Spin hydrodynamics and causality problem

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ExHIC-p workshop on polarization phenomena in nuclear collisions

March 16, 2024

Based on:

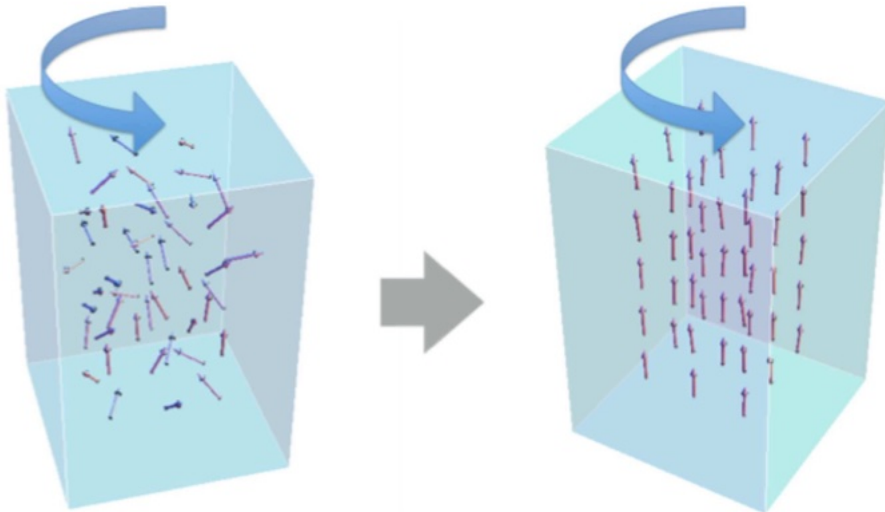
- D.L. Wang, SP, **Phys.Rev.D (Lett),** 109 (2024) 3, L031504
- X.Q. Xie, C. Yang, D.L. Wang, SP, **Phys.Rev.D** 108 (2023) 9, 094031

Outline

- **Spin polarization in HIC**
- **General discussion for causality and stability**
- **Causality analysis for spin hydrodynamics**
- **New improved causality criterion**
- **Summary and outlook**

Spin polarization in HIC

Barnet and Einstein-de Hass effects



Barnett effect:

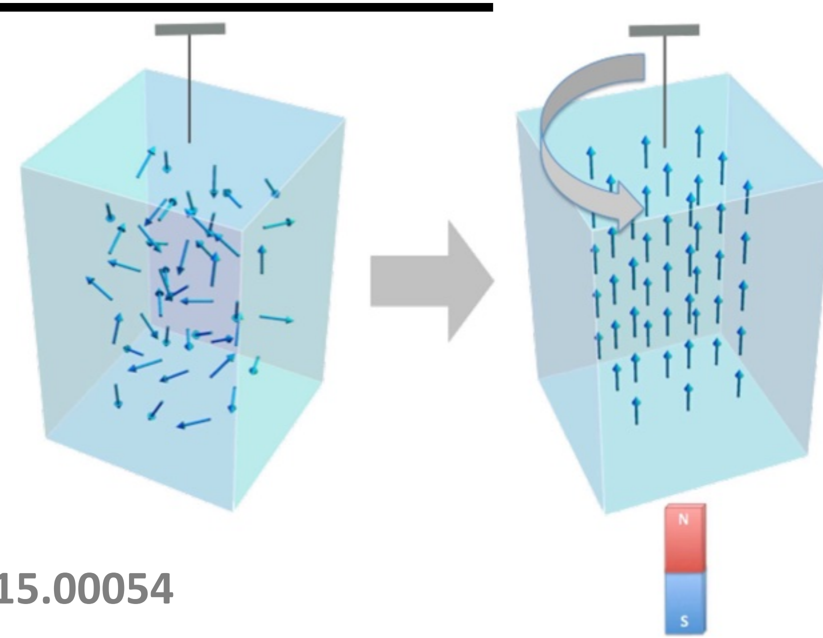
Rotation $\Longrightarrow$ Magnetization

Barnett, Magnetization by rotation, Phys Rev. (1915) 6:239–70.

Einstein-de Haas effect:

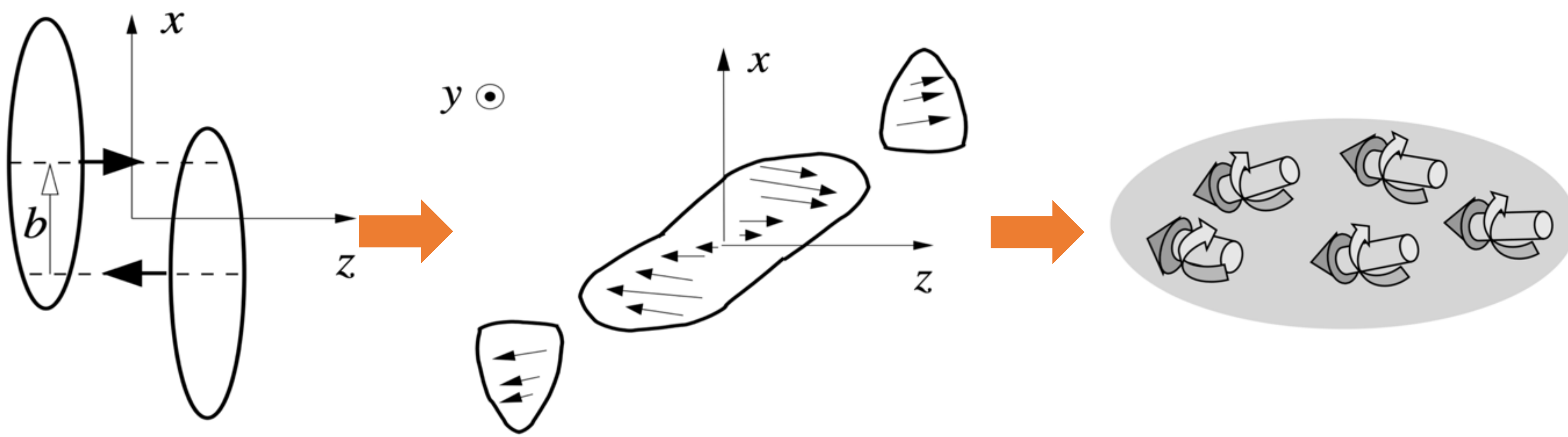
Magnetization $\Longrightarrow$ Rotation

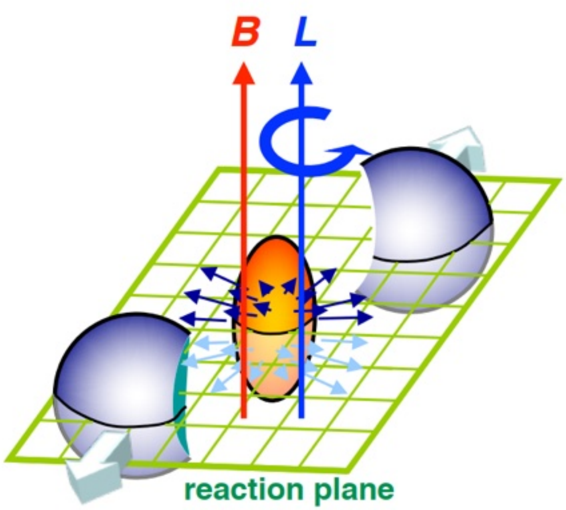
Einstein, de Haas, Experimental proof of the existence of Ampere's molecular currents. Verh Dtsch Phys Ges. (1915) 17:152.



Figures: copy from paper doi: 10.3389/fphy.2015.00054

OAM to polarization in HIC



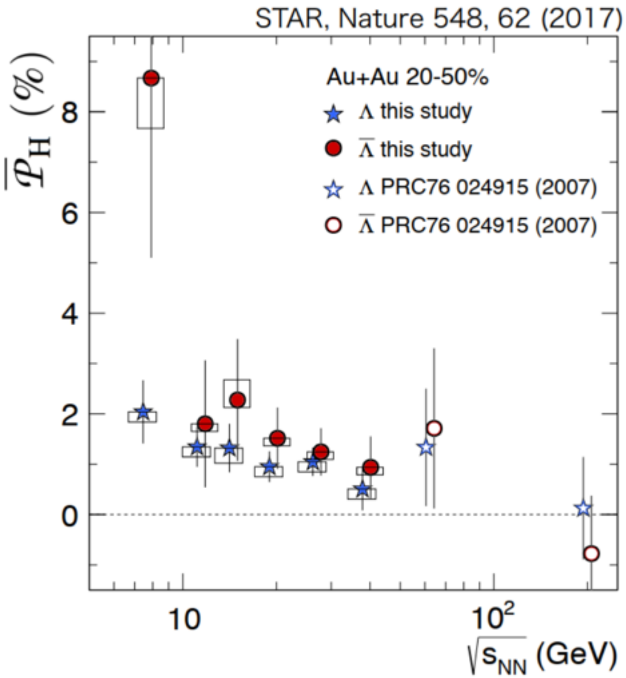


- **Huge global orbital angular momenta ($L \sim 10^5 \hbar$) are produced in HIC.**

- **Global orbital angular momentum leads to the polarizations of Λ hyperons and vector mesons through spin-orbital coupling.**

  Liang, Wang, PRL (2005); PLB (2005);
  Gao, Chen, Deng, Liang, Wang, Wang, PRC (2008)

Global polarization for Λ and $\bar{\Lambda}$ hyperons

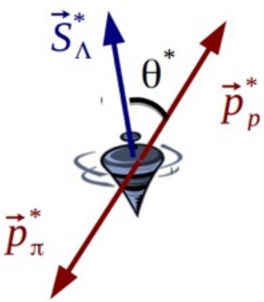


parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

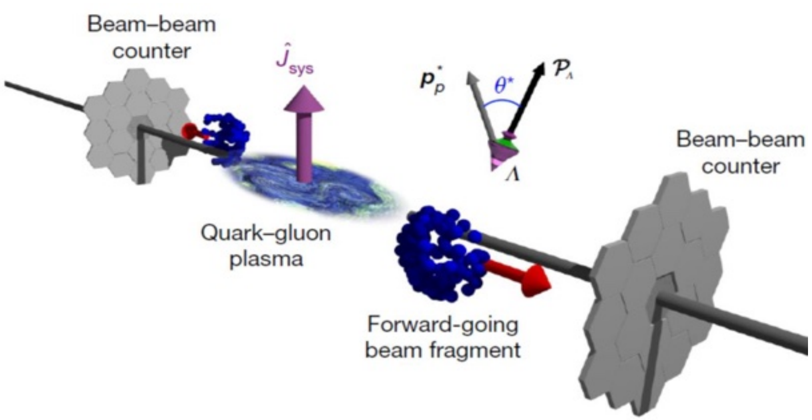
$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P_\Lambda} \cdot \mathbf{p_p^*})$$

α: Λ decay parameter (=0.642±0.013)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame



$$\Lambda \to p + \pi^+$$

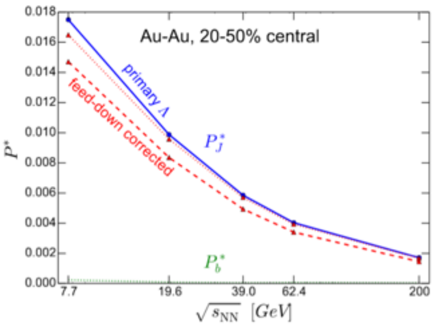
(BR: 63.9%, c τ ~7.9 cm)



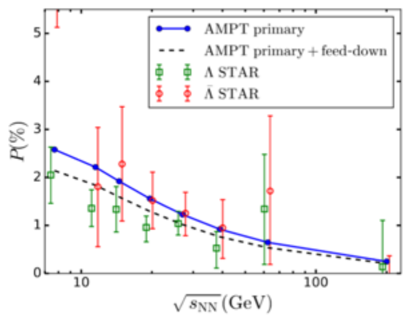
- **The vorticity of QGP can be as large as $(9 \pm 1)\text{x}10^{21}$/s。**
- **It is the most vortical fluid so far.**

Liang, Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)
Fang, Pang, Q. Wang, X. Wang, PRC (2016)
…

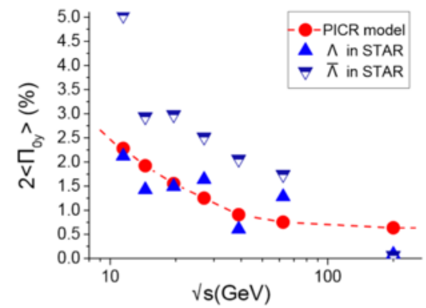
Phenomenological models for global polarization



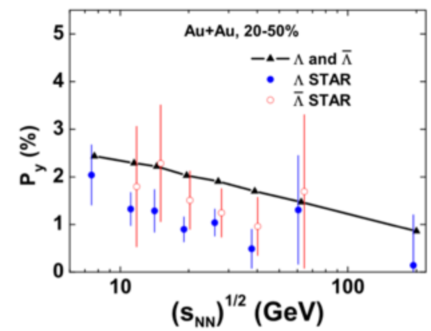
Karpenko, Becattini, EPJC(2017)



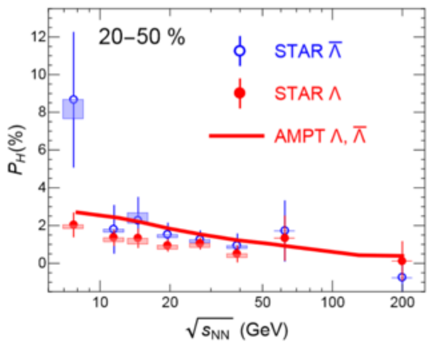
Li, Pang, Wang, Xia PRC(2017)



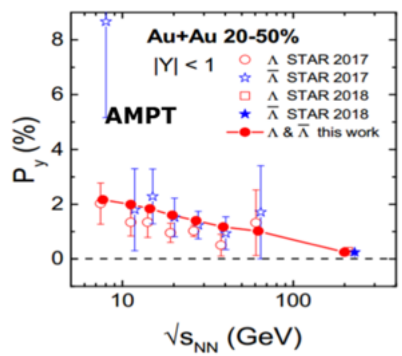
Xie, Wang, Csernai, PRC(2017)



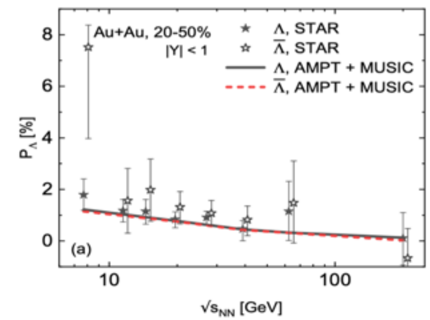
Sun, Ko, PRC(2017)



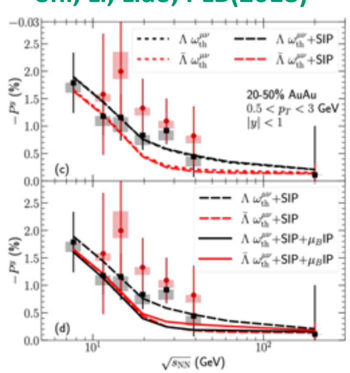
Shi, Li, Liao, PLB(2018)



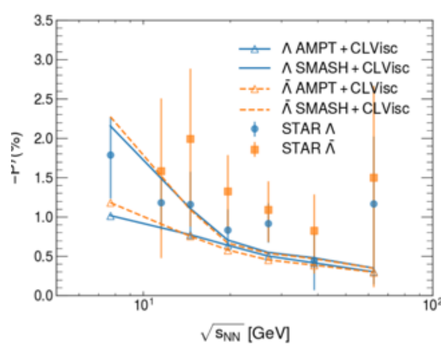
Wei, Deng, Huang, PRC(2019)



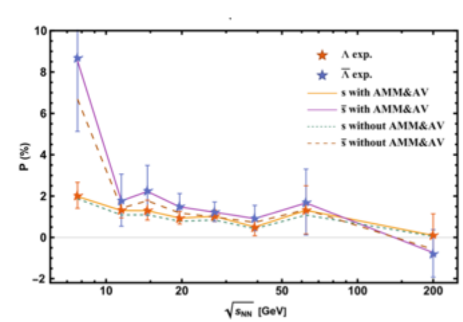
Fu, Xu, Huang, Song, PRC (2021)



S. Ryu, V. Jupic, C. Shen, PRC (2021)



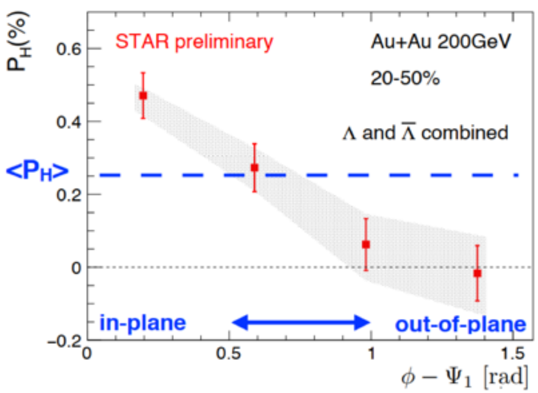
Y.X. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

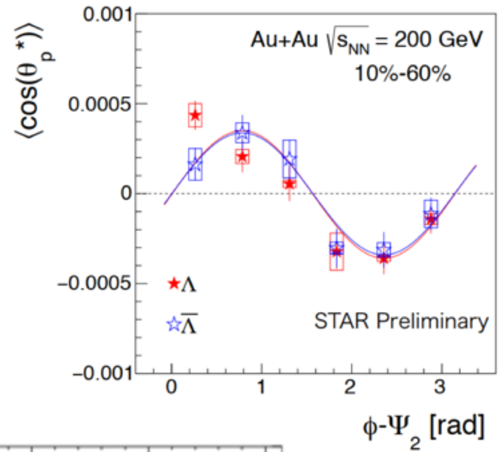


Xu, Lin, Huang, Huang, PRDL (2022)

Local polarization

$$\mathcal{S}^{\mu}_{\rm shear}(\mathbf{p}) = -\frac{\hbar}{4m_{\Lambda}N}\int d\Sigma\cdot p f_V^{(0)}(1-f_V^{(0)})\frac{\epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}}{(u\cdot p)T}\frac{1}{2}\{p^{\sigma}(\partial_{\sigma}u_{\nu}+\partial_{\nu}u_{\sigma})-Du_{\nu}\}$$

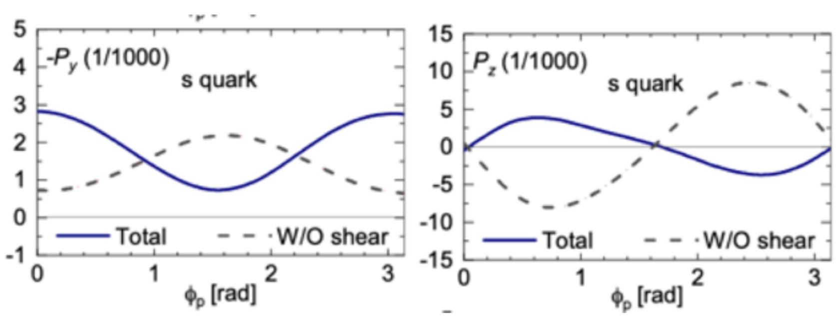


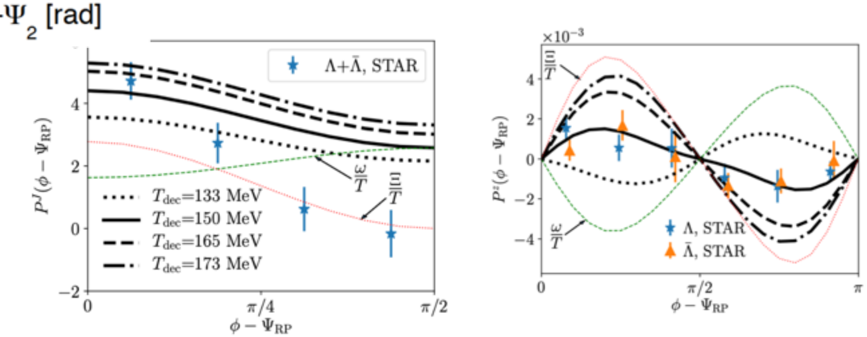


Early works:
(thermal vorticity only)
- **UrQMD :**

Becattini, Karpenko, PRL (2018)
- **AMPT:**

Xia, Li, Tang, Wang, PRC (2018)





s quark scenarios (Thermal vorticity + shear)

Fu, Liu, Pang, Song, Yin, PRL 2021

Isothermal equilibrium
(Thermal vorticity + shear)

Becattini, Buzzegoli, Palermo, Inghirami, Karpenko, PRL 2021

Also see:

Yi, Pu, Yang, PRC (2021); Yi, Wu, Qin, Pu, PRC (2022)
Ryu, Jupic, Shen, PRC (2021)

Theoretical developments

- **Spin hydrodynamics (macroscopic approach)**
  - Florkowski, Friman, Jaiswal, Ryblewski, Speranza (2017-2018);
  - Montenegro, Tinti, Torrieri (2017-2019);
  - Hattori, Hongo, Huang, Matsuo, Taya PLB(2019) ; arXiv: 2201.12390; arXiv: 2205.08051
  - Fukushima, SP, Lecture Note (2020); PLB(2021); Wang, Fang, SP, PRD(2021); Wang, Xie, Fang, SP, PRD (2022); …
  - S.Y. Li, M.A Stephanov, H.U Yee, arXiv:2011.12318
  - D. She, A. Huang, D.F. Hou, J.F Liao, arXiv: 2105.04060
  - Weickgenannt, Wanger, Speranze, Rischke, PRD 2022; PRD 2022; Weickgennatt, Wanger, Speranza, PRD 2022; arXiv:2306.05936;
  - Peng, Zhang, Sheng, Wang, CPL 2021

- **Quantum kinetic theory with collisions (microscopic approach)**
  - Weickgenannt, Sheng, Speranza, Wang, Rischke, PRD 100, 056018 (2019)
  - Hattori, Hidaka, Yang, PRD100, 096011 (2019); Yang, Hattori, Hidaka, arXiv: 2002.02612.
  - Liu, Mameda, Huang, arXiv:2002.03753.
  - Gao, Liang, PRD 2019
  - Wang, Guo, Shi, Zhuang, PRD100, 014015 (2019) ; Z.Y. Wang, arXiv:2205.09334;
  - Li ,Yee, PRD100, 056022 (2019)
  - Hou, Lin, arXiv: 2008.03862; Lin, arXiv: 2109.00184; Lin, Wang, arXiv:2206.12573
  - Fang, SP, Yang, PRD (2022)

- **Other approaches:**
  - **Side-jump effect** Liu, Sun, Ko PRL(2020)
  - **Mesonic mean-field** Csernai, Kapusta, Welle, PRC(2019)
  - **Using different vorticity** Wu, Pang, Huang, Wang, PRR (2019)

- **Recent reviews:**
  - Gao, Ma, SP, Wang, NST (2020)
  - Gao, Liang, Wang, IJMPA (2021)
  - Hidaka, SP, Yang, Wang, PPNP (2022)
  - Becattini, Buzzegoli, Niida, SP, Tang, Wang, arXiv:2402.04540

General discussion for causality and stability

Causality and stability for relativistic systems

- **Causality:**
  The speed of propagating signal cannot be larger than the speed of light.

- **Stability:**
  The small perturbation near the equilibrium (or the solutions of differential equations) must decay with time.

Linear modes analysis (I)

- **Relativistic hydrodynamics:**

  Energy-momentum and currents conservation equations

- **In the linear mode analysis, one considers the perturbations of independent macroscopic variables within the system, e.g. energy density e , number density ρ , etc., near the equilibrium.**

$$\partial_t \varphi(t,\vec{x}) + \mathrm{M}(\partial)\varphi(t,\vec{x}) = 0,$$

$$\varphi(t,\vec{x}) = (\delta e, \delta\rho, ...)^{\mathrm{T}}$$

$$\mathrm{M}(\partial) = \textstyle\sum_{i=0}^{N} \mathrm{M}^{(i)} \partial_{i_1}\partial_{i_2}...\partial_{i_N}$$

Linear mode analysis (II)

- **We usually consider a plane-wave type perturbation:**

$$\varphi = \varphi_0 e^{-i\omega t + i\vec{k}\cdot\vec{x}}, \quad \varphi_0 = \text{const.}.$$

- **The differential equations in linear mode analysis becomes**

$$0 = \mathcal{P}(\omega, \vec{k}) \equiv \det[\omega + i\mathrm{M}(\vec{k})].$$

- **Stability: perturbation decays with time**

$$\text{Im } \omega \leq 0, \qquad \text{for } \vec{k} \in \mathbb{R}^3,$$

- **Causality: group velocity of perturbation is smaller than 1.**

$$\lim_{|\vec{k}|\to+\infty} \left\{ \frac{|\text{Re } \omega|}{|\vec{k}|} \leq 1, \ |\omega/\vec{k}| \text{ is bounded} \right\}, \ \vec{k} \in \mathbb{R}^3.$$

E. Krotscheck and W. Kundt, Communications in Mathematical Physics 60, 171 (1978)

Applications to relativistic hydro (I)

- **The conventional relativistic hydrodynamics up to the first order in gradient expansion is acausal and unstable.**

- **Relativistic hydrodynamics have been extended to**
  - **Second order hydro:**

    Müller-Israel-Stewart (MIS) theory

    Baier-Romatschke-Son-Starinets-Stephanov (BRSSS) theory

    Denicol-Niemi-Molnar- Rischke (DNMR) theory
  - **Generalized first order causal hydro**

    Bemfica-Disconzi-Noronha-Kovtun (BDNK) theory

Applications to relativistic hydro (II)

- Asymptotic causality condition :

$$\frac{\text{Shear viscosity}}{\text{Relaxation time for shear viscous tensor} \times (\text{Energy density} + \text{Pressure})} \leq \frac{3}{4}\left(1 - \text{Speed of sound}^2\right)$$

$$\frac{\text{剪切粘滞系数}}{\text{弛豫时间} \times (\text{能量密度} + \text{压强})} \leq \frac{3}{4}\left[1 - \text{声速}^2\right]$$

SP, Koide, Rischke, Phys. Rev. D 81, 114039 (2010)

- Does stability of relativistic dissipative fluid dynamics imply causality?

Applications to spin hydrodynamics

X.Q. Xie, C. Yang, D.L. Wang, SP, **Phys.Rev.D** 108 (2023) 9, 094031



Basic conservation equations in canonical form

- **Total angular momentum conservation**

$$\partial_\alpha J^{\alpha\mu\nu}_{\mathrm{can}} = 0 \qquad J^{\lambda\mu\nu} = \underbrace{x^\mu \Theta^{\lambda\nu} - x^\nu \Theta^{\lambda\mu}}_{\text{Orbital part}} + \underbrace{\Sigma^{\lambda\mu\nu}}_{\text{Spin tensor}},$$

$$\rightarrow \quad \boxed{\partial_\lambda \Sigma^{\lambda\mu\nu} = -2\Theta^{[\mu\nu]},}$$

- **Energy-momentum conservation**

$$\partial_\mu \Theta^{\mu\nu} = 0,$$

- **Currents conservation**

$$\partial_\mu j^\mu = 0,$$

Spin tensor, spin density and chemical potential

Spin density:
has 6 independent components
S^{ij} 3 rotating; S^{0i} 3 boosting

μ, ν is anti-symmetic

$$\Sigma^{\alpha\mu\nu} = u^\alpha S^{\mu\nu} + \Sigma_{(1)}^{\alpha\mu\nu}$$

spin tensor

Parallel to fluid velocity u^μ; Leading order

Perpendicular to fluid velocity u^μ; Higher order

Thermodynamic relations

$$e + p = Ts + \mu n + \omega_{\mu\nu} S^{\mu\nu}$$

energy density

pressure

temperature X entropy density

spin chemical potential

spin density

6-d.o.f Spin hydrodynamics

- **By using entropy principle, one can get**

$$
\begin{aligned}
\Theta^{\mu\nu} &= (e+p)u^{\mu}u^{\nu} - pg^{\mu\nu} + 2h^{(\mu}u^{\nu)} + 2q^{[\mu}u^{\nu]} + \pi^{\mu\nu} + \phi^{\mu\nu}, \\
q^{\mu} &= \lambda[(u\cdot\partial)u^{\mu} + \frac{1}{T}\Delta^{\mu\nu}\partial_{\nu}T - 4\omega^{\mu\nu}u_{\nu}], \\
\phi^{\mu\nu} &= 2\gamma[T\omega_{th}^{\mu\nu} + 2(g^{\mu\alpha} - u^{\mu}u^{\alpha})(g^{\nu\beta} - u^{\nu}u^{\beta})\omega_{\alpha\beta}]/T.
\end{aligned}
$$

Spin hydrodynamics:

Florkowski, Friman, Jaiswal, Ryblewski, Speranza (2017-2018);

Montenegro, Tinti, Torrieri (2017-2019);

Hattori, Hongo, Huang, Matsuo, Taya PLB(2019) ; arXiv: 2201.12390; arXiv: 2205.08051

Fukushima, SP, Lecture Note (2020); PLB(2021); Wang, Fang, SP, PRD(2021); Wang, Xie, Fang, SP, PRD (2022)

S.Y. Li, M.A Stephanov, H.U Yee, arXiv:2011.12318

D. She, A. Huang, D.F. Hou, J.F Liao, arXiv: 2105.04060Weickgenannt, Wanger, Speranze, Rischke, PRD 2022; PRD 2022; Weickgennatt, Wanger, Speranza, PRD 2022; arXiv:2306.05936

Recent review:

SP, X.G. Huang, “Relativistic spin hydrodynamics”, Acta Phys.Sin. 72 (2023) 7, 071202

1^{st} order spin hydrodynamics

- **We now consider the small perturbations on top of static equilibrium,**

$$\varphi = \{\delta e, \delta u^i, \delta S^{\mu\nu}\} \qquad \varphi = \varphi_0 e^{-i\omega t + i\vec{k}\cdot\vec{x}}, \quad \varphi_0 = \mathrm{const.}.$$

- **Main linearized equations for spin hydro becomes**

$$\mathcal{M}_1 \delta\tilde{X}_1 = 0,$$

$$\delta\tilde{X}_1 \equiv (\delta\tilde{e}, \delta\tilde{\vartheta}^x, \delta\tilde{S}^{0x}, \delta\tilde{\vartheta}^y, \delta\tilde{S}^{0y}, \delta\tilde{S}^{xy}, \delta\tilde{\vartheta}^z, \delta\tilde{S}^{0z}, \delta\tilde{S}^{xz}, \delta\tilde{S}^{yz})^{\mathrm{T}},$$

$$\mathcal{M}_1 \equiv \begin{pmatrix} M_1 & 0 & 0 & 0 \\ A_1 & M_2 & 0 & 0 \\ A_2 & 0 & M_2 & 0 \\ A_3 & 0 & 0 & M_3 \end{pmatrix},$$

Propagating modes in 1^{st} order spin hydro

- **Large k->infinity limit**

$$
\begin{aligned}
\omega &= -4iD_b\gamma_{\|}^{-1}\lambda'^{-1}k^{-2}+O(k^{-3}),\\
\omega &= -ic_s^{2/3}\gamma_{\|}^{1/3}k^{4/3}+O(k),\\
\omega &= (-1)^{1/6}c_s^{2/3}\gamma_{\|}^{1/3}k^{4/3}+O(k),\\
\omega &= (-1)^{5/6}c_s^{2/3}\gamma_{\|}^{1/3}k^{4/3}+O(k),\\
\omega &= -2iD_b+O(k^{-1}),\\
\omega &= 2iD_s\gamma_\perp(\gamma'+\gamma_\perp)^{-1}+O(k^{-1}),\\
\omega &= \pm ik\sqrt{2\lambda'^{-1}(\gamma'+\gamma_\perp)}+O(k^0),
\end{aligned}
$$

$$\omega = i(\gamma'+\gamma_\perp)k^2 \text{ as } k\to\infty,$$

Breaks causality criteria

$$\lim_{k\to\infty}\left|\frac{\omega}{k}\right| \text{ is bounded.}$$

1^{st} order spin hydrodynamics is always unstable and acausal!

Minimal extension to 2nd order hydro

- **We add the relaxation time term to the spin hydro**

$$\tau_q \Delta^{\mu\nu} \frac{d}{d\tau} q_\nu + q^\mu = \lambda[T^{-1}\Delta^{\mu\alpha}\partial_\alpha T + (u\cdot\partial)u^\mu - 4\omega^{\mu\nu}u_\nu],$$

$$\tau_\phi \Delta^{\mu\alpha}\Delta^{\nu\beta} \frac{d}{d\tau}\phi_{\alpha\beta} + \phi^{\mu\nu} = 2\gamma_s \Delta^{\mu\alpha}\Delta^{\nu\beta}(\partial_{[\alpha}u_{\beta]} + 2\omega_{\alpha\beta}),$$

$$\tau_\pi \Delta^{\alpha<\mu}\Delta^{\nu>\beta} \frac{d}{d\tau}\pi_{\alpha\beta} + \pi^{\mu\nu} = 2\eta\partial^{<\mu}u^{\nu>},$$

Shear viscous tensor

$$\tau_\Pi \frac{d}{d\tau}\Pi + \Pi = -\zeta\partial_\mu u^\mu,$$

Bulk viscous pressure

Also see: Y.C. Liu and X.G. Huang, Nucl. Sci. Tech. 31, 56 (2020), 2003.12482.

Causality conditions

- **Causality can be satisfied if the following inequalities are fulfilled:**

$$0 \leq \frac{b_1^{1/2} \pm (b_1 - b_2)^{1/2}}{6(2\tau_q - \lambda')\tau_\pi \tau_\Pi} \leq 1 \text{ and } 0 \leq \frac{2\tau_q(\gamma'\tau_\pi + \gamma_\perp \tau_\phi)}{(2\tau_q - \lambda')\tau_\pi \tau_\phi} \leq 1,$$

$$b_1 = \{8\gamma_\perp \tau_q \tau_\Pi + \tau_\pi[2\tau_q(3\gamma_{||} - 4\gamma_\perp) + 3\tau_\Pi c_s^2(3\lambda' + 2\tau_q)]\}^2,$$

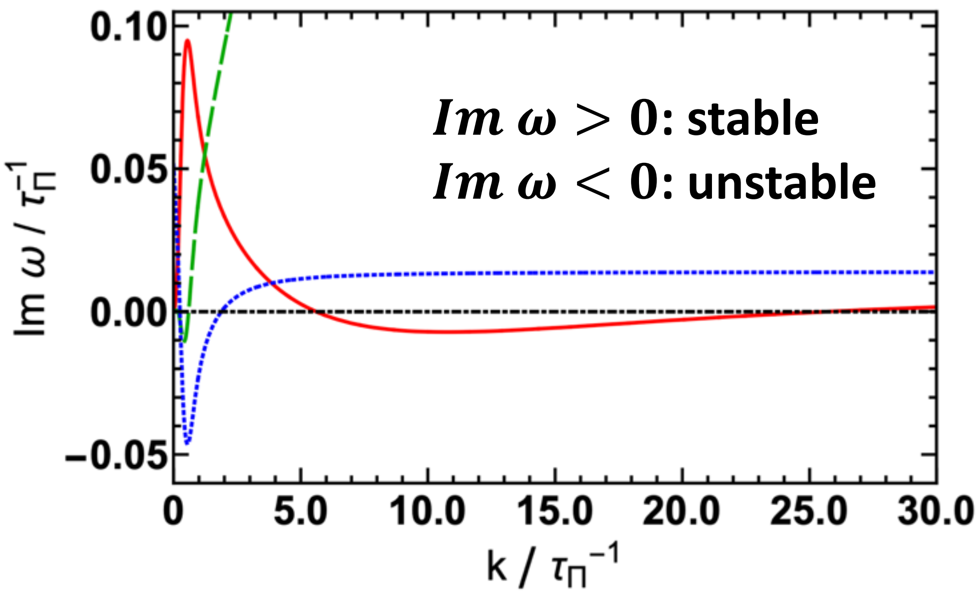
$$b_2 = 12 c_s^2 \lambda'(2\tau_q - \lambda')\tau_\pi \tau_\Pi[\tau_\pi(3\gamma_{||} - 4\gamma_\perp) + 4\gamma_\perp \tau_\Pi].$$

Non-trivial Stability conditions

- We implement the conventional stability conditions and derive the following inequality:

$$\tau_q > \lambda'/2,$$
$$D_s > 0, \quad D_b < -4c_s\lambda\gamma_{\|}^{-1}\left|\chi_e^{0x}\right| \le 0,$$
$$b_1 > b_2 > 0, \quad \frac{c_2}{c_3} > 0.$$

- The above conditions can make the system be stable at k->0 and k->infinity limits. But, the system is unstable for finite k!



Conventional stability criteria fails ?!!

Improved Causality and stability criteria in linear response theory

D.L. Wang, SP, **Phys.Rev.D (Lett),** 109 (2024) 3, L031504

Problems in conventional analysis

- **A practical challenge arises:**
  Need to check conditions in different frames

- **A concern arises:**
  Conventional causality criterion is incomplete.

- **An question arises:**
  What constitutes the relationship between the stability and the causality criteria?

A practical challenge arises

- Commonly, the causality and stability conditions are first derived from the conventional criteria in the rest frame.
- Then, the verification of these criteria in other reference frames follows.
- However, this process of examining conditions across different frames is frequently burdensome.

A concern arises

- **Causality: group velocity of perturbation is smaller than 1.**

$$\lim_{|\vec{k}|\to+\infty}\left\{\frac{|\mathrm{Re}\ \omega|}{|\vec{k}|}\leq 1,\ |\omega/\vec{k}|\ \text{is bounded}\right\},\ \vec{k}\in\mathbb{R}^3.$$

- **Conventional causality criterion fails sometimes.**

L. Gavassino, M. M. Disconzi, and J. Noronha, 2307.05987

A question arises

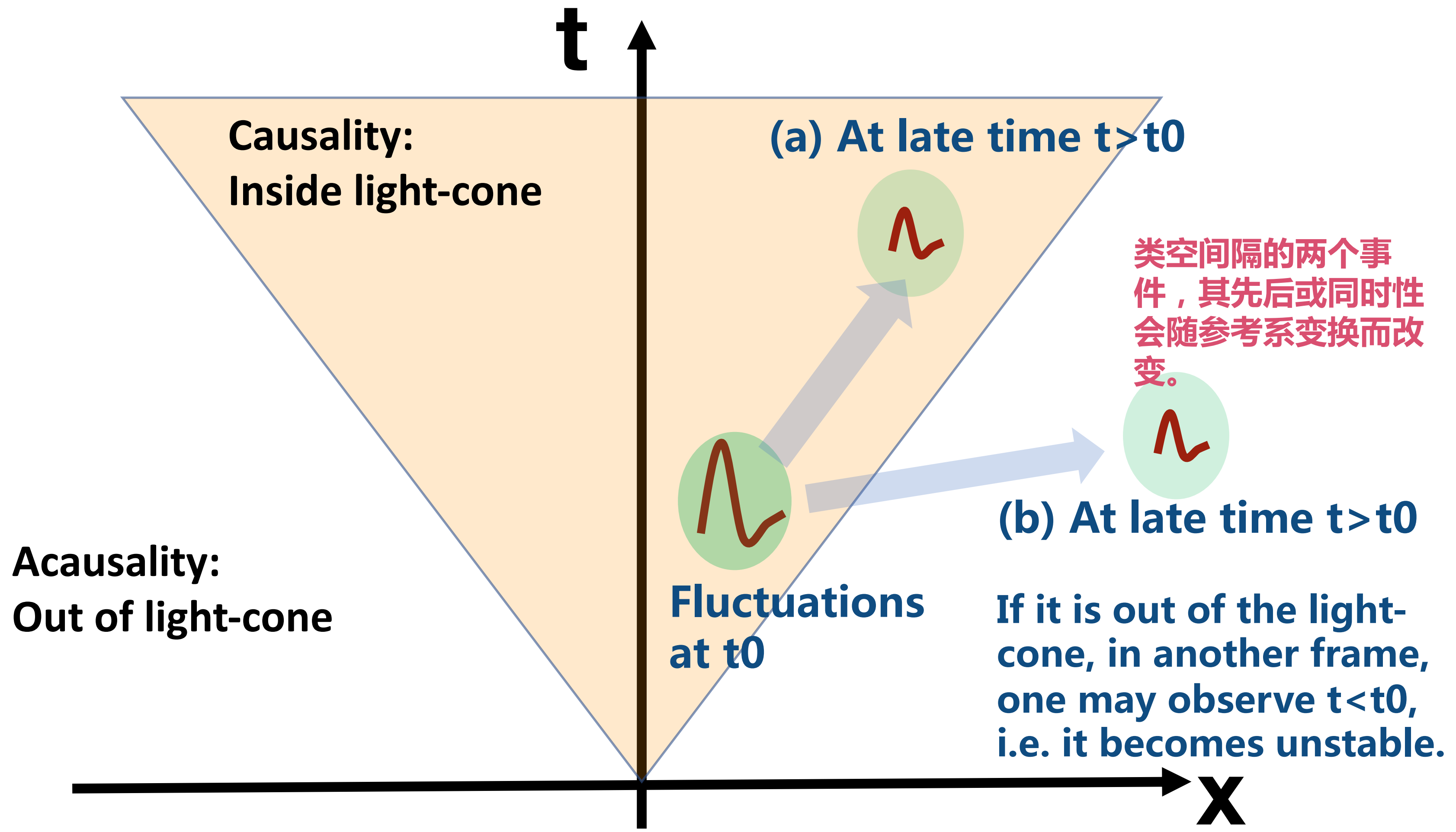
- **Does stability imply causality? Furthermore, what constitutes the relationship between the stability and the causality criteria?**

Does stability of relativistic dissipative fluid dynamics imply causality?

SP, Koide, Rischke, Phys. Rev. D 81, 114039 (2010)

Connection between causality and stability

- **Acausal propagating can lead to unstable.**



L. Gavassino, Phys. Rev. X 12, 041001 (2022)

Updated stability criterion

- **The improved stability condition for a 3 + 1 dimensional relativistic system is,**

$$\mathrm{Im}\ \omega \leq |\mathrm{Im}\ \vec{k}|, \quad \text{for } \vec{k} \in \mathbb{C}^3.$$

Complex

M. P. Heller, A. Serantes, M. Spaliński, and B. Withers, 2212.07434.

L. Gavassino, Phys. Lett. B 840, 137854 (2023).

- **The imaginary part of k comes from the Lorentz transformation.**

  Assuming the system is stable in one frame, i.e. $\mathrm{Im}\,\omega < 0$. Then, if we transform them to another frame, $(\omega, k) \to (\omega', k')$ by Lorentz transformation, the k' will also have a imaginary part.

Extending stability criterion to all frames

Theorem 1. The stability criterion holds true across all IFR if it is satisfied in a single IFR.

$$\mathrm{Im}\ \omega \leq |\mathrm{Im}\ \vec{k}|, \quad \text{for } \vec{k} \in \mathbb{C}^3.$$

Dong-ling Wang, SP, Phys. Rev. D (Lett), 109 (2024) 3, L031504

Improved causality criterion (I)

Theorem 2. *Suppose that the initial data $\varphi(0,\vec{x})$ for differential equations (1) is smooth with respect to $\vec{x}$, and the volume of the support of $\varphi(0,\vec{x})$ is both finite and non-vanishing. If two constants $R>0$ and $b\in\mathbb{R}$ exist such that*

$$\text{Im}\ \omega \leq |\text{Im}\ \vec{k}| + b,\ \ \textit{for}\ |\vec{k}| > R, \tag{8}$$

then the influence of the initial data propagates with sub-luminal speed.

Simplified version:

$$\text{Im}\ \omega \leq |\text{Im}\ \vec{k}| + b',\ \ \text{for}\ \vec{k} \in \mathbb{C}^3.$$

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Theorem 3. *The causality criterion (8) or (9) holds true across all IFR if it is fulfilled in a single IFR.*

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Improved causality criterion (II)

- Consider the dispersion relation ω = k(1 + i)/2 satisfying the conventional causality condition

$$\lim_{|\vec{k}|\to+\infty}\left\{\frac{|\mathrm{Re}\ \omega|}{|\vec{k}|}\leq 1,\ |\omega/\vec{k}|\ \text{is bounded}\right\},\ \vec{k}\in\mathbb{R}^3.$$

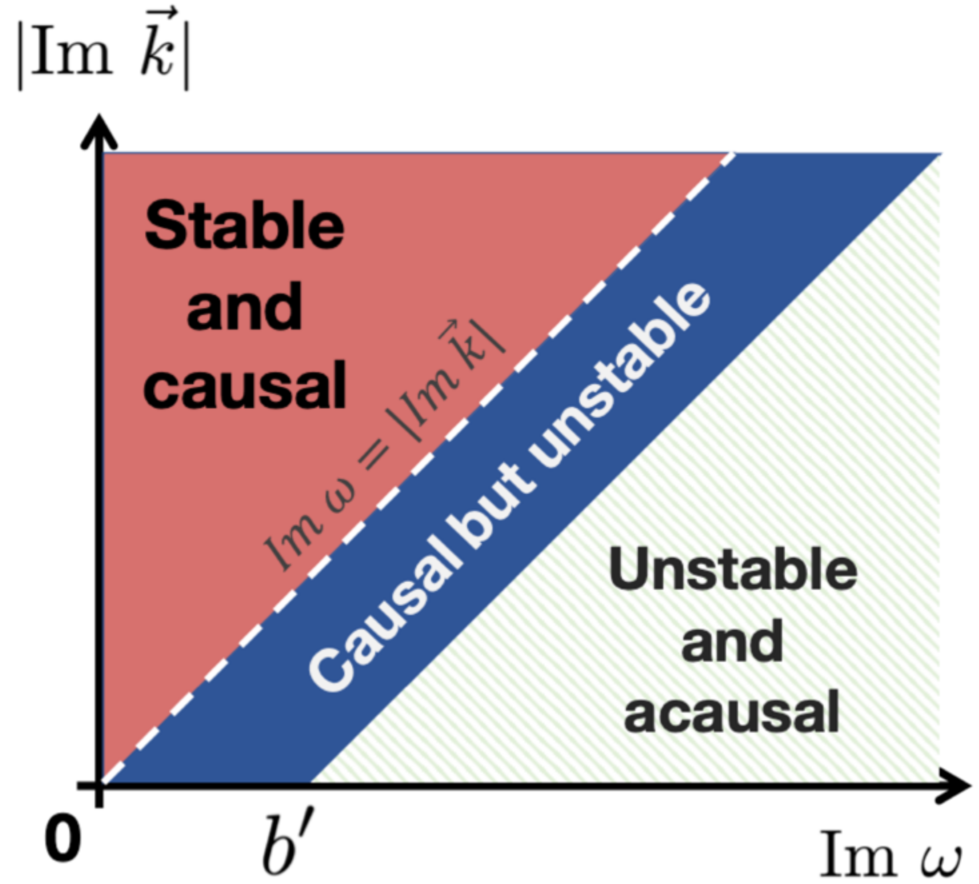
does not obey

$$\mathrm{Im}\ \omega\leq|\mathrm{Im}\ \vec{k}|+b',\ \text{for}\ \vec{k}\in\mathbb{C}^3.$$

- We know that the above desperation relations are proved to be acausal.

P. D. Lax, Hyperbolic Partial Differential Equations Courant Lecture Notes (American Mathematical Society/Courant Institute of Mathematical Sciences, 2006)

Stability means causality



Conclusion:

Stability in all inertial frame of reference means causality in linear mode analysis.

Easter egg

R.E. Hoult, P. Kovtun, , Phys.Rev.D 109, 046018 (2024).

Note added: As we were preparing the final version of the manuscript, we received a preliminary version of the preprint [10], whose results have overlap with ours, and appears on arXiv on the same day.

Ref. [10]: Dong-ling Wang, SP, Phys.Rev.D (Lett), 109 (2024) 3, L031504

What is the **necessary** and **sufficient** causality criteria?

1. $$\text{Im}\ \omega \leq |\text{Im}\ \vec{k}|, \quad \vec{k} \in \mathbb{C}^3.$$

Heller, Serantes, Spalinski, Withers, Phys.Rev.Lett. 130, 261601 (2023).
Gavassino, Phys.Lett.B 840, 137854 (2023).
Gavassino, Disconzi, Noronha, arXiv:2307.05987.

2. $$\text{Im}\ \omega \leq |\text{Im}\ \vec{k}| + b', \ \text{for}\ \vec{k} \in \mathbb{C}^3.$$

Dong-ling Wang, SP, Phys.Rev.D (Lett),109 (2024) 3, L031504

3. $$0 \leq \lim_{|\vec{k}|\to\infty} \frac{|\text{Re}\ \omega|}{|\vec{k}|} \leq 1, \quad \lim_{|\vec{k}|\to\infty} \frac{\text{Im}\ \omega}{|\vec{k}|} = 0, \quad \vec{k} \in \mathbb{R}^3,$$

$$\mathcal{O}_\omega \left[F(\omega, \vec{k} \neq 0) \right] = \mathcal{O}_{|\vec{k}|} \left[F(\omega = a|\vec{k}|, \vec{k}) \right].$$

Hoult, Kovtun, Phys.Rev.D 109, 046018 (2024).

It is not the end of the story, but merely the beginning of it.

Summary and outlook

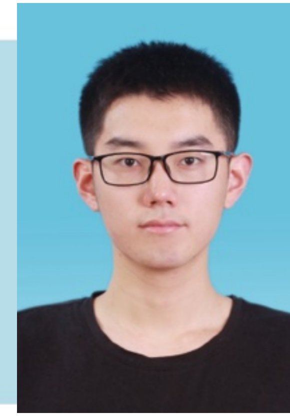
Summary

- We study the causality and stability for spin hydrodynamics. We derived the causality conditions and find the conventional stability criterion (fails) cannot make the system be stable for finite wave length limit.

- We introduced and proved an improved causality criteria. By the new criteria, we find that stability in all inertial frame of reference means causality in linear mode analysis.

Also see: the last talk in this workshop (March 17, 15:30 – 16:00)

The effects of self-energies in spin polarization and spin alignment



Shuo Fang (方 硕)

University of Science and Technology of China (USTC)

In collaboration with: Shi Pu, Di-Lun Yang

ExHIC-p workshop on polarization phenomena in nuclear collisions
Mar. 14-17, 2024, Taiwan

Thank you!

Backup

尾声2: what happens in spin hydro?

- **For a differential equations,**

$$\partial_t \psi = \mathrm{P}(-i\partial)\psi, \qquad \mathrm{P}(-i\partial) \equiv \sum_{j=0}^{m} \mathrm{P}_{(j)}^{i_1, i_2, \ldots, i_j} \partial_{i_1} \partial_{i_2} \ldots \partial_{i_j},$$

- **Stability condition should be**

$$\lambda_{\max}\left(\mathrm{P}^{\dagger}(\vec{k}) + \mathrm{P}(\vec{k})\right) \leq 0 \quad \text{for } \vec{k} \in \mathbb{R}^3,$$

- **For conventional hydrodynamics,**

$$P(\mathbf{k}) = -iM^i k_i - N$$ M, N are real symmetric matrices

$$\Rightarrow \quad \frac{1}{2}\lambda_{\max}\left(\mathrm{P}^{\dagger}(\vec{k}) + \mathrm{P}(\vec{k})\right) = \max\left(\mathrm{Im}\ \omega(0)\right) \leq 0 \quad \text{for } \vec{k} \in \mathbb{R}^3$$

In preparation

Propagating modes in 1st order spin hydro

- **Small k->0 limit**

$$\omega = \pm c_s k + \frac{i}{2}(\gamma_{\|} \mp 4c_s\lambda\chi_e^{0x}D_b^{-1})k^2 + O(k^3),$$ **Sound mode**

$$\omega = (-i \pm \sqrt{4D_b\lambda' - 1})\lambda'^{-1} + O(k),$$

$$\omega = i\gamma_{\perp}k^2 + O(k^3),$$ **Shear mode**

$$\omega = 2iD_s + O(k^2).$$

- **If we neglect the shear mode, then causality requires:**

$$D_s > 0, \boxed{\lambda' < 0}, \; D_b < -4c_s\lambda\gamma_{\|}^{-1}|\chi_e^{0x}| \leq 0.$$

Contradict with the entropy principle!