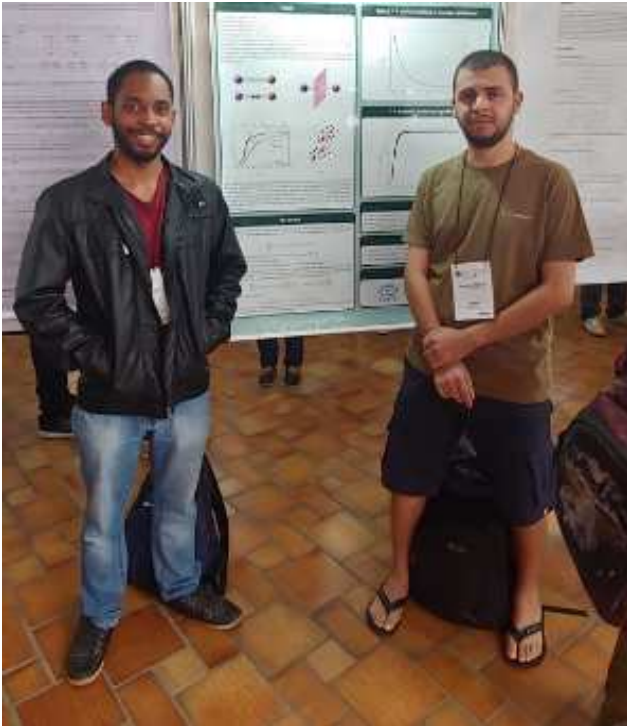


## Spin 1 resonances as probes of spin-vorticity dynamics



2305.02985 (PRD), 2104.12941 (PRC) with Kayman Jhosef Carvalho  
Goncalves, Paulo Henrique de Moura

## Synopsis

**The theoretical necessity** of spin-vorticity non-equilibrium

**The experimental necessity** of spin 1 and higher (qubits vs qutrits)

**From qutrits** to non-equilibrium to data

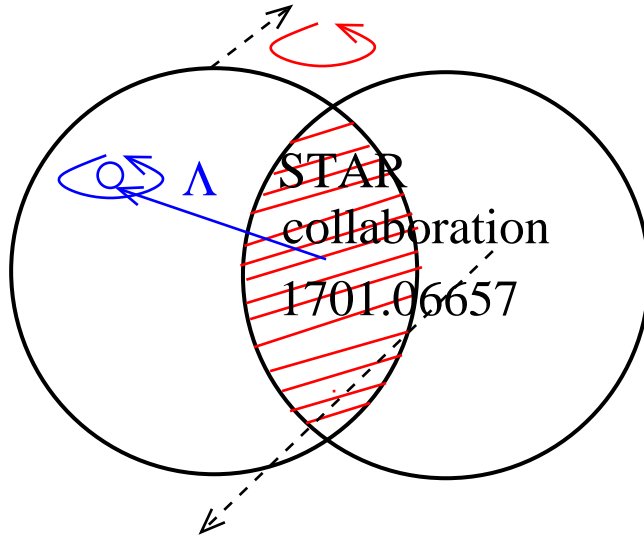
**Coalescence** of light vector mesons

**Quarkonia** mass shifts

**Melting** a new mechanism for alignment  
Correlate abundance and vorticity?

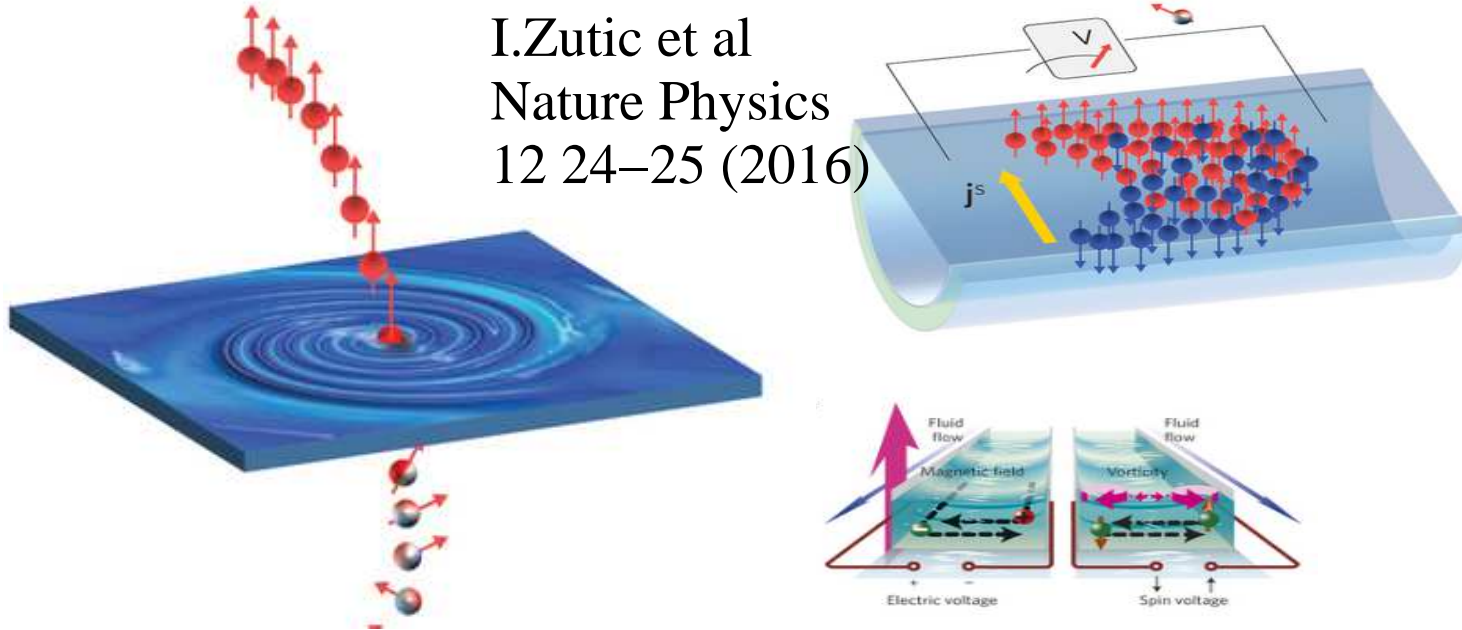
**What to measure** to get non-equilibrium

## Hydrodynamics with spin



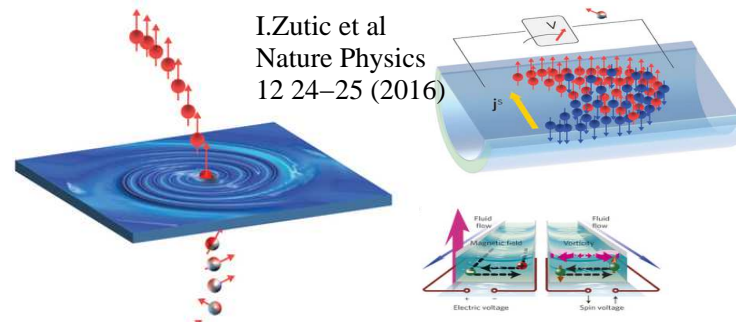
A remarkable experimental discovery, which opened a fascinating field of theoretical investigation.

I.Zutic et al  
Nature Physics  
12 24–25 (2016)



What does a relativistic system look like where

- Close to local equilibrium (strongly coupled, many DoFs)
- Dofs have spin (not "colliding balls" but "shapes")



Unresolved very non-trivial statistical mechanics problem. Spin **not the same** as "small vortex".

**Spin** quantum microstate, how angular momentum is shared determines macroscopic entropy

**Vorticity** A classical collective excitation carrying angular momentum

So **spin hydrodynamics** means backreaction of microscopic DoFs on macroscopic perturbations!

## Phenomenology so far: Becattini et al, 1303.3431

GC ensemble with angular momentum as a conserved quantity, fermions (1 species)

$$\exp\left(-\frac{p_\alpha u^\alpha}{T}\right) \rightarrow \exp\left(-\frac{p_\alpha u^\alpha}{T}\right) (\bar{u}, \bar{v}) \exp\left[\frac{\Sigma_{\mu\nu}\omega^{\mu\nu}}{T}\right] \begin{pmatrix} u \\ v \end{pmatrix}$$

And Fermi-Dirac statistics. Here

- $\omega^{\mu\nu}$  vorticity tensor
- $\Sigma^{\mu\nu}$  spin projection tensor  $\sim \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$

local equilibrium isentropic particle production with spin

**Fits**  $\Lambda$  global polarization

**Doesen't fit** local polarization (wrong phase).

- use T-vorticity (**why** ?) only vorticity that makes theoretical sense (Angular momentum, conserved circulation) is  $\nabla \times [(e + p)\vec{u}]$
- Symmetric shear and isothermal freeze-out (**but** theoretical justification not clear...)

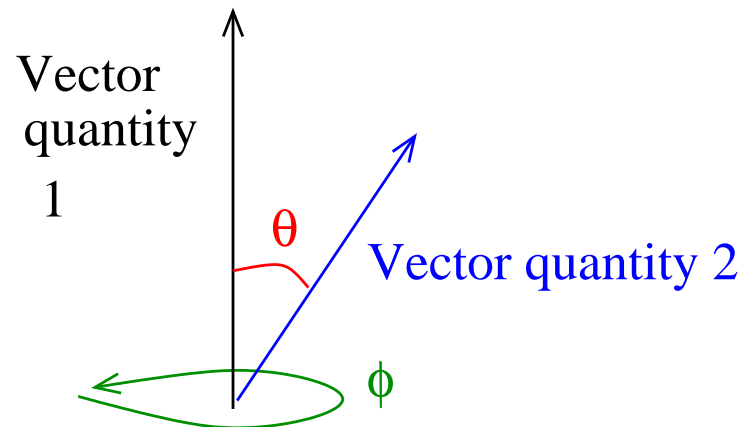
**Underestimates** and wrong sign for some spin-1 particles,  $\phi, J/\Psi$

**Investigation ongoing** Toroidal vortex rings, jet-induced polarization, ...

But... and this is the main point!

Theoretically **This can not be the whole story!** Perfect equilibrium between vorticity and spin acausal!

GT+Montenegro,1807.02796,GT, Montenegro, Tinti,1701.08263



**Vorticity+spin** must be aligned at equilibrium for entropy to have a well-defined minimum

**But** this cannot occur instantaneously in a dynamical system



$$\tau_Y u_\alpha \partial^\alpha s_{\beta\gamma} + s_{\beta\gamma} = \omega_{\beta\gamma}$$

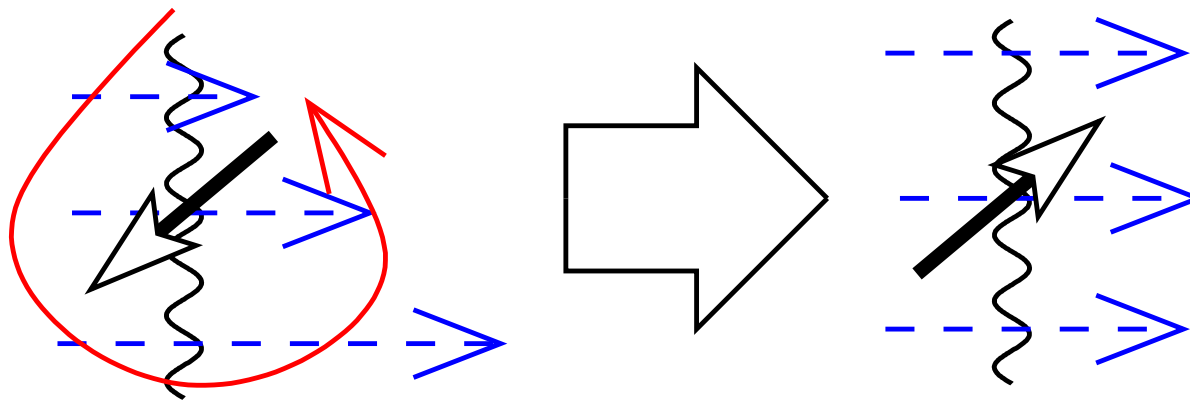
**GT, Tinti, Montenegro** dispersion relation quartic, always non-causal (Ostrogradski's theorem). **GT, Montenegro** :  $\tau_Y$  Kramers-Konig dual to vortical susceptibility

**Nora Weickgenannt et al** transport theory with spin has non-local collision term

**E.Speranza et al** non-linear stability analysis

Qualitative picture :GT,D Montenegro, 1807.02796 lower limit to  $\eta/s$  in polarizable fluids (anti-”ferrovortetic” or phase transition)

$$\tau_Y^2 \geq \frac{8c\chi^2(b_o, 0)}{(1 - c_s^2)b_o F'(b_o)} \quad , \quad \frac{\eta}{s} \geq T\tau_Y$$



Many attempts at quantitative picture [Stephanov, Hongo, Florkowski, Ryblewski, Torrieri](#)  
spin/vorticity nonequilibrium theoretically inevitable, **Phenomenology needed!**

## Spin and vorticity: classical-quantum interaction

$$\rho_{spin} = \text{Tr}_{bath} \left[ \rho_{spin} \times \underbrace{\rho_{bath}}_{vorticity} \right]$$

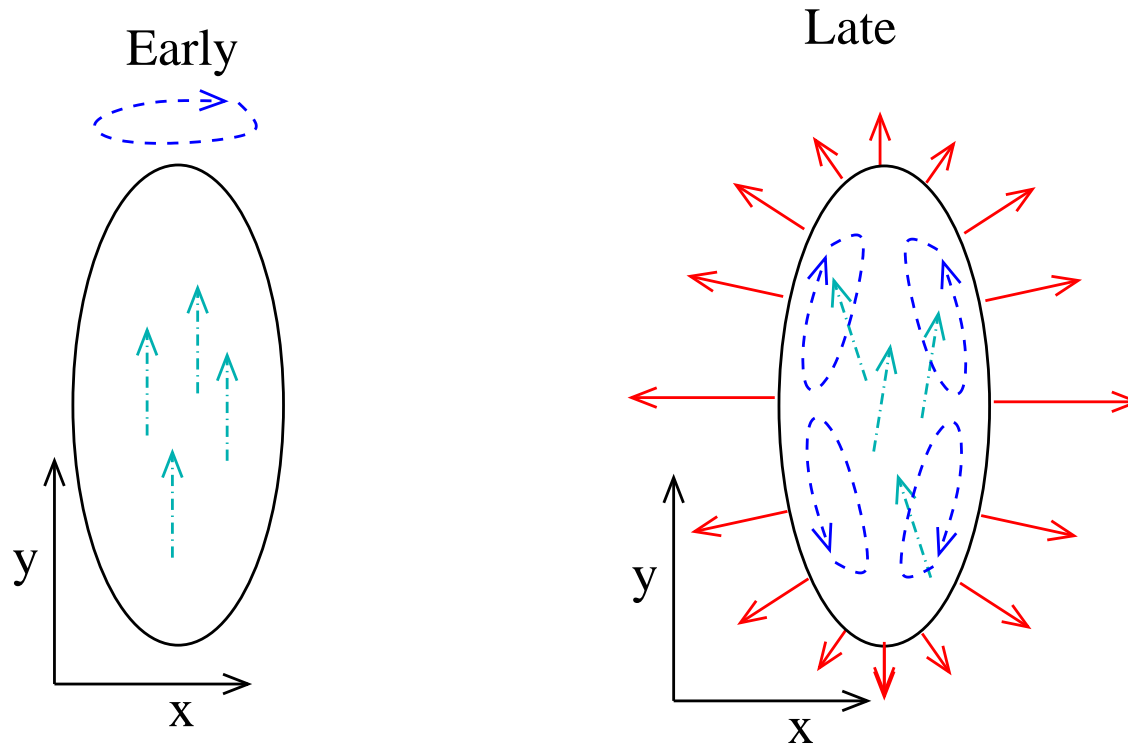
$\rho_{spin}$  Mixed, evolves under Lindblad equation

$\rho_{bath}$  Maximally mixed, evolves under classical equation of motion

$H_{int}$  spin-orbit coupling

**Cooper-Frye** limit of maximal mixing

Need model-building for this



Global vorticity formed earlier, local vorticity later. Thus the former should be more equilibrated with vorticity than the latter! **important consequences for  $\Lambda$ , vector meson**. But we need more quantitative observables!

Why spin 1 resonances can help: spin 1/2 is a qbit of information

$$\rho = \begin{pmatrix} \rho_{1,1} & \rho_{1,-1} \\ \rho_{1,-1}^* & 1 - \rho_{1,1} \end{pmatrix}, \quad \underbrace{U^{-1}\rho U = \text{Diag}(\alpha, 1 - \alpha)}_{dU = \hat{I} - \delta\theta \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - i\delta\phi \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}$$

1 parameter, 2 angles Whatever mechanism combines spin ( $\equiv \phi$ ) and angular momentum ( $\equiv \theta$ ) will result in some superposition of +1, -1

$$\frac{dN_{decay}}{d\theta} = 1 + \alpha \cos \theta, \quad \alpha \sim \rho_{1,1}$$

Thus in practice any state indistinguishable from equilibrium  $\rho_{equilibrium} \propto \begin{pmatrix} e^{w/T} & 0 \\ 0 & e^{-w/T} \end{pmatrix}$  Can study Dependence of  $\alpha$  on  $y, p_T$  but need to fit many parameters. Need qualitative signature!

Spin 1 contains more information even if parity conserved!

$$\frac{dN}{d\theta d\phi} \propto \cos^2 \theta \rho_{00} + \sin^2 \theta \left( \frac{1 - \rho_{00}}{2} + r_{1,-1} \cos(2\phi) + \alpha_{1,-1} \sin(2\phi) \right) +$$

$$+ \sin(2\theta) (r_{10} \cos \phi + \alpha_{10} \sin \phi)$$

Where

Variable	Element	coefficient $\times \frac{3}{4\pi}$
$\rho_{00}$	$\rho_{00}$	$\cos^2 \theta$
$\frac{1 - \rho_{00}}{2}$	$\frac{\rho_{11} + \rho_{-1-1}}{2}$	$\sin^2 \theta$
$r_{10}$	$Re[\rho_{-10} - \rho_{10}]$	$\sin(2\theta) \cos(\phi)$
$\alpha_{10}$	$Im[-\rho_{-10} + \rho_{10}]$	$\sin(2\theta) \sin(\phi)$
$r_{1,-1}$	$Re[\rho_{1,-1}]$	$\sin^2 \theta \cos(2\phi)$
$\alpha_{1,-1}$	$Im[\rho_{1,-1}]$	$\sin^2 \theta \sin(2\phi)$

**Small system** Beam direction, momentum three parameters

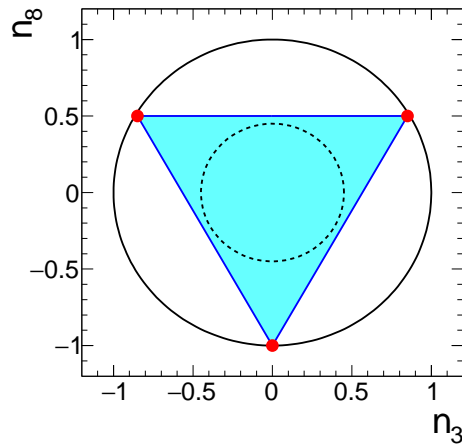
$$\rho_{00} \equiv \frac{1 + \lambda_\theta}{3 + \lambda_\theta} \quad , \quad \text{Re}[\rho_{1,-1}] \equiv \frac{\lambda_\phi}{3 + \lambda_\theta} \quad , \quad \text{Re}[\rho_{-10} - \rho_{10}] \equiv \frac{\lambda_{\theta\phi}}{3 + \lambda_\theta}$$

**Large system** Beam direction, momentum and impact parameter

2 additional parameters,  $\text{Im}[\rho_{1,-1}]$ ,  $\text{Im}[\rho_{-10} - \rho_{10}]$  **Crucial information about phases**

(**Wild idea** : Perhaps can get new axis from cumulants in small systems?)

Spin-1 is a qutrit! Two vectors! Can be mixed  $\rho^2 \neq \rho$  in all frames!



$$\rho_8(n_3, n_8) = \frac{1}{3} U^{-1}(\theta, \phi) \begin{pmatrix} 1 + \sqrt{3} n_3 + n_8 & 0 & 0 \\ 0 & 1 - \sqrt{3} n_3 + n_8 & 0 \\ 0 & 0 & 1 - 2n_8 \end{pmatrix} U(\theta, \phi)$$

Two parameters, two angles so generic state neither pure nor thermal! But what is angle  $\phi$ ? Need “objective” ebye definition! beam axis



## From angular distributions to purity

$$\frac{dN}{d\theta d\phi} \propto \cos^2 \theta \rho_{00} + \sin^2 \theta \left( \frac{1 - \rho_{00}}{2} + r_{1,-1} \cos(2\phi) + \alpha_{1,-1} \sin(2\phi) \right) + \\ + \sin(2\theta) (r_{10} \cos \phi + \alpha_{10} \sin \phi)$$

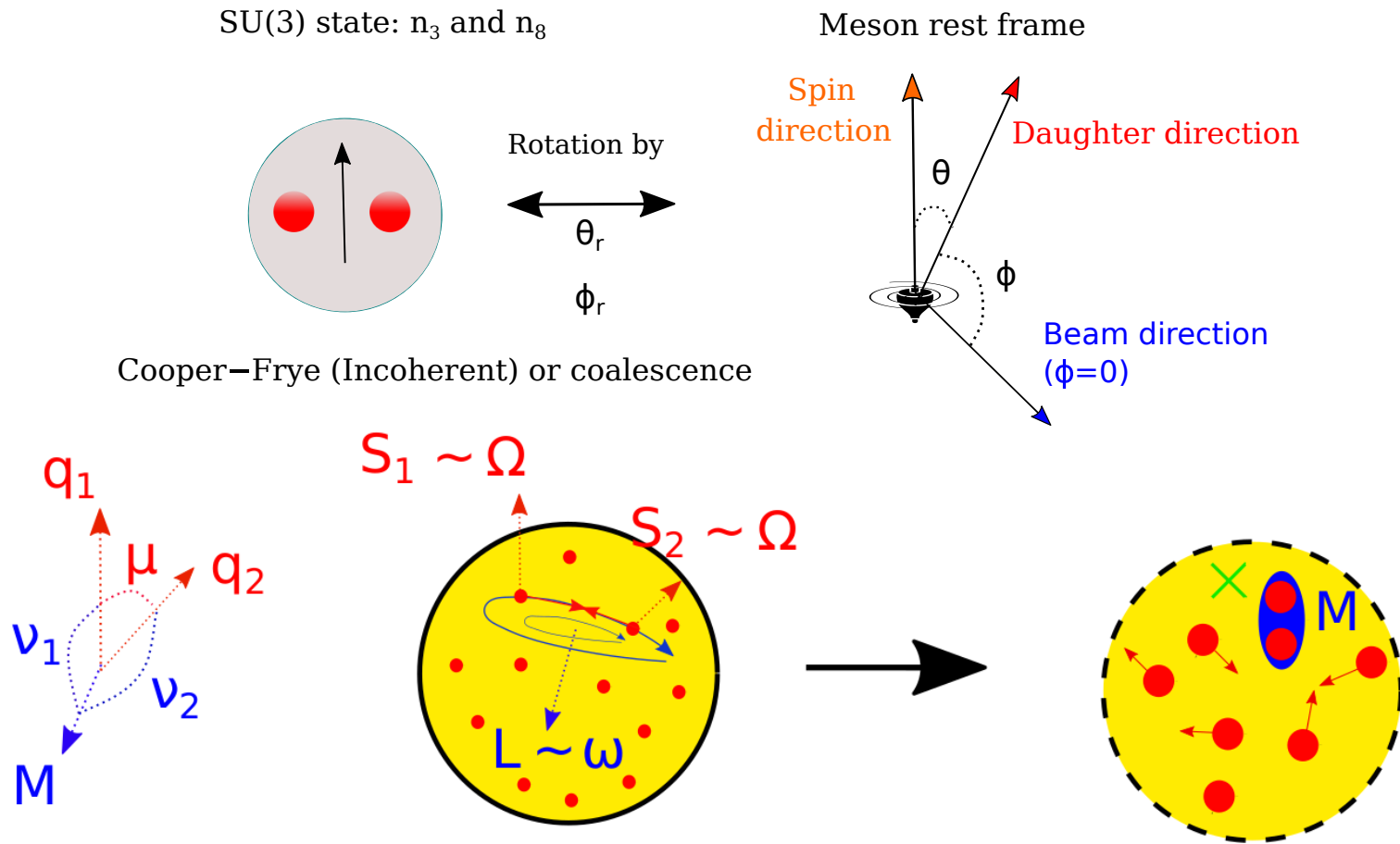
## Matrix elements and purity directly related

$$\frac{1}{12} \left( 3 \left( n_8 - \sqrt{3} n_3 \right) \cos(2\theta_r) - \sqrt{3} n_3 + n_8 + 4 \right) = \rho_{00}$$

$$\frac{(n_8 - \sqrt{3} n_3) \sin(\theta_r) \cos(\theta_r) \cos(\phi_r)}{\sqrt{2}} = r_{10}$$

$$-\frac{(\sqrt{3} n_3 + 3n_8) \sin(\theta_r) \sin(\phi_r)}{3\sqrt{2}} = \alpha_{10} \quad , \quad \phi_r = -\frac{1}{2} \tan^{-1} \left( \frac{\alpha_{1,-1}}{r_{1,-1}} \right)$$

# A concrete example: Coalescence $K^*, \phi$



**Cooper-Frye limit** for some  $\theta$

$$r\hat{h}_o = \frac{1}{N}U(\theta)^+ \begin{pmatrix} e^{-w/T} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{w/T} \end{pmatrix} U(\theta)$$

So all quantities related to  $\phi_r$  compatible with zero

**Non-equilibrium** Non-trivial  $\rho_{i \neq j}$  with two well-defined axes,  $\theta_r, \phi_r$ , whose exact nature depends on mechanism combining spin and vorticity

## Coalescence in a rotating medium

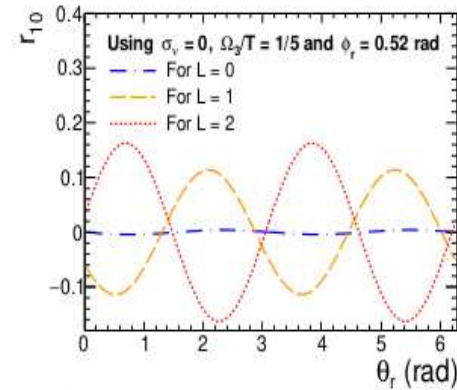
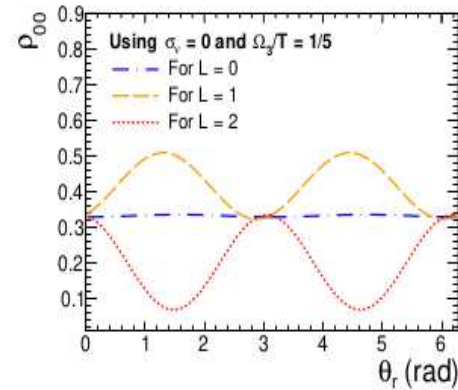
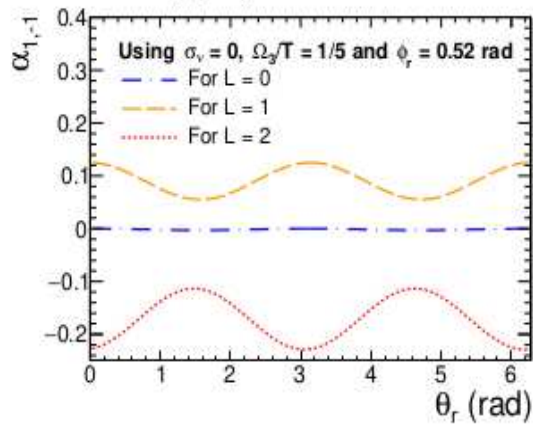
The expression of the vector meson density matrix in terms of the quark density matrices and the vorticity is straight-forwardly

$$\begin{aligned} (\hat{\rho}^M)_{mn} = & \sum_{ijkl} (P_{12}^L)_{ijklmn} U_S(\phi_r, \theta_r) (U_\omega(\mu_1, \nu_1) \rho^1(\Omega) U_\omega^{-1}(\mu_1, \nu_1))_{ij} \times \\ & \times (U_\omega(\mu_2, \nu_2) \rho^2(\Omega) U_\omega^{-1}(\mu_2, \nu_2))_{kl} U_S^{-1}(\phi_r, \theta_r) \end{aligned}$$

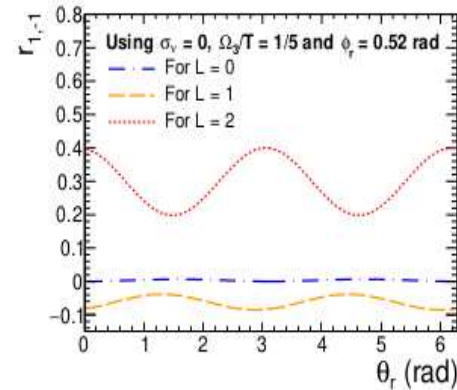
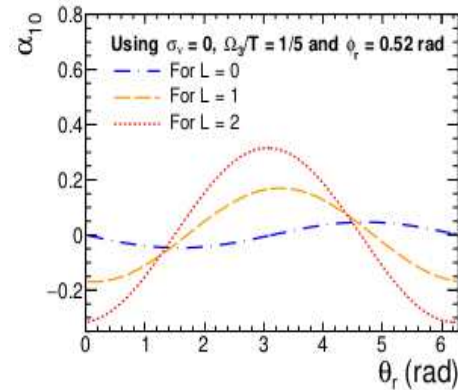
Where  $P_L(w)$  is the (unknown) probability to acquire a spin quantum number from vorticity and the rest are 6-j and C-G coefficients!

**Big approximation** : non-relativistic. But with constituent quarks and moderate  $p_T, y$  not bad

## Golden signature: off-diagonal elements



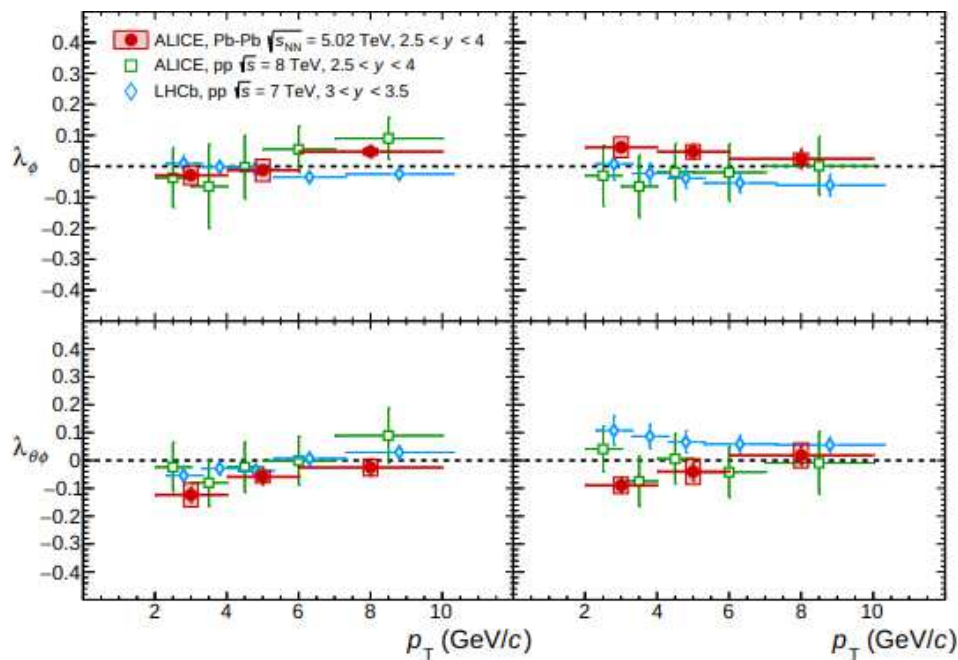
Kayman Jhosef  
GT (PRC)  
2104.12941



If ebye  $\phi$  coefficients  $\simeq 0$ , Cooper-Frye, otherwise coalescence

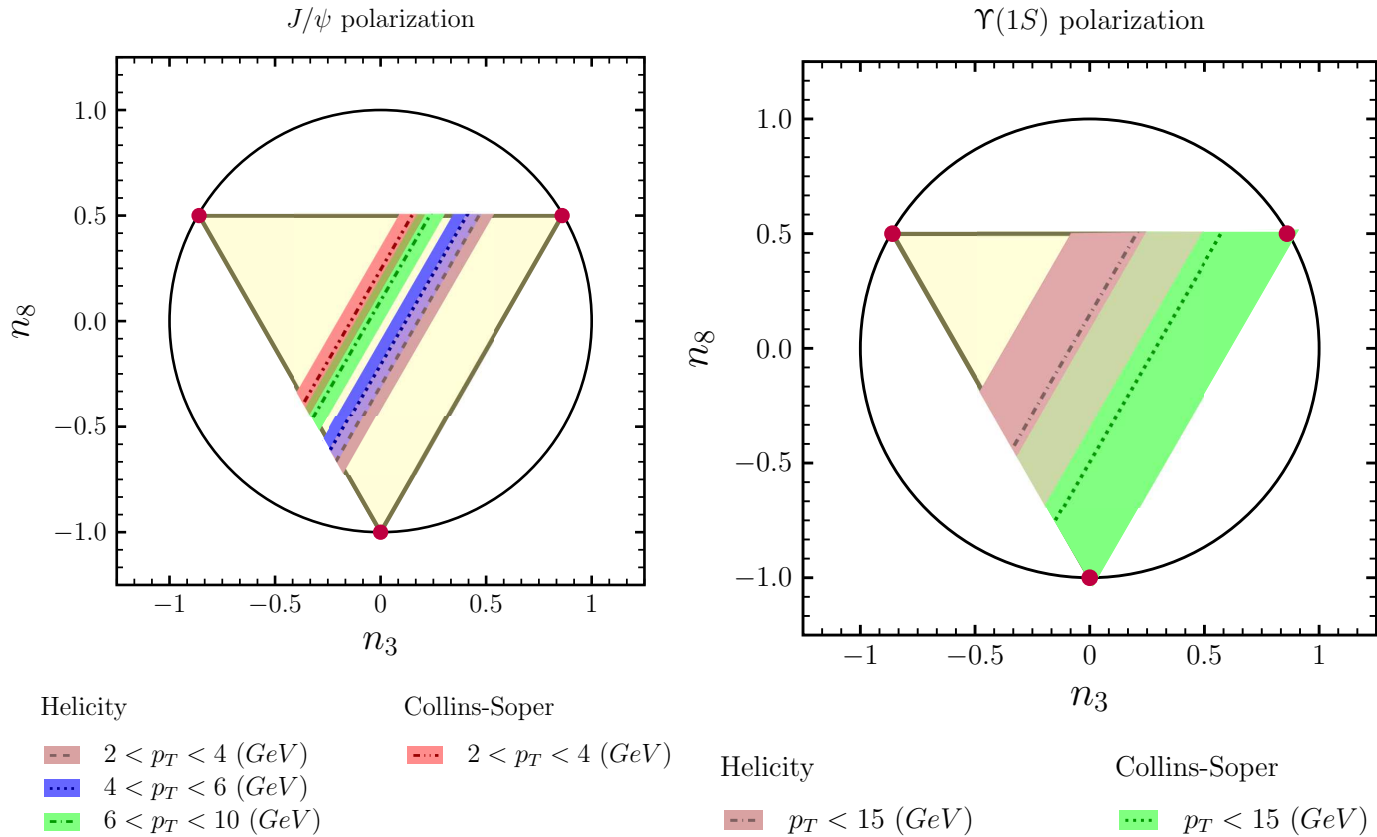
And then we discovered it was all measured... for quarkonium!

$$\rho_{00} = \frac{1 + \lambda_\theta}{3 + \lambda_\theta} \quad , \quad r_{1,-1} = \frac{\lambda_\phi}{3 + \lambda_\theta} \quad , \quad r_{10} = \frac{\lambda_{\theta\phi}}{3 + \lambda_\theta}$$

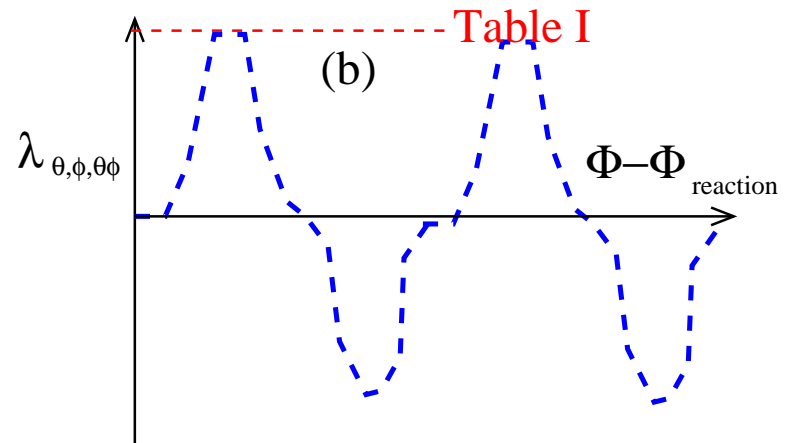
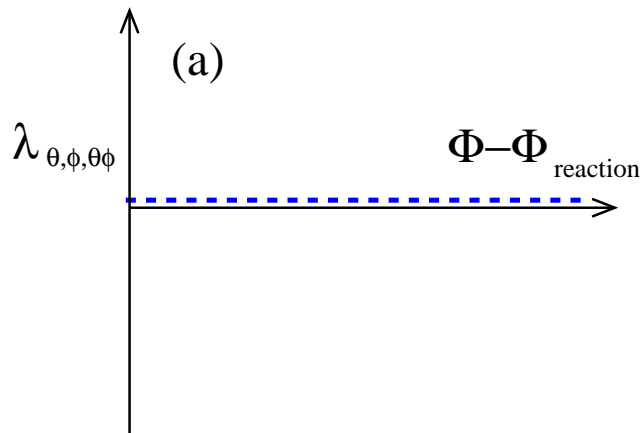


ALICE  
collaboration  
2005.11128  
PLB

Solving for  $n_{1,8}$  gives close to maximally mixed state



Off-diagonal elements compatible with zero, Cooper-Frye! **but...**

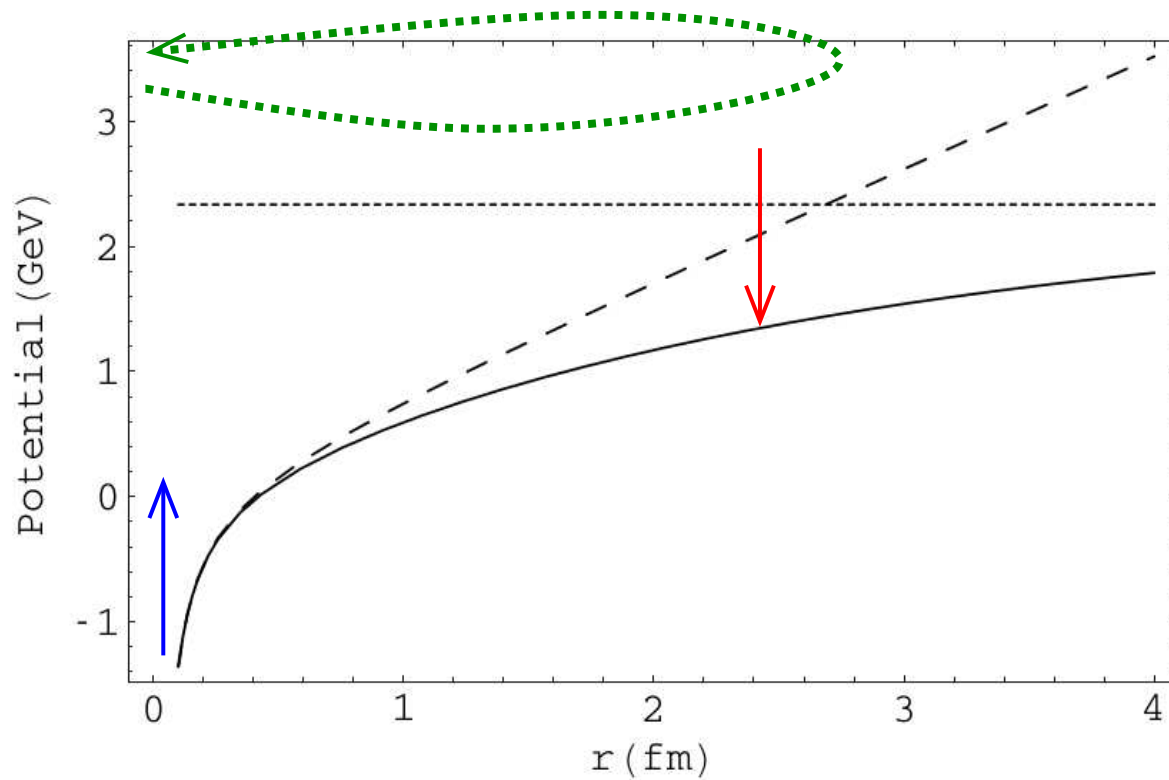


Remember spin-orbit non-equilibrium is all about interplay between early (global) and late (local) axes. **Need modulation in angle from reaction plane of  $\lambda$ s!**

**Qualitatively one expects diagonal elements to be "local", and any harmonic function averages to zero. Is it zero in every azimuthal bin?**



Getting more quantitative for Quarkonium  
Rotating Cornell potential with spin orbit interaction



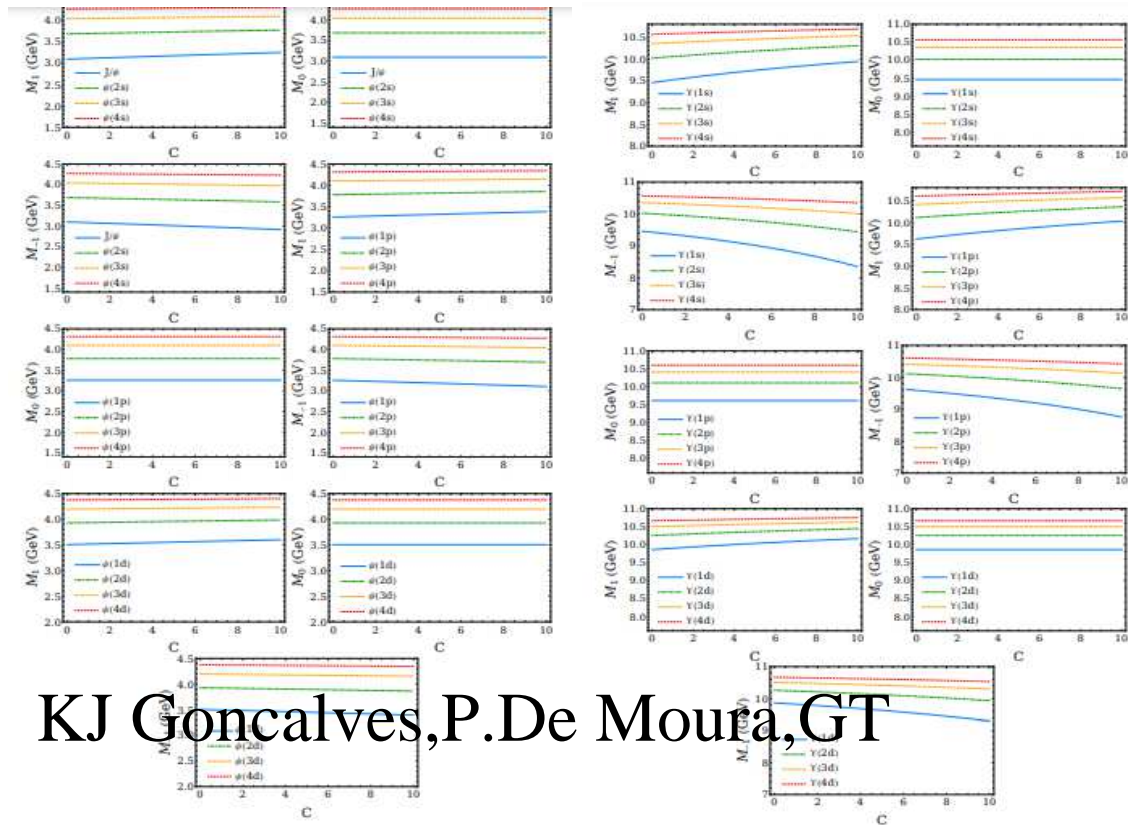
## Rotating Cornell potential with spin orbit interaction

$$\left\{ \begin{array}{l} \mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2, \\ \mu = \frac{m_1 m_2}{m_1 + m_2}, \\ \mathbf{r}_1 = \mathbf{R} + \frac{m_2}{m_1 + m_2} \mathbf{r}, \end{array} \right. \quad \begin{array}{l} \mathbf{p} = \mu \left( \frac{\mathbf{p}_1}{m_1} - \frac{\mathbf{p}_2}{m_2} \right) \\ \mathbf{r}_2 = \mathbf{R} - \frac{m_1}{m_1 + m_2} \mathbf{r} \end{array}$$

$$\mathcal{H} = \frac{P^2}{2M} + \frac{p^2}{2\mu} - \mathbf{P} \cdot (\boldsymbol{\omega} \times \mathbf{R}) - \mathbf{p} \cdot (\boldsymbol{\omega} \times \mathbf{r}) - \boldsymbol{\omega} \cdot (\mathbf{S}_1 + \mathbf{S}_2) + V(\mathbf{r})$$

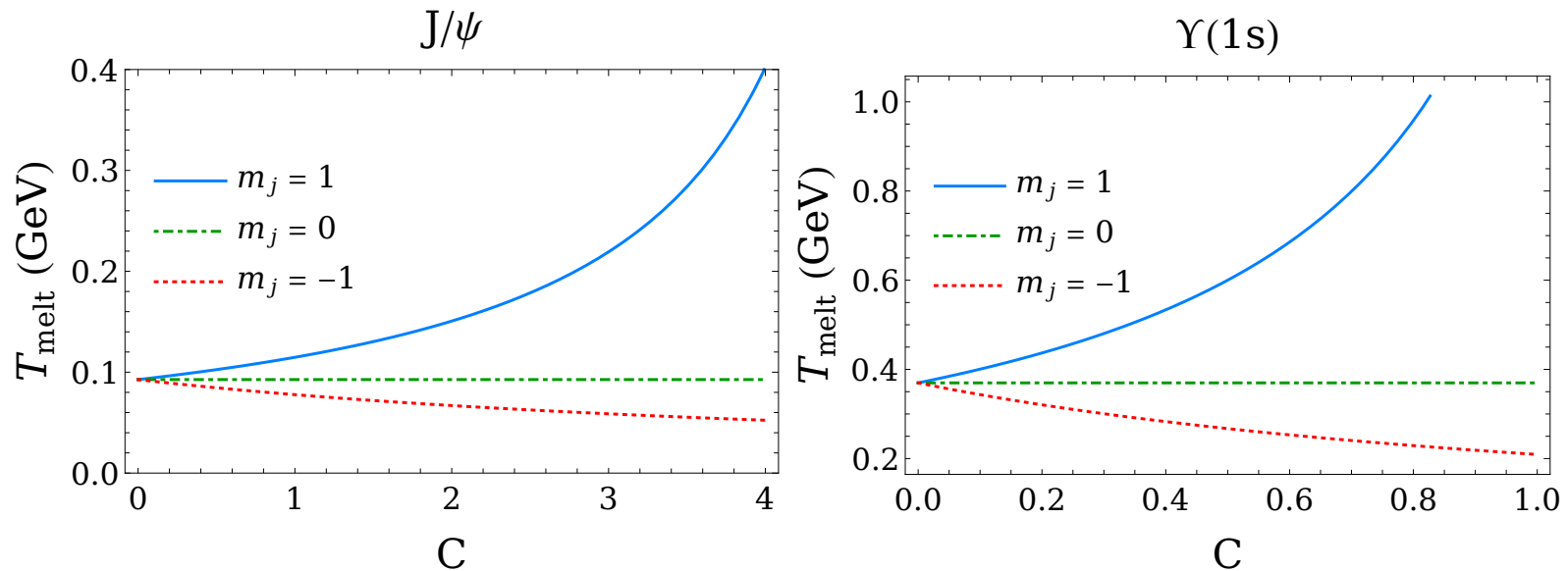
With a Cornell-type potential. This way vorticity and spin interactions accounted for. Mass correction  $\Delta E_{i,j}$  and Melting temperature from Debye formula. **More realistic QFT/Open QM models?**

Mass becomes sensitive to relative direction of spin and vorticity



This is experimentally undetectable (widening), but...

One can get melting temperature from Debye formula!



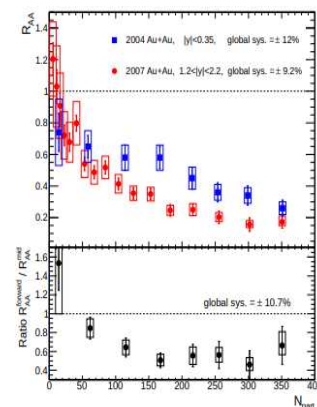
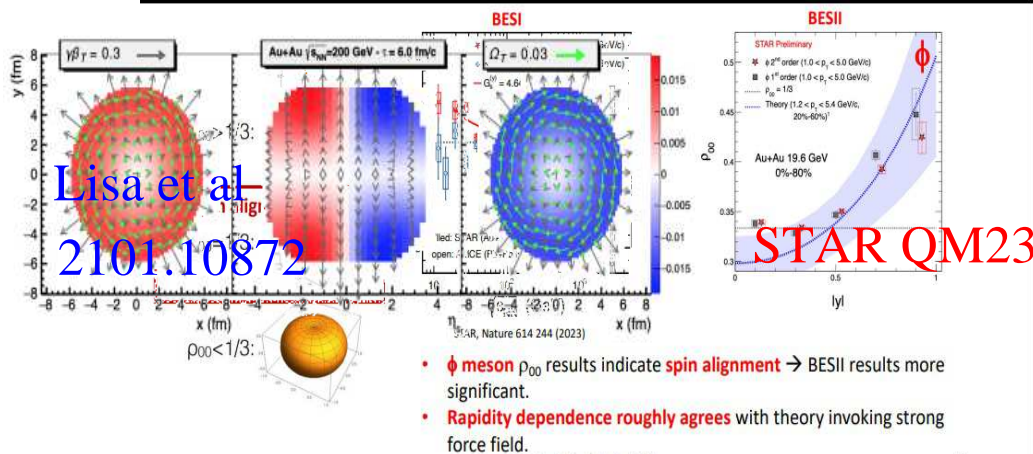
Melting temperature depends non-monotonically on rotation and (anti)polarization. Could such “distillation” explain  $\phi$  spin alignment?  $\phi$  is quarkonium and only polarized survive! If confirmed by more realistic models (QFT, openQM) need abundance vs alignment scans with rapidity.

Can also have to do with quarkonium being more suppressed at higher rapidity (Which is also higher vorticity)

## New Insights into Global Spin Alignment in Heavy-Ion Collisions: Measurements of $\phi$

Tues 930  
B. Xi  
ID 353

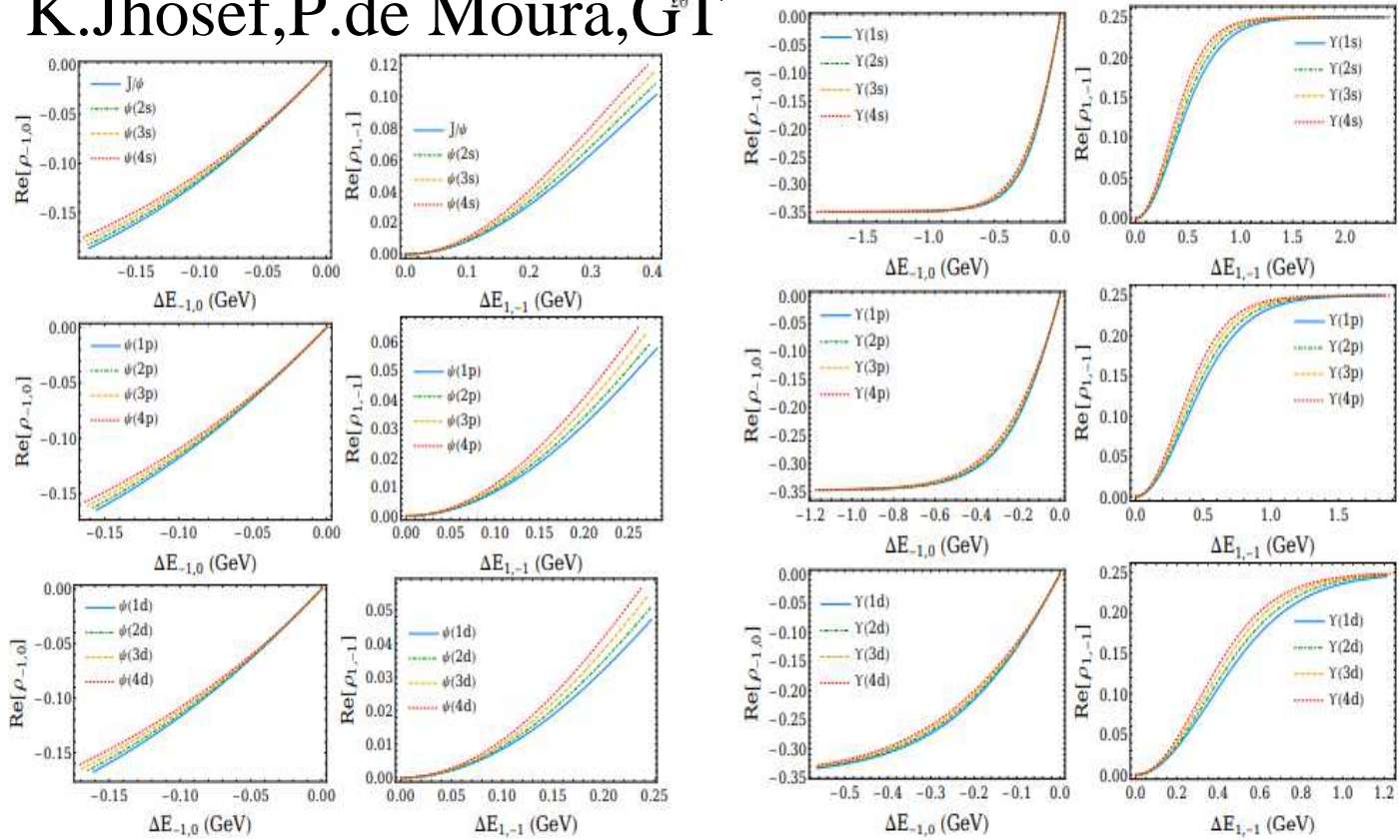
PHENIX  
1103.6269



“Distillation” is an important effect qualitatively different from Cooper-Frye  
Need: SUppression vs alignment in rapidity!

# Correlation between invariant mass and off-equilibrium $\rho$ !

K.Jhosef, P.de Moura, GT



## Conclusions

**Non-equilibrium** between spin and vorticity theoretically well-established, need phenomenology

**Spin and vorticity** different objects

**Cooper-Frye** could be misleading

**Spin1** vector mesons can serve as such a link because of a rich structure in their density matrix. **Coalescence in vector mesons, potential models in quarkonia**

**To do** baryon coalescence (can coalescence explain local polarization?),  $\Omega$  (Spin 3/2), etc.

SPARE SLIDES