



The effects of self-energies in spin polarization and spin alignment

Shuo Fang (方 硕)

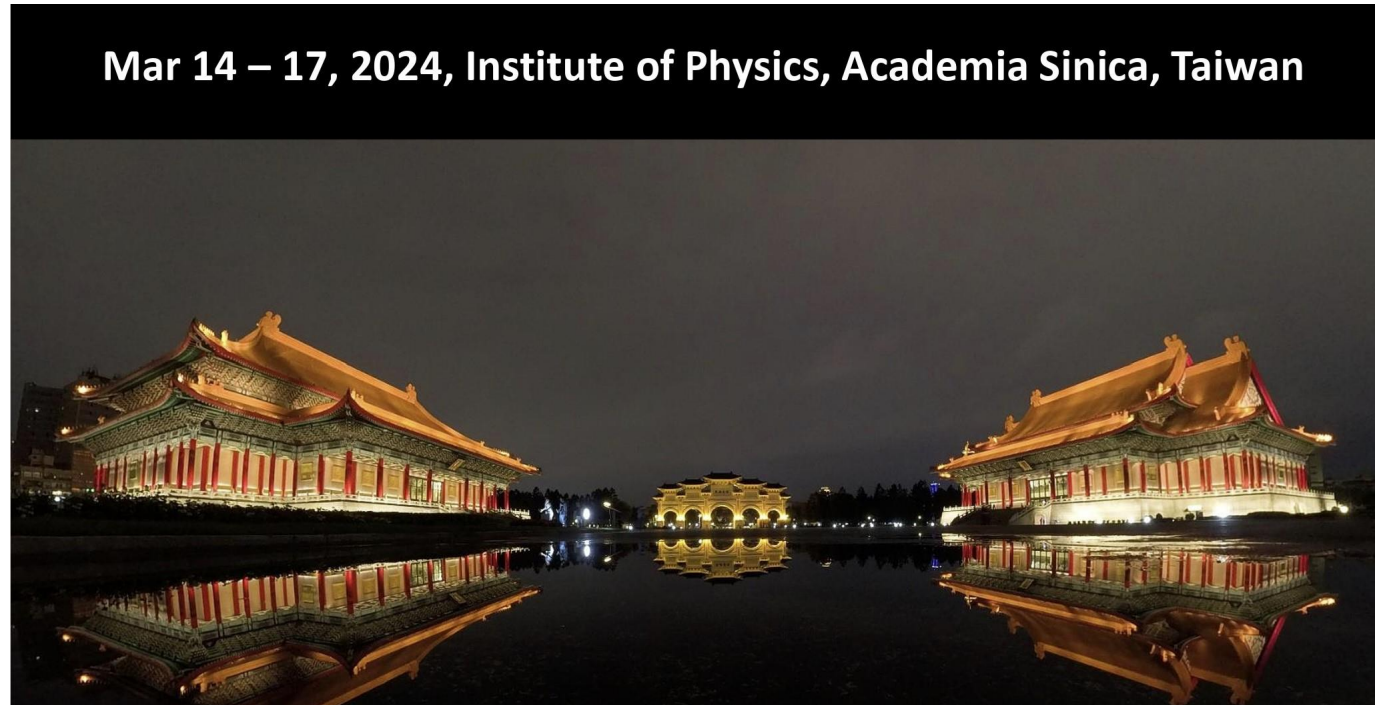
**University of Science and Technology of
China (USTC)**

In collaboration with: Shi Pu, Di-Lun Yang

**ExHIC-p workshop on polarization phenomena in
nuclear collisions**

Mar. 14-17, 2024, Taiwan

Mar 14 – 17, 2024, Institute of Physics, Academia Sinica, Taiwan





Outline

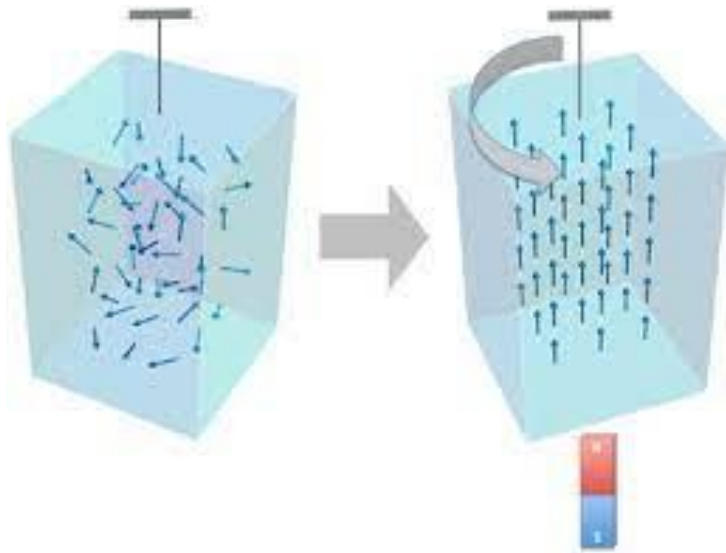
- Spin polarization and alignment in heavy ion collisions
- Quantum kinetic theory with collisions and self-energies
- Self-energy corrections to spin polarization and alignment
- Summary



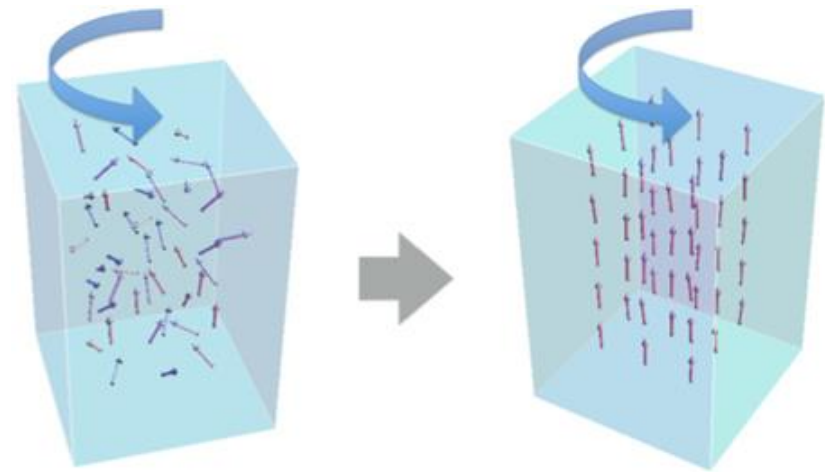
Outline

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Spin-orbit coupling in many-body system



Einstein de-Haas effect
Polarization -> rotation

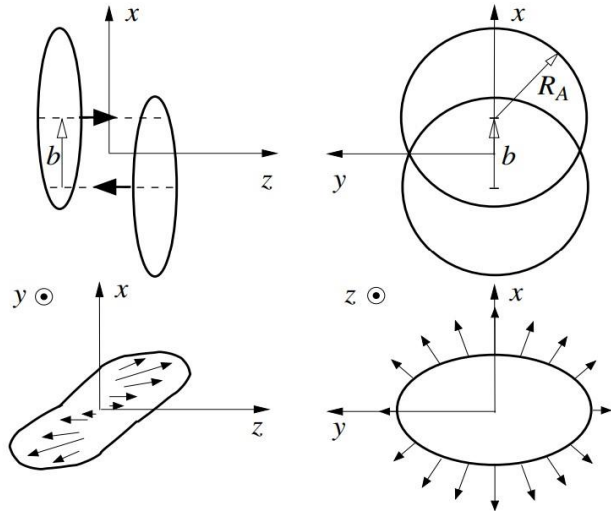


Barnett effect
Rotation -> Polarization

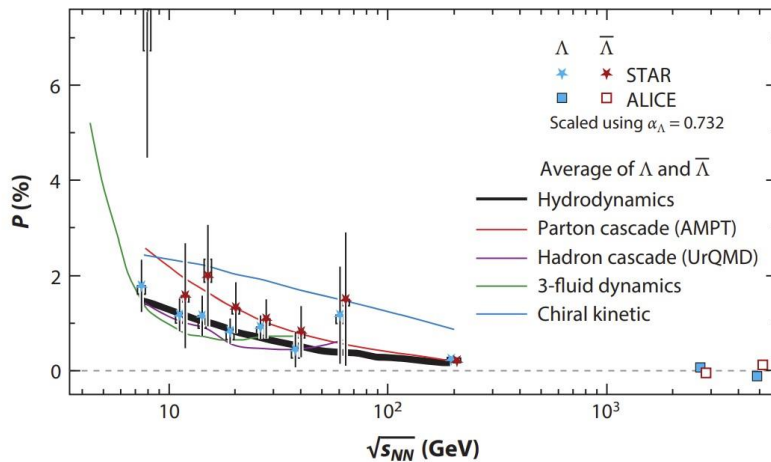
S. J. Barnett, Phys. Rev. 6, 239 (1915)

A. Einstein and W. J. de Haas, Verh. d. Deutsch. Phys. Ges. 17, 152 (1915)

Global polarization



- Large initial orbit angular momentum (OAM) of order of $10^5 \hbar$ is produced in peripheral heavy ion collisions.
- The deconfined quarks can be polarized along the direction of the initial OAM via **spin-orbit coupling**, and the hadrons in final state can be further polarized from quark coalescence mechanism.
- QGP is **the most vortical matter in nature**, $\omega \sim 10^{22} s^{-1}$.



Z. T. Liang, X. N. Wang, PRL(2005)

B. Betz, M. Gyulassy, G. Torrieri, PRC (2007)

F. Becattini, F. Piccinini, J. Rizzo PRC (2008)

G.-H. Gao, S.-W. Chen, W.-T. Deng, Z.-T. Liang, Q. Wang PRC (2008)

L. Adamczyk et al. [STAR Collaboration], Nature 548, 62(2017)

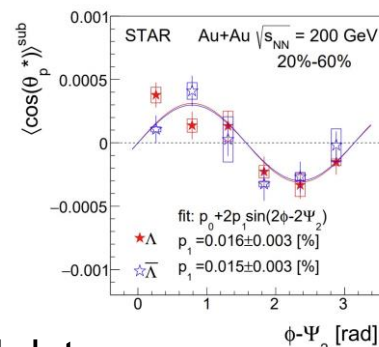
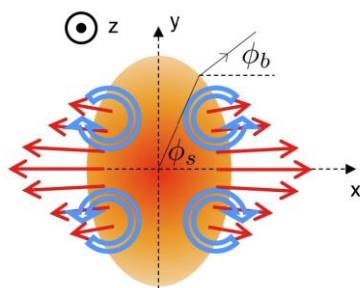
F. Becattini, M. A. Lisa, ARNPS 70 (2020) 395-423

Recent review: Becattini, Buzzegoli, Niida, Pu, Tang, Wang, 2402.04540.

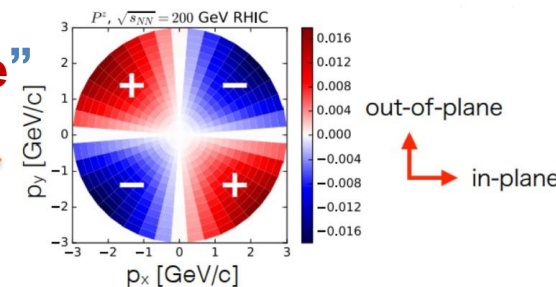


Sign puzzle in local polarization

- Stronger in-plane expansion of QGP due to spacetime anisotropy can induce local polarization along the beam line.



“sign puzzle”



- In the past, the theories in the market **CANNOT** explain the local polarization well, even gives opposite results.

Experimental data

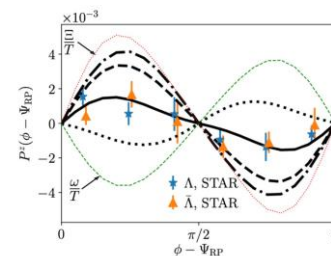
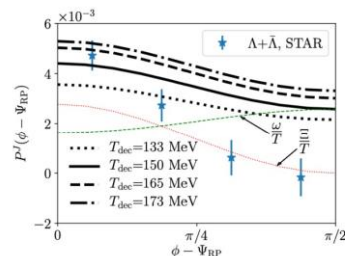
[STAR Collaboration], PRL(2019)

Phenomenological model

Becattini, Karpenko, PRL(2018)

- The shear effects can be important to local polarization, and even quantitatively explain the experiment by proper choice of parameter.

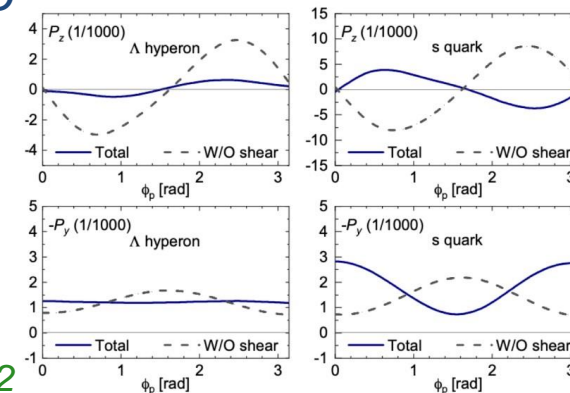
✓ Shear induced polarization:
An “Intrinsic Hall effect” in HIC



Fu, Liu, Pang et al, PRL. 127 (2021) 14, 142301

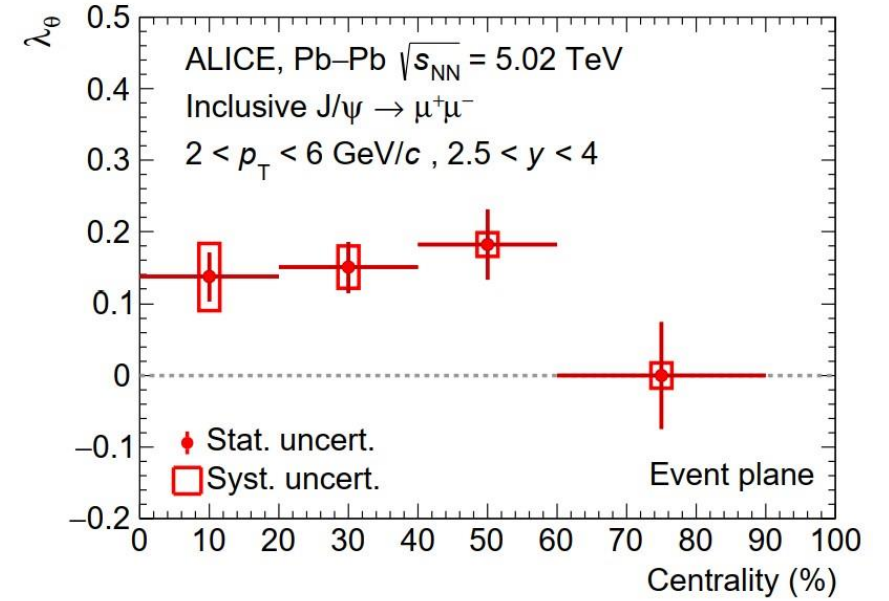
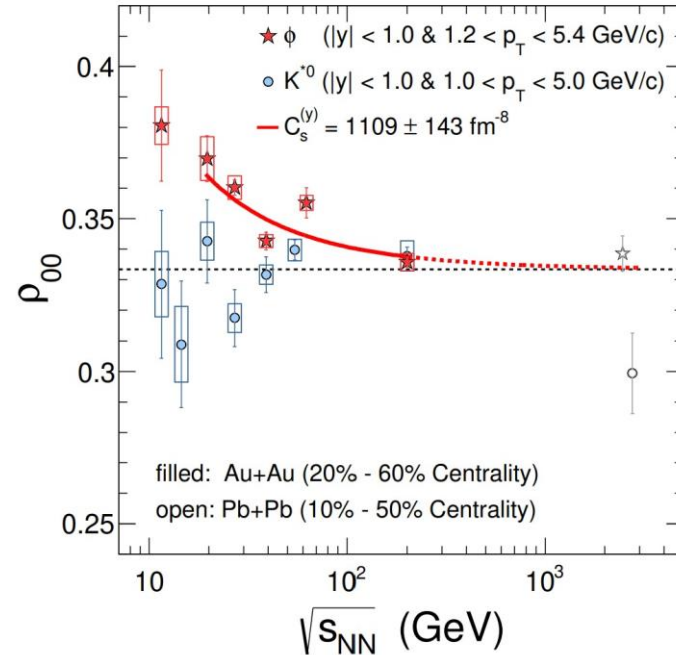
Becattini, Buzzegoli, Palermo, PRL. 127 (2021) 27, 272302

Yi, Pu, and Yang, PRC. 104 (2021) 6, 064901



Global spin alignment

Physics Mechanisms	$\langle \rho_{00} \rangle$
\mathbf{c}_Λ : Quark coalescence vorticity & magnetic field ^[1]	$< 1/3$ (Negative $\sim 10^{-5}$)
\mathbf{c}_ε : E-comp. of Vorticity tensor ^[1]	$< 1/3$ (Negative $\sim 10^{-4}$)
\mathbf{c}_E : Electric field ^[2]	$> 1/3$ (Positive $\sim 10^{-5}$)
\mathbf{c}_F : Fragmentation ^[3]	$> \text{ or } < 1/3$ ($\sim 10^{-5}$)
\mathbf{c}_L : Local spin alignments ^[4]	$< 1/3$
\mathbf{c}_A : Turbulent color field ^[5]	$< 1/3$
\mathbf{c}_ϕ : Vector meson strong force field ^[6]	$> 1/3$ (Can accommodate large positive signal)
\mathbf{c}_g : Glasma fields + effective potential	could be significant



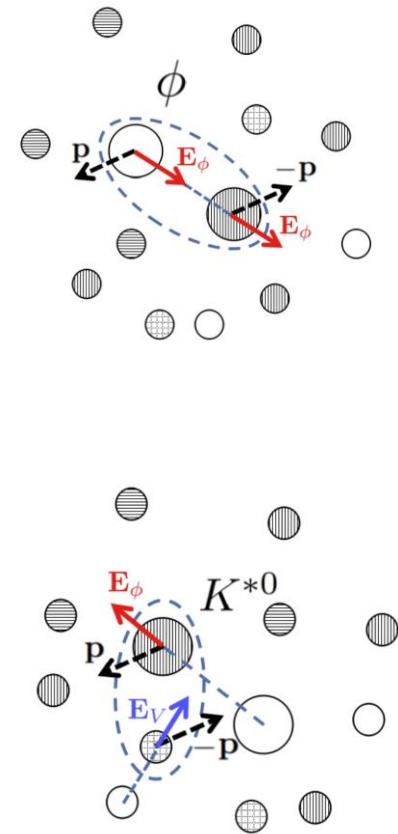
[STAR Collaboration], Nature (2023); [ALICE Collaboration], PRL (2023);
 Sheng, Oliva, Wang PRD(2020); Sheng, Wang, Wang, PRD(2020)
 Sheng, Oliva, Liang, Wang, Wang, PRL (2023); PRD(2024);
 Kumar, Muller, Yang, 2212.13354; 2304.04181;
 Muller, Yang, PRD(2022); Yang, JHEP(2022);
 Wagner, Weickgenannt, Speranza, PRR(2023);
 Li, Liu, 2206.11890; Dong, Yin, Sheng, Yang, Wang, 2311.18400;

Table taken from
 Prof. **Ai-hong Tang's** slides

Question: how can we understand spin polarizations and alignments emerged in heavy ion collisions?

1. Roughly speaking, the **hyperon** spin polarization inherits from quarks, quantitatively describing the **spin polarization of quarks** in the **many-body system** needs a spin transport equation;
2. Spin transport equation of vector fields is also necessary to understand the spin properties of vector mesons;
3. **Vector meson** spin alignment also reflects the correlations of the polarization of quarks: also need the input from the spin transport of quarks.

Question: Microscopically, how to implement the **spin evolution** into the evolution of quarks and vector mesons?





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- Spin polarization and alignment in heavy ion collisions
- **Quantum kinetic theory with collisions and self-energies**
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Present efforts on quantum kinetic theory

Up to now there are plenty of investigations on relativistic quantum kinetic theory and spin transport equations of fermions:

See Prof. **Qun Wang's** talk for a review

- Chiral kinetic theory:

D. T. Son, N. Yamamoto, PRL(2012); PRD(2013); S. Lin, A. Shukla, JHEP (2019) ; M.A. Stephanov, Y. Yin PRL (2012) ; J.-Y. Chen, D. T. Son, M. A. Stephanov, H.-U. Yee, Y. Yin, PRL (2014) ; J.-W. Chen, J.-Y. Pang, S. Pu, Q. Wang PRD (2014) ; J.-Y. Chen, D. T. Son, M. A. Stephanov, PRL (2015); J.-H. Gao, Z.-T. Liang, S. Pu, Q. Wang, X.-N. Wang PRL(2012); J.-W. Chen, S. Pu, Q. Wang, X.-N. Wang, PRL (2013) ; Y. Hidaka, S. Pu, D.-L. Yang, PRD (2017) ; A. Huang, S. Shi, Y. Jiang, J. Liao, P. Zhuang, PRD (2018) ; N. Mueller, R. Venugopalan, PRD(2018) ; PRD 96 (2017); Y.-C. Liu, L.-L. Gao, K. Mameda, X.-G. Huang, PRD(2019); S.-Z. Yang, J.-H. Gao, Z.T.-Liang, Q. Wang, PRD(2020); K. Mameda, PRD(2023); N. Yamamoto, D.-L. Yang, APJ(2020); PRD(2021), PRL(2023), 2308.08257;

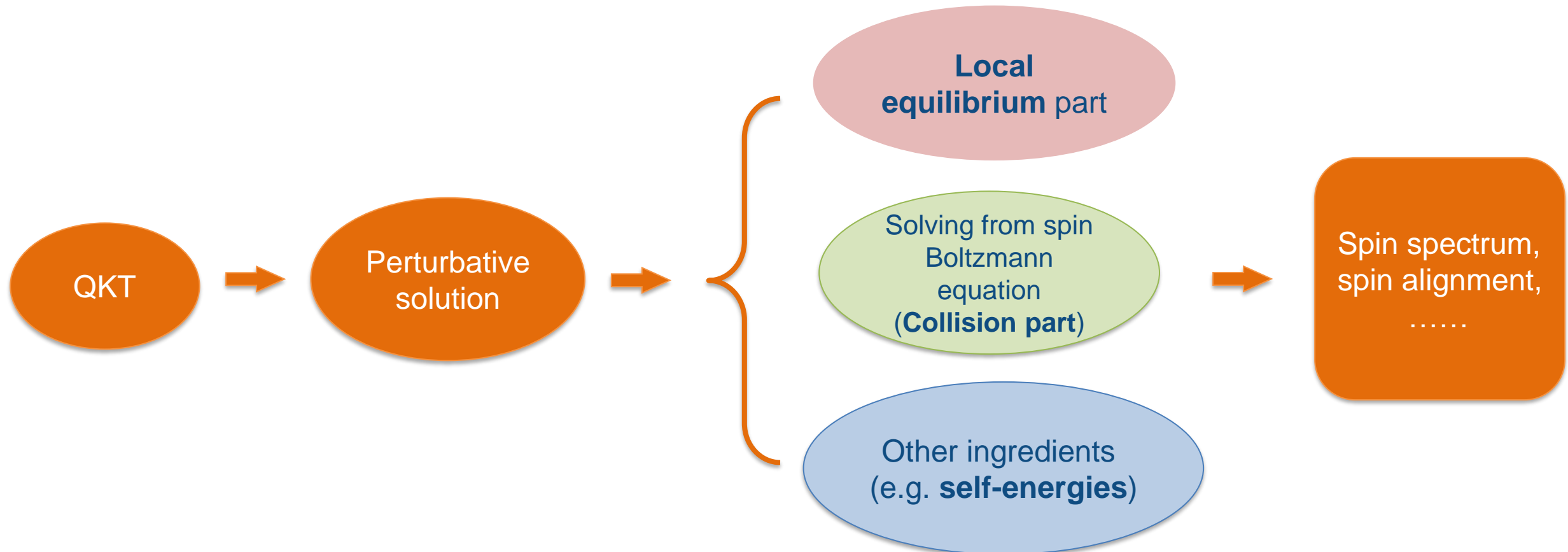
- Quantum kinetic theory including quantum corrections:

- Quantum kinetic theory without collisions: *J.-H. Gao, Z.-T. Liang, PRD(2019); N. Weickgenannt, X.-L. Sheng, E. Speranza, Q. Wang, D. H. Rischke PRD(2019); K. Hattori, Y. Hidaka, D.-L. Yang PRD(2019); Y.-C. Liu, K. Mameda, X.-G. Huang CPC (2021); Z. Wang, X. Guo, S. Shi, P. Zhuang PRD(2019); D.-L. Yang, JHEP(2022); S.-X. Ma, J.-H. Gao, 2209.10737;*
- Spin transport theory with **collisions** based on QKT: *N. Weickgenannt, X.-L. Sheng, E. Speranza, Q. Wang, D. H. Rischke PRL(2021); PRD(2021); PRD(2021); D.-L. Yang, K. Hattori, Y. Hidaka, JHEP(2020); Z. Wang, X. Guo, P. Zhuang, EPJC(2021); Z. Wang, P. Zhuang, 2105.00915; N. Weickgenannt, D. Wagner, E. Speranza, D. H. Rischke, PRD(2022), PRDL(2022); SF, S. Pu, D.-L. Yang PRD(2022); Z. Wang, PRD(2022); S. Lin, PRD(2022); S. Lin, Z. Wang, JHEP(2022);*

Also see Y. Hidaka, S.Pu, Q. Wang, D.-L. Yang, **PPNP(2022)** for a recent **review on CKT&QKT**

Spin polarization from interactions

The **dynamical** evolution of spin DoFs incorporate the **effects of interactions** to spin polarization density S^μ , which further relates to the spin spectrum of quarks and spin density matrix of vector mesons.



Why QKT with self-energies?

An example in relativistic **classical** kinetic theory:

- In classical kinetic theory, the **leading order contribution** of interactions lies in the **mean field approximation**:

$$\begin{aligned}\Sigma(x, y) &= \Sigma_{\text{MF}}(x) \delta^{(4)}(x, y) + \Sigma^>(x, y) \Theta(x_0, y_0) + \Sigma^<(x, y) \Theta(y_0, x_0), \\ P(x, y) &= P_{\text{MF}}(x) \delta^{(4)}(x, y) + P^>(x, y) \Theta(x_0, y_0) + P^<(x, y) \Theta(y_0, x_0), \\ \Pi(x, y) &= \Pi_{\text{MF}}(x) \delta^{(4)}(x, y) + \Pi^>(x, y) \Theta(x_0, y_0) + \Pi^<(x, y) \Theta(y_0, x_0).\end{aligned}$$

$$\begin{aligned}\left(\left[\left(p_\mu + \frac{i}{2} \partial_\mu \right) \gamma^\mu - M - \Sigma_{\text{MF}}(X) + \frac{i}{2} \partial_\mu \Sigma_{\text{MF}}(X) \partial_p^\mu \right] G^\cong(X, p) \right)_{\alpha\beta} \\ = (\Sigma^\cong(X, p) G^-(X, p) + \Sigma^+(X, p) G^\cong(X, p))_{\alpha\beta},\end{aligned}$$

A comprehensive paper on the Walecka model: *S. Mrowczynski, U. W. Heinz, Annals Phys. 229, 1(1994)*: In the pairing approximation, corrections of vacuum meson mass,

$$M^*(X) = M - g_s \langle \phi(X) \rangle.$$

And the **effective Lorentz force** induced by background mean mesonic field,

An effective Lorentz force

$$\begin{aligned}\frac{[p^\mu \partial_\mu - g_v p^\mu \langle F_{\mu\nu}(X) \rangle \partial_p^\nu - g_s M^*(X) \partial_\mu \langle \phi(X) \rangle \partial_p^\mu] G^\cong(X, p)}{=} -\frac{1}{2} g_s \partial_\mu \langle \phi(X) \rangle \{ \gamma^\mu, G^\cong(X, p) \} - \frac{1}{2} g_v \langle F_{\mu\nu}(X) \rangle \{ \gamma^\mu \gamma^\nu, G^\cong(X, p) \},\end{aligned}$$

Similar applications in **high-temperature gauge theory**: *J. P. Blaizot & E. Iancu, NPB 557, 183(1999); Phys. Report. 359, 355(2002)*.

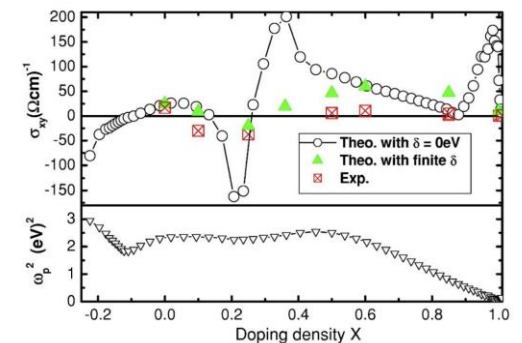
Why QKT with collisions?

- Collision terms serve as **dynamical** sources of quark spin polarization with **higher order couplings**: the **collisions** between quarks bring **off-equilibrium effects**.
- Similar insights from condensed matter physics: the anomalous Hall effects, spin Hall effects etc from **electron scattering and impurity scattering**.

$$\mathcal{S}^\mu(\mathbf{p}) = \frac{\int d\Sigma \cdot p \mathcal{J}_5^\mu(p, X)}{2m_\Lambda \int d\Sigma \cdot \mathcal{N}(p, X)},$$

- Spin polarization phenomena: **momentum spectrum** of **spin Hall effects** for the hot dense matter. The **collisional effects** must play a role.

Extrinsic anomalous Hall effects from collisions in condensed matter physics



D. Xiao, M.-C. Chang, Q. Niu RMP 82, 1959
N. A. Sinitsyn 2008, J. Phys.: Condens. Matter 20,023201
Y. Yao, et al. PRB 75, 020401
Hirsch, J. E., 1999, PRL 83, 1834.

QKT with self-energies and collisions (I)

- We define the gauge invariant 2-point Wigner function for fermions,

$$S_{\alpha\beta}^{\leftarrow}(x, p) = - \int d^4 y e^{-ip \cdot y} \langle : \bar{\psi}_{\beta}(x + \frac{y}{2}) e^{\frac{y}{2} \cdot \overleftarrow{D}(x)} \otimes e^{-\frac{y}{2} \cdot \overrightarrow{D}(x)} \psi_{\alpha}(x - \frac{y}{2}) : \rangle$$

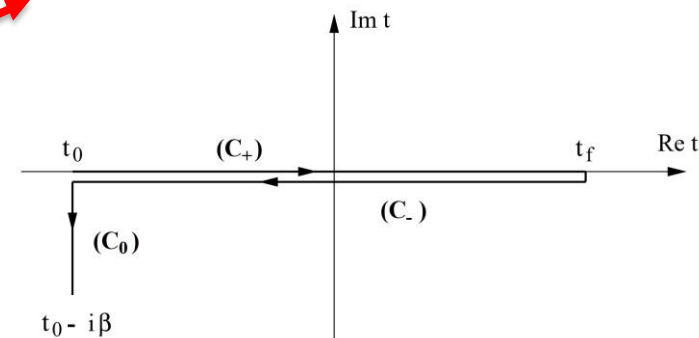
- Along the Schwinger-Keldysh contour, one derives the Kadanoff-Baym equations up to $O(\hbar)$,

$$\Sigma(x, y) = -i\hbar^{-1} \Sigma^{\delta}(x) \delta_C^{(4)}(x, y) + \theta_C(x_0, y_0) \Sigma^{\rightarrow}(x, y) + \theta_C(y_0, x_0) \Sigma^{\leftarrow}(x, y), \quad \text{Retarded and advanced quantities}$$

$$\left[\frac{i\hbar}{2} \gamma^{\mu} \nabla_{\mu} + \gamma^{\mu} \Pi_{\mu} - m + \Sigma^{\delta}(X) \star \right] S^{\leftarrow}(q, X) = -\hbar \left[\Sigma_g^r \star S^{\leftarrow} + \Sigma_g^{\leftarrow} \star S^a \right],$$

$$S^{\leftarrow} \left(-\frac{i\hbar}{2} \gamma^{\mu} \overleftarrow{\nabla}_{\mu} + \gamma^{\mu} \overleftarrow{\Pi}_{\mu} - m \right) + S^{\leftarrow} \star \Sigma^{\delta}(X) = -\hbar \left[S^r \star \Sigma_g^{\leftarrow} + S^{\leftarrow} \star \Sigma_g^a \right],$$

Gauge-invariant Moyal product



- From the analytical properties of r/a quantities,

Blaizot, Iancu, Phys.Rept.(2002); Hidaka, Pu, Yang, PRDL(2017); Hattori, Hidaka, Yang, JHEP(2020); Yamamoto, Yang, 2308.08257; SF, Pu, Yang, 2311.15197.

$$\mathcal{O}^r(q, X) = \text{Re}\mathcal{O}^r(q, X) + i \frac{\mathcal{O}^{\rightarrow}(q, X) - \mathcal{O}^{\leftarrow}(q, X)}{2},$$

$$\mathcal{O}^a(q, X) = \text{Re}\mathcal{O}^r(q, X) - i \frac{\mathcal{O}^{\rightarrow}(q, X) - \mathcal{O}^{\leftarrow}(q, X)}{2}.$$

- ✓ $\text{Re}S^r = 0$, perfect quasi-particle $\Rightarrow \delta(p^2 - m^2)$
- $\text{Re}S^r \neq 0$, nonzero-width

We will work on quasi-particle approximation but $\text{Re}\Sigma^r \neq 0$.

QKT with self-energies and collisions (II)

- Work in the Clifford basis,

$$\bar{\Sigma}_g(q, X) = \Sigma^\delta(X) + \text{Re}\Sigma_g^r$$

Self-energy with tadpole and retarded

Collisional self-energy

$$S^{\lessgtr} = \mp \left[\mathcal{F}^{\lessgtr} + iP^{\lessgtr}\gamma^5 + \gamma^\mu \mathcal{V}_\mu^{\lessgtr} + \gamma^5 \gamma^\mu \mathcal{A}_\mu^{\lessgtr} + \frac{1}{2} \mathcal{S}_{\mu\nu}^{\lessgtr} \gamma^{\mu\nu} \right],$$

$$\Sigma^{\lessgtr} = \mp \left[\Sigma_F^{\lessgtr} + i\Sigma_P^{\lessgtr}\gamma^5 + \gamma^\mu \Sigma_{V,\mu}^{\lessgtr} + \gamma^5 \gamma^\mu \Sigma_{A,\mu}^{\lessgtr} + \frac{1}{2} \Sigma_{T,\mu\nu}^{\lessgtr} \gamma^{\mu\nu} \right]$$

- Based on the **quantum nature** of **spin polarization**, we adopt the power counting,

$$\mathcal{V}^\mu \sim \mathcal{O}(\hbar^0), \mathcal{A}^\mu \sim \mathcal{O}(\hbar^1),$$

From EoMs, $\mathcal{P} \sim \mathcal{O}(\hbar^2)$, $\mathcal{S}^{\mu\nu} \sim \mathcal{O}(\hbar^1)$, $\mathcal{F} \sim \mathcal{O}(\hbar^0)$, and similar for the self-energies

- Then we obtain the master equations for the Clifford components of Wigner functions:
 - Kinetic equations for Wigner functions;
 - Constraint equations:

$$\tilde{q}_{[\mu} \mathcal{V}_{\nu]}^< = \mathcal{O}(\hbar^2), \quad (\tilde{q}^2 - \tilde{m}^2) \mathcal{A}_\mu^< = \frac{\hbar}{2} \epsilon_{\mu\nu\alpha\beta} \tilde{q}^\nu \tilde{\mathcal{D}}^\alpha \mathcal{V}^{<,\beta} + \tilde{m} \epsilon_{\mu\nu\alpha\beta} \mathcal{V}^{<,\nu} \bar{\Sigma}_T^{\alpha\beta}$$

$$(\tilde{q}^2 - \tilde{m}^2) \mathcal{V}_\mu^< = \mathcal{O}(\hbar^2), \quad + 2(\bar{\Sigma}_A \cdot \tilde{q}) \mathcal{V}_\mu^< - 2\bar{\Sigma}_{A,\mu} (\tilde{q} \cdot \mathcal{V}^<) + \mathcal{O}(\hbar^3)$$

$$\tilde{q}^\mu \mathcal{A}_\mu^< = \bar{\Sigma}_A^\mu \mathcal{V}_\mu^< + \mathcal{O}(\hbar^3) \quad \tilde{\mathcal{D}}^\mu \mathcal{V}_\mu^< = -\frac{1}{\tilde{m}} \tilde{q}_\mu \widehat{\Sigma}_F \mathcal{V}^\mu - \frac{1}{\tilde{m}} [\Sigma_F (\tilde{q}_\mu \mathcal{V}^{<,\mu})]_{\text{P.B.}} + \mathcal{O}(\hbar^2),$$

K. Hattori, Y. Hidaka, D.-L. Yang, PRD 100, 096011 (2019)

D.-L. Yang, K. Hattori, Y. Hidaka, JHEP 07 (2020) 070

SF, S. Pu, D.-L. Yang, 2311.15197

QKT with self-energies and collisions (III)

- From the master equations, we find the mass and momentum are modified by the self-energies,

$$\tilde{m} = m - \bar{\Sigma}_F, \quad \tilde{q}_\mu = q_\mu + \bar{\Sigma}_{V,\mu},$$

- The perturbative solutions are modified in dispersion relations, solution structure, etc,

Distribution of vector charge

$$\mathcal{V}_\mu^<(q, X) = 2\pi\epsilon(q_0)\delta(\tilde{q}^2 - \tilde{m}^2)\tilde{q}_\mu f_V^<(q, X) + \mathcal{O}(\hbar^2),$$

Modified spin tensor

$$\mathcal{A}_\mu^<(q, X) = \mathcal{A}_{(\text{dy})\mu}^<(q, X) + \mathcal{A}_{(\text{non})\mu}^<(q, X) + \mathcal{A}_{(\text{mag})\mu}^<(q, X)$$

$$S_n^{\mu\nu}(\tilde{q}, \tilde{m}) = \frac{\epsilon^{\mu\nu\rho\sigma}\tilde{q}_\rho n_\sigma}{2(\tilde{q} \cdot n + \tilde{m})},$$

$$= 2\pi\epsilon(q_0)\left\{ \delta(\tilde{q}^2 - \tilde{m}^2) \left[a_\mu f_A^< + \hbar S_{n,\mu\nu}(\tilde{q}, \tilde{m}) \tilde{\mathcal{D}}^\nu f_V^< - \bar{\Sigma}_{A,\mu} f_V^< \right] - \delta'(\tilde{q}^2 - \tilde{m}^2) f_V^< \right.$$

Effective spin 4-vector
in phase space

$$\times \left[\frac{\hbar}{2} \epsilon_{\mu\nu\alpha\beta} \tilde{q}^\nu (QeF^{\alpha\beta} + 2\nabla^{[\alpha} \bar{\Sigma}^{\beta]}) + \tilde{m} \epsilon_{\mu\nu\alpha\beta} \tilde{q}^\nu \bar{\Sigma}_T^{\alpha\beta} + 2(\bar{\Sigma}_A \cdot \tilde{q}) \tilde{q}_\mu - 2\tilde{m}^2 \bar{\Sigma}_{A,\mu} \right] \Big\}$$

Effective field strength tensor

Other sources of spin polarizations from self-energies

- These new ingredients related to the **self-energies** and their **gradients** in the axial-vector Wigner function can serve as **new sources of quark spin polarization**.

QKT with self-energies and collisions (IV)

- We then obtain the scalar kinetic equation for fermion distributions (the **Boltzmann** equation),

$$0 = 2\pi\epsilon(q_0)\delta(\tilde{q}^2 - \tilde{m}^2) \left\{ \tilde{q}_\mu \tilde{\nabla}^\mu f_V^< + \tilde{q}_\mu \widehat{\Sigma}_V^\mu f_V + \tilde{m} \widehat{\Sigma}_F f_V \right. \\ \left. - \tilde{m} [(\nabla_\alpha \bar{\Sigma}_F) \partial_q^\alpha f_V^< - (\partial_{q,\alpha} \bar{\Sigma}_F) \partial_X^\alpha f_V^<] \right\} + \mathcal{O}(\hbar^2),$$

And the axial kinetic equation for the quark effective **spin vector** with **modifications from self-energies**,

$$\square_{a,\mu}^{(n)} \mathcal{A}^< + \tilde{q}_\mu \square_q^{(n)} \mathcal{A}^< + \tilde{m} \square_{m,\mu}^{(n)} \mathcal{A}^< = \mathcal{C}_{1,\mu}^{(n)} + \hbar(\tilde{q}_\mu \mathcal{C}_{2,\mu}^{(n),q} + \tilde{m} \mathcal{C}_{2,\mu}^{(n),m}).$$

with e.g.

$$\square_{a,\mu}^{(n)} \mathcal{A}^< = \delta(\tilde{q}^2 - \tilde{m}^2) \left\{ \tilde{q} \cdot \tilde{\nabla} (a_\mu f_A^<) + Qe\bar{F}_{\mu\nu} (a^\nu f_A^<) + 2\epsilon_{\mu\nu\alpha\beta} \frac{\bar{\Sigma}_A^\nu}{\hbar} \tilde{q}^\alpha a^\beta f_A^< \right. \\ \left. + \frac{\tilde{q} \cdot \nabla \bar{\Sigma}_F}{\tilde{m}} (a_\mu f_A^<) - \tilde{q}_\mu \frac{\nabla^\nu \bar{\Sigma}_F}{\tilde{m}} (a_\nu f_A^<) \right\},$$

- It is found that, there exists the terms similar to the **relaxation time** even **in the absence of collision kernel**; AKE serves as a **generalized BMT equation** with **effective “Lorentz forces”** originating from the spin tensor and self-energies.

Spin Boltzmann equation with collision terms (I)

- We firstly consider the **massless** fermions and **neglect the self-energies** for simplicity. The independent EoMs of the Wigner functions are simplified to,

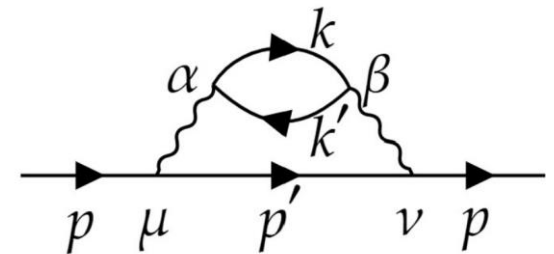
$$\begin{aligned}\Delta \cdot \mathcal{V}^< &= \Sigma_V^< \cdot \mathcal{V}^> - \Sigma_V^> \cdot \mathcal{V}^<, \\ \Delta \cdot \mathcal{A}^< &= \Sigma_V^< \cdot \mathcal{A}^> - \Sigma_V^> \cdot \mathcal{A}^< - \Sigma_A^< \cdot \mathcal{V}^> + \Sigma_A^> \cdot \mathcal{V}^<.\end{aligned}$$

- For massless fermions, the **equilibrium conditions** are,

$$\begin{aligned}\text{global equilibrium} &: \partial_{(\mu}(\beta u_{\nu)}) = 0, \omega_s^{\mu\nu} = -\Omega^{\mu\nu} = -\partial^{[\mu}(\beta u^{\nu]} \\ \text{local equilibrium} &: \omega_s^{\mu\nu} = -\Omega^{\mu\nu}. \quad \text{Spin chemical potential} \quad \text{Thermal vorticity}\end{aligned}$$

- We consider the evolution of a hard probe electron in a **thermalized QED plasma**, and evaluate the collision kernels for **a realistic gauge theory**.

$$e^-(p) + e_{eq}^-(k) \leftrightarrow e^-(p') + e_{eq}^-(k'),$$



Spin Boltzmann equation with collision terms (II)

- A spin Boltzmann equation (SBE) with explicit collision kernel is derived using **HTL approximation**:

$$(p \cdot \partial) f_V^<(x, p) = \mathcal{C}_V^{\text{HTL}}[f_V] + \mathcal{O}(\hbar^2),$$

$$(p \cdot \partial) f_A^<(x, p) + \hbar \partial_\mu S_{(u)}^{\mu\nu} \partial_\nu f_V^<(x, p) = \mathcal{C}_A^{\text{HTL}}[f_V, f_A] + \mathcal{O}(\hbar^2),$$

- It is found that, when the vector and axial **distribution functions of probes are in local equilibrium**, the **collision kernels are zero**: Detailed balance exists for quantum modes.
- For certain cases, we find the **spin relaxation rate can be much slower** than particle relaxation rate.

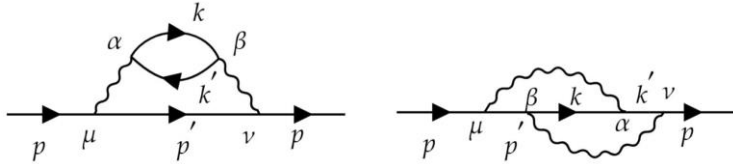
$$\mathcal{C}_V[f_V] = \frac{e^4 \delta(p^2)}{24\beta^2} \ln \frac{T}{m_D} \left[2f_V^<(p)f_V^>(p) + |\mathbf{p}|F(p)\hat{p}_{\perp,\alpha}\partial_{p_\perp}^\alpha f_V^<(p) - |\mathbf{p}|\frac{1}{\beta}(\partial_{p_\perp} \cdot \partial_{p_\perp})f_V^<(p) \right]$$

$$\begin{aligned} \mathcal{C}_A[f_V, f_A] = & -\frac{e^4 \delta(p^2)}{8\pi^2 |\mathbf{p}|} \ln \frac{T}{m_D} \left\{ \frac{2\pi^2}{3\beta^2} |\mathbf{p}|F(p)f_A^<(p) + \frac{\pi^2}{3\beta^2} |\mathbf{p}|^2 F(p)[(\hat{p}_\perp \cdot \partial_{p_\perp}) - \frac{1}{\beta}(\partial_{p_\perp} \cdot \partial_{p_\perp})]f_A^<(p) \right. \\ & - \frac{2\pi^2}{3\beta^2} |\mathbf{p}|^2 f_A^<(p)(\hat{p}_\perp \cdot \partial_{p_\perp})f_V^<(p) + \hbar F(p)|\mathbf{p}|H_{3,\alpha}\partial_{p_\perp}^\alpha f_V^<(p) \\ & - \hbar \frac{\pi^2}{12\beta^2} F(p)|\mathbf{p}|e^{\rho\alpha\nu\beta}\hat{p}_{\perp,\nu}u_\beta\partial_{p_\perp,\rho}\partial_\alpha f_V^<(p) + \hbar \frac{\pi^2}{6\beta^3} e^{\rho\alpha\nu\beta}\hat{p}_{\perp,\rho}u_\beta\partial_{p_\perp,\nu}\partial_\alpha f_V^<(p) \\ & + \hbar \frac{\pi^2}{6\beta^2} e^{\mu\xi\lambda\kappa}p_\lambda u_\kappa \partial_\xi f_V^<(p)\partial_{p_\perp,\mu} f_V^<(p) \\ & \left. - \hbar \frac{\pi^2}{12\beta^3} |\mathbf{p}|e^{\rho\alpha\nu\beta}\hat{p}_{\perp,\nu}u_\beta\hat{p}_{\perp,(\gamma}g_{\lambda)\rho}\hat{p}_{\perp,\lambda}\partial_{p_\perp}^\lambda\partial_{p_\perp}^\gamma\partial_\alpha f_V^<(p) \right\} + \mathcal{O}(\hbar^2). \end{aligned}$$

Solving SBE: Gradient expansion

- In the **Navier-Stokes limit**, we can derive the solution of SBE when the vector distribution f_V is in **local equilibrium** for the Moller scattering process,

$$\begin{aligned}
 & \delta f_A^<(x, p) \\
 = & -\frac{\hbar}{2} f_{V, \text{leq}}^<(x, p) f_{V, \text{leq}}^>(x, p) \frac{48\pi\beta^2}{e^4 \ln \frac{T}{m_D}} \left\{ -\left(-\frac{27\zeta(3)}{\pi^2\beta} + E_{\mathbf{p}}\right) \frac{2 \ln 2}{3} (\omega_\rho \nabla^\rho \beta - \beta \nabla_\alpha \omega^\alpha) \right. \\
 & + \left(-\frac{27\zeta(3)}{\pi^2\beta} + E_{\mathbf{p}}\right) \left(\frac{5}{3} - \frac{360\zeta(3)}{7\pi^4} \ln 2\right) \beta \omega_\rho \nabla^\rho \alpha \\
 & + \left[-\frac{45\zeta(3) - 7\pi^2 \ln 2}{14\pi^2 \ln 2} \epsilon^{\mu\nu\alpha\beta} u_\beta \nabla_\nu \alpha \nabla_\mu \beta - \frac{3\pi^2}{10 \ln 2} \beta \sigma^{\mu\alpha} \omega_\mu \right] p_{\langle\alpha} \\
 & + \left[-\frac{\beta}{4} \left(\omega^{\langle\alpha} \nabla^{\rho} \beta + \frac{1}{2} \beta \nabla^{\langle\rho} \omega^{\alpha} + \epsilon^{\mu\nu\sigma\rho} \sigma_\mu^\alpha u_\sigma \nabla_\nu \beta \right) + \frac{3645\zeta^2(3) + 7\pi^6}{1512\pi^4\zeta(3)} \beta^2 \epsilon^{\mu\nu\sigma\rho} \sigma_\mu^\alpha u_\sigma \nabla_\nu \alpha \right. \\
 & \left. + \frac{10935\zeta^2(3) - 14\pi^6}{756\pi^4\zeta(3)} \beta^2 \omega^{\langle\alpha} \nabla^{\rho} \alpha \right] p_{\langle\alpha} p_{\rho\rangle} + \frac{7\pi^4}{10125\zeta(5)} \beta^3 \omega^{\langle\rho} \sigma^{\mu\lambda} p_{\langle\mu} p_{\rho} p_{\lambda\rangle} \left. \right\},
 \end{aligned}$$



- We notice that, the corrections are proportional to combinations of hydrodynamic fluctuations and, therefore, lead to a **higher order corrections** to the spin polarization **if f_V is in local equilibrium**.

Spin polarization with off-equilibrium corrections

- In terms of spin Cooper-Frye formula,

$$\begin{aligned}\delta\mathcal{P}^\mu(t; \mathbf{p}) &= -\mathcal{P}_{\text{leq}}^\mu(t; \mathbf{p}) \frac{1}{N} \int_{\Sigma} d\Sigma_{\alpha} p^{\alpha} \delta f_{\text{V}}^{\leq}(x, p) + \frac{\hbar}{2N} \int_{\Sigma} d\Sigma \cdot p p^{\mu} \delta f_{\text{A}}^{\leq}(x, p) \\ &= -\mathcal{P}_{\text{leq}}^\mu (\mathcal{P}_{\text{V, shear}} + \mathcal{P}_{\text{V, chem}}) + \mathcal{P}_{\text{A, vort-}\nabla\text{T}}^\mu + \mathcal{P}_{\text{A, }\nabla\text{vort}}^\mu + \mathcal{P}_{\text{A, vort-chem}}^\mu \\ &\quad + \mathcal{P}_{\text{A, vort-shear}}^\mu + \mathcal{P}_{\text{A, chem-}\nabla\text{T}}^\mu + \mathcal{P}_{\text{A, shear-}\nabla\text{T}}^\mu + \mathcal{P}_{\text{A, shear-chem}}^\mu,\end{aligned}$$

where we have, for example,

$$\begin{aligned}\mathcal{P}_{\text{A, vort-}\nabla\text{T}}^\mu &= -\frac{\hbar^2 \beta_0^2}{4N e^4 \ln \frac{1}{e}} \int_{\Sigma} d\Sigma \cdot p f_{\text{V, leq}}^{\leq}(p) f_{\text{V, leq}}^{\geq}(p) \\ &\quad \times p^{\mu} \left[d_2 \left(E_{\mathbf{p}} - d_1 \frac{1}{\beta_0} \right) \omega^{\alpha} \nabla_{\alpha} \beta_0 - d_6 \beta_0 p_{\langle \alpha} p_{\rho \rangle} \omega^{\alpha} \nabla^{\rho} \beta_0 \right], \\ \mathcal{P}_{\text{A, }\nabla\text{vort}}^\mu &= \frac{\hbar^2 \beta_0^2}{4N e^4 \ln \frac{1}{e}} \int_{\Sigma} d\Sigma \cdot p f_{\text{V, leq}}^{\leq}(p) f_{\text{V, leq}}^{\geq}(p) \\ &\quad \times p^{\mu} \left[\left(E_{\mathbf{p}} - d_1 \frac{1}{\beta_0} \right) d_2 \beta_0 \nabla^{\alpha} \omega_{\alpha} + d_6 \frac{1}{2} \beta_0^2 \nabla^{\alpha} \omega^{\rho} p_{\langle \alpha} p_{\rho \rangle} \right],\end{aligned}$$

Becattini, Chandra, Del Zanna, Grossi, Annals Phys (2013);
Fang, Pang, Wang, Wang, PRC(2016);
Yi, Pu, Yang, PRC(2021);
SF, Pu, et al in preparation



Outline

- Spin polarization and alignment in heavy ion collisions
- Quantum kinetic theory with collisions and self-energies
- **Self-energy corrections to spin polarization and alignment**
- Summary

Self-energy: widely used in nuclear physics

The crucial role of self-energy has been realized in nuclear physics:

- The **mean field approximations** in low-energy nuclear physics: nuclear EoS, binding energy,...

See: *B. D. Serot, Rept. Prog. Phys. 55, 1855(1992); B. D. Serot and J. D. Walecka, Int. J. Mod. Phys. E 6, 515(1997).*

- The **mean field approximations** used in QCD phase transition: for example, in the use of NJL model,...

See: *S. P. Klevansky, RMP 64,649(1992); T. Hatsuda, T. Kunihiro, Phys. Rept. 247 (1994) 221-367.*

Actually, people use the contributions of self-energy widely when they deal with spin polarization phenomena.....

- The **external electromagnetic** (EM) field induces the difference of spin polarization of $\Lambda/\bar{\Lambda}$: In the background field gauge, the EM field are the **classical fields (mean fields)**; *Muller, Schafer, PRD(2018);*

- Spin alignment of ϕ mesons from **spacetime dependent** mean ϕ fields

at freezeout:

$$g_{\phi} F_{\phi}^{\mu\nu} = g_{\phi} (\partial^{\mu} \langle \phi^{\nu} \rangle - \partial^{\nu} \langle \phi^{\mu} \rangle)$$

*Guo, Shi, Feng, Liao, PLB(2019);
Xu, Lin, Huang, Huang, PRDL(2022);
Peng, Wu, Wang, She, Pu, PRD(2023);*

- Spin alignment of J/ψ from initial-stage **classical color fields**.

Sheng, Oliva, Liang, Wang, Wang, PRL (2023); PRD(2024); Muller, Yang, PRD(2022); Yang, JHEP(2022).

Spin polarization from one-point potential

- Consider a quark-meson interaction,

$$\mathcal{L}_{\text{int}} = g_\phi \phi_\mu \bar{\psi} \gamma^\mu \psi,$$

Up to the leading order in coupling constant (with Hatree/tadpole diagram), we derive, $\bar{\Sigma}_V^\mu = g_\phi \langle \phi_\mu \rangle$, and the contribution to the axial vector Wigner function reads,

$$\delta \mathcal{A}_s^{<, \mu} = -2\pi \delta'(\tilde{q}^2 - \tilde{m}^2) \frac{\hbar}{2} \epsilon_{\mu\nu\alpha\beta} \tilde{q}^\nu g_\phi F_\phi^{\alpha\beta} f_V^{<},$$

then up to the linear order in self-energy and working in the **global equilibrium** case, we obtain

$$\delta \mathcal{P}_s^\mu(\mathbf{q}, x) = \frac{\int \frac{dq_0}{2\pi} \delta \mathcal{A}_s^{<, \mu}(q, x)}{2 \int \frac{dq_0}{2\pi} \mathcal{F}_s^{<}(q, x)} = -\frac{\hbar g_\phi}{4mT} B_\phi^\mu (1 - f_V^{<}),$$

which induces the spin polarization of strange quarks.

- Similarly, one can obtain the quark polarization from other effective fields via a tadpole diagram.
- In our formalism, the **quantum part of gauge fields** and the **background color fields** are neglected in the **background field gauge**.

Spin polarization from a thermal QCD background (I)

- Omitting the external EM field and background color field, the **leading order contribution** from **gauge** fields to quark spin polarization comes from the **Fock diagram**.
- Due to a **lack** of knowledge the **quantum part of gluonic Wigner function** in $O(\hbar)$, we firstly neglect the corresponding corrections,

$$\delta\mathcal{A}_{\mu,SE}^{\leq}(q, X) = 2\pi\epsilon(q_0)\partial_q^\nu\delta(q^2 - m^2)f_V^{\leq}\frac{\hbar}{2}\epsilon_{\mu\nu\beta\alpha}\partial_X^\beta\text{Im}\Sigma_V^{++,\alpha}$$

- For the weakly coupling QGP, we obtain,

$$\left\{ \begin{array}{l} m_f^2 = C_F \frac{g^2}{8} \left(\frac{\mu^2}{\pi^2} + T^2 \right) \\ \mathcal{Q}_0(x) = \frac{1}{2} \ln \left| \frac{x+1}{x-1} \right|, \\ \mathcal{Q}_1(x) = x\mathcal{Q}_0(x) - 1, \end{array} \right. \quad \begin{aligned} & \partial_\beta \text{Im}\Sigma_{V,\alpha}^{++}(q, X) \\ &= C_F \frac{g^2}{4} \left(\frac{\mu}{\pi^2} \partial_\beta \mu + T \partial_\beta T \right) \left[\frac{u_\alpha}{|q_\perp|} \mathcal{Q}_0 \left(\frac{q_0}{|q_\perp|} \right) + \frac{q_{\perp,\alpha}}{|q_\perp|^2} \mathcal{Q}_1 \left(\frac{q_0}{|q_\perp|} \right) \right] \\ &+ m_f^2 \left\{ \frac{\partial_\beta u_\alpha}{|q_\perp|} \left[\mathcal{Q}_0 \left(\frac{q_0}{|q_\perp|} \right) - \frac{q_0}{|q_\perp|} \mathcal{Q}_1 \left(\frac{q_0}{|q_\perp|} \right) \right] - u_\alpha q_\perp^\gamma (\partial_\beta u_\gamma) \frac{2}{|q_\perp|^2} \mathcal{Q}_1 \left(\frac{q_0}{|q_\perp|} \right) \right. \\ &\left. + q_{\perp,\alpha} q_\perp^\gamma (\partial_\beta u_\gamma) \frac{1}{|q_\perp|^3} \left[\mathcal{Q}_0 \left(\frac{q_0}{|q_\perp|} \right) - 3 \frac{q_0}{|q_\perp|} \mathcal{Q}_1 \left(\frac{q_0}{|q_\perp|} \right) \right] \right\}, \end{aligned}$$

*M. L. Bellac, Thermal Field Theory (2011);
SF, S. Pu, D.-L. Yang, 2311.15197*

Spin polarization from a thermal QCD background (II)

- In the canonical pseudo-gauge, a modification to the spin Cooper-Frye formula reads,

$$\begin{aligned}\delta\mathcal{P}^\mu(t, \mathbf{q}) &= \hbar \frac{\int_\Sigma q \cdot d\sigma \int \frac{dq_0}{2\pi} \delta\mathcal{A}^{<,\mu}(q, X)}{2 \int_\Sigma q \cdot d\sigma \int \frac{dq_0}{2\pi} \mathcal{F}^{<}(q, X)} \\ &= \delta\mathcal{P}_{\text{therm}}^\mu(t, \mathbf{q}) + \delta\mathcal{P}_{\text{shear}}^\mu(t, \mathbf{q}) + \delta\mathcal{P}_{\text{chem}}^\mu(t, \mathbf{q}) + \delta\mathcal{P}_{\text{acc}}^\mu(t, \mathbf{q}) + \delta\mathcal{P}_{\text{vor}}^\mu(t, \mathbf{q}),\end{aligned}$$

where $\delta\mathcal{P}_{\text{therm}}^\mu(t, \mathbf{q}) = -\frac{\hbar^2}{2mN} \int_\Sigma q \cdot d\sigma G_T(E_q, \mathbf{q}) \frac{m_f^2 T}{E_q^3} \epsilon^{\mu\nu\alpha\beta} q_\nu \partial_\alpha \left(\frac{u_\beta}{T} \right),$

Becattini, Chandra, Del Zanna, Grossi, Annals Phys (2013);

$\delta\mathcal{P}_{\text{shear}}^\mu(t, \mathbf{q}) = -\frac{\hbar^2}{4mN} \int_\Sigma q \cdot d\sigma G_{\omega_1}(E_q, \mathbf{q}) \frac{m_f^2}{E_q^3} \frac{\epsilon^{\mu\nu\rho\sigma} q_\rho u_\sigma}{E_q} q^\gamma \sigma_{\nu\gamma},$

Fang, Pang, Wang, Wang, PRC(2016);

Yi, Pu, Yang, PRC(2021);

SF, Pu, Yang, 2311.15197

$\delta\mathcal{P}_{\text{chem}}^\mu(t, \mathbf{q}) = -\frac{\hbar^2}{4mN} \int_\Sigma q \cdot d\sigma G_T(E_q, \mathbf{q}) \frac{C_F g^2 \mu T}{4\pi^2 E_q^2} \frac{\epsilon^{\mu\nu\rho\sigma} q_\rho u_\sigma}{E_q} \nabla_\nu \left(\frac{\mu}{T} \right),$

$\delta\mathcal{P}_{\text{acc}}^\mu(t, \mathbf{q}) = \frac{\hbar^2}{4mN} \int_\Sigma q \cdot d\sigma G_T(E_q, \mathbf{q}) \frac{3m_f^2}{E_q^3} \epsilon^{\mu\nu\rho\sigma} q_\rho u_\sigma D u_\nu,$

$\delta\mathcal{P}_{\text{vor}}^\mu(t, \mathbf{q}) = \frac{\hbar^2}{4mN} \int_\Sigma q \cdot d\sigma \frac{m_f^2}{E_q^2} \left[\omega^\mu \left(4G_T(E_q, \mathbf{q}) - \frac{|q_\perp|^2}{E_q^2} G_{\omega_1}(E_q, \mathbf{q}) + 2G_{\omega_2}(E_q, \mathbf{q}) \right) \right.$

$\left. - \frac{(\omega \cdot q)}{E_q} \left(6u^\mu G_T(E_q, \mathbf{q}) + \frac{q_\perp^\mu}{E_q} G_{\omega_1}(E_q, \mathbf{q}) \right) \right],$

Here,

$N \equiv \int_\Sigma q \cdot d\sigma f_V^{<}(E_q, X)$

Spin polarization from a thermal QCD background (III)

- Comments:
 - ✓ An additional term proportional to **kinetic vorticity appears** even **in global equilibrium**;
 - ✓ The distribution function f_V can be off-equilibrium in general;
 - ✓ Such **interaction-dependent contributions** of the first order in gradients may be **dissipationless** as they exist even when the collision terms are zero.

- An order-of-magnitude approximation with $g^2 = \frac{4\pi}{3}$, $N_c = 3$, $T = 0.165$ GeV, and $m_s = 0.3$ GeV, yields,

	$ q_\perp = 0.5$ GeV	$ q_\perp = 1.0$ GeV	$ q_\perp = 2.0$ GeV
$ \delta \mathcal{J}_{\text{therm}}^{5,\mu} / \mathcal{J}_{\text{therm,leq}}^{5,\mu} $	0.325	0.098	0.024
$ \delta \mathcal{J}_{\text{shear}}^{5,\mu} / \mathcal{J}_{\text{shear,leq}}^{5,\mu} $	0.081	0.028	0.007
$ \delta \mathcal{J}_{\text{vor}}^{5,\mu} / \mathcal{J}_{\text{therm,leq}}^{5,\mu} $	0.177	0.103	0.030

$$\mathcal{J}_{\text{therm,leq}}^{5,\mu}(\mathbf{q}, x) = \frac{\hbar}{8E_q} f_{V,\text{leq}}^< f_{V,\text{leq}}^> \epsilon^{\mu\nu\alpha\beta} q_\nu \partial_\alpha \left(\frac{u_\beta}{T} \right),$$

$$\mathcal{J}_{\text{shear,leq}}^{5,\mu}(\mathbf{q}, x) = -\hbar \frac{1}{4E_q^2 T} f_{V,\text{leq}}^< f_{V,\text{leq}}^> \epsilon^{\mu\nu\rho\sigma} q_\rho u_\sigma q^\alpha \sigma_{\nu\alpha},$$

Spin alignment from quark self-energies

- Possible applications to vector meson spin alignment:

$$\rho_{00} \approx \frac{1 - \hat{\Pi}^{yy}(\mathbf{q} = \mathbf{0})}{3 - \sum_{i=x,y,z} \hat{\Pi}^{ii}(\mathbf{q} = \mathbf{0})},$$

where

$$\hat{\Pi}^{ii}(\mathbf{q}) = \frac{4 \int_{\Sigma} d\sigma \cdot q \left[\mathcal{J}_{5q,a}^i\left(\frac{\mathbf{q}}{2}, x\right) + \delta \mathcal{J}_{5q,SE}^i\left(\frac{\mathbf{q}}{2}, x\right) \right] \left[\mathcal{J}_{5\bar{q},a}^i\left(\frac{\mathbf{q}}{2}, x\right) + \delta \mathcal{J}_{5\bar{q},SE}^i\left(\frac{\mathbf{q}}{2}, x\right) \right]}{\int_{\Sigma} d\sigma \cdot q f_{Vq}^<(E_{\mathbf{q}/2}, \frac{\mathbf{q}}{2}) f_{V\bar{q}}^<(E_{\mathbf{q}/2}, \frac{\mathbf{q}}{2})}$$

- If one determine the $\delta J_{5,SE}$ from quark self-energies, a nontrivial contribution to spin alignment is obtained. One of the most important result is the **spin alignment of ϕ from an effective ϕ meson field**.
- In the framework of **chiral quark model**, the presence of **effective meson fields** can be used to explain the **spin alignment of** other mesons e.g. $K^{*,0}$.

Kumar, Muller, Yang, PRD(2023);

Sheng, Oliva, Liang, Wang, Wang, PRL (2023); PRD (2024)

SF, Pu, Yang, 2311.15197



Outline

- Spin polarization and alignment in heavy ion collisions
- Quantum kinetic theory with collisions and self-energies
- Self-energy corrections to spin polarization and alignment
- Collisional effects on spin polarization
- **Summary**



Summary

1. We have derived the **QKT with self-energies and collisions** for spin-half fermions. It is found that the dispersion relation, perturbative solutions, and kinetic equations are modified by self-energies.
2. The **spin polarization** of quarks from **self-energies** are **of leading order in couplings and gradients**. Our results provide a theoretical support to the s-quark polarization from effective meson fields; we also derive a **modification of the spin Cooper-Frye formula** from quark self-energies; it has important contributions to spin alignment.
3. **Collisional effects** is discussed from a spin Boltzmann equation based on HTL approximation; the **off-equilibrium corrections** are derived.

Thanks for your attention!

Mar 14 – 17, 2024, Institute of Physics, Academia Sinica, Taiwan



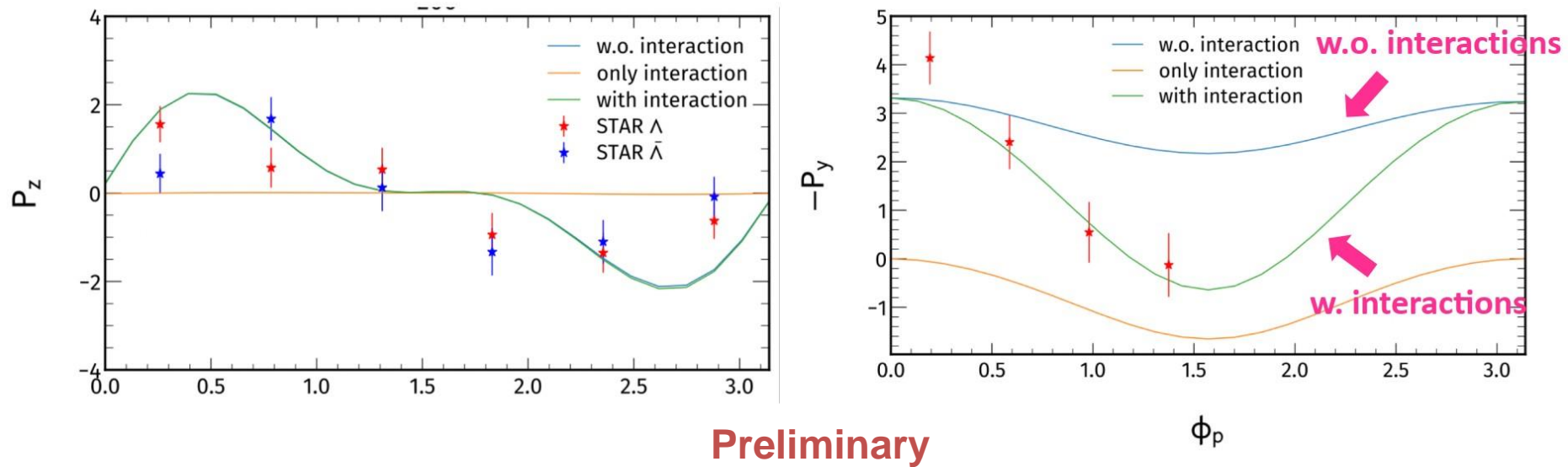


Backup slides

Numerical Estimation of off-equilibrium effects by solving SBE

- The off-equilibrium effect can be described using numerical simulation,
$$S^\mu(\mathbf{p}) = \frac{\int d\Sigma \cdot p \mathcal{J}_5^\mu(p, X)}{2m_\Lambda \int d\Sigma \cdot \mathcal{N}(p, X)},$$

200GeV, CLVisc hydrodynamics + AMPT initial + EoS: NEOS-BQS



Preliminary

SF, S. Pu, et al in preparation
C. Yi, SF, S. Pu, et al. in preparation