Timescales of spin transport

David Wagner
in collaboration with
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based mainly on

DW, N. Weickgenannt, DHR, Phys.Rev.D 106 (2022) 11, 116021
DW, MS, DHR, in preparation (2024)

Polarization phenomena in nuclear collisions | 16.03.2024

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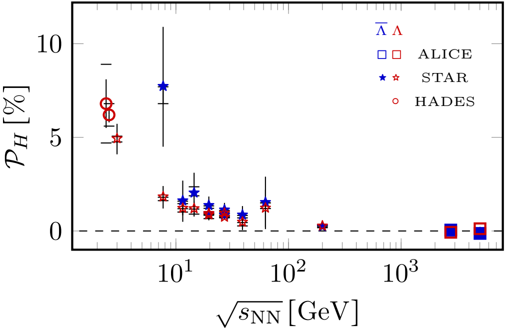
ELEMENTS

Global Λ-polarization



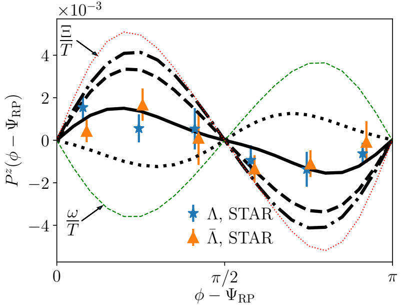
- Global polarization: polarization of Λ-hyperons along angular-momentum direction
  - Can be well explained by considering **thermal vorticity** on freeze-out hypersurface $S^\mu_\varpi = -\epsilon^{\mu\nu\alpha\beta} k_\nu \frac{\int \mathrm{d}\Sigma_\lambda k^\lambda f_0(1-f_0)\varpi_{\alpha\beta}}{8m \int \mathrm{d}\Sigma_\lambda k^\lambda f_0}$

$$\varpi_{\mu\nu} := -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu),\ \beta^\mu := u^\mu/T,\ f_0 = [\exp(u^\mu k_\mu/T) + 1]^{-1}$$

Local Λ-polarization

- Local polarization: Angle-dependent polarization of Λ-hyperons along beam-direction
  - Could only be explained recently by incorporating shear effects (neglecting temperature gradients)

$$S^{\mu}_{\xi} = -\epsilon^{\mu\nu\alpha\beta} k_\nu \frac{\int \mathrm{d}\Sigma_\lambda k^\lambda f_0 (1-f_0) \hat{t}_\alpha \frac{k^\gamma}{k^0} \Xi_{\gamma\beta}}{4mT \int \mathrm{d}\Sigma_\lambda k^\lambda f_0}$$



F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, A. Palermo, PRL 127 (2021) 272302

$\omega_{\mu\nu} := \frac{1}{2}(\partial_\mu u_\nu - \partial_\nu u_\mu)$, $\Xi_{\mu\nu} := \frac{1}{2}(\partial_\mu u_\nu + \partial_\nu u_\mu)$, $\Delta^{\mu\nu} := g^{\mu\nu} - u^\mu u^\nu$

Open questions

$$S^{\mu}_{\varpi} = -\epsilon^{\mu\nu\alpha\beta} k_\nu \frac{\int \mathrm{d}\Sigma_\lambda k^\lambda f_0(1-f_0)\varpi_{\alpha\beta}}{8m \int \mathrm{d}\Sigma_\lambda k^\lambda f_0}$$

- Traditional approaches to computing the polarization
  - assume equilibrated spin degrees of freedom
  - neglect dissipative quantities

Open questions

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- Traditional approaches to computing the polarization
  - assume equilibrated spin degrees of freedom
  - neglect dissipative quantities
- Not clear so far:
  - (I) How fast do spin degrees of freedom equilibrate?
  - (II) How do dissipative effects influence polarization?
- This talk: Provide an estimate for the answer to question (I)

- Hydrodynamics is based on conservation laws
  - Consider a system of uncharged fields
    - $\rightarrow$ Should conserve energy-momentum and total angular momentum

Conservation laws

$$\partial_\mu T^{\mu\nu} = 0 \tag{1a}$$

$$\partial_\lambda J^{\lambda\mu\nu} =: \hbar \partial_\lambda S^{\lambda\mu\nu} + T^{[\mu\nu]} = 0 \tag{1b}$$

Spin Hydrodynamics: Basics

- Hydrodynamics is based on conservation laws
  - Consider a system of uncharged fields
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- **10** equations for **16**+**24** quantities
- Additional information about dissipative quantities has to be provided
  - Possibilities: Microscopic theories (e.g., kinetic theory), gradient expansion, ...
- This talk: Consider ideal spin hydrodynamics
  (with input from kinetic theory)

$A^{[\mu}B^{\nu]} := A^\mu B^\nu - A^\nu B^\mu$

Ideal spin hydrodynamics

Definition

Ideal (spin) hydrodynamics is characterized by the fact that the conservation laws (together with an equation of state) completely determine the evolution of the conserved currents.

- What are the quantities at our disposal?
  - Symmetric part of $T^{\mu\nu}$ characterized by energy density ε, pressure P, and four-velocity u^μ
  - Spin tensor characterized by spin potential $\Omega^{\mu\nu} = -\Omega^{\nu\mu}$
    - Decomposition: $\Omega^{\mu\nu} = u^{[\mu}\kappa^{\nu]} + \epsilon^{\mu\nu\alpha\beta}u_\alpha\omega_\beta$

Building blocks

- Scalars: ε, P
- Vectors: u^μ, κ^μ
- Pseudovectors: ω^μ

Interlude: Equilibria

Global equilibrium

- The inverse four-temperature $\beta^\mu := u^\mu/T$ is a Killing vector, $\partial^\mu\beta^\nu + \partial^\nu\beta^\mu = 0$
- The spin potential is equal to the thermal vorticity, $\Omega^{\mu\nu} = \varpi^{\mu\nu} := -\frac{1}{2}\left(\partial^\mu\beta^\nu - \partial^\nu\beta^\mu\right)$
- Entropy is exactly conserved, $\partial_\mu S^\mu = 0$

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Local equilibrium

- No restriction on the inverse four-temperature
- No restriction on the spin potential
- Entropy is conserved up to (small) quantum corrections, $\partial_\mu S^\mu \approx 0$

- Ideal hydrodynamics is based on the concept of **local equilibrium**

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- Demand that the spin tensor is linear in the spin potential

Form of $S^{\lambda\mu\nu}$

$$
\begin{aligned}
S^{\lambda\mu\nu} &= A u^\lambda \Omega^{\mu\nu} + B u^\lambda u_\alpha \Omega^{\alpha[\mu} u^{\nu]} + C u^\lambda \Omega^{\alpha[\mu} \Delta^{\nu]}{}_\alpha \\
&\quad + D u_\alpha \Omega^{\alpha[\mu} \Delta^{\nu]\lambda} + E \Delta^\lambda{}_\alpha \Omega^{\alpha[\mu} u^{\nu]} \\
&= (A - B - C)\, u^\lambda u^{[\mu} \kappa^{\nu]} + E u^{[\mu} \epsilon^{\nu]\lambda\alpha\beta} u_\alpha \omega_\beta \\
&\quad + (A - 2C)\, u^\lambda \epsilon^{\mu\nu\alpha\beta} u_\alpha \omega_\beta + D \kappa^{[\mu} \Delta^{\nu]\lambda}
\end{aligned}
$$

- The objects $A, \ldots, E$ are functions of ε, have to be fixed by microscopic theory.

Decomposition of $T^{\mu\nu}$

- Split into symmetric and antisymmetric part, $T^{\mu\nu} = \frac{1}{2}T^{(\mu\nu)} + \frac{1}{2}T^{[\mu\nu]}$

Form of $T^{\mu\nu}$

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - P\Delta^{\mu\nu} + \frac{1}{2}T^{[\mu\nu]} ,$$

$$T^{[\mu\nu]} = -\hbar^2\Gamma^{(\kappa)}\left(u^{[\mu}\kappa^{\nu]} - u_\alpha \varpi^{\alpha[\nu}u^{\mu]}\right) + \hbar^2\Gamma^{(\omega)}\left(\epsilon^{\mu\nu\alpha\beta}u_\alpha\omega_\beta - \varpi^{\langle\mu\rangle\langle\nu\rangle}\right)$$

- Features of $T^{[\mu\nu]}$:
  - Vanishes in global equilibrium
  - Follows from quantum kinetic theory
  - Factor of $\hbar^2$ $\rightarrow$ second-order quantum effect

$A^{\langle\mu\rangle} := \Delta^{\mu\nu}A_\nu$

Equations of motion and setup

- Goal: include quantum effects to **leading order** $\rightarrow \mathcal{O}(\hbar)$
- Since $T^{[\mu\nu]} \sim \mathcal{O}(\hbar^2)$, we can neglect it for energy-momentum conservation

Conservation equations

$$\partial_\mu T^{(\mu\nu)} = 0 + \mathcal{O}(\hbar^2) \; , \tag{2a}$$

$$\partial_\lambda S^{\lambda\mu\nu} = \frac{1}{\hbar} T^{[\nu\mu]} + \mathcal{O}(\hbar^2) \; . \tag{2b}$$

- Spin potential does not enter the conservation of energy and momentum at this order!
  - $\rightarrow$ No backreaction of spin on fluid evolution, fluid profile serves as input for spin potential
- Choice in this work: **fluid at rest**, $u^\mu = \text{const.}$, $\varepsilon = \text{const.}$, $P = \text{const.}$

Spin equations of motion in a fluid at rest

- Project equations of motion for spin tensor to obtain evolution equations for the components of the spin potential

Equations of motion for the spin potential

$$(A-B-C)\dot{\kappa}^{\langle\mu\rangle} = E\epsilon^{\mu\nu\alpha\beta}u_\nu\nabla_\alpha\omega_\beta + \hbar\Gamma^{(\kappa)}\kappa^\mu \,, \tag{3a}$$

$$(A-2C)\dot{\omega}^{\langle\mu\rangle} = D\epsilon^{\mu\nu\alpha\beta}u_\nu\nabla_\alpha\kappa_\beta - \hbar\Gamma^{(\omega)}\omega^\mu \,. \tag{3b}$$

- Go to fluid rest frame, $\kappa^\mu \equiv (0,\boldsymbol{\kappa})$, $\omega^\mu \equiv (0,\boldsymbol{\omega})$

$$\tau_\kappa\dot{\boldsymbol{\kappa}} + \boldsymbol{\kappa} = \mu_\kappa \boldsymbol{\nabla}\times\boldsymbol{\omega} \,, \tag{4a}$$

$$\tau_\omega\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} = -\mu_\omega \boldsymbol{\nabla}\times\boldsymbol{\kappa} \,, \tag{4b}$$

- Quantities $A,\ldots,E$, $\Gamma^{(\kappa)}$, $\Gamma^{(\omega)}$ are constant due to the assumptions on the fluid

$\tau_\kappa := -\frac{A-B-C}{\hbar\Gamma^{(\kappa)}}$, $\mu_\kappa := -\frac{E}{\hbar\Gamma^{(\kappa)}}$, $\tau_\omega := \frac{A-2C}{\hbar\Gamma^{(\omega)}}$, $\mu_\omega := -\frac{D}{\hbar\Gamma^{(\omega)}}$.

Spin waves

- $\boldsymbol{\kappa}$ and $\boldsymbol{\omega}$ follow coupled relaxation equations
  $\rightarrow$ Disentangle longitudinal and transverse components

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Longitudinal components: Decay

$$\tau_\kappa \frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{\nabla}\cdot\boldsymbol{\kappa}) = -\boldsymbol{\nabla}\cdot\boldsymbol{\kappa}\ , \tag{5a}$$

$$\tau_\omega \frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{\nabla}\cdot\boldsymbol{\omega}) = -\boldsymbol{\nabla}\cdot\boldsymbol{\omega}\ , \tag{5b}$$

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Transverse components: Damped waves

$$\ddot{\boldsymbol{\kappa}} + a\dot{\boldsymbol{\kappa}} + b\boldsymbol{\kappa} - c_{\mathfrak{s}}^2 \Delta \boldsymbol{\kappa} = 0 \,, \tag{6a}$$

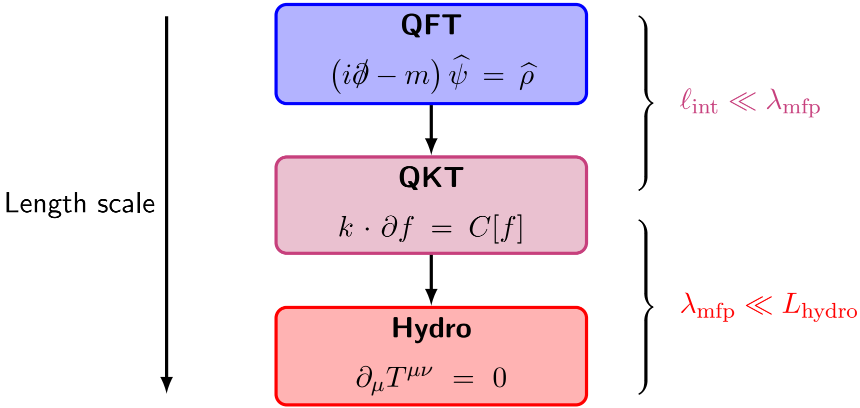
$$\ddot{\boldsymbol{\omega}} + a\dot{\boldsymbol{\omega}} + b\boldsymbol{\omega} - c_{\mathfrak{s}}^2 \Delta \boldsymbol{\omega} = 0 \,, \tag{6b}$$

- For $\tau_\omega, \tau_\kappa \rightarrow \infty$ the damping vanishes

V. E. Ambrus, R. Ryblewski, R. Singh, Phys. Rev. D 106 (2022) 1, 014018

$a := \frac{\tau_\kappa + \tau_\omega}{\tau_\kappa \tau_\omega}$, $b := \frac{1}{\tau_\kappa \tau_\omega}$, $c_{\mathfrak{s}}^2 := \frac{\mu_\kappa \mu_\omega}{\tau_\kappa \tau_\omega}$.

Microscopic approach: concept



Kinetic theory with spin: basics

- Quantum kinetic theory: Effective description of the underlying QFT in the limit where the fields behave as scattering quasiparticles
  $\rightarrow$ Quantum corrections can be treated perturbatively
- Distribution of particles characterized by distribution function $f(x, k, \mathfrak{s})$
  - Arguments: position x, momentum k, continuous "spin" variable $\mathfrak{s}$

Boltzmann equation

$$k^\mu \partial_\mu f(x, k, \mathfrak{s}) = \frac{1}{2} \int \mathrm{d}\Gamma_1 \mathrm{d}\Gamma_2 \mathrm{d}\Gamma' \delta^{(4)}(k_1 + k_2 - k - k') \widetilde{\mathcal{W}} \times \left[f(x + \Delta_1 - \Delta, k_1, \mathfrak{s}_1) f(x + \Delta_2 - \Delta, k_2, \mathfrak{s}_2) - f(x, k, \mathfrak{s}) f(x + \Delta' - \Delta, k', \mathfrak{s}') \right] \quad (7)$$

$$\mathrm{d}\Gamma := \mathrm{d}K \mathrm{d}S , \qquad \mathrm{d}K := \frac{\mathrm{d}^3 k}{(2\pi\hbar)^3 k^0} , \qquad \mathrm{d}S := \frac{m}{\sqrt{3}\pi} \mathrm{d}^4 \mathfrak{s} \delta(k^\alpha \mathfrak{s}_\alpha) \delta(\mathfrak{s}^2 + 3).$$

Connection to hydrodynamics

- We can express the conserved quantities in terms of integrals over the distribution function

Conserved currents

$$\frac{1}{2}T^{(\mu\nu)} = \int \mathrm{d}\Gamma k^\mu k^\nu f \; , \tag{8}$$

$$S^{\lambda\mu\nu} = \frac{1}{2m}\int \mathrm{d}\Gamma k^\lambda \epsilon^{\mu\nu\alpha\beta} k_\alpha \mathfrak{s}_\beta f \; . \tag{9}$$

$$T^{[\mu\nu]} = \frac{1}{2}\int [\mathrm{d}\Gamma]\widetilde{\mathcal{W}}\Delta^{[\mu}k^{\nu]}\left(f_1 f_2 - f f'\right) \tag{10}$$

- Given a distribution $f(x, k, \mathfrak{s})$, we can compute these expressions and read off the needed coefficients

$[\mathrm{d}\Gamma] \coloneqq \mathrm{d}\Gamma_1\,\mathrm{d}\Gamma_2\,\mathrm{d}\Gamma\,\mathrm{d}\Gamma'$

Obtaining the coefficients

- Ideal spin hydrodynamics $\equiv$ local equilibrium in kinetic theory

Local-equilibrium distribution function

$$f(x, k, \mathfrak{s}) = e^{-\beta E_{\mathbf{k}}} \left(1 - \frac{\hbar}{4m} \epsilon^{\mu\nu\alpha\beta} \Omega_{\mu\nu} k_\alpha \mathfrak{s}_\beta \right) . \tag{11}$$

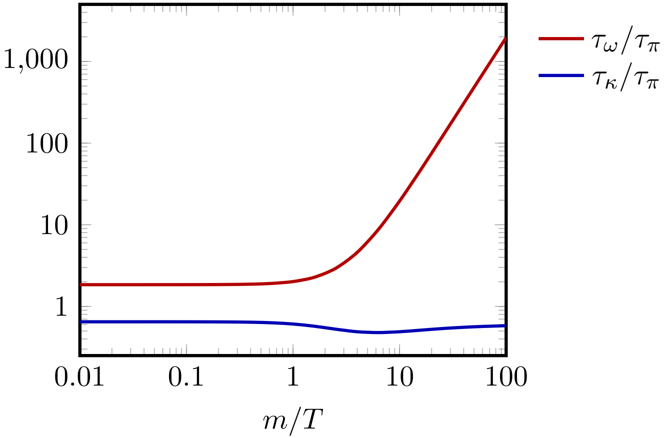
Spin-wave coefficients from kinetic theory

$$\tau_\kappa := \frac{I_{31}}{2m^2 \Gamma^{(\kappa)}} , \quad \mu_\kappa := \frac{I_{31}}{4m^2 \Gamma^{(\kappa)}} ,$$
$$\tau_\omega := \frac{I_{30} - I_{31}}{4m^2 \Gamma^{(\omega)}} , \quad \mu_\omega := \frac{I_{31}}{4m^2 \Gamma^{(\omega)}} . \tag{12}$$

- $\Gamma^{(\kappa)}$ and $\Gamma^{(\omega)}$ are given through collision integrals that contain the spacetime shift Δ^μ
  - $\rightarrow$ *Nonlocal collisions cause relaxation of the spin potential towards thermal vorticity!*

Spin relaxation timescales

- We need to specify a microscopic interaction
  - → Choose NJL-type model, $\mathcal{L}_{\text{int}} := G(\bar{\psi}\psi)^2$
- Compare with the relaxation timescale of the shear-stress tensor τ_π



Conclusion & Outlook

- In ideal spin hydrodynamics, the components of the spin potential fulfill damped wave equations
- Their relaxation timescales can be computed from quantum kinetic theory
  - τ_κ is on the order of the timescales of usual dissipative processes
  - τ_ω can be orders of magnitude larger! (depending on m/T)
- This implies that the timescale of spin equilibration can be much longer than the one of usual dissipative processes

  $\rightarrow$ Implication for simulations: spin hydrodynamics should be used to capture polarization dynamics correctly!

Appendix

Appendix: Quantum kinetic theory

- Spin is a quantum property
  - → Start from quantum field theory
  - → Use **Wigner-function formalism**

Wigner function (Spin 1)

$$W_{\alpha\beta}(x,k) := \frac{1}{(2\pi\hbar)^4}\int \mathrm{d}^4 v e^{-ik\cdot y/\hbar}\langle : \bar{\psi}_\beta(x+y/2)\psi_\alpha(x-y/2) : \rangle$$

- Determines a **quantum phase-space distribution function**
- Equations of motion follow from field equations
  - Determined by Lagrangian $\mathcal{L}_0 + \mathcal{L}_{\text{int}}$
- Independent components: scalar $\mathcal{F}$, axial vector $\mathcal{A}^\mu$

$\mathcal{F} := \mathrm{Tr} W \,, \qquad \mathcal{A}^\mu := \mathrm{Tr}\gamma^\mu\gamma_5 W$

Appendix: Extending phase space

Boltzmann equations

- Not one, but four equations in (x, k)-phase space

$$k \cdot \partial \mathcal{F}(x,k) = \mathcal{C}_{\mathcal{F}} \;, \quad k \cdot \partial \mathcal{A}^\mu(x,k) = \mathcal{C}^\mu_{\mathcal{A}}$$

- Way to compactify this: **Enlarge** phase space from (x, k) to $(x, k, \mathfrak{s})$
- Measure $\mathrm{d}S := \frac{3m}{2\sigma\pi}\mathrm{d}^4\mathfrak{s}\,\delta[\mathfrak{s}^2 + \sigma^2]\delta(k \cdot \mathfrak{s})$

Boltzmann equation in extended phase space

$$\mathfrak{f}(x,k,\mathfrak{s}) := \frac{1}{2}\left(\mathcal{F} - \mathfrak{s}_\mu \mathcal{F}^\mu\right) \tag{13}$$

- Only on-shell parts $\mathfrak{f}(x,k,\mathfrak{s}) = \delta(k^2 - m^2) f(x,k,\mathfrak{s})$ contribute

$$k \cdot \partial f(x,k,\mathfrak{s}) = \mathfrak{C}[f] \tag{14}$$

$\mathfrak{C} := \frac{1}{2}\left(\mathcal{C}_{\mathcal{F}} - \mathfrak{s}_\mu \mathcal{C}^\mu_{\mathcal{A}}\right)$

Appendix: Equilibrium

- Local equilibrium distribution function fulfills $\mathfrak{C}[f_{\text{eq}}] = 0$
- Has to depend on the **collisional invariants**
  $\rightarrow$ Charge, four-momentum and total angular momentum

Local-equilibrium distribution function

$$f_{\text{eq}}(x, k, \mathfrak{s}) = \exp\left(-\beta_0 E_{\mathbf{k}} + \frac{\hbar}{2}\Omega_{\mu\nu}\Sigma_{\mathfrak{s}}^{\mu\nu}\right) \qquad (15)$$

- Necessary conditions on Lagrange multipliers $\beta_0 u^\mu$, $\Omega^{\mu\nu}$ for a vanishing collision term: $\partial^{(\mu}(\beta_0 u^{\nu)}) = 0$, $\Omega^{\mu\nu} = -\frac{1}{2}\partial^{[\mu}(\beta_0 u^{\nu]})$
- Same conditions as for **global** equilibrium, where $k \cdot \partial f_{\text{eq}} = 0$
- **However**, we can relax these constraints if we only demand that the **local** part of the collision term vanishes!

$\Sigma_{\mathfrak{s}}^{\mu\nu} := -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}k_\alpha \mathfrak{s}_\beta$, $E_{\mathbf{k}} := k \cdot u$