New Insights on the Lepton Angular Distribution in Drell-Yan and Vector Boson Production

Jen-Chieh Peng

University of Illinois at Urbana-Champaign

National Central University

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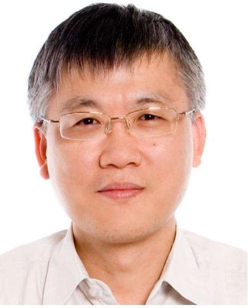
Based on the papers with Wen-Chen Chang,
Evan McClellan, Oleg Teryaev
(and Daniel Boer for one paper)



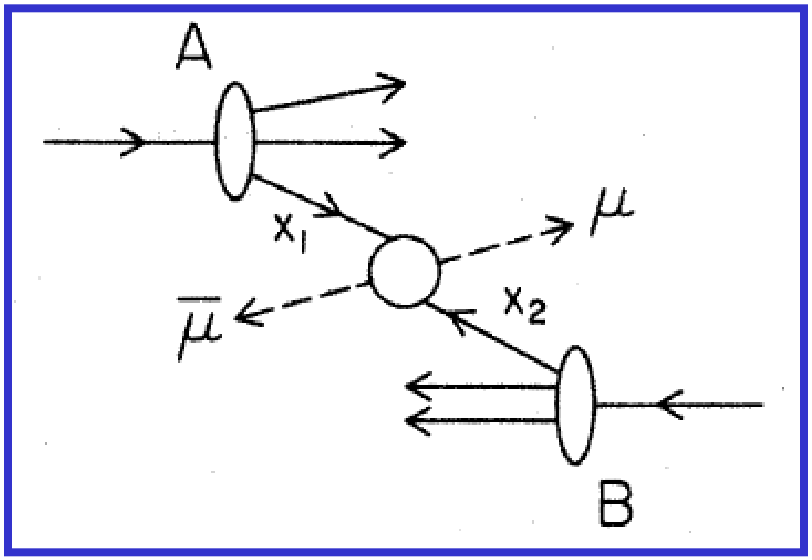
Phys. Lett. B758 (2016) 384;
Phys. Rev. D 96 (2017) 054020;
Phys. Lett. B789 (2019) 356;
Phys. Rev. D 99 (2019) 014032
Phys. Lett. B797 (2019) 134895
Phys. Rev. D 103 (2021) 034011

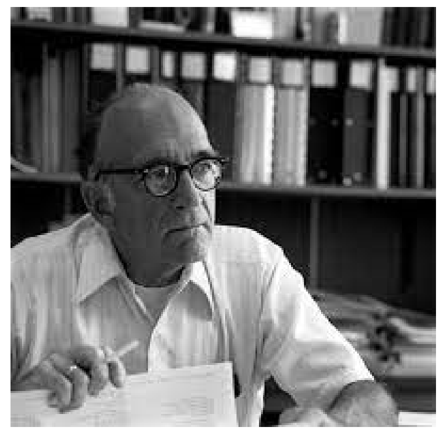
The Drell-Yan Process

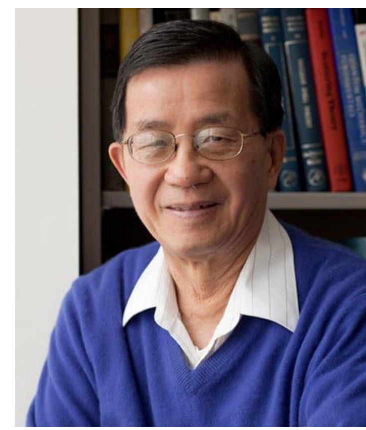
MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 May 1970)

Cited ~1800 times







$$\left(\frac{d^2\sigma}{dx_1 dx_2}\right)_{D.Y.} = \frac{4\pi\alpha^2}{9sx_1x_2}\sum_a e_a^2\left[q_a(x_1)\bar{q}_a(x_2)+\bar{q}_a(x_1)q_a(x_2)\right]$$

Angular Distribution in the "Naïve" Drell-Yan

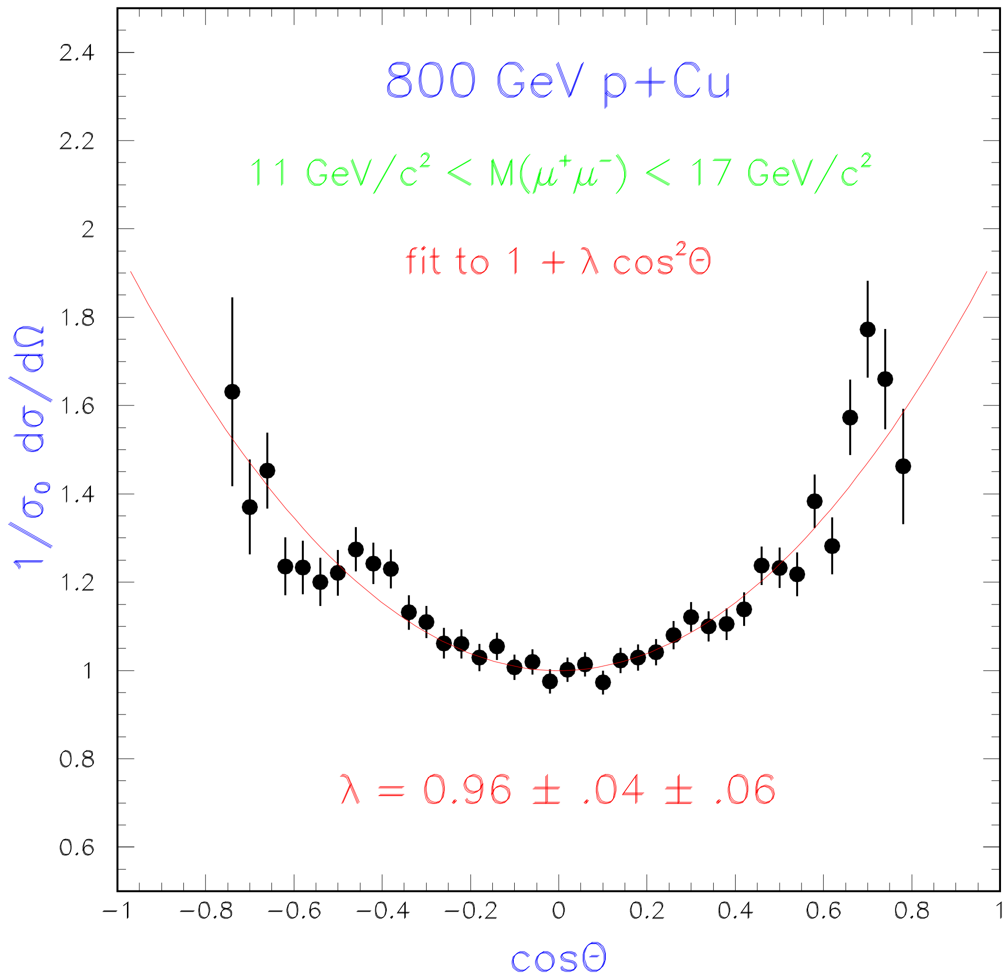
VOLUME 25, NUMBER 5 PHYSICAL REVIEW LETTERS 3 AUGUST 1970

(3) The virtual photon will be predominantly transversely polarized if it is formed by annihilation of spin-$\frac{1}{2}$ parton-antiparton pairs. This means a distribution in the di-muon rest system varying as $(1+\cos^2\theta)$ rather than $\sin^2\theta$ as found in Sakurai's[10] vector-dominance model, where θ is the angle of the muon with respect to the timelike photon momentum. The model used in Fig.

Drell-Yan angular distribution

Lepton Angular Distribution of "naïve" Drell-Yan:

$$\frac{d\sigma}{d\Omega} = \sigma_0 (1 + \lambda \cos^2 \theta); \quad \lambda = 1$$

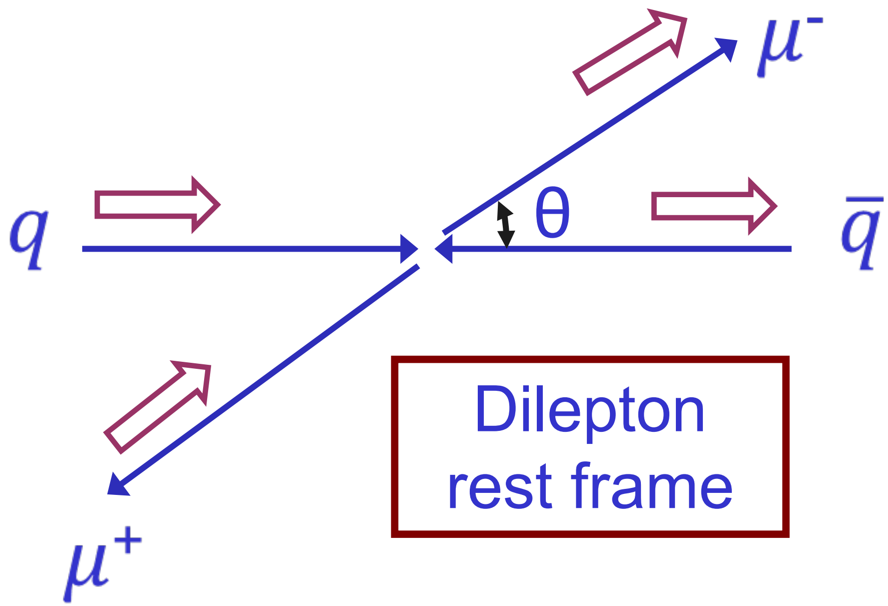


Data from Fermilab E772

(Ann. Rev. Nucl. Part. Sci. 49 (1999) 217-253)

Why is the lepton angular distribution $1 + \cos^2\theta$?

Helicity conservation and parity



$RL \rightarrow RL$

$$d\sigma \sim (1+\cos\theta)^2$$

$RL \rightarrow LR$

$$d\sigma \sim (1-\cos\theta)^2$$

$LR \rightarrow LR$

$$d\sigma \sim (1+\cos\theta)^2$$

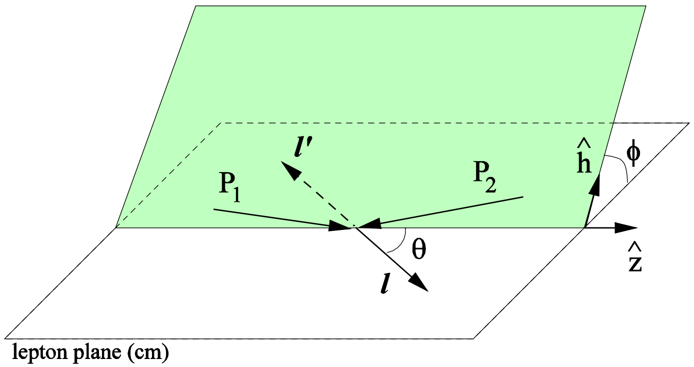
$LR \rightarrow RL$

$$d\sigma \sim (1-\cos\theta)^2$$

Adding all four helicity configurations:

$$d\sigma \sim 1+\cos^2\theta$$

Drell-Yan lepton angular distributions for $p_T > 0$



Θ and Φ are the decay polar and azimuthal angles of the μ^- in the dilepton rest-frame

Collins-Soper frame

A general expression for Drell-Yan decay angular distributions:

$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right)=\left[\frac{3}{4\pi}\right]\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right]$$

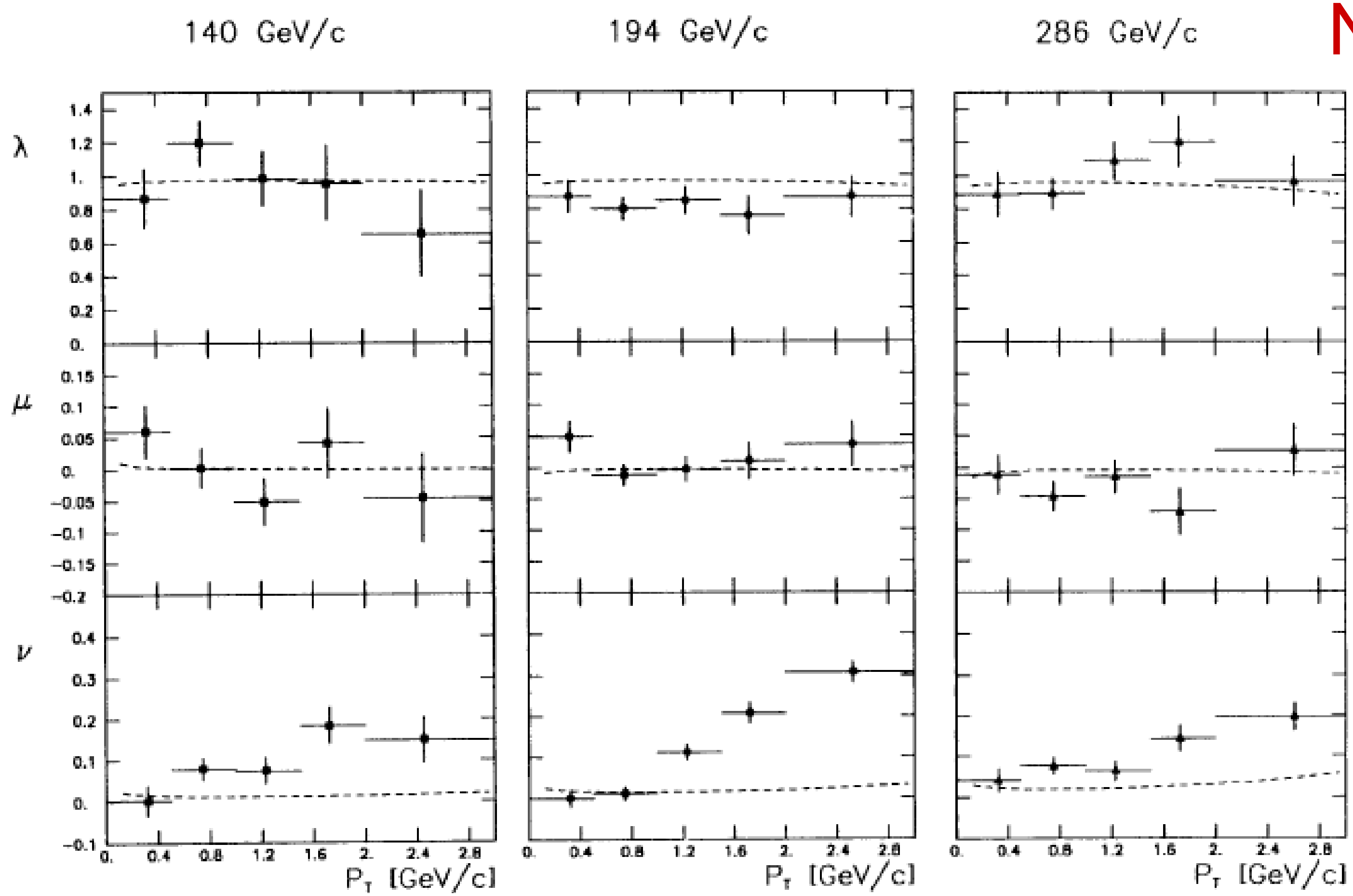
Lam-Tung relation: $1-\lambda=2\nu$

- Reflect the spin-1/2 nature of quarks (analog of the Callan-Gross relation in DIS)
- Insensitive to QCD - corrections

Decay angular distributions in pion-induced Drell-Yan

$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right)=\left[\frac{3}{4\pi}\right]\left[1+\lambda\cos^2\theta+\mu\sin2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos2\phi\right]$$

NA10 $\pi^- + W$



Z. Phys.

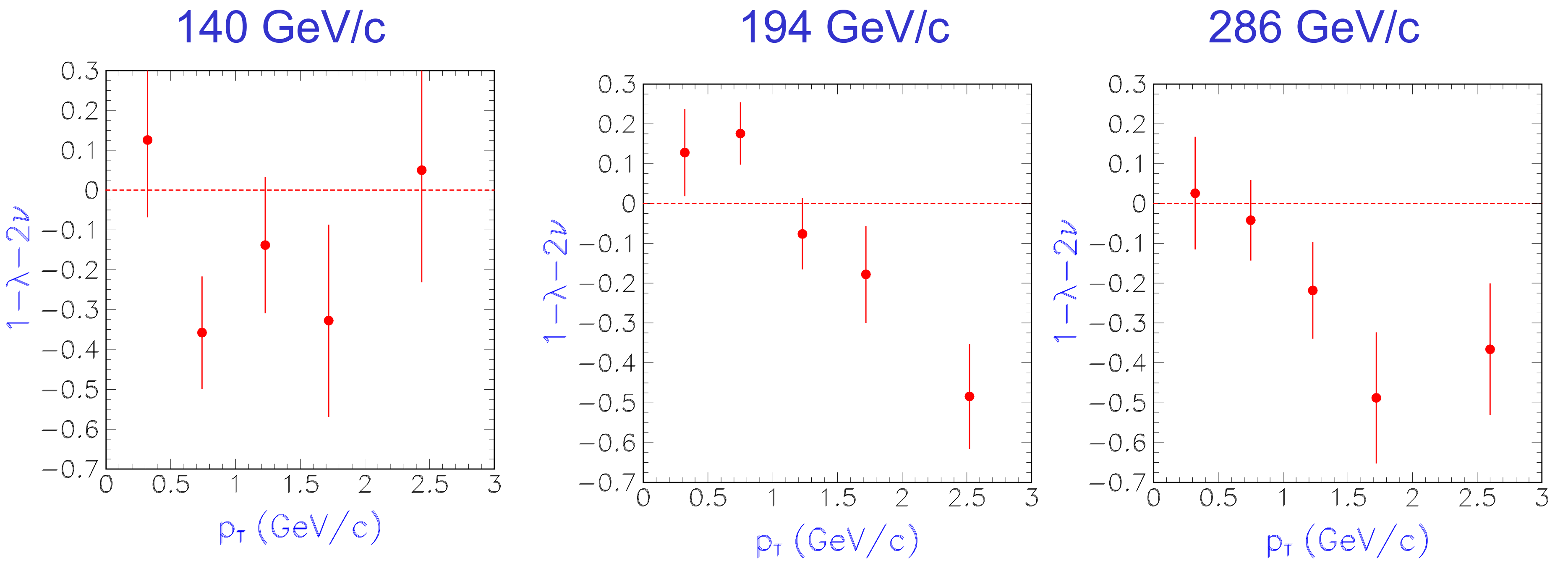
37 (1988) 545

Dashed curves are from pQCD calculations

$\nu \neq 0$ and ν increases with p_T

Decay angular distributions in pion-induced Drell-Yan

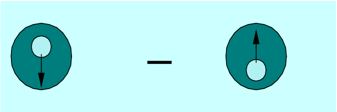
Is the Lam-Tung relation (*1-λ-2ν=0*) violated?



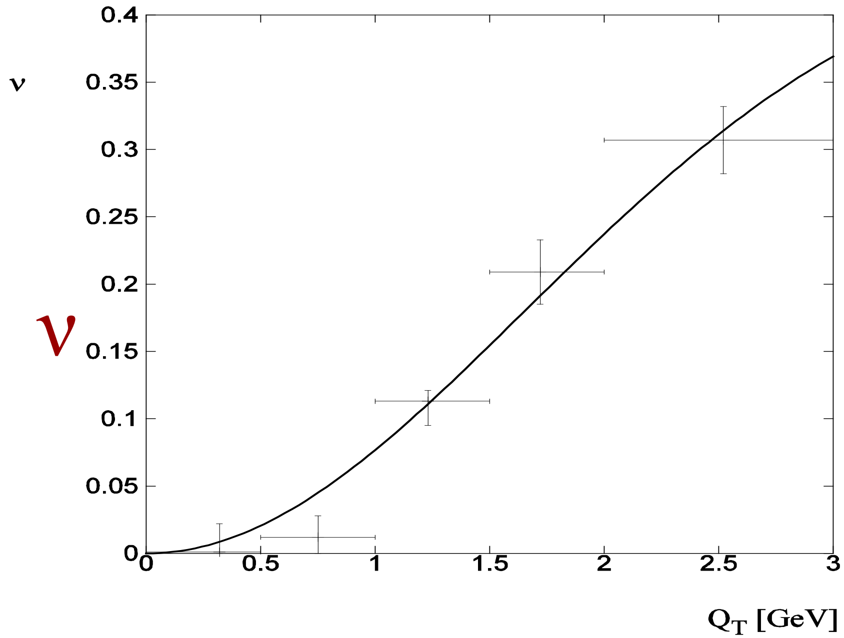
Data from NA10 (Z. Phys. 37 (1988) 545)

Violation of the Lam-Tung relation in NA10 and E615 suggests interesting new origins (Brandenburg, Nachtmann, Mirkes, Brodsky, Khoze, Müller, Eskolar, Hoyer,Väntinnen, Vogt, etc.)

Boer-Mulders function $h_1^{\perp}$



- Boer pointed out that the cos2φ dependence can be caused by the presence of the Boer-Mulders function.
- $h_1^{\perp}$ can lead to an azimuthal dependence with $\nu \propto \left(\frac{h_1^{\perp}}{f_1}\right)\left(\frac{\bar{h}_1^{\perp}}{\bar{f}_1}\right)$



Boer, PRD 60 (1999) 014012

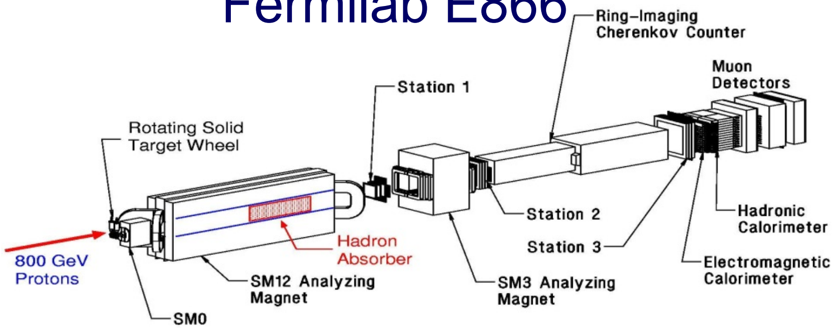
The violation of the Lam-Tung relation is due to the presence of the Boer-Mulders TMD function

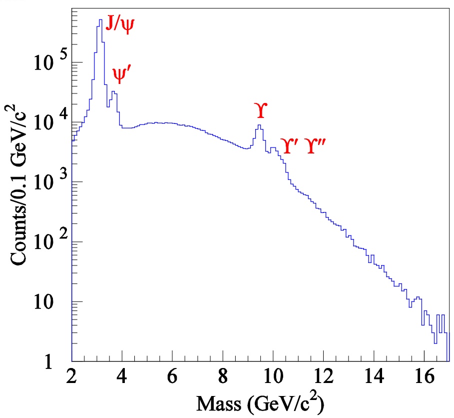
The puzzle is resolved. It also leads to the first extraction of the Boer-Mulders function

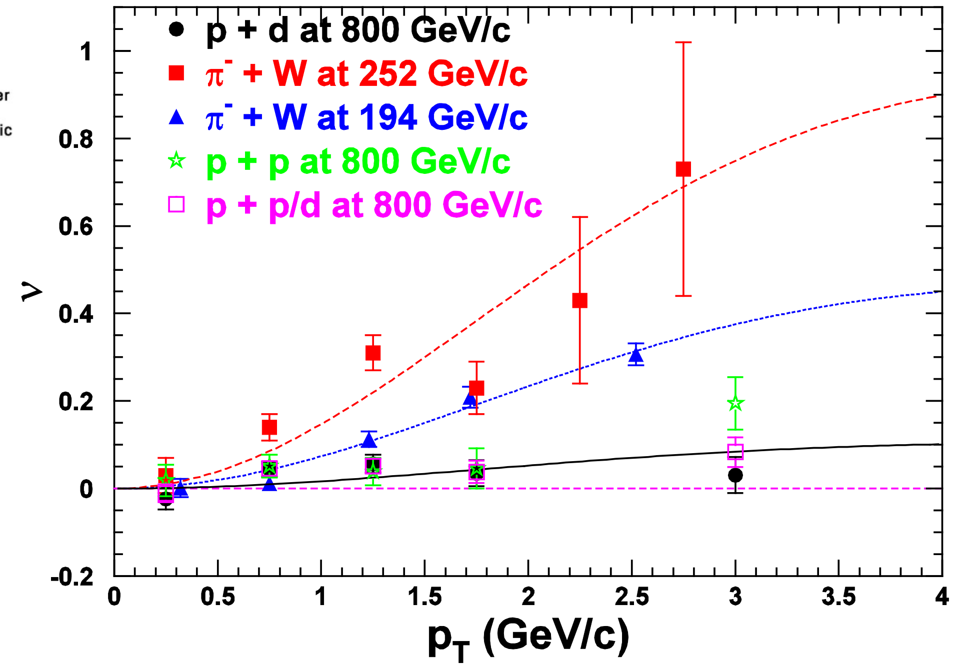
Azimuthal cos2Φ Distribution in p+d Drell-Yan

Lingyan Zhu, JCP et al., PRL 99 (2007) 082301; PRL 102 (2009) 182001

Fermilab E866







With Boer-Mulders function $h_1^{\perp}$:

$\nu(\pi^- W \rightarrow \mu^+\mu^- X) \sim$ [valence $h_1^{\perp}(\pi)$] * [valence $h_1^{\perp}(p)$]

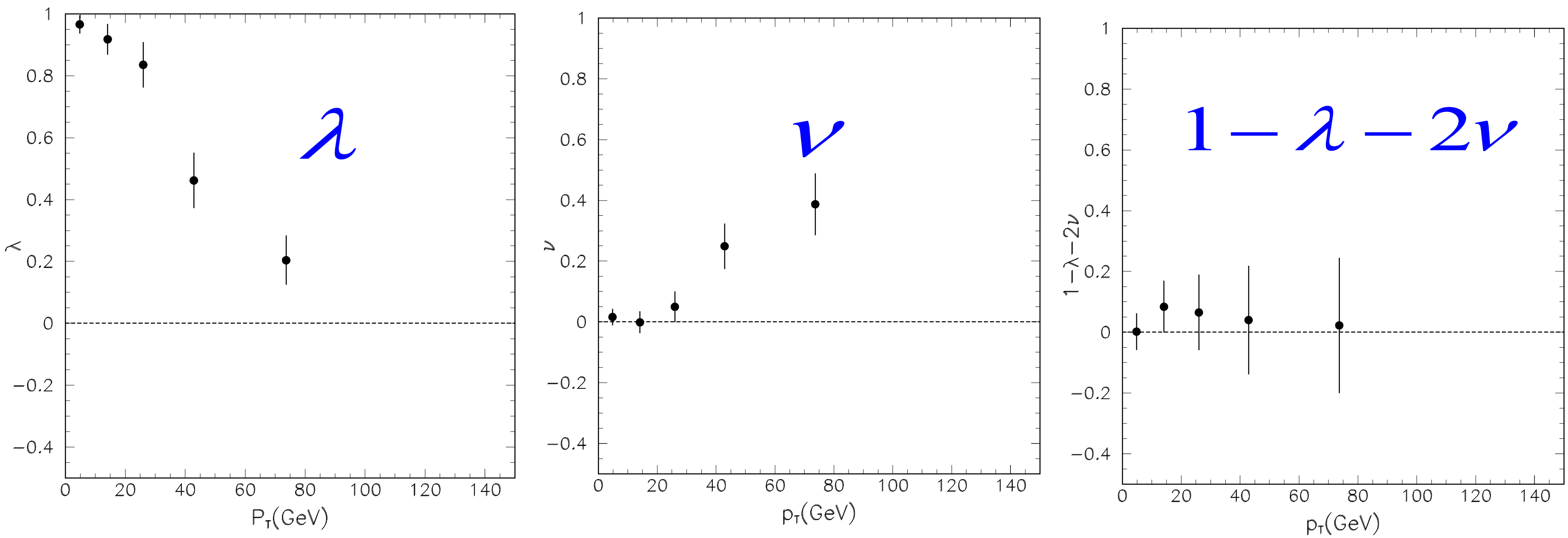
$\nu(pd \rightarrow \mu^+\mu^- X) \sim$ [valence $h_1^{\perp}(p)$] * [sea $h_1^{\perp}(p)$]

Sea-quark BM function is much smaller than valence BM function

Angular distribution data from CDF Z-production

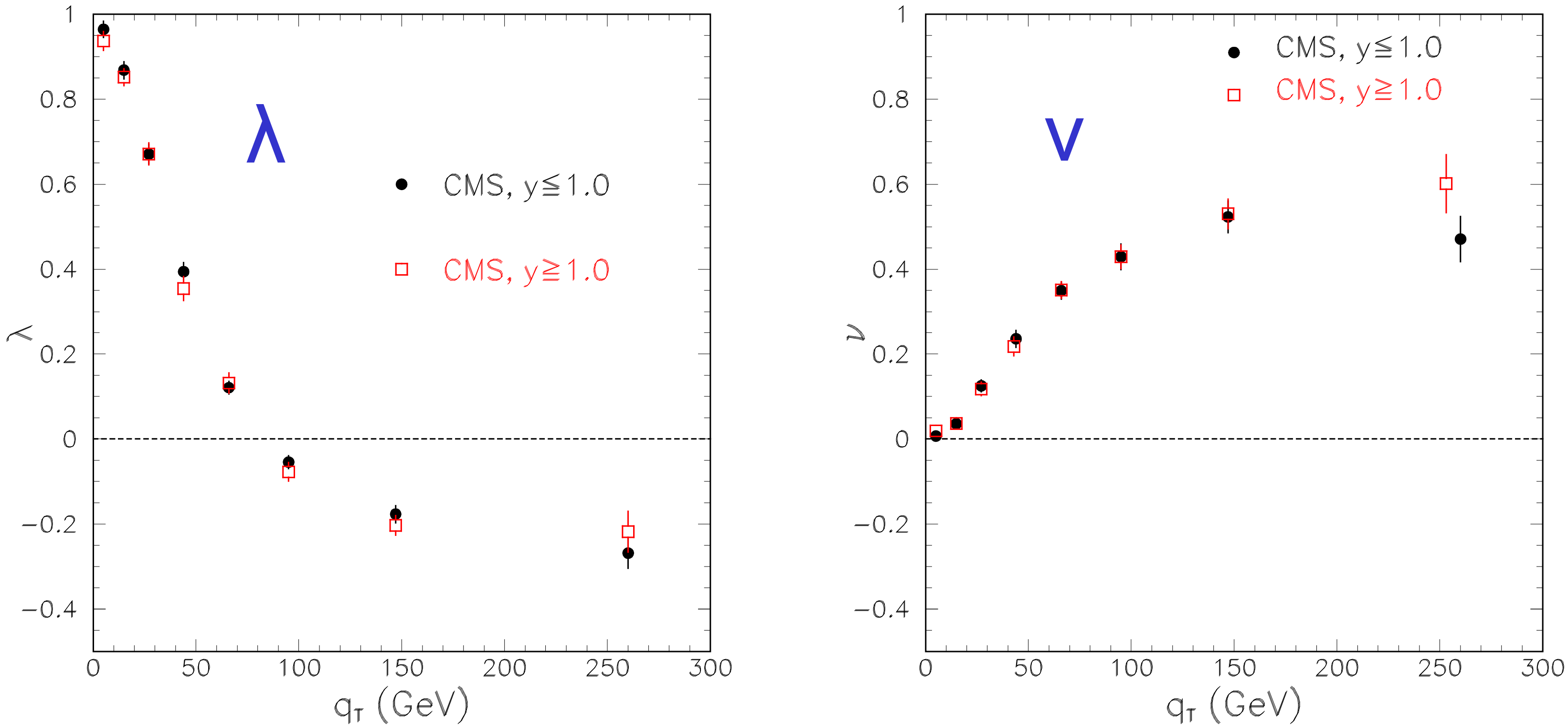
$$p + \overline{p} \rightarrow e^{+} + e^{-} + X \;\; \text{at} \sqrt{s} = 1.96\,\text{TeV}$$

arXiv:1103.5699 (PRL 106 (2011) 241801)



- Strong p_T (q_T) dependence of λ and ν
- Lam-Tung relation ($1-\lambda = 2\nu$) is satisfied within experimental uncertainties (TMD is not expected to be important at large p_T)

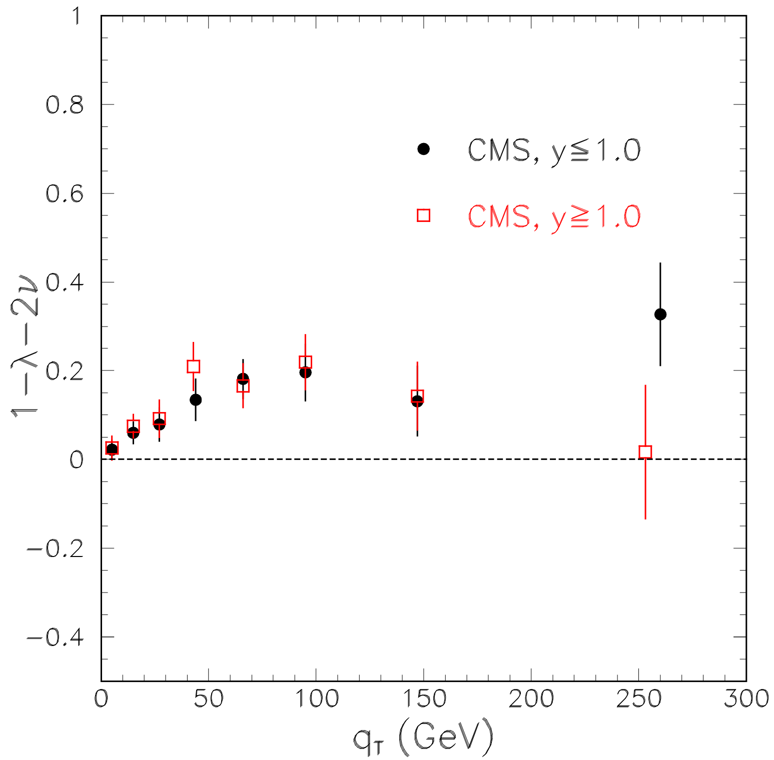
CMS (ATLAS) data for *Z*-boson production in *p*+*p* collision at 8 TeV



(arXiv:1504.03512, PL B 750 (2015) 154)

- Striking q_T (p_T) dependencies for λ and ν were observed at two rapidity regions
- Is Lam-Tung relation violated?

Recent data from CMS for *Z*-boson production in *p*+*p* collision at 8 TeV



- Yes, the Lam-Tung relation is violated ($1-\lambda > 2\nu$)!
- Can one understand the origin of the violation of the Lam-Tung relation (It cannot be due to the Boer-Mulders function)?

Interpretation of the CMS Z-production results

$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta) + A_1 \sin 2\theta \cos\phi$$
$$+ \frac{A_2}{2}\sin^2\theta \cos 2\phi + A_3 \sin\theta\cos\phi + A_4 \cos\theta$$
$$+ A_5 \sin^2\theta \sin 2\phi + A_6 \sin 2\theta \sin\phi + A_7 \sin\theta \sin\phi$$

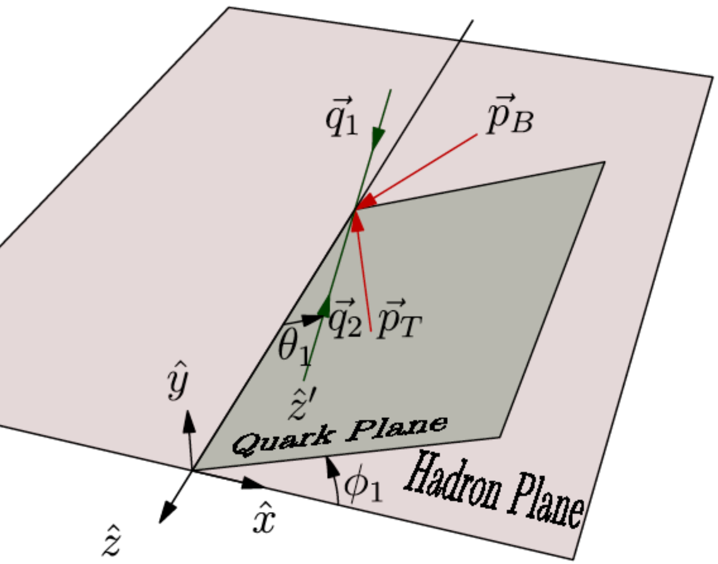
Questions:

- How is the above expression derived?
- Can one express $A_0 - A_7$ in terms of some quantities?
- Can one understand the q_T dependence of $A_0, A_1, A_2,$ etc?
- Can one understand the origin of the violation of Lam-Tung relation?

$\lambda = \frac{2-3A_0}{2+A_0}$; $\nu = \frac{2A_2}{2+A_0}$; L-T relation, $1-\lambda = 2\nu$, becomes $A_0 = A_2$

How is the angular distribution expression derived?

Define three planes in the Collins-Soper frame



1) Hadron Plane

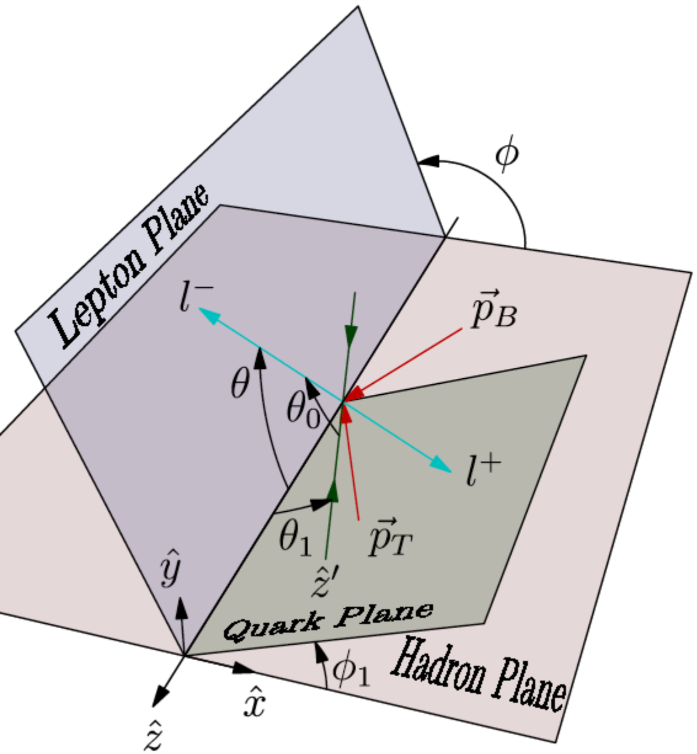
- Contains the beam $\vec{P}_B$ and target $\vec{P}_T$ momenta
- Angle β satisfies the relation $\tan\beta = q_T / Q$

2) Quark Plane

- q and $\overline{q}$ have head-on collision along the $\hat{z}'$ axis
- $\hat{z}'$ and $\hat{z}$ axes form the quark plane
- $\hat{z}'$ axis has angles θ_1 and ϕ_1 in the C-S frame

How is the angular distribution expression derived?

Define three planes in the Collins-Soper frame



1) Hadron Plane

- Contains the beam $\vec{P}_B$ and target $\vec{P}_T$ momenta
- Angle β satisfies the relation $\tan\beta = q_T / Q$

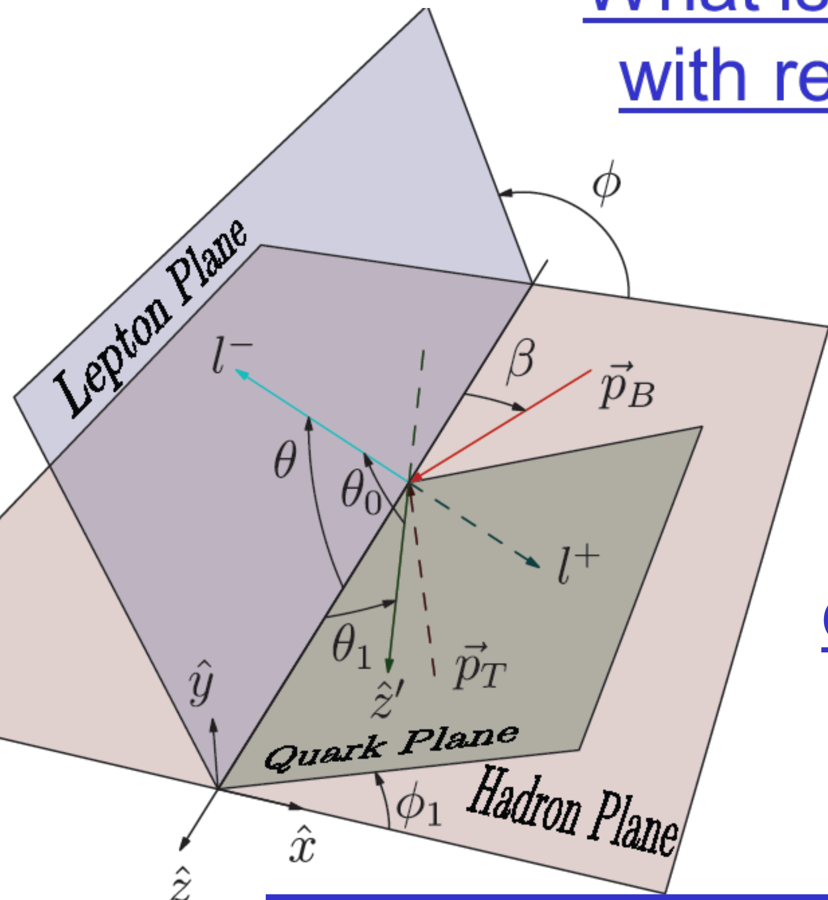
2) Quark Plane

- q and $\overline{q}$ have head-on collision along the $\hat{z}'$ axis
- $\hat{z}'$ axis has angles θ_1 and ϕ_1 in the C-S frame

3) Lepton Plane

- l^- and l^+ are emitted back-to-back with equal $|\vec{P}\,|$
- l^- and $\hat{z}$ form the lepton plane
- l^- is emitted at angle θ and ϕ in the C-S frame

How is the angular distribution expression derived?



What is the lepton angular distribution with respect to the $\hat{z}'$ (natural) axis?

$$\frac{d\sigma}{d\Omega} \propto 1 + a\cos\theta_0 + \cos^2\theta_0$$

Azimuthally symmetric !

How to express the angular distribution in terms of θ and ϕ?

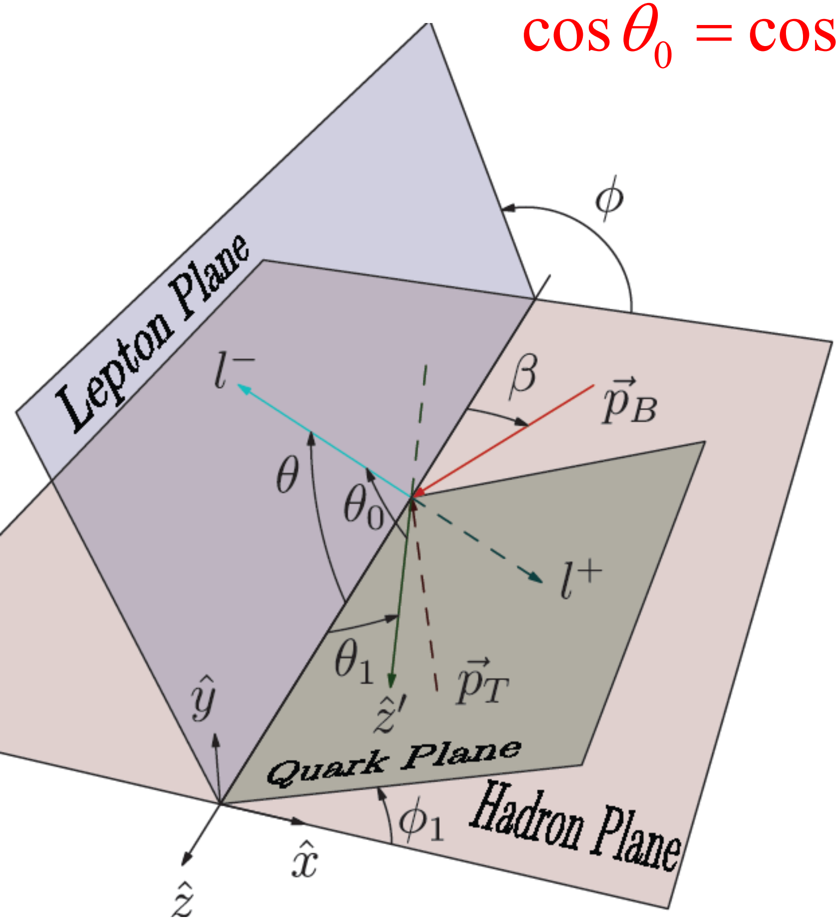
Use the following relation (addition theorem):

$$\cos\theta_0 = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos(\phi - \phi_1)$$

How is the angular distribution expression derived?

$$\frac{d\sigma}{d\Omega} \propto 1 + a\cos\theta_0 + \cos^2\theta_0$$

$$\cos\theta_0 = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos(\phi - \phi_1)$$



$$\begin{aligned}
\frac{d\sigma}{d\Omega} \propto\ & (1+\cos^2\theta) + \frac{\sin^2\theta_1}{2}(1-3\cos^2\theta) \\
& + (\frac{1}{2}\sin 2\theta_1\cos\phi_1)\sin 2\theta\cos\phi \\
& + (\frac{1}{2}\sin^2\theta_1\cos 2\phi_1)\sin^2\theta\cos 2\phi \\
& + (a\sin\theta_1\cos\phi_1)\sin\theta\cos\phi + (a\cos\theta_1)\cos\theta \\
& + (\frac{1}{2}\sin^2\theta_1\sin 2\phi_1)\sin^2\theta\sin 2\phi \\
& + (\frac{1}{2}\sin 2\theta_1\sin\phi_1)\sin 2\theta\sin\phi \\
& + (a\sin\theta_1\sin\phi_1)\sin\theta\sin\phi .
\end{aligned}$$

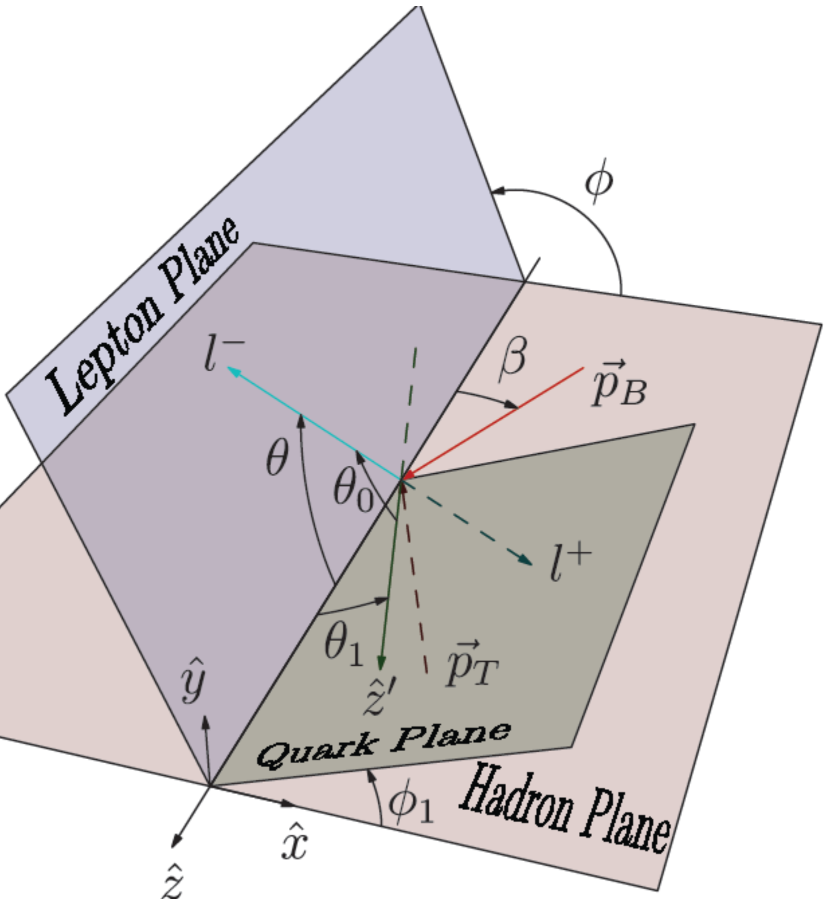
All eight angular distribution terms are obtained!

$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{\sin^2\theta_1}{2}(1-3\cos^2\theta)$$
$$+ (\frac{1}{2}\sin 2\theta_1 \cos\phi_1)\sin 2\theta\cos\phi$$
$$+ (\frac{1}{2}\sin^2\theta_1 \cos 2\phi_1)\sin^2\theta\cos 2\phi$$
$$+ (a\sin\theta_1\cos\phi_1)\sin\theta\cos\phi + (a\cos\theta_1)\cos\theta$$
$$+ (\frac{1}{2}\sin^2\theta_1 \sin 2\phi_1)\sin^2\theta\sin 2\phi$$
$$+ (\frac{1}{2}\sin 2\theta_1 \sin\phi_1)\sin 2\theta\sin\phi$$
$$+ (a\sin\theta_1\sin\phi_1)\sin\theta\sin\phi.$$

$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta)$$
$$+ A_1\sin 2\theta\cos\phi$$
$$+ \frac{A_2}{2}\sin^2\theta\cos 2\phi$$
$$+ A_3\sin\theta\cos\phi + A_4\cos\theta$$
$$+ A_5\sin^2\theta\sin 2\phi$$
$$+ A_6\sin 2\theta\sin\phi$$
$$+ A_7\sin\theta\sin\phi$$

$A_0 - A_7$ are entirely described by θ_1, ϕ_1 and a

Angular distribution coefficients $A_0 - A_7$



$$A_0 = \langle \sin^2\theta_1 \rangle$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle$$

$$A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a\langle \sin\theta_1 \cos\phi_1 \rangle$$

$$A_4 = a\langle \cos\theta_1 \rangle$$

$$A_5 = \frac{1}{2}\langle \sin^2\theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2}\langle \sin 2\theta_1 \sin\phi_1 \rangle$$

$$A_7 = a\langle \sin\theta_1 \sin\phi_1 \rangle$$

Some implications of the angular distribution coefficients $A_0 - A_7$

$$A_0 = \langle \sin^2\theta_1 \rangle$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle$$

$$A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a\langle \sin\theta_1 \cos\phi_1 \rangle$$

$$A_4 = a\langle \cos\theta_1 \rangle$$

$$A_5 = \frac{1}{2}\langle \sin^2\theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2}\langle \sin 2\theta_1 \sin\phi_1 \rangle$$

$$A_7 = a\langle \sin\theta_1 \sin\phi_1 \rangle$$

- $A_0 \geq A_2$ (or $1 - \lambda - 2\nu \geq 0$)
- Lam-Tung relation ($A_0 = A_2$) is satisfied when $\phi_1 = 0$
- Forward-backward asymmetry, a, is reduced by a factor of $\langle \cos\theta_1 \rangle$ for A_4
- Some equality and inequality relations among $A_0 - A_7$ can be obatined

Some implications of the angular distribution coefficients $A_0 - A_7$

$$A_0 = \left\langle \sin^2 \theta_1 \right\rangle$$

$$A_1 = \frac{1}{2} \left\langle \sin 2\theta_1 \cos \phi_1 \right\rangle$$

$$A_2 = \left\langle \sin^2 \theta_1 \cos 2\phi_1 \right\rangle$$

$$A_3 = a \left\langle \sin \theta_1 \cos \phi_1 \right\rangle$$

$$A_4 = a \left\langle \cos \theta_1 \right\rangle$$

$$A_5 = \frac{1}{2} \left\langle \sin^2 \theta_1 \sin 2\phi_1 \right\rangle$$

$$A_6 = \frac{1}{2} \left\langle \sin 2\theta_1 \sin \phi_1 \right\rangle$$

$$A_7 = a \left\langle \sin \theta_1 \sin \phi_1 \right\rangle$$

Some bounds on the coefficients can be obtained

$$0 < A_0 < 1$$

$$-1/2 < A_1 < 1/2$$

$$-1 < A_2 < 1$$

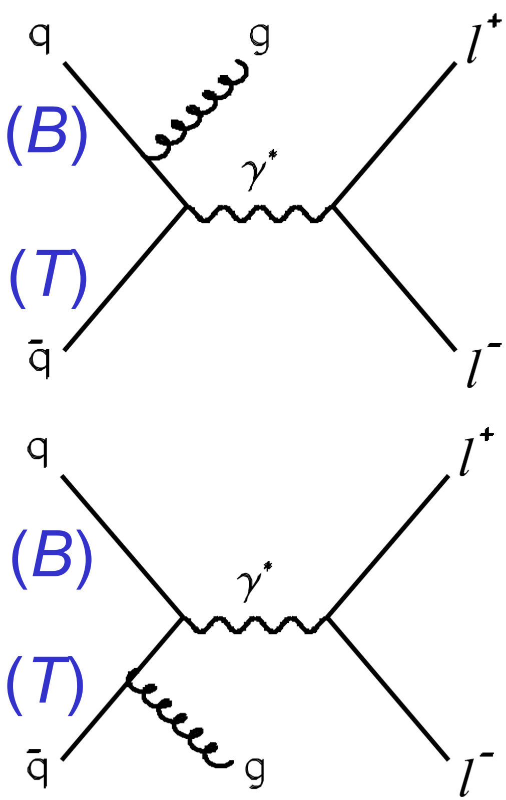
$$-a < A_3 < a$$

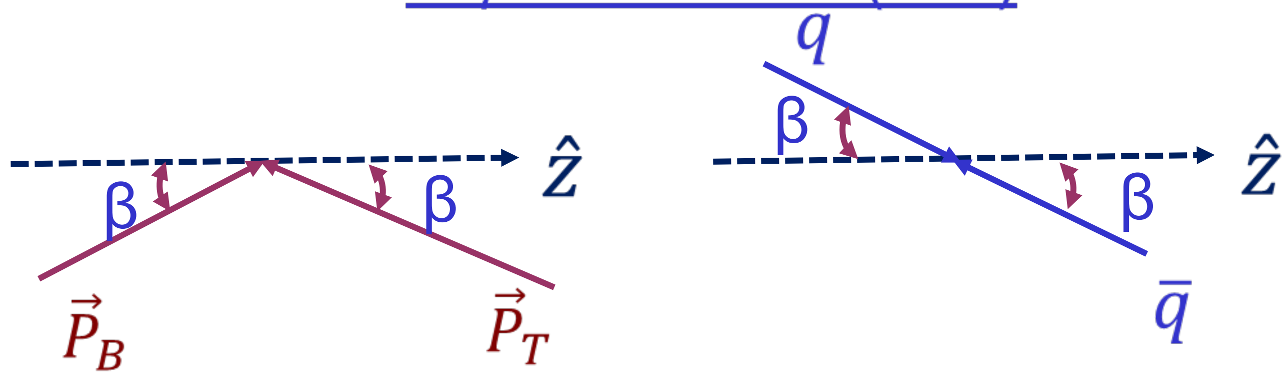
$$-a < A_4 < a$$

What are the values of θ_1 and ϕ_1 at order α_s?

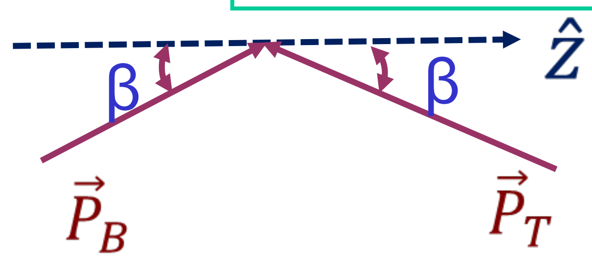
1) $q\bar{q} \to \gamma^*(Z^0)g$

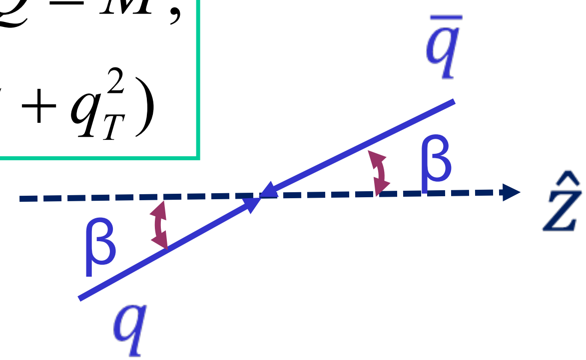
In γ^* rest frame (C-S)





$$\tan\beta = q_T / Q; \quad Q = M;$$
$$\sin^2\beta = q_T^2 / (Q^2 + q_T^2)$$





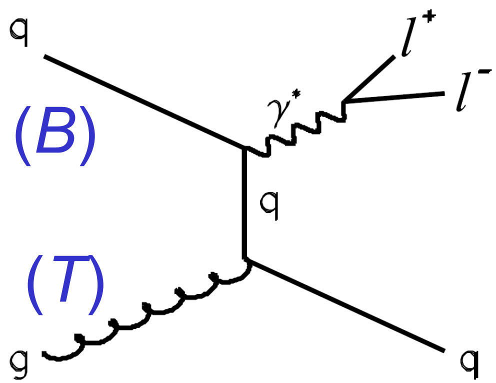
$\theta_1 = \beta$ and $\phi_1 = 0$; $A_0 = A_2 = \sin^2\beta$

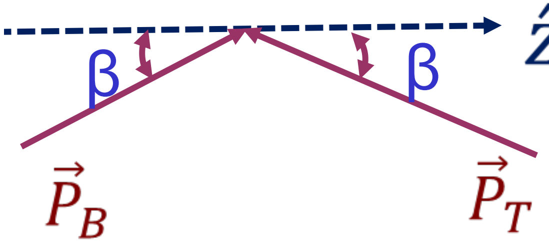
$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2}; \qquad \nu = \frac{2A_2}{2 + A_0} = \frac{2q_T^2}{2Q^2 + 3q_T^2}$$

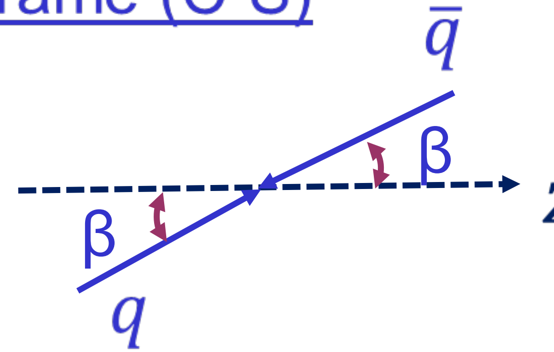
What are the values of θ_1 and ϕ_1 at order α_s?

2) $qg \rightarrow \gamma^*(Z^0)q$

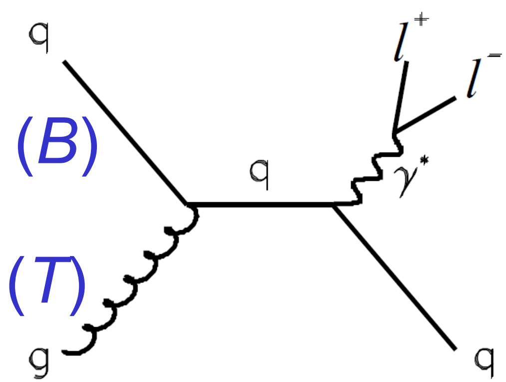
In γ^* rest frame (C-S)

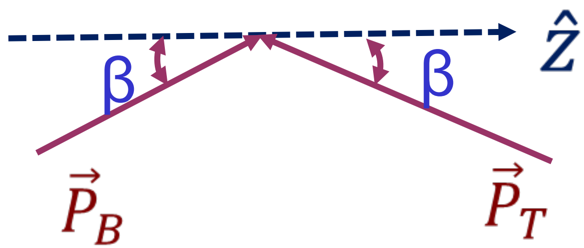


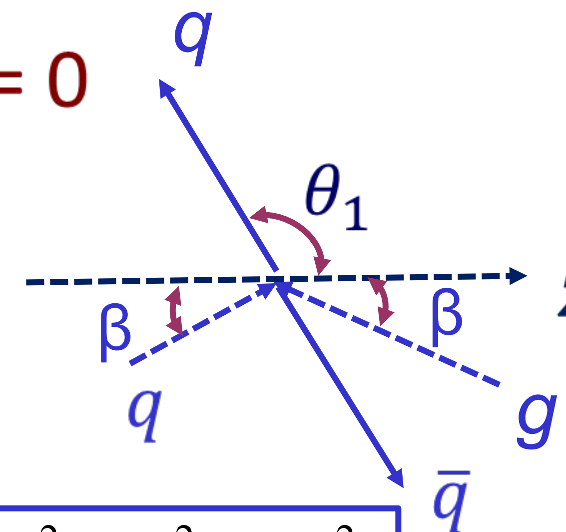




$\theta_1 = \beta$ and $\phi_1 = 0$





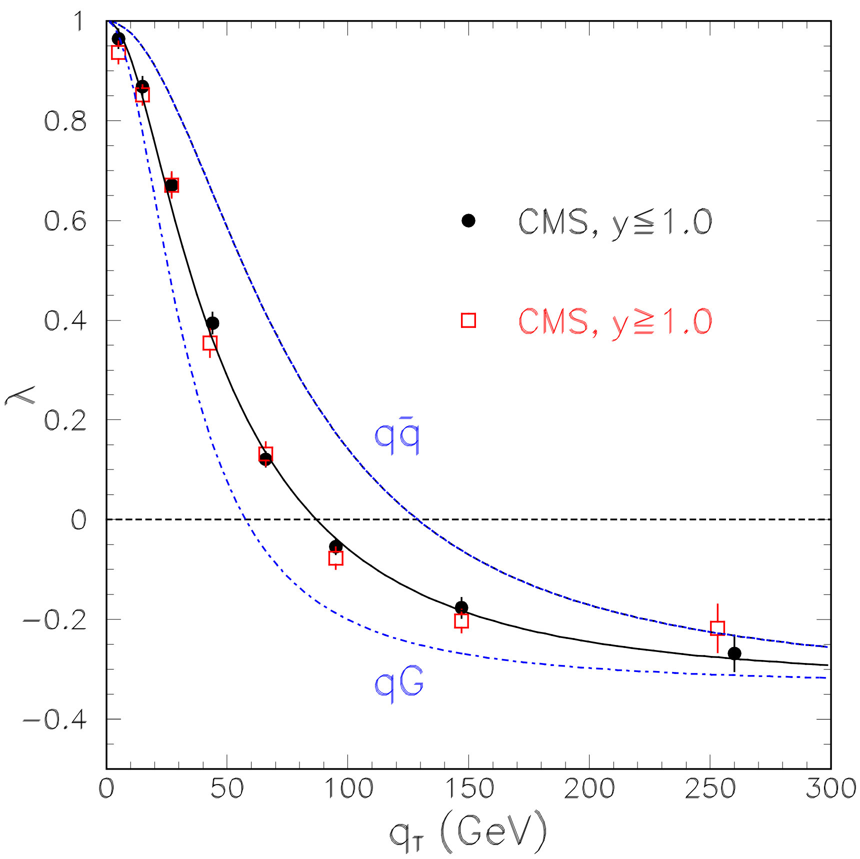


$\theta_1 > \beta$ and $\phi_1 = 0$; $A_0 = A_2 \approx 5q_T^2/(Q^2 + 5q_T^2)$

$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2}; \qquad \nu = \frac{2A_2}{2 + A_0} = \frac{10q_T^2}{2Q^2 + 15q_T^2}$$

Compare with CMS data on λ

(Z production in p+p collision at 8 TeV)



$$\lambda = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2} \quad \text{for} \quad q\bar{q} \to Zg$$

$$\lambda = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2} \quad \text{for} \quad qG \to Zq$$

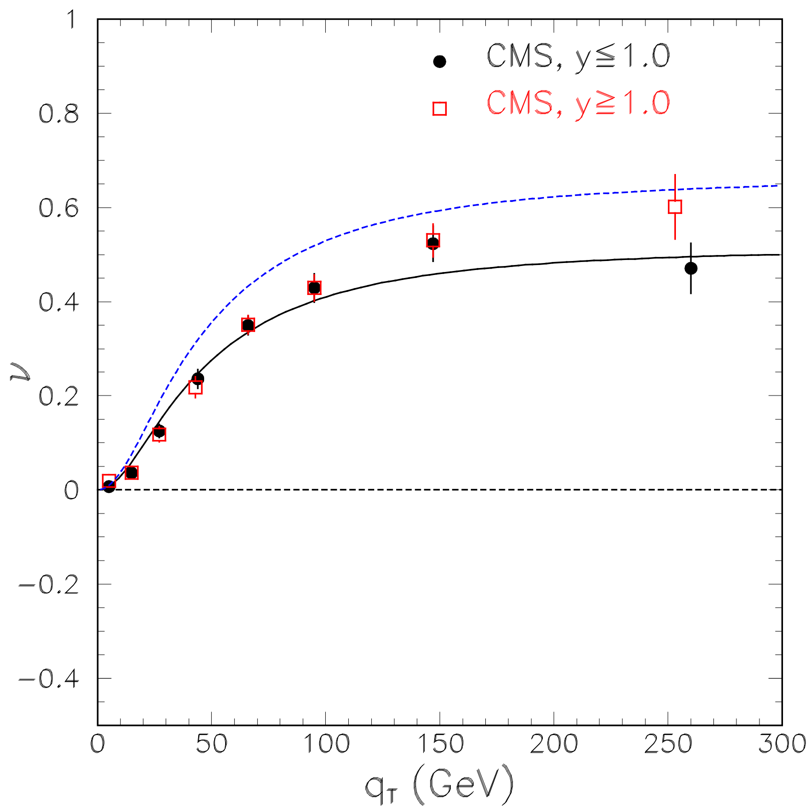
For both processes
$\lambda = 1$ at $q_T = 0$ ($\theta_1 = 0^o$)
$\lambda = -1/3$ at $q_T = \infty$ ($\theta_1 = 90^o$)

Data can be well described with a mixture of 58.5% qG and 41.5% $q\bar{q}$ processes

The scaling variable is q_T / Q
Q is the mass of dilepton

Compare with CMS data on ν

(*Z* production in *p*+*p* collision at 8 TeV)



$$\nu = \frac{2A_2}{2+A_0};\ A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle;\ A_2 = \langle \sin^2\theta_1 \rangle$$

when $\phi_1 = 0$, then

$$\nu = \frac{2q_T^2}{2Q^2 + 3q_T^2} \quad \text{for} \quad q\bar{q} \to Zg$$

$$\nu = \frac{10q_T^2}{2Q^2 + 15q_T^2} \quad \text{for} \quad qG \to Zq$$

Dashed curve corresponds to a mixture of 58.5% qG and 41.5% $q\bar{q}$ processes (and $\phi_1 = 0$)

Solid curve corresponds to

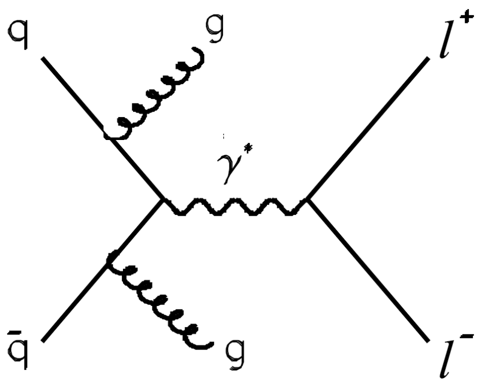
$$\langle \sin^2\theta_1 \cos 2\varphi_1 \rangle / \langle \sin^2\theta_1 \rangle = 0.77 \quad (\phi_1 \neq 0)$$

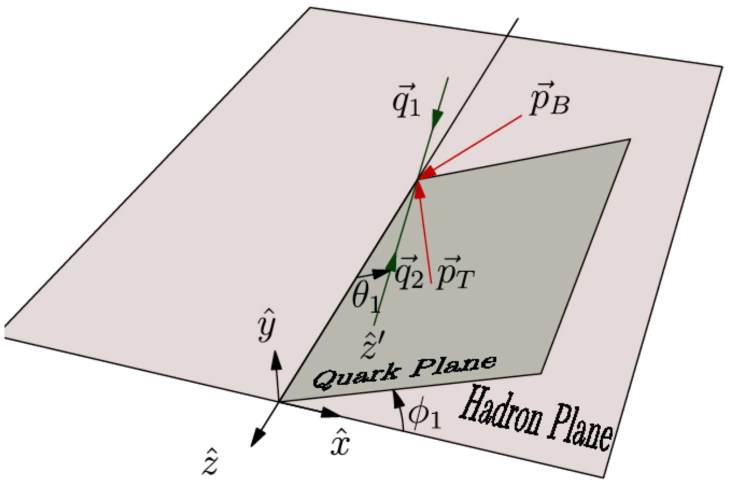
$\phi_1 \neq 0$ implies that the q - $\bar{q}$ axis is not on the hadron plane

What can cause $\phi_1 \neq 0$?

Origins of the non-coplanarity

1) Processes at order α_s^2 or higher

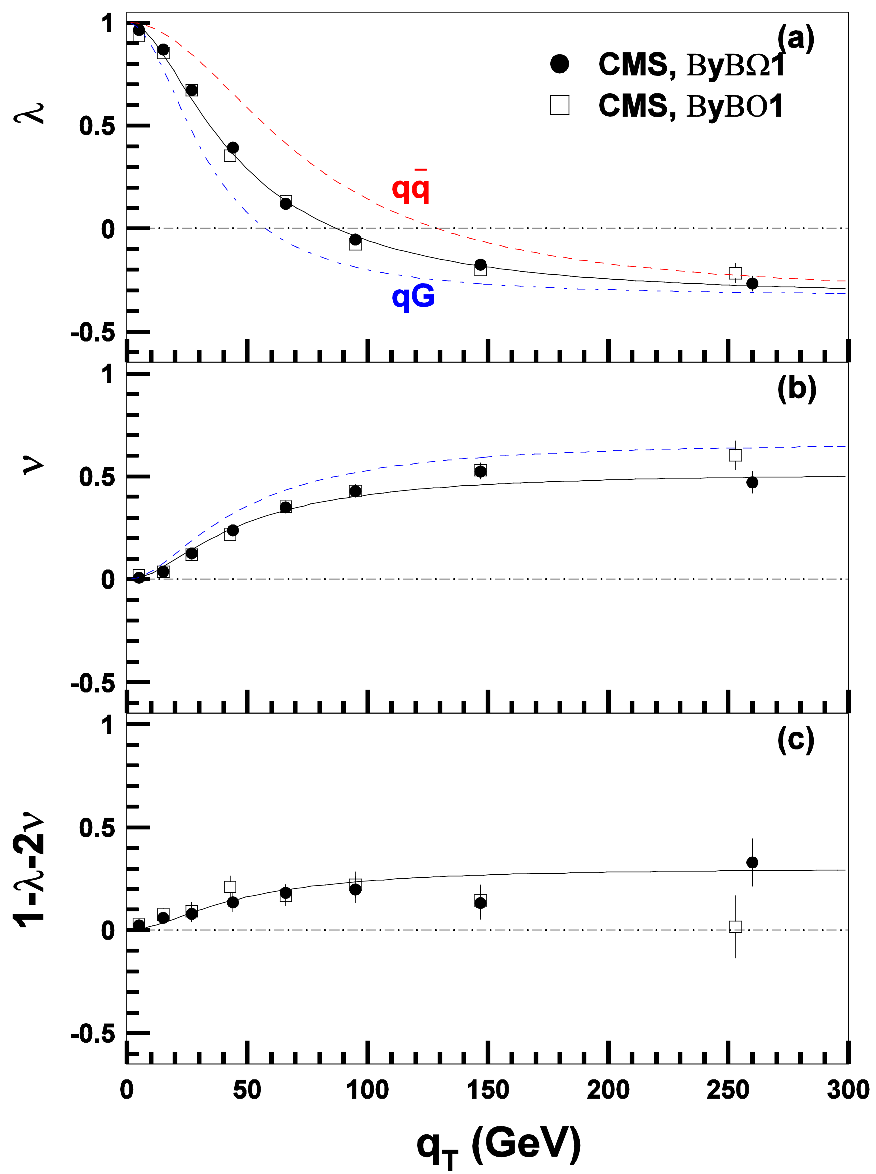




2) Intrinsic k_T from interacting partons

(Boer-Mulders functions in the beam and target hadrons)

Compare with CMS data on Lam-Tung relation

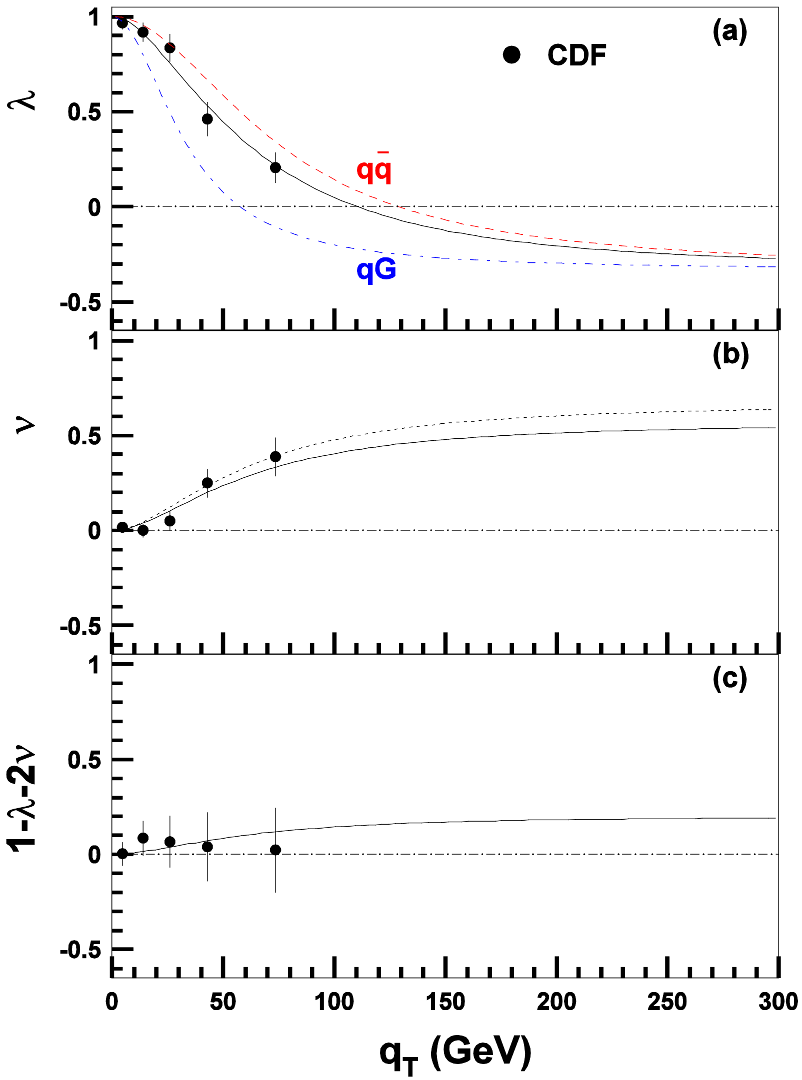


Solid curves correspond to a mixture of 58.5% qG and 41.5% $q\bar{q}$ processes, and $\langle \sin^2\theta_1 \cos 2\phi_1 \rangle / \langle \sin^2\theta_1 \rangle = 0.77$

Violation of Lam-Tung relation is well described with a finite non-coplanarity angle

Compare with CDF data

(Z production in $p+\bar{p}$ collision at 1.96 TeV)



Solid curves correspond to a mixture of 27.5% qG and 72.5% $q\bar{q}$ processes, and

$$\langle \sin^2\theta_1 \cos 2\varphi_1 \rangle / \langle \sin^2\theta_1 \rangle = 0.85$$

$$(\phi_1 \neq 0)$$

Violation of Lam-Tung relation is not ruled out

Geometric interpretations on the rotational invariance of some quantities

On the Rotational Invariance and Non-Invariance of Lepton Angular Distributions in Drell-Yan and Quarkonium Production

Jen-Chieh Peng[a], Daniël Boer[b], Wen-Chen Chang[c], Randall Evan McClellan[a,d], Oleg Teryaev[e]

(Phys Lett B789 (2019) 352)

Quantities invariant under rotations along the y-axis (Faccioli et al.)

$$\mathcal{F} = \frac{1+\lambda+\nu}{3+\lambda}$$

$$\mathcal{F} = \frac{1+\lambda_0 - 2\lambda_0 \sin^2\theta_1 \sin^2\phi_1}{3+\lambda_0} = \frac{1+\lambda_0 - 2\lambda_0 y_1^2}{3+\lambda_0}$$

$$\tilde{\lambda} = \frac{2\lambda+3\nu}{2-\nu}$$

$$\tilde{\lambda} = \frac{\lambda_0 - 3\lambda_0 \sin^2\theta_1 \sin^2\phi_1}{1+\lambda_0 \sin^2\theta_1 \sin^2\phi_1} = \frac{\lambda_0 - 3\lambda_0 y_1^2}{1+\lambda_0 y_1^2}$$

$$\tilde{\lambda}' = \frac{(\lambda-\nu/2)^2 + 4\mu^2}{(3+\lambda)^2}$$

$$\tilde{\lambda}' = \frac{\lambda_0^2 (z_1^2 + x_1^2)^2}{(3+\lambda_0)^2} = \frac{\lambda_0^2 (1-y_1^2)^2}{(3+\lambda_0)^2}$$

$y_1 = \sin\theta_1 \sin\phi_1$ is the component of $\hat{z}'$ along the y-axis in the dilepton rest frame; invariant under rotation along y-axis

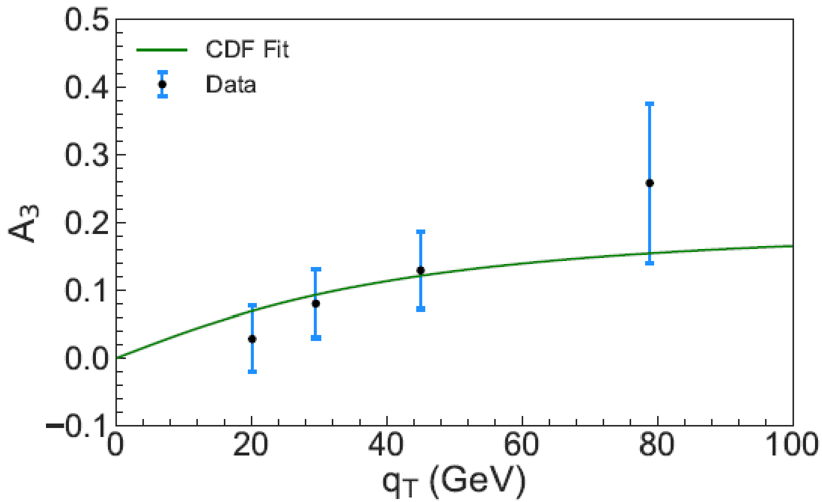
Other implications

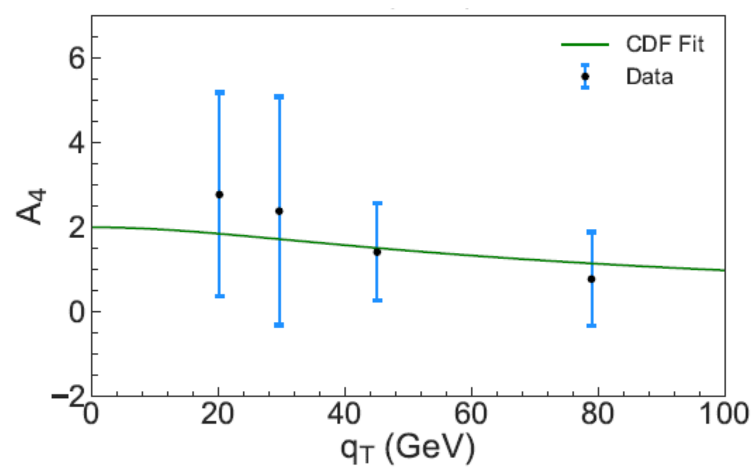
Extend this study to W-boson production at CDF

PHYSICAL REVIEW D **103,** 034011 (2021)

Lepton angular distribution of *W* boson productions

Yang Lyu,[1,2] Wen-Chen Chang,[3] Randall Evan McClellan,[1,4] Jen-Chieh Peng,[1] and Oleg Teryaev[5]



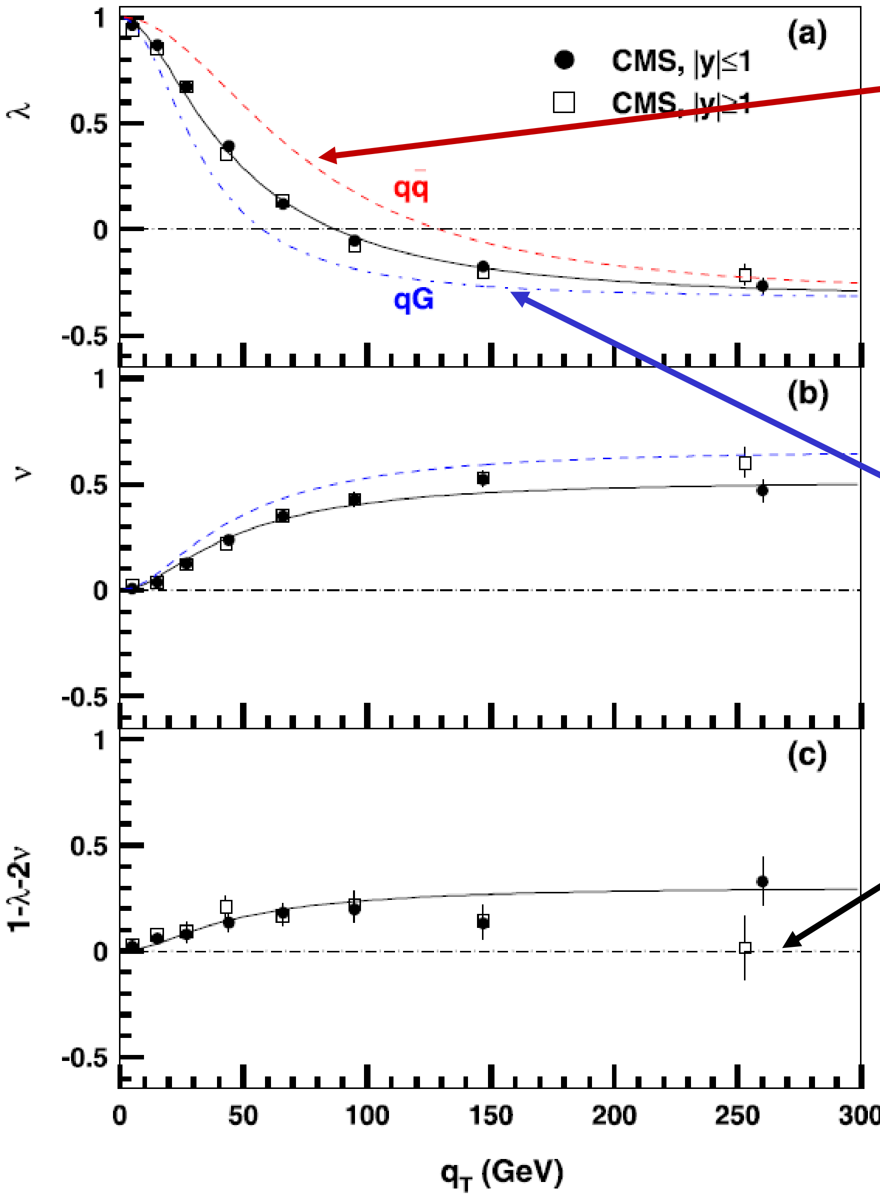


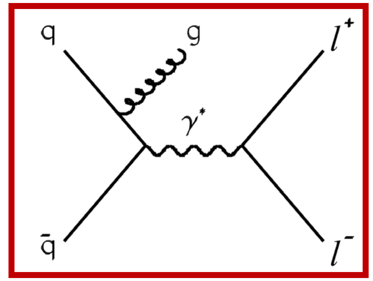
W-boson production in p-pbar collision from CDF

Other implications

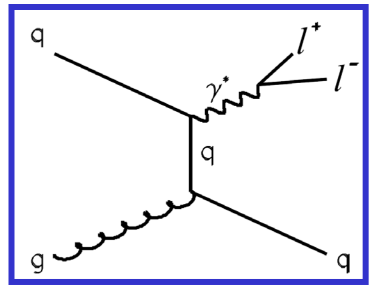
- Extend this study to Z plus jets data at LHC
  - The angular distribution coefficients are expected to be different, in general, for Z plus single jet and Z plus multi-jets events
  - Lam-Tung relation is expected to be satisfied by Z plus single jet events, but badly violated by Z plus two or more jets.
  - The q_T dependence of A_0 would be different for Z plus a single quark jet events and Z plus a single gluon jet events (can lead to the validation of various algorithms for quark/gluon jets separation)
  - Would be great to have these data from LHC!

Expected Z plus jets results





Z plus single gluon jet



Z plus single quark jet

Phys. Lett. B797 (2019) 134895

Z plus single jet

Very different results for Z plus single jet, depending on whether it is a quark jet or gluon jet.

Summary

- A "geometric model" is developed to understand many features of the lepton angular distribution in Drell-Yan and quarkonium productions in hadron collisions
- The lepton angular distribution coefficients $A_0 - A_7$ can be described in terms of the polar and azimuthal angles of the $q-\overline{q}$ axis (natural axis)
- Violation of the Lam-Tung relation is due to the acoplanarity of the $q-\overline{q}$ axis and the hadron plane. This can come from order α_s^2 or higher processes or from intrinsic k_T
- This approach predicts different behavior in the lepton angular distributions for Z plus quark jet, Z plus gluon jet, and Z plus multiple jets, to be tested with future anlaysis of LHC data