Short-time Laplace Transform Analysis for Gravitational Wave from Black Hole Quasinormal Mode

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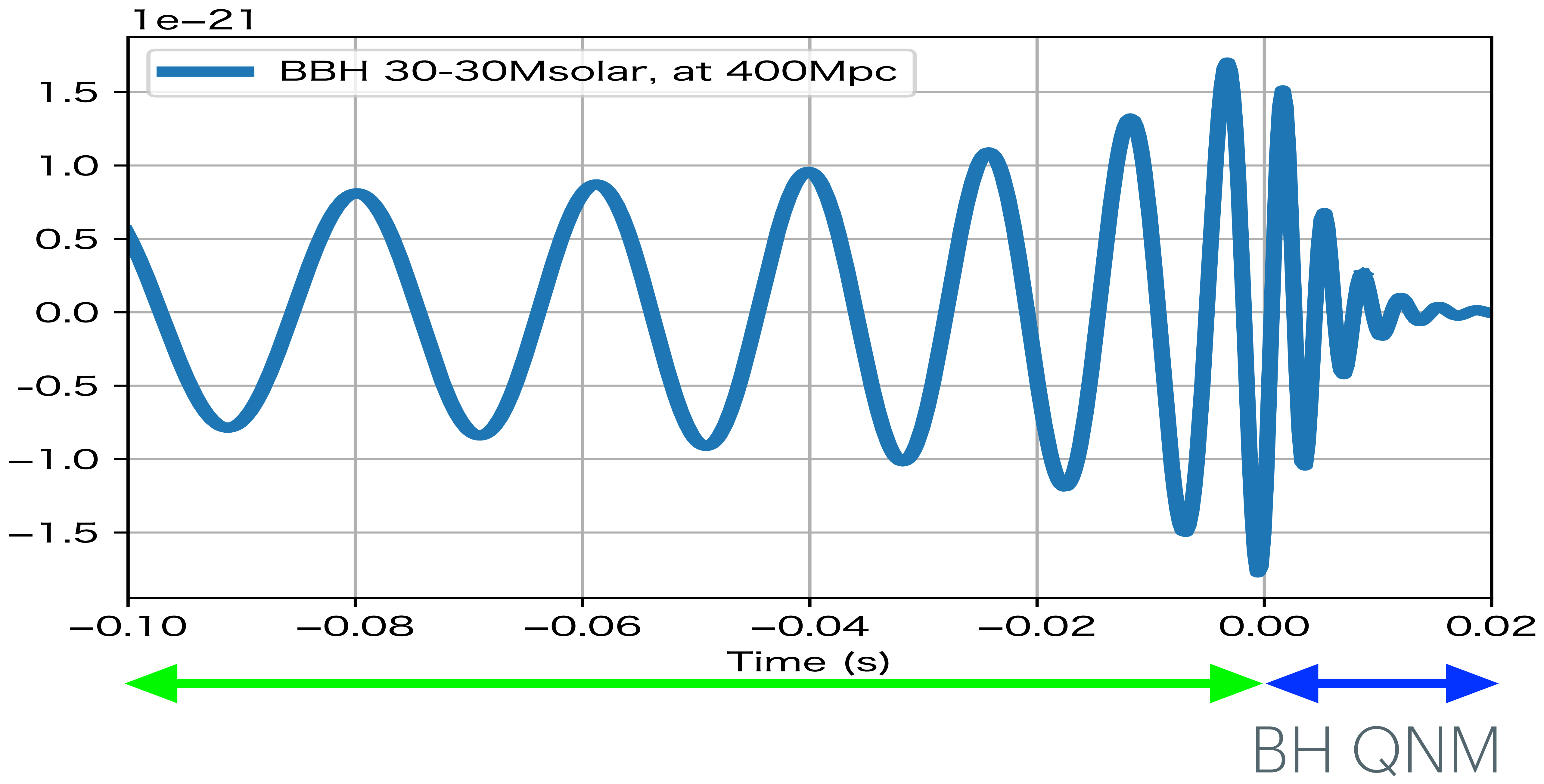
Outline

- Blackhole Quasinormal mode
- Laplace Transform : Idea & Motivation
- Implementation of Laplace transfom for Numerical Analysis of Gravitational Waveform
    - with Simulation waveform
    - Short-time Laplace Transform
- Real observed gravitational waveform : GW150914
    - extracting ringdown waveform
    - estimation of blackhole mass and Kerr parameter

Blackhole Quasinormal mode

Perturbation of BH space-time has particular oscillation mode.

- Blackhole Quasinormal modes (BH-QNMs) are dumped-sinusoidal ("ringdown") gravitational wave (GW) form.
- BH mass and angular momentum determine its frequency and decay time.
- How to identify QNM, especially higher modes(index l,m) and overtones(n) are interested problem.



$$h_+ + \mathrm{i}h_\times = -\frac{2}{r^4}\int_{-\infty}^{+\infty}\frac{d\omega}{\omega^2}e^{\mathrm{i}\omega t}\sum_{lm} S_{lm}\left(\iota,\beta\right)R_{lm\omega}(r)\,.$$

Berti, Cardoso, Will, Phys.Rev.D73:064030,2006

$$h(f_c, Q, t_0, \phi_0; t) = e^{-\frac{\pi f_c(t-t_0)}{Q}}\cos(2\pi f_c(t-t_0)-\phi_0)$$

central frequency :

$$f_c = \frac{1}{2\pi M_{BH}}\left[1.5251 - 1.1568(1-\alpha)^{0.1292}\right]$$

$$= 538.4\left(\frac{M}{60M_\odot}\right)^{-1}\left[1.5251 - 1.1568(1-\alpha)^{0.1292}\right]$$

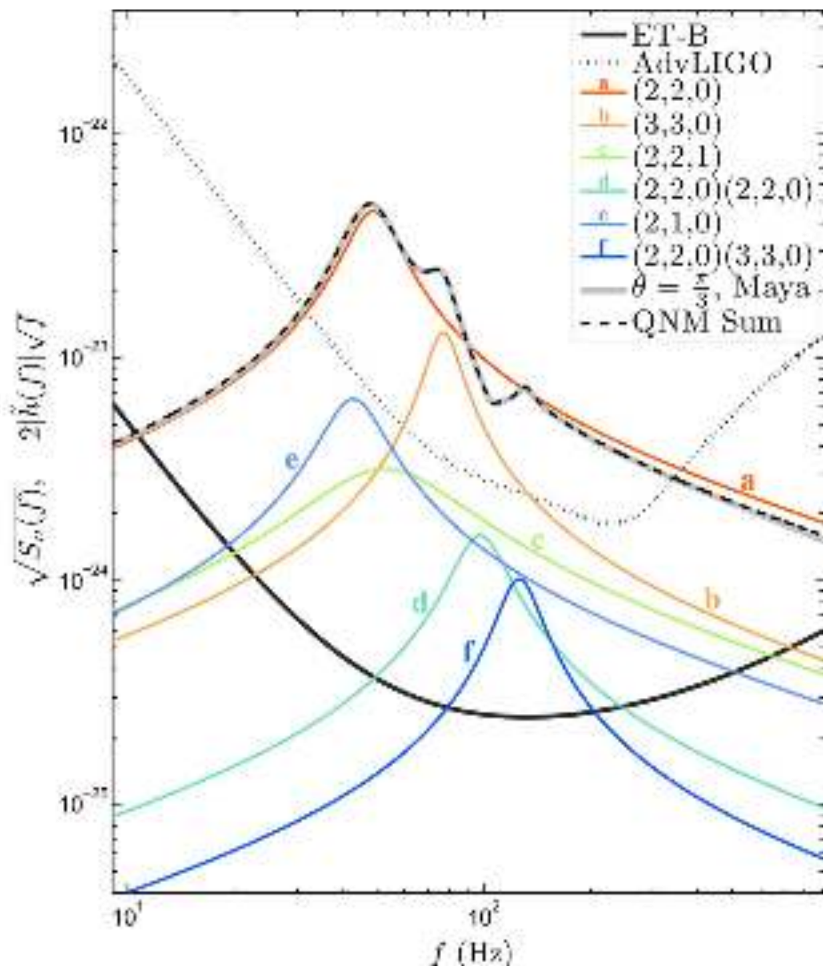
Q value :

$$Q = 0.7000 + 1.4187\,(1-\alpha)^{-0.4990}$$

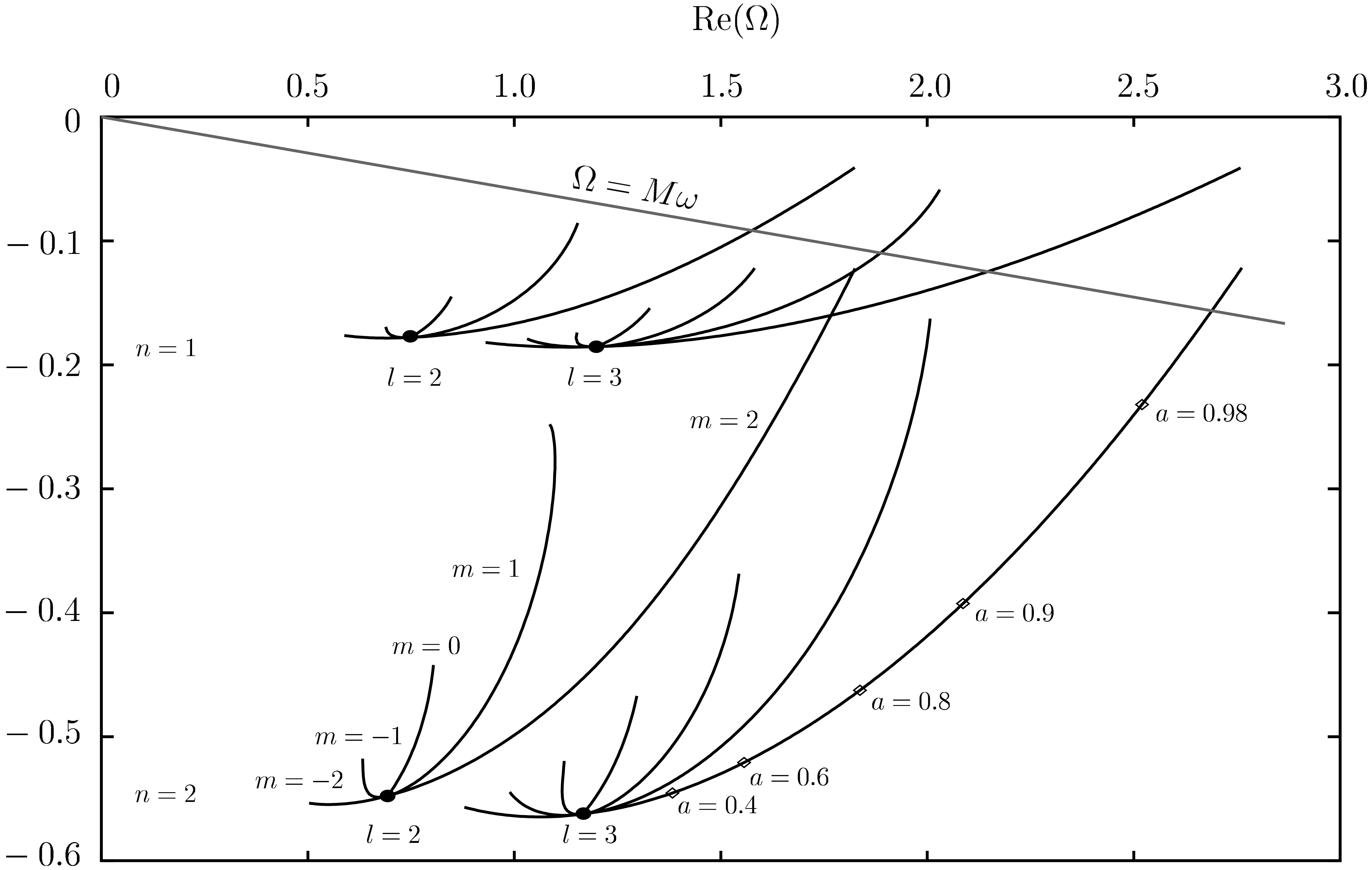
Kerr parameter : α

BH mass : M

Nakano, Tanaka, Nakamura, Phys.Rev.D92:064003,2015



Example of expected GW spectrum, Phys. Rev. D 90, 124032



BH QNM on complex plane of frequency

Class.Quant.Grav.21:787-804,2004

Laplace Transform : Idea & Motivation

Laplace Transform : time series in real ==> complex frequency domain

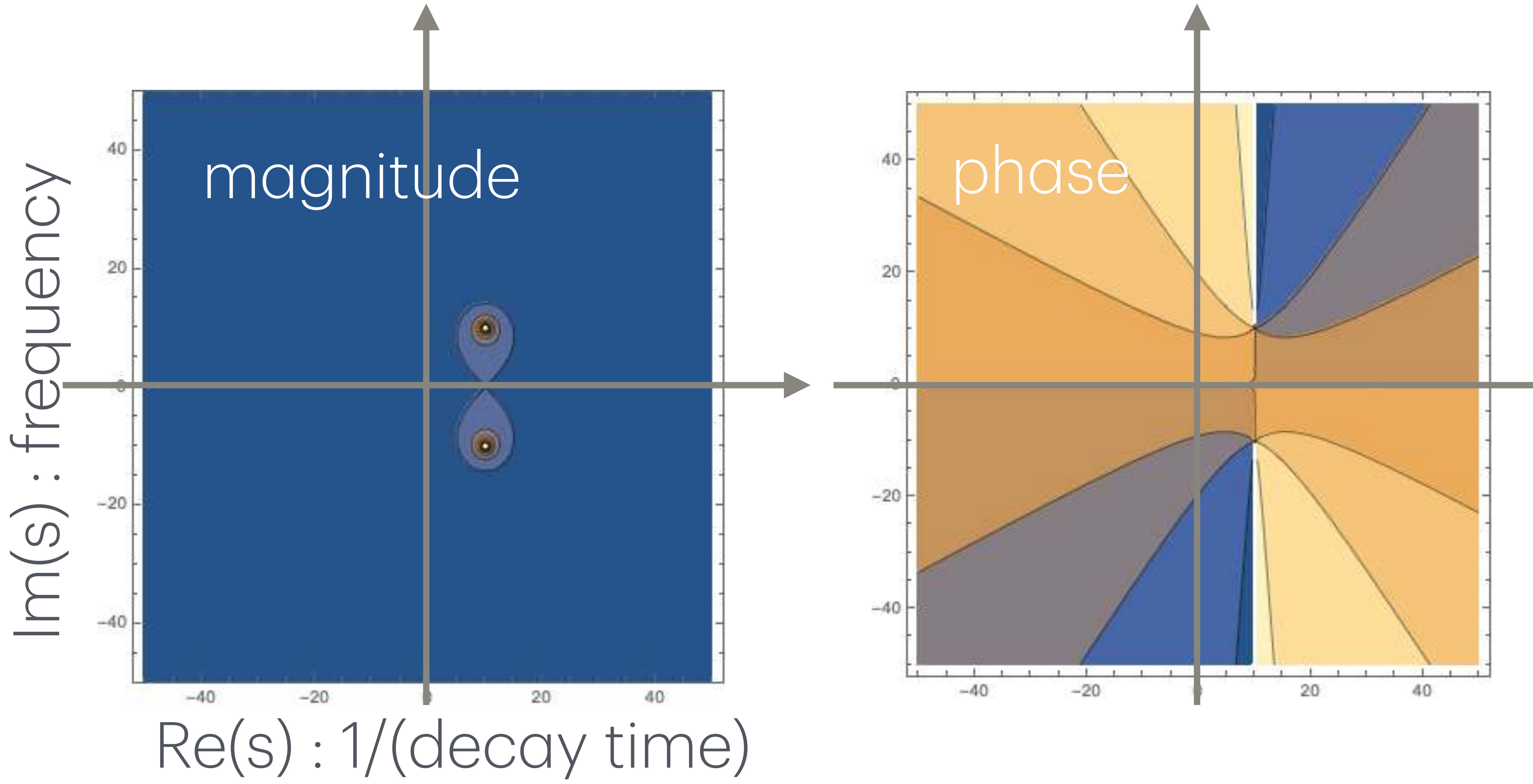
$$H(s) = \mathcal{L}[h](s) = \int_0^\infty h(t) e^{-st} dt$$
$$= \int_0^\infty h(t) e^{-(b+i\omega)t} dt$$

h(t) : time series
H(s) : Laplace transform of *h(t)*
s : complex frequency
b : Re(s), ω : Im(s)

- clear and simple definition,
- well known its behavior for typical time signals in electric circuit.

Dumped sinusoidal wave will be represented as 'pole' in complex plane.

$$\mathcal{L}[e^{bt}\cos\omega t] = \frac{s-b}{(s-b)^2+\omega^2}$$
$$\mathcal{L}[e^{bt}\sin\omega t] = \frac{\omega}{(s-b)^2+\omega^2}$$



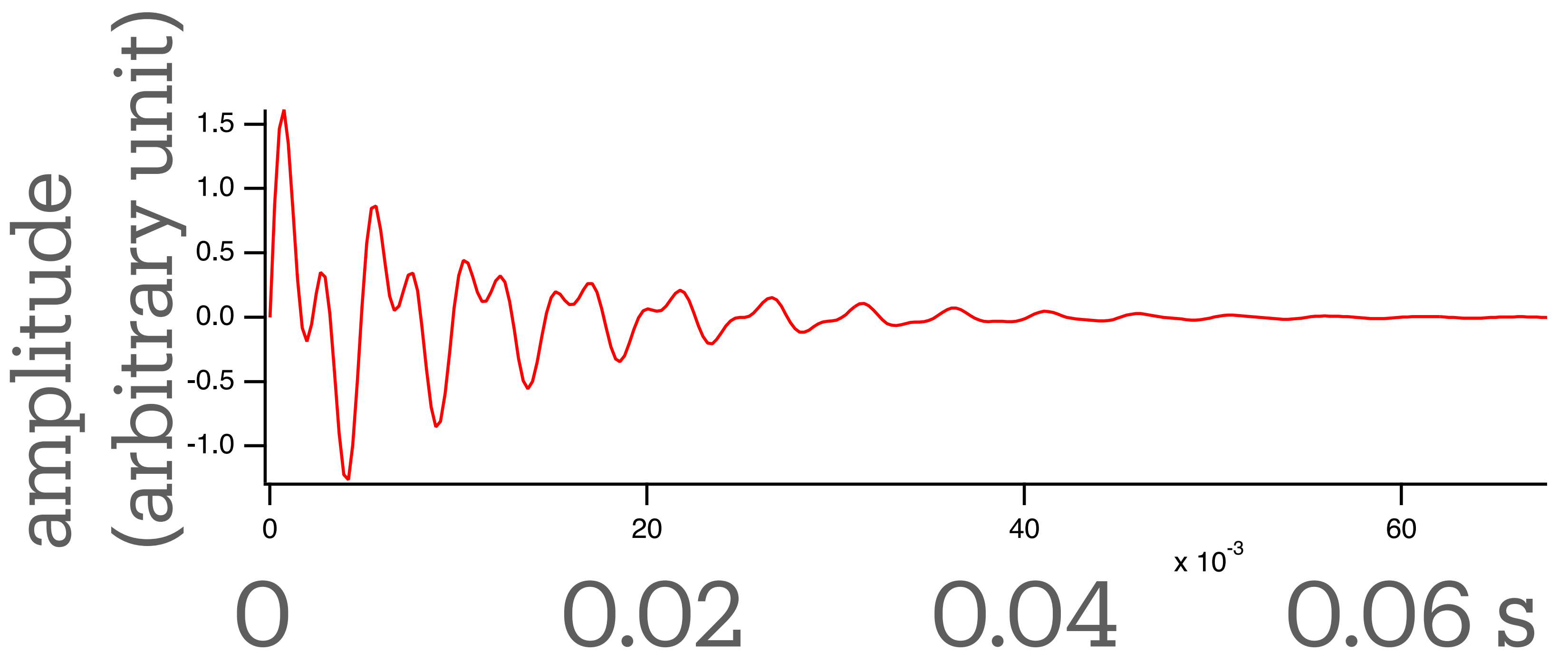
This property is expected to be **suitable for viewing BH QNM**.

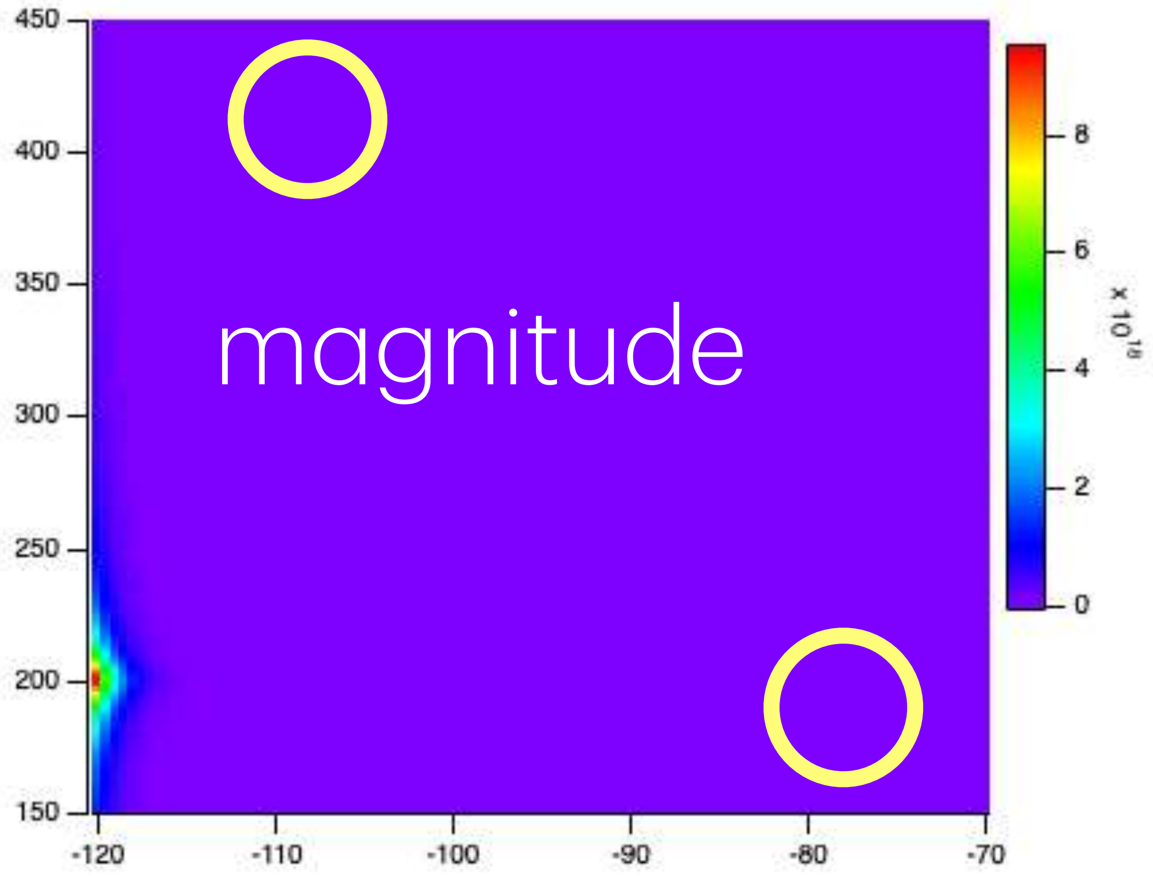
Implementation of Laplace transfom for Numerical Analysis of Gravitational Waveform

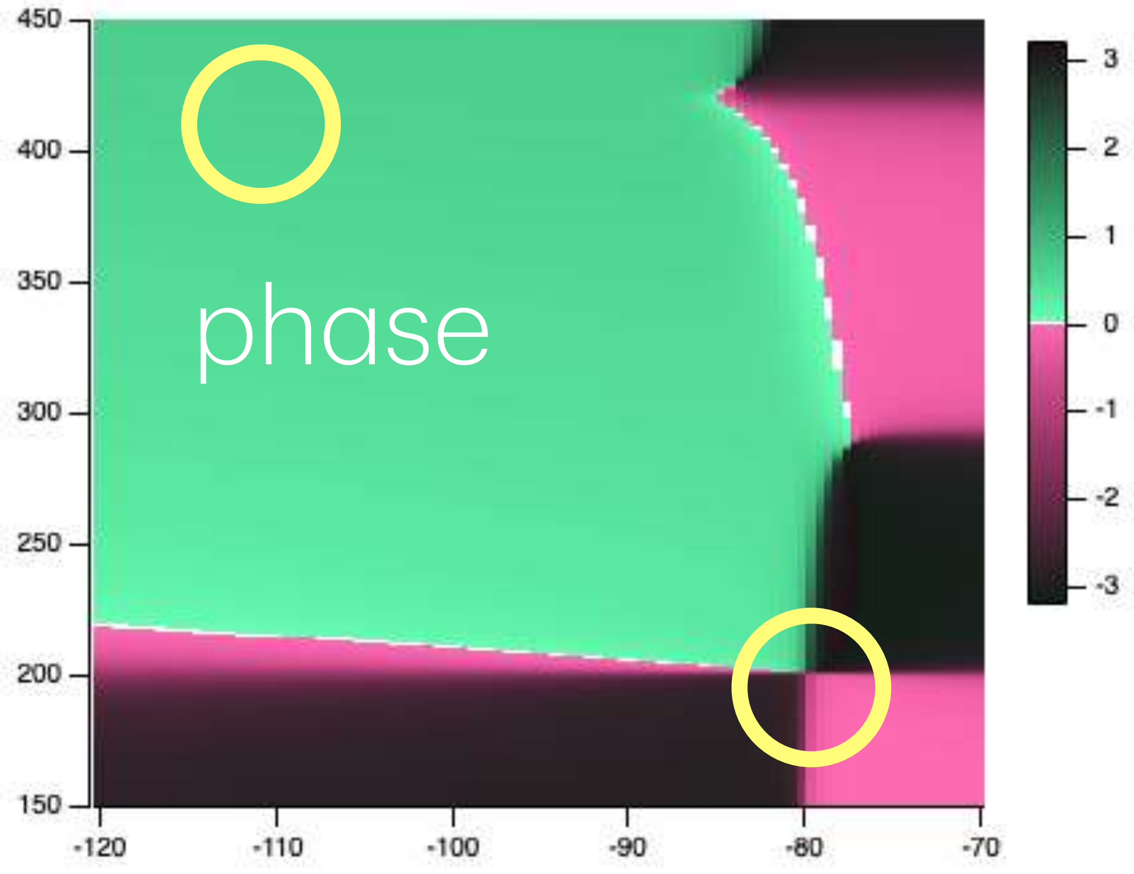
$$H(s) = \mathcal{L}[h](s) = \int_0^\infty h(t) e^{-st} dt$$
$$= \int_0^\infty h(t) e^{-(b+i\omega)t} dt$$

- Laplace transform is implemented as Fourier transform of $h(t)e^{bt}$.
- We employ fast Fourier transform (FFT) for numerical calculation.
- With scanning parameter b (= inverse of decay time = real part of complex frequency s), we got Laplace transform.

example : double exponential decay time series



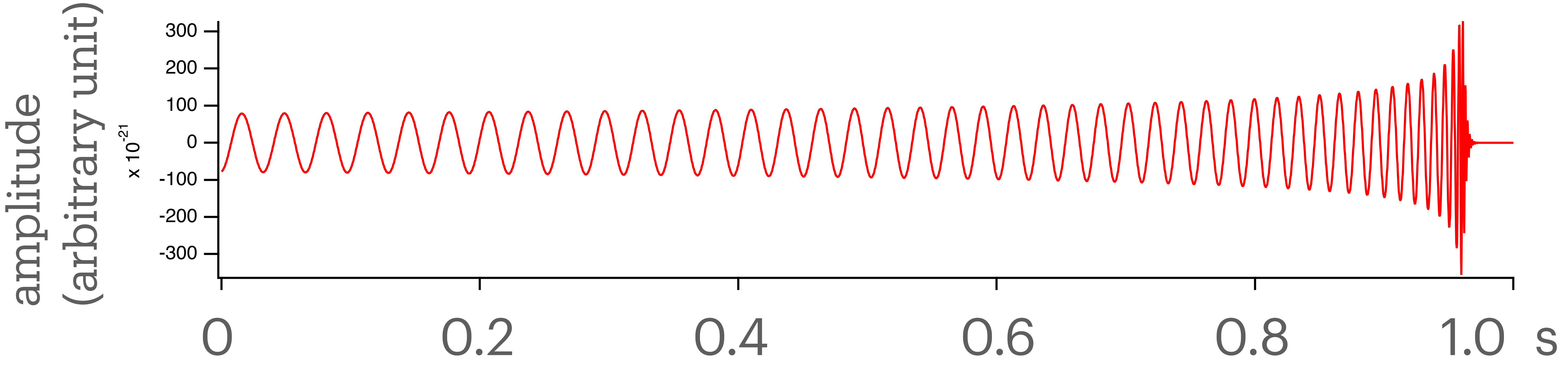




Pole that has smaller b is hard to find in this example, but phase map suggest two poles.

with Simulation waveform

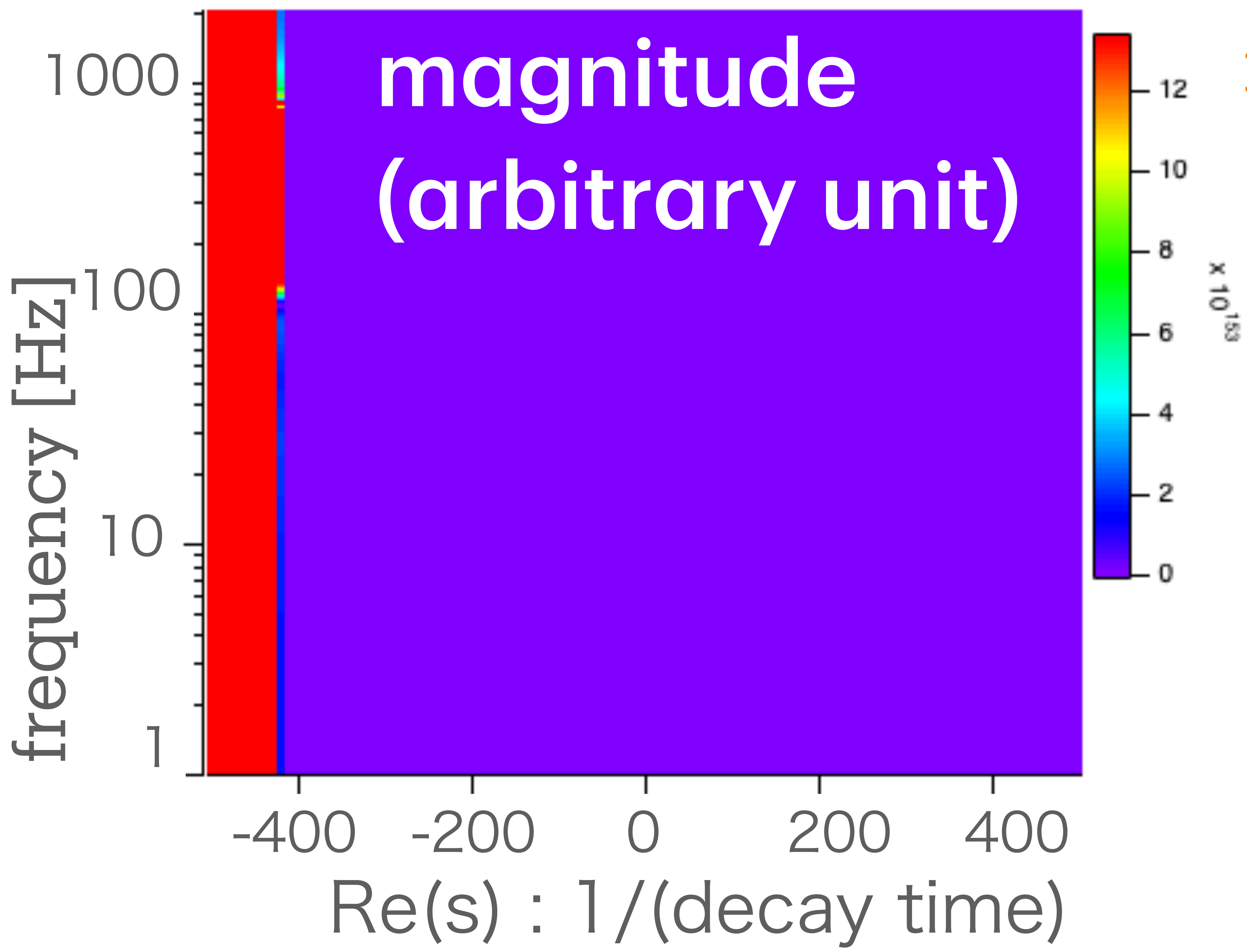
GW waveform by IMRphenom (SEOBNRv4_opt, pycbc)

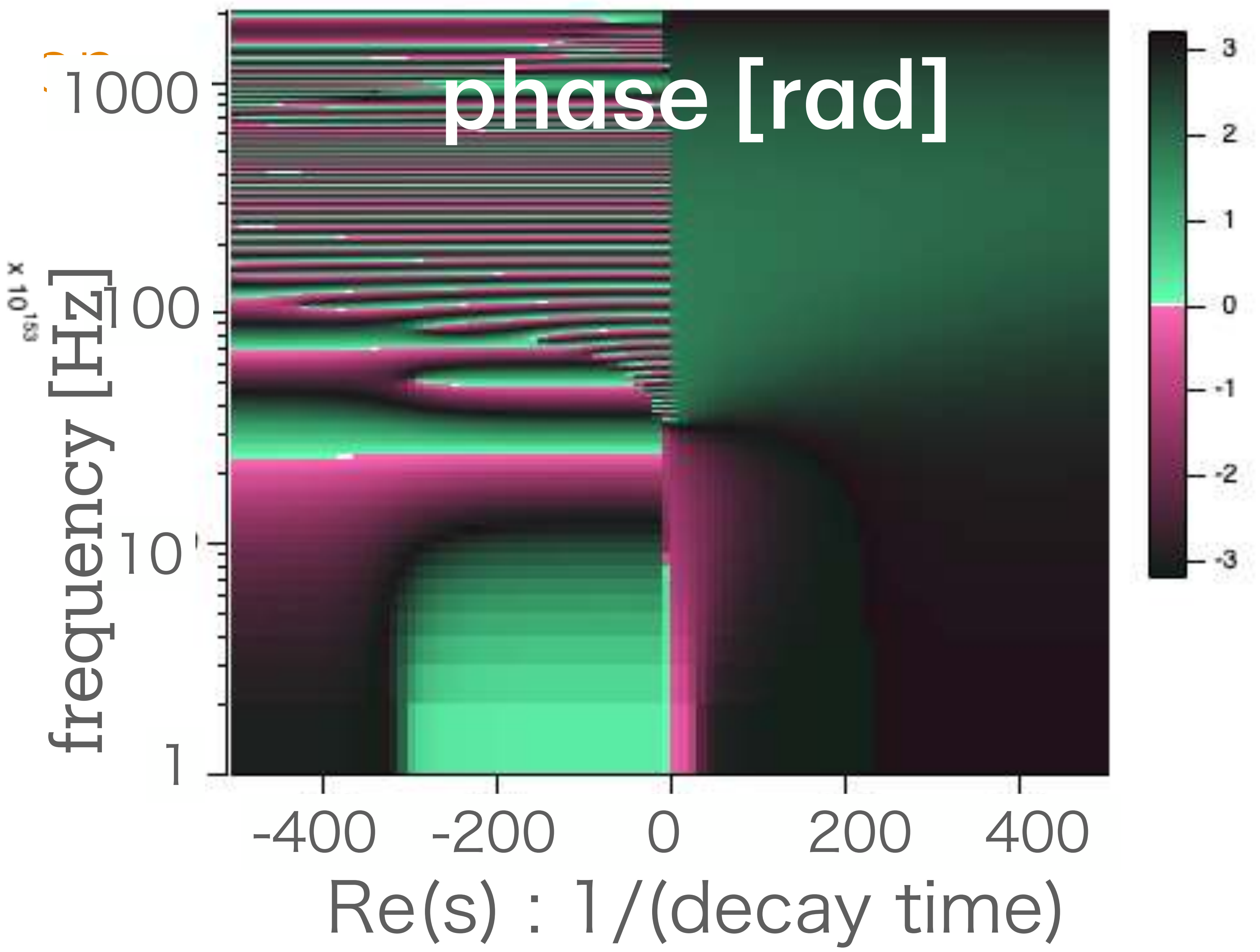


Initial Blackhole masses $15M_\odot - 15M_\odot$

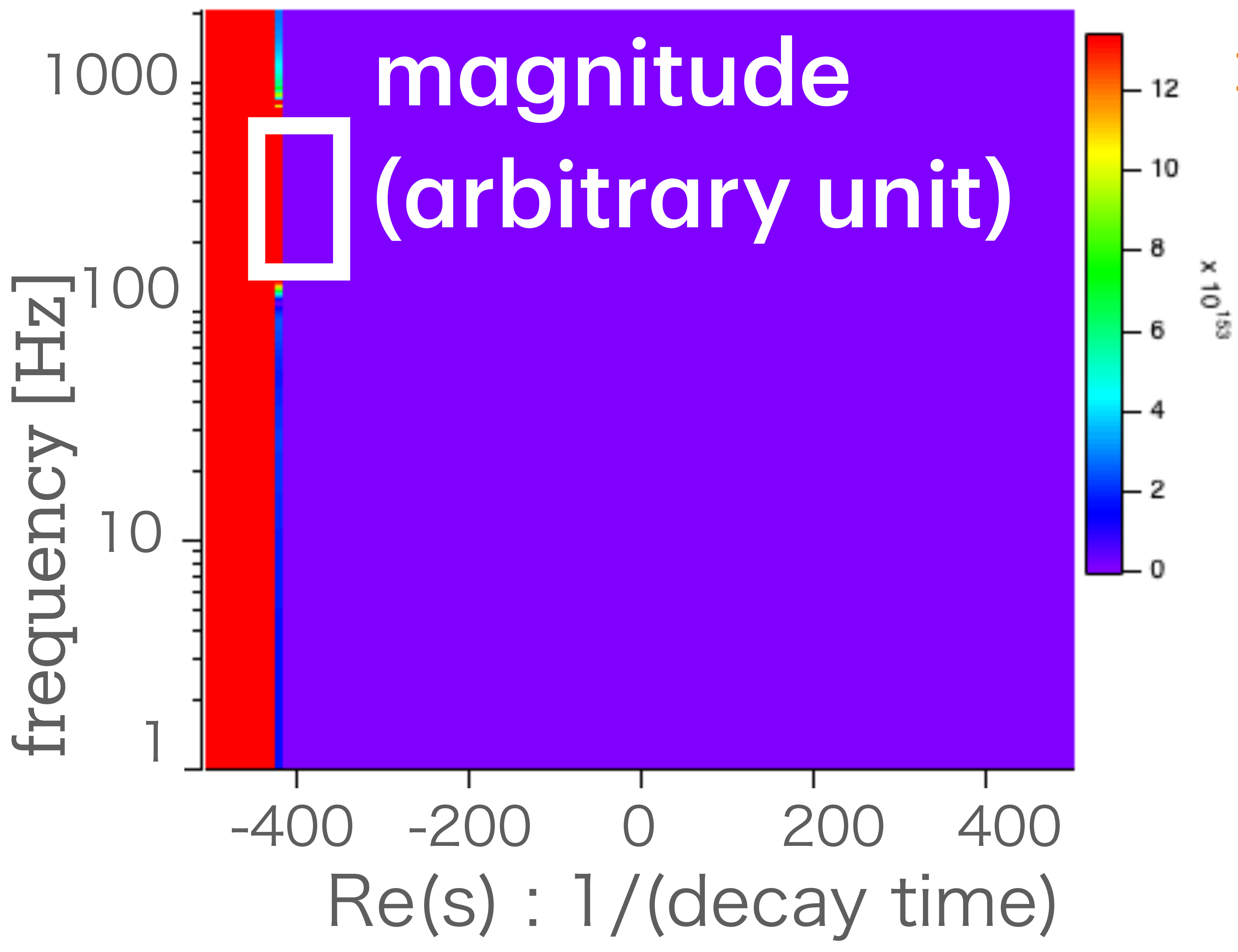
$$\left.\frac{H(b,f)}{\frac{1}{f_{\mathrm{MAX}}}\int_0^{f_{\mathrm{MAX}}} H(b,f)\,df}\right|_{a=a_0}$$

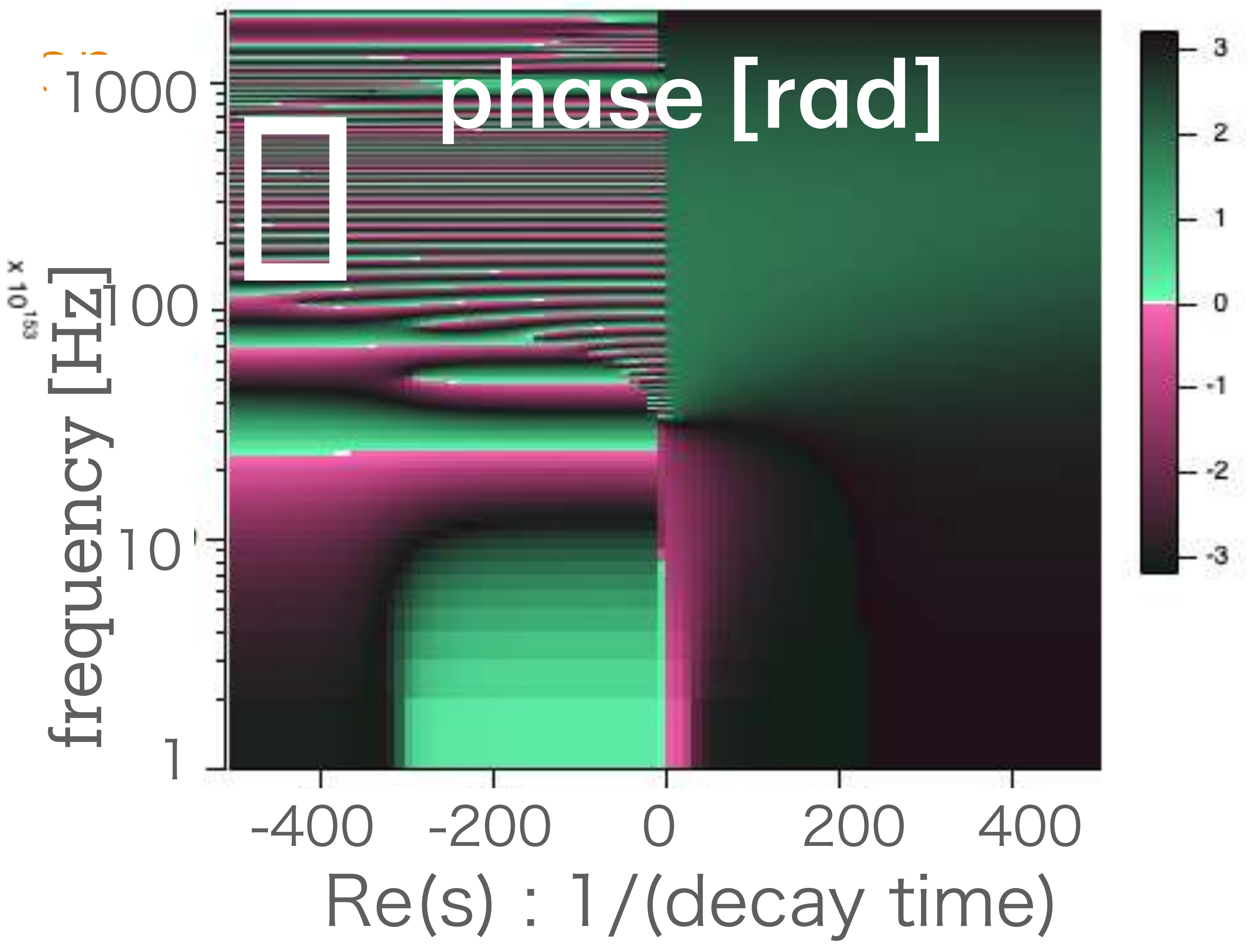
with Simulation waveform (con'd)



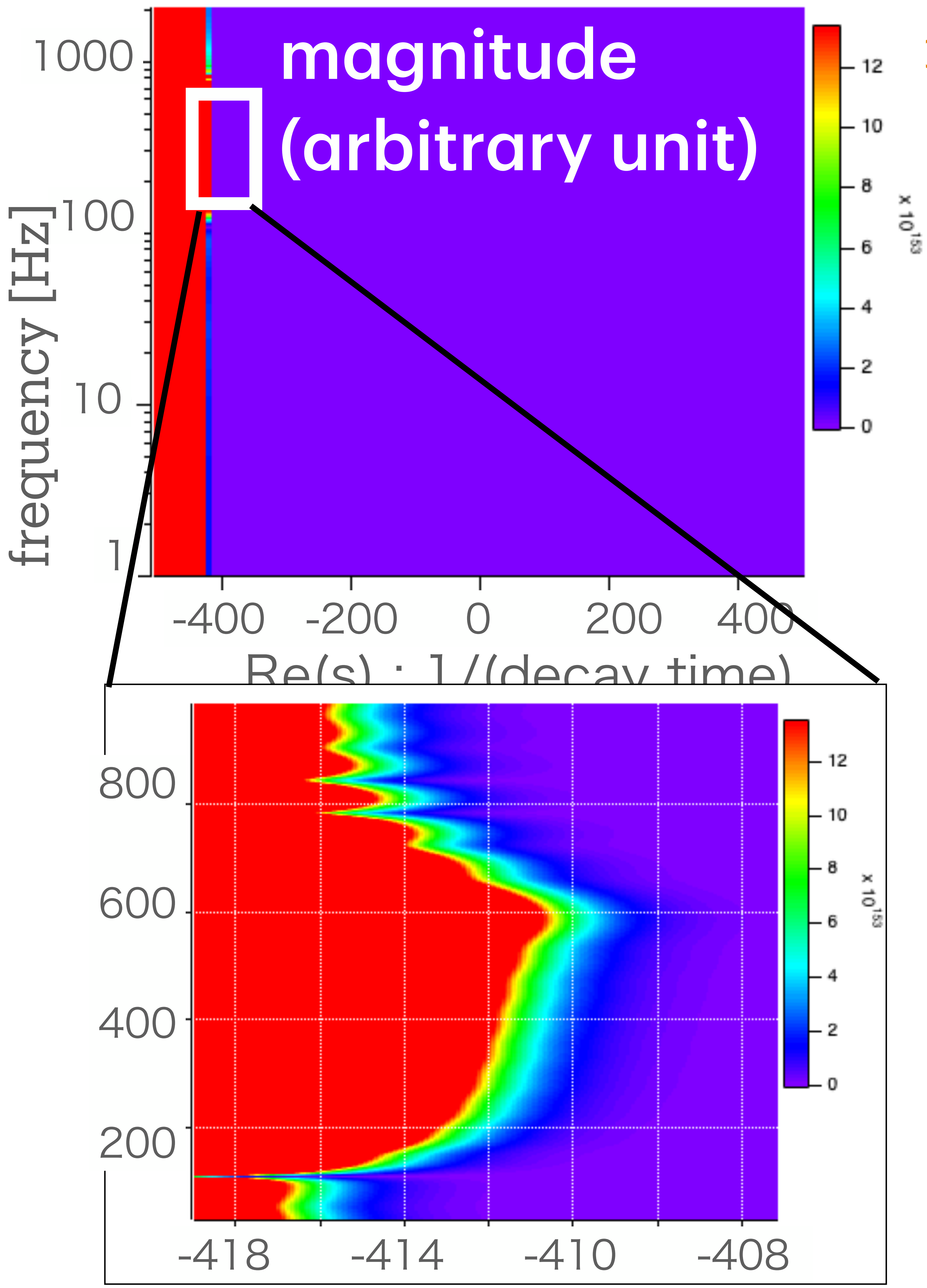


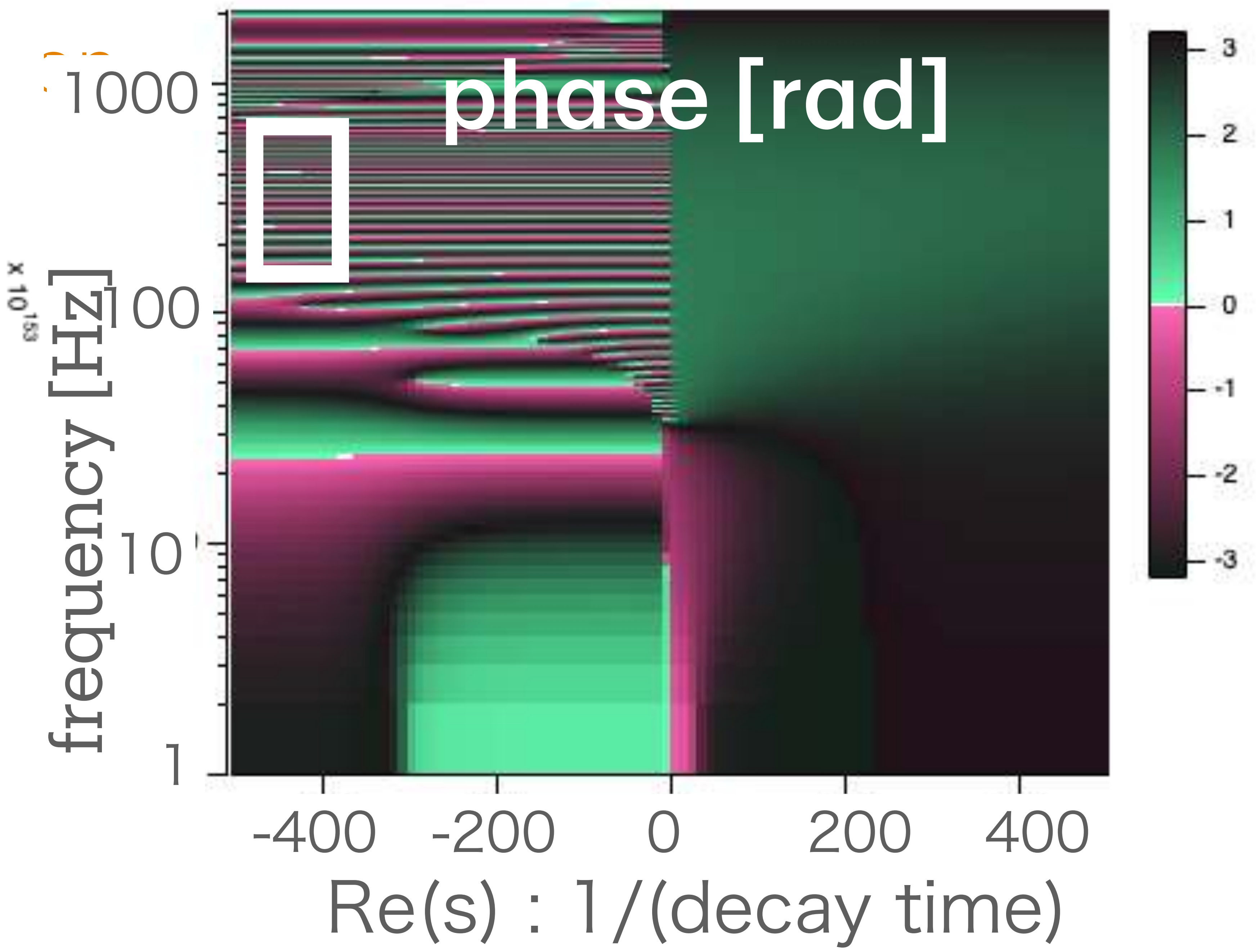
with Simulation waveform (con'd)



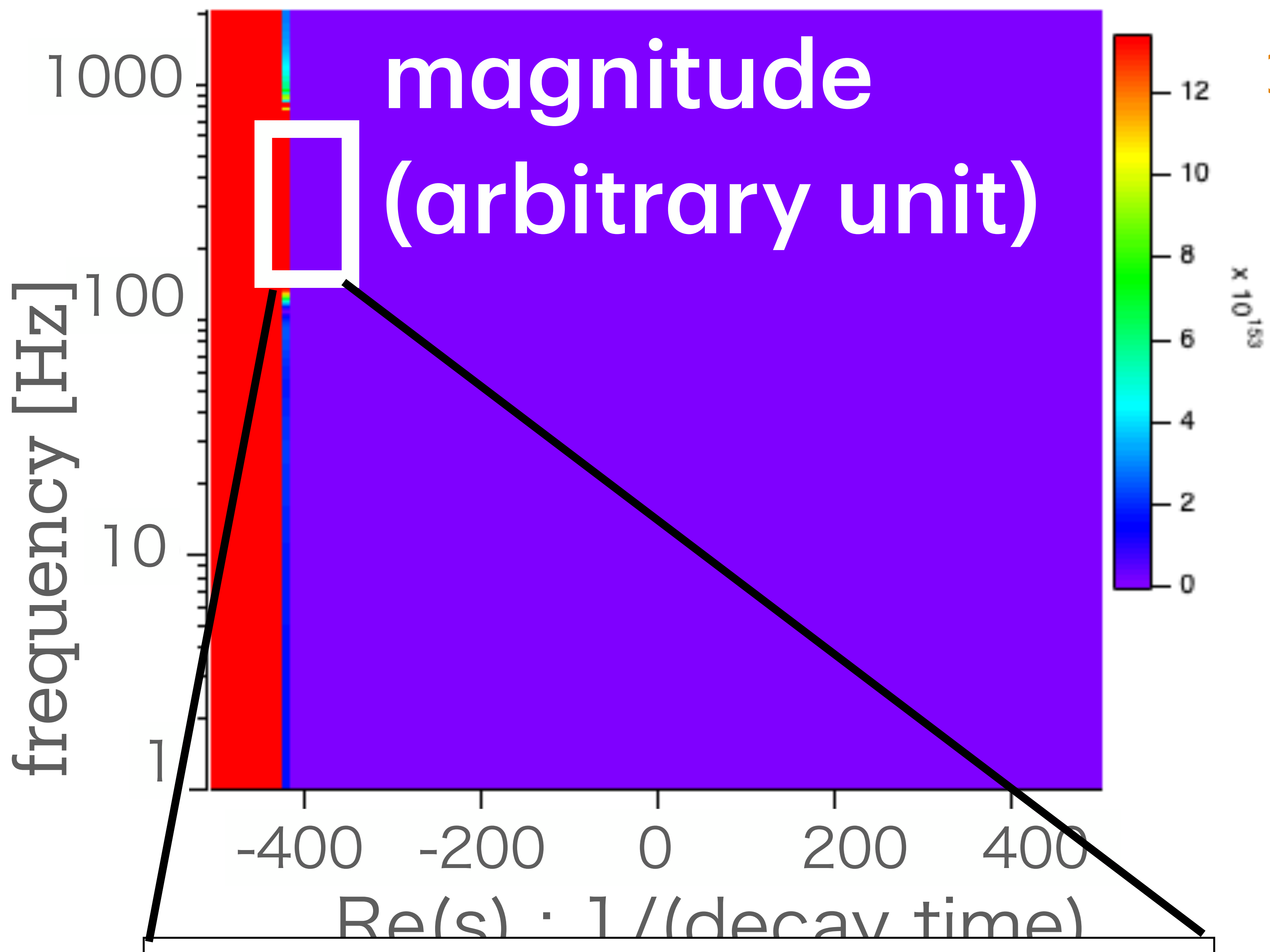


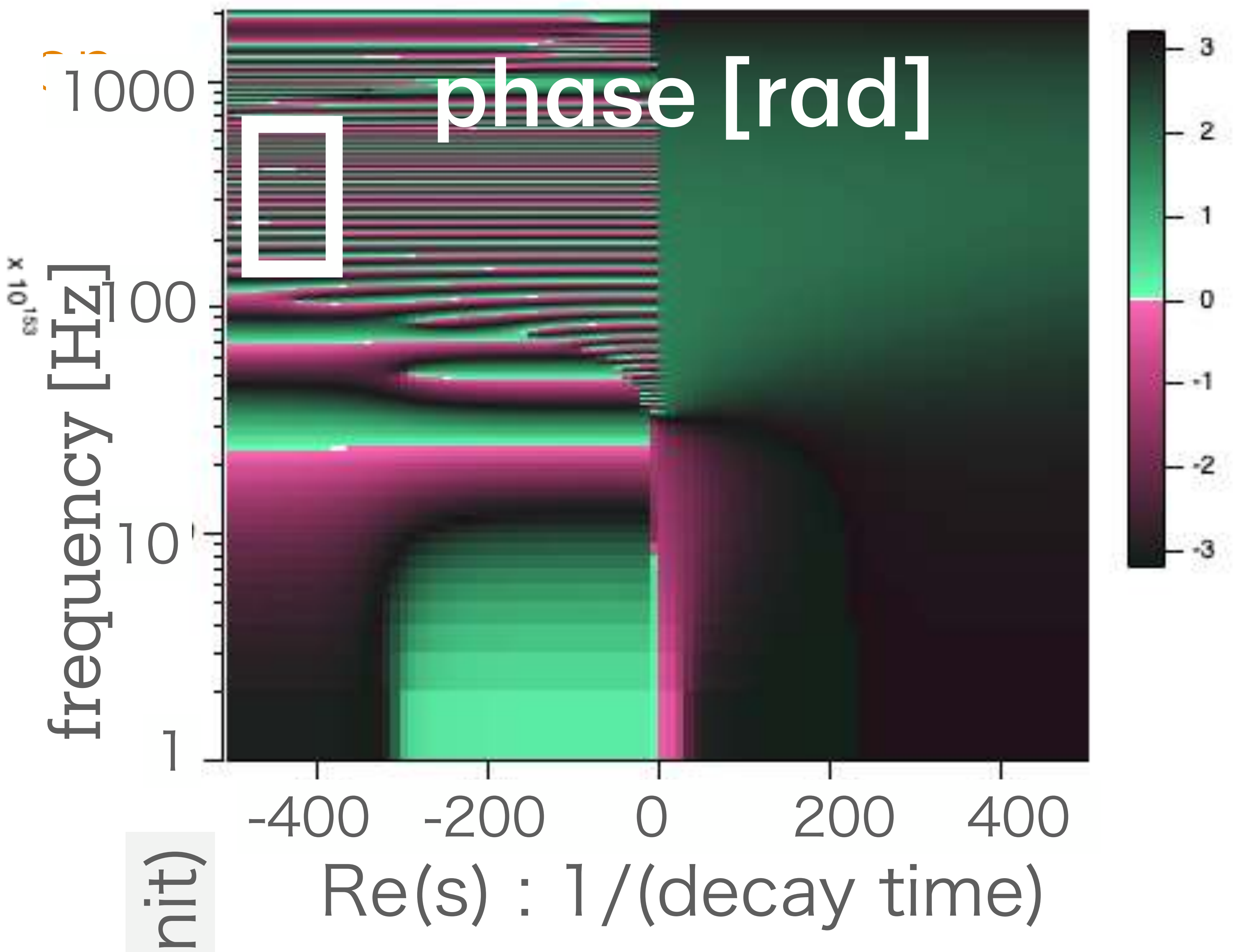
with Simulation waveform (con'd)

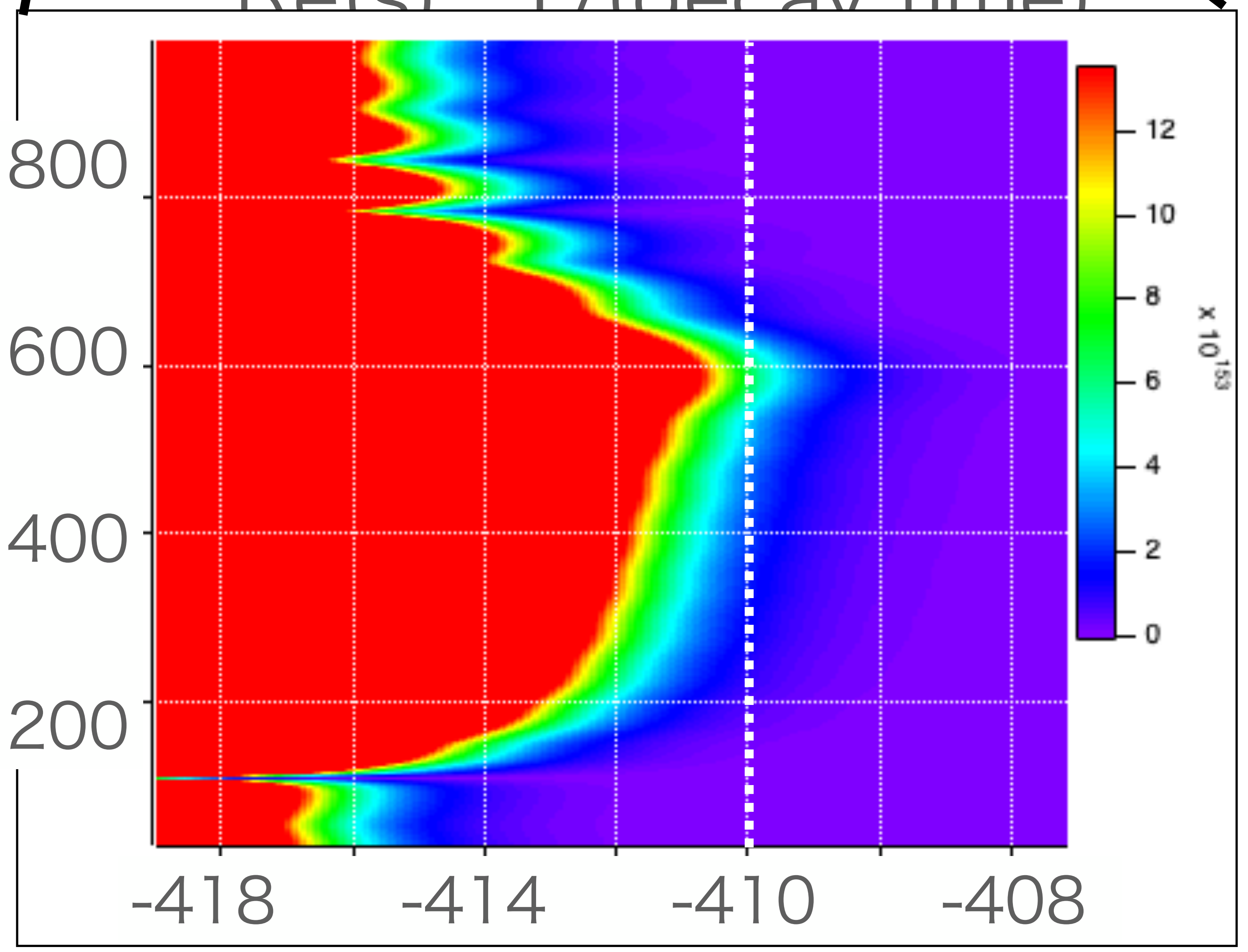


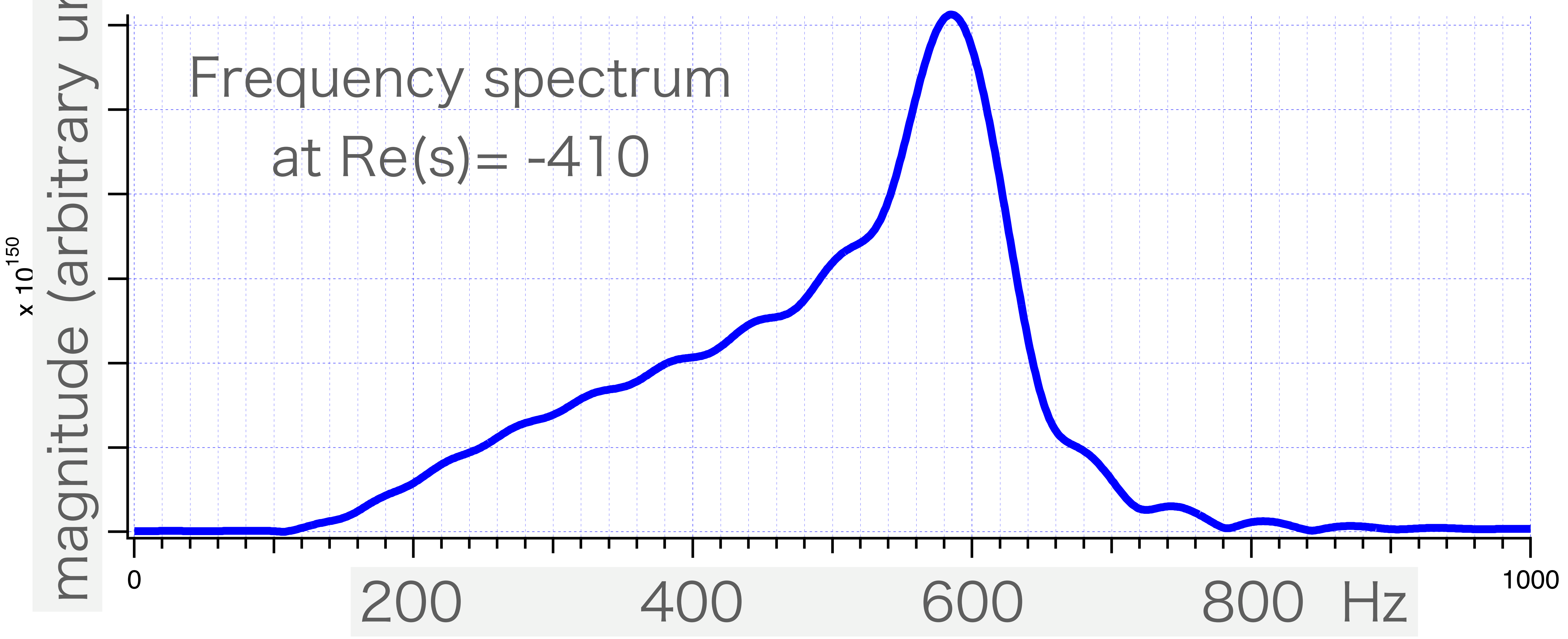


with Simulation waveform (con'd)

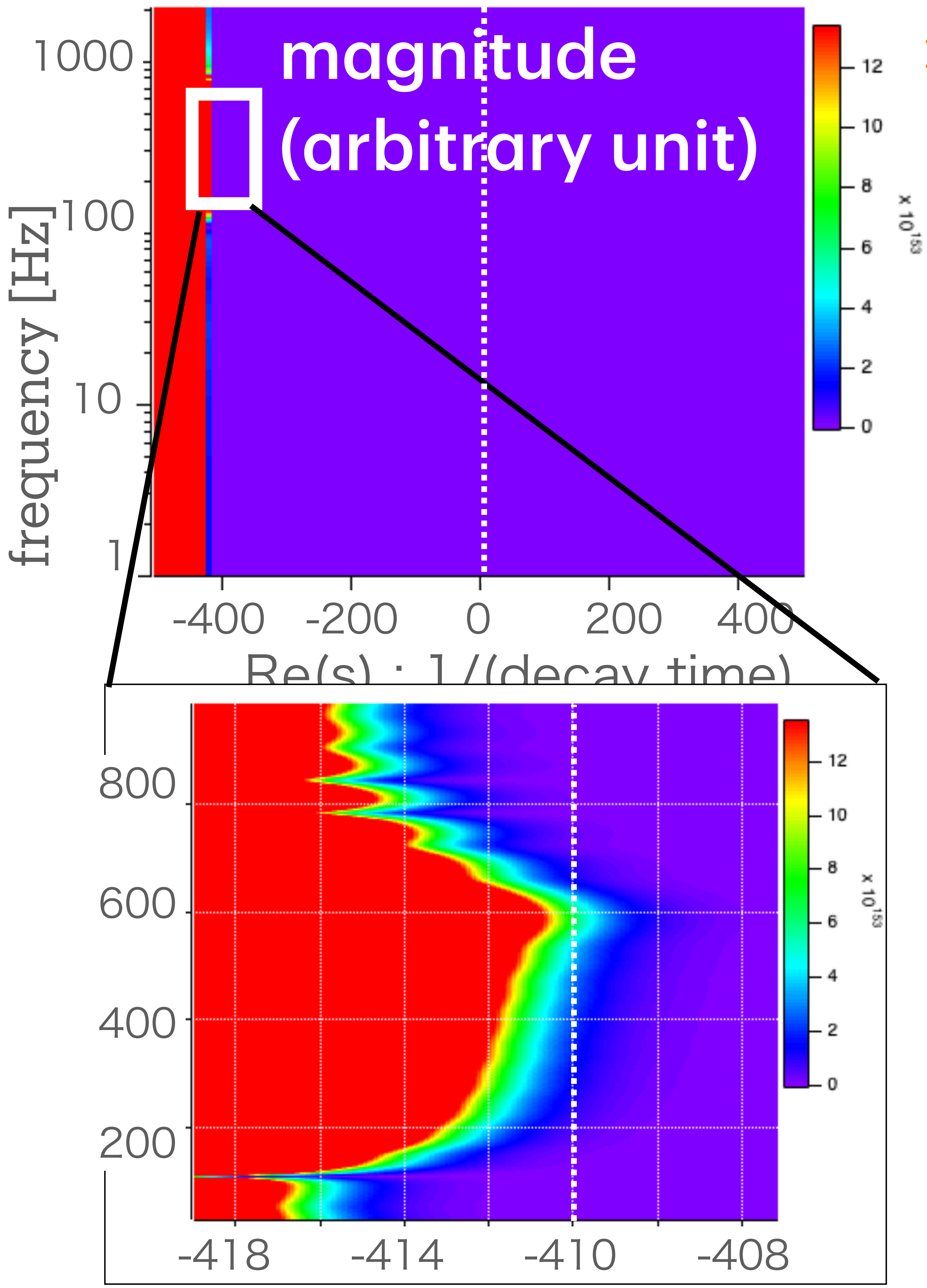


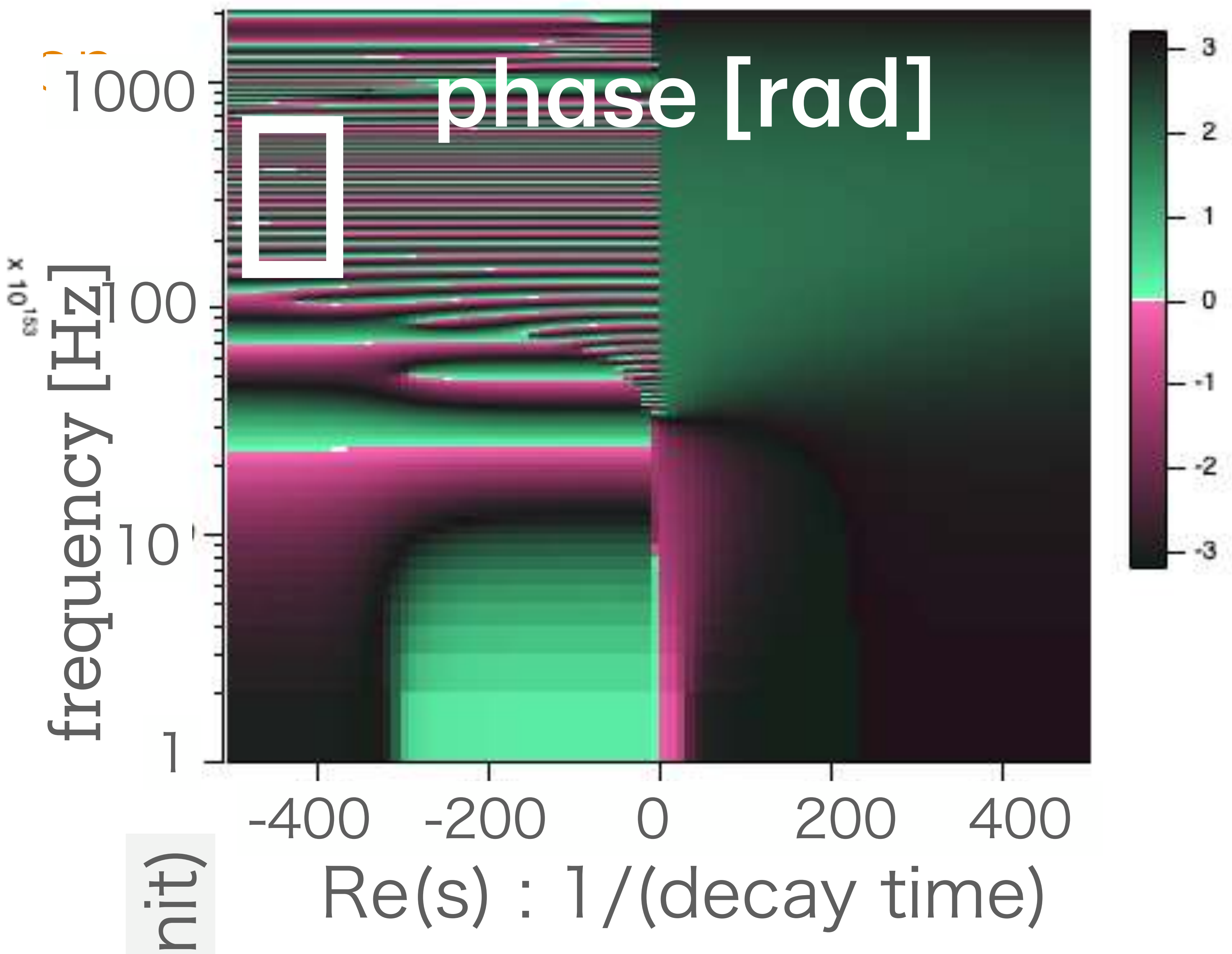


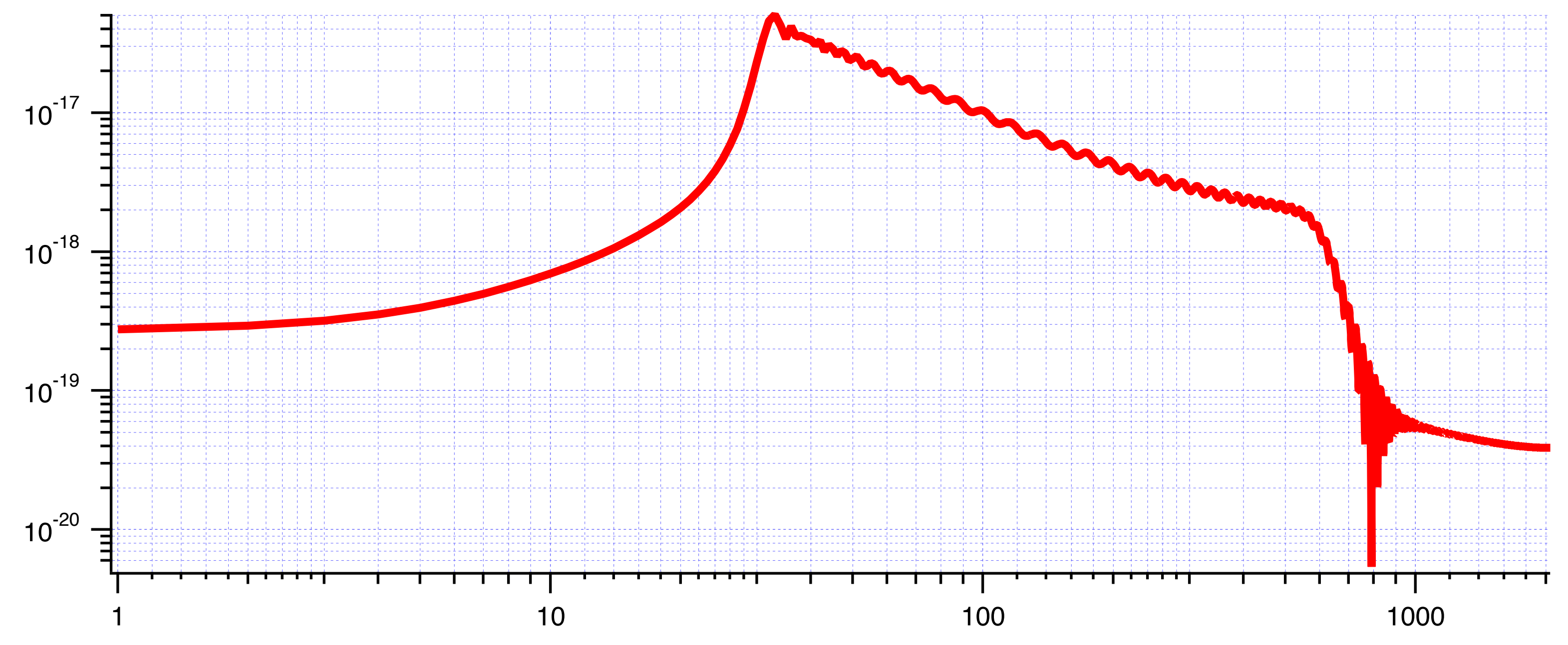




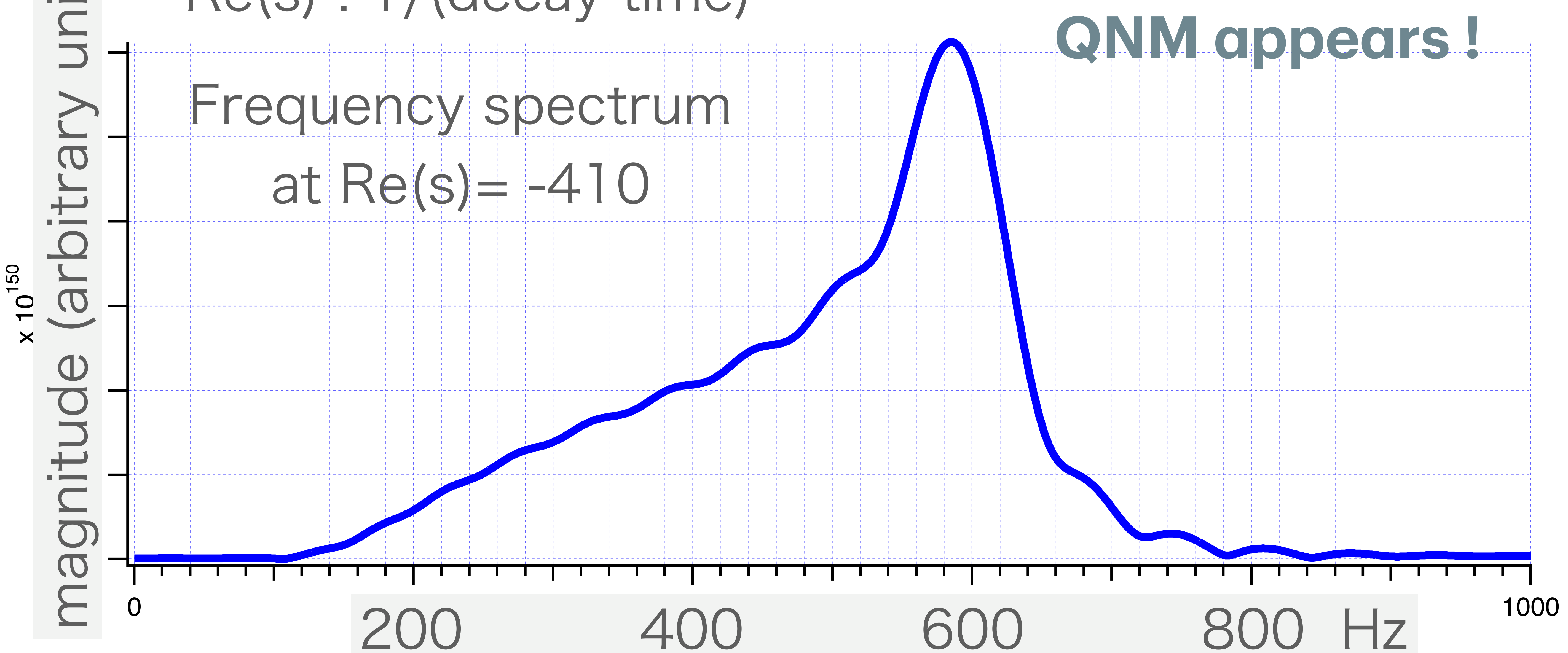
with Simulation waveform (con'd)





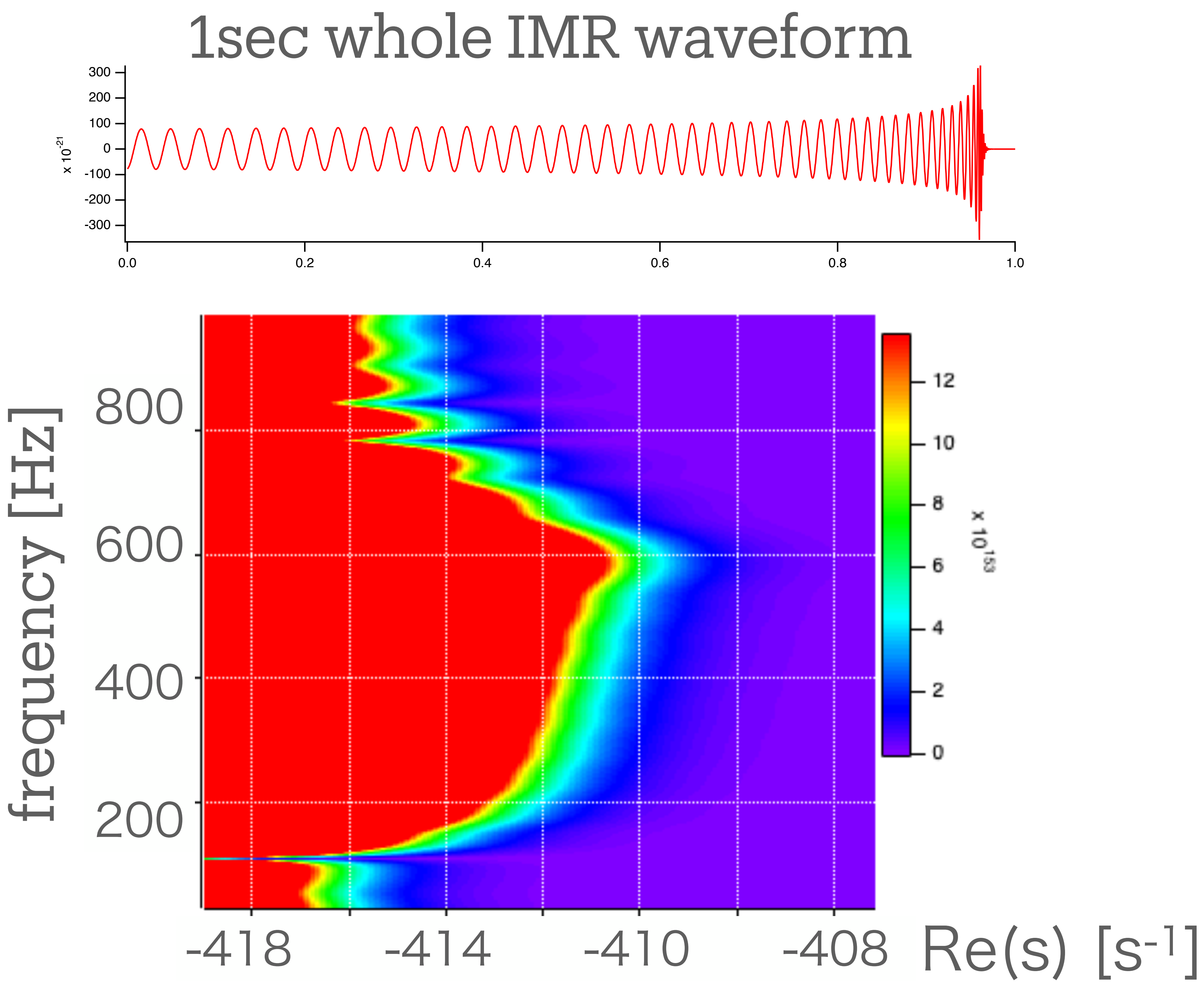


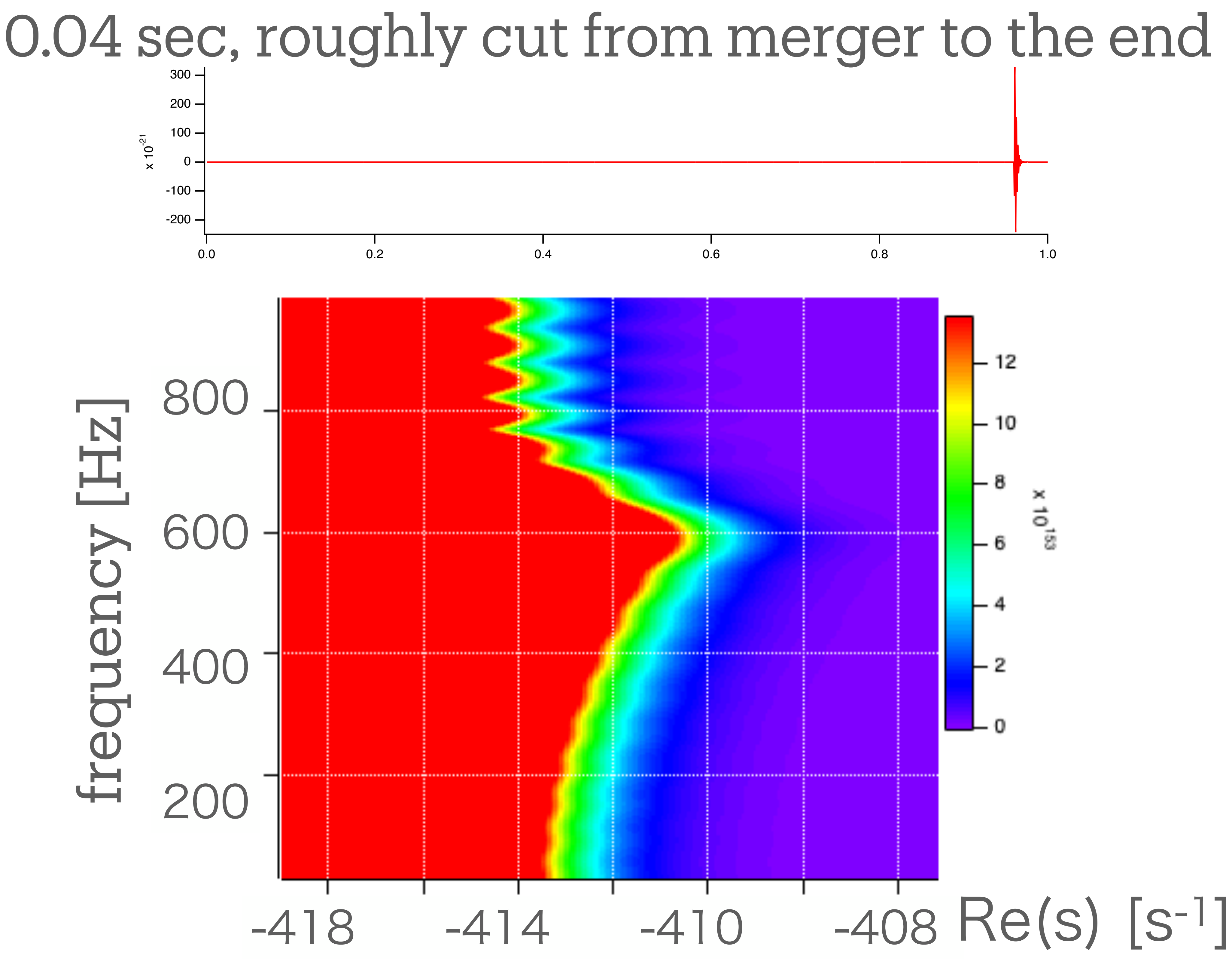
cf: spectrum at Re(s)=0
(same to Fourier tr.)



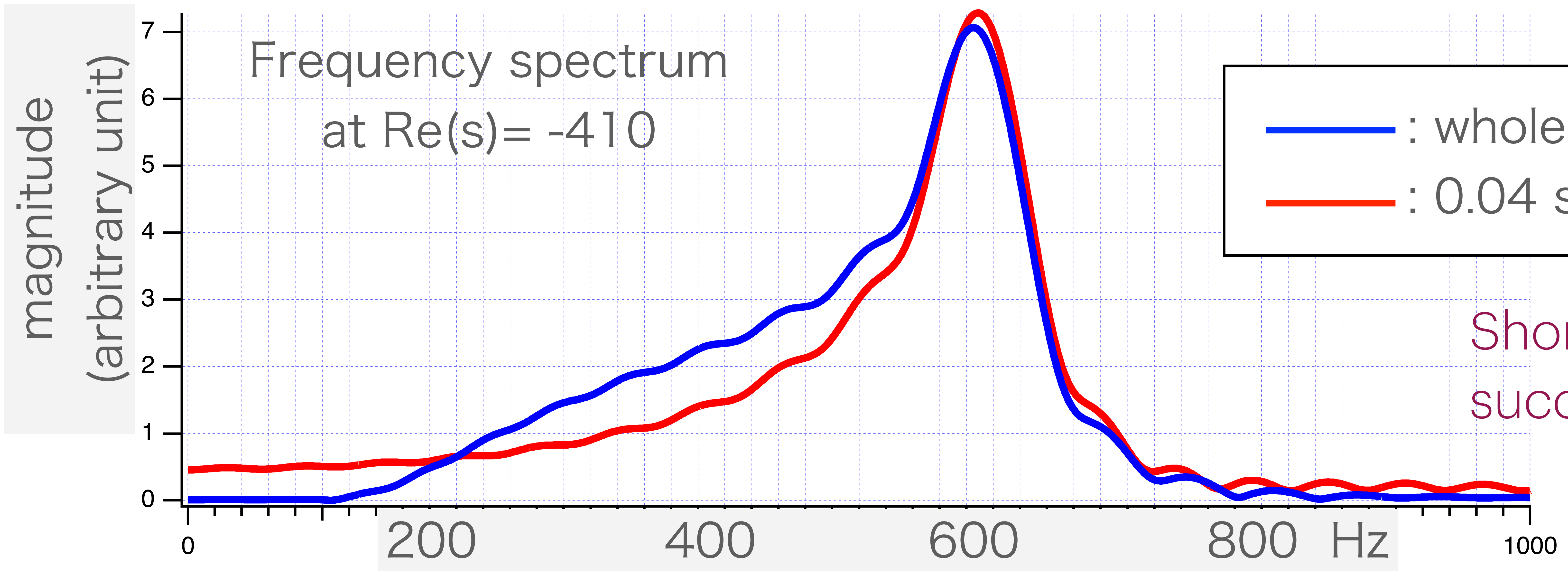
Short-time Laplace Transform

- With time slice, we can supress waveform components of non-QNM (i.e, chirp, merger).
- However, no longer needed to cut-out strictly around QNM.

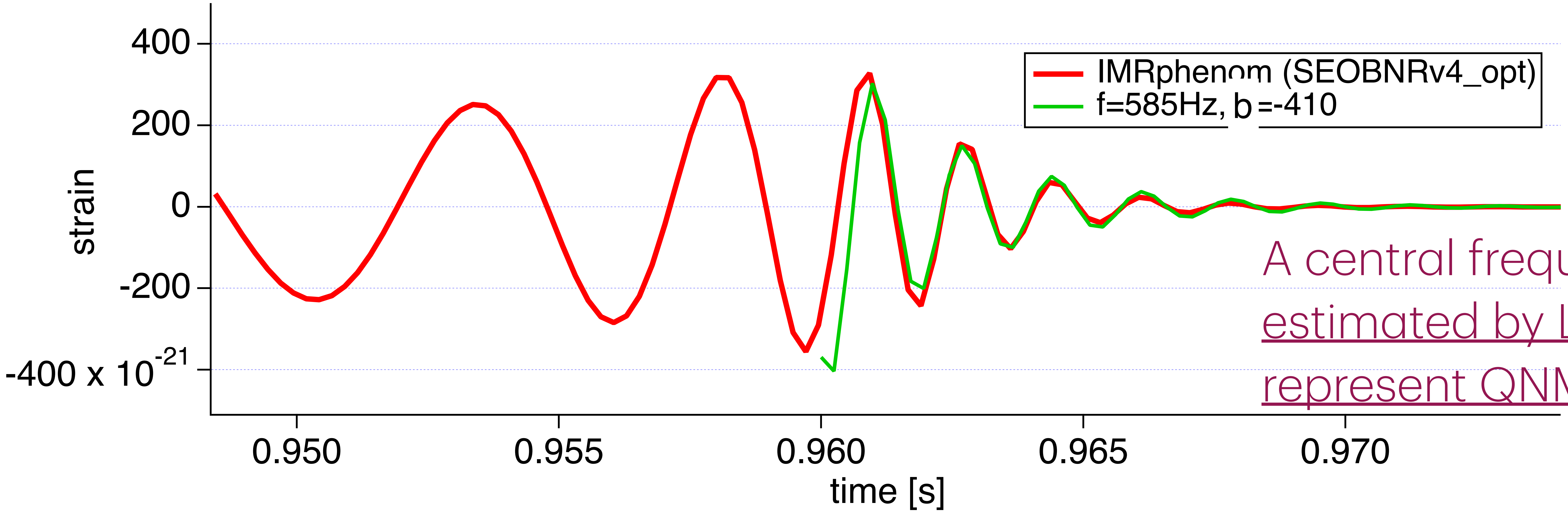




Short-time Laplace Transform (cont'd)



Short-time Laplace transform succeeded to enhance QNM.

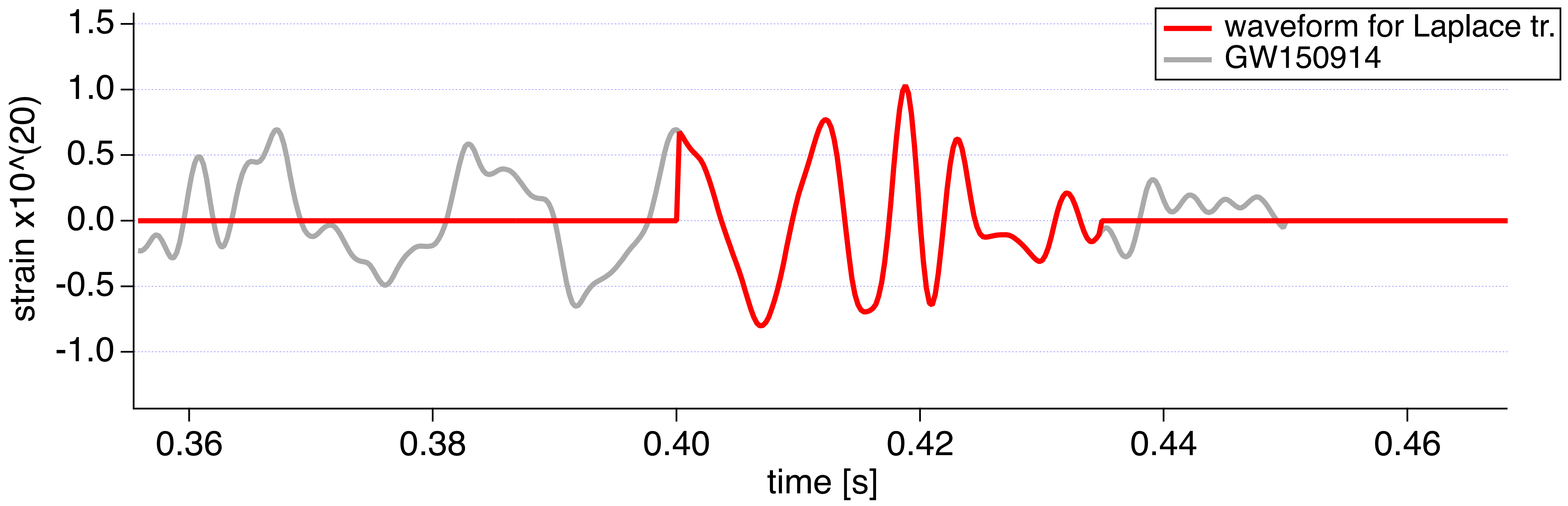


A central frequency and decay time estimated by Laplace transform can represent QNM mode in time series.

Real observed gravitational waveform

GW150914

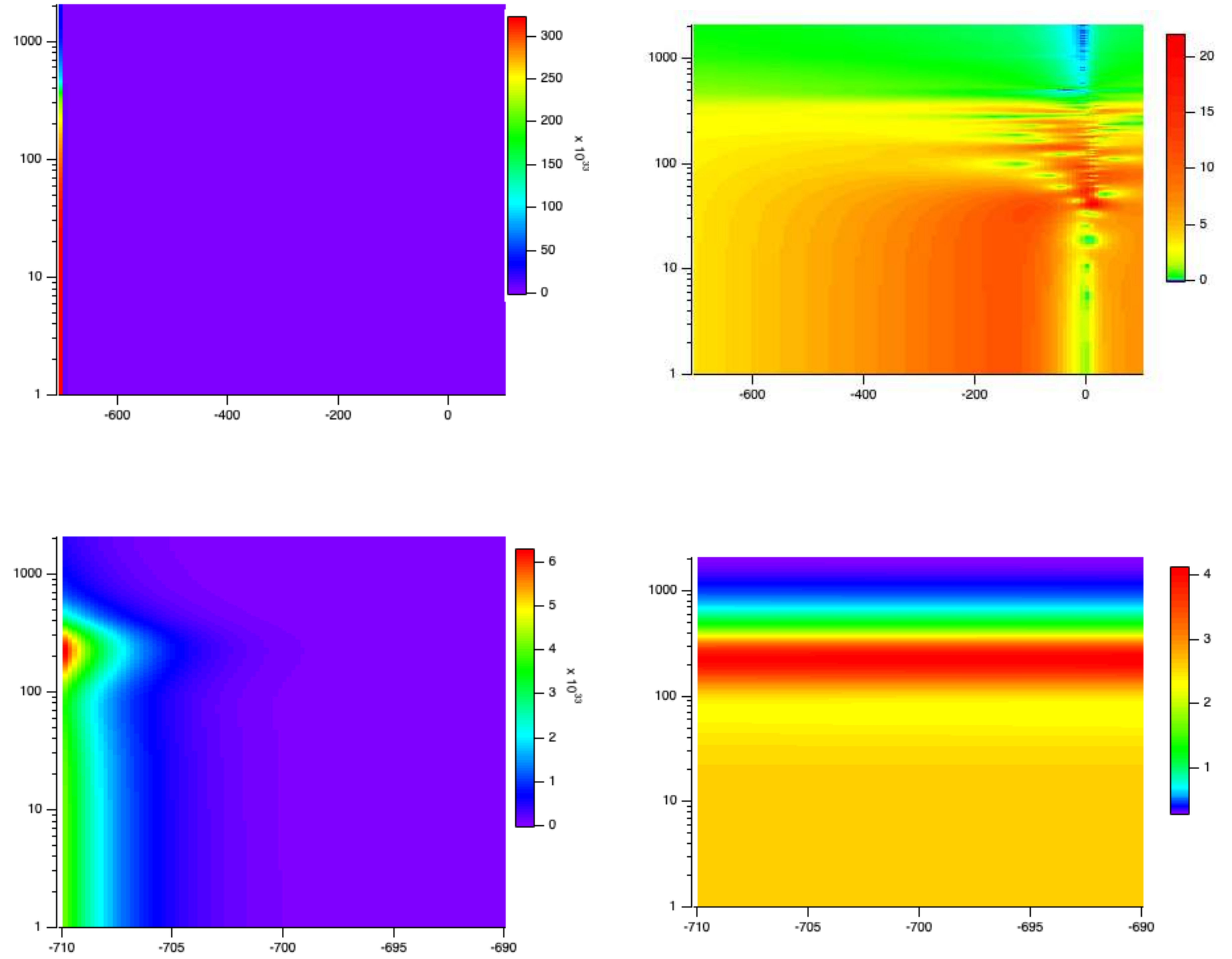
all waveform



short-chunk
around QNM

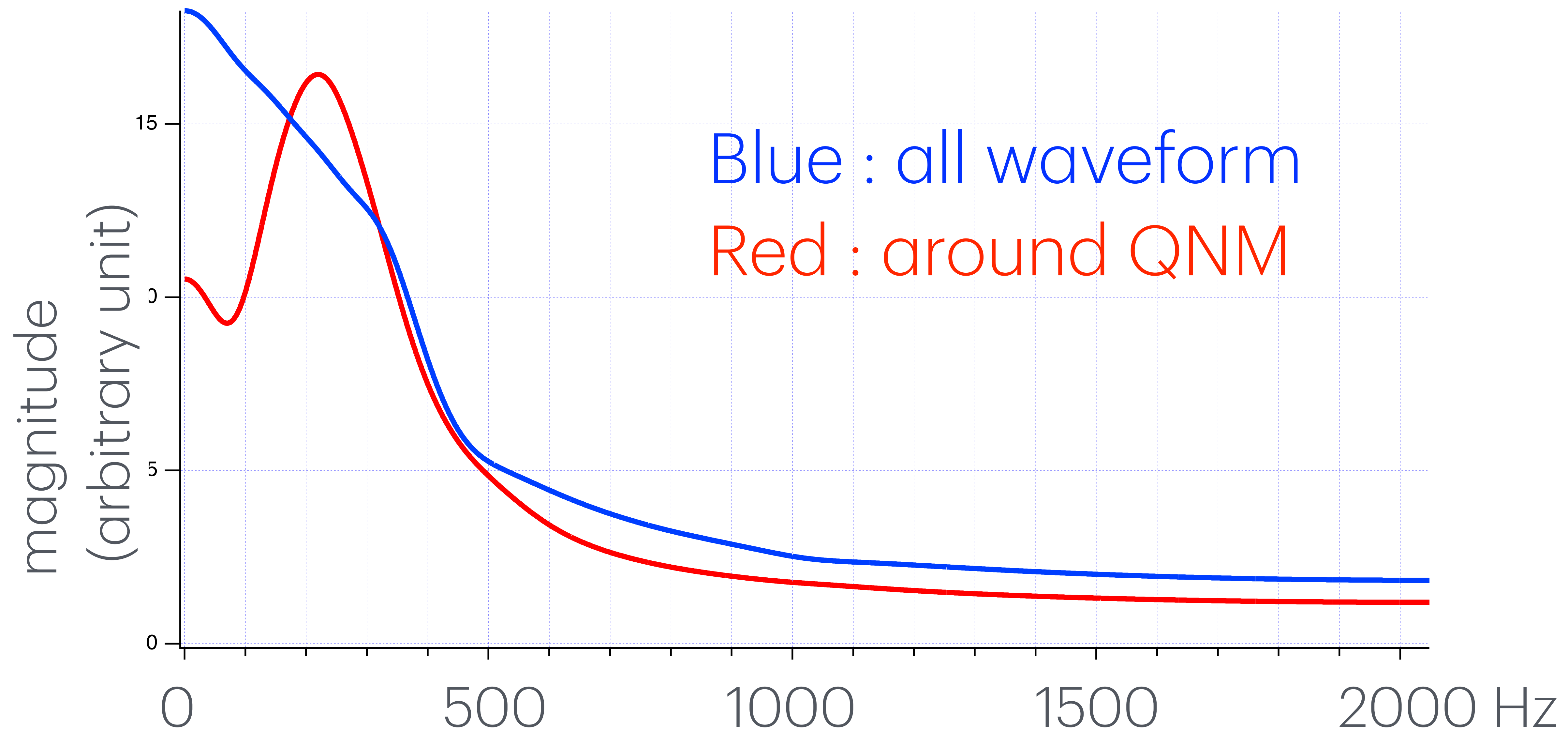
Laplace tr.

magnitude (arbitrary unit) normalized magnitude

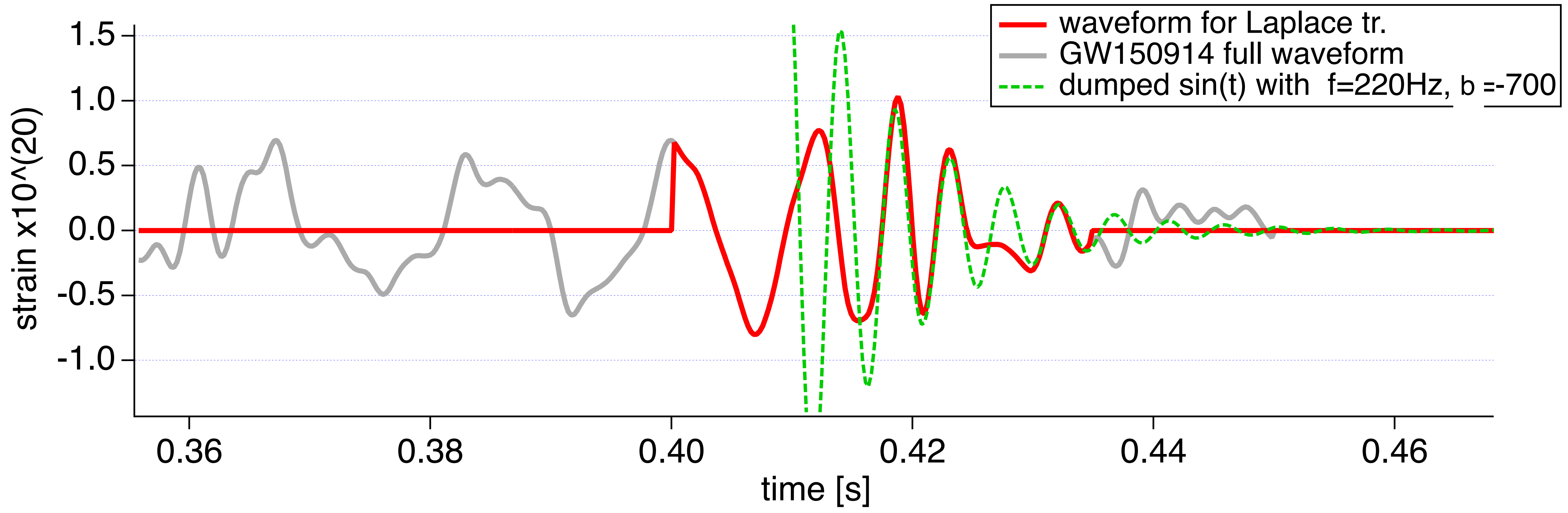


Real observed gravitational waveform : GW150914 (cont'd)

Spectrum at Re(s)=-700



Time series of original GW150914 and estimated QNM



central frequency : $f_0 \sim 220\text{Hz}$

1/(decay time) : $Re(s) \sim -700$

Quality factor : $Q \sim 3.1$

Kerr parameter : $\alpha \sim 0.65$

Total mass at detector frame :

$M(1+z) \sim 75.7 M_{\odot}$

-> $M \sim 68.4 M_{\odot}$ with redshift 0.107

Final mass (M_sun)	$63.1^{+3.4}_{-3.0}$
Final spin	$0.69^{+0.05}_{-0.04}$

GWTC-1 PE for GW150914

Summary

- We employ Laplace transform for the analysis of gravitational waves from blackhole quasinormal mode.
  - QNMs may be appear as poles in complex plane.
  - One of key merit of Laplace transform : no need to explicitly give the QNM part strictly.
- Checking with simulation waveform
  Laplace transform extract QNM
- Demonstration with GW150914 waveform
  We got consistent result (mass, Kerr parameter) of another analysis.
- To do :
  - Error of estimated parameters
  - Try with much more observed waveforms