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Gravitational wave radiometry with Stokes parameters

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Outline

We are developing a new method to detect gravitational wave backgrounds…

Target	The circular polarization of astrophysical gravitational wave backgrounds
Method	Gravitational wave radiometry and Stokes parameters
Result	We tried to simulate wave source locations and test if the polarization can be detected correctly

Contents

1. Background and Purpose

Why do we want to observe the circular polarized gravitational wave background?

the circular polarized Cosmological Gravitational Wave Background (CGWB)

generated by ***parity violation*** of the gravitational interaction in the early universe

- the existence Chern-Simons term   N. Seto, Phys. Rev. Lett. **97**, 151101 (2006)
- the coupling of axion and gravity   S. Kanno *et al.*, arXiv:2304.03944 [hep-th](2023)

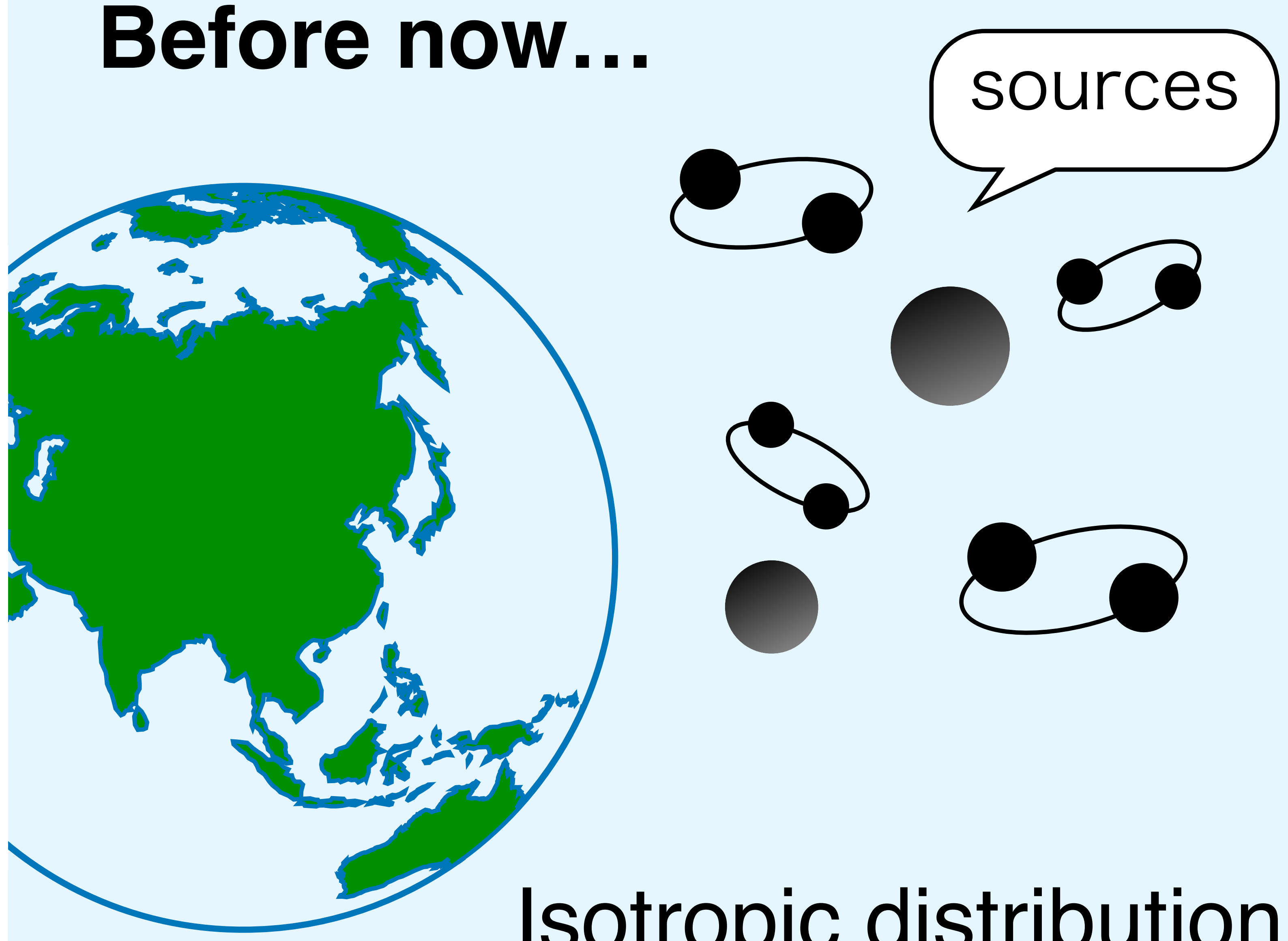
This could provide one of the important hints about ***what theories the early universe followed***.

1. Background and Purpose

Why do we want to observe the circular polarized gravitational wave background?

the circular polarized Astrophysical Gravitational Wave Background (AGWB)

Before now…



Isotropic distribution

➡Unpolarized…?

Recently… L. Valbusa Dall'Armi *et al.*, Phys. Rev. Lett. 131, 041401 (2023)

Number of wave sources may fluctuate
due to observation time limitation and detector sensitivity.

➡AGWB also could have the polarization.
SNR will be affected by a 20% error if we observe AGWB

The circular polarized AGWB may be foreground for circular polarized CGWB

1. Background and Purpose

Purpose of this study

To develop the method for all sky search for possible circular polarized AGWB

Gravitational wave radiometry × Stokes parameters

The method using spherical harmonics basis

- The advantage in depicting large scale structures in all sky.
- The resolution comparable to the radiometry needs $l > 200$.

Gravitational wave radiometry

- Good at searching for point sources
- Easily responds to increases in observation time

Best suited for searching for AGWB

Principle of Gravitational Wave Radiometry

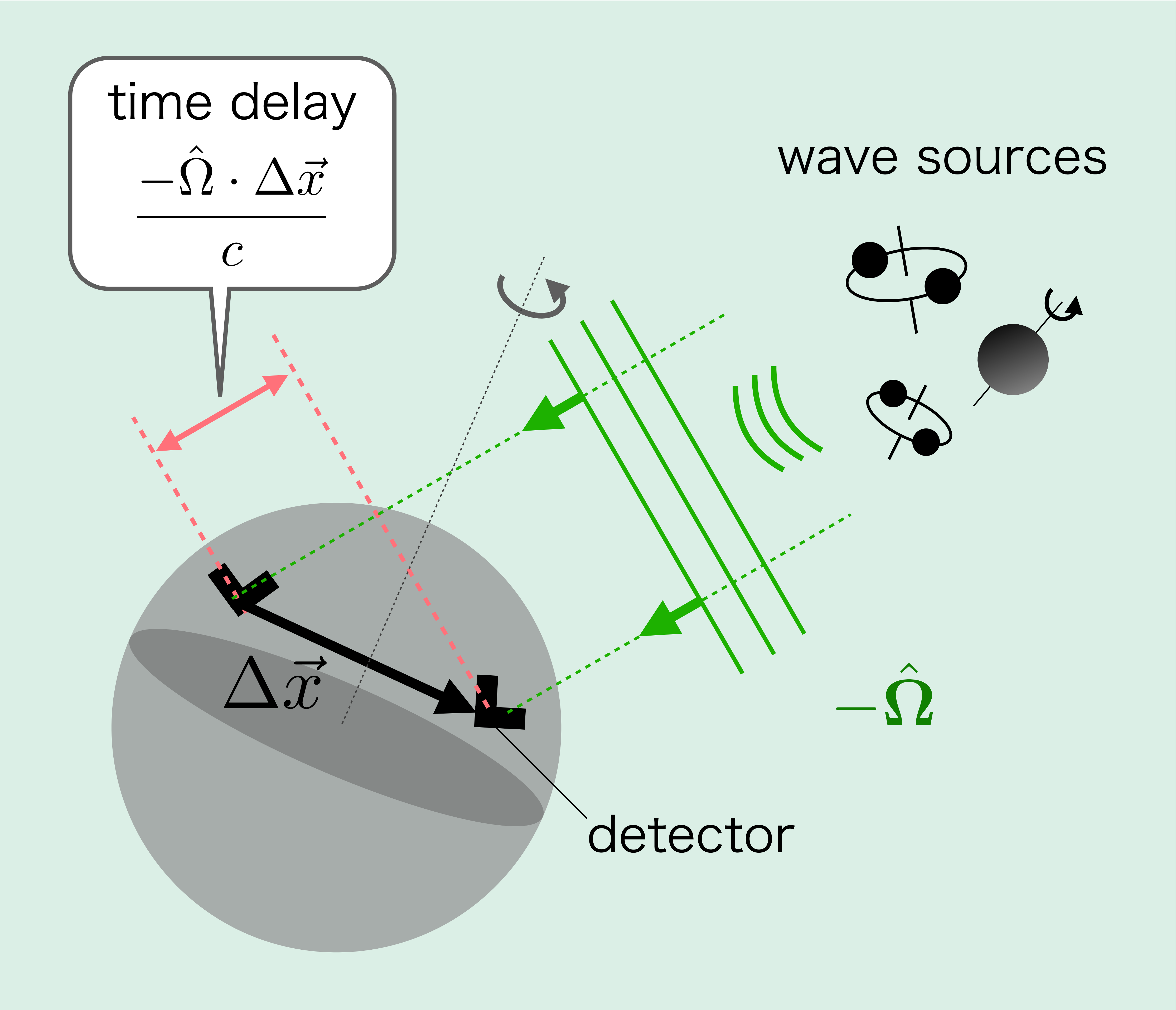
- Locate the wave sources by **arrival time delay**
- **All sky mapping** is possible by choosing appropriate time delay

How to deal with the change of arrival time delay due to Earth rotation?

Tracking it using **radiometry filter**:

$$\tilde{R}(\hat{\mathbf{\Omega}}, t, f) = \lambda \frac{\gamma^*(\hat{\mathbf{\Omega}}, t, f) H(f)}{P_a(t; |f|) P_b(t; |f|)}$$

and calculate it par **"segments"** of Δt.



2. Method

Stokes parameters in Gravitational wave

Gravitational wave background needs to be correlated and statistically characterized.

In plus and cross polarization basis:

$$I \equiv \langle h_+ h_+^* \rangle + \langle h_\times h_\times^* \rangle$$
$$Q \equiv \langle h_+ h_+^* \rangle - \langle h_\times h_\times^* \rangle$$
$$U \equiv \langle h_+ h_\times^* \rangle + \langle h_\times h_+^* \rangle$$
$$V \equiv -i \left(\langle h_+ h_\times^* \rangle - \langle h_\times h_+^* \rangle \right)$$

In circular polarization basis:

$$e^R \equiv \frac{e^+ + ie^\times}{\sqrt{2}}$$
$$e^L \equiv \frac{e^+ - ie^\times}{\sqrt{2}}$$

$$I \equiv \langle h_R h_R^* \rangle + \langle h_L h_L^* \rangle$$
$$Q \equiv \langle h_R h_L^* \rangle + \langle h_L h_R^* \rangle$$
$$U \equiv -i \left(\langle h_R h_L^* \rangle - \langle h_L h_R^* \rangle \right)$$
$$V \equiv \langle h_R h_R^* \rangle - \langle h_L h_L^* \rangle$$

I mode : Intensity of gravitational wave

Q mode : Intensity of linear polarization

U mode : Intensity of elliptically polarization

V mode : Intensity of circular polarization

V mode > 0 : Right hand

V mode < 0 : Left hand

3. Gravitational Wave Radiometry with Stokes parameters

How to pick out polarization and direction dependent component?

Assuming variable separations:

$$I(f,\hat{\mathbf{\Omega}}) = 2H_I(f)\mathcal{P}_I(\hat{\mathbf{\Omega}}) \qquad U(f,\hat{\mathbf{\Omega}}) = 2H_U(f)\mathcal{P}_U(\hat{\mathbf{\Omega}})$$

$$Q(f,\hat{\mathbf{\Omega}}) = 2H_Q(f)\mathcal{P}_Q(\hat{\mathbf{\Omega}}) \qquad V(f,\hat{\mathbf{\Omega}}) = 2H_V(f)\mathcal{P}_V(\hat{\mathbf{\Omega}})$$

A point source exists in a given direction in all sky:

$$\mathcal{P}_\alpha(\hat{\mathbf{\Omega}}) = \delta(\hat{\mathbf{\Omega}}' - \hat{\mathbf{\Omega}})$$

Modify radiometry filter $\tilde{R}$ and pick out Stokes parameters

Dependence on antenna pattern:

$$\Gamma(\hat{\Omega},t) = F_1^+(\hat{\Omega},t)F_2^+(\hat{\Omega},t) + F_1^\times(\hat{\Omega},t)F_2^\times(\hat{\Omega},t)$$

$$\Gamma^I \equiv F_1^+(\hat{\Omega},t)F_2^+(\hat{\Omega},t) + F_1^\times(\hat{\Omega},t)F_2^\times(\hat{\Omega},t)$$

$$\Gamma^Q \equiv F_1^+(\hat{\Omega},t)F_2^+(\hat{\Omega},t) - F_1^\times(\hat{\Omega},t)F_2^\times(\hat{\Omega},t)$$

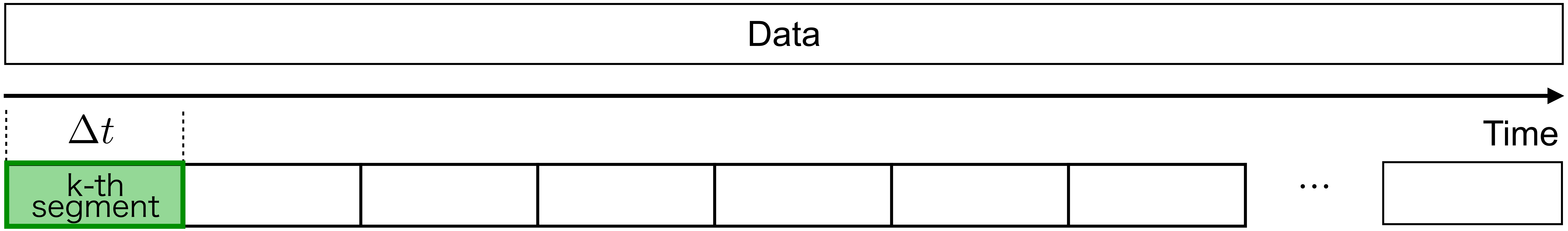
$$\Gamma^U \equiv F_1^+(\hat{\Omega},t)F_2^\times(\hat{\Omega},t) + F_1^\times(\hat{\Omega},t)F_2^+(\hat{\Omega},t)$$

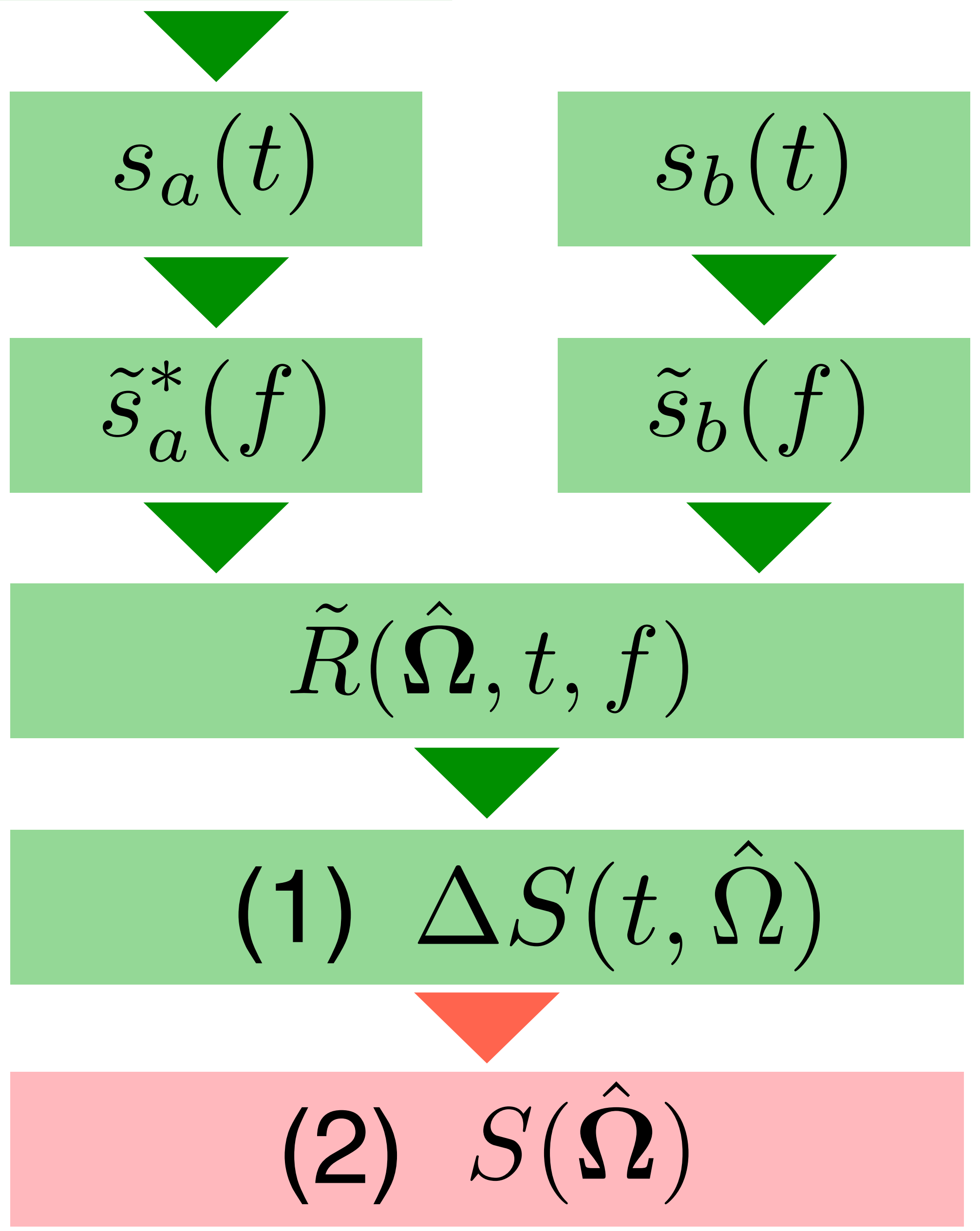
$$\Gamma^V \equiv i\left[F_1^+(\hat{\Omega},t)F_2^\times(\hat{\Omega},t) - F_1^\times(\hat{\Omega},t)F_2^+(\hat{\Omega},t)\right]$$

Separated into 4 corresponding Stokes parameters

3. Gravitational Wave Radiometry with Stokes parameters

How to pick out polarization and direction dependent component?





Output of a detector in k-th segment: $s_k(t) = h_k(t) + n_k(t)$

(1) $\Delta S(t, \hat{\Omega}) = \int_{-\infty}^{\infty} df \; \tilde{s}^*_1(f; t)\tilde{s}_2(f; t)\tilde{R}(\hat{\Omega}, t, f)$

(2) $S_\alpha(\hat{\mathbf{\Omega}}) = \dfrac{\int_0^T \Delta S_\alpha(\hat{\mathbf{\Omega}}, t)\sigma^{-2}(\hat{\mathbf{\Omega}}, t)\, dt}{\int_0^T \sigma^{-2}(\hat{\mathbf{\Omega}}, t)\, dt}$

(3) $s(\hat{\Omega}) \equiv \langle S(\hat{\Omega}) \rangle = \sum_\alpha \int_{S^2} d\hat{\Omega}' \, \mathcal{P}_\alpha(\hat{\Omega}') B^\alpha(\hat{\Omega}, \hat{\Omega}')$

Point spread function:

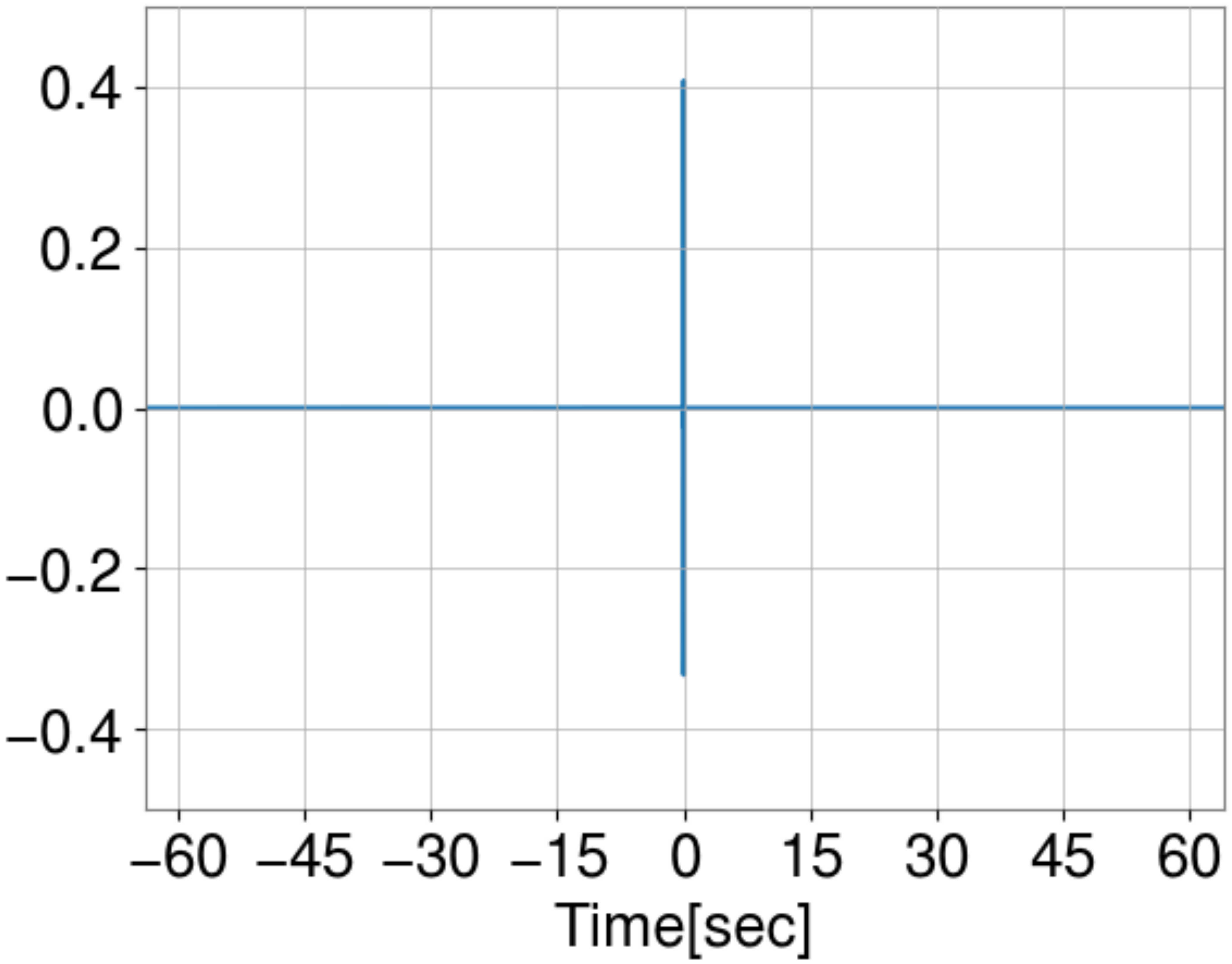
Represents the spreading of a point source due to diffraction

4. Simulation analysis

Can we detect polarization correctly?

We created the all sky map of $S(\hat{\Omega})$ with ***a point source*** from specific injection location.

Injection signal / 1 segment



- **Detector** KAGRA / Virgo
- **Gravitational wave signal (no detector noise)**

  Injection locations: the direction of galactic center
  $(\alpha, \delta) = (-5\text{h}\,42\text{m}\,38\text{s},\ -25^\circ\,28')$

  Injection signal: sine-Gaussian wave
  (injectied every 128 sec)

  Frequency: 500 Hz **Quality factor**: 5

- **Calculation and Mapping**

  1 segment: 128 sec **Observation time**: 3600 × 24 sec

  All sky map: Aitoff's projection, $1\text{pixel} = 0.5^\circ \times 0.5^\circ$

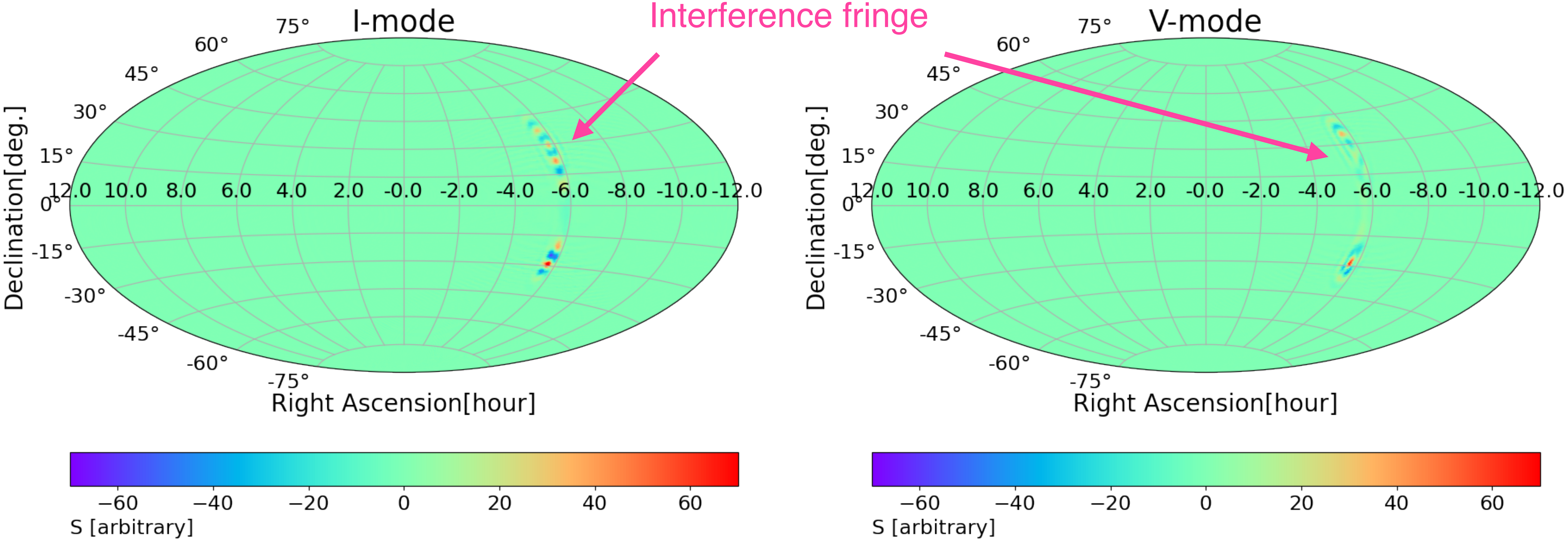
4. Simulation analysis

Can we detect polarization correctly?

Injection signal: Right-hand wave

$$(\alpha, \delta) = (-5\text{h}\,42\text{m}\,38\text{s},\ -25^\circ\,28')$$

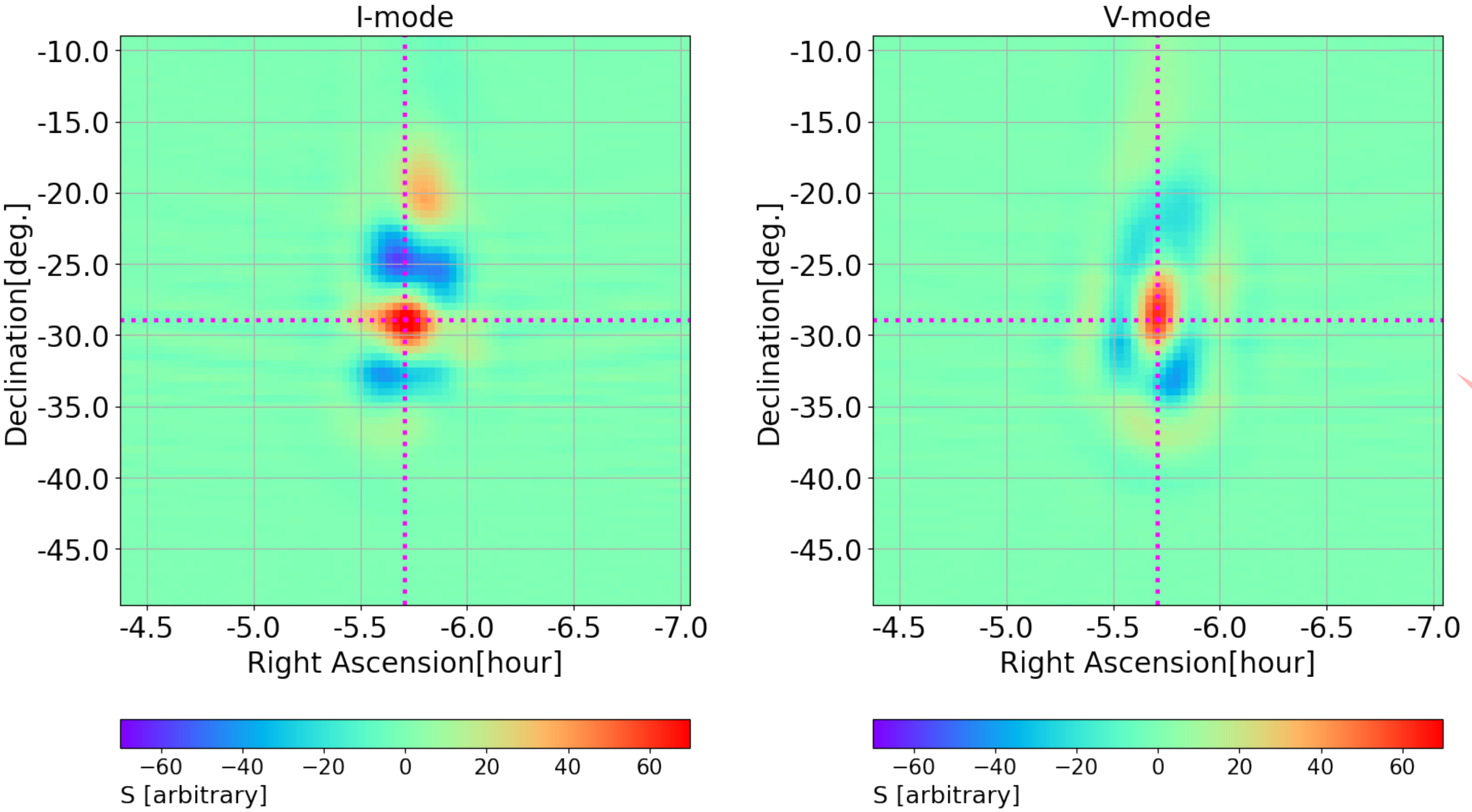
Interference fringe



4. Simulation analysis

Can we detect polarization correctly?

Injection signal: Right-hand wave



Diffraction limit

$\sim \lambda/D \sim 3.9$

(for 500 Hz)

➡ roughly same size

$V > 0$

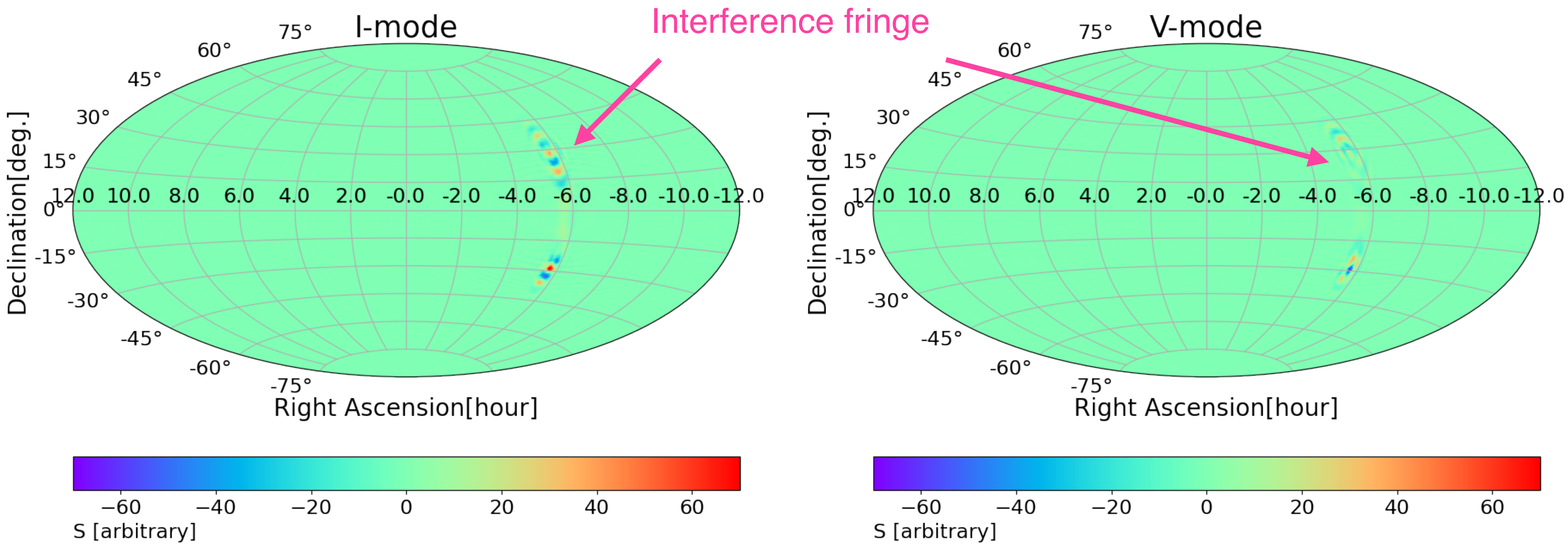
➡Right-hand wave

4. Simulation analysis

Can we detect polarization correctly?

Injection signal: Left-hand wave

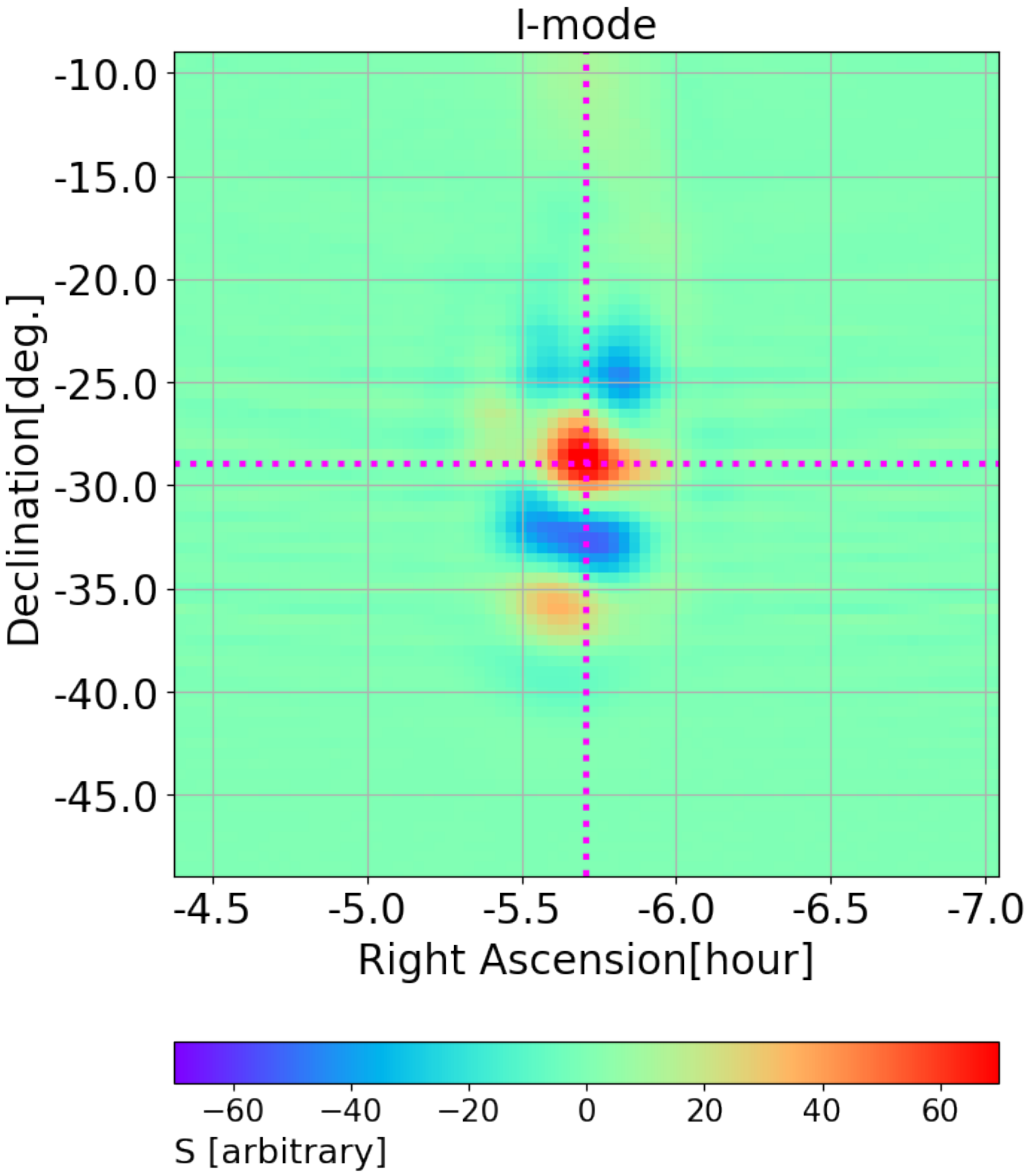
$$(\alpha, \delta) = (-5\text{h}\,42\text{m}\,38\text{s},\ -25^\circ\,28')$$

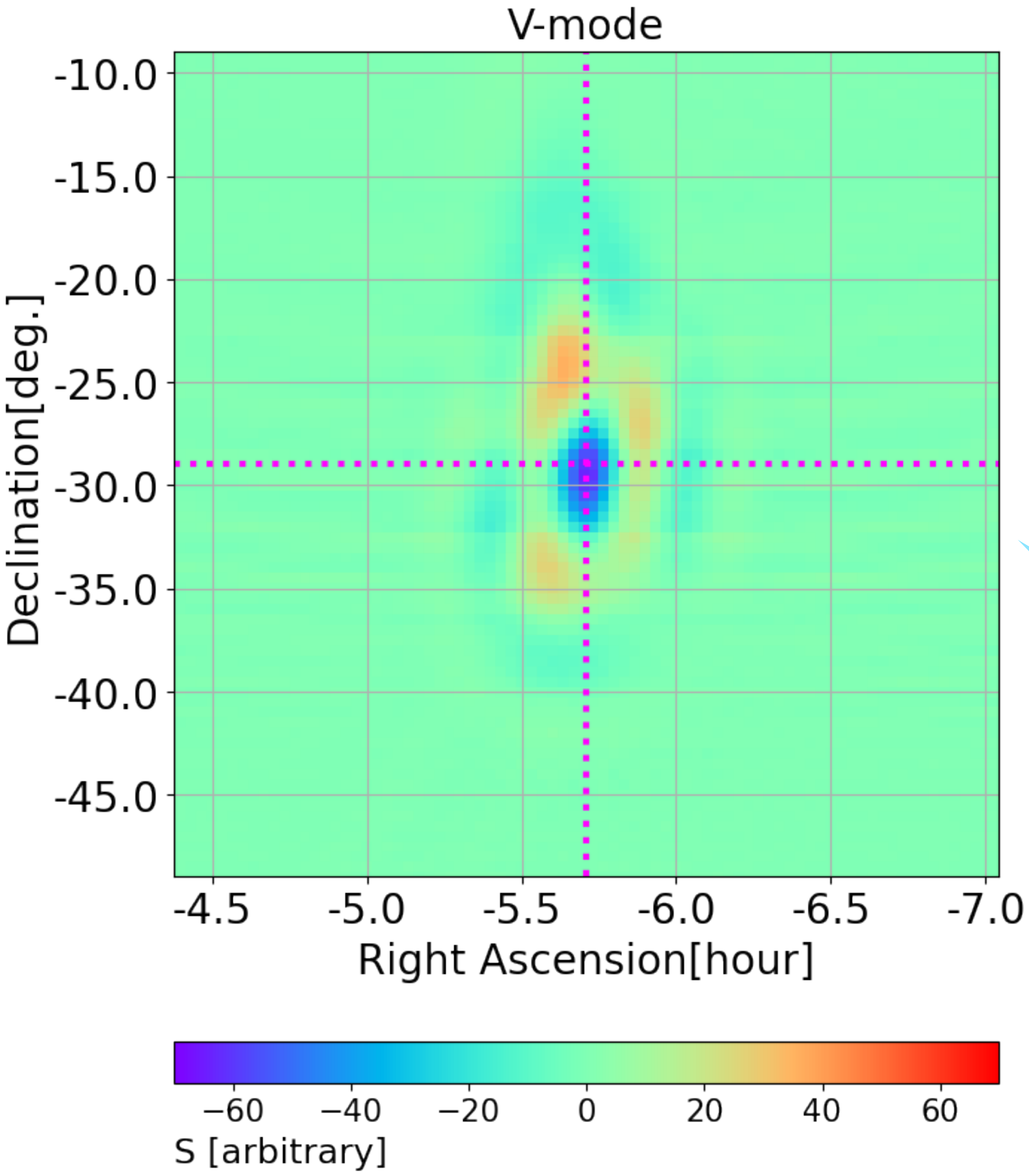


4. Simulation analysis

Can we detect polarization correctly?

Injection signal: Left-hand wave





Diffraction limit

$\sim \lambda / D \sim 3.9$

(for 500 Hz)

➡ roughly same size

$V < 0$

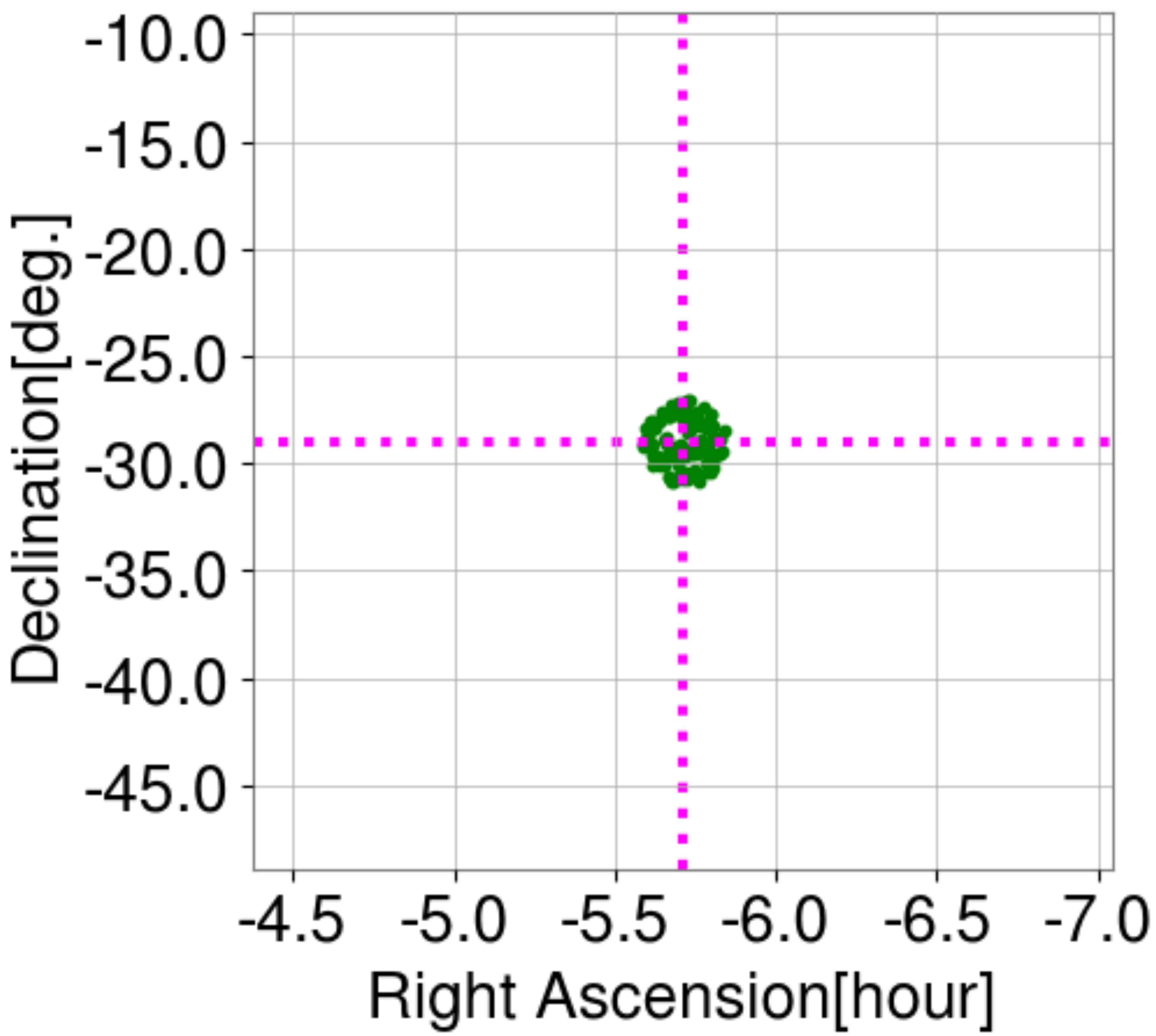
➡Left-hand wave

4. Simulation analysis

Can we detect polarization correctly?

We also analyzed signals from ***multiple wave sources*** to investigate point spreading.

Locations of wave sources



- **Detector** KAGRA / Virgo
- **Gravitational wave signal (no detector noise)**

  Number of wave sources: 100
  (set their coordinates generated by uniform random numbers)

  Injection locations: the direction of galactic center

  $$(\alpha, \delta) = (-5\mathrm{h}\,42\mathrm{m}\,38\mathrm{s},\ -25^\circ\,28') \pm (2^\circ,\ 2^\circ)$$

  Injection signal: sine-Gaussian wave (Injectied every 128 sec)

- **Calculation and Mapping**

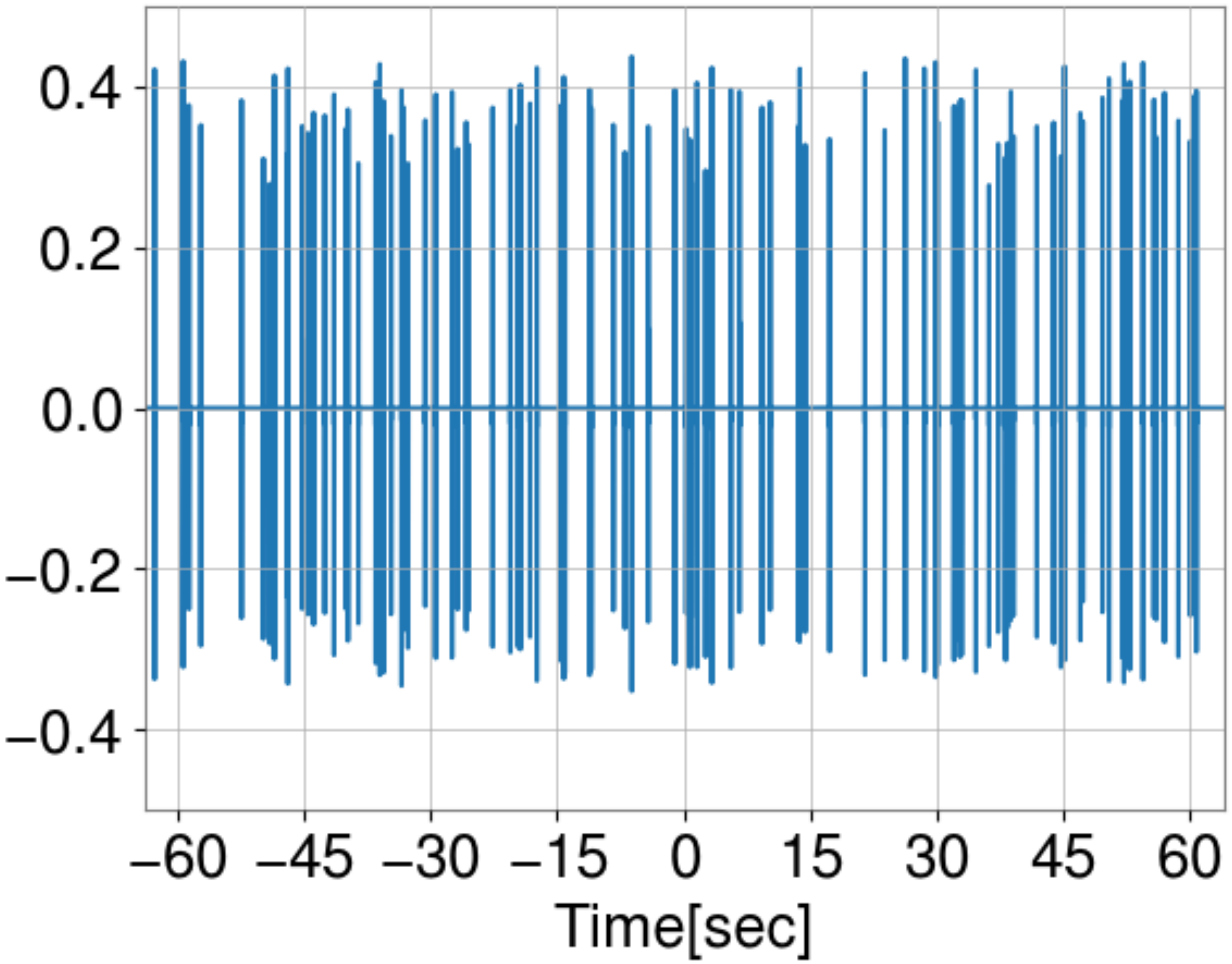
  1 segment: 128 sec **Observation time**: 3600×24 sec

  All sky map: Aitoff's projection, $1\mathrm{pixel} = 0.5^\circ \times 0.5^\circ$

4. Simulation analysis

Can we detect polarization correctly?

Injection signal / 1 segment



Gravitational wave signal (no detector noise)

Number of wave sources: 100
(set their injection time generated by uniform random numbers)

Injection signal: sine-Gaussian wave
(Injectied every 128 sec)

Frequency: given uniform random for 450-550 Hz

Quality factor: 5

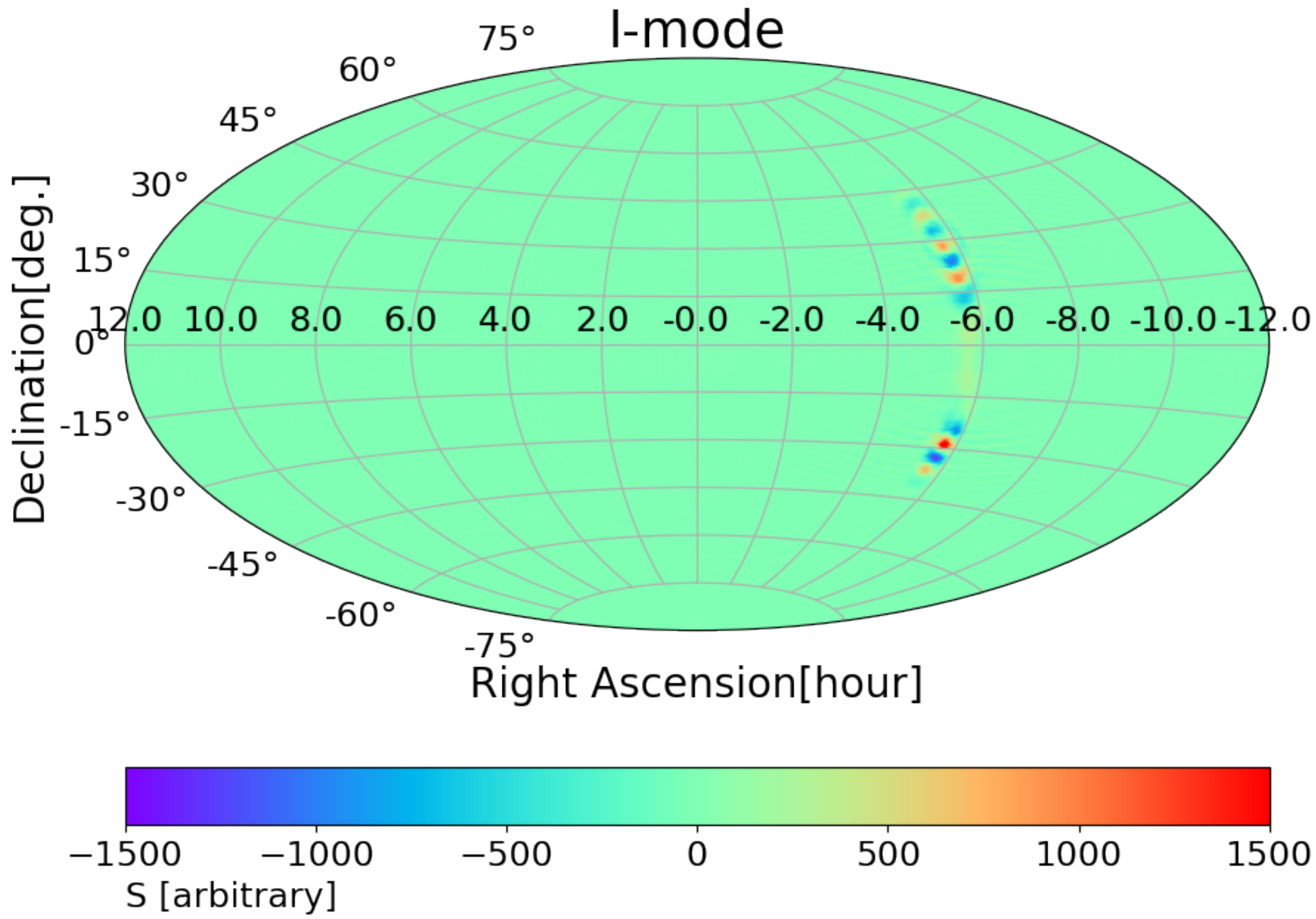
4. Simulation analysis

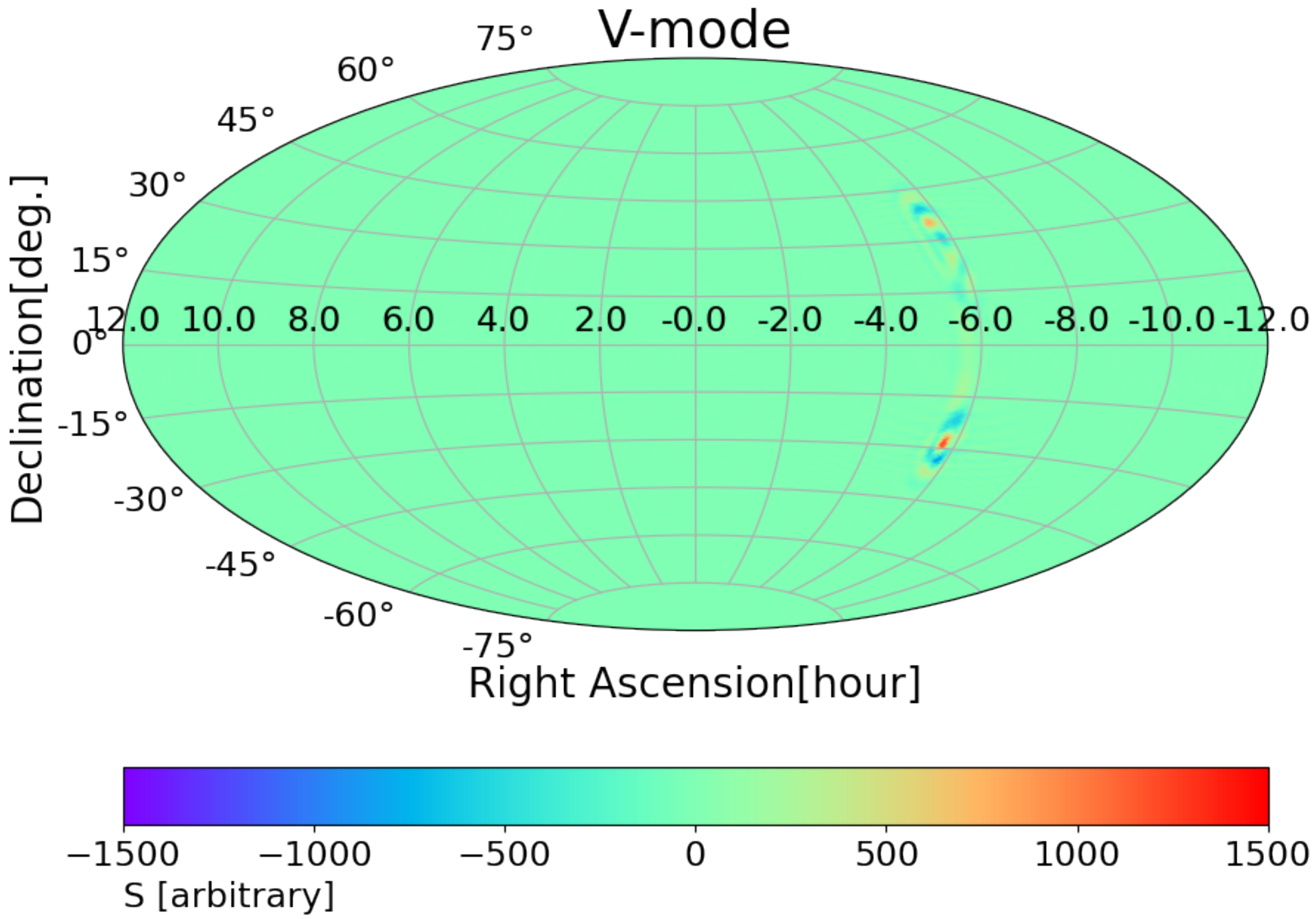
- ***Can we detect polarization correctly?***

Injection signal: Right-hand wave

Number of sources: 100

$$(\alpha, \delta) = (-5\text{h}\,42\text{m}\,38\text{s},\ -25^\circ\,28') \pm (2^\circ,\ 2^\circ)$$

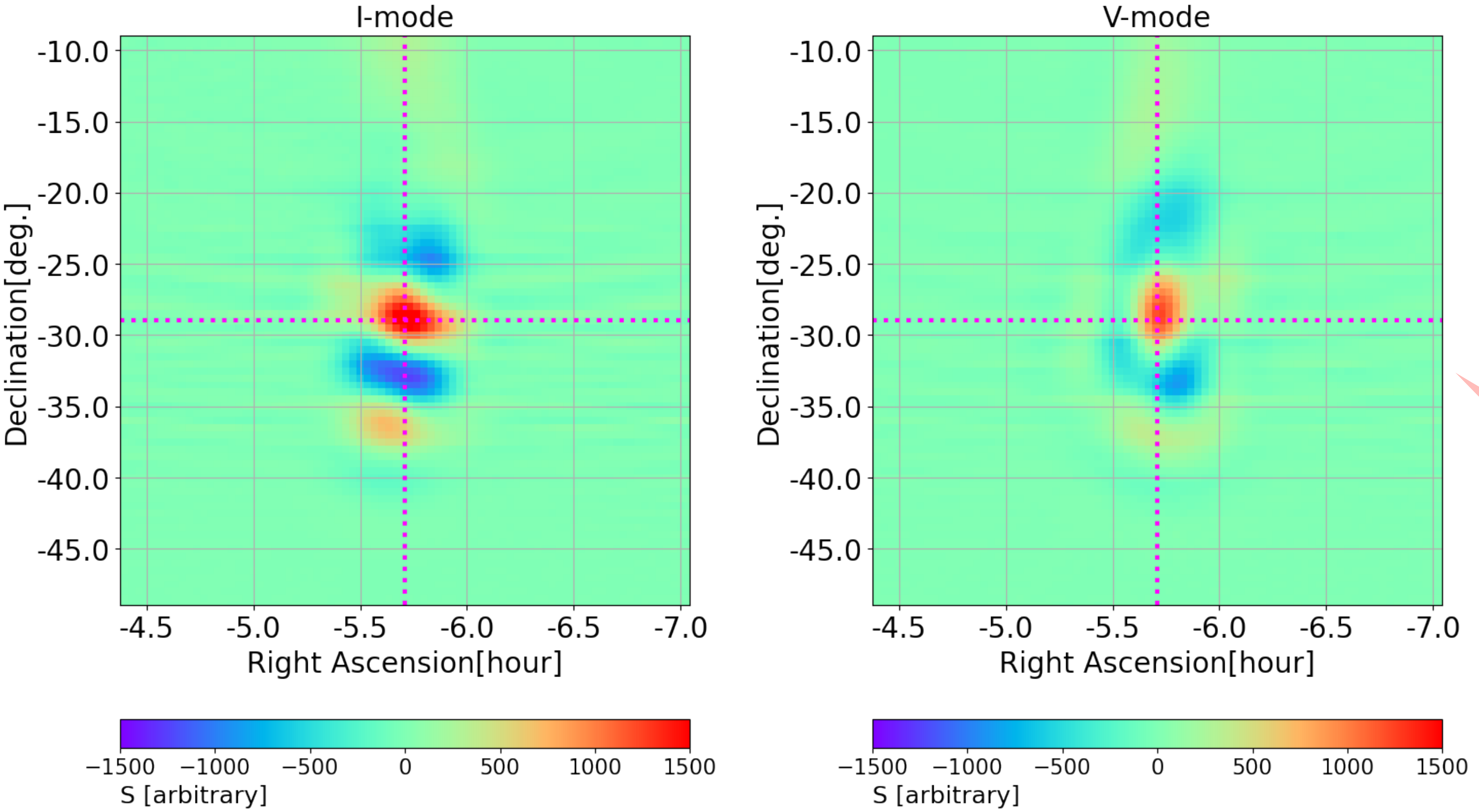




4. Simulation analysis

Can we detect polarization correctly?

Injection signal: Right-hand wave



Diffraction limit

$\sim \lambda/D \sim 3.9$

(for 500 Hz)

➡ roughly same size

$V > 0$

➡Right-hand wave

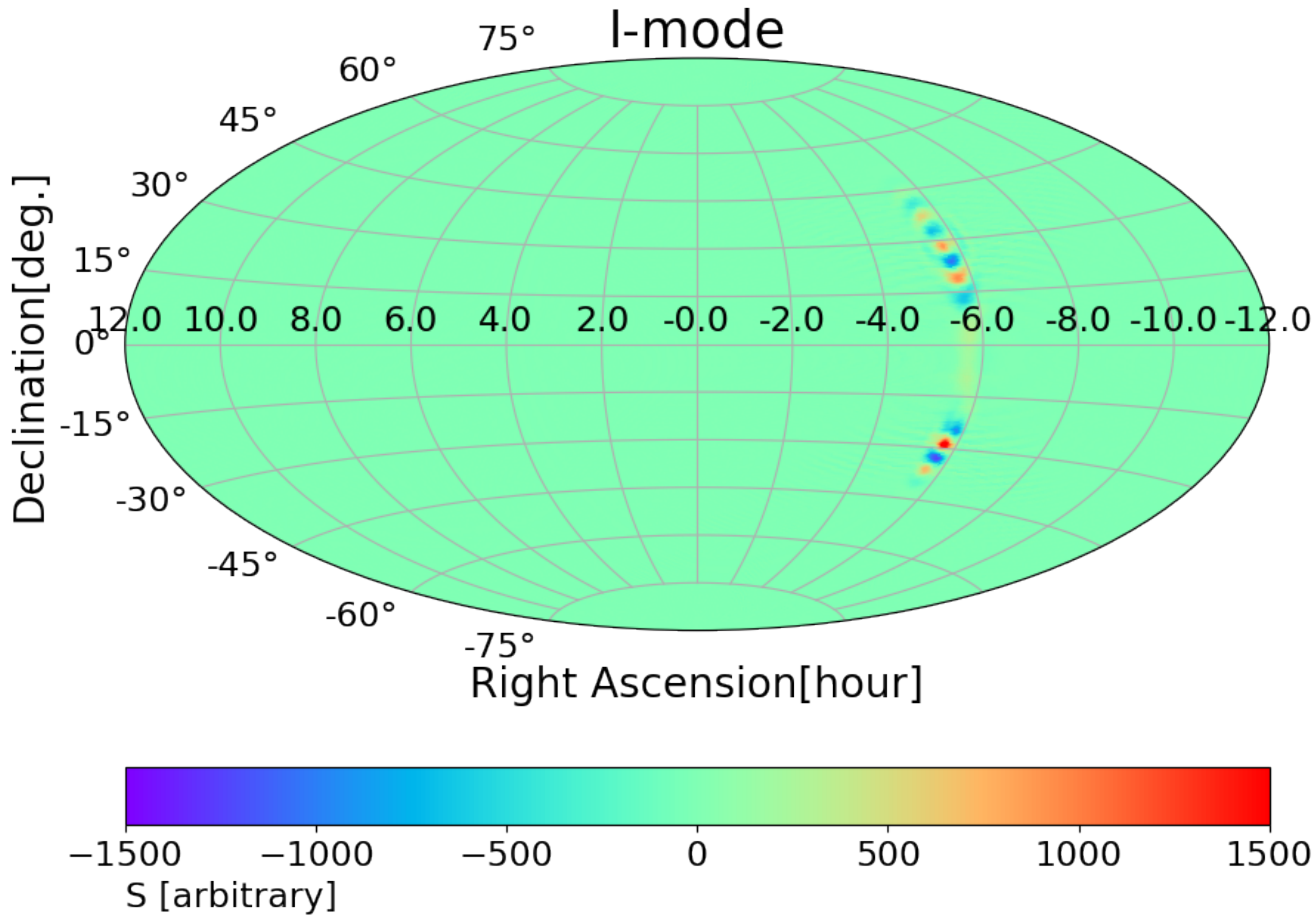
4. Simulation analysis

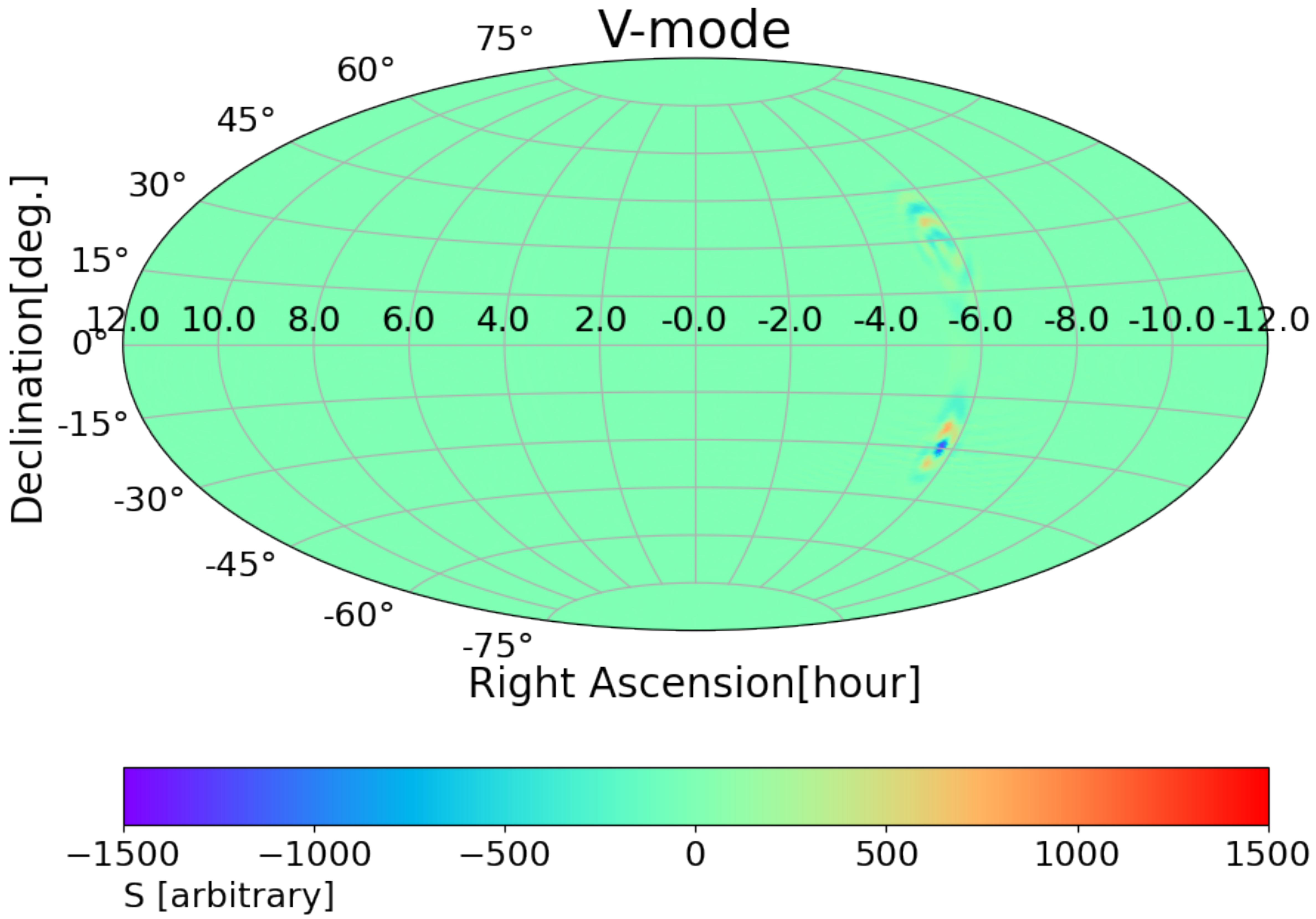
Can we detect polarization correctly?

Injection signal: Left-hand wave

Number of sources: 100

$$(\alpha, \delta) = (-5\text{h}\,42\text{m}\,38\text{s},\ -25^\circ\,28') \pm (2^\circ,\ 2^\circ)$$

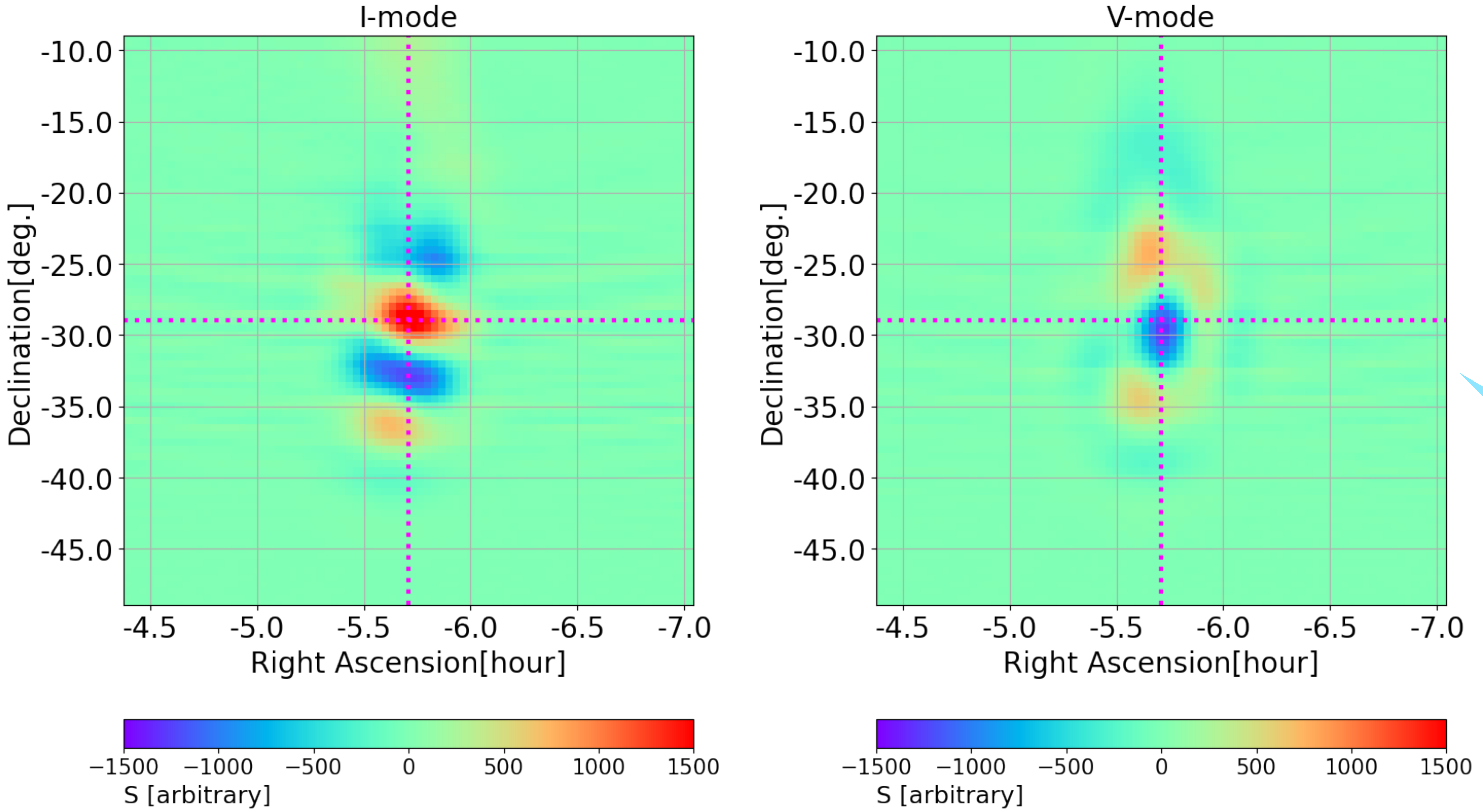




4. Simulation analysis

Can we detect polarization correctly?

Injection signal: Left-hand wave



Diffraction limit

$\sim \lambda/D \sim 3.9$

(for 500 Hz)

➡ roughly same size

$V < 0$

➡Left-hand wave

5. Summary

- To detect the circular polarized astrophysical gravitational wave background, **we developed a new method with gravitational wave radiometry** to search all sky.
- In this method, **the modified radiometry filter can distinguish the circular polarization and other polarizations.**

Next step:

- much more sources
- signals including detectors noise
- real data for O4