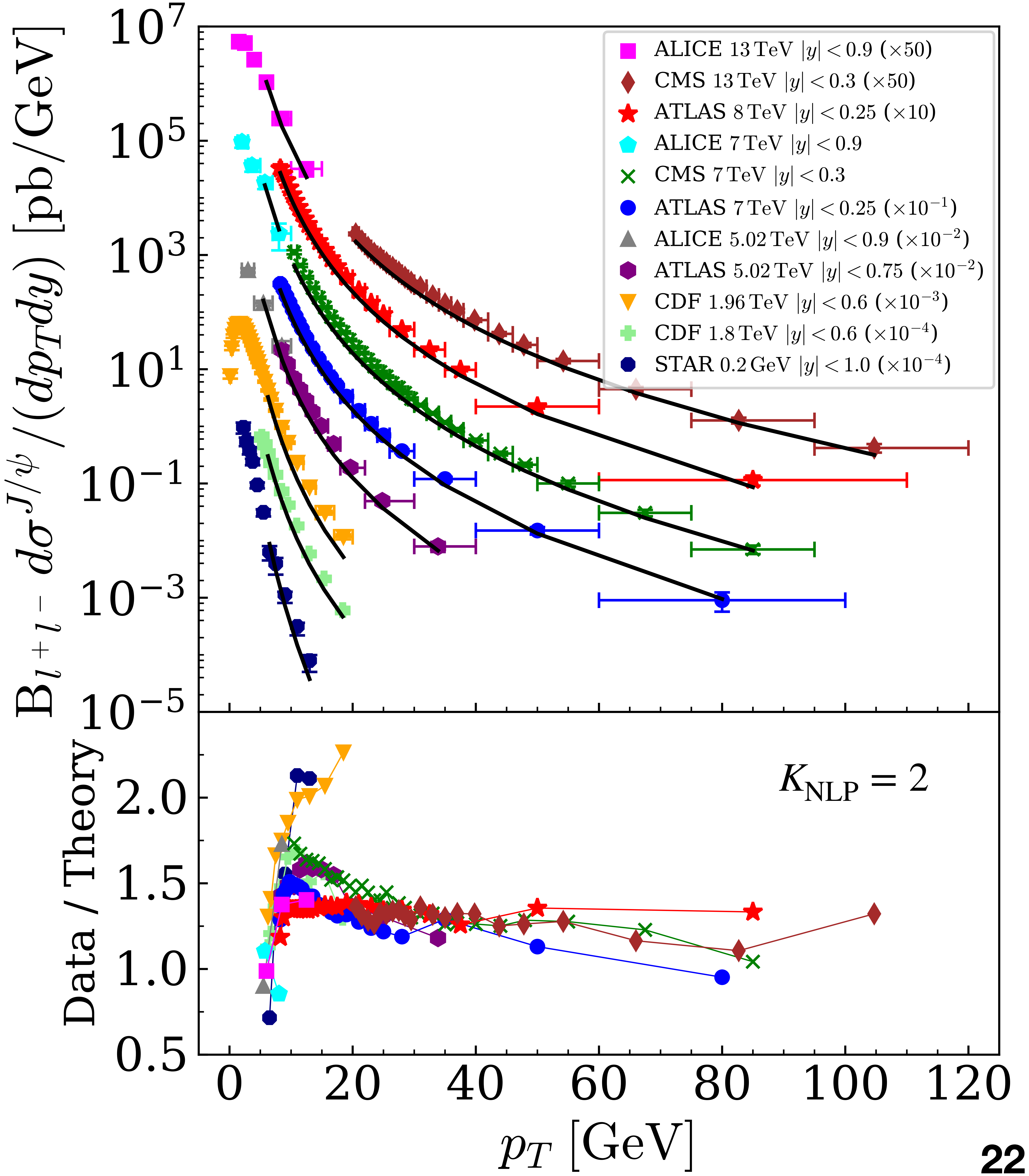
$$D_{f\to H}(z;m,\mu_0) = \sum_{[Q\bar{Q}(n)]} \pi\alpha_s \left\{ \hat{d}^{(1)}_{f\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \frac{\alpha_s}{\pi}\hat{d}^{(2)}_{f\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \mathcal{O}(\alpha_s^2) \right\} \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m^{2L+3}}$$

LDMEs

$$D_{[Q\bar{Q}(\kappa)]\to H}(z;m,\mu_0) = \sum_{[Q\bar{Q}(n)]} \left\{ \hat{d}^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \frac{\alpha_s}{\pi}\hat{d}^{(1)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \mathcal{O}(\alpha_s^2) \right\} \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m^{2L+1}}$$

$\mu_0 = \mathcal{O}(2m)$: input scale, $\mu_\Lambda = \mathcal{O}(m)$: NRQCD factorization scale

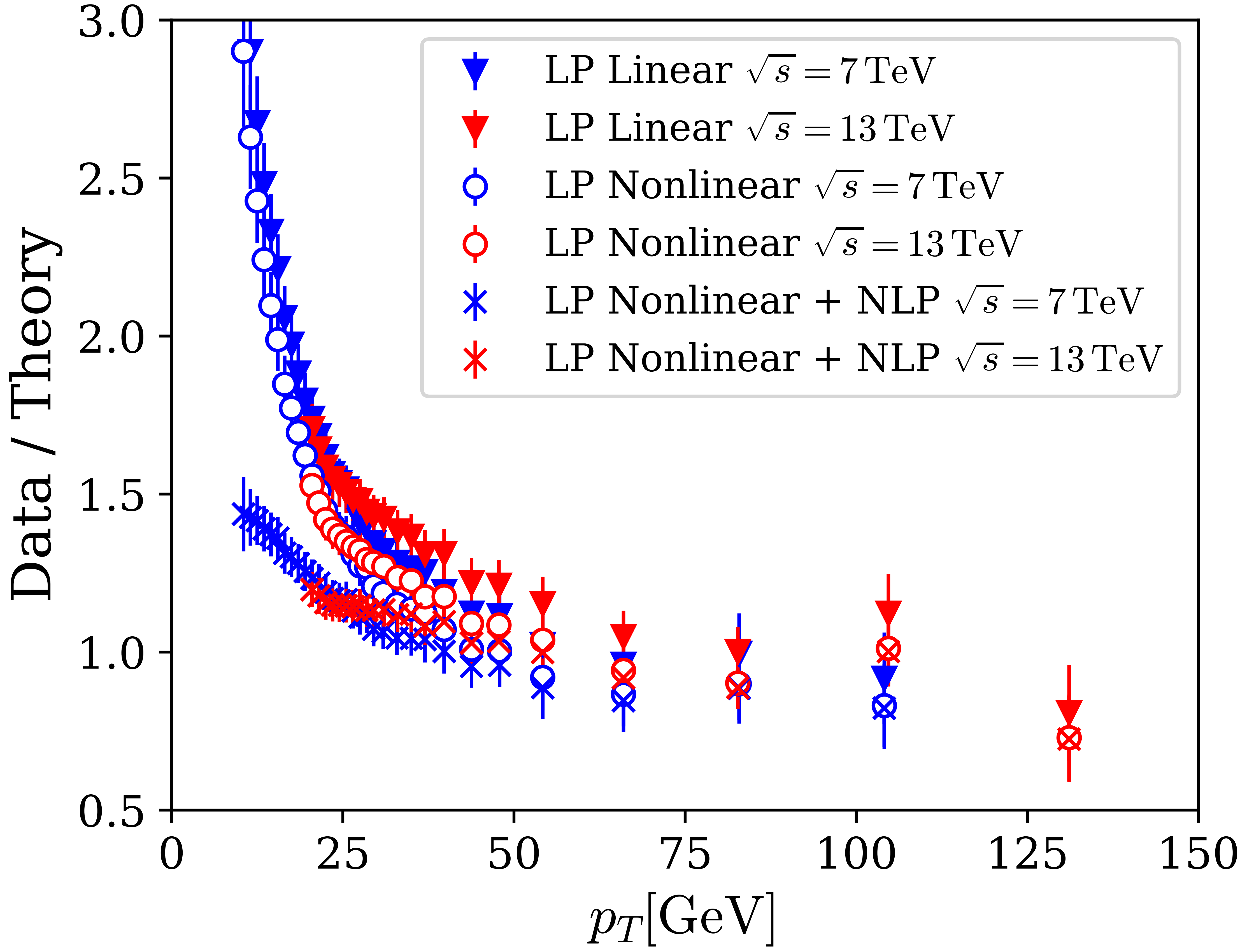
$\kappa = v^{[c]}, a^{[c]}, t^{[c]}, \quad n = {}^{2S+1}L_J^{[c]}$



${}^1S_0^{[8]}$ dominant scenario

Lee, Qiu, Sterman, **KW**, SciPost Phys. Proc.8, 143 (2022)

- Fitting the LP formalism with the linear DGLAP evolution eq. to CMS data on high p_T prompt J/ψ at $\sqrt{s} = 7, 13\,\mathrm{TeV}$ in the bin, $|y| < 1.2$.
- Only the ${}^1S_0^{[8]}$ channel is considered, yielding unpolarized J/ψ. Combining LP and NLP could overshoot data for the other two color octet channels.
- $\langle \mathcal{O}({}^1S_0^{[8]}) \rangle / \mathrm{GeV}^3 = 0.1286 \pm 5.179 \cdot 10^{-3}$ fitted by high p_T data is similar to the one extracted using fixed order NRQCD at NLO. Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 (2012)



The power corrections do not vanish even at the highest p_T, giving 10-30% corrections. At $p_T = 30\,\mathrm{GeV}$ and below, the NLP corrections become significant.

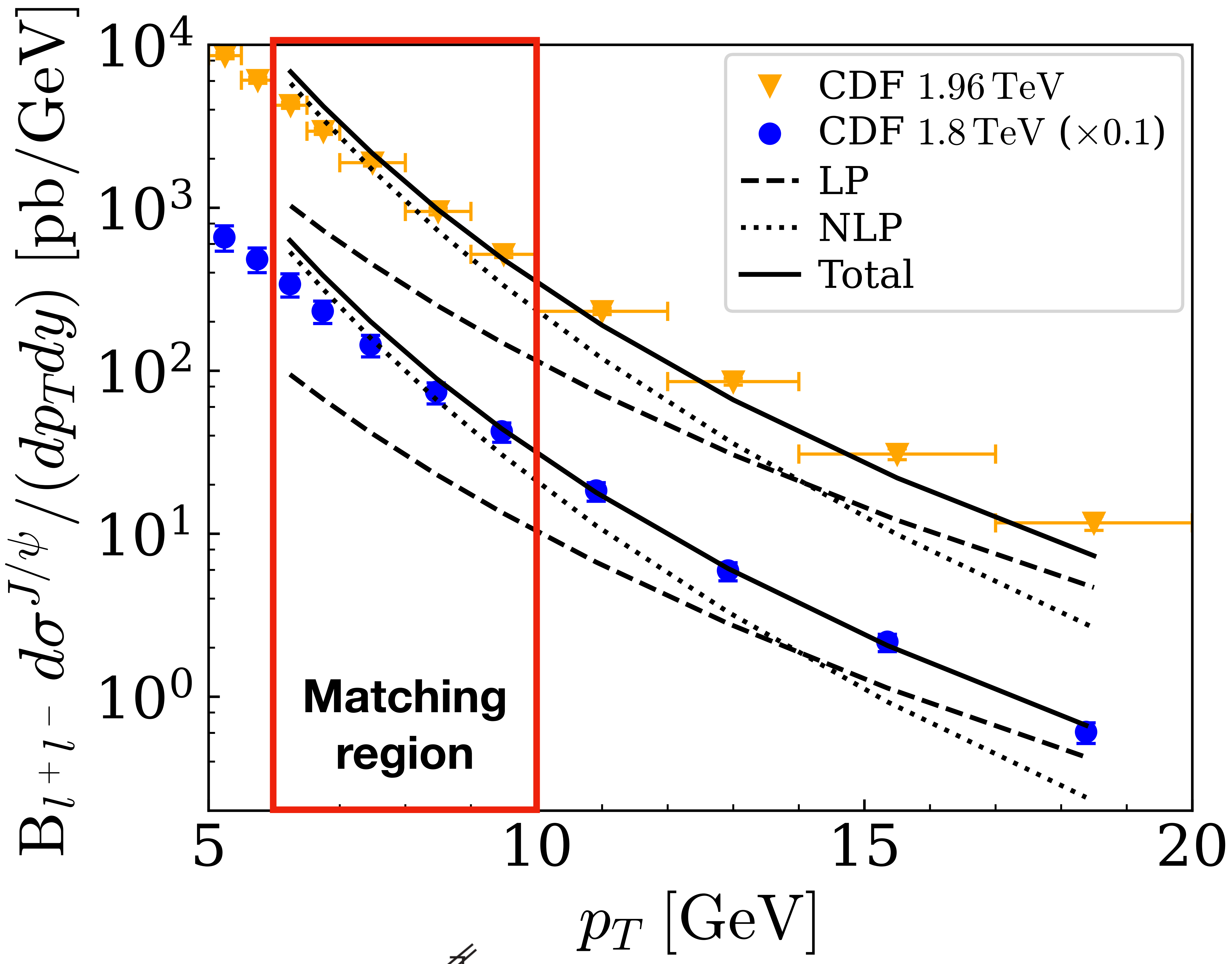
Toward the matching to NRQCD

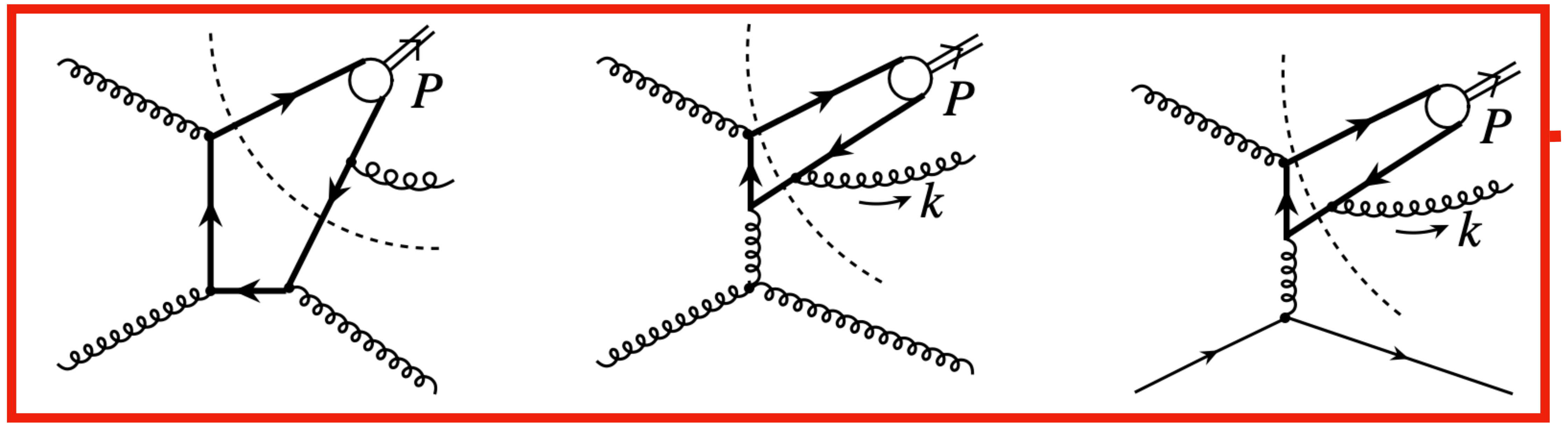
Lee, Qiu, Sterman, **KW**, arXiv:2211.12648 [hep-ph]

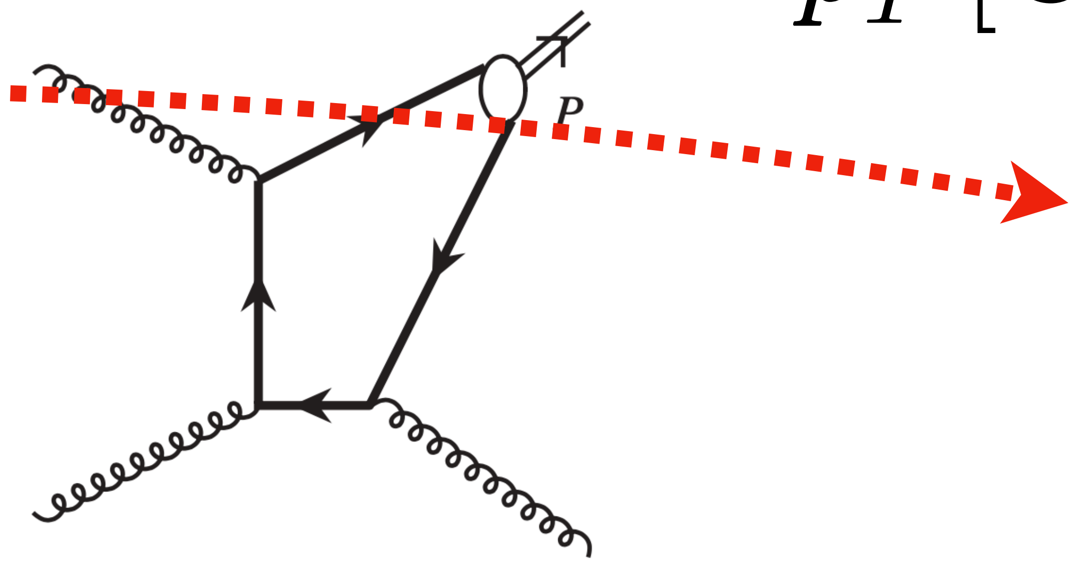
1. $\ln(p_T^2/m^2)$-type logarithmically enhanced contributions start to dominate when $p_T \gtrsim 5 \times (2m_c) \sim 15\,\mathrm{GeV}$, where the LP is significant, power corrections are small.

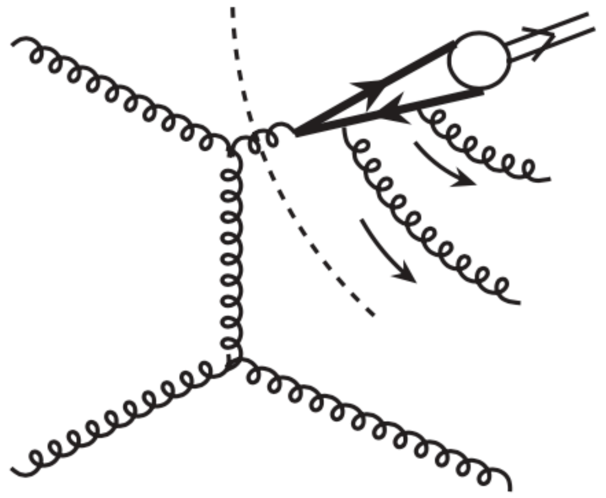
2. The NLP contribution is important at $p_T \lesssim 10\,\mathrm{GeV} = \mathcal{O}(2m_c)$, where matching between QCD factorization and NRQCD factorization can be made.

3. Further exploration of the shape of the FFs at large-z would help us understand the quarkonium production mechanism.

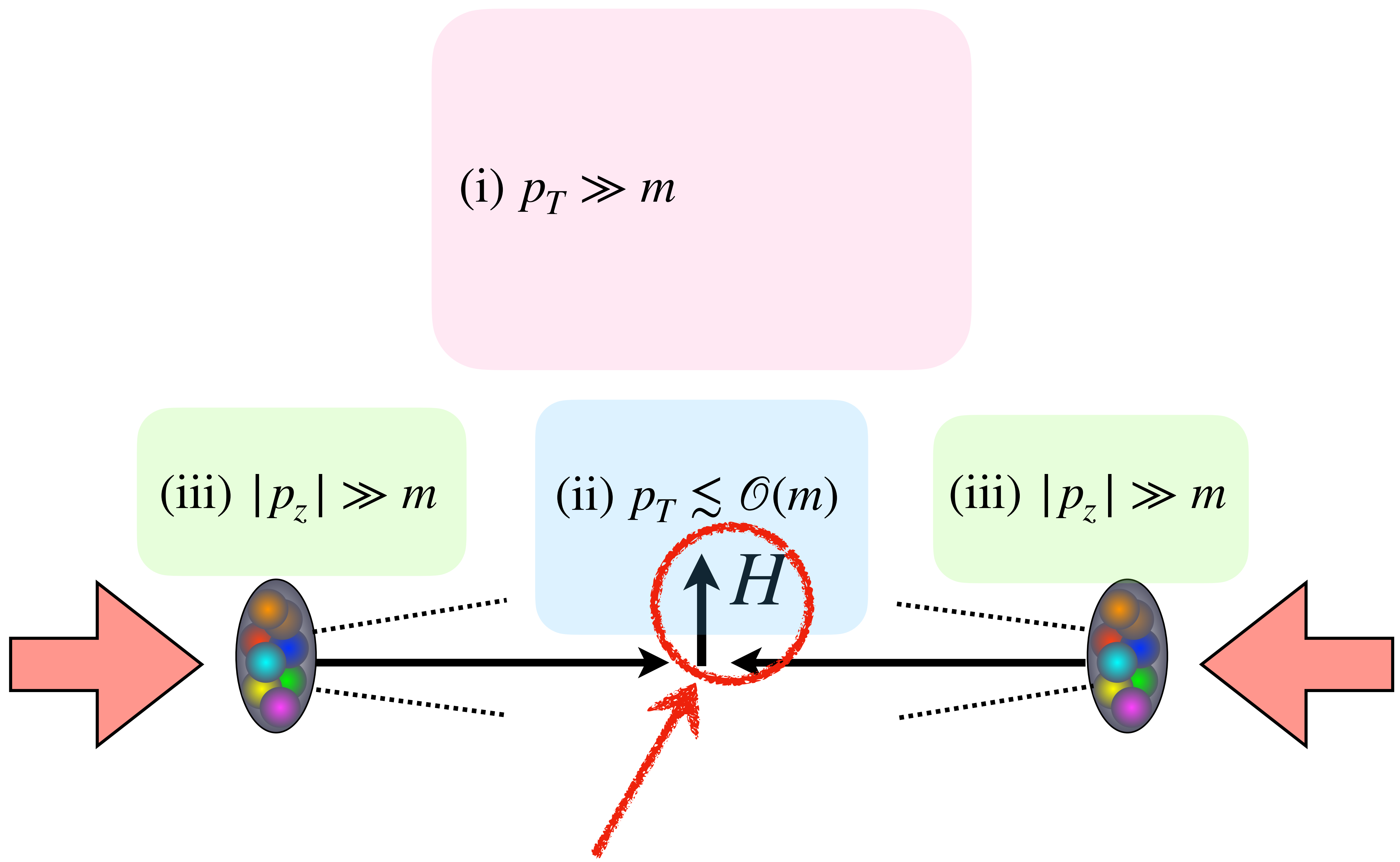






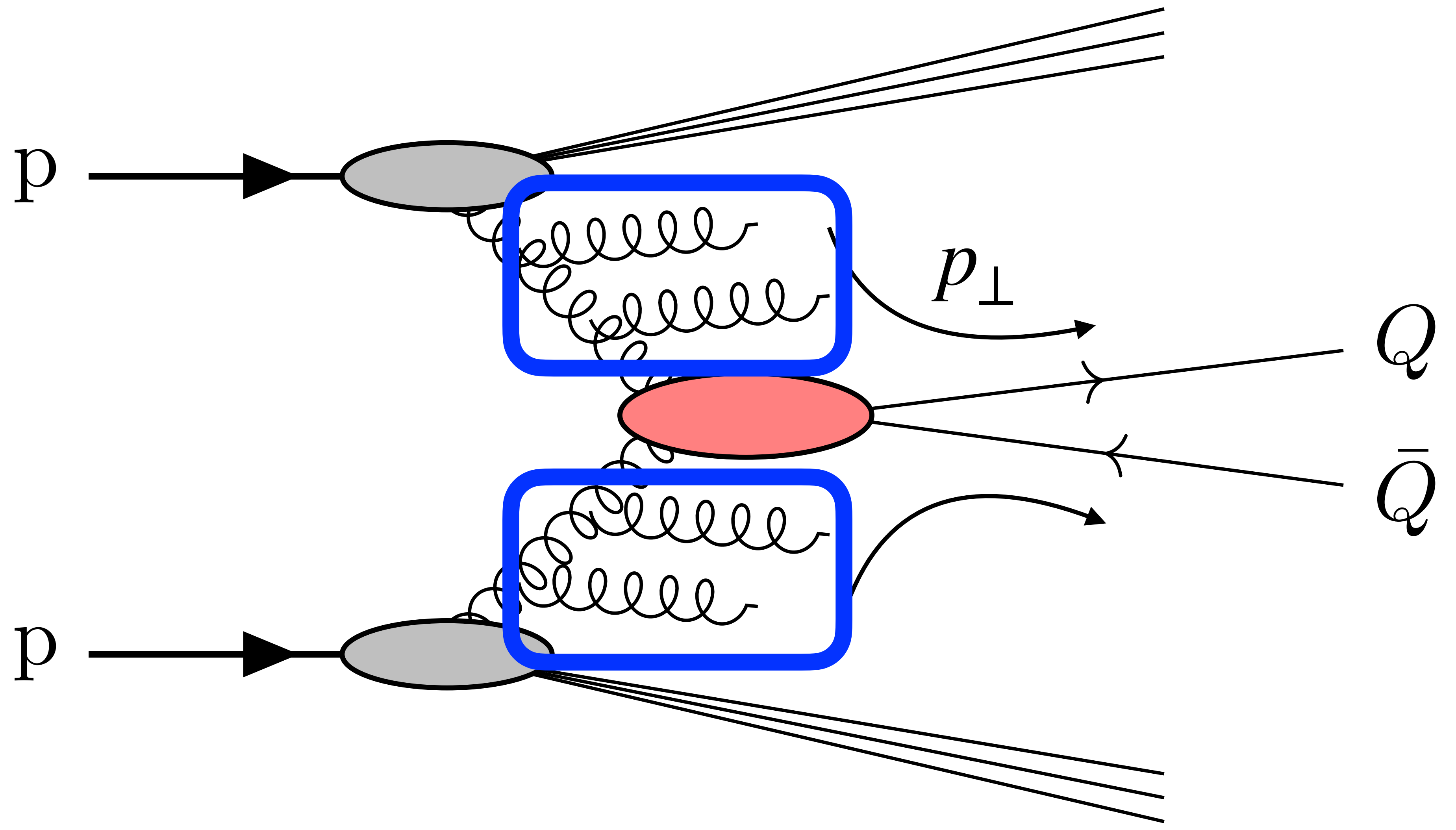


II-2. Hadronic quarkonium production of low p_T

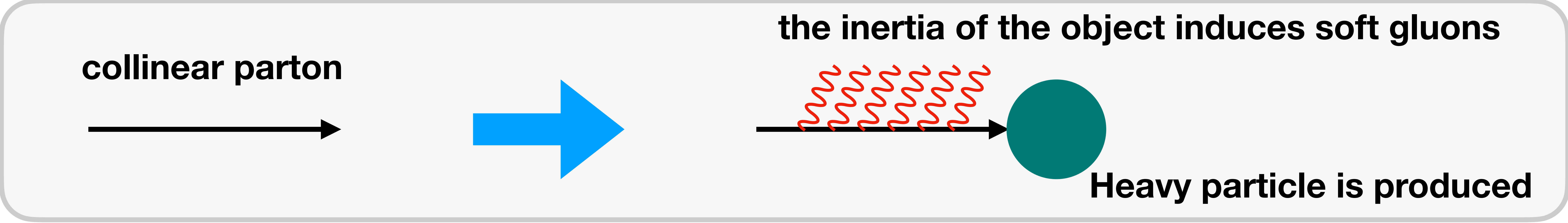


Heavy quark pair production of low p_T

- When $M^2 \sim (2m)^2 \gg p_T^2 \gg \Lambda_{\rm QCD}^2$:
  - ✔ The short distance part for heavy quark pair production is calculated at LO in α_s: less recoil particles.
  - ✔ $M^2 \gg p_T^2 \gg \Lambda_{\rm QCD}^2$ opens the phase space for soft gluon radiations: $\ln(M^2/p_T^2)$ -type logarithmic enhancement needs to be resummed.
  - ✔ The heavy quark pair's p_T is provided by the initial-state gluon shower, not recoil gluons.



Initial-state soft-collinear gluon shower → Sudakov form factor



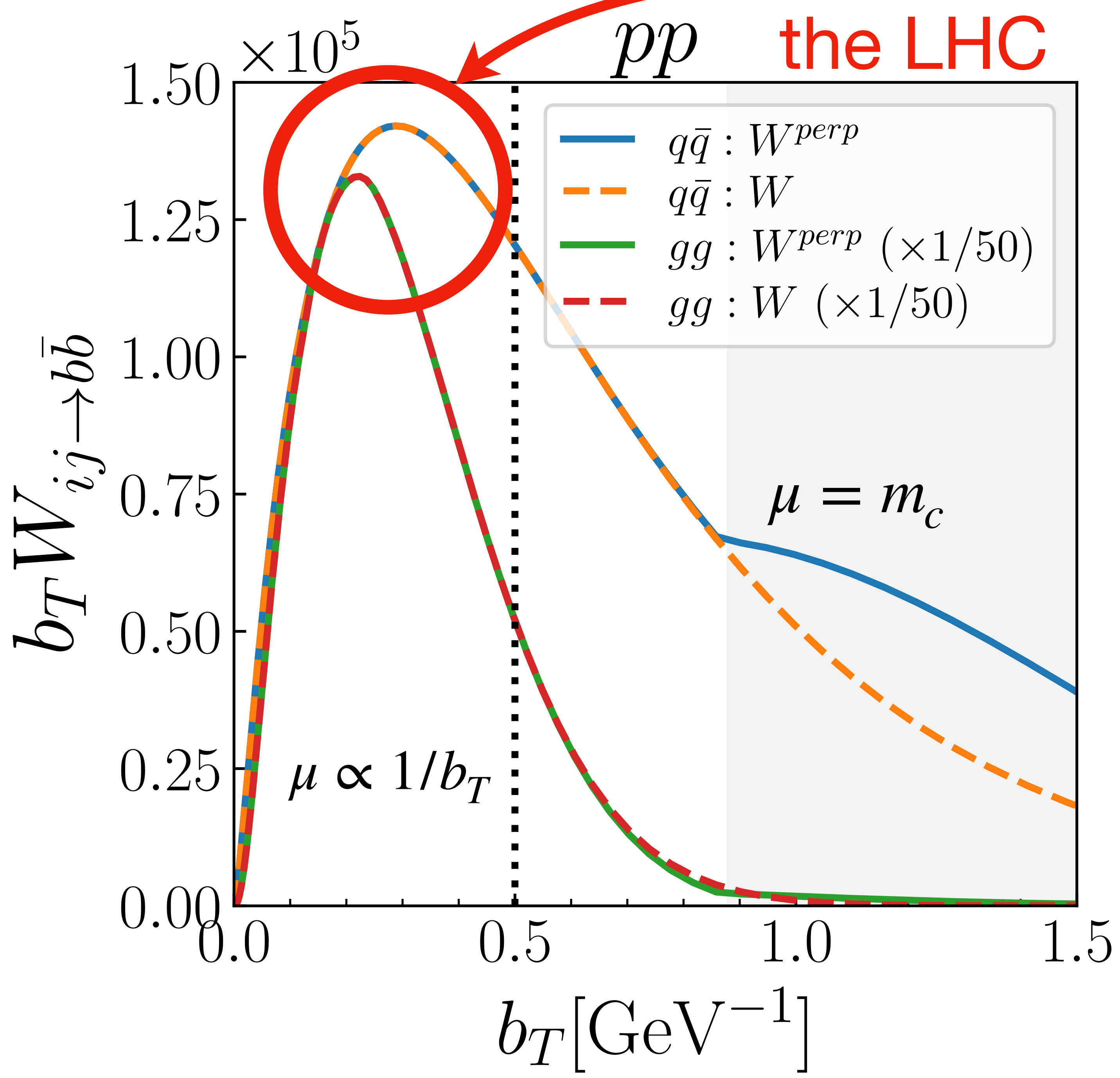
Υ production

NB: J/ψ is a light quarkonium, so nonperturbative gluon shower is significant with weak predictive power.

Collins-Soper-Sterman (CSS) formalism:

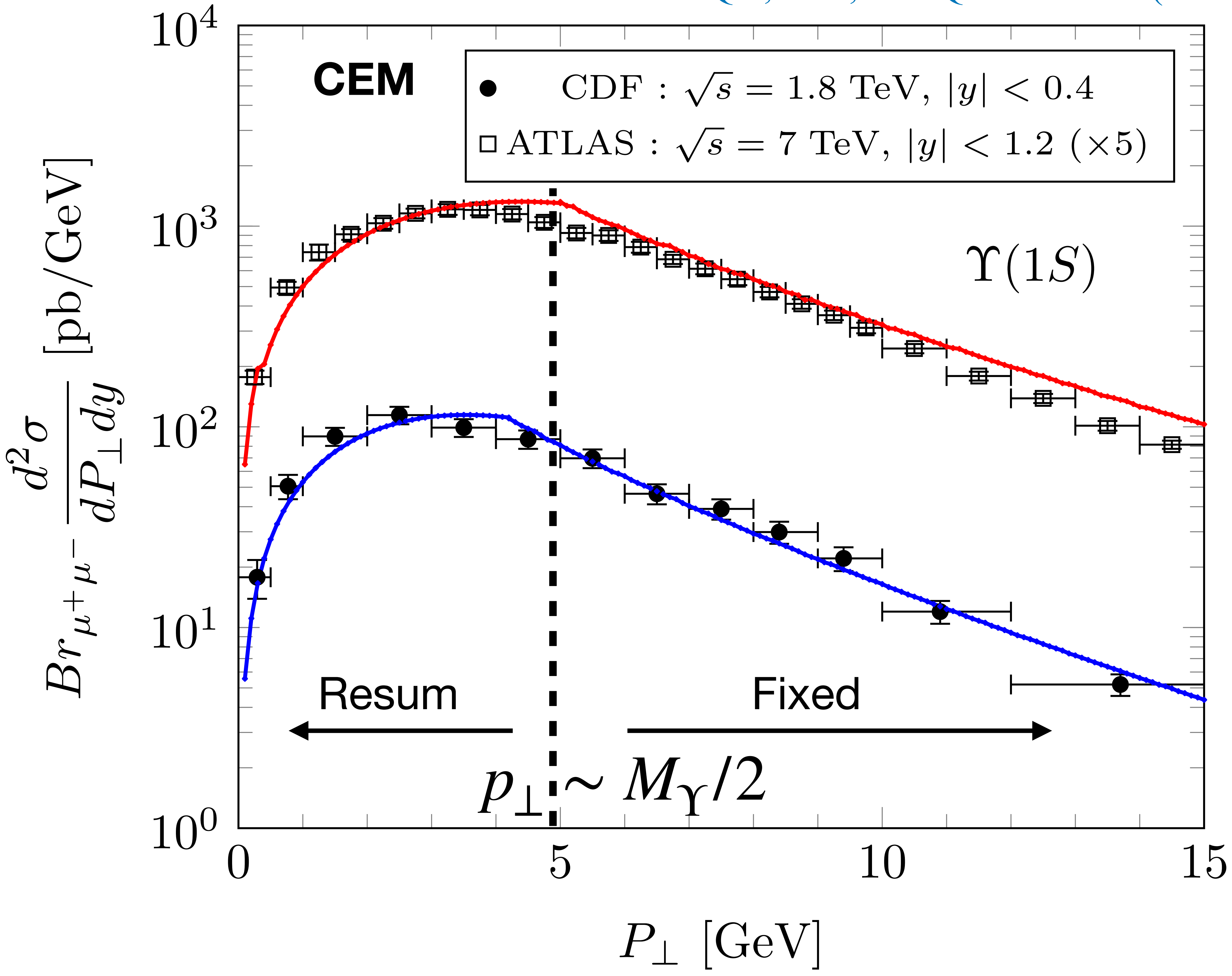
$$\left.\frac{d\sigma_{A+B\to Q\bar{Q}+X}}{d^2p_\perp^2 dy}\right|_{\text{resum}} = \int \frac{db_\perp}{2\pi} J_0(P_\perp b_\perp)\left(\sum_q b_\perp W_{q\bar{q}} d\hat{\sigma}_{q\bar{q}\to Q\bar{Q}} + b_\perp W_{gg} d\hat{\sigma}_{gg\to Q\bar{Q}}\right)$$

Perturbative gluon shower at the LHC



Fourier Transformation

Qiu, **KW**, PoS QCDEV2017 (2017) 024

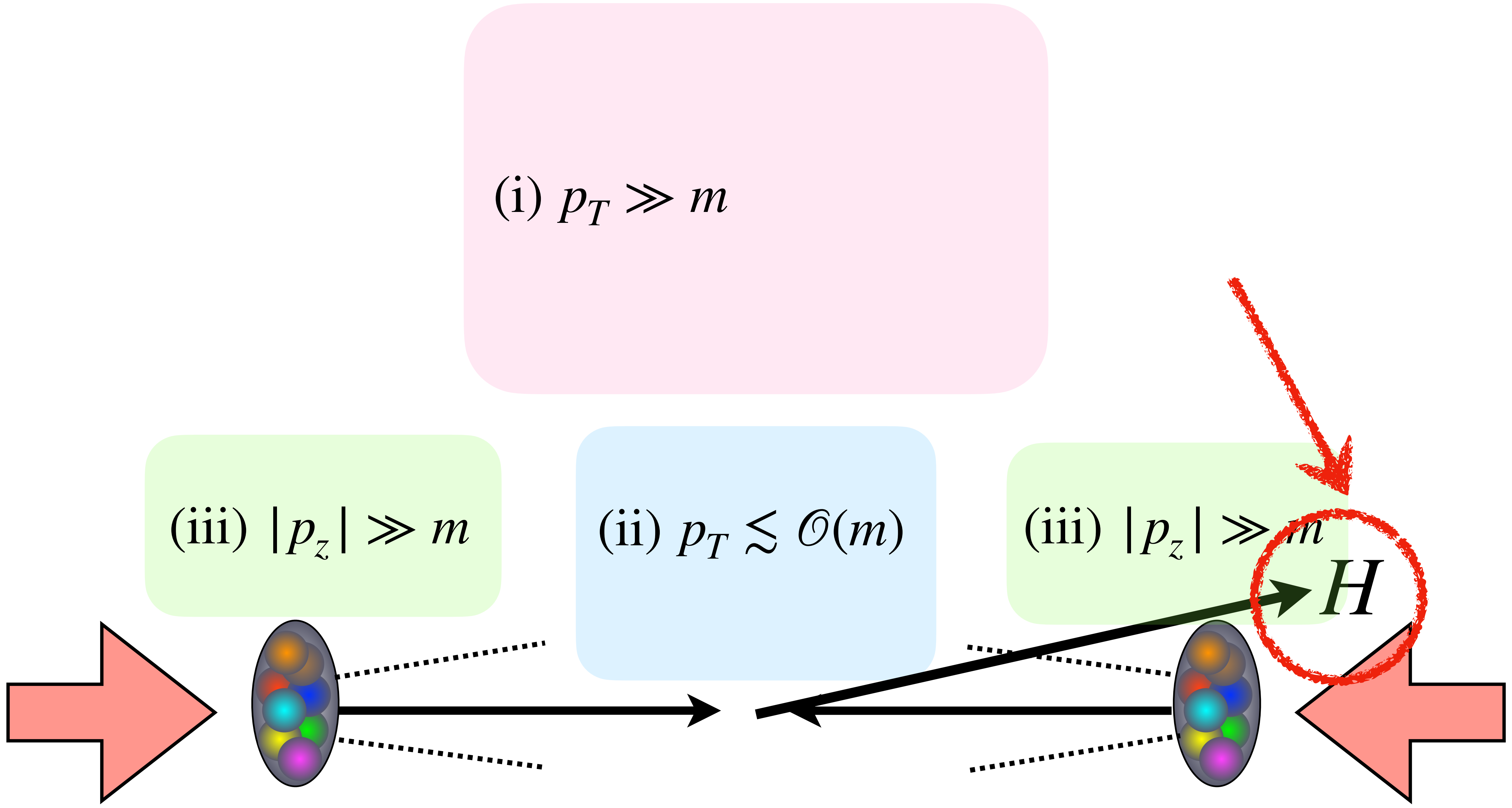


Matching condition

$$\frac{d\sigma_{A+B\to\psi+X}(m\neq 0)}{d^2p_T dy} = \frac{d\sigma^{\text{Resum}}_{A+B\to\psi+X}(m\neq 0)}{d^2p_T dy} + \frac{d\sigma^{\text{NRQCD}-(n)}_{A+B\to\psi+X}(m\neq 0)}{d^2p_T dy} - \frac{d\sigma^{\text{Asym}-(n)}_{A+B\to\psi+X}(m\neq 0)}{d^2p_T dy}$$

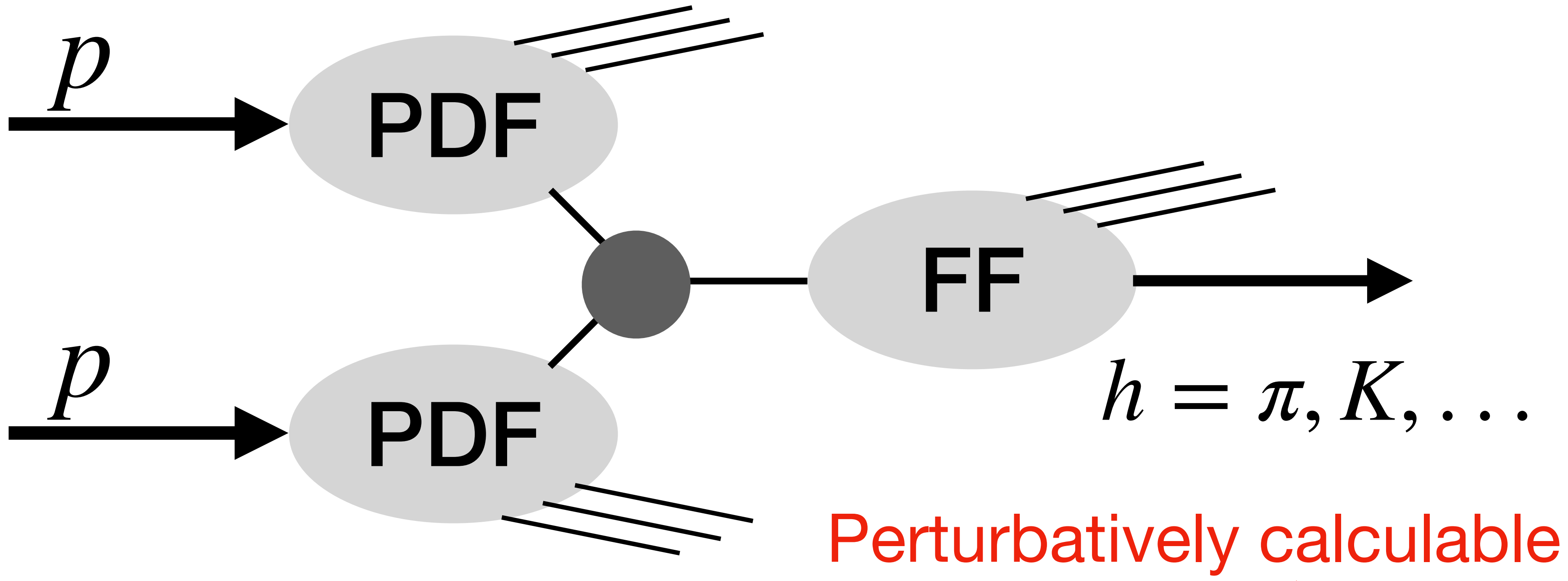
$p_T \ll M$ $\quad$ $p_T \gtrsim \mathcal{O}(M)$ $\quad$ subtract double counting

III. Forward production and nuclear effects

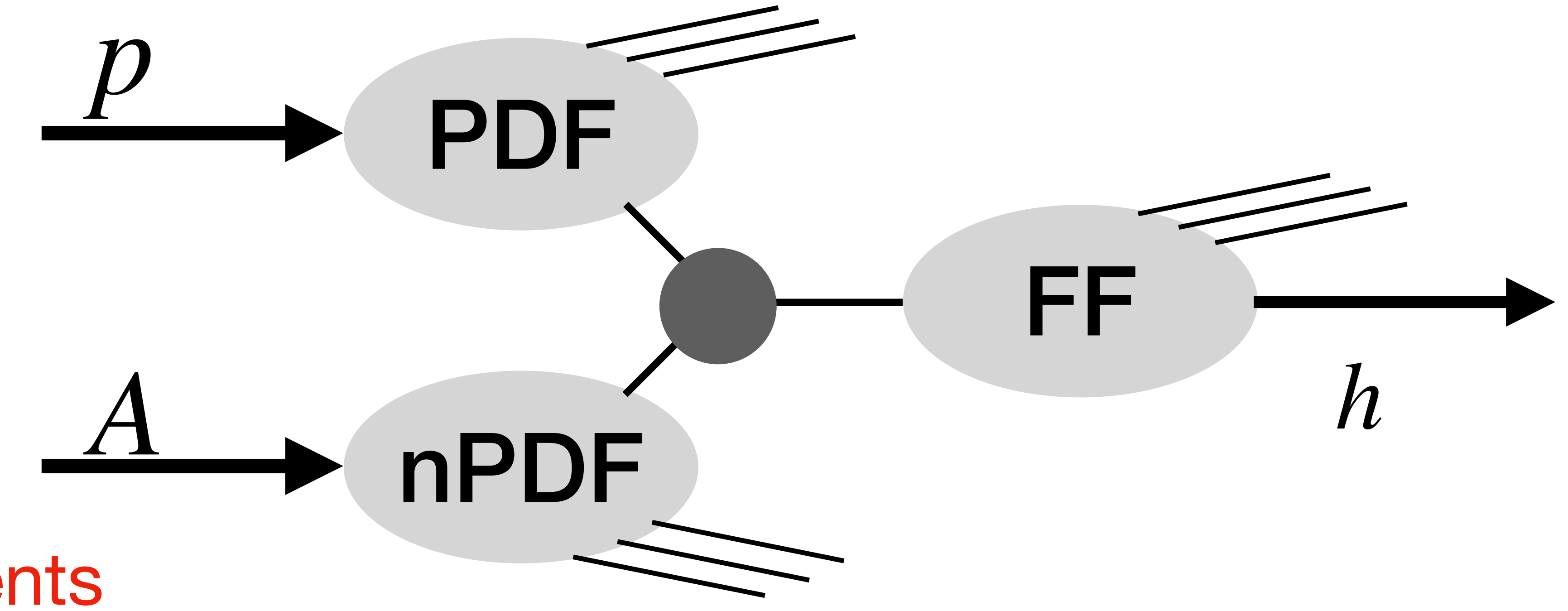


Naive changeover from pp to pA

Hadronic collision: $p + p \to h + X$



hadron-ion collision: $p + A \to h + X$



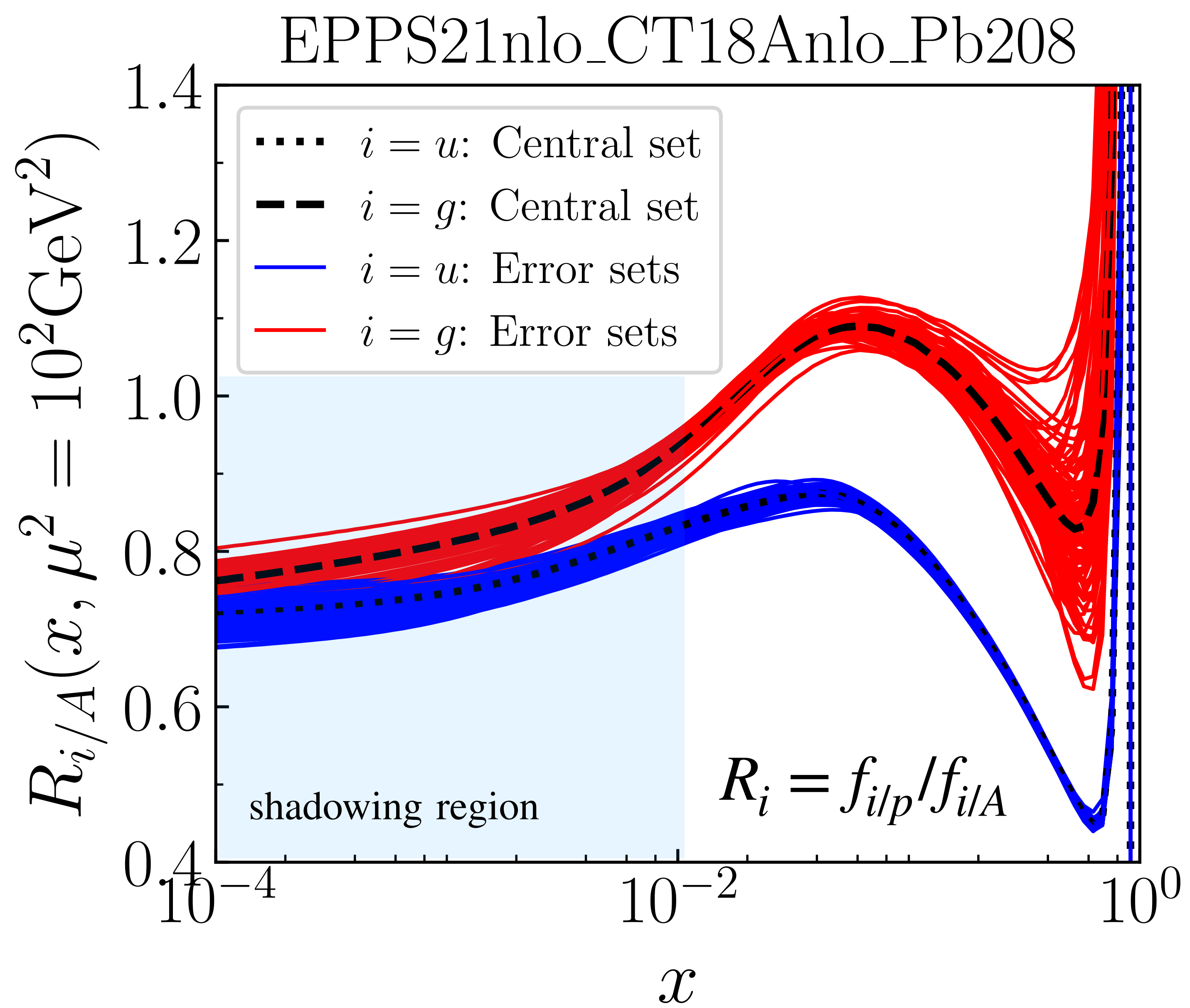
Perturbatively calculable coefficients

$$d\sigma_{p+p\to h+X} \approx f_{i/p} \otimes f_{j/p} \otimes D_k^h \otimes C$$

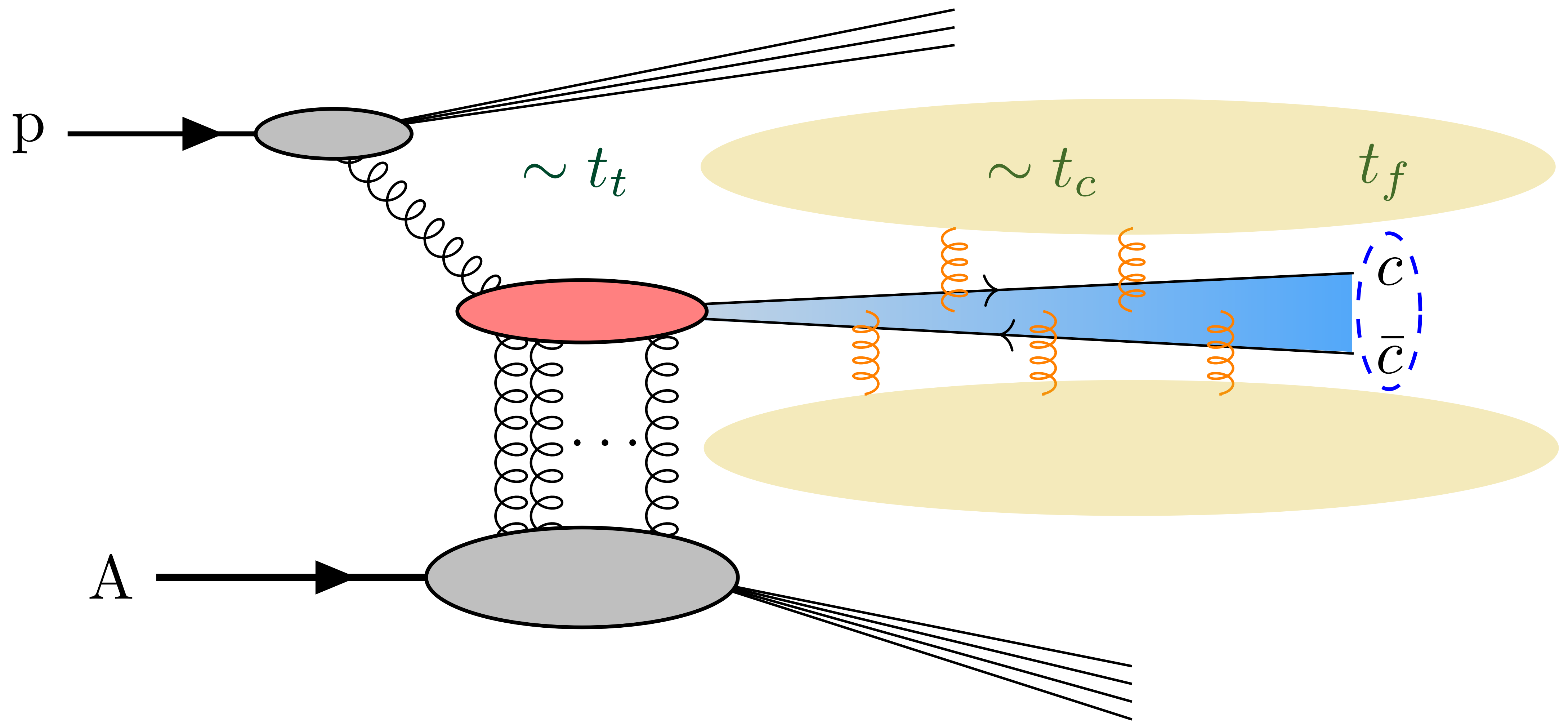
Nonperturbative universal functions

$$d\sigma_{p+A\to h+X} \approx f_{i/p} \otimes A f_{j/A} \otimes D_k^h \otimes C$$

- Proton's leading twist PDFs are replaced with nuclear PDFs.
- Furthermore, nuclear multiple scattering can change:
  1. the p_T distribution (Cronin effect)
  2. the invariant mass of $Q\bar{Q}$ (nuclear broadening)



Factorization and its breaking



$Q\bar{Q}$ can hadronize outside the nuclear matter at forward rapidity.
Only the multiple scattering of incoming partons and $Q\bar{Q}$ (not quarkonium) does matter.

- At backward rapidity, a quarkonium bound state could be formed inside the nucleus. The multiple scattering can interfere with the hadronization: no factorization.
- If $p_T \sim mv \sim Mv/2 \rightarrow v$-expansion in NRQCD is not ensured.

Brodsky and Mueller, PLB206, 685-690 (1988)

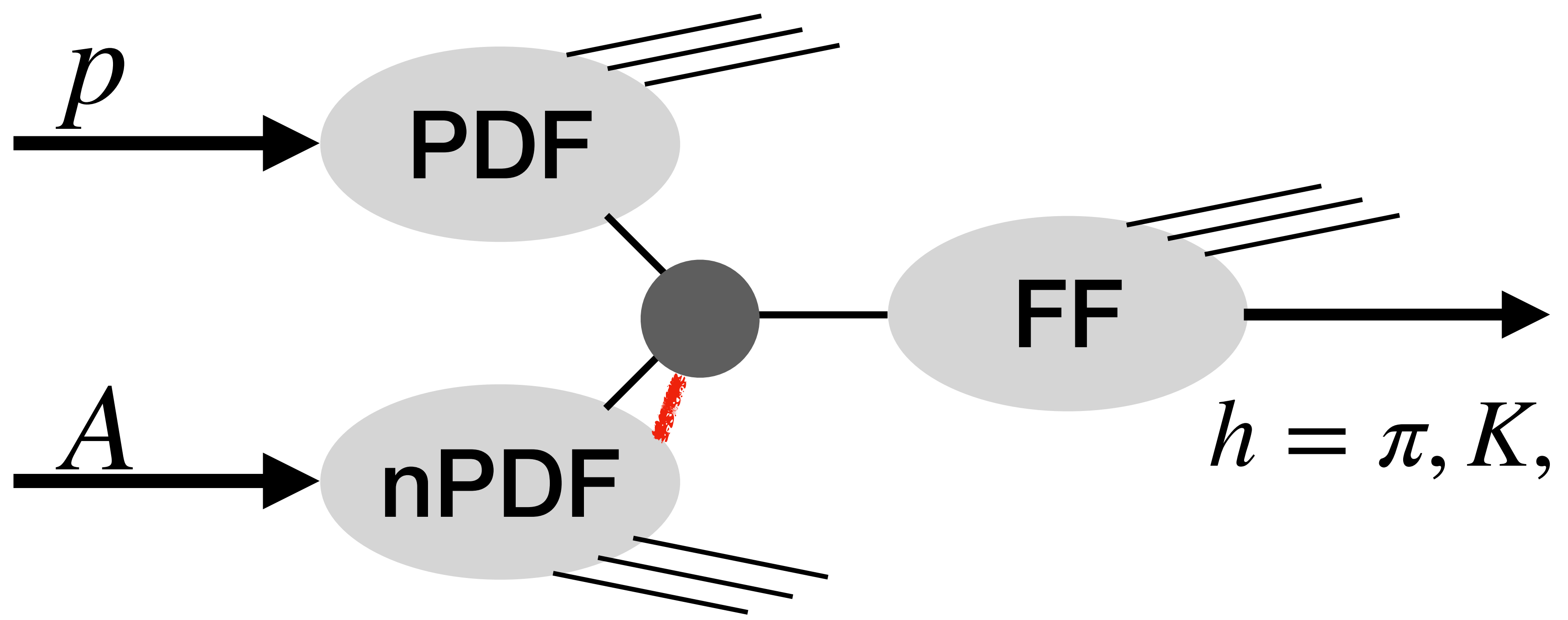
- At forward rapidity, spectator interactions could interfere with the hadronization: break of factorization.
- However, due to Lorentz time dilation, the hadronization of $Q\bar{Q}$ can be effectively frozen when it passes through the nucleus:

$$\frac{1}{mv}\frac{p_z}{M} \gg \frac{1}{p_T} \quad \text{or} \quad y \gg \ln\frac{Mv}{p_T}$$

Qiu, Sun, Xiao and Yuan, PRD89, 034007 (2014)
Kharzeev and Tuchin, NPA770, 40 (2006)
Ma, Venugopalan, **KW**, Zhang, PRC97 (2018) 1, 014909

Profound multiple scattering effects

hadron-ion collision: $p + A \to h + X$



$h = \pi, K, \ldots$

amplified by nuclear size

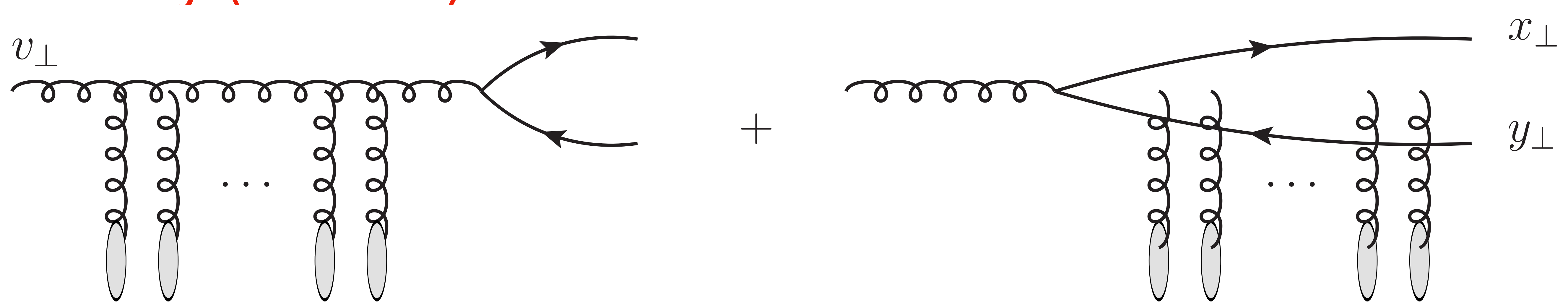
$$d\sigma_{p+A\to h+X} \approx f_{i/p} \otimes A f_{j/A} \otimes D_k^h \otimes C + \mathcal{O}\left(\frac{A^{1/3}}{Q^2}\right) + \cdots$$

hard interactions involving more than one nucleon are suppressed by powers of Q^2

- Each scattering in the nuclear target is too soft to calculate perturbatively.
- One can reorganize perturbation theory in terms of a weak coupling gauge field.

$\to$ Color Glass Condensate effective field theory (small-x)

- Phase rotation: interactions with semi-classical fields.

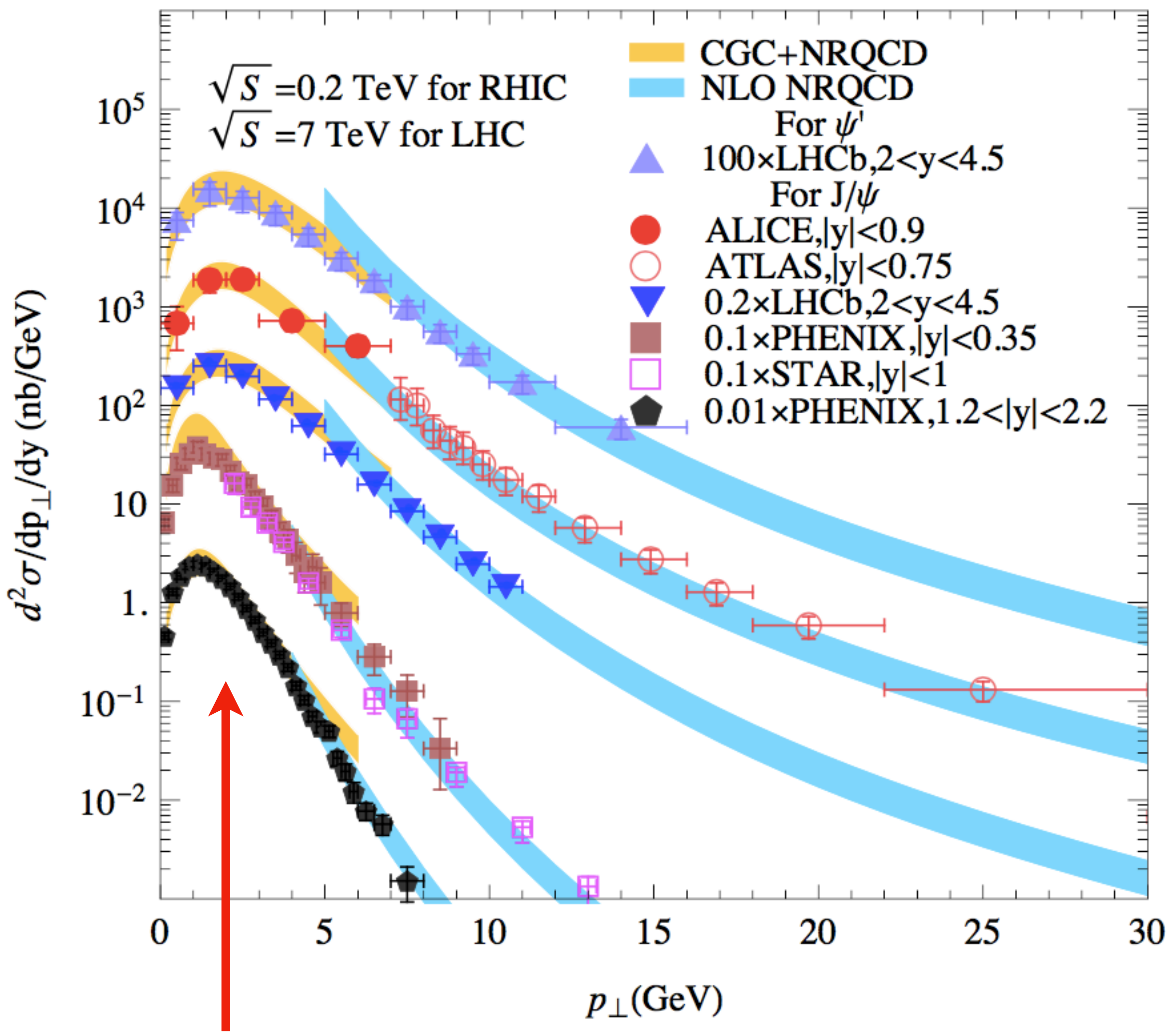


$$U(x_\perp) \equiv \mathcal{P}_+ \exp\left[ig \int dx^+ t^a A_a^-(x^+, x_\perp)\right] = 1 + ig \int dx^+ t^a A_a^-(x^+, x_\perp) + \cdots$$

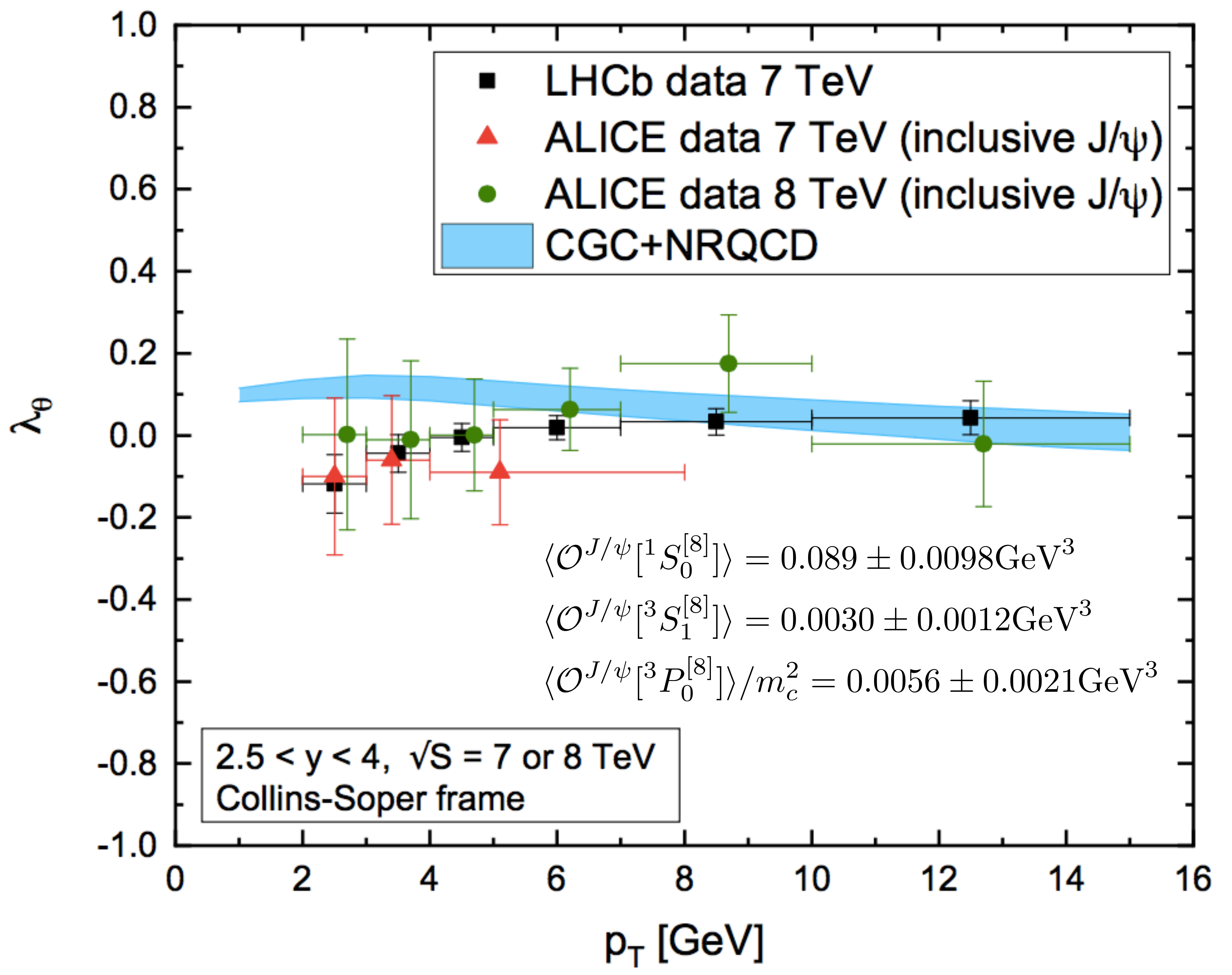
- Resum coherent multiple scattering in the eikonal approximation.

Forward J/ψ production and polarization in pp

Ma, Venugopalan, PRL113, 19, 192301 (2014)



Ma, Stebel and Venugopalan, JHEP12, 057 (2018)

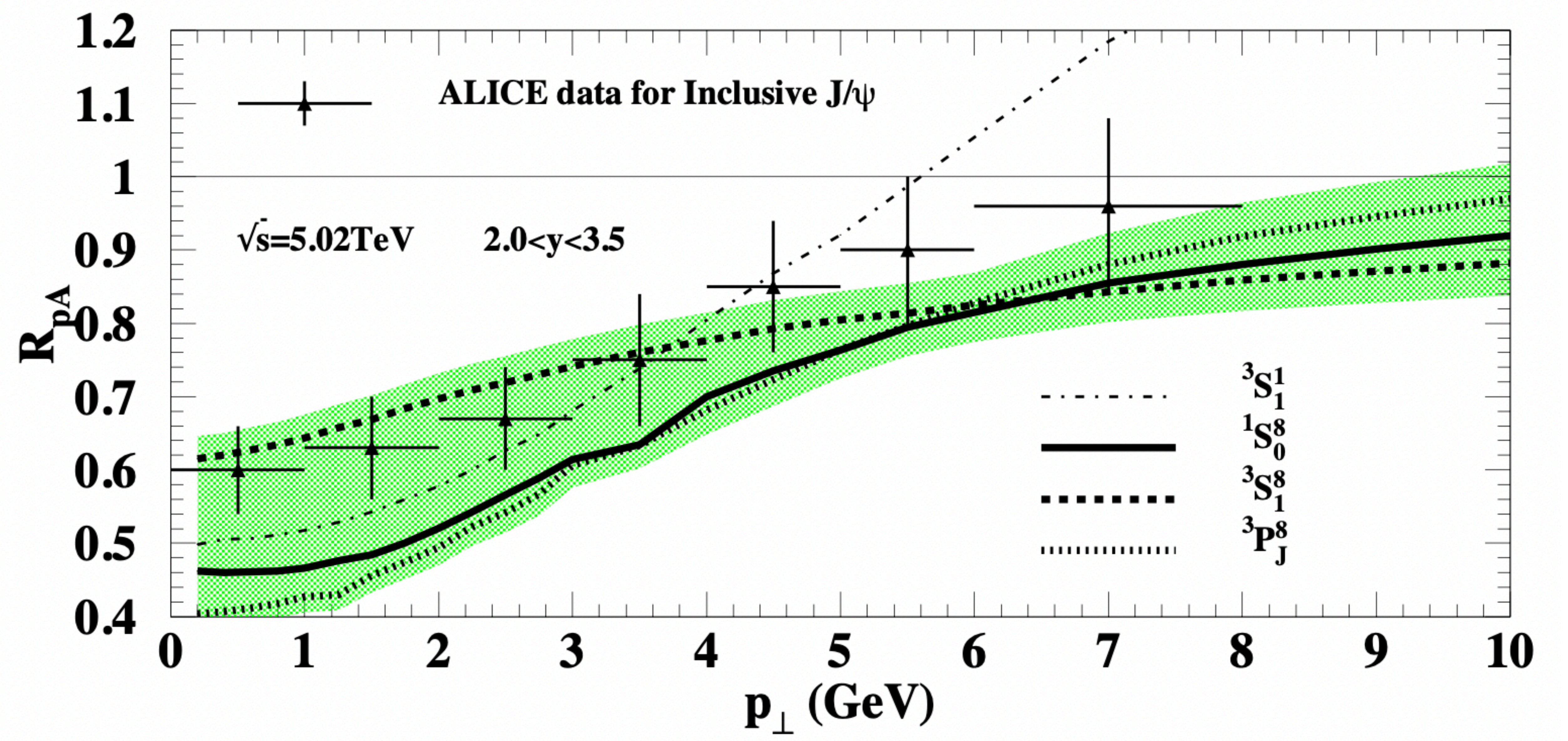


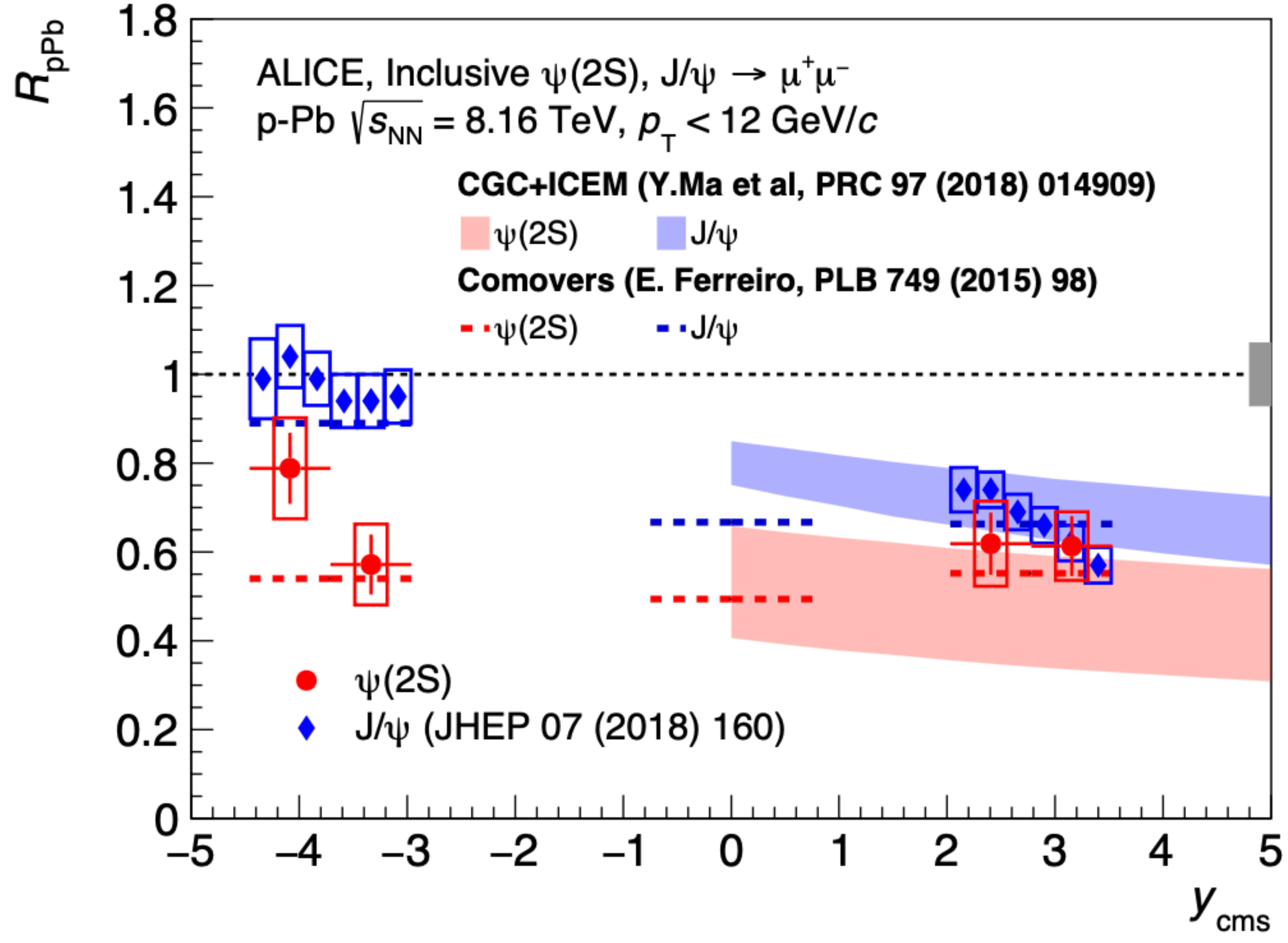
➢ The MV-model + JIMWLK evolution gives a good parametrization of the unpolarized gluon distribution at small-x.

➢ Both NRQCD and ICEM reproduce the p_T spectrum. J/ψ is unpolarized! [Ma, Venugopalan, **KW**, Zhang, PRC97 (2018) 1, 014909]

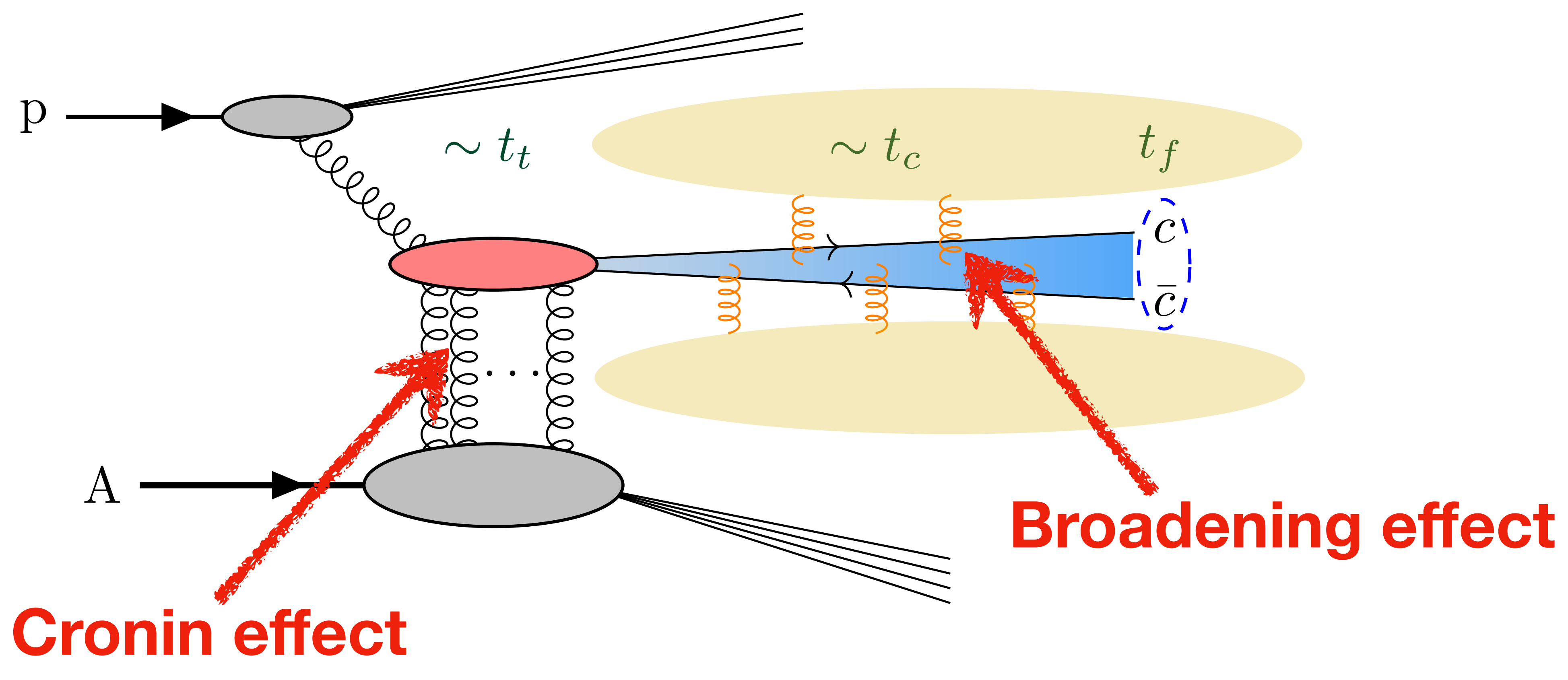
Nuclear dependence

Ma, Venugopalan and Zhang, PRD92, 071901 (2015)





➢ Relative weights between LDMEs control the strength of the nuclear suppression.



➢ Multiple scattering of the pair broads the relative momentum of the pair: $M^2 \to M^2 + \Delta k^2$

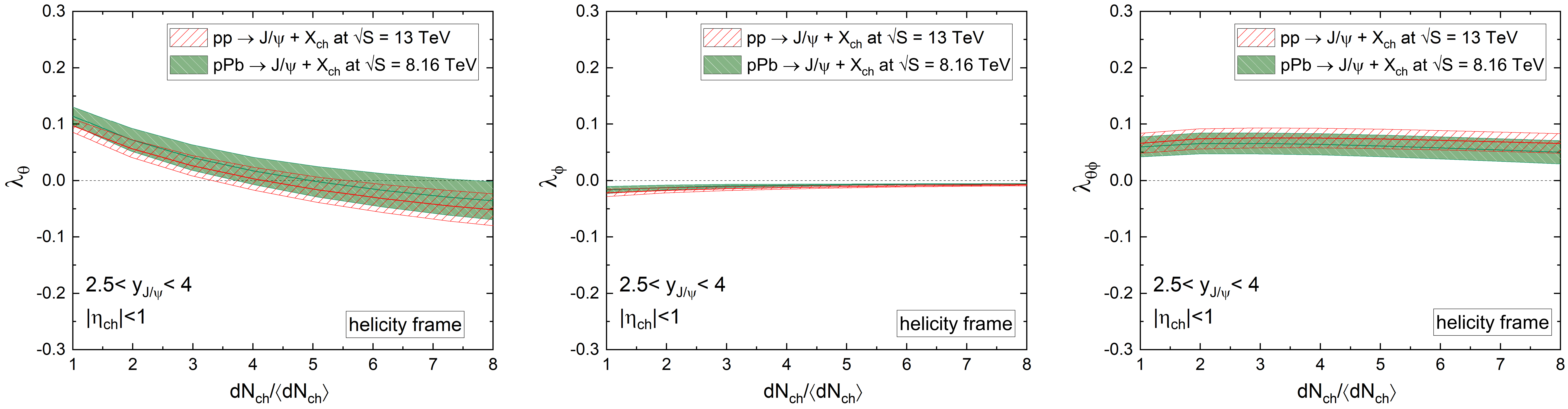
➢ $c\bar{c}$ of large M^2 has less chance to form a quarkonium, e.g. $\psi(2S)$, in pA collisions.

Polarization of forward J/ψ vs. $N_{\rm ch}$

$$\frac{d\sigma^{J/\psi(\to l^+l^-)}}{d\Omega} \propto 1 + \lambda_\theta \cos^2\theta + \lambda_\phi \sin^2\theta \cos 2\phi + \lambda_{\theta\phi} \sin 2\theta \cos\phi$$

CGC + NRQCD with initial fluctuations

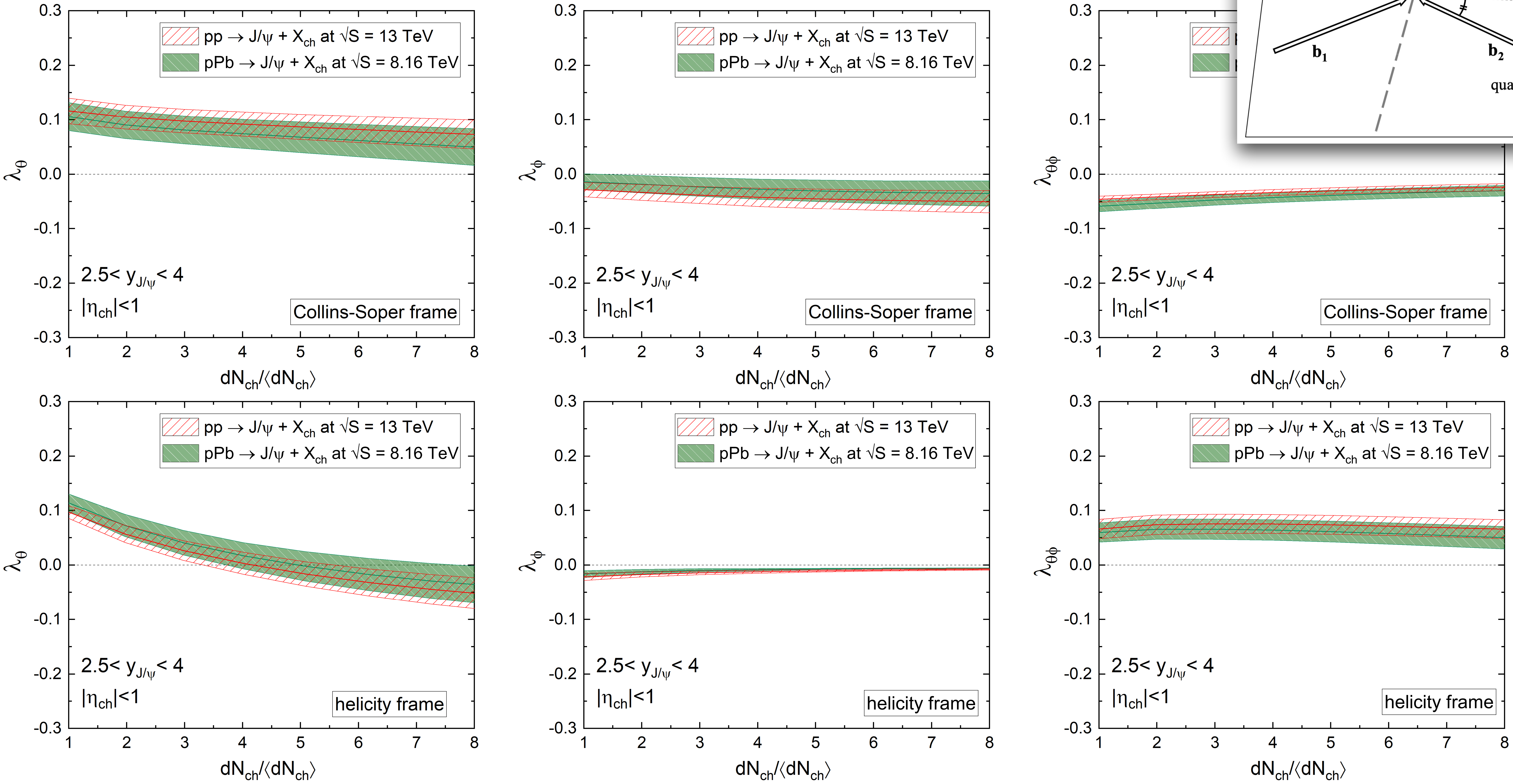
Stebel and **KW**, PRD104, no.3, 034004 (2021)

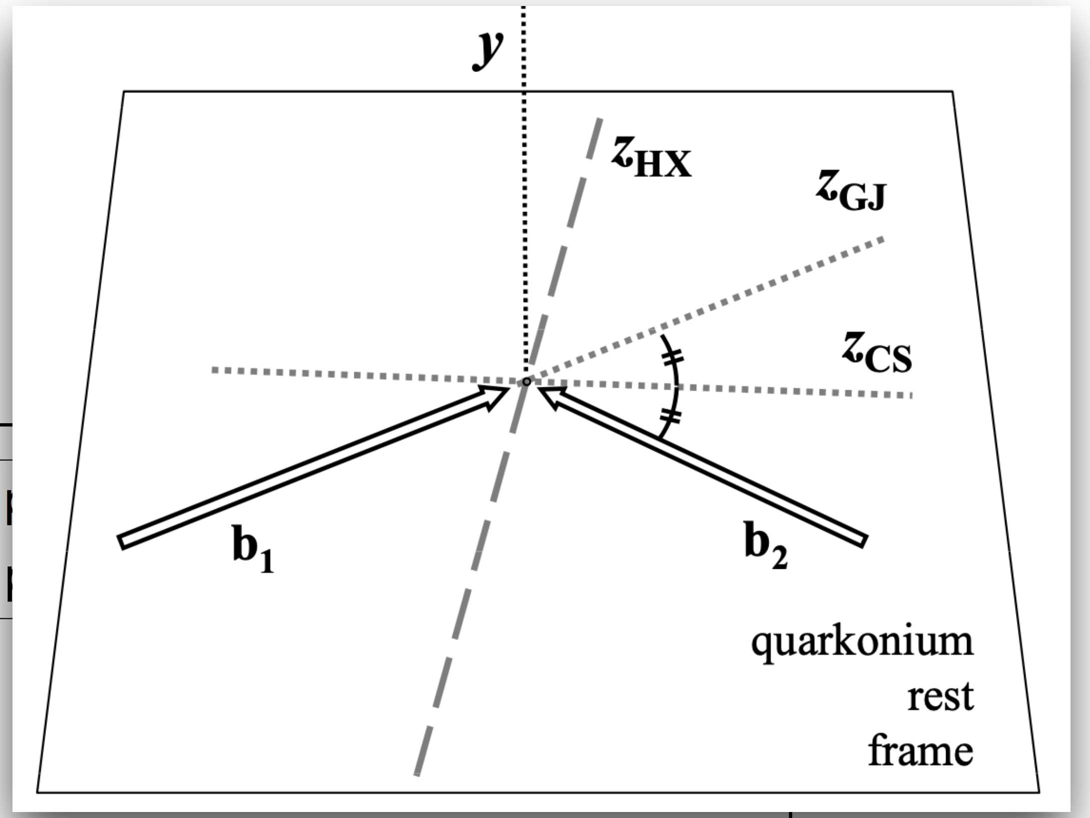


- ➢ J/ψ gets more unpolarized with $N_{\rm ch}$ due to the multiple rescattering at a short distance.
- ➢ Lack of collision energy and system size dependence.
- ➢ Better models can be explored.

Polarization: frame-dependence

Stebel and **KW**, PRD104, no.3, 034004 (2021)





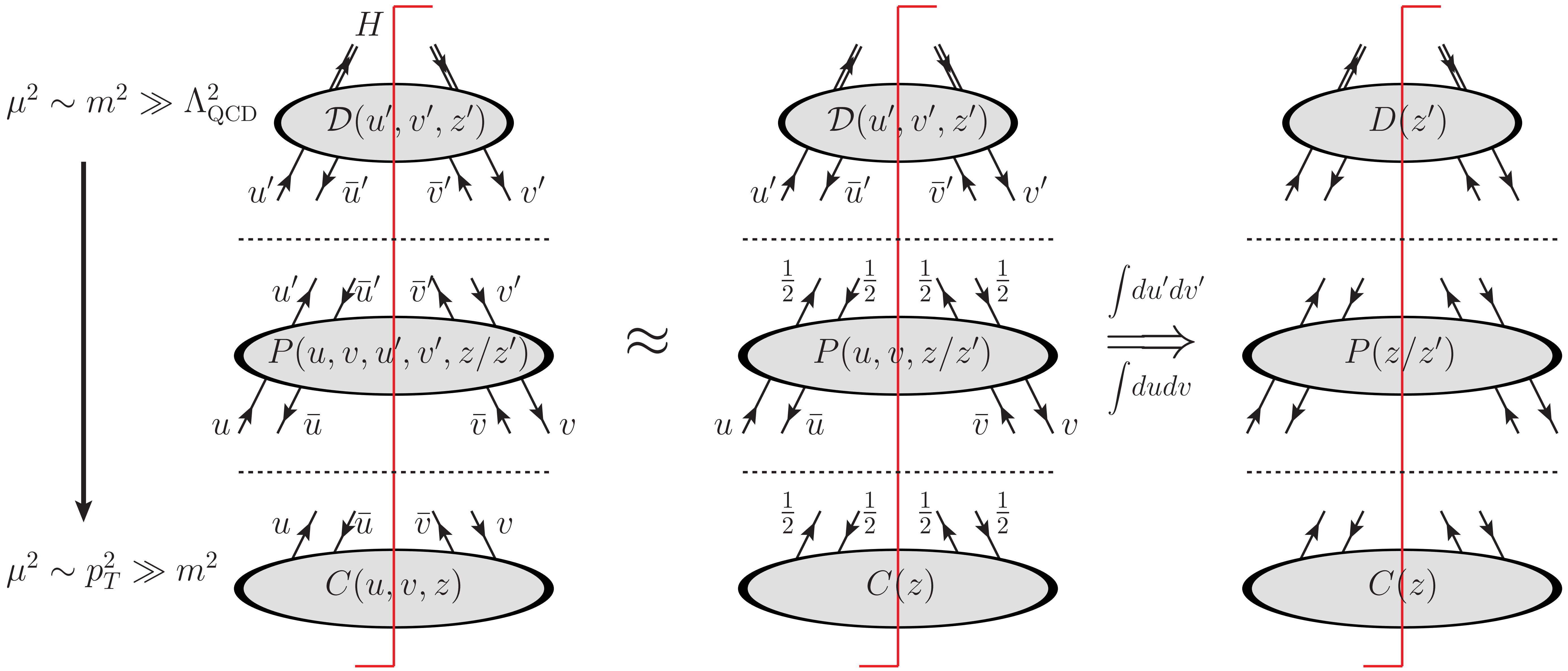
Summary

- The emergence of heavy quarkonium from a produced heavy quark pair has been a big challenge to QCD since November 1974 (a half-century ago!).
- Polarization provides crucial information on the quarkonium production mechanism; the p_T spectrum may not discriminate the production models.
- The collider data indicate that a produced J/ψ is unpolarized in pp collisions; lack of system and frame dependence.
- The ${}^1S_0^{[8]}$ dominance scenario describes a suite of data shown here, but no clear theoretical consensus has been achieved yet.
- Controllable medium dependence in pA collisions could provide further information on the emergence of quarkonium from a produced heavy quark pair.

Thank you!

Backup

Evolution equations in a simplified situation



- The produced heavy quark pair is dominated by its on-shell state at high p_T.
- We may expand the SDCs and evolution kernels on lower virtuality sides at each evolution step around $u = v = 1/2$.
- This can be a reasonable approximation suggested by the evolution of DP FFs in u, v-space. S-to-S channels are not dominant at high p_T.

$$\frac{d\sigma^H_{\rm NLP}}{dyd^2p_T} = \int dzdudv\, C_{[Q\bar{Q}]}(p_Q, p_{\bar{Q}}, \mu)\, \mathscr{D}_{[Q\bar{Q}]\to H}(u, v, z, \mu) \approx \int dz\, C_{[Q\bar{Q}]}(\hat{p}^+_Q = \frac{1}{2}p^+_c, \hat{p}^+_{\bar{Q}} = \frac{1}{2}p^+_c, \mu) \underbrace{\int dudv\, \mathscr{D}_{[Q\bar{Q}]\to H}(u, v, z, \mu)}_{\equiv D_{[Q\bar{Q}]\to H}(z,\mu)}$$

$$\frac{\partial D_{[Q\bar{Q}(\kappa)]\to H}(z, \mu)}{\partial \ln \mu^2} \approx \sum_n \int_z^1 \frac{dz'}{z'} \int_0^1 du \int_0^1 dv\, \Gamma_{[Q\bar{Q}(n)]\to[Q\bar{Q}(\kappa)]}\left(u, v, u' = \frac{1}{2}, v' = \frac{1}{2}, \frac{z}{z'}\right) D_{[Q\bar{Q}(\kappa)]\to H}\left(z', \mu\right),$$

$$\frac{\partial D_{f\to H}(z, \mu)}{\partial \ln \mu^2} \approx \frac{\alpha_s}{2\pi} \sum_{f'} \int_z^1 \frac{dz'}{z'} P_{f\to f'}(z/z') D_{f'\to H}(z') + \frac{\alpha_s^2(\mu)}{\mu^2} \sum_{[Q\bar{Q}(\kappa)]} \int_z^1 \frac{dz'}{z'} P_{f\to[Q\bar{Q}(\kappa)]}\left(u' = \frac{1}{2}, v' = \frac{1}{2}, \frac{z}{z'}\right) D_{[Q\bar{Q}(\kappa)]\to H}\left(z', \mu\right)$$

Input FFs

Ma, Qiu, Zhang, PRD89, no.9, 094029, ibid. 094030 (2014)
Lee, Qiu, Sterman, **KW**, SciPost Phys. Proc.8, 143 (2022)

$$D_{f\to H}(z;m,\mu_0) = \sum_{[Q\bar{Q}(n)]} \pi\alpha_s \left\{ \hat{d}^{(1)}_{f\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \frac{\alpha_s}{\pi}\hat{d}^{(2)}_{f\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \mathcal{O}(\alpha_s^2) \right\} \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m^{2L+3}}$$

LDMEs

$$D_{[Q\bar{Q}(\kappa)]\to H}(z;m,\mu_0) = \sum_{[Q\bar{Q}(n)]} \left\{ \hat{d}^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \frac{\alpha_s}{\pi}\hat{d}^{(1)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \mathcal{O}(\alpha_s^2) \right\} \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m^{2L+1}}$$

$\mu_0 = \mathcal{O}(2m)$: input scale, $\mu_\Lambda = \mathcal{O}(m)$: NRQCD factorization scale $\qquad \kappa = v^{[c]}, a^{[c]}, t^{[c]}, \quad n = {}^{2S+1}L_J^{[c]}$

Perturbative SDCs of input FFs in α_s and v expansion in the NRQCD are reliable only when SDCs $\ll \mathcal{O}(1)$. Indeed, the NRQCD factorization is not reliable as $z \to 1$ where SDCs $\hat{d}(z)$ include the following terms:

1. $\delta(1-z)$ at LO in α_s expansion
2. $f(z)\ln(1-z)$ with $f(z)$ being a regular function
3. $\dfrac{f(z)}{[1-z]_+}, f(z)\left[\dfrac{\ln(1-z)}{1-z}\right]_+$ due to the perturbative cancelation of IR divergences

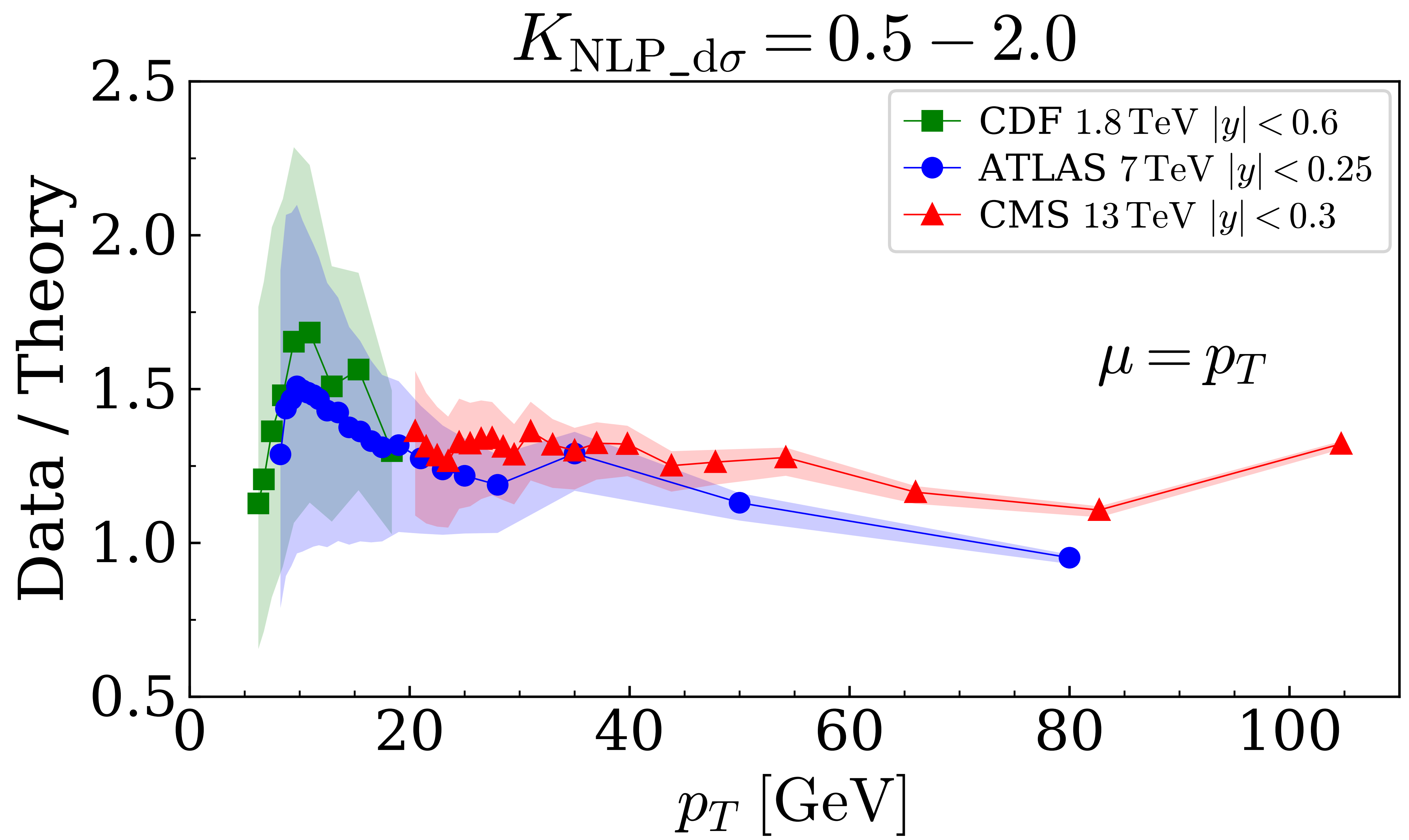
In our current analysis, we use analytic results if those vanish as $z \to 1$, otherwise, singular or negative input FFs are cast into

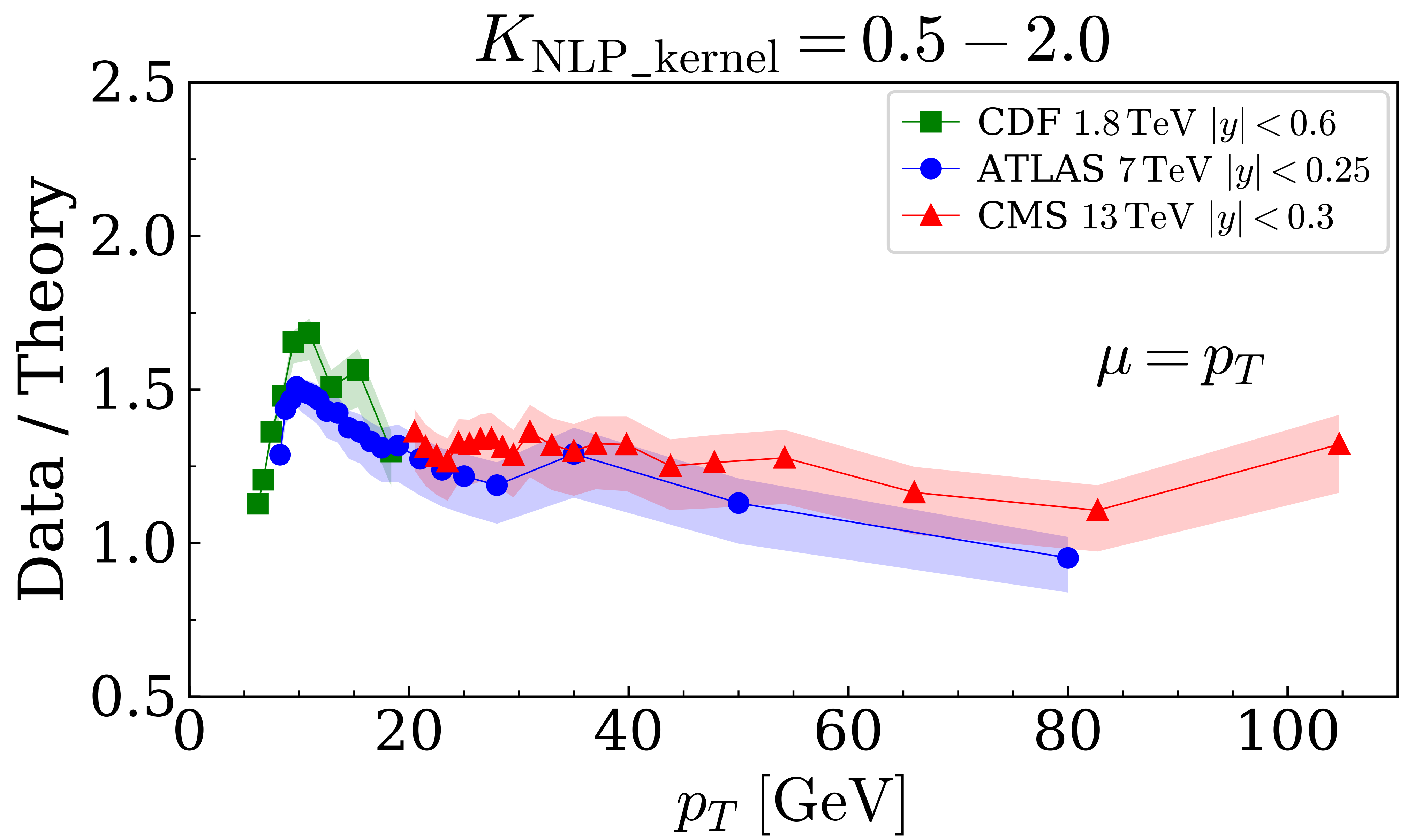
$$D_{[Q\bar{Q}(n)]}(z) = C_{[Q\bar{Q}(n)]}(\alpha_s)\frac{z^\alpha(1-z)^\beta}{B[1+\alpha,1+\beta]} \qquad (\alpha \gg 1,\ 1 > \beta > 0)$$

→ to be tuned, imitating δ-function at LO.

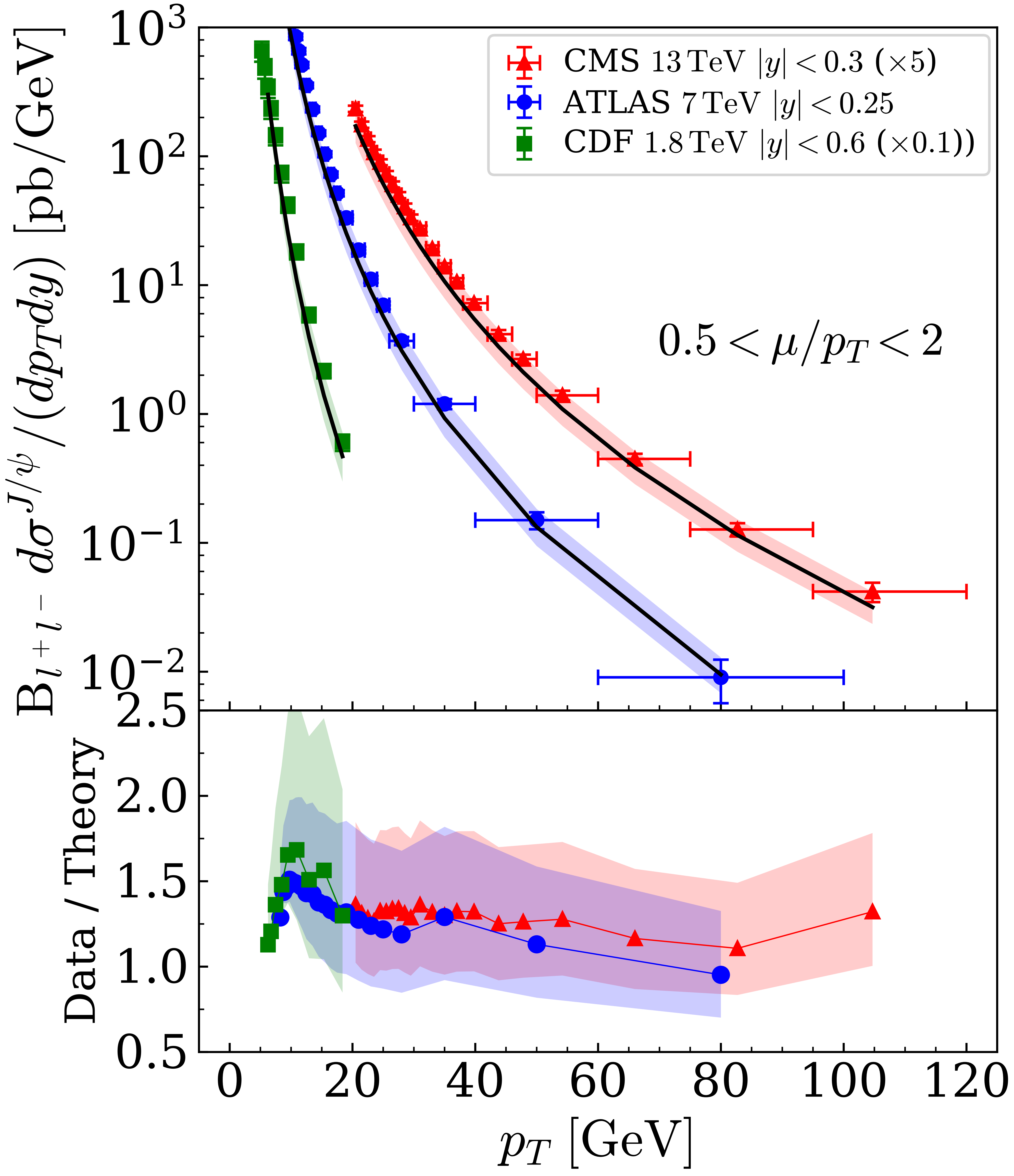
$C_{[Q\bar{Q}(n)]}$: abs. value of the first moment

Uncertainty of theoretical calculations

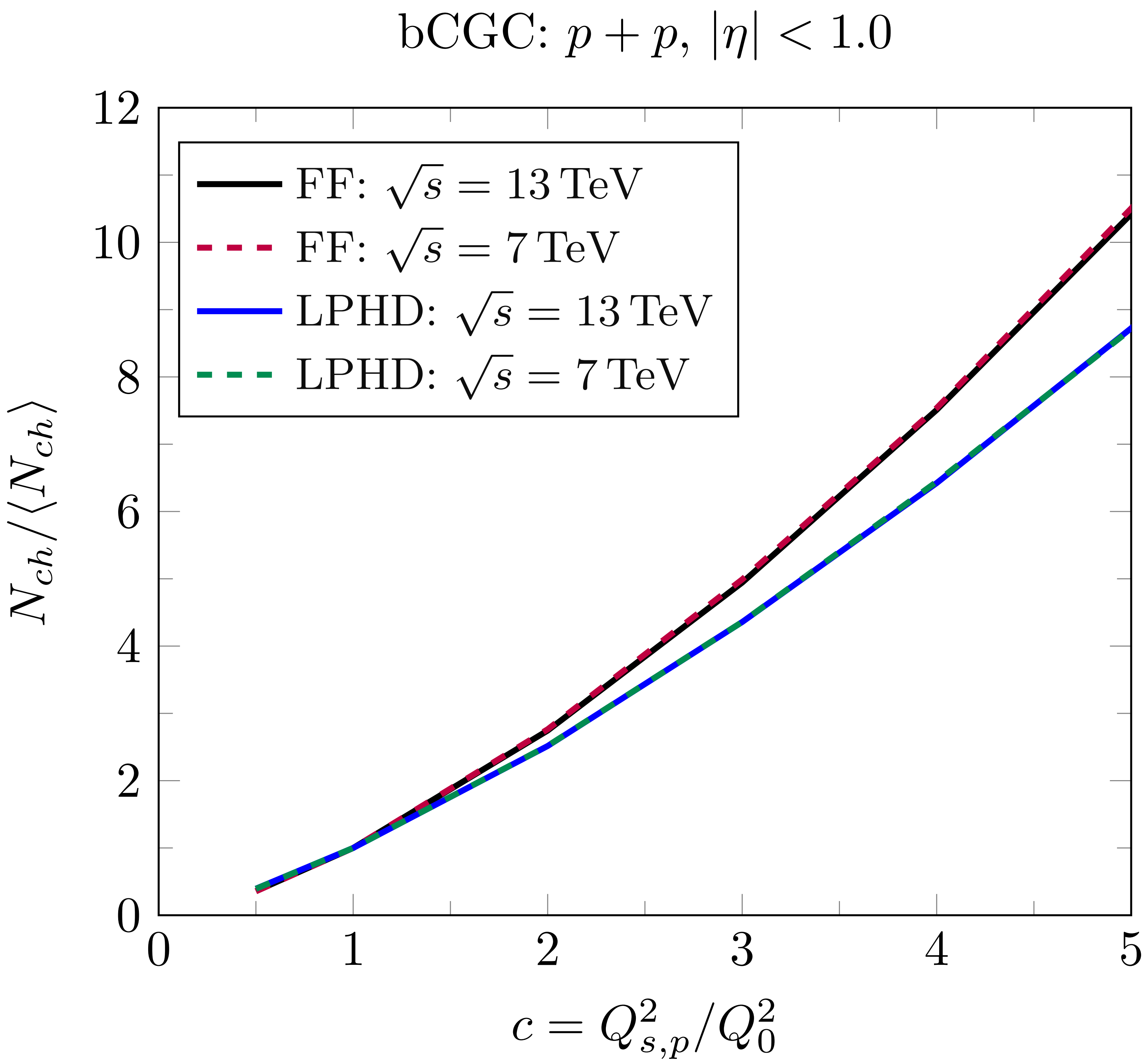


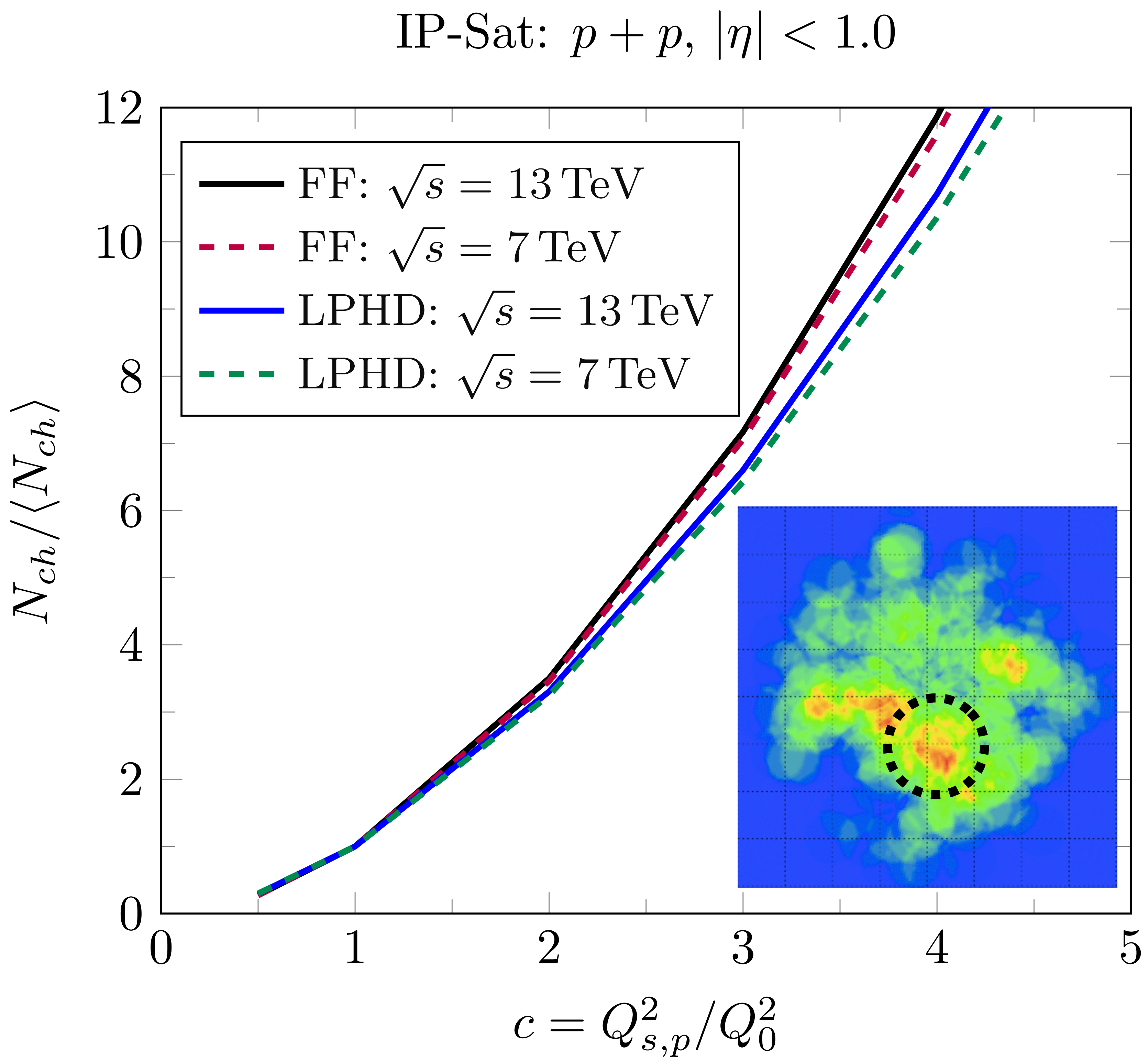


Choice of factorization/renormalization scale



Initial state fluctuations: a simple model setup





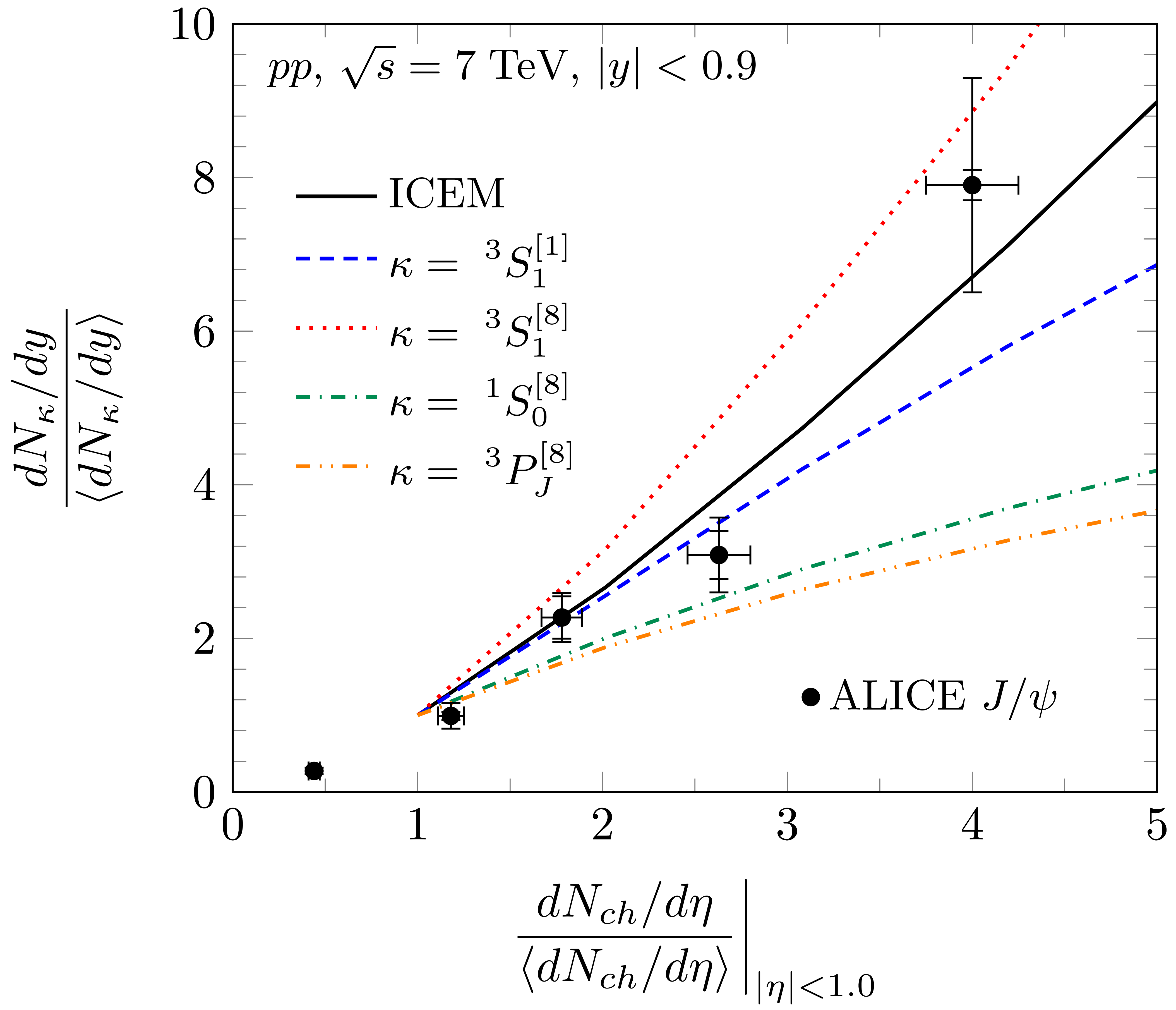
- The difference between FF and LPHD is about <15% (<20%) at $c = 4$ in bCGC (IP-Sat).
- About 30% difference between bCGC and IPSat comes from the energy dependence of Q_s.
- Geometrical fluctuation off.

➢ High multiplicity events $N_{ch} \gg \langle N_{ch}\rangle$: In pp collisions, $Q_{s,p} = cQ_0^2$, $c \geq 1$. In pA collisions, implementation gets more complicated due to the fluctuation from N_{coll}. Nevertheless, we shall set $Q_{s,A} = c\xi Q_0^2$, $c \geq 1$ and $\xi \sim 2\,\text{or}\,3$ for heavy targets.

Levin and Rezaeian, PRD82, 014022 (2010)
Dusling and Venugopalan, PRD87, no.9, 094034 (2013)

Note: These dense gluon configurations could have eccentric shapes whose final state interactions can in principle generate flow.

New constraint on LDMEs?



Ma, Tribedy, Venugopalan, **KW**, PRD98, 7, 074025 (2018)
Ma, Tribedy, Venugopalan, **KW**, NPA982, 747-750 (2019)

${}^3S_1^{[8]}$ state is favored.

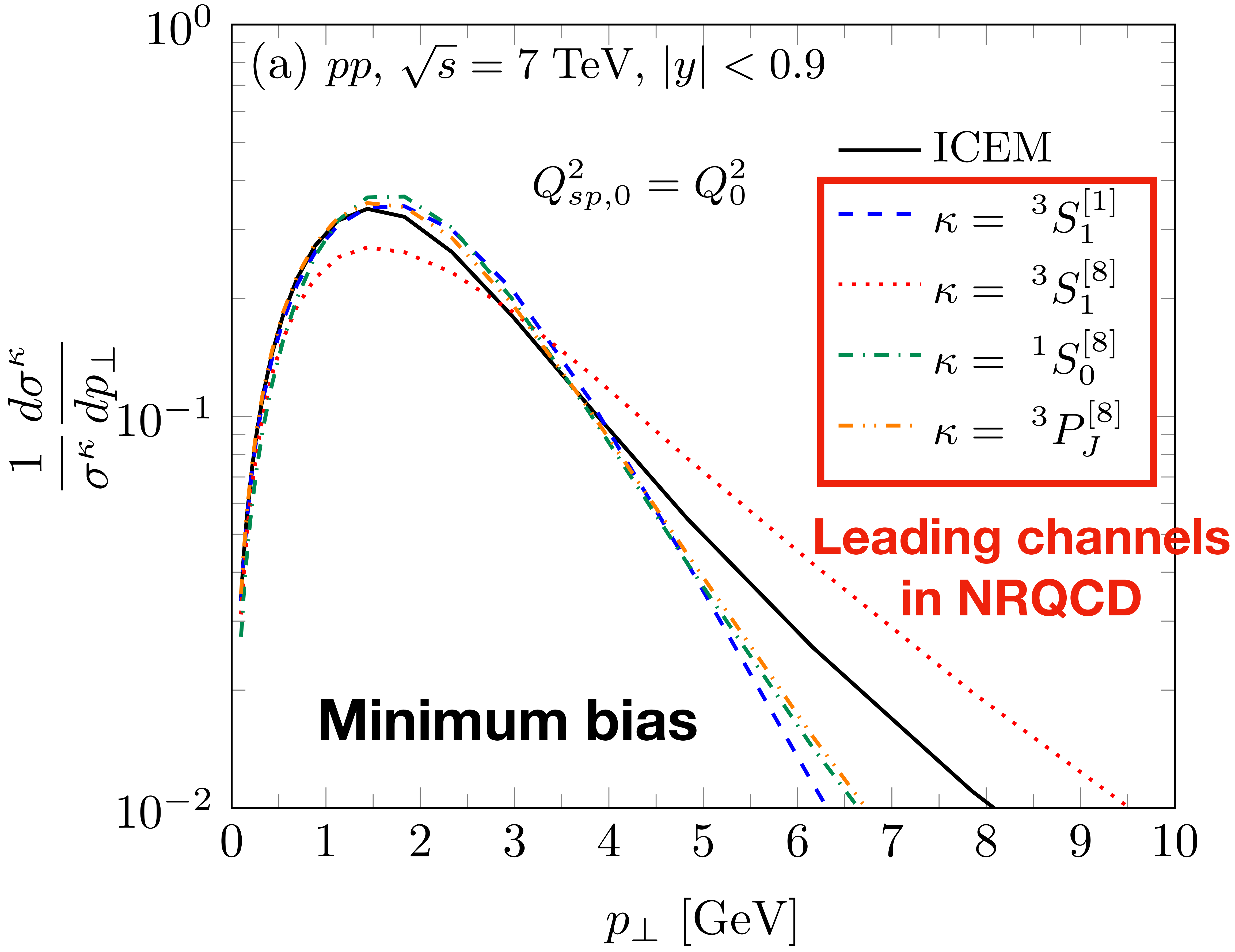
- Consistent with the universality requirement from BELLE e^+e^- data:

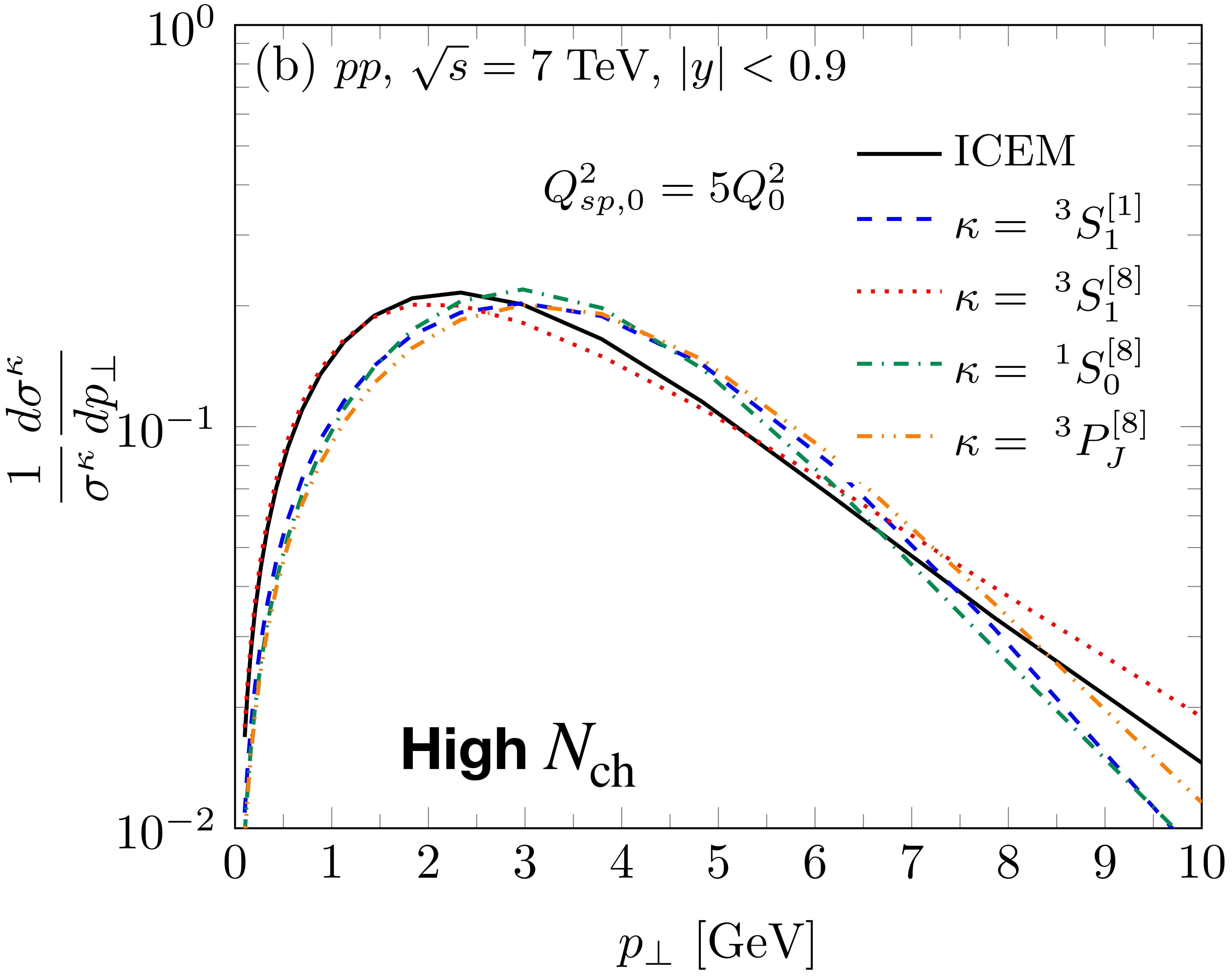
$$\langle \mathcal{O}^{J/\psi}[{}^1S_0^{[8]}] \rangle + 4.0 \langle \mathcal{O}^{J/\psi}[{}^3P_0^{[8]}] \rangle / m^2 < 2.0 \pm 0.6 \times 10^{-2}\,\mathrm{GeV}^3$$

Zhang, Ma, Wang, Chao, PRD**81** (2010)

- **Caveat**: Tevatron and LHC data tell that ${}^1S_0^{[8]}$ has a large weight at high p_T.

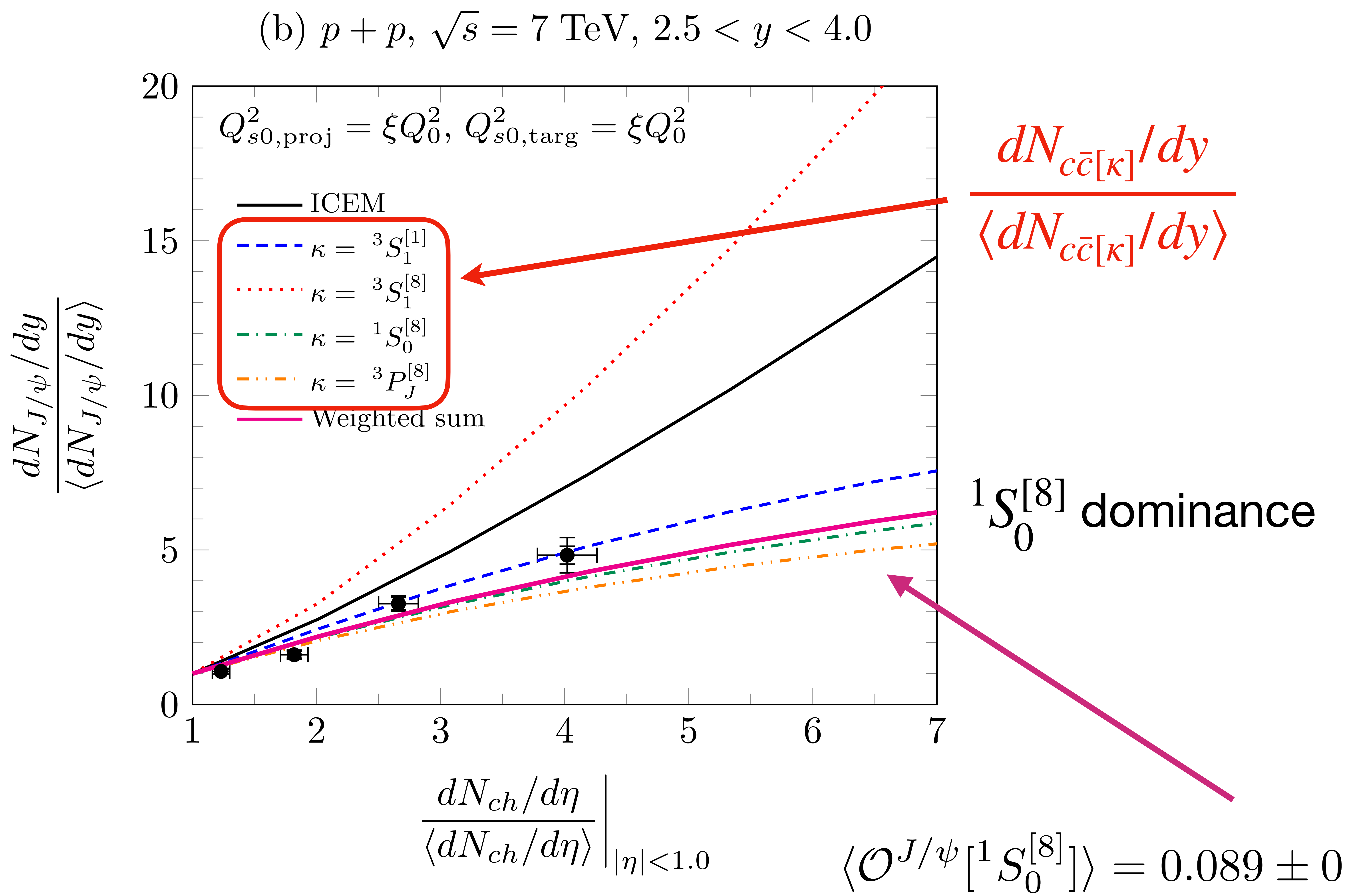
Quarkonium's p_T spectrum and fluctuation





- ➢ Both Improved-CEM and NRQCD give similar $p_\perp$ slopes at low $p_\perp$, even when $N_{\rm ch}$ is much high.

Forward production: weighted sum with LDMEs



$$\langle \mathcal{O}^{J/\psi}[{}^1S_0^{[8]}] \rangle = 0.089 \pm 0.0098 \mathrm{GeV}^3$$

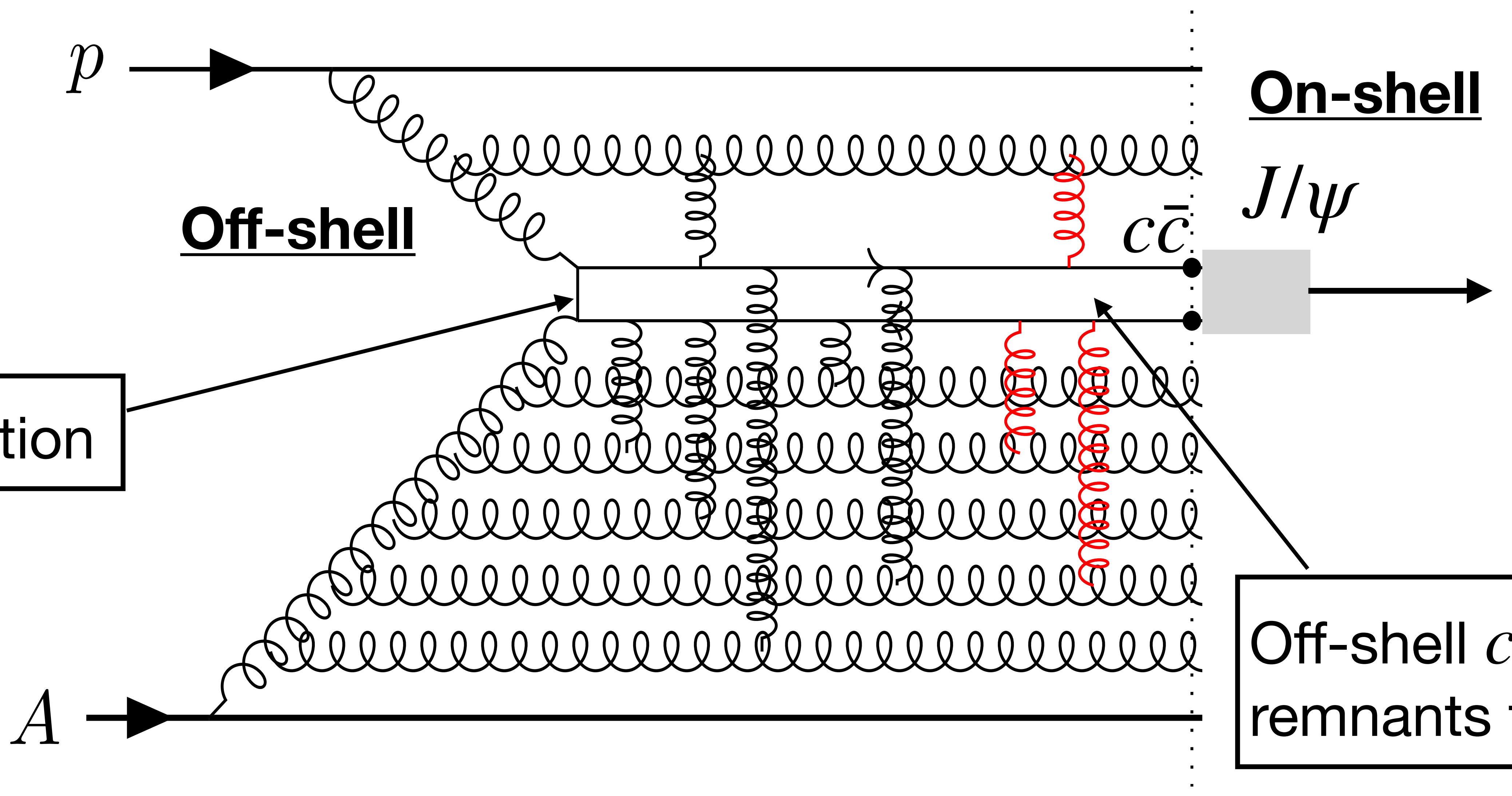
Our choice $\langle \mathcal{O}^{J/\psi}[{}^3S_1^{[8]}] \rangle = 0.0030 \pm 0.0012 \mathrm{GeV}^3$

$$\langle \mathcal{O}^{J/\psi}[{}^3P_0^{[8]}] \rangle / m_c^2 = 0.0056 \pm 0.0021 \mathrm{GeV}^3$$

Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 (2012).

An initial state effect scenario

Long lived soft partons are radiated more in the forward direction.



Hard scattering: $c\bar{c}$ production

Off-shell $c\bar{c}$ interacts with remnants from beams.

- Does coherent multiple scattering effect dominate over final state incoherent scattering?
- Can we distinguish between these effects as one goes from large to small systems?

1. Semi-hard multiple rescattering of high occupied gluons: $k \sim \mathcal{O}(Q_s)$
   - $c\bar{c}$ production yield is enhanced at high multiplicity.

Ma, Venugopalan, **KW**, Zhang (2017)

2. Nuclear enhanced soft colors transfer from spectators: $k \sim \mathcal{O}(\Lambda_{QCD}) \sim \Delta E_{J/\psi}$
   - The soft color exchange effect is seen in ψ' stronger suppression in MB p+A collisions as ψ' is a loosely binding system. J/ψ is a stronger bound state but can be broken in high N_{ch}.

→ Multiple rescattering effects can be studied in a dense-dense Glasma system.

→ It's possible to model final state incoherent rescattering effects by matching our results to kinetic theory or open quantum system descriptions.

Tanji, Berges (2017)
Yao, Mehen (2018)
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