Spin-1 quarkonia in a rotating frame and their spin contents

ref) PLB 843 (2023) 137986

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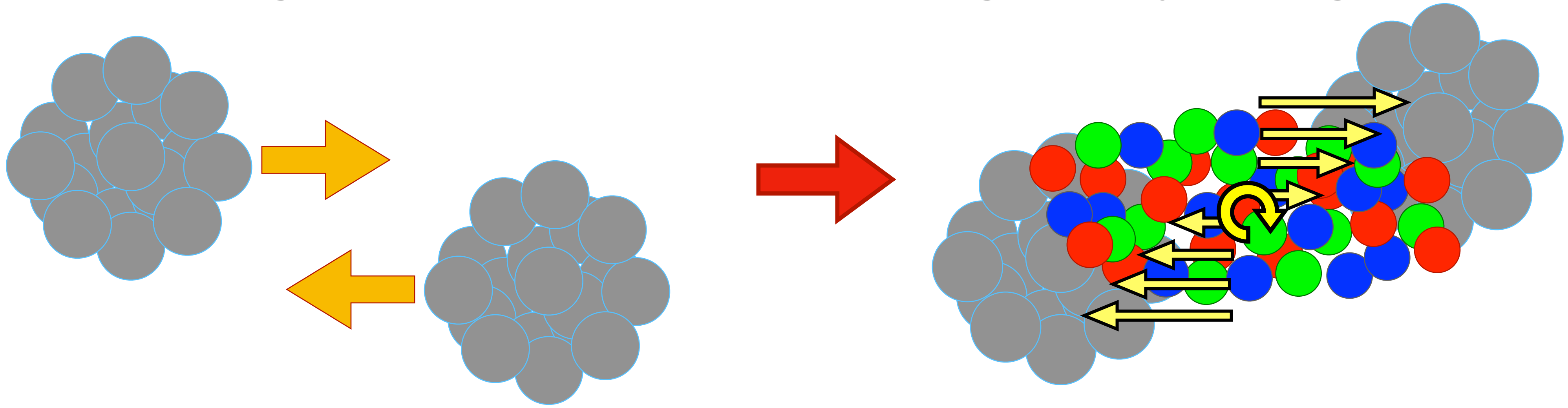
@ExHIC-P workshop, Academia Sinica, Taiwan, 2024.03.15

Introduction

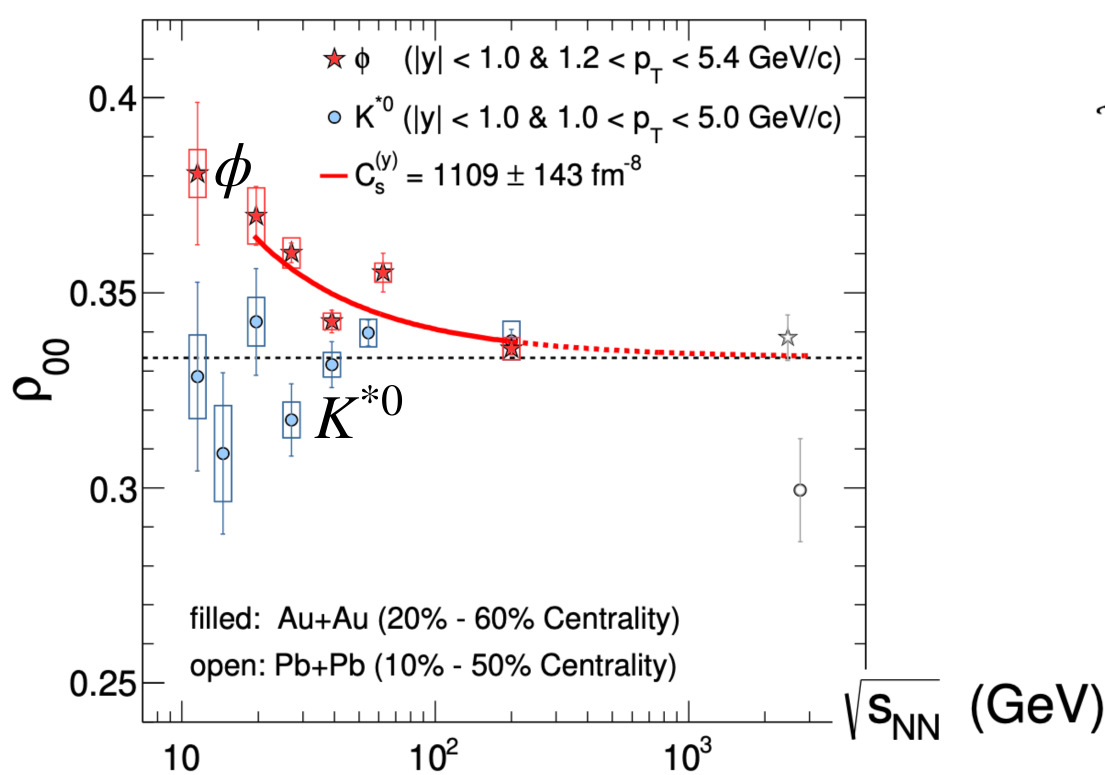
In non-central HICs,

initial angular momentum

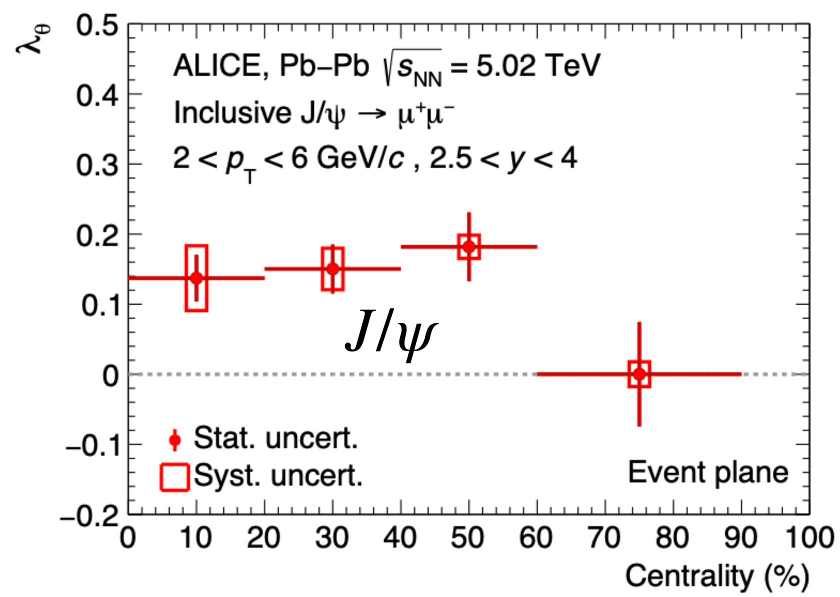
Large vorticity & Strong B field



STAR, arXiv:2204.02302 (2022)



ALICE, arXiv:2204.10171 (2022)

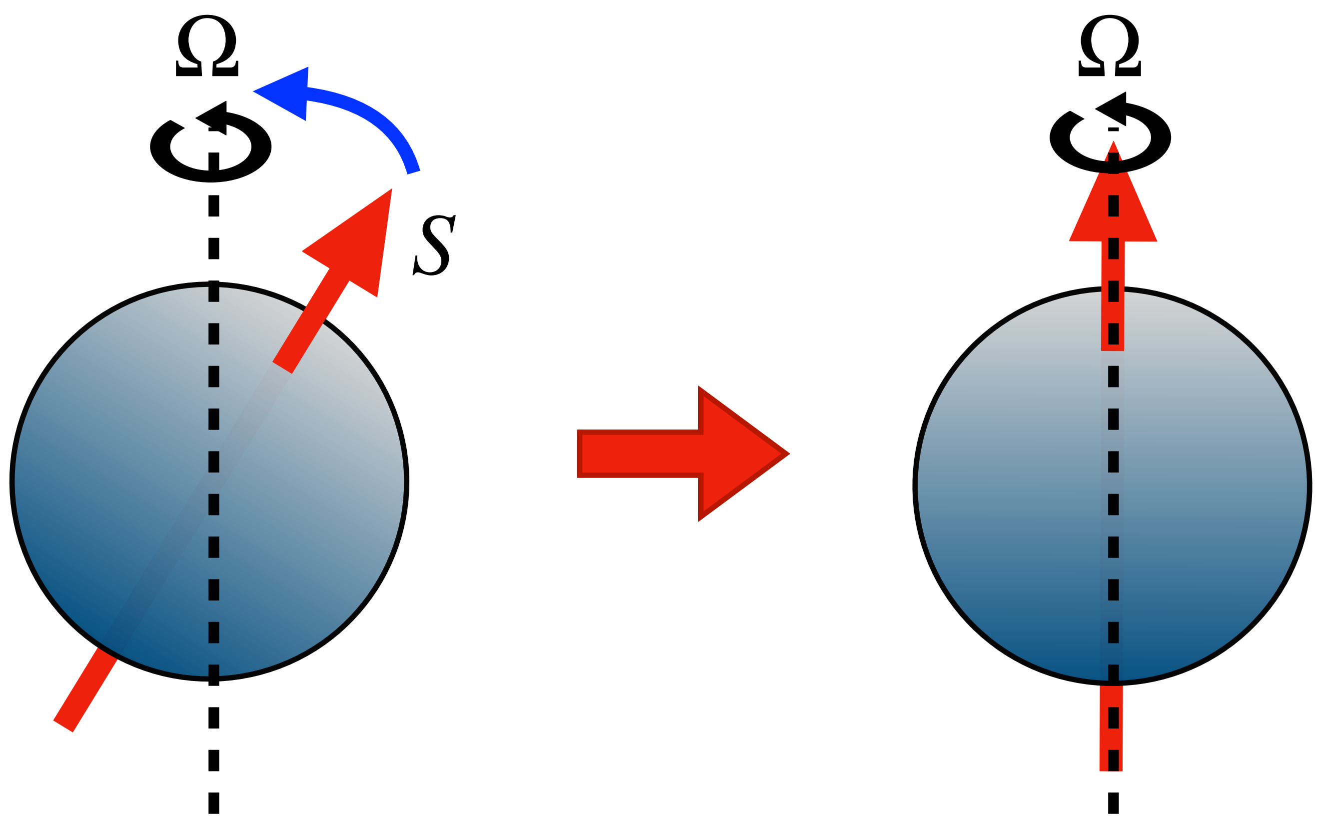


=> Detailed mechanisms are complex and still not clearly understood

Introduction

Spin-Rotation Coupling(SRC)

the most fundamental theory for spin alignment in a rotating medium



$$H_{\mathrm{SRC}} = H_{\mathrm{r}} - H_{\mathrm{i}} = -S \cdot \Omega$$

H_i : Inertial frame energy
H_r : Rotating frame energy
S : Spin
Ω : Angular velocity

Introduction

Q. Does $H_{\mathrm{SRC}} = - S \cdot \Omega$ hold true for all particles?

- Spin1/2 : Dirac eq. in a rotating frame using G.R.

$$\left[i\partial\!\!\!/_x + gA\!\!\!/(x) + \mathbf{\Sigma} \cdot \mathbf{\Omega} - m \right] \Psi(x) = 0 \text{ where } \mathbf{\Sigma} = \gamma^0(\mathbf{S} + \mathbf{L})$$

- Spin1 : No strict derivation based on G.R. until recently

- Phys.Rev.D 102 (2020) 12, 125028 - J.Kapusta, E.Rrapaj, S.Rudaz
  - Proca eq. for massive spin-1 particle using G.R.
  - $H_{\mathrm{SRC}} = -\frac{1}{2} S \cdot \mathbf{\Omega}$ for spin-1!
  - contradictory to naive expectation and quark model

- **Motivation**

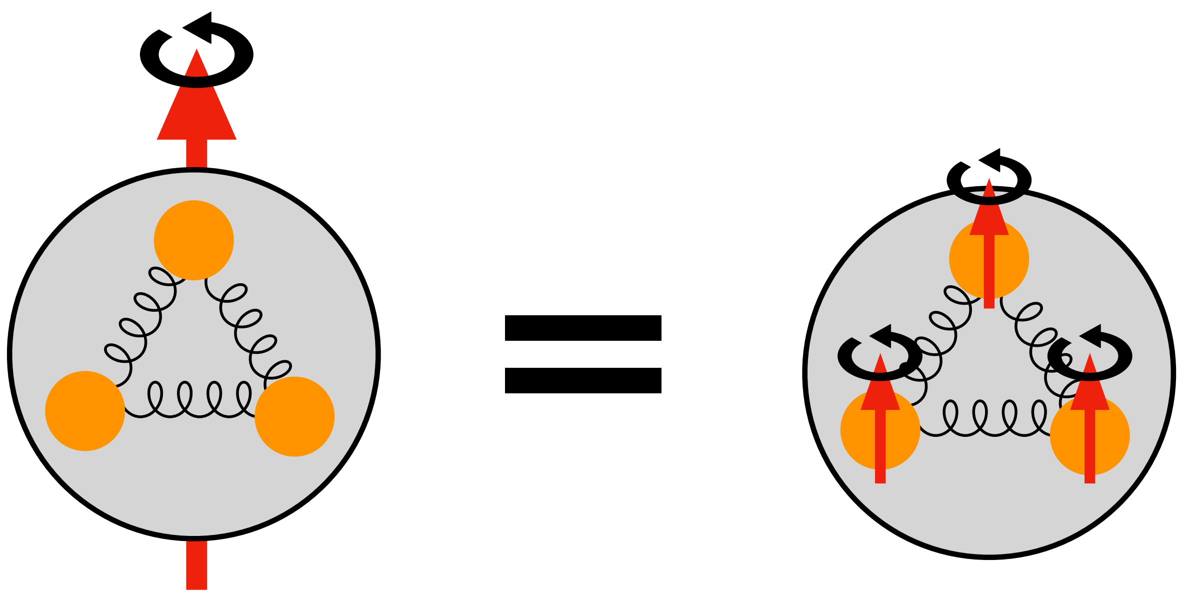
  To clarify the strength of SRC for spin-1 particle in a different way!

Outline

- We study SRC of spin-1 heavy $Q\bar{Q}$ system
- Introduce a free parameter "g_Ω" which indicates the strength of SRC,

$$H_{\text{SRC}} = -\, g_\Omega \, S \cdot \Omega$$

- "Total SRC of the system = All quark + gluon in a rot frame"



- We prove that $g_\Omega = g_\Omega^{\text{quark}}(Q^2) + g_\Omega^{\text{gluon}}(Q^2) = 1$ for spin-1 $Q\bar{Q}$ system
- Each component of g_Ω carried by quarks and gluons = Spin content
- We study spin contents of J/ψ, $\Upsilon(1S)$, χ_{c1}, χ_{b1}

How to extract g_Ω?

1. **Consider the correlation function in a rotating frame**

$$\Pi^{\mu\nu}(q) = i \int d^4x e^{iqx} \langle 0 | \mathrm{T}\{\mathrm{j}^\mu(\mathrm{x}) \mathrm{j}^\nu(0)\} | 0 \rangle$$

2. **Put the system at the center of the rotation** => no external OAM
   Pick out a right circularly polarized state => $\Pi^+(\omega) = \epsilon^+_\mu \epsilon^{+*}_\nu \Pi^{\mu\nu}(\omega, 0)$

$q_\mu = (\omega, \vec{0})$
$\epsilon^+_\mu = (0,1,i,0)/\sqrt{2}$

3. **Up to linear terms in Ω**

$$\Pi^+(\omega) = \omega^2 \Pi^{vac}(\omega^2) + \omega \Pi^{rot}(\omega^2) \Omega + \mathcal{O}(\Omega^2)$$

   Π^{vac} : ordinary vacuum invariant ftn. vacuum properties ex) mass

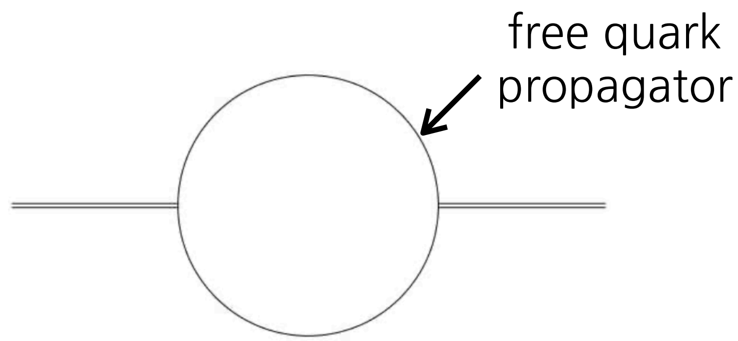
   Π^{rot} : new function appearing in a rotating frame. spin information

4. **Extract g_Ω by comparing two different descriptions of Π^{rot}**
   (a) Direct computation in a rotating frame
   (b) Phenomenological derivation from Π^{vac}

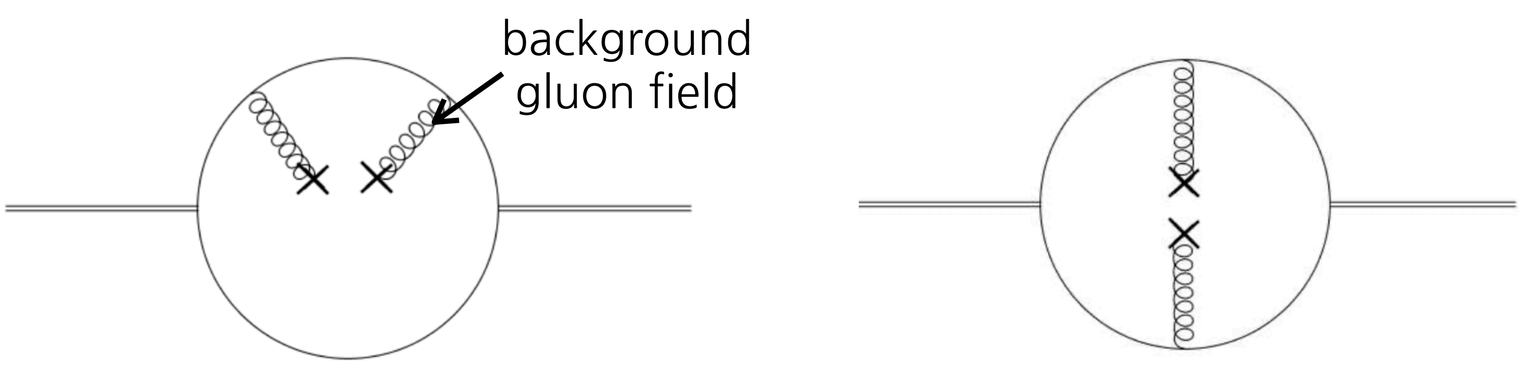
(a) Direct computation in a rotating frame

Feynman diagrams in Operator Product Expansion(OPE)

- Leading perturbative diagram



- Leading non-perturbative diagrams : Gluon condensates



- Compute in an inertial frame → Ω independent terms → Π^{vac}
- Compute in a rotating frame → collect Ω linear terms → Π^{rot}

Quarks in a rotating frame

- Recall Dirac eq. in a rotating frame

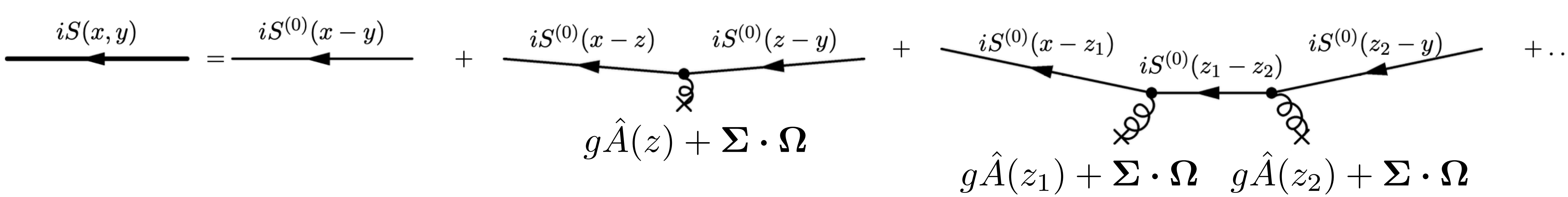
$$\left[i\partial\!\!\!/_x + gA\!\!\!/(x) + \mathbf{\Sigma}\cdot\mathbf{\Omega} - m\right]\Psi(x) = 0 \quad \text{where} \quad \mathbf{\Sigma} = \gamma^0(\mathbf{S}+\mathbf{L})$$

- Quark propagator in a rotating frame

$$\left[i\partial\!\!\!/_x + gA\!\!\!/(x) + \mathbf{\Sigma}\cdot\mathbf{\Omega} - m\right] S(x) = \delta(x)$$

  - difficult to find full propagator
  - expansion in terms of 'g' and '$\mathbf{\Omega}$'

$$S^{\text{full}} \approx S^{(0)} + S^{(0)}\left[gA\!\!\!/ + \Sigma\cdot\Omega\right]S^{(0)} + S^{(0)}\left[gA\!\!\!/ + \Sigma\cdot\Omega\right]S^{(0)}\left[gA\!\!\!/ + \Sigma\cdot\Omega\right]S^{(0)} + \cdots$$



Gluons in a rotating frame

- Covariant derivatives in curved space-time(Γ^a_{bc} : Christoffel symbols)

$$D_c G_{ab} = \partial_c G_{ab} - \Gamma^d_{ca} G_{db} - \Gamma^d_{cb} G_{ad}$$

- Fock-Schwinger gauge($x^\mu A_\mu = 0$) in curved space-time

$$A_\mu(x) = -\frac{1}{2} x^\nu G_{\mu\nu} - \frac{1}{3} x^\nu x^\alpha \partial_\alpha G_{\mu\nu} + \cdots$$

$$= -\frac{1}{2} x^\nu G_{\mu\nu} - \frac{1}{3} x^\nu x^\alpha D_\alpha G_{\mu\nu} - \frac{1}{3} x^\nu x^\alpha (\Gamma^d_{\alpha\mu} G_{d\nu} + \Gamma^d_{\alpha\nu} G_{\mu d}) + \cdots$$

additional contribution in curved space-time

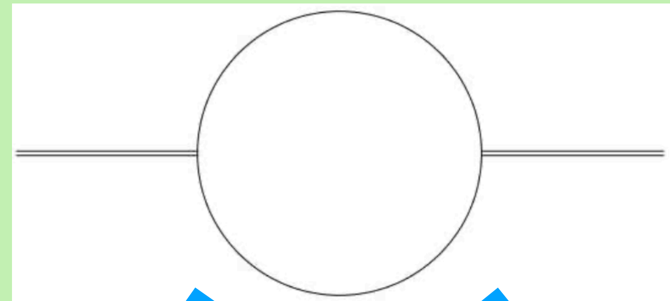
- $\Gamma^2_{01} = \Omega, \Gamma^1_{02} = -\Omega$ in a rotating frame.

$$\not{A}^{\mathrm{new}}(x) = -\frac{1}{3} x^\nu x^\alpha \gamma^\mu (\Gamma^d_{\alpha\mu} G_{d\nu} + \Gamma^d_{\alpha\nu} G_{\mu d}) \propto \vec{x} \times (\vec{E} \times \vec{B}) \cdot \Omega = J_g \cdot \Omega$$

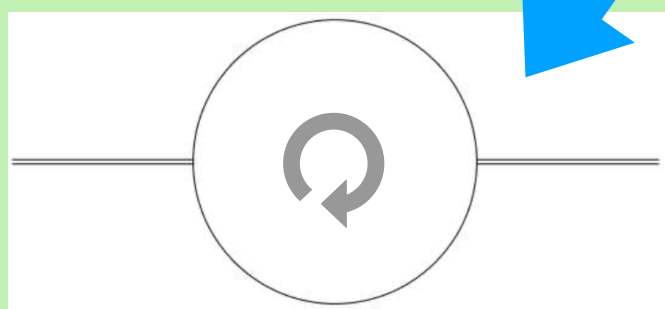
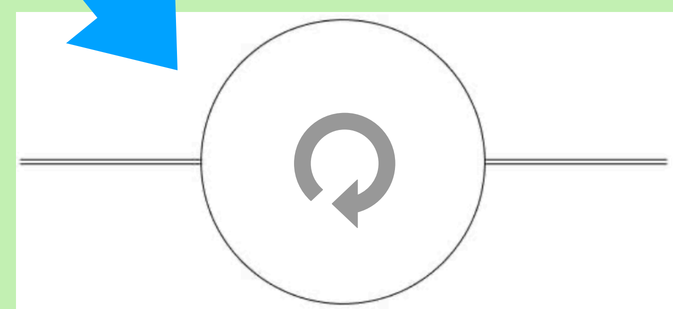
- Kapusta et al. thought that $D_c G_{ab} = \partial_c G_{ab}$ in a rotating frame.
  (Their result might be wrong)

Summary of method (a)

perturbative

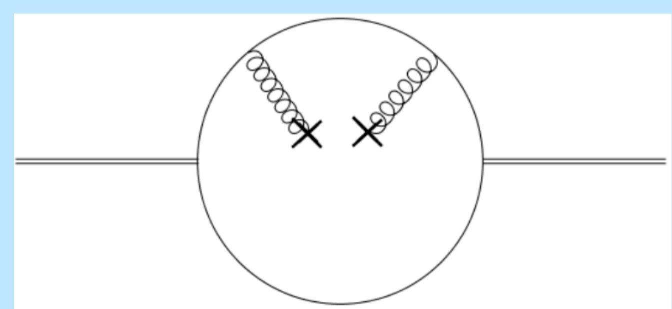
$\Pi^{vac} =$ 

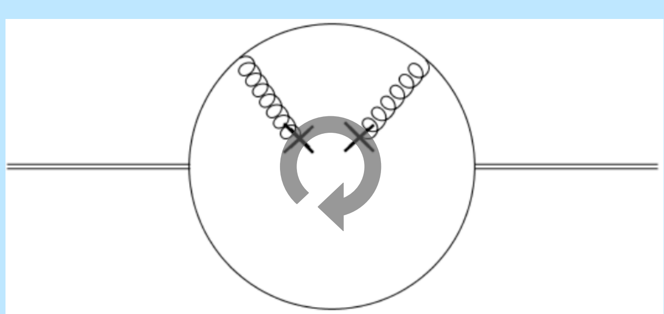
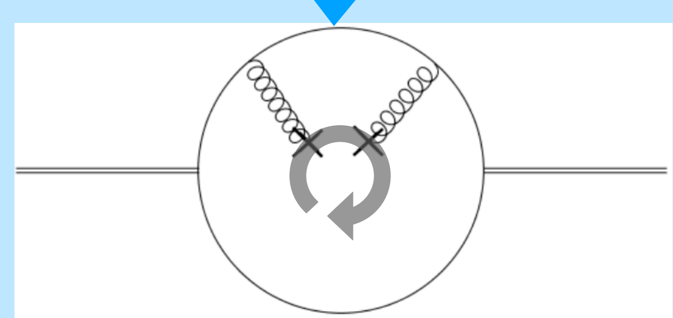
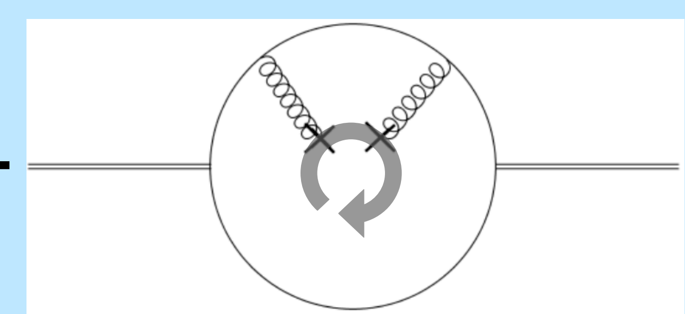
decomposition into spin and orbital
a.m. according to their origin

$\Pi^{rot}_{(a)} =$  $+$ 

quark spin | quark orbital

non-perturbative

$\Pi^{vac} =$ 

$\Pi^{rot}_{(a)} =$  $+$  $+$ 

quark spin | quark orbital | gluon total

(b) Phenomenological derivation

- In an inertial frame

$$\Pi^{+}(\omega) = \epsilon_{+}^{\mu}\epsilon_{+}^{\nu *}\Pi_{\mu\nu}(\omega,0) = \omega^2\,\Pi^{vac}(\omega^2)$$

- Energy shift of all right circularly polarized state in a rotating frame

=> $\omega \to \omega - g_\Omega\,\Omega \qquad H_{\mathrm{SRC}} = -\,g_\Omega\,S\cdot\Omega$

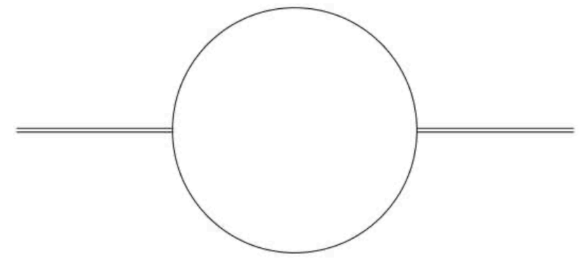
$$\begin{aligned}\Pi^{+}(\omega + g_\Omega\Omega) &= (\omega + g_\Omega\Omega)^2\,\Pi^{vac}((\omega + g_\Omega\Omega)^2)\\ &= \omega^2\,\Pi^{vac}(\omega^2) + \omega\,\Pi^{rot}(\omega^2)\,\Omega + \mathcal{O}(\Omega^2)\end{aligned}$$

- Simple expression of rotating part in terms of vacuum invariant ftn.

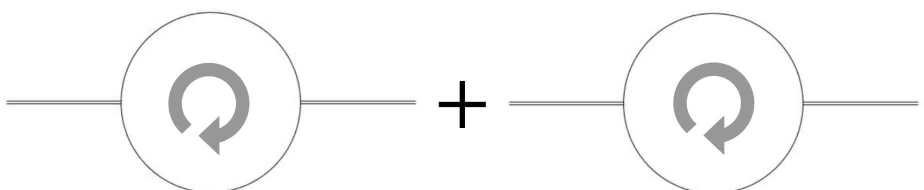
$$\Pi^{rot}_{(b)}(\omega^2) = 2g_\Omega\left\{\Pi^{vac}(\omega^2) + \omega^2\frac{\partial\Pi^{vac}(\omega^2)}{\partial\omega^2}\right\}$$

=> We can directly derive Π^{rot} from Π^{vac} but it includes unknown g_Ω

g_Ω in perturbative region

$\Pi^{vac}_{pert} =$ 

method (a)
direct computation in a rot frame
all responses of quarks in a rot frame
quark's spin + orbital AM

$\Pi^{rot}_{(a)} =$ 

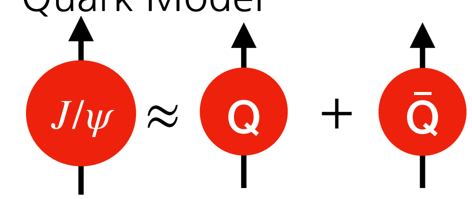
method (b)
phen. derivation based on the energy shift of total system by "-$g_\Omega \Omega$".

$$\Pi^{rot}_{(b)} = 2g_\Omega \left\{ \Pi^{vac}(\omega^2) + \omega^2 \frac{\partial \Pi^{vac}(\omega^2)}{\partial \omega^2} \right\}$$

$g_\Omega = 1$ in the perturbative region

When two free quarks form a spin-1 state in a rel. way, they follow $H_{SRC} = -S \cdot \Omega$

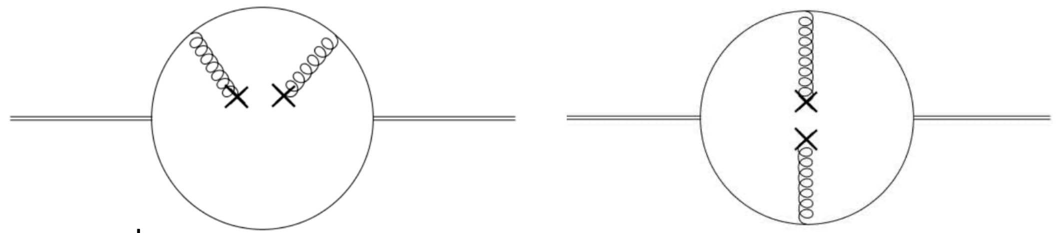
Quark Model



$$|11\rangle \sim |\frac{1}{2}\frac{1}{2}\rangle + |\frac{1}{2}\frac{1}{2}\rangle$$

$$H_r = m_{J/\psi} - \Omega$$

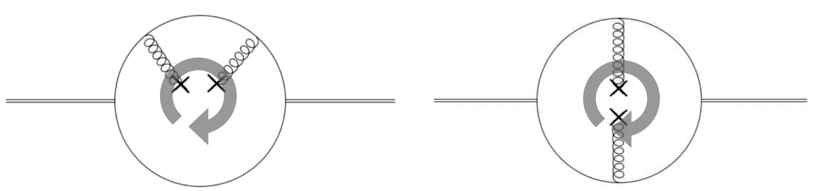
g_Ω in non-perturbative region

$\Pi^{vac}_{G0} =$ 

G0 : gluon condensate

method (a)
direct computation in a rot frame
all responses of quarks, gluons in a rot frame

$\Pi^{rot}_{(a)}$ = quark spin + quark orbit + gluon of



method (b)
phen. derivation based on the energy shift of total system by "-$g_\Omega \Omega$".

$$\Pi^{rot}_{(b)} = 2 g_\Omega \left\{ \Pi^{vac}(\omega^2) + \omega^2 \frac{\partial \Pi^{vac}(\omega^2)}{\partial \omega^2} \right\}$$

$g_\Omega = 1$ in the non-perturbative region

Even in non-pert region, spin-1 system follows $H_{SRC} = - S \cdot \Omega$

Physical meaning of $g_\Omega = 1$?

Method (b)

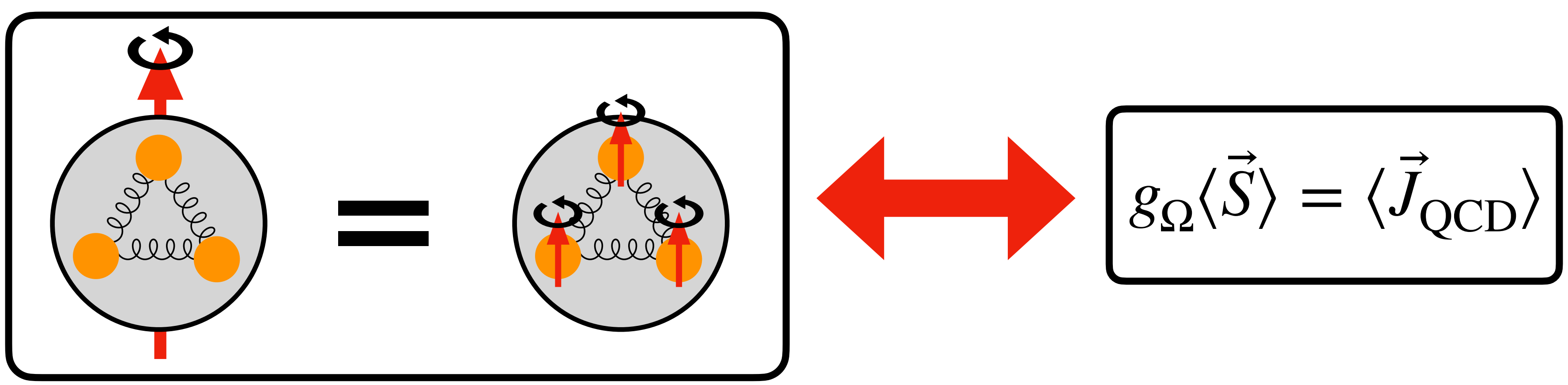
= SRC of the total system

= $g_\Omega \langle \vec{S} \rangle$ where $\vec{S}$ is spin-1 operator where $\langle \cdots \rangle = \int d^4x e^{iq\cdot x} \langle 0 | T[j(x) \cdots j(0)] | 0 \rangle$

Method (a)

= Ω linear terms in all responses of quarks and gluon in a rotating frame

= $\langle \vec{J}_{\mathrm{QCD}} \rangle$ where $\vec{J}_{\mathrm{QCD}} = \int d^3x \left(\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma_5\psi + \psi^\dagger(\vec{x} \times (-i\vec{D}))\psi + \vec{x} \times (\vec{E} \times \vec{B})\right)$



Therefore, we can conclude that $g_\Omega = \langle \vec{J}_{\mathrm{QCD}} \rangle / \langle \vec{S} \rangle$

- $g_\Omega = 1$ means $\langle \vec{S} \rangle = \langle \vec{J}_{\mathrm{QCD}} \rangle$
- This should be valid for any Feynman diagram ($\because$ AM conservation)

Application - g_Ω of ground states

From Kallen-Lehmann(or spectral) rep,

"$g_\Omega = 1$" is universal for all physical states that can couple to $j^+(x)$.

If we can extract the ground state,

=> Fraction of g_Ω carried by each a.m. inside the ground state

$$g_\Omega^{\mathrm{ground}} = \frac{\langle J_{\mathrm{QCD}} \rangle}{\langle S_{\mathrm{tot}} \rangle} = \frac{\langle S_q \rangle + \langle L_k \rangle + \langle L_p \rangle + \langle J_g \rangle}{\langle S_{\mathrm{tot}} \rangle} = 1$$

$S_q = \frac{1}{2}\gamma^1\gamma^2$: quark spin,

$L_k = r \times p$: kinetic part of quark orbital a.m,

$L_p = r \times gA$: potential part of quark orbital a.m,

$J_g = r \times (E \times B)$: gluon total a.m.

=> Spin content of the ground state

Result - spin contents of spin-1 quarkonia

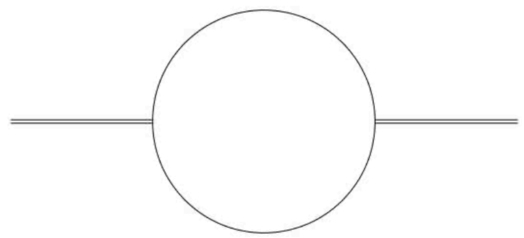
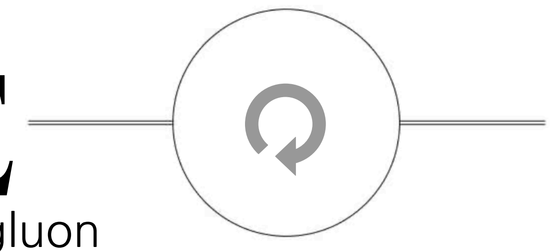
With the help of 'QCD sum rule' + simple 'pole+continuum' ansatz.

			Vector (%)			Axial (%)		
			S-wave	$\Upsilon(1S)$	J/ψ	P-wave	χ_{b1}	χ_{c1}
Quark	spin	S_q	100	92	88	50	43	40
Quark	$r \times p$	L_k	0	7.6	11	50	57	61
Quark	$r \times gA$	L_p	0	0.003	0.2	0	-0.001	0.08
Gluon	$r \times (E \times B)$	J_g	0	0.015	0.8	0	-0.005	-1.5

- Total sum of 4 pieces becomes 1
- Classical picture from the naive Q.M.
  S-wave: quark spin(100%) , P-wave: quark spin(50%) quark oam(50%)
- Spin contents are slightly different from the classical picture.
  As the quark mass becomes lighter, spin contents deviate more from the classical picture
  ex) J/ψ is considered as S-wave but quarks do not carry all of the total spin
  $\Upsilon(1S)$ is still comparable with the classical picture

Summary

- For a given Feynman diagram in an inertial frame, there is a counterpart in a rotating frame

$\Pi^{vac} =$  $\Pi^{rot} = \sum_{\text{quark, gluon}}$ 

$$\Pi^{rot} = 2\left\{ \Pi^{vac}(\omega^2) + \omega^2 \frac{\partial \Pi^{vac}(\omega^2)}{\partial \omega^2} \right\} \text{ with } g_\Omega = 1$$

$$\langle \vec{S} \rangle = \langle \vec{J}_{\mathrm{QCD}} \rangle$$

1. We prove that spin-1 composite systems follow $H_{\mathrm{SRC}} = -S \cdot \Omega$
2. We examine spin contents of ground states in a relativistic way using QCD Sum Rules

Back up

Future plans, possible extensions

1. Light quark system

vector mesons

	Q.M.	$\Upsilon(1S)$	J/ψ	ρ, ω, ϕ
S_q	100	92	88	?
L_k	0	7.6	11	?
L_p	0	0.003	0.2	?
J_g	0	0.015	0.8	?

nucleons

	Q.M.	p, n
S_q	100	?
L_k	0	?
L_p	0	?
J_g	0	?

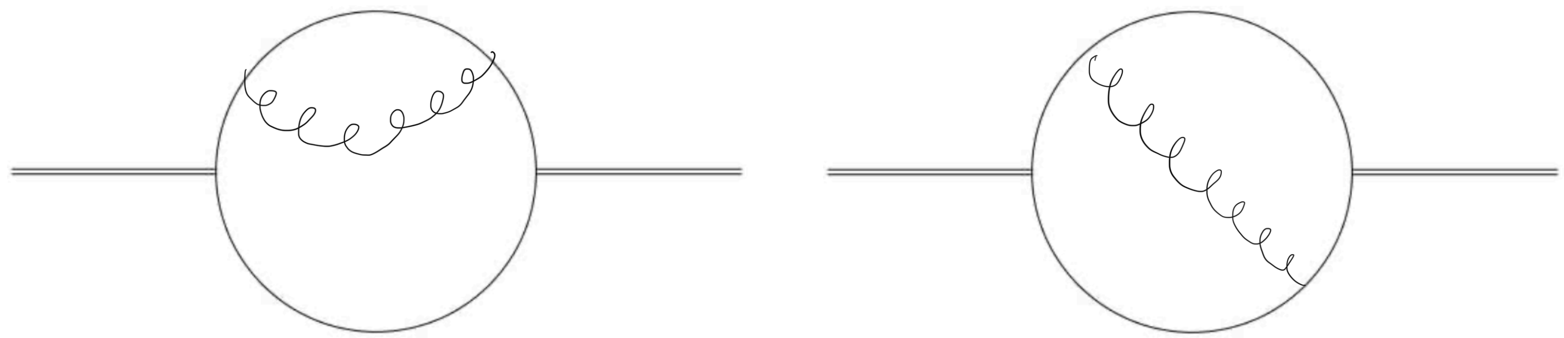
2. vacuum -> medium

$\langle 0 | \cdots | 0 \rangle \rightarrow \langle \Omega | \cdots | \Omega \rangle$ Lorentz symmetry broken

3. uniform rotation -> local vorticity

4. away from the center, 3-momentum, finite size effect, etc

Q. α_s-correction?

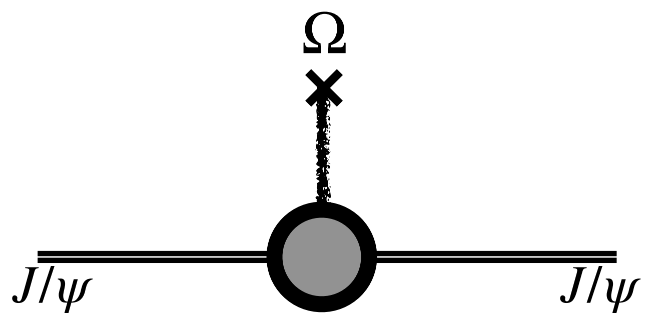


- In perturbative region, $L_p = r \times gA$ and $J_g = r \times (E \times B)$ start to contribute from α_s-corrections. Therefore, more accurate analysis requires α_s-correction of Π^{rot}

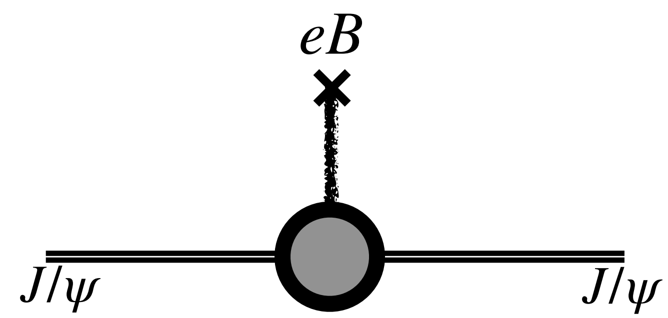
- Signs of L_p and J_g will become more clear

Ω vs B

- Linear term



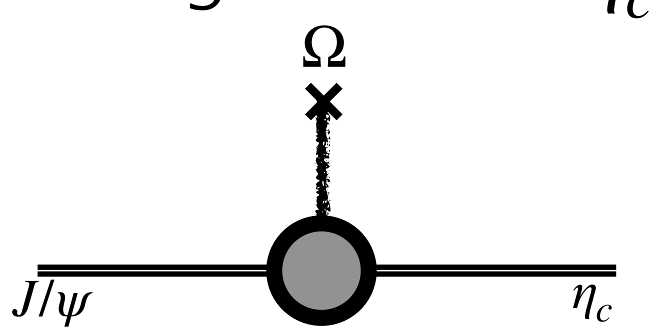
$\neq 0$



$= 0$

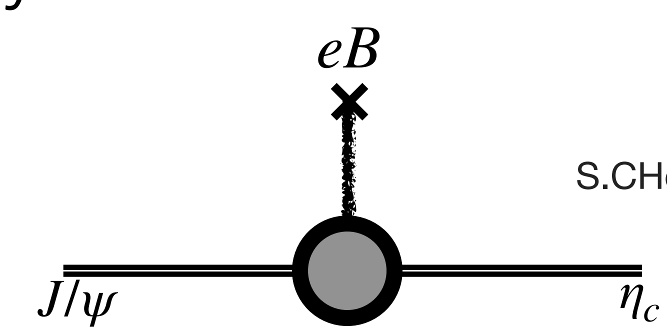
(at least LO)

- Mixing between η_c and J/ψ by Ω?



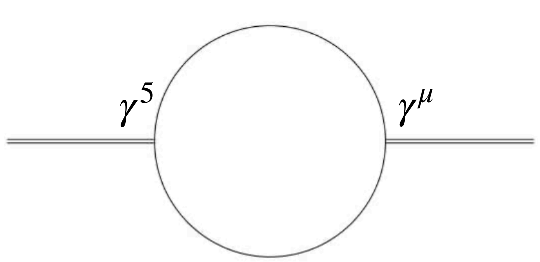
$= 0$

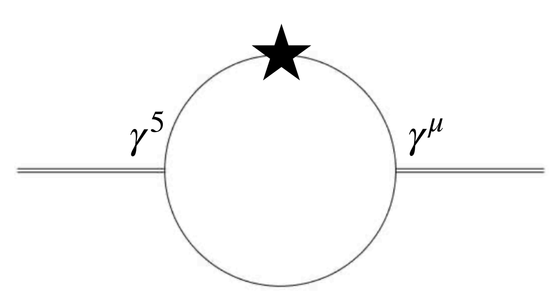
S.CHo,K.Hattori,S.H.Lee et al. PRL113.172301



$\neq 0$

This can be simply understood by $\Pi^{rot}(\omega^2) = 2\left\{ \Pi^{vac}(\omega^2) + \omega^2 \frac{\partial \Pi^{vac}(\omega^2)}{\partial \omega^2} \right\}$

Since 

$= 0$, 

$= 0$

★ indicates insertion of Ω linear terms

Dirac eq in a rotating frame

- By EEP, non-inertial frame ~ curved space-time
- Dirac eq in curved space-time

$$\left[i\gamma^a e_a^\mu \left(\partial_\mu - iqA_\mu - \frac{i}{4}\omega_\mu^{bc}\sigma^{bc}\right) - m\right]\Psi = 0$$

$\mu, \nu, \ldots$: curved space-time,

$a, b, \ldots$: flat space-time

$e_a^\mu(x)$: vierbein s.t. $g^{\mu\nu}(x) = e_a^\mu(x) e_b^\nu(x) \eta^{ab}$

$\omega_\mu^{ab} = e_\nu^a(\partial_\mu + \Gamma_{\mu\sigma}^\nu e^{\sigma b})$: spin connection,

- In a rotating frame,

$$ds^2 = (-1 + (\boldsymbol{\Omega} \times \mathbf{r})^2)dt^2 + 2(\boldsymbol{\Omega} \times \mathbf{r})d\mathbf{r}dt + d\mathbf{r}^2$$

➡ $$\left[i\partial\!\!\!/_x + gA\!\!\!/(x) - m - \gamma^0 \left\{ \frac{1}{2}\begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix} + \vec{x} \times (i\vec{\partial}_x + g\vec{A}(x)) \right\} \cdot \vec{\Omega}\right]\Psi(x) = 0$$

➡ $$\left[i\partial\!\!\!/_x + gA\!\!\!/(x) - m + \gamma^0(L_q + S_q) \cdot \Omega\right]\Psi(x) = 0$$